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NONLINEAR OPTICS
or OPTICAL PHYSICS

Shining light on an old problem

Daniele Faccio and Fabio Biancalana

The evolution of modulational instability sidebands in an optical fibre are shown to provide new insights into Fermi-Pasta-Ulam-Tsingou recurrences.

In this issue of *Nature Photonics*, Mussot et al. [1] demonstrate a technique that exploits nonlinear optics in a glass fibre to provide new insights into an old and famous problem in physics that was first encountered by Enrico Fermi, John Pasta, Stanislaw Ulam and Mary Tsingou in the 1950s. The so-called Fermi-Pasta-Ulam-Tsingou problem relates to the counter-intuitive situation where highly complex physical systems are seen to exhibit unexpected periodic behaviour with a flow of energy that oscillates between states on a regular basis. Interestingly, the initial study that revealed this strange behaviour played a pivotal role in the birth of computational physics, exploiting the nascent computing power developed by the Manhattan project (the US initiative to develop a nuclear weapon during World War II).

In 1953, Fermi, Pasta, Ulam and Tsingou used the Los Alamos MANIAC computer (previously used for performing calculations on thermonuclear processes) to simulate a one-dimensional chain of coupled masses connected by springs (see figure 1) with a linear restoring force plus a weak nonlinear correction – a problem that could not be solved analytically. Their expectation was to verify Fermi's statistical prediction that the presence of the nonlinear correction would ultimately lead to an equipartition of the energy across all of the oscillating modes of the system – a phenomenon known as thermalization.

In hindsight, it is hard to decide whether this numerical experiment was more important for the very puzzling result that emerged from it or for the fact that it effectively ushered in the age of computational physics (or experimental mathematics, depending on your viewpoint). Today, physicists and engineers very much take for granted the ability to be able to perform numerical simulations and often expect or demand to see a numerical comparison with experimental results. But before 1953, this was an unheard-of practice and the attempt to numerically observe thermalization behaviour was truly pioneering.

The result from this simulation however, showed a very unexpected behaviour, which in turn has been the founding pillar for other discoveries and studies that continue to date. Instead of thermalizing, the system seemed to spread energy out and then periodically return to its initial state with energy flowing back to the more ordered configuration. These oscillations now come under the name of FPU recurrences (Fermi-Pasta-Ulam) after the trio of names on the 1955 report [2,3] which did not acknowledge Tsingou's contribution in the programming of MANIAC.

The stage was then set for one of the greatest puzzles in modern physics: why was the system not thermalizing as it should do, and what was the physics behind the recurrences? The quest to answer these questions has led to many discoveries that have branched off in other directions and have become entire research fields of their own. In 1965, Kruskal and Zabusky [4] recognised the connection to the Korteveg De-Vries (KdV) equation and the existence of localised particle-like states called solitons that propagate invariantly though the system and thus break the ergodicity of the problem.
It was later discovered that increasing the interaction strength of the system leads to chaotic behaviour and then eventually to the long-sought after thermalization.

More recently, attention has reverted back to experiments with the aim of studying FPU recurrences and physics in real systems. One of the goals is to verify the universality of the FPU problem and flesh out the details of any deviations and connections to solitons and chaos.

One of the major obstacles so far has been the presence of losses in experimental systems: dissipation dampens oscillations and can very rapidly quench any evidence of FPU recurrences. Alongside this difficulty with loss is that real experiments are typically hindered by technical restraints that prevent easy access to all of the system parameters and details, thus providing only a partial picture of the dynamics. A range of systems have been analysed in the past such as water waves (where the major issue is indeed losses) [5] and optical fibre systems [6]. Optical fibres have actually been one of the preferred systems for studying a whole range of physical concepts that are driven by weak nonlinearities and have provided the first ever experimental evidence for example of rogue waves [7] and soliton solutions such as the Peregrine soliton [8].

Now Mussot et al., have devised a fibre-based system that provides an effectively lossless setting to study FPU dynamics whilst keeping track of the full details of the evolution with remarkable precision [1]. Losses are overcome by pumping the fibre with a second, counter-propagating laser beam that provides gain that is largest towards the end of the fibre where losses would otherwise be the highest. By carefully balancing the amount of power in the counter-propagating laser beam, they can effectively eliminate losses. At the same time, Mussot et al. keep track of the evolution of the system by monitoring the very small back-reflected (due to Rayleigh scattering) signal along the fibre length. This weak signal is interfered using a reference laser beam (that acts as a local oscillator for a homodyne detection measurement), thus making it possible to monitor the full amplitude and phase of the propagating beam in the fibre with a precision of 20 m along a total fibre length of 7.7 km.

Mussot et al. also inject an intense laser beam into their optical fibre: this beam plays the role of the initial oscillation mode in the FPU problem. They then inject a much weaker set of sidebands, which correspond to adding a small amount of oscillation in other modes in the FPU chain of oscillators (see Fig. 1). The final part of their ingenious technical solution is a clever pulse shaping approach that allows them to precisely control the amplitude and phase of the sidebands, thus allowing the sidebands to act as either an amplitude or as a frequency modulation of the main beam. These modulations in turn stimulate a process that in fibre optics is known as modulation instability (MI), whereby weak optical nonlinearity transfers energy from the main beam to sidebands that gradually increase in number and amplitude. Sure enough, rather than continuously spreading out and `thermalizing', the energy is seen to periodically oscillate back and forth, refocusing back into the main beam and then out again into the sidebands.

Controlling the nature of the initial state allows Mussot et al. to experimentally observe a feature that had never been observed before in optics, namely a symmetry breaking analogous to the famous Anderson-Higgs mechanism in particle physics and in condensed matter physics. In fact, the dynamics of the MI can be simplified to a three-wave mixing process treating the MI sidebands as single peaks that exchange energy with the pump. The dynamics of this model shows a transition between a parabolic potential to a Mexican-hat potential, depending on whether the frequency considered is outside or inside the MI sideband range, respectively.

Qualitatively different types of FPU recurrences are predicted, depending on the trajectories of the evolution of the system in the phase space. The experimental techniques used by Mussot et al. make it possible to effectively map out the phase space during the evolution of MI along the fibre and thus demonstrate that indeed the amplitude and frequency modulation input conditions lead to very different phase-space trajectories that are separated by a so-called homoclinic crossing point.

The system proposed by Mussot et al. represents a strong step forward in the study of FPU physics, providing a robust system that allows access to the full phase space and details of the recurrences. A key point that remains to be seen is the total distance over which this technique can be extended. In the present work only two recurrences are observed rather than a long scale evolution. Expanding this scale will be key for future studies aimed, for example, at investigating the transition to chaos around the homoclinic structure as well as the link to rogue wave formation and the evolution towards final thermalization.

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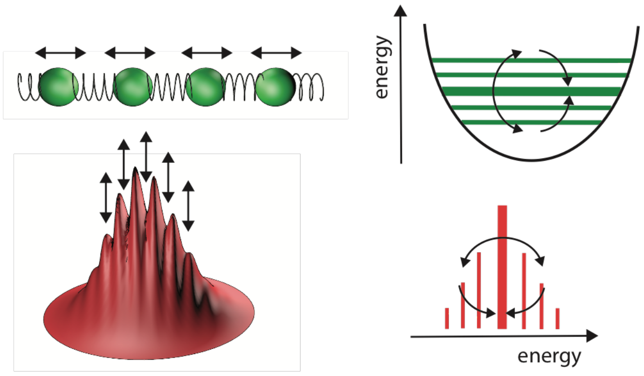

Fig1: Schematical over of original FPU problem, the chain of coupled oscillators, together with the energy levels and recurrent coupling out from the input energy mode and then back in from the outer energy levels. These modes correspond to different oscillation frequencies in the chain of coupled masses.
The laser pulse can also exhibit different oscillation frequencies superimposed on its envelope. These correspond to a comb of modes that are observed in the spectrum and FPU recurrences occur in the form of energy transferring back and forth from the principal input mode into outer modes and back again.

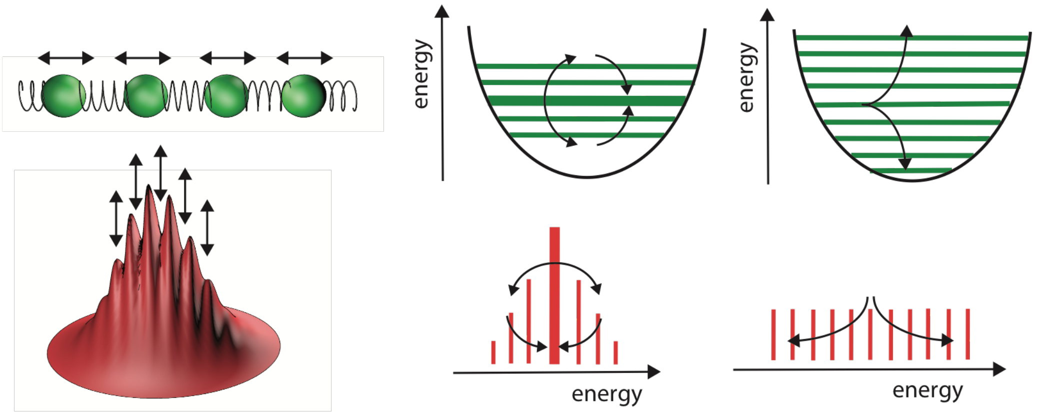

Fig1: Schematical over of original FPU problem, the chain of coupled oscillators, together with the energy levels and recurrent coupling out from the input energy mode and then back in from the outer energy levels. These modes correspond to different oscillation frequencies in the chain of coupled masses. The far right energy level diagrams show the equipartition (thermalisation) of energy across all levels that FPUT where expecting to observe. The laser pulse can also exhibit different oscillation frequencies superimposed on its envelope. These correspond to a comb of modes that are observed in the spectrum and FPU recurrences occur in the form of energy transferring back and forth from the principal input mode into outer modes and back again.