Volatility and Correlation Timing: The Role of Commodities

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Abstract

This paper examines the role of commodities from the perspective of dynamic asset allocation. We model conditional second moments of stock, bond and commodity futures and examine their impact on the portfolio choice decision of a risk-averse investor in a mean-variance framework. Findings suggest that adding commodities in the opportunity set enhances portfolio risk-return characteristics and offers diversification benefits. Moreover, there is substantial economic value in both volatility and correlation timing strategies. Results are robust across various sub-periods and rebalancing strategies, alternative correlation dynamics specifications, short-sale constraints and transaction costs under both in- and out-of-sample settings.

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1. Introduction

Over the last several years, commodity markets have experienced dramatic fluctuations. Significant amounts of funds allocated to commodity futures and index funds, made the sector very popular in the mid-2000s among institutional investors of versatile risk attitudes; either as a pure speculation instrument or as a diversification tool. The statistical features of commodity returns arise from the underlying demand and supply dynamics, yet the price formation function across commodities is diverse and this might result in substantial diversification potential.

Investors' interest in commodities is primarily motivated by the belief that commodities offer a hedge against inflation (Bodie, 1983; Irwin and Landa, 1987; Edwards and Park, 1996) and form an alternative asset class which can bestow diversification gains to investors. In particular, while equity returns tend to be impacted adversely during periods of inflation, commodity prices increase and, thus, long positions in commodity futures realize profits. This is consistent with efficient diversification against downturns in traditional assets such as equity and bond markets (see Gorton and Rouwenhorst, 2006; Büyükşahin et al., 2010; Chong and Miffre, 2010). The diversification benefits of commodities have been examined by Jensen et al. (2000), Belousova and Dorfleitnerr (2012) and You and Daigler (2013), among others. For example, Bodie and Rosansky (1980) conduct a comprehensive analysis of 23 individual commodities during the period from 1950 to 1976 and find that, by switching from a stock only portfolio to one that contained 60% stocks and 40% commodities, investors could have reduced their risk by 30% without giving up any returns. Georgiev (2001) performs a similar study over the period 1995 to 2005 and demonstrates that adding a commodity component to a diversified portfolio leads to enhanced Sharpe ratios. Similar are the results of Conover et al. (2010) who report that commodity exposure improves portfolio returns in periods of increasing interest rates; consistent with the view that commodities serve as an inflation hedge.

Another branch of the literature (e.g., Tang and Xiong, 2012; Lombardi and Ravazzolo, 2016; Silvennoinen and Thorp, 2013) argues that the correlation of commodities with stocks and bonds has strengthened. As such, their effectiveness as an alternative risk diversification channel[1] diminishes, as a consequence of financialization of the commodity markets. For example, Daskalaki and Skiadopoulos (2011) challenge their return and risk advantages and find that a mean-variance investor is not better off by allocating a portion of their capital to commodities compared to a portfolio that consists of traditional assets, consistent with the empirical evidence on the increasing financialization of commodities. Similarly, Cotter et al., (2017) implement different strategies and conclude that commodities do not improve the opportunity set of an investor with an existing portfolio of stocks, bonds and T-bills.

Much of the previous research reports mixed evidence on the merits of commodity investment as part of a diversified portfolio. In essence, these gains are hard to predict and can vary significantly across commodities, throughout time or with respect to the business cycle. Belousova and Dorfleitnerr (2012) confirm that there is a strong variation in the diversification contribution across individual commodities and commodity sectors. This can be attributed to

[1] Silvennoinen and Thorp (2013) present evidence favoring commodity and financial market integration and document that correlations between stock returns and returns to the majority of commodity futures have increased. This implies that there might be variables with the capacity to predict both commodity and equity returns (e.g., see Hong and Yogo, 2012). For instance, Asness et al. (2013) find common factors able to explain the pooled cross-section of various asset classes including commodities. On the contrary, some earlier studies – prior to the 2007-2009 financial crisis (e.g., Chong and Miffre, 2010; Büyükşahin et al., 2010) - challenge the view of increased integration and argue that commodity returns are affected by commodity-specific variables. Hence, equity asset pricing factors cannot explain the cross-section of commodity futures suggesting market segmentation (e.g., Bessembinder and Chan, 1992; Erb and Harvey, 2006).

the unique fundamentals of each commodity sector which makes them uncorrelated with one another. In other words, it is more meaningful to consider them as a market of separate assets rather than a homogeneous market (e.g., see Erb and Harvey, 2006). In addition, Büyükşahin et al. (2010) find that the alleged benefits commodities could bring to equity investors did not materialize when they would have helped the most. This time-variation in the diversification value is further confirmed by Adams and Glück (2015) who argue that commodities provide less loss protection after 2008. After the financial crisis, a new channel transmitting stock market shocks to commodities has opened, especially when the latter exhibit high volatility. In effect, whether commodities add economic value in asset allocation seems to be linked to the business cycle and market conditions. For example, Gorton and Rouwenhorst (2006) assert that commodities improve the risk-return profile of stock and bond portfolios and the effect can be more pronounced in late expansion and early recession phases. Furthermore, Jensen et al. (2000) find that during restrictive phases of the monetary cycle, commodity futures can lead to significant portfolio return enhancement. Finally, Cheung and Miu (2010) also report that the diversification gains of commodities are regime-dependent with the overall long-run benefits being a result of the infrequent episodes of outbursts in the commodity markets.

Another reason for conflicting results in the literature might be attributed to the various research designs. The majority of studies analyzing the contribution of commodity investment in a portfolio of traditional assets is based on an in-sample setting. However, in-sample analyses implicitly entail forward looking information and, therefore, tend to overstate the achievable gains. For example, Daskalaki and Skiadopoulos (2011) find that, commodities contribute only in-sample, but do not add value out-of-sample. Bessler and Wolff (2015) test different asset allocation strategies and report that the attainable benefits of commodities are much smaller than suggested by previous studies and depend on the type of commodity. Other studies conclude that commodities enhance the out-of-sample performance of optimized portfolios (Gao and Nardari, 2018; Daskalaki et al., 2017; You and Daigler, 2013). Given the diverse conclusions, the out-of-sample contribution of commodities remains ambiguous; this constitutes an additional motivation to explore whether the benefits ascribed to commodities have been exaggerated or not, and investigate the means to practically exploit them.

The aim of this paper is to empirically examine the impacts of considering commodity investments while at the same time exploit asset volatility and correlation dynamics from the perspective of dynamic portfolio management. We consider an active portfolio manager who uses forecasts from dynamic volatility and correlation models to rebalance a portfolio that contains traditional assets (stocks, bonds and cash) and a pool of 14 commodities traded on the CME Group as well as a diversified commodity index. To this end, we compare the performance of different models of forecasting covariances in terms of optimizing mean-variance efficient portfolios; (a) sample covariance, (b) constant conditional correlation (Bollerslev, 1990), (c) dynamic conditional correlation (Engle, 2002), (d) mixed data sampling conditional correlation (Colacito et al., 2011) and (e) regime switching dynamic correlation (Pelletier, 2006). A more accurate set of volatility and/or correlation predictions will render the investors a way to adaptively adjust their positions so as to achieve a higher utility level. Our analysis aims to provide market participants with information that can be used to fine tune risk attitudes and support the decision making process.

The contributions of this article are several. First, we revisit the role of commodities in asset allocation and their capacity to provide diversification benefits in a case study which examines portfolio risk-return characteristics. Results are validated in terms of Sharpe ratios and risk-adjusted abnormal realized returns (Modigliani and Modigliani, 1997). Optimal portfolios derived from either the traditional asset classes alone (equities, bonds and cash) or augmented with different commodity investments. More importantly, we consider both static and several dynamic asset allocation strategies, and therefore, offer additional insights; whether or not the portfolio benefits of commodities depend on the implemented asset allocation approach. In doing so, we investigate individual commodities and a diversified commodity index separately, thereby evaluating their potential impact from a portfolio management perspective.

Second, we systematically address the issue under the prism of short-horizon volatility and correlation timing strategies. This way, asset allocation efficiency, in terms of risk minimization and return maximization, is directly linked to predictions of volatilities and correlations. To the best of our knowledge, this is one of a few studies that explicitly takes into account predictability of second moments in forming optimal portfolios. This aspect has been largely neglected by asset allocation studies that consider commodities which mainly rely on constant historical estimators (e.g., Bodie and Rosansky, 1980; Jensen et al., 2000; Belousova and Dorfleitner, 2012) or rolling-sample estimators (e.g., Daskalaki and Skiadopoulos, 2011; Bessler and Wolff, 2015). An exception is Gao and Nardari (2018) who consider dynamic forward looking strategies. As it is widely agreed that the covariance structure of asset class returns varies substantially across periods and market conditions, this might have an effect on the diversification value which is itself time-varying.

Third, our analysis focuses not only on whether volatility timing is able to generate economic value compared to a benchmark strategy; but also on any additional value that can be bestowed to the investor when timing both correlations and volatility. Thus, for the first time to our knowledge, we assess the impact of dynamic correlations separately from that of volatility and provide a comprehensive analysis of the extent to which dynamic correlations affect optimal portfolio choice. To capture the trade-off between risk and return and derive the economic value of dynamic strategies we measure the fees mean-variance risk averse investors will be willing to pay to switch from one model to another based on the postulated utility gains (performance or switching fee); for applications, see Fleming et al. (2001, 2003), Corte et al. (2009) and Chou and Liu (2010), among others.

Forth, we assess the robustness of our conclusions to the choice of parameters such as different specifications for correlation dynamics, rebalancing frequency, estimation period (sub-periods) and transaction costs. We also consider how sensitive our results are to different investment styles, i.e., whether there is any impact on the diversification value of commodities if short selling is not permitted. In addition, since existing studies that support the inclusion of commodities in the opportunity set are mainly based on in-sample assessments, we also rely on out-of-sample performance evaluations. Finally, the mean-variance setting is also contrasted with optimization of alternative risk measures that focus on tail-risk (conditional value-at-risk).

The structure of the paper is as follows. The next section describes the methodology employed to construct optimum portfolios and quantify volatility and correlation timing gains. Section 3 introduces the econometric methodology and variance-covariance predictive models. Section 4 presents the data and presents the model estimation results. Section 5 offers the main empirical results on dynamic portfolio management and provides portfolio performance comparisons based on different models of the conditional second moments. Finally Section 6 concludes the paper.

2. Optimal portfolio selection

In this section we first formulate the asset allocation problem using mean-variance analysis. Then, we present the performance evaluation framework. The details of the methodology are as follows

2.1. Asset allocation in a mean-variance framework

Our objective is to determine whether there is economic value in conditioning trading strategies on volatility and correlation, and if so, which specification works best. For this reason, the standard Markowitz (1952) mean-variance portfolio analysis is employed. Let r_{t+1} represent the $Nx1$ vector of risky asset returns, with conditional expectation $\mu_{t+1|t} = E_t[r_{t+1}]$ and conditional covariance $H_{t+1|t} = E_t\left[\left(r_{t+1} - \mu_{t+1|t}\right)\left(r_{t+1} - \mu_{t+1|t}\right)'\right]$. For each date t, the investor constructs portfolios through the following optimization:

$$\min_{w_t}\left\{\left(\sigma_p^*\right)^2 = w_t' H_{t+1|t} w_t\right\},$$

$$s.t.\ \mu_p^* = w_t'\mu_{t+1|t} + (1 - w_t'\mathbf{1})r_f, \tag{1}$$

where w_t is a $Nx1$ vector of portfolio weights on the risky assets and r_f is the return on the risk free asset; μ_p^*, is the target rate of return. We impose no constraints on short positions since futures can be easily shorted in practice. Solving the above quadratic problem results in the following optimum weights:

$$w_t = \frac{(\mu_p^* - r_f)H_{t+1|t}^{-1}(\mu_{t+1|t} - r_f\mathbf{1})}{(\mu_{t+1|t} - r_f\mathbf{1})' H_{t+1|t}^{-1}(\mu_{t+1|t} - r_f\mathbf{1})}, \tag{2}$$

Applying standard no-arbitrage arguments under the cost of carry model - since futures contracts do not involve any up-front costs investment - the futures return equals the spot return minus the risk-free rate. Consequently, Eq. (2) can be simplified to

$$w_t = \frac{(\mu_p^*)H_{t+1|t}^{-1}(\mu_{t+1|t})}{(\mu_{t+1|t})' H_{t+1|t}^{-1}(\mu_{t+1|t})}. \tag{3}$$

Optimal portfolios can alternatively be constructed using other objective functions. We consider also a maximum expected return rule which leads to a portfolio allocation on the efficient frontier for a given target volatility σ_p^*. The investor's optimization problem and its solution can then be represented by the following Eq. (4) and (5), respectively

$$\max_{w_t} \left\{ \mu_{p,t+1} = w_t' \mu_{t+1|t} + (1 - w_t' \mathbf{1}) r_f \right\},$$

$$s.t.\ (\sigma_p^*)^2 = w_t' H_{t+1|t} w_t. \tag{4}$$

$$w_t = \frac{\sigma_p^* H_{t+1|t}^{-1} (\mu_{t+1|t} - r_f \mathbf{1})}{\sqrt{(\mu_{t+1|t} - r_f \mathbf{1})' H_{t+1|t}^{-1} (\mu_{t+1|t} - r_f \mathbf{1})}}, \tag{5}$$

Again, applying standard no-arbitrage arguments,

$$w_t = \frac{\sigma_p^* H_{t+1|t}^{-1} (\mu_{t+1|t})}{\sqrt{(\mu_{t+1|t})' H_{t+1|t}^{-1} (\mu_{t+1|t})}}. \tag{6}$$

The mean–variance framework above is used to devise trading strategies that identify the dynamically rebalanced portfolio with (i) minimum variance for any choice of expected return or (ii) maximum expected return for any choice of variance.

2.2. Performance measurement

To quantify the value of volatility and correlation timing, we follow Fleming et al. (2001; 2003) and compare dynamic strategies to that of the unconditional mean-variance efficient static strategies that have the same target expected return and volatility. In particular, the investor's realized utility in period $t + 1$ can be written as

$$U(W_{t+1}) = W_t R_{p,t+1} - 0.5\lambda W_t^2 (R_{p,t+1})^2,$$

where W is the investor's wealth, R_p the gross portfolio return and λ an absolute relative risk aversion coefficient. We hold the investor's degree of relative risk aversion, $\delta_t = \lambda W_t / (1 - \lambda W_t)$, equal to a fixed value δ. Thus, one can use the average realized utility, $\bar{U}(\cdot)$, to consistently estimate the expected utility generated by a given level of the initial wealth W_0 (West et al., 1993; Fleming et al., 2001, 2003; Corte et al., 2009)

$$\bar{U}(\cdot) = W_0 \left(\sum_{t=1}^{T} R_{p,t+1} - 0.5\delta (1 + \delta)^{-1} (R_{p,t+1})^2 \right).$$

We standardize the investor problem by assuming she allocates \$1 in every time period. Note that, by fixing δ rather than λ, we are interpreting quadratic utility as an approximation to a non-quadratic utility function with the approximating choice of λ dependent on wealth. Our evaluation focuses on the fee, Φ, an investor is willing to pay for switching from one modelling strategy to another. This is equivalent to finding the value of Φ that satisfies:

$$\sum_{t=0}^{T}\left\{\left(R_{p,t+1}^{*}-\Phi\right)-\frac{\delta}{2(1+\delta)}\left(R_{p,t+1}^{*}-\Phi\right)^{2}\right\}=\sum_{t=0}^{T}\left\{R_{p,t+1}-\frac{\delta}{2(1+\delta)}\left(R_{p,t+1}\right)^{2}\right\}, \quad (7)$$

where $R_{p,t+1}^{*}$ the gross portfolio return constructed using the expected return, volatility and correlation forecasts from a certain model and $R_{p,t+1}$ a benchmark's gross return.

3. Econometric models

Finding the optimal portfolio allocation requires information of the variability of individual asset classes and their co-movements. Traditionally, Autoregressive Conditional Heteroscedasticity (ARCH) models (Engle, 1982 and Bollerlev, 1986) - have been widely used to describe the volatility of asset prices, due to their flexibility. These models have been extended to multivariate models[2] to study the co-movements of asset returns; this is of paramount importance since the covariance/correlation structure is an indispensable parameter in asset pricing, asset allocation and risk management decisions.

We begin our formal description of the econometric models by letting $r_t=(r_{1t},r_{2t},\dots r_{Nt})'$ represent the returns of N assets at time t

$$r_t=\mu_t+\varepsilon_t$$

$$\varepsilon_t=z_tH_t^{1/2}, \quad (8)$$

[2] For a comprehensive survey of multivariate GARCH models the reader is referred to Bauwens et al. (2006).

where $\mu_t = (\mu_{1t}, \mu_{2t}, \dots \mu_{Nt})'$ the vector of conditional means, H_t the conditional covariance matrix, and ε_t a vector of innovations; z_t denote the standardized residuals. As the primary focus of our study is the effect of dynamic volatility and correlation on asset allocation, our analysis assumes a constant conditional mean $\mu_t = \mu$. This is equivalent to specifying a random walk model for the (log) asset prices, e.g., see Fleming et al. (2001, 2003) and Chou and Liu (2010). By construction, in this setting, optimal weights will vary across models only to the extent that forecasts of the conditional volatility and correlations will vary. Note also that changes in expected returns are hard to detect while the volatility is far more predictable (Merton, 1980).

As for the second conditional moments, models of conditional correlations are based on the partition of the variance-covariance matrix (see Bollerslev, 1990)

$$H_t = D_t P_t D_t,$$
$$D_t = \mathrm{diag}\left(h_{1t}^{1/2}, h_{2t}^{1/2}, \dots, h_{Nt}^{1/2}\right). \tag{9}$$

D_t is the NxN diagonal matrix of volatilities and $P_t = [\rho_{ij,t}]$ a positive definite correlation matrix with $\rho_{ii,t} = 1$, for i = 1, 2… N, for every t. This means that the off-diagonal elements of the conditional covariance matrix are defined as $[H_t]_{ij} = h_{it}^{1/2} h_{jt}^{1/2} \rho_{ij,t}$, for $i \neq j$. This decomposition allows for separate formulation of individual volatilities and cross-correlation matrices.

We assume that individual variance processes are driven by a GARCH(1,1) model (Engle, 1982; Bollerlev, 1986). The conditional variance of each asset *i* is given by:

$$h_{it} = \omega_i + \alpha_i (r_{it-1} - \mu_{it})^2 + \beta_i h_{it-1}, \tag{10}$$

with $\omega_i > 0$ and $a_i, \beta_i \geq 0$ to guarantee nonnegative variance and $a_i + \beta_i < 1$ so that the variance process is stationary and unconditional long-run variance of asset *i* can be defined as $\omega_i / (1 - \alpha_i - \beta_i)$.

Following Engle (2002), a two-stage estimation procedure is employed. The first step involves the estimation of univariate models for conditional variances; in the second step we estimate the conditional correlations dynamics. Under the assumption of normally distributed innovations, the log-likelihood estimator can be written as

$$lnL = -\frac{1}{2}\sum_{t=1}^{T}[Nlog(2\pi) + 2log(|D_t|) + log(|P_t|) + z_t' P_t^{-1} z_t], \tag{11}$$

where z_t are the standardized residuals $z_t = D_t^{-1}\varepsilon_t \sim N(0, P_t)$ with $\varepsilon_t = r_t - \mu_t$. Our empirical applications consider four models, the Constant Conditional Correlation (CCC; Bollerslev, 1990), the Dynamic Conditional Correlation (DCC; Engle, 2002), the Dynamic Component Conditional Correlation (MDC; Colacito et. al, 2011), and the Regime Switching Correlation (RSC; Pelletier, 2006) ; these are briefly described next.

3.1. The Constant Conditional Correlation model

The CCC model (Bollerslev, 1990) assumes constant correlations but dynamic volatilities. The following decomposition of the conditional covariance matrix is assumed

$$H_t^{CCC} = D_t \bar{P}_t D_t. \tag{12}$$

$\bar{P}_t$ is set equal to the unconditional correlation matrix $\bar{P}$ and D_t contains the GARCH(1,1) volatilities. The main feature of the CCC model is that, as correlations are constant, the dynamics of covariances are governed exclusively by the dynamics of volatilities as $[H_t^{CCC}]_{ij} = h_{it}^{1/2} h_{jt}^{1/2} \bar{\rho}_{ij}$.

3.2.The Dynamic Conditional Correlation model

The DCC model (Engle, 2002) combines dynamic correlations and the GARCH model. The correlation structure can be represented by

$$H_t^{DCC} = D_t P_t D_t,$$

$$P_t = (diag\{Q_t\})^{-1/2} Q_t (diag\{Q_t\})^{-1/2},$$

$$\mathrm{Q_t} = (1 - a - \beta)\bar{P} + a z_{t-1} z'_{t-1} + \beta \mathrm{Q_{t-1}}, \tag{13}$$

where P_t is the NxN symmetric matrix of dynamic conditional correlations, and Q_t is an NxN symmetric positive-definite matrix, a and β are non-negative parameters. The process in Eq. (13) is mean-reverting, on the condition that $a + \beta < 1$.

3.3.The Dynamic Component Conditional Correlation model

The MDC model (MIDAS-DCC; Colacito et al., 2011) is a dynamic conditional correlation model (Engle, 2002) with mixed data sampling (MIDAS). It decomposes the correlation process into a long-run and a short-run component with the general process described as follows

$$H_t^{MDCC} = D_t P_t D_t,$$

$$P_t = (diag\{Q_t\})^{-1/2} Q_t (diag\{Q_t\})^{-1/2},$$

$$\mathrm{Q_t} = (1 - a - \beta)\bar{P}_t + a z_{t-1} z'_{t-1} + \beta \mathrm{Q_{t-1}}. \tag{14}$$

Essentially, the long-run component of correlations $\bar{P}_t$ can be filtered from some empirical proxies given by weighted averages of cross-products of residuals $\mathrm{c}_{ij,t}$. Let $\bar{P}_t = [\bar{\rho}_{ij,t}]$, K_c the number of lags of realized correlations considered and N_c the number of daily non-overlapping returns needed to compute each realized correlation, respectively. The long run correlation component is

$$\bar{\rho}_{ij,t} = \sum_{l=1}^{K_c} \varphi_l(\omega_r)\, c_{ij,t-1}$$

$$\mathrm{c}_{ij,t-1} = \frac{\sum_{k=t-N_c^{ij}}^{\mathrm{t}} z_{i,k} z_{j,k}}{\sqrt{\sum_{k=t-N_c}^{\mathrm{t}} z_{i,k}^2}\sqrt{\sum_{k=t-N_c}^{\mathrm{t}} z_{j,k}^2}} \tag{15}$$

where φ_l denotes the weight in the weighting scheme; $c_{ij,t-1}$ and corresponds to the block sampling scheme; and ω_r the rate of decay in the weighting scheme. For a complete description of the model we refer to Colacito et al. (2011).

3.4.The Regime Switching Correlation model

The RSC model (Pelletier, 2006) assumes that correlations switch stochastically over time, among a finite number of regimes. In this case, following the decomposition of the conditional covariance matrix as P_t process is driven by

$$H_t^{RSC} = D_t P_t D_t,$$

$$\mathrm{P_t} = \sum_{v=1}^{V} I_{\{S_t=v\}}\,\mathrm{P}_v \tag{16}$$

where I is the indicator function, S_t is an unobserved Markov chain process independent of ε_t which can take $\{1, 2, \ldots v\}$ possible values and P_v are regime dependent correlation matrices. Regime switches in the state variable, S_t, are assumed to be governed by a VxV transition probability matrix; we set $V = 2$ (for more details on the estimation procedure, we refer to Pelletier, 2006). Transition probabilities between states are assumed to follow a first order Markov chain and remain constant through time

$$p_{ij} = \Pr(S_t = j|S_{t-1} = i, St-2 = l, \ldots) = \Pr(S_t = j|S_{t-1} = i) \tag{17}$$

4. Data and estimation results

The data set for this study comprises daily closing futures prices collected from Datastream. The sample period spans from December 19, 1994 to January 3, 2012, resulting in 4,301 observations after adjusting for US bank holidays. We consider S&P 500 and 30 year Treasury bonds to proxy the performance of traditional assets, namely the stock and bond market. We also use the 3-month Treasury bill rate to substitute for the risk free rate (cash). For the asset class of commodities we use the S&P Goldman Sachs Commodity Index (GSCI), as well as a set of individual commodity futures contracts written on major commodities: from the energy complex we consider West Texas Intermediate (NCL) Crude Oil and Henry Hub Nat. Gas (NNG); for metals, Gold (NGC), Silver (NSL) and High Grade Copper (NHG); from the agricultural sector, Wheat (CW), Corn (CO), Soybeans (CS) and Orange Juice (NJO); the soft commodities, Coffee (NKC), Cocoa (NCC), Sugar (NSB) as well as Live Cattle (CCL) and Cotton (NCT). It is assumed that the investor will roll over to the front month contract the first day of the expiry months March, June, September and December which constitute common expiration months among all futures considered. Hence, although contracts trade under dissimilar expiry schedules[3], switching contracts among different assets takes place on the same day. To adjust for rollover artificial gains/losses on rollover days, the appropriate one-day overlapping prices of each contract are used to calculate returns.

Panel A of Table 1 reports summary statistics for the futures returns over the period of the analysis. The statistics show the diversity of the risk-return profile of different assets. Commodity futures exhibit higher volatility levels than financial assets. In the relative high volatility group, e.g. more than 30% per annum (p.a.), we can classify WTI, Nat. Gas, Silver, Wheat, Cocoa, Coffee and Sugar. On the other hand, Gold and Live Cattle are the least volatile commodities and comparable to financials. Moreover, non-negligible skewness and excess kurtosis signify that the unconditional distribution of asset returns is not normal. Based on the Ljung-Box (1978) Q statistics the autocorrelation structure reveals strong persistence. Engle's (1982) ARCH test, carried out as the Q statistic on the squared returns' series, indicates the

[3] For instance, NYMEX WTI contracts are traded for all consecutive month deliveries within the current and the next 5 years. On the other hand, S&P 500 futures are listed for eight months in the March quarterly cycle.

existence of heteroscedasticity. This provides preliminary evidence in support for the use of time-varying conditional variance.

Panel B of Table 1 presents the risk-return profiles of portfolios constructed based on the naïve diversification 1/N rule; in which a fraction 1/N of wealth is allocated to each of the N assets available for investment at each rebalancing date. We also present the risk-return profile of value-weighted portfolios; in which the fraction of wealth allocated to each of the assets available for investment at each rebalancing date, is determined by the market value of the individual contracts (this strategy invests in each asset proportional to its market value). The same table shows the annualized mean, volatility and Sharpe ratios (SR) for the entire sample and the last 7 years. For the equally weighted portfolios of stocks, bonds and commodity futures, it is only GSCI, WTI and Gold that manage to outperform the traditional portfolio, with Gold having the ability to reduce portfolio volatility by more than 150 annual basis points. For value-weighted portfolios, in addition to GSCI, WTI and Gold some benefits can be exploited when investing in Silver, Copper and Soybeans as well (entire sample). Yet, for the last 7 years of our sample, these gains are not preserved for WTI; although Silver, Copper, Soybeans, Sugar and Orange Juice seem to gain rank in terms of maximizing SRs mainly due to enhanced returns; results, during the last 7 years of our sample period are consistent across both portfolio strategies. Note that, several studies find that simple portfolio strategies, such as an equally-weighted portfolio, often outperform the mean-variance optimal portfolio especially in an out-of-sample setting (e.g., DeMiguel et al., 2009). Moreover, these strategies are not prone to estimation errors as they do not require forecast models or optimization techniques and are easily implementable. To this end, we can use these preliminary figures as reference for benchmarking purposes against our model dependent asset allocation results, presented in the ensuing analysis.

4.1.Conditional covariance estimates

Parameters of univariate GARCH(1,1) models appear in the last three columns of Table 1 (Panel A). Results are standard for financial data. At conventional levels, ARCH coefficients are significant and range from 0.028 to 0.092, while GARCH coefficients are significant at 1% level and range from 0.901 to 0.967. Moreover, the conditional variance process is stationary, $\alpha + \beta < 1$ in all cases, and strongly persistent as the sum is close to 1.

The Table reports also the sample return correlation with stock, $\rho_{s,i}$, and bond returns, $\rho_{b,i}$. The correlation between stock and bond returns, $\rho_{s,b}$, is negative (-0.211), whereas for stock and commodity returns is positive and significant within the range of 3.4% (Nat. Gas) to 22.6% (Copper), apart from Gold which is -3.4%. On the other hand, the correlation between bond and commodity returns is negative and significant within the range of -3.7% (Silver) to -17% (Copper) with the exception of Gold which is 4.3%.

The parameter estimates, along with standard errors, of the dynamic models, i.e., DCC, MDC and RSC, are presented in Table 2. For both DCC and MDC, α coefficients, measuring the sensitivities of asset correlations to market shocks, are statistically significant in all equations with figures ranging between 0.016-0.027 and 0.024-0.039, for the DCC and MDC. Estimates β, measuring the sensitivity of current correlation to past values, range from 0.968-0.981 and 0.884-0.959, for the DCC and MDC, and with all parameters being statistically significant. Moreover, $\alpha + \beta$ is less than one but close to unity, i.e., the conditional correlations are stationary and persistent. This finding has implications in risk and portfolio management as the impacts of asset-specific market shocks have prolonged effects on the subsequent dependence structure. Persistent co-movements lend support to the presence of predictable patterns and reflect slow mean reversion in correlations due to the existence of transitory trends. MDC models produce marginally less persistence in conditional correlations than DCC (0.995 to 0.997 vs. 0.926 to 0.983). Finally, the MIDAS filter parameter (ω_r) is significant in nearly all cases and ranges between 1.012 to 3.136.

Turning next to the regime switching model (RSC), correlations are clearly differentiated between two regimes. State correlations between stock and bond returns, $\rho_{s,b}$, are significant in all cases, while they are negative in state 1 and positive in state 2. Commodities display a quite different pattern. In state 1, correlations are significant in nearly all cases and $\rho_{s,c} > 0$ and $\rho_{b,c} < 0$ with only exception Gold where the relationship with bond returns is positive and significant. On the other hand, $\rho_{s,c} = \rho_{b,c} = 0$ in state 2; at 5% significance level. From the estimated transition probabilities we can calculate the duration of being in each regime, e.g., for state 1, this is $\sum_{i=1}^{\infty} iP_{11}^{i-1}(1 - P_{11}) = (1 - P_{11})^{-1}$. The figures presented for state 1 (2) correspond to approx. 6.5 (4.5) months, while both regimes are highly persistent; all probabilities of staying in a specific state are high. As implied by the transition probabilities, the commodities' potential to offer diversification gains are time-varying and depend on the regime that the market is in. Also, markets switch between periods of significant and zero correlations with higher tendency on the former. This is important as identifying the phase of the business cycle encloses information on how and if commodities can act as an efficient diversification tool.

Figure 1 plots the estimated conditional correlations between commodity futures returns to the stock (left) and bond (right) returns. The figure displays average conditional correlations (across the three models DCC, MDC and RSC). To offer a collective view, the first row of the figure shows the average conditional correlation, across the fifteen commodity assets under examination, along with the interquartile range (25th and 75th percentiles) for each estimate at each point in time. Inspection of the individual stock-commodity correlations reveals several interesting features. We can see diverse dynamics across the individual commodities supporting the view that commodities constitute a market of individual dissimilar assets rather than a homogeneous market (e.g., see Erb and Harvey, 2006). In addition, before the financial crisis correlations oscillate around zero, while increases and decreases are frequently observed within the range of -20% to 20%.

Previous studies note that the behavior of commodities appears to have changed somewhere between 2004 and the 2007–2009 financial crisis (see, among others, Tang and Xiong, 2012; Daskalaki and Skiadopoulos, 2011; Daskalaki et. al, 2017). The average stock-commodity correlation (Figure 1 at the top) marks a structural change during and after the 2008 financial crisis. This also holds for the individual correlation estimates for all commodities; apart from Nat. Gas and to a certain extent Gold. For GSCI and WTI, a gradual upward shift in the individual correlation estimates is noted, as soon as 2005. Afterwards, during and following the 2007-2008 period, correlation anchors at higher levels. The bond-commodity average correlation displays a similar pattern, with the expected opposing sign interpretation (as yield and bond prices are inversely related).

In retrospect, it is only after 2008 that correlation remained at high levels compared to the history of the series; consistent to Büyükşahin and Robe (2014) and Adams and Glück (2015), among others. Commodities as an asset class have become popular to institutional investors (e.g., see Büyükşahin and Robe, 2014) and much of this trend is fuelled by the belief that commodities offer consistent diversification benefits; especially against downturns in stock markets (e.g., Gorton and Roubenworst, 2006). From 2004 onwards, the unprecedented inflow of funds into commodities is believed to have generated linkages between commodities and traditional assets. Our findings corroborate Büyükşahin et al. (2010), among others, i.e., prior to 2008, the large-scale capital inflows into commodities and the presence of institutional investors was not accompanied by an increase in correlations of commodities with traditional assets. During the financial crisis, however, correlations significantly increased; see also Cheung and Miu (2010), Daskalaki and Skiadopoulos (2011) and

Silvennoinen and Thorp (2013), among others. For example, Adams and Glück (2015) suggest that the financial crisis may have initiated and amplified the occurrence of risk spillovers between commodities and other assets. As a result, financial markets serve as a channel transmitting outside shocks to commodities which in turn are also determined by the aggregate investor risk appetite for financial assets and the investment behavior of commodity investors, in addition to supply and demand dynamics (Tang and Xiong, 2012).

Furthermore, Figure 2 reports the average (across the three models DCC, MDC and RSC) conditional correlation, after splitting the sample based on asset volatility percentiles, i.e., 90%, 75% and 50% for the right (dark colour bars) and left (light colour bars) tails of the volatility distributions; the time series' of conditional volatilities are obtained from the GARCH model estimates (see Table 1). The first two plots at the top (first row) represent the role of commodity volatility to the formation of stock-commodity (left) and bond-commodity (right) correlation. The two plots at the bottom (second row), portray role of financial market volatility, i.e., stock (left) and bond (right) volatility, respectively.

For most commodities, correlations with stock returns rise in high commodity volatility states. An exception to this is Coffee where the relationship is reversed, while for Nat. Gas and Orange Juice there does not seem to be a strong link to commodity volatility. Concerning the effect of stock market volatility, same conclusions can be drawn, albeit more pronounced. In particular, correlations of commodities with stock returns rise in high stock volatility regimes which is indicative of a certain degree of interconnectedness. Gold constitutes an exception to this, i.e., high stock market volatility is associated with high negative correlations. Turning next to the impact on bond correlations, similar conclusions can be drawn but with the expected opposite sign. Relatively high volatilities positively affect correlations (in absolute value). When considering commodity (bond) volatility, for Cocoa and Coffee (Silver) this relationship is rather weak, while for Gold, high (low) asset volatility is associated with positive (negative) correlations. In conclusion, we find that, similar to Silvennoinen and Thorp (2013), closer integration emerges around high volatility states indicating contagion in extreme market conditions; in line also with Büyükşahin et al. (2010) who argue that, from a portfolio perspective (at least for passive investment strategies), the diversification role of commodities is significantly reduced in periods of turmoil.

5. Empirical results

The objective of this article is to examine the benefits of (i) augmenting a portfolio of traditional assets (stocks and bonds) with commodities, and (ii) implementing diverse dynamic structures for the asset returns variances and covariances/correlations in portfolio construction. This is achieved through an investment exercise which employs the covariance matrix prediction models presented in Section 3. The economic value of short-horizon volatility and correlation timing is assessed by analyzing the performance of the dynamically rebalanced portfolios constructed using the set of candidate multivariate models. We focus on the realized Sharpe ratios (SR) and performance fees (Φ), a risk averse investor with a degree of relative risk aversion of $\delta = 6$, is willing to pay for switching from one model to another (see Section 2.1). Our approach also requires a benchmark stock, bond and cash only mean-variance efficient portfolio to measure the effect of excluding commodities from the opportunity set. This section discusses the results in terms of in- and out-of-sample tests.

5.1.In-Sample Portfolio Performance

The setup of our in-sample numerical experiments is as follows. We use a history of data covering the period December 1994 to January 2012 to estimate the parameters of the CCC, DCC, MDC and RSC models. This period contains 4,300 daily return observations for each asset. We then construct optimal portfolios of four assets (futures): S&P 500, US Bond, cash and an individual commodity (or index). Then, two portfolios are constructed: a minimum volatility portfolio (MinV) with a target annual return of $\mu_p^* = 10\%$ (Eq. 3) and a maximum return portfolio (MaxR) with target volatility of $\sigma_p^* = 12\%$ p.a. (Eq. 5). Given the optimized weights we calculate returns on the portfolio for a holding period of 1 trading day.

In Table 3, Panel A, we initially report the results of a stock, bond and cash only portfolio. We find that there is substantial economic value associated with volatility timing. This is evident from both SRs and the performance fees that CCC models with GARCH volatilities generate compared to the benchmark sample covariance model (Static[4]); note that covariances of this model, and hence optimized weights, are governed exclusively by the dynamics of volatilities as correlations are constant. Relative to the Static approach, CCC produces higher SRs; 5.8% (14.7%) improvement in the SR of the MinV (MaxR) rule. Moreover, MinV (MaxR) portfolio performance fee, Φ, for switching from the Static to the CCC amounts to 30 (219) annual basis points (bps). On the other hand, the fee for switching from Static to the conditional correlation models with dynamic GARCH volatilities increases to 60 (250) bps, for RSC model. Therefore, in addition to the economic value associated with timing volatility, there is also value specifically due to correlation timing.

To investigate whether the above results are preserved, and possibly enhanced, if we add commodity exposure in our opportunity set, we document portfolio performance of stocks, bonds, GS commodity index and cash in Table 3, Panel B. When timing conditional second

[4] The benchmark Static model is the only empirical model that assumes constant covariance matrix. Therefore, the in-sample optimal weights for this trading strategy remain constant over time. However, to implement a more realistic strategy we perform the optimizations every year separately, i.e., weights change on an annual basis; note that this actually improves Static method Sharpe ratios.

moments, results are similar to those of a portfolio of traditional assets. For example, the MinV (MaxR) strategy implemented by DCC (MDC) outperforms the alternatives with an improvement in SR close to 10% (22.5%) compared to the Static approach and a fee Φ of 22 (325) annual bps. The benefits added to investors interested in maximizing returns are higher than those of minimizing volatility. Moreover, the set of dynamic correlation models leads to similar results, improving the further the SR of the CCC method by approx. 2.6% and yielding higher annual fees.

To formally assess the magnitude of the gains that can actually be realized by an investor when adding a commodity in a portfolio of stocks, bonds and cash we compute the $M2$ measure of Modigliani and Modigliani (1997) which evaluates the abnormal return a strategy would have earned if it had the same risk as some benchmark. As benchmark, we consider the portfolio in Panel A of Table 3 (stock, bond and cash only). $M2$ is essentially a risk-adjusted abnormal return and is directly related to the SR:

$$M2 = \frac{\sigma_{bench}}{\sigma_p}\left(\mu_p - r_f\right) - \left(\mu_{bench} - r_f\right) = \sigma_{bench}\left(SR_p - SR_{bench}\right) \tag{18}$$

From Table 3, Panel B, the reported $M2$ measures are all positive and considerable. Adding GSCI in our portfolio, the MinV (MaxR) objective yields 798 (550) bps of risk-adjusted abnormal returns, without considering rebalancing (Static). When we apply a dynamic strategy $M2$ demonstrates a potential to rise as high as 863 bps p.a. (DCC) when the goal is MinV and 719 bps p.a. (MDC) when the goal is MaxR. Further, diversification prospective of GSCI is high as $\sigma_p < 6.3\%$, while the stock, bond, cash only portfolio yields $\sigma_p > 10.3\%$, which translates to an average 88% increase in the SR, from 0.9 to 1.7. Similarly, GSCI has the potential to produce a return of $\mu_p = 25.69\%$ (MDC), as opposed to the stock, bond, cash only portfolio which has a ceiling at $\mu_p = 17.88\%$ (RSC); the former yields SR = 2.103 whereas the latter drops to 1.495, i.e., a decrease of 28.9%.

To check the robustness of the obtained results, we consider also investing in individual commodity futures. The goal is to take advantage of the heterogeneity in terms of commodity risk-return characteristics seeking to maximize diversification gains. Table 3 presents the results for energy commodities (Panels C and D) and metals (Panels E to G); Table 4 shows the results for agricultural commodities including live cattle and cotton. Interestingly, we find that the risk-adjusted abnormal returns as measured by $M2$ are all positive suggesting that economic gains are robust and investors are better off allocating a certain portion of their wealth to commodities. In terms of magnitude, for the MinV strategy, $M2$ is on average 449, within the range of 74 to 1003. For the MaxR, $M2$ has an average value of 413 ranging from 201 to 671. Considering also the fact that SR across all commodities and strategies lies between 0.924 to 2.056 (vs. 0.855 to 1.495 for the stock, bond and cash portfolio), we can conclude that commodities offer a substantial source of diversification. These gains are more pronounced when the optimization goal is to maximize return (average SR across commodities is 1.785 vs. 1.333 for MinV).

Figure 3 illustrates the yearly average (across models) abnormal returns ($M2$) in annualized bps. In particular, the chart demonstrates the evolution of $M2$ from 1995 to 2011; Panel (a) depicts minimum volatility while Panel (b) maximum return portfolios. Clearly, our previous results are robust in the sub-period analysis. It is only in 1995 and 1996 that some deviations can be observed, for Nat. Gas and Silver (1995) and Copper, Wheat, Soybeans, Corn and Sugar (1996). In all other cases, i.e. 248 out of 255 (15 commodity futures; 17 years), abnormal returns are positive; that is more than 97% of the time.

Across all commodities and strategies SR lies between 0.924 to 1.776 for the Static approach; on average, SR is 1.23 (1.56) under MinV (MaxR). CCC (no correlation timing) generates SRs in the range of 0.993 to 2 (average of 1.32 and 1.80 for the MinV and MaxR).

The corresponding figures for the dynamic correlation models are from 1.014 to 2.056 (averages of 1.36 and 1.86).

Furthermore, we find that, for most commodities volatility and correlation timing gains, as measured by Φ, are positive (13 out of 14 cases). Under MinV strategies only Coffee shows negative fees. In total, Φ ranges between -38 to 110 (average of 27) annual bps. Benefits are maximized for Copper, Soybeans, Sugar and Orange Juice for which Φ can reach levels in excess of 50 bps. Regarding model choice, while the benefits over timing volatility point towards an average 16 annual bps fee (incl. of the benchmark portfolio and the portfolio of GSCI), DCC improves this to 28, MDC to 30 and RSC to 34. On the other hand, MaxR strategies are more fruitful as all commodity cases generate positive Φ (14 out of 14 cases). Φ ranges between 139 to 423 (average of 260) annual bps, while benefits are maximized for Crude oil, Gold, Soybeans and Sugar for which Φ can be in excess of 300. This also holds for GSCI (DCC and MDC). Still, timing both volatilities and correlations implies superior performance with average fees of 273 (DCC), 274 (MDC) and 261 (RSC) which are more than the 236 fee of CCC (incl. of the benchmark portfolio and the portfolio of GSCI). Therefore, our results suggest again that that economic gains are robust and investors are better off when timing the second moments of portfolio components.

To get a sense of the economic value of volatility and correlation timing across years, Figures 4 and 5 show the performance fees (Φ) in annualized bps; from 1995 to 2011. Figure 4 depicts MinV while Figure 5 MaxR portfolios. Interestingly, Φ depends not only on the particular year but the specific strategy as well. For example, in 1996 (1995 and 1997) timing both correlation and volatilities provides the maximum (minimum) benefits when considering MinV portfolios; in all cases but Silver and Soybeans (Wheat, Corn, Cocoa). Timing only volatilities results in maximum (minimum) benefits when considering MinV portfolios in 1995 (1996 and 1997); 9 out of 15 cases (12 out of 15 cases). Concerning the MaxR strategies in

Figure 5, timing only volatilities provides the maximum (minimum) gains in 1995 (1997) in all cases (in all cases but Coffee and Cotton). For timing both correlations and volatilities minimum gains coincide with the CCC model but the maximum gains occurred in 1995 (4 cases) as well as 2002-2003 (7 cases) and 2008 (2 cases; Silver and Copper). Finally, performance fees are positive 87% of the time (82% for the CCC and 92% for the MDC) with most negative fees during 1996-1999 for CCC but 1997-1999 for MDC.

In conclusion, the in-sample analysis designates commodities as a substantial source of diversification, providing robust economic gains with average, across commodities, abnormal returns in excess of 4% p.a., irrespective of the optimization objective when compared to the traditional portfolio. We also compare different forecasting models to judge which method improves our ability to construct optimal portfolios. For static portfolios, abnormal returns are on average close to 3.8%, for volatility timing strategies 4.25% and for correlation and volatility timing this increases to more than 4.4%. Finally, we find that a risk averse investor facing commodity risk will pay a performance fee of about 1.25% p.a. for volatility timing and a further 0.25% p.a. for correlation timing.

5.2.Out-of-sample Performance

The results so far suggest a key role for commodity investment and volatility and correlation timing in asset-allocation decisions. However, our analysis was based on in-sample performance. Studies such as Inoue and Kilian (2006) show that in-sample tests have higher power, and therefore, tend to be more credible than out-of-sample tests. Still, relying solely on in-sample performance might not capture the forecasting power a practitioner might have had in real time. For example, Daskalaki and Skiadopoulos (2011) find that the alleged diversification benefits of commodities hold under the in-sample setting, but are not preserved out-of-sample.

To this end, we also implement a real time forecasting exercise. The setup of our experiment is as follows. We use a history of data covering the period December 1994 to January 2005 to estimate the parameters of the sample covariance, CCC, DCC, MDC and RSC models. This period contains 2,540 daily return observations for each asset. We then construct mean-variance efficient portfolios of stocks, bonds, commodities and cash and a benchmark stock, bond and cash portfolio. Given covariance one-day ahead forecast estimates we calculate optimized weights and compute realized returns on the portfolio for a holding period of 1 day. We assume three rebalancing frequencies: daily, weekly and monthly. Then, using a rolling window of 2,540 observations, estimation and optimization procedures are repeated until the dataset is exhausted. This exercise produces 1,760 out-of-sample observations that cover a period of 7 years, from January 2005 to January 2012.

Tables 5 and 6 show the out-of-sample results for weekly rebalances[5]. First, we examine portfolio performance in terms of the value added when our portfolio is augmented with a commodity. We can see that the optimal portfolios formed based on the traditional investment opportunity set yield lower SRs than the corresponding portfolio strategies based on the expanded opportunity set. Some exceptions occur, i.e., Cocoa and Live Cattle for which no strategy or model preserves the in-sample gains as well as Copper and Wheat (MinV) and Orange Juice (MaxR). These results are confirmed by the $M2$ measure which is negative in these instances. However, investing in commodities generates abnormal returns of 142 annual bps on average, resulting in an average SR of more than 0.47. This is higher than the max of 0.44 (MDC) of the stock, bond and cash only portfolio. More importantly, SR has the potential

[5] For brevity we report the case of weekly rebalancing frequency; results on daily and monthly frequencies are available from the authors upon request.

to reach a value in excess of 0.9 (Nat. Gas, CCC and Gold, RSC). As for GSCI, this generates SRs in excess of 0.63, as long as a dynamic strategy is considered. Abnormal returns in this case are limited to 102 and 22 bps when comparing the Static approaches of the MinV and MaxR strategies. Yet, their average values across models are 430 and 334 respectively; but for both strategies they exceed 430 bps in more than one cases.

Next, we examine the effect of rebalancing frequency, i.e., daily and monthly. Figure 6 illustrates the average (across the dynamic models; CCC, DCC, MDC and RSC) abnormal returns during the out-of-sample period. The chart reports the $M2$ measure with the stock, bond, cash portfolio as benchmark. The three columns correspond to three different rebalancing frequencies, i.e., daily (black), weekly (grey) and monthly (white). Overall, rebalancing strategies are close, producing equivalent gains. Under MinV, the magnitude of average annual abnormal returns reaches levels of 99, 141 and 172 bps, for daily, weekly and monthly rebalancing. Under MaxR, these figures are 135, 143 and 130 bps. On aggregate, weekly (monthly) rebalancing proves better in 13 (12) cases; 7 (8) out of 15 for MinV and 6 (4) out of 15 for MaxR strategies. Therefore, we can conclude that our results are robust.

We now focus on the economic value of volatility and correlation timing. Tables 5 and 6 report the performance fees (Φ) for all considered portfolios. It appears that the in-sample gains of timing volatility and correlations are preserved. Clearly, all dynamic strategies generate added value. MinV strategies yield fees within the range of 22 to 1,031 annual bps (average of 436) and MaxR strategies within the range of 119 to 577 annual bps (average of 327). CCC computes structures that realize the highest fees when interested in minimizing volatility (9 out of 16 cases). The second best model is the RSC (6 out of 16 cases). In terms of maximizing return RSC ranks first (9 out of 16 cases) and MDC second (7 cases).

Finally, we incorporate transaction costs, as their impact is indispensable from assessing the profitability of trading rules in an out-of-sample setting. In particular, if any gain does not cover the extra cost, less accurate but less variable weighting strategies would prove superior. Based on Marquering and Verbeek (2004), we subtract transaction costs from the net portfolio return ex-post. Although mean-variance portfolios are no longer optimal in the presence of transaction costs, this approximation maintains simplicity and tractability in the mean-variance setting. The net of transaction costs return, $R_{p,t+1}^{*,net}$, is calculated as

$$R_{p,t+1}^{*,net} = R_{p,t+1}^{*}\left(1 - tc\sum_{i=1}^{N}\left|w_{i,t+1} - w_{i,t}\right|\right) \tag{19}$$

where tc the proportional transaction cost. The cost of each trade over N assets, can be represented by portfolio turnover $tc\sum_{i=1}^{N}\left|w_{i,t+1} - w_{i,t}\right|$; the fraction of the portfolio value that is liquidated or reallocated at rebalancing points. Once the return is adjusted, Φ is re-calculated. Transactions costs are set to 50 bps per transaction which is consistent to DeMiguel et al., (2009) and Gao and Nardari, (2018); and conservative with respect to Bessler and Wolff (2015).

Results on the relative cost of rebalancing strategies implied by the different prediction models are presented in Tables 5 and 6 under the column $\Phi_{\delta=6}^{tc=50}$. It appears that the MinV strategies require a higher proportion of the portfolio to be restructured at each rebalancing point which imposes a higher transaction cost. In particular, fees drop by 38.5% on average (from 436 to 269) while for MaxR strategies this figure is 22% (from 327 to 256) with corresponding ranges -213 to 835 and 48 to 504, respectively. Yet, negative - after transaction costs - fees are observed in only two cases Silver (DCC, MDC) and Copper (DCC) MinV strategies; this confirms the robustness of our previous analysis as Φ is consistently positive. Moreover, it seems that all dynamic strategies' specifications require similar proportion of the portfolio to be restructured at each rebalancing point which imposes comparable transaction costs. The expected drop in Φ after incorporating transaction costs and based on the model considered is, on average, between 166-169 bps in the MinV and 66-75 bps in the

MaxR case. Therefore, transaction costs are compensated for the dynamic weighting strategies.

Figure 7 consolidates information on out-of-sample performance fees, with and without transaction costs. The shadowed area shows the annual switching fees from static allocation to a volatility (CCC) and a volatility/correlation timing strategy (black line; maximum of DCC, MDC, RSC). Portfolios are ranked clockwise, according to their performance. For traders engaging in timing conditional moments and in the presence of 50 bps costs per transaction investors can still benefit in all cases. The same holds for monthly rebalancing, thus, dynamic strategies' results are robust. Weekly rebalances perform better, as more than 90% (75%) of the time are superior to daily (monthly) rebalances. For MinV, CCC proves slightly better[6] while ifor MaxR dynamic correlation models are superior.

In summary, out-of-sample results corroborate the in-sample analysis, yet with a reasonable reduction in gains. Including a commodity in our portfolio, abnormal returns, depending on the rebalancing frequency, are on average 1%-1.5% p.a. as compared to the traditional portfolio, while the risk-adjusted abnormal return of the commodity index portfolio is more than 3.4% if we apply a dynamic strategy. Further, although for static portfolios abnormal returns can be negative, for volatility timing strategies they lie within 1.6%-2.2% and for volatility/correlation timing 1.9%-2.4%, depending on the rebalancing strategy. Performance fees for volatility timing are, on average, within the range of 3.5%-4% while for volatility/correlation timing 3.8%-4.3%, depending on how often rebalancing occurs. After transaction costs, these figures are 2.3%-2.8% and 2.6%-3.1%, respectively.

[6] Note that, out-of-sample, CCC involves, to a certain extent, correlation timing. Despite CCC in-sample optimal weights change only due to volatility, out-of-sample weights will vary because of correlation as well since every day we re-estimate the correlation matrix of the model using a rolling window forecasting scheme.

5.2.1. Additional robustness checks: shorting restricted portfolios

Although futures contracts can be easily shorted in practice, margin requirements, collaterals, or fiduciary rules often put in place restrictions on short selling. It is thus important to assess the impact of short-sale constraints on the diversification gains of commodities and the examined volatility and correlation timing rules; given that the unconstrained optimizer does not necessarily produce well diversified portfolios (Black and Litterman, 1992) and may lead to unstable and extreme portfolio weights. To this end, it may be desirable to impose nonnegativity constraints to circumvent the effects of estimation errors (see Michaud, 1989; Eichhorn et al., 1998; Jagannathan and Ma, 2003). Constraints are useful in real-time practical applications[7] and can provide a hedge against estimation error, often leading to improved performance (Board and Sutcliffe, 1994).

Out-of-sample results of shorting-restricted weekly rebalanced portfolios are presented in Table 7. The portfolios based on the traditional investment opportunity set, still yield lower SRs than the corresponding strategies based on the expanded opportunity set. The only exception is Live Cattle for which no strategy or model outperforms the stock, bond and cash portfolio strategies, as well as Copper and Cocoa (MinV); see M2 measure. Augmenting the portfolios with commodities generates an average SR of more than 0.52; this is higher than

[7] Jagannathan and Ma (2003) show that, excluding short sales in a minimum-variance portfolio problem is equivalent to downward adjusting the large elements of the covariance matrix. Yet, this shrinkage-like effect may induce specification errors since it reduces the covariance when this is relatively large. Hence, if the estimation errors are larger than the specification errors, prohibiting short sales would potentially improve out-of-sample performance. If expected returns and the covariance matrix estimators are error-free, constraining short sales can act adversely, as certain trades (e.g., bearish views) are precluded. Still, it is inevitable to accept some estimation error since optimization inputs (expected returns and the covariance matrix) are essentially unknown.

the 0.47 average SR of the unconstrained strategies in Tables 5 and 6. In general, all shorting-restricted portfolios perform marginally better in terms of SRs.

When excluding short sales, commodity augmented portfolios generate abnormal returns of 106 annual bps on average, which is lower than the 142 bps for long-short portfolios. This is mainly driven by the better performance of long-only stock, bond and cash portfolio with SRs of 0.282 to 0.549 (MinV) and 0.297 to 0.536 (MaxR) as opposed to a maximum achieved SR of 0.442 for the unconstrained portfolios (Tables 5 and 6). However, most of the results are similar to the ones obtained with no restrictions on the portfolio weights with GSCI, WTI and Gold being the most noticeable examples. Moreover, CCC still computes structures that realize the highest fees when interested in minimizing volatility (12 out of 16 cases). The second best model is the RSC (4 out of 16 cases). In terms of maximizing return RSC ranks first (10 out of 16 cases) and MDC second (5 cases).

MinV strategies yield fees within the range of 87 to 1,054 annual bps (average of 579, i.e., 143 bps higher than the unconstrained portfolios). MaxR strategies within the range of 150 to 576 annual bps (average of 364, i.e., 37 bps higher than the unconstrained portfolios). When transaction costs are considered, benefits from imposing short-selling restrictions are relatively lower, i.e., average performance fee in annual bps is 370 (MinV) and 294 (MaxR), as opposed to 269 (MinV) and 256 (MaxR) for the unconstrained portfolios. Since performance fees of dynamic models are even higher than those observed in Tables 5 and 6 in more than 70% of the cases considered, we can conclude that volatility and correlation timing works well under both constrained and unconstrained optimization schemes.

En masse, the out-of-sample analysis excluding short sales validates our previous findings in Section 5.2, yet with reasonable deviations. Including a commodity in our portfolio, abnormal returns, depending on the rebalancing frequency, are on average 0.6%-1% p.a. (daily and monthly rebalancing detailed results are not reported here and are available upon request) as compared to the traditional portfolio, while the risk-adjusted abnormal return of the commodity index portfolio is more than 1.85% if we apply a dynamic strategy. Further, although for static portfolios abnormal returns can be negative failing to outperform the stock-bond-cash portfolio, for volatility timing strategies they lie within 0.4%-0.8% and for volatility/correlation timing 1.2%-1.5%, depending on the rebalancing strategy. Performance fees for volatility timing are, on average, within the range of 4.4%-5.2% while for volatility/correlation timing 4.2%-4.9%, depending on how often rebalancing occurs. After transaction costs, these figures are 3.1%-3.9% and 2.9%-3.5%, respectively.

5.2.2. Additional robustness checks: mean-CVaR optimal portfolios

So far, we have restricted our analysis to mean-variance approach. Nevertheless, it would seem prudent to evaluate the efficiency of the traditional mean-variance approach by conducting some alternative analysis. In unreported work, we have explored the possibility of potential additional benefits when tail risk is considered. For this reason, we repeat the out-of-sample exercise by minimizing conditional value-at-risk (CVaR)[8] and setting the target return to 10% per annum, consistent with the mean-variance case; for details on mean-CVaR optimizations, we refer to Rockafellar and Uryasev (2000). While optimizing, instead of imposing a distributional assumption on the asset return dynamics, we use the empirical

[8] VaR is the maximum portfolio loss one expects to suffer at a specific confidence level and time horizon. CVaR is the conditional expectation of losses exceeding VaR. We focus on CVaR rather than VaR as the former has more attractive properties in many respects. It focuses on both the frequency and size of losses in case of extreme events, it is sub-additive and convex (Rockafellar and Uryasev, 2000) and satisfies all statistical axioms of a coherent measure of risk in the sense of Artzner et al. (1999). Moreover, the minimization of CVaR usually leads to near optimal solutions in VaR terms because VaR never exceeds CVaR.

distribution of the asset returns. We note that, as shown by Rockafellar and Uryasev (2000), for normal loss distributions portfolios constructed in the mean-variance framework are also mean-CVaR optimal portfolios. Findings (available from the authors upon request) indicate that dynamic mean-variance strategies outperform mean-CVaR portfolios, in terms of SRs, in all cases apart from Copper and Cocoa, while the mean-CVaR method outperforms the static strategy in all cases apart from Gold and Cotton. Results are robust for alternative performance measures, i.e., ratio of average excess returns divided by negative returns' volatility (Sortino), VaR and CVaR.

It has to be noted that criticisms against the mean-variance framework stress that it is appropriate only for normally distributed returns or for investors having quadratic preferences. However, studies such as Levy and Markowitz (1979), Pulley (1981), Kroll et al. (1984) and Hlawitschka (1994) show that mean-variance portfolio selection results are very similar to those obtained from a direct optimization of expected utility for various utility functions and historical distributions of returns, suggesting that higher moments in practice play a secondary role; particularly for short holding periods (Pulley, 1981) which could extend to a year (Kroll et al., 1984). Moreover, Chambers and Quiggin (2005), prove that much of the standard mean-standard deviation analysis can be extended to general invariant preferences, without requiring the preferences to be neutral with respect to higher moments. Han (2006) also provides justification for using a conditional mean-variance framework with stochastic volatility.

6. Conclusions

The empirical literature in financial economics has long determined that accurate forecasts of volatilities and correlations are critical for asset allocation. This paper provides a comprehensive evaluation of the economic value of dynamic strategies that invest in the commodities market in addition to the traditional opportunity set (stocks, bonds and cash). We address the issue of time-varying second moments of asset returns and concentrate on their impacts in terms of portfolio construction and commodity diversification effects. Our analysis focuses on the commodities market by making use of 17 years of daily returns data from 14 major commodities and a diversified commodity index.

We find that risk averse investors are better off including commodities in their portfolio with average, across commodities, abnormal returns in excess of 4% p.a., irrespective of the optimization objective, compared to the traditional portfolio. Results are confirmed out-of-sample, yet with a reasonable reduction in gains. Depending on the rebalancing frequency, abnormal returns are on average 1%-1.5% p.a. We also utilize different methods of covariance predictions to judge which model improves the ability to construct optimal portfolios. Allowing for rich correlation structures such as regime switching (RSC) or mixed data sampling (MDC) conditional correlations performs equally well and is slightly better than the baseline dynamic conditional correlation model (DCC). A mean-variance investor facing commodity risk will pay a performance fee of about 1.25% per year for volatility timing and a further 0.25% per year for correlation timing. Out-of-sample net of transaction costs fees for volatility timing are, on average, within the range of 2.3%-2.8% while for correlation and volatility timing 2.6%-3.1%, depending on the rebalancing frequency. Our results are robust to the presence of short-sales constraints; when imposing such restrictions portfolios are marginally better. In conclusion, both volatility and correlation timing matter to an investor, and it pays to take dynamic volatilities and correlations into consideration when devising portfolio strategies.

As this is the first study to comprehensively assess the economic value of volatility and correlation timing for a range of commodities, there is scope to potentially extend our analysis. For example, various studies attempt to incorporate the higher moments (conditional skewness and conditional kurtosis) in asset pricing and portfolio analysis; see, for example, Jondeau and Rockinger (2012) and Gao and Nardari (2018), among others. Since, we have restricted our analysis to the mean-variance criterion, future research should look at the potential economic gains of commodity-augmented portfolios using higher-moment dynamic strategies that would allow distributional timing. Measuring the economic value of such strategies requires sophisticated models to accurately capture the temporal evolution of the conditional distributions. Moreover, given the increasing emphasis on risk management, there is a proliferation of measures capturing different types of risk (see for example, Rockafellar and Uryasev, 2000). Creating diversified portfolios using alternative risk objectives, albeit an important research question, is left for future research.

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Table 1
Risk-return characteristics

Panel A: Descriptive statistics and GARCH estimates

Future Contract	(Ticker)	μ	σ	Skew	Kurt	$Q(6)$	$Q^2(6)$	Unc. Correlation $\rho_{s,i}$	Unc. Correlation $\rho_{b,i}$	GARCH(1,1) Coefficients ω_i	GARCH(1,1) Coefficients α_i	GARCH(1,1) Coefficients β_i
Financials												
· S&P500	(ISP)	4.126	20.76	-0.102***	9.193***	44.72***	1,797***	1	-0.211***	0.015***	0.092***	0.901***
· 30y US Bond	(CUS)	5.054	10.11	-0.234***	1.937***	5.765	230.1***	-0.211***	1	0.002**	0.037***	0.957***
Com. Index												
· GS Com. Ind.	(GSCI)	8.530	22.75	-0.236***	3.021***	14.26**	730.6***	0.194***	-0.145***	0.006*	0.045***	0.953***
Energies												
· WTI Crude oil	(NCL)	11.85	32.73	-0.267***	2.683***	11.07*	725.5***	0.170***	-0.131***	0.036**	0.043***	0.948***
· Natural gas	(NNG)	-13.04	43.27	0.034	2.025***	4.722	260.4***	0.034**	-0.024	0.122***	0.063***	0.922***
Metals												
· Gold 100oz	(NGC)	5.262	17.13	0.077**	6.632***	16.01**	439.8***	-0.034**	0.043***	0.002	0.040***	0.960***
· Silver 5,000oz	(NSL)	7.235	30.47	-0.675***	6.197***	7.969	534.6***	0.067***	-0.037**	0.012*	0.042***	0.955***
· HG Copper	(NHG)	7.030	28.84	-0.223***	3.876***	24.48***	1,142***	0.226***	-0.170***	0.025**	0.041***	0.952***
Agricultural												
· Wheat	(CW)	-9.680	30.03	0.067*	2.104***	5.321	498.5***	0.114***	-0.091***	0.024**	0.042***	0.951***
· Soybeans	(CS)	3.931	23.89	-0.202***	2.297***	12.90**	631.3***	0.123***	-0.099***	0.034***	0.065***	0.921***
· Corn	(CC)	-5.153	26.57	0.007	2.106***	18.62***	616.9***	0.117***	-0.079***	0.028***	0.067***	0.924***
· Cocoa	(NCC)	-1.903	30.15	-0.137***	2.594***	4.212	154.4***	0.062***	-0.060***	0.016	0.028***	0.968***
· Coffee	(NKC)	-3.808	37.08	0.071*	4.664***	23.88***	545.3***	0.096***	-0.070***	0.188**	0.056***	0.908***
· Sugar #11	(NSB)	5.684	32.10	-0.257***	2.551***	16.05**	306.4***	0.086***	-0.069***	0.011	0.033***	0.964***
· Orange Juice	(NJO)	-6.123	27.46	-0.176***	4.668***	28.11***	196.9***	0.051***	-0.024	0.013	0.030*	0.966***
Other												
· Live Cattle	(CLC)	0.284	13.83	-0.112***	1.202***	18.20***	383.0***	0.102***	-0.077***	0.010***	0.043***	0.943***
· Cotton #2	(NCT)	-7.317	27.32	-0.014	1.472***	14.95**	661.9***	0.117***	-0.079***	0.012**	0.041***	0.955***

Panel B: Risk-return profiles of equally and value weighted portfolios

	Equally-weighted portfolios						Value-weighted portfolios					
	Entire sample			2005-2012			2005-2012			2005-2012		
	μ_p	σ_p	SR	μ_p	σ_p	SR	μ_p	σ_p	SR	μ_p	σ_p	SR
Financials only:	4.59	10.54	0.435	2.99	10.86	0.275	3.37	14.05	0.240	0.96	14.90	0.065
Financials plus:												
· GS Com. Ind.	5.90	10.95	**0.539**	4.57	12.99	**0.352**	3.90	13.20	**0.295**	1.75	14.69	**0.119**
· WTI Crude oil	7.01	13.58	**0.516**	2.55	15.38	0.166	3.57	13.77	**0.259**	0.28	15.15	0.018
· Natural gas	-1.29	16.18	-0.080	-12.5	16.45	-0.761	0.90	13.68	0.066	-5.06	14.62	-0.346
· Gold 100oz	4.81	9.00	**0.535**	7.34	10.25	**0.716**	3.62	12.52	**0.289**	2.97	12.78	**0.233**
· Silver 5,000oz	5.47	12.63	0.433	8.16	15.37	**0.531**	3.57	13.72	**0.260**	2.46	15.13	**0.163**
· HG Copper	5.40	12.68	0.426	7.27	15.07	**0.482**	3.42	13.82	**0.248**	1.89	15.04	**0.125**
· Wheat	-0.17	12.62	-0.013	0.62	14.85	0.042	2.45	13.61	0.180	0.10	14.50	0.007
· Soybeans	4.37	11.00	0.397	4.99	12.30	**0.406**	3.21	13.24	**0.243**	1.38	14.08	**0.098**
· Corn	1.34	11.73	0.114	2.94	13.88	0.212	3.02	13.65	0.221	0.81	14.50	0.056
· Cocoa	2.43	12.45	0.195	2.33	13.15	0.177	2.95	13.58	0.217	0.71	14.43	0.049
· Coffee	1.79	14.59	0.123	3.00	13.33	0.225	2.25	13.70	0.164	0.82	14.33	0.057
· Sugar #11	4.95	13.10	0.378	5.46	14.58	**0.374**	3.27	13.76	0.238	1.16	14.66	**0.079**
· Orange Juice	1.02	11.75	0.087	3.60	12.49	**0.289**	2.78	13.61	0.204	0.97	14.47	**0.067**
· Live Cattle	3.15	8.65	0.365	0.52	9.10	0.057	2.99	13.09	0.229	0.40	13.83	0.029
· Cotton #2	0.62	11.93	0.052	2.02	13.33	0.152	2.45	13.44	0.183	0.88	14.53	0.061

This table presents summary statistics of daily futures returns (Panel A) and the performance of ad-hoc portfolios that include stocks, bonds and commodities (Panel B). The sample spans from December 19, 1994 to January 3, 2012. In Panel A, the annualized percent mean and percent volatility are denoted by μ and σ. Skew and Exc. Kurt measure the coefficients of skewness and excess kurtosis, respectively i.e. the centralised third and fourth moments of the data, denoted $\hat{a}_3$ and $(\hat{a}_4 - 3)$, respectively; their asymptotic distributions under the null are $\sqrt{T}\hat{a}_3 \sim N(0,6)$ and $\sqrt{T}(\hat{a}_4 - 3) \sim N(0,24)$. $\rho_{s,i}$ is the correlation coefficient of each futures contract with the S&P 500 futures; $\rho_{b,i}$ is the correlation coefficient of each futures contract with the US Bond futures. $Q(6)$ and $Q^2(6)$ are Ljung-Box (1978) tests for 6th order autocorrelation in the level and squared series, respectively. The statistics are $\chi^2(6)$ distributed. Asterisks ***, ** and * indicate significance at 1%, 5% and 10% level. In Panel B, the annualized percent mean, percent volatility and Sharpe ratio for the considered portfolios are denoted by μ_p, σ_p, and SR, respectively. The portfolios reported are the 1/N *equally-weighted* diversification strategy (in which a fraction 1/N of wealth is allocated to each of the N assets available for investment at each rebalancing date) and a *value-weighted strategy* (in which weights are based on the futures contracts' market value at each rebalancing date). SRs in bold indicate higher SR compared to the one achieved by the traditional stock - bond portfolio.

Table 2
Estimation results of dynamic conditional correlation models

	DCC		MDC			RSC							
						State 1				State 2			
	a	β	a	β	ω	$\rho_{s,b}$	$\rho_{s,c}$	$\rho_{b,c}$	p_{11}	$\rho_{s,b}$	$\rho_{s,c}$	$\rho_{b,c}$	p_{22}
Com. Index													
CGS	0.024***	0.972***	0.034***	0.938***	2.099***	-0.411***	0.198***	-0.179***	0.991***	0.369***	-0.042	0.026	0.989***
	(0.004)	(0.005)	(0.003)	(0.009)	(0.679)	(0.040)	(0.045)	(0.039)	(0.003)	(0.091)	(0.036)	(0.028)	(0.005)
Energies													
NCL	0.027***	0.968***	0.039***	0.930***	2.139***	-0.405***	0.168***	-0.156***	0.992***	0.385***	-0.057	0.025	0.988***
	(0.005)	(0.006)	(0.003)	(0.009)	(0.623)	(0.041)	(0.051)	(0.035)	(0.003)	(0.096)	(0.040)	(0.030)	(0.005)
NNG	0.018***	0.979***	0.025***	0.953***	1.423***	-0.386***	0.021	-0.048**	0.993***	0.428***	0.020	0.038	0.988***
	(0.003)	(0.004)	(0.003)	(0.010)	(0.355)	(0.034)	(0.020)	(0.020)	(0.003)	(0.057)	(0.030)	(0.026)	(0.004)
Metals													
NGC	0.018***	0.978***	0.033***	0.938***	1.631***	-0.366***	-0.013	0.077***	0.994***	0.449***	-0.066*	-0.104**	0.989***
	(0.003)	(0.004)	(0.003)	(0.006)	(0.288)	(0.036)	(0.022)	(0.022)	(0.003)	(0.053)	(0.037)	(0.045)	(0.004)
NSL	0.022***	0.973***	0.032***	0.940***	1.807***	-0.375***	0.086***	0.005	0.994***	0.437***	-0.031	-0.083***	0.989***
	(0.004)	(0.006)	(0.003)	(0.007)	(0.385)	(0.029)	(0.018)	(0.022)	(0.003)	(0.047)	(0.031)	(0.032)	(0.004)
NHG	0.022***	0.973***	0.024***	0.959***	1.563***	-0.386***	0.277***	-0.197***	0.993***	0.422***	0.015	-0.046*	0.988***
	(0.005)	(0.006)	(0.003)	(0.010)	(0.598)	(0.037)	(0.025)	(0.028)	(0.003)	(0.066)	(0.035)	(0.025)	(0.004)
Agricultural													
CW	0.019***	0.977***	0.028***	0.954***	1.012	-0.392***	0.126***	-0.092***	0.992***	0.419***	-0.031	-0.007	0.988***
	(0.003)	(0.004)	(0.010)	(0.031)	(1.431)	(0.032)	(0.022)	(0.022)	(0.003)	(0.055)	(0.032)	(0.028)	(0.004)
CS	0.019***	0.978***	0.027***	0.951***	1.384***	-0.402***	0.149***	-0.135***	0.992***	0.396***	-0.017	-0.001	0.989***
	(0.003)	(0.004)	(0.003)	(0.010)	(0.439)	(0.038)	(0.027)	(0.031)	(0.003)	(0.074)	(0.032)	(0.031)	(0.004)
CC	0.021***	0.975***	0.027***	0.953***	1.220**	-0.394***	0.125***	-0.101***	0.992***	0.416***	0.001	0.020	0.988***
	(0.003)	(0.005)	(0.005)	(0.016)	(0.602)	(0.031)	(0.020)	(0.027)	(0.003)	(0.051)	(0.030)	(0.031)	(0.004)
NCC	0.016***	0.980***	0.027***	0.941***	1.816***	-0.379***	0.082***	-0.059***	0.993***	0.439***	-0.049	-0.047	0.988***
	(0.003)	(0.004)	(0.006)	(0.021)	(0.496)	(0.036)	(0.019)	(0.023)	(0.003)	(0.064)	(0.035)	(0.035)	(0.004)
NKC	0.017***	0.980***	0.026***	0.951***	1.547***	-0.385***	0.125***	-0.084***	0.993***	0.430***	0.057*	-0.018	0.988***
	(0.003)	(0.004)	(0.003)	(0.011)	(0.272)	(0.034)	(0.022)	(0.024)	(0.003)	(0.058)	(0.030)	(0.028)	(0.004)
NSB	0.018***	0.979***	0.042***	0.884***	3.136***	-0.384***	0.074***	-0.081***	0.993***	0.432***	0.018	-0.003	0.988***
	(0.003)	(0.004)	(0.005)	(0.022)	(0.801)	(0.037)	(0.020)	(0.021)	(0.003)	(0.066)	(0.038)	(0.033)	(0.004)
NJO	0.016***	0.981***	0.028***	0.934***	2.188***	-0.387***	0.075***	-0.019	0.993***	0.428***	-0.021	-0.015	0.988***
	(0.003)	(0.004)	(0.006)	(0.028)	(0.486)	(0.033)	(0.021)	(0.023)	(0.003)	(0.055)	(0.032)	(0.028)	(0.004)
Other													
CLC	0.019***	0.978***	0.032***	0.932***	2.006***	-0.385***	0.087***	-0.087***	0.993***	0.431***	0.015	-0.025	0.988***
	(0.003)	(0.004)	(0.004)	(0.014)	(0.519)	(0.036)	(0.018)	(0.022)	(0.003)	(0.061)	(0.030)	(0.026)	(0.004)
NCT	0.019***	0.977***	0.032***	0.932***	1.697***	-0.381***	0.110***	-0.079***	0.993***	0.437***	-0.022	-0.029	0.987***
	(0.003)	(0.005)	(0.003)	(0.011)	(0.465)	(0.038)	(0.019)	(0.022)	(0.003)	(0.063)	(0.036)	(0.033)	(0.004)

This table reports the maximum likelihood estimates of the dynamic conditional correlation (DCC), dynamic component conditional correlation with mixed data sampling (MDC) and regime switching correlation (RSC) models. Figures in $(\cdot)$ denote the estimated standard errors. Asterisks ***, ** and * indicate significance at 1%, 5% and 10% level, respectively. The estimation period covers daily data from December 1994 to January 2012.

Table 3

In-sample portfolio performance: commodity index, energy and metals

	Minimum Volatility ($\mu_p^* = 10\%$)					Maximum Return ($\sigma_p^* = 12\%$)				
	μ_p	σ_p	SR	$\Phi_{\delta=6}$	$M2$	μ_p	σ_p	SR	$\Phi_{\delta=6}$	$M2$
Panel A: Stock, Bond and Cash only										
Static	9.21	10.78	0.855			15.40	12.17	1.265		
CCC	9.48	10.49	0.904	30		17.58	12.11	1.452	219	
DCC	9.32	10.33	0.903	15		17.75	11.87	1.495	238	
MDC	9.39	10.31	0.911	22		17.82	11.89	1.498	245	
RSC	9.86	10.42	0.947	68		17.88	11.96	1.495	250	
Panel B: Stock, Bond, Cash and GC Com. Ind.										
Static	9.99	6.27	1.595		798	22.54	13.12	1.718		550
CCC	10.13	5.92	1.711	15	847	25.22	12.51	2.017	275	684
DCC	10.15	5.84	1.738	18	863	25.60	12.21	2.097	316	714
MDC	10.19	5.83	1.747	22	862	25.69	12.22	2.103	325	719
RSC	10.12	5.83	1.736	15	822	25.30	12.26	2.064	285	680
Panel C: Stock, Bond, Cash and WTI Crude Oil										
Static	9.99	6.85	1.459		652	21.78	13.00	1.676		500
CCC	10.21	6.51	1.568	24	697	23.78	12.39	1.919	206	566
DCC	10.32	6.35	1.624	35	744	24.67	12.18	2.026	297	630
MDC	10.47	6.36	1.645	50	757	25.02	12.21	2.050	332	656
RSC	10.09	6.43	1.569	12	648	23.63	12.18	1.940	193	532
Panel D: Stock, Bond, Cash and Natural Gas										
Static	10.02	5.97	1.680		890	23.27	13.11	1.776		621
CCC	10.42	5.60	1.860	42	1003	24.87	12.44	2.000	168	664
DCC	10.27	5.53	1.857	28	985	25.04	12.27	2.041	186	649
MDC	10.29	5.54	1.858	30	976	25.08	12.29	2.040	190	645
RSC	10.45	5.53	1.891	45	983	24.91	12.11	2.056	175	671
Panel E: Stock, Bond, Cash and Gold										
Static	9.93	7.27	1.366		552	19.51	12.89	1.514		302
CCC	9.95	6.69	1.488	5	612	23.09	12.70	1.819	360	445
DCC	10.19	6.61	1.544	30	662	23.70	12.50	1.896	423	475
MDC	10.19	6.61	1.542	30	650	23.02	12.28	1.875	358	448
RSC	10.19	6.66	1.530	29	607	23.16	12.52	1.850	369	424
Panel F: Stock, Bond, Cash and Silver										
Static	9.97	8.80	1.134		301	18.51	12.94	1.431		201
CCC	9.58	8.58	1.117	-38	223	20.71	12.53	1.652	224	243
DCC	10.39	8.43	1.233	44	340	21.38	12.16	1.759	295	313
MDC	10.22	8.42	1.213	27	312	21.28	12.20	1.744	285	293
RSC	9.97	8.46	1.179	2	241	20.90	12.33	1.695	245	239
Panel G: Stock, Bond, Cash and Copper										
Static	9.99	8.28	1.206		379	20.48	13.12	1.561		360
CCC	10.38	8.16	1.272	40	387	22.91	12.62	1.816	249	441
DCC	10.64	7.94	1.339	67	451	23.09	12.16	1.898	271	479
MDC	10.67	7.94	1.344	71	446	23.07	12.19	1.892	269	469
RSC	10.50	7.98	1.316	53	384	23.19	12.32	1.882	279	462

The table reports the in-sample portfolio performance of selected minimum volatility and maximum return portfolio strategies investing in the S&P 500 futures, US Bond futures, cash and commodity futures. Static is the benchmark strategy using the full sample covariance estimates, CCC is a dynamic strategy using the constant conditional correlation model. DCC, MDC and RSC are strategies that employ dynamic conditional correlations (see notes in Table 2). The annualized percent mean (in excess of the risk free rate), percent volatility and Sharpe ratio are denoted by μ_p, σ_p, and SR, respectively. σ_p^* and μ_p^* correspond to the *target* annualized volatilities and returns. The performance fee $\Phi_{\delta=6}$ denotes the amount an investor with quadratic utility and degree of relative risk aversion δ equal to 6 is willing to pay for switching from Static to one of the dynamic models and is reported in annual bps. For comparison, we also report the performance of a Stock, Bond and Cash only strategy in Panel A. *M2* is the Modigliani and Modigliani (1997) measure of the abnormal return a strategy would have earned if it had the same risk as the stock, bond and cash portfolio. The sample period spans from December 1994 to January 2012.

Table 4
In-sample portfolio performance: agricultural and other commodities

	Minimum Volatility ($\mu_p^* = 10\%$)					Maximum Return ($\sigma_p^* = 12\%$)				
	μ_p	σ_p	SR	$\Phi_{\delta=6}$	$M2$	μ_p	σ_p	SR	$\Phi_{\delta=6}$	$M2$
Panel A: Stock, Bond, Cash and Wheat										
Static	9.93	7.74	1.283		462	19.75	12.68	1.557		355
CCC	9.94	7.28	1.365	4	484	21.77	12.28	1.773	206	389
DCC	9.97	7.15	1.395	8	508	22.20	12.15	1.828	251	395
MDC	9.96	7.14	1.395	7	499	22.24	12.20	1.824	254	387
RSC	10.14	7.14	1.421	25	494	22.09	12.05	1.834	240	405
Panel B: Stock, Bond, Cash and Soybeans										
Static	9.96	10.54	0.945		98	19.93	12.68	1.572		373
CCC	10.76	8.83	1.219	93	330	22.70	12.40	1.832	280	460
DCC	10.73	8.75	1.227	92	335	22.80	12.09	1.887	293	465
MDC	10.79	8.75	1.233	97	332	22.87	12.14	1.884	300	459
RSC	10.92	8.76	1.247	110	312	22.99	12.21	1.884	311	465
Panel C: Stock, Bond, Cash and Corn										
Static	10.00	8.68	1.152		320	18.79	12.57	1.494		279
CCC	9.95	8.45	1.177	-3	287	20.15	12.32	1.636	139	223
DCC	10.21	8.29	1.232	24	339	20.73	12.15	1.707	198	251
MDC	10.16	8.30	1.224	19	323	20.88	12.20	1.712	213	254
RSC	10.29	8.29	1.241	32	306	20.77	12.10	1.718	203	266
Panel D: Stock, Bond, Cash and Cocoa										
Static	9.98	7.99	1.249		425	19.50	12.58	1.550		347
CCC	10.18	7.62	1.336	22	453	21.93	12.36	1.774	245	391
DCC	10.13	7.54	1.343	18	454	22.22	12.09	1.838	277	407
MDC	10.12	7.55	1.340	16	442	22.23	12.16	1.829	277	393
RSC	10.03	7.54	1.331	8	399	22.04	12.08	1.825	260	395
Panel E: Stock, Bond, Cash and Coffee										
Static	9.98	10.49	0.952		105	19.10	12.94	1.476		257
CCC	9.56	9.63	0.993	-35	94	20.78	12.36	1.681	174	278
DCC	9.80	9.51	1.030	-10	132	21.39	12.13	1.764	237	319
MDC	9.86	9.49	1.039	-4	132	21.52	12.15	1.771	251	325
RSC	9.73	9.56	1.018	-18	74	21.14	12.19	1.735	212	287
Panel F: Stock, Bond, Cash and Sugar										
Static	9.99	7.67	1.302		482	19.16	12.95	1.479		260
CCC	10.15	7.23	1.404	19	525	22.26	12.49	1.783	316	401
DCC	10.23	7.11	1.440	28	555	22.29	12.18	1.831	322	399
MDC	10.20	7.10	1.437	25	542	22.24	12.27	1.812	315	374
RSC	10.63	7.13	1.490	67	565	22.73	12.26	1.854	365	430
Panel G: Stock, Bond, Cash and Orange Juice										
Static	9.99	7.04	1.421		610	20.05	12.68	1.582		385
CCC	10.36	6.64	1.560	39	688	22.81	12.47	1.828	278	456
DCC	10.25	6.56	1.563	29	682	22.71	12.20	1.862	271	435
MDC	10.28	6.57	1.565	31	674	22.71	12.23	1.857	271	427
RSC	10.64	6.56	1.622	67	703	23.15	12.23	1.893	315	476
Panel H: Stock, Bond, Cash and Live Cattle										
Static	9.99	8.69	1.149		317	19.91	12.63	1.576		378
CCC	9.97	8.46	1.179	0	288	22.17	12.38	1.790	229	410
DCC	10.19	8.31	1.226	23	334	22.69	12.17	1.864	283	438
MDC	10.32	8.29	1.244	36	343	22.85	12.20	1.872	299	445
RSC	10.10	8.35	1.209	14	273	22.34	12.11	1.844	249	418
Panel I: Stock, Bond, Cash and Cotton										
Static	9.97	10.79	0.924		74	20.76	12.64	1.642		458
CCC	9.80	9.83	0.997	-8	98	22.79	12.42	1.835	206	464
DCC	9.92	9.75	1.018	4	118	22.79	12.13	1.878	209	455
MDC	9.89	9.75	1.014	1	106	22.78	12.15	1.874	207	448
RSC	9.98	9.78	1.021	10	77	22.89	12.18	1.879	218	460

See notes in Table 3.

Table 5
Out-of-sample portfolio performance: commodity index, energy and metals

	Minimum Volatility ($\mu_p^* = 10\%$)						Maximum Return ($\sigma_p^* = 12\%$)					
	μ_p	σ_p	SR	$\Phi_{\delta=6}$	$\Phi_{\delta=6}^{tc=50}$	$M2$	μ_p	σ_p	SR	$\Phi_{\delta=6}$	$\Phi_{\delta=6}^{tc=50}$	$M2$
Panel A: Stock, Bond and Cash only												
Static	6.00	24.19	0.248				3.55	13.64	0.260			
CCC	9.98	23.53	0.424	412	135		4.60	11.94	0.385	123	52	
DCC	10.09	23.46	0.430	424	149		4.89	11.85	0.413	155	84	
MDC	10.56	23.89	0.442	462	178		4.96	11.86	0.418	161	90	
RSC	10.09	23.52	0.429	422	146		4.91	11.85	0.414	155	85	
Panel B: Stock, Bond, Cash and GC Com. Ind.												
Static	5.32	18.34	0.290			102	3.99	14.45	0.276			22
CCC	10.67	15.81	0.675	572	447	590	9.02	11.60	0.778	536	468	470
DCC	9.86	15.36	0.642	497	379	497	8.89	12.02	0.740	518	445	387
MDC	9.71	15.42	0.630	482	363	450	8.77	12.25	0.716	503	428	353
RSC	10.18	15.73	0.647	523	399	513	9.49	12.09	0.785	577	504	440
Panel C: Stock, Bond, Cash and WTI Crude Oil												
Static	4.26	15.90	0.268			49	2.49	14.42	0.173			-119
CCC	7.31	13.64	0.536	334	241	263	6.69	11.40	0.587	454	389	242
DCC	6.74	13.16	0.512	282	195	193	6.71	11.83	0.567	450	380	182
MDC	6.35	13.21	0.481	242	155	92	6.39	12.06	0.530	417	344	133
RSC	7.24	13.58	0.533	326	234	244	7.16	11.81	0.606	497	427	229
Panel D: Stock, Bond, Cash and Natural Gas												
Static	6.04	21.50	0.281			80	5.91	13.87	0.426			227
CCC	13.83	19.26	0.718	818	632	691	10.20	11.08	0.921	459	397	641
DCC	12.55	19.01	0.660	693	511	539	10.24	11.53	0.888	458	392	563
MDC	12.13	19.01	0.638	652	470	468	10.55	11.89	0.887	486	415	556
RSC	12.87	19.18	0.671	723	539	570	10.27	11.46	0.896	462	396	572
Panel E: Stock, Bond, Cash and Gold												
Static	8.88	15.20	0.584			812	8.10	13.96	0.580			436
CCC	12.30	15.11	0.814	343	230	917	9.54	10.98	0.869	177	116	578
DCC	11.26	15.09	0.746	239	126	741	9.51	11.53	0.825	168	102	488
MDC	11.02	15.12	0.729	216	102	685	9.65	11.84	0.815	179	109	471
RSC	12.07	15.14	0.797	320	206	866	10.35	11.49	0.901	253	187	577
Panel F: Stock, Bond, Cash and Silver												
Static	5.46	21.94	0.249			3	4.48	14.09	0.318			79
CCC	13.00	20.70	0.628	777	563	480	8.84	11.45	0.772	464	398	463
DCC	10.91	20.54	0.531	570	358	237	8.10	11.87	0.682	386	315	318
MDC	10.57	20.52	0.515	536	325	174	8.05	12.13	0.664	379	305	292
RSC	12.43	20.75	0.599	719	504	401	9.05	11.89	0.761	481	410	412
Panel G: Stock, Bond, Cash and Copper												
Static	2.18	20.21	0.108			-339	2.12	13.39	0.158			-140
CCC	5.24	22.12	0.237	272	25	-441	5.97	14.01	0.426	379	280	50
DCC	2.66	21.59	0.123	22	-213	-721	3.88	14.15	0.274	168	68	-165
MDC	3.20	21.33	0.150	83	-147	-697	4.38	14.16	0.309	218	117	-129
RSC	5.94	21.92	0.271	344	102	-372	6.62	14.27	0.464	440	338	60

The table reports the out-of-sample portfolio performance of selected minimum volatility and maximum return portfolio strategies investing in the S&P 500 futures, US Bond futures, cash and different commodity futures. Models are estimated using a rolling window forecasting scheme of 2,540 daily returns. The out-of-sample period covers data from January 2005 to January 2012 (1,760 daily observations) and the rebalancing frequency is set to weekly. See also notes in Table 3.

Table 6
Out-of-sample portfolio performance: agricultural and other commodities

	Minimum Volatility ($\mu_p^* = 10\%$)						Maximum Return ($\sigma_p^* = 12\%$)					
	μ_p	σ_p	SR	$\Phi_{\delta=6}$	$\Phi_{\delta=6}^{tc=50}$	$M2$	μ_p	σ_p	SR	$\Phi_{\delta=6}$	$\Phi_{\delta=6}^{tc=50}$	$M2$
Panel A: Stock, Bond, Cash and Wheat												
Static	5.15	21.12	0.244			-9	2.30	15.62	0.147			-155
CCC	7.44	19.18	0.388	262	78	-85	3.92	11.42	0.343	211	146	-50
DCC	6.49	18.82	0.345	173	-6	-199	4.55	11.95	0.381	270	199	-38
MDC	6.85	18.86	0.363	207	27	-190	5.30	12.41	0.427	340	263	11
RSC	7.01	19.01	0.369	221	40	-141	4.48	11.97	0.374	262	191	-47
Panel B: Stock, Bond, Cash and Soybeans												
Static	4.00	22.49	0.178			-170	3.62	14.02	0.258			-2
CCC	10.19	20.54	0.496	656	445	170	5.91	11.02	0.536	261	200	181
DCC	10.04	20.53	0.489	640	429	138	6.36	11.71	0.543	299	230	153
MDC	10.09	20.55	0.491	644	433	116	6.63	12.09	0.548	322	249	154
RSC	10.66	20.57	0.518	701	490	210	6.65	11.70	0.568	328	260	183
Panel C: Stock, Bond, Cash and Corn												
Static	5.87	17.68	0.332			203	3.65	13.56	0.269			12
CCC	8.96	16.65	0.538	324	185	267	4.67	11.94	0.391	119	48	7
DCC	8.20	16.31	0.503	253	120	170	5.46	12.35	0.442	195	119	34
MDC	7.74	16.32	0.474	207	73	76	5.92	12.70	0.466	237	156	57
RSC	8.67	16.46	0.527	299	163	232	5.30	12.25	0.433	181	106	23
Panel D: Stock, Bond, Cash and Cocoa												
Static	0.90	22.40	0.040			-504	1.37	14.08	0.097			-222
CCC	5.59	20.71	0.270	500	286	-364	3.80	11.09	0.343	276	215	-49
DCC	5.63	20.63	0.273	507	294	-368	4.23	11.66	0.363	313	245	-60
MDC	5.77	20.68	0.279	520	306	-389	4.56	11.99	0.380	343	271	-45
RSC	6.28	20.85	0.301	567	350	-301	4.65	11.76	0.395	354	285	-22
Panel E: Stock, Bond, Cash and Coffee												
Static	5.48	15.38	0.356			260	3.14	12.38	0.254			-8
CCC	7.24	13.90	0.521	195	99	227	4.88	10.79	0.452	189	131	81
DCC	7.35	13.67	0.538	210	116	253	5.81	11.24	0.517	279	215	123
MDC	7.22	13.68	0.528	197	103	206	6.20	11.57	0.536	314	247	140
RSC	7.59	13.80	0.550	231	136	284	5.74	11.19	0.513	271	209	117
Panel F: Stock, Bond, Cash and Sugar												
Static	4.35	23.26	0.187			-147	4.11	14.74	0.279			26
CCC	14.02	19.75	0.710	1031	835	671	8.65	11.28	0.767	492	428	457
DCC	13.30	19.64	0.677	961	768	580	8.93	11.83	0.755	515	445	405
MDC	13.34	19.67	0.678	963	769	562	9.36	12.30	0.761	552	476	406
RSC	13.23	19.83	0.667	950	753	559	8.97	11.95	0.751	518	446	400
Panel G: Stock, Bond, Cash and Orange Juice												
Static	2.72	21.57	0.126			-294	0.58	14.55	0.040			-300
CCC	8.65	17.72	0.488	656	499	149	4.10	11.09	0.370	391	329	-17
DCC	7.69	17.52	0.439	564	410	21	4.53	11.68	0.388	428	360	-30
MDC	7.12	17.58	0.405	507	352	-88	4.57	12.09	0.378	428	355	-47
RSC	8.14	17.51	0.465	610	457	86	4.80	11.66	0.412	455	387	-2
Panel H: Stock, Bond, Cash and Live Cattle												
Static	3.06	21.37	0.143			-254	2.26	13.92	0.162			-134
CCC	4.60	18.79	0.245	198	23	-422	3.43	11.10	0.309	148	87	-90
DCC	4.88	18.69	0.261	227	53	-397	3.98	11.74	0.339	197	128	-88
MDC	5.12	18.69	0.274	253	79	-401	4.27	12.10	0.353	222	149	-77
RSC	5.34	18.75	0.285	273	98	-338	4.22	11.70	0.361	222	154	-62
Panel I: Stock, Bond, Cash and Cotton												
Static	5.77	14.33	0.403			374	3.98	12.69	0.314			73
CCC	7.78	12.94	0.601	217	133	416	5.51	11.53	0.478	164	98	111
DCC	7.69	12.67	0.607	212	132	416	6.14	11.90	0.516	224	153	122
MDC	7.78	12.65	0.615	220	140	412	6.66	12.24	0.544	273	198	150
RSC	8.06	12.80	0.630	247	166	474	6.38	11.84	0.539	249	178	148

See notes in Tables 3 and 5.

Table 7
Out-of-sample portfolio performance: Shorting-restricted mean-variance portfolios

	Min. Volatility ($\mu_p^* = 10\%$)				Max. Return ($\sigma_p^* = 12\%$)			
	SR	$\Phi_{\delta=6}$	$\Phi_{\delta=6}^{tc=50}$	$M2$	SR	$\Phi_{\delta=6}$	$\Phi_{\delta=6}^{tc=50}$	$M2$
Panel A: Stock, Bond and Cash only								
Static	0.282				0.297			
CCC	0.549	614	313		0.536	241	173	
DCC	0.473	418	125		0.497	198	129	
MDC	0.460	387	93		0.484	186	116	
RSC	0.482	439	147		0.491	191	122	
Panel B: Stock, Bond, Cash and GC Com. Ind.								
Static	0.290			20	0.276			-28
CCC	0.675	572	447	310	0.778	536	468	283
DCC	0.642	498	380	409	0.741	520	447	287
MDC	0.626	475	357	403	0.713	500	425	272
RSC	0.647	523	399	399	0.784	576	502	344
Panel C: Stock, Bond, Cash and WTI Crude Oil								
Static	0.268			-33	0.173			-170
CCC	0.536	334	241	-31	0.587	454	388	60
DCC	0.515	287	200	102	0.574	460	389	91
MDC	0.478	239	151	43	0.530	416	343	55
RSC	0.533	326	234	123	0.605	495	425	134
Panel D: Stock, Bond, Cash and Natural Gas								
Static	0.158			-299	0.151			-200
CCC	0.589	955	715	99	0.602	478	418	78
DCC	0.530	820	582	137	0.569	472	405	85
MDC	0.512	783	544	126	0.571	494	422	104
RSC	0.573	922	681	221	0.609	519	452	139
Panel E: Stock, Bond, Cash and Gold								
Static	0.584			726	0.580			389
CCC	0.815	345	232	654	0.870	178	118	391
DCC	0.750	244	131	669	0.828	171	104	390
MDC	0.732	219	106	658	0.817	178	109	396
RSC	0.796	319	205	761	0.898	249	183	479
Panel F: Stock, Bond, Cash and Silver								
Static	0.264			-43	0.338			56
CCC	0.639	766	551	220	0.789	457	391	296
DCC	0.558	592	381	205	0.724	409	338	267
MDC	0.550	574	363	216	0.716	414	340	275
RSC	0.614	718	503	321	0.785	484	412	346
Panel G: Stock, Bond, Cash and Copper								
Static	0.216			-158	0.289			-10
CCC	0.300	192	-55	-612	0.513	326	227	-26
DCC	0.254	87	-148	-530	0.444	232	132	-62
MDC	0.289	160	-69	-414	0.484	288	188	1
RSC	0.338	274	31	-349	0.558	400	297	79
Panel H: Stock, Bond, Cash and Wheat								
Static	0.266			-38	0.303			9
CCC	0.615	753	511	162	0.669	330	271	155
DCC	0.561	629	389	212	0.650	346	280	179
MDC	0.547	600	360	211	0.656	379	307	204
RSC	0.592	704	461	266	0.669	371	304	209

	Min. Volatility ($\mu_p^* = 10\%$)				Max. Return ($\sigma_p^* = 12\%$)			
	SR	$\Phi_{\delta=6}$	$\Phi_{\delta=6}^{tc=50}$	$M2$	SR	$\Phi_{\delta=6}$	$\Phi_{\delta=6}^{tc=50}$	$M2$
Panel I: Stock, Bond, Cash and Soybeans								
Static	0.206			-182	0.292			-7
CCC	0.520	642	430	-72	0.578	261	200	50
DCC	0.494	586	375	49	0.554	264	195	67
MDC	0.500	600	388	97	0.568	297	224	100
RSC	0.538	681	469	137	0.604	324	255	133
Panel J: Stock, Bond, Cash and Corn								
Static	0.271			-27	0.315			25
CCC	0.622	764	518	179	0.683	329	270	172
DCC	0.567	639	395	228	0.662	343	277	194
MDC	0.550	602	357	218	0.661	368	297	210
RSC	0.596	709	462	275	0.686	374	308	229
Panel K: Stock, Bond, Cash and Cocoa								
Static	0.076			-496	0.138			-219
CCC	0.442	792	571	-264	0.478	369	307	-67
DCC	0.436	777	557	-91	0.499	414	346	3
MDC	0.437	780	559	-57	0.514	445	373	36
RSC	0.450	813	589	-78	0.513	435	366	26
Panel L: Stock, Bond, Cash and Coffee								
Static	0.267			-36	0.317			28
CCC	0.598	724	476	121	0.662	305	246	148
DCC	0.554	622	375	196	0.660	342	276	192
MDC	0.538	586	339	189	0.656	359	288	204
RSC	0.575	677	426	226	0.661	350	282	200
Panel M: Stock, Bond, Cash and Sugar								
Static	0.187			-228	0.279			-24
CCC	0.722	1054	859	424	0.782	509	445	288
DCC	0.686	977	784	514	0.766	527	457	316
MDC	0.688	984	790	553	0.774	567	491	344
RSC	0.673	963	766	463	0.760	528	456	316
Panel N: Stock, Bond, Cash and Orange Juice								
Static	0.276			-15	0.319			30
CCC	0.620	740	499	173	0.686	329	270	176
DCC	0.559	603	363	207	0.650	328	261	180
MDC	0.538	558	318	189	0.638	338	266	182
RSC	0.597	694	452	279	0.691	376	309	236
Panel O: Stock, Bond, Cash and Live Cattle								
Static	0.143			-334	0.162			-185
CCC	0.246	201	25	-744	0.311	150	89	-263
DCC	0.268	242	68	-495	0.350	209	140	-173
MDC	0.285	273	99	-424	0.367	239	166	-138
RSC	0.286	276	100	-474	0.361	221	153	-153
Panel P: Stock, Bond, Cash and Cotton								
Static	0.267			-35	0.309			17
CCC	0.615	768	515	161	0.677	330	272	165
DCC	0.562	646	394	215	0.660	349	283	192
MDC	0.543	603	351	201	0.655	368	297	204
RSC	0.593	723	468	269	0.681	378	311	223

The table reports the out-of-sample portfolio performance of selected minimum volatility and maximum return portfolio strategies with short-selling restrictions, i.e., non-negative portfolio weights. See also notes in Table 3.

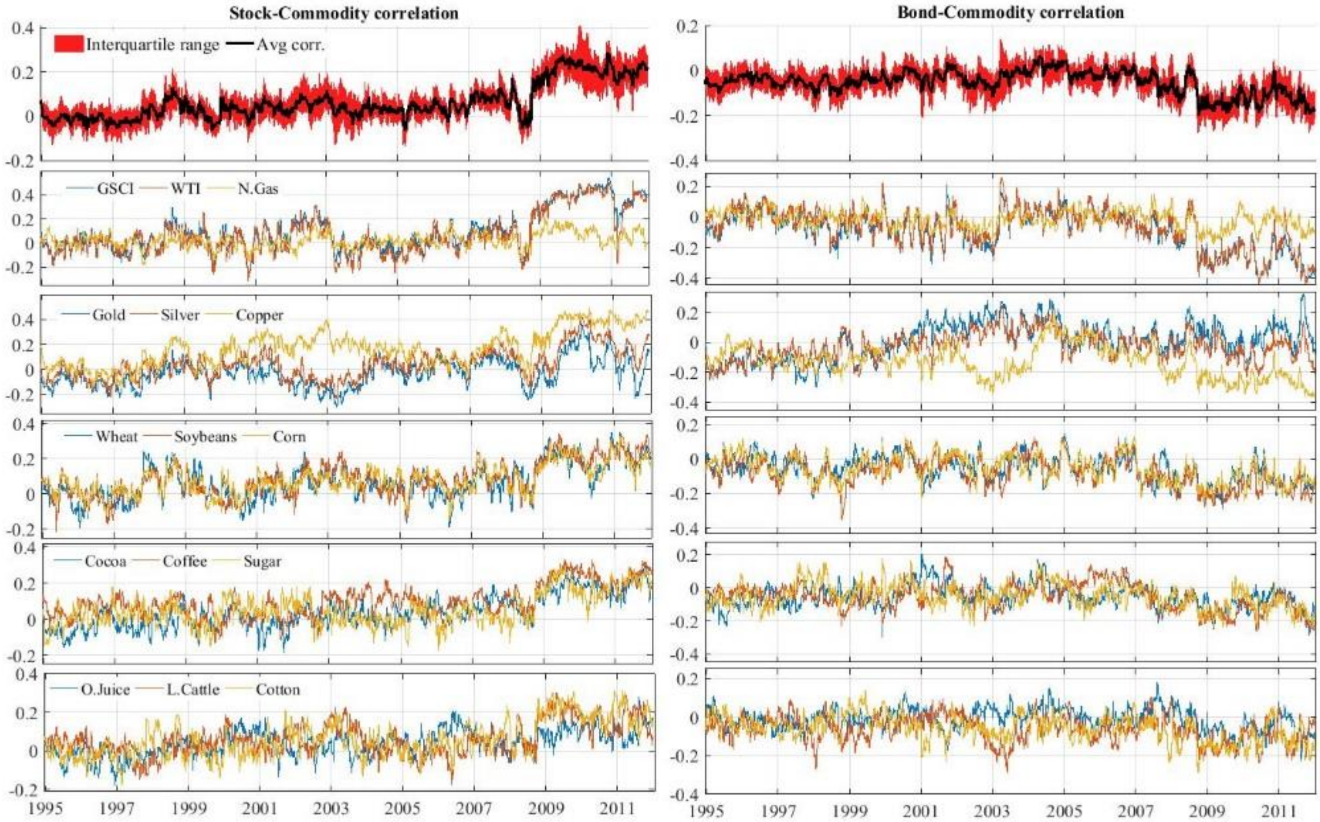

Figure 1: Correlation dynamics. This figure plots the time-varying correlation between commodity and stock returns (left) and commodity and bond returns (right) correlation dynamics. The displayed estimates are based on the average correlation at each point in time, across the dynamic conditional correlation models (DCC, MDC and RSC). The first two plots at the top, portray the average, across commodities (15 in total), correlation along with the interquartile range of the estimate (25% and 75% percentiles).

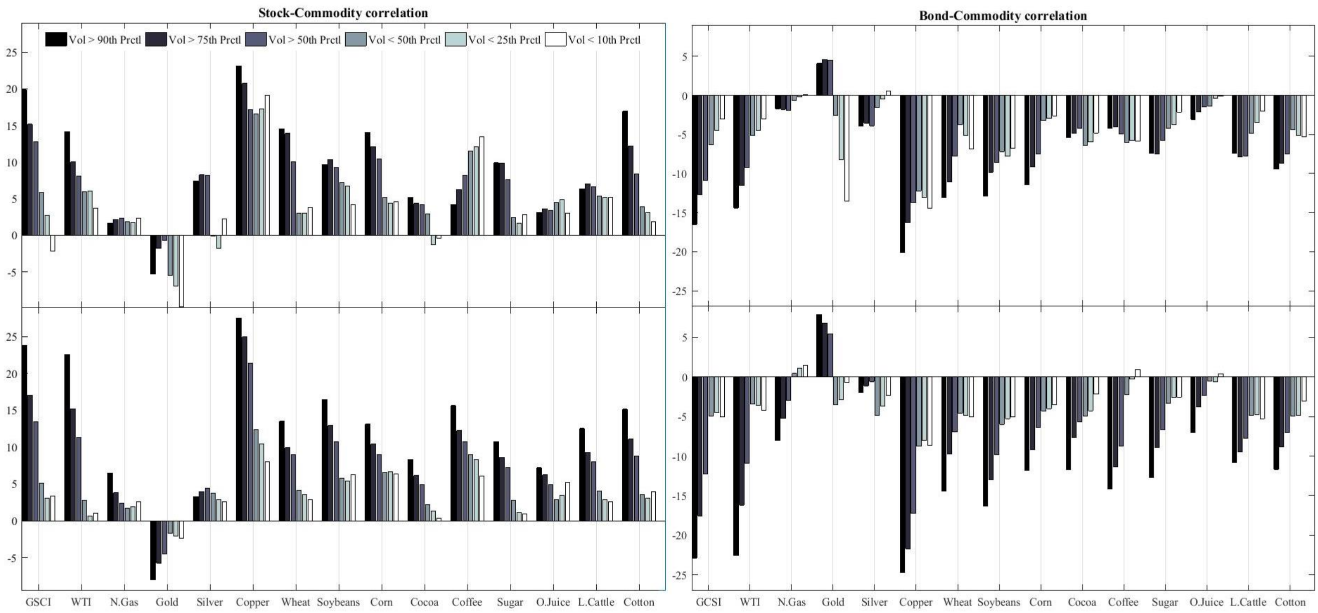

Figure 2: Correlation and volatility percentiles. This figure sh
ows the conditional stock-commodity (left) and bond-commodity (right) correlation (average across DCC, MDC, RSC models). Barplots at the top compute the mean value of correlation after splitting the sample based on commodity volatility percentiles; barplots at the bottom split the sample based on financial market volatility percentiles, i.e., stock (left) and bond (right) volatility. Volatilities used are GARCH(1,1) estimates.

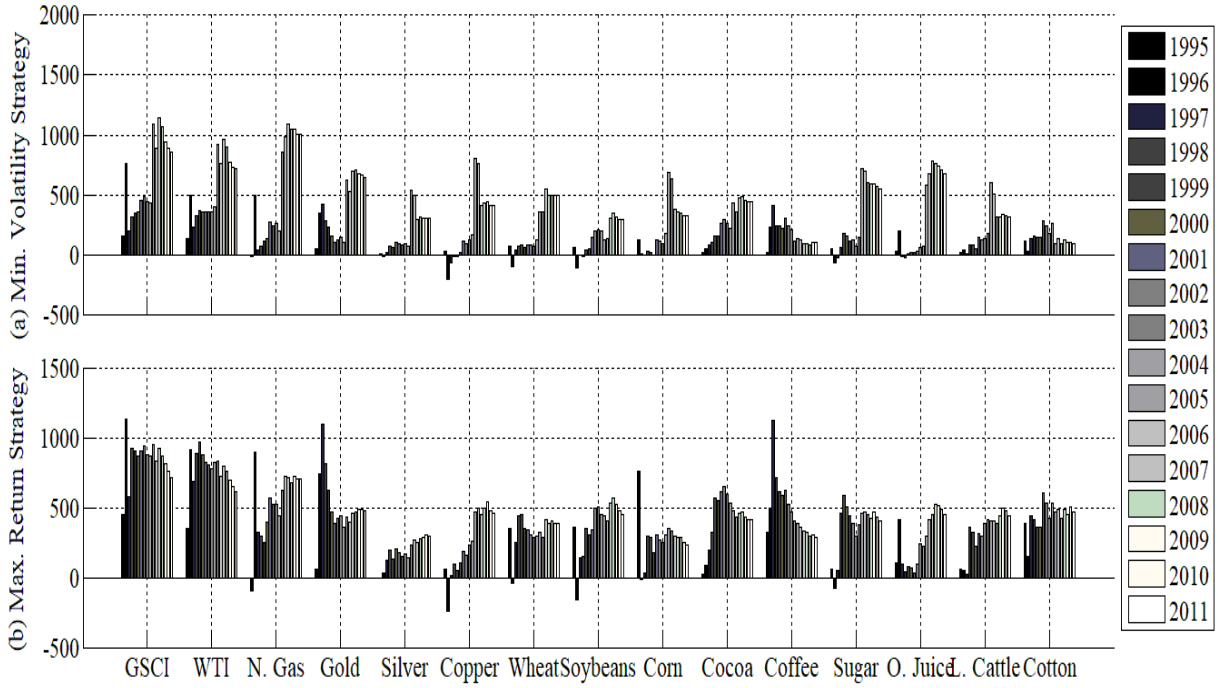

Figure 3: Risk-adjusted abnormal returns 1995-2011. This figure illustrates the evolution of the - average across strategies - $M2$ measure (Eq. 18), in annual bps from 1995 to 2011. $M2$ (Modigliani and Modigliani, 1997) quantifies the abnormal return a portfolio comprising stock, bond, commodity and cash would have earned if it had the same risk as the benchmark stock, bond and cash only portfolio. Results of daily portfolio optimizations use optimum weights that either (a) minimize volatility while setting a target expected return of 10 percent or (b) maximize return subject to a target volatility level of 12 percent.

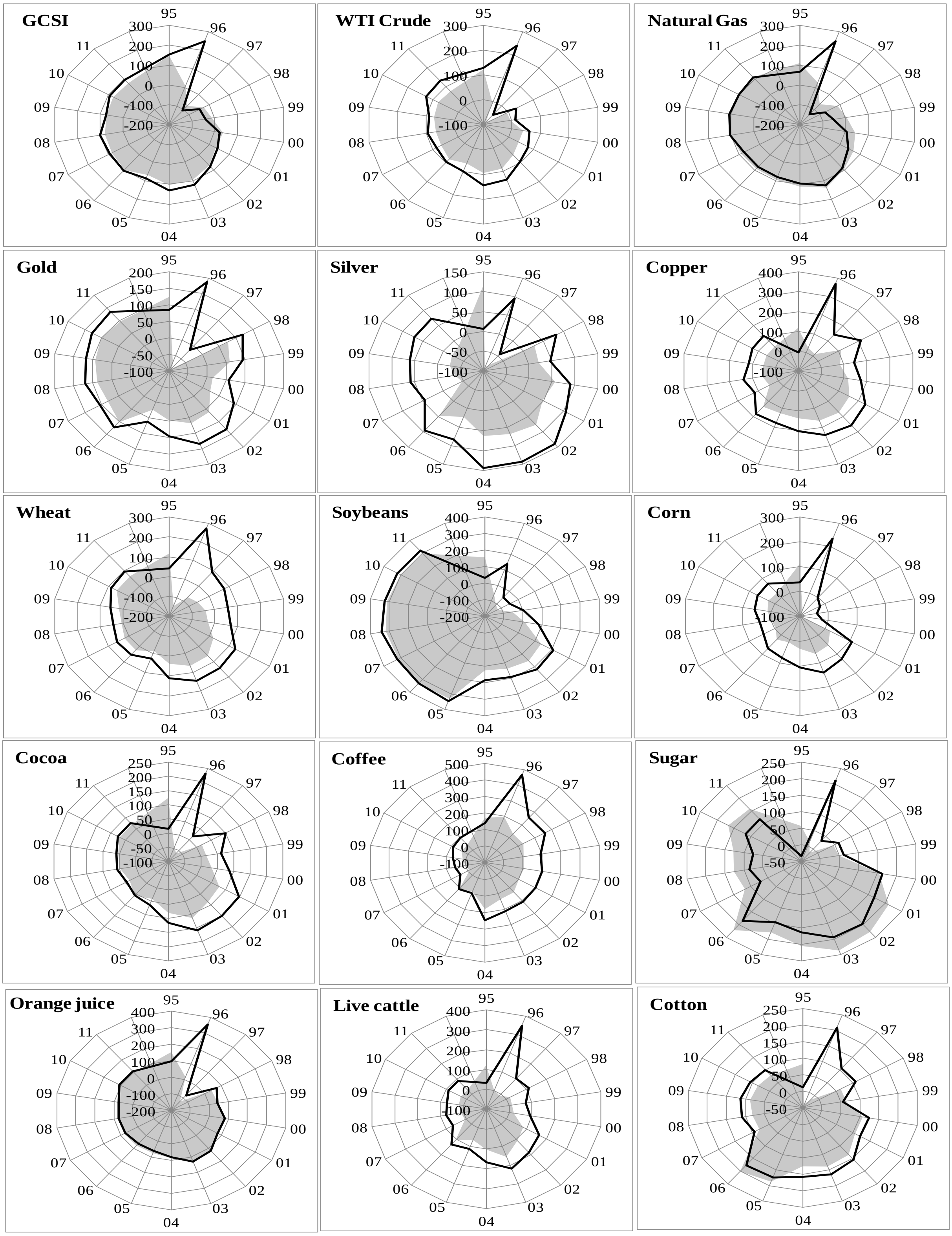

Figure 4: Minimum volatility performance fees. This figure illustrates the changes in the performance fees (in annual bps) per year from 1995 to 2011, for minimum volatility mean-variance efficient portfolios containing three assets (stock, bond, and commodity) and cash; results of daily portfolio optimizations use optimum weights that minimize volatility while setting a target expected return of 10 percent. The shadowed area shows the annual fees an investor is willing to pay for switching from a static allocation strategy to a volatility timing strategy. The black line represents the corresponding fees when timing both volatility and correlation.

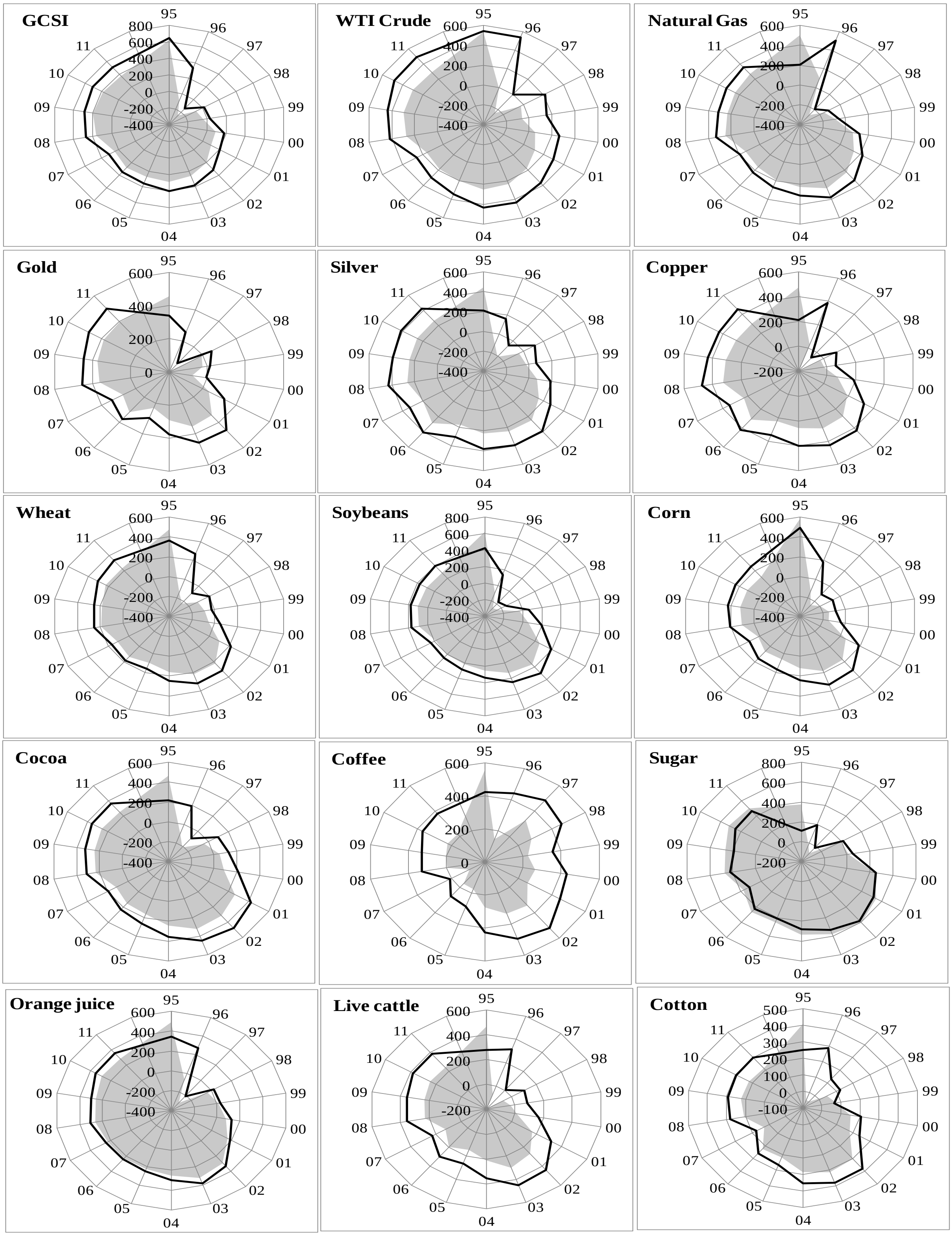

Figure 5: Maximum expected return performance fees. This figure illustrates the changes in the performance fees (in annual bps) per year from 1995 to 2011, for maximum return mean-variance efficient portfolios containing three assets (stock, bond, and commodity) and cash; results of daily portfolio optimizations use optimum weights that maximize expected return while setting a target conditional volatility of 12 percent. The shadowed area shows the annual fees an investor is willing to pay for switching from a static allocation strategy to a volatility timing strategy. The black line represents the corresponding fees when timing both volatility and correlation.

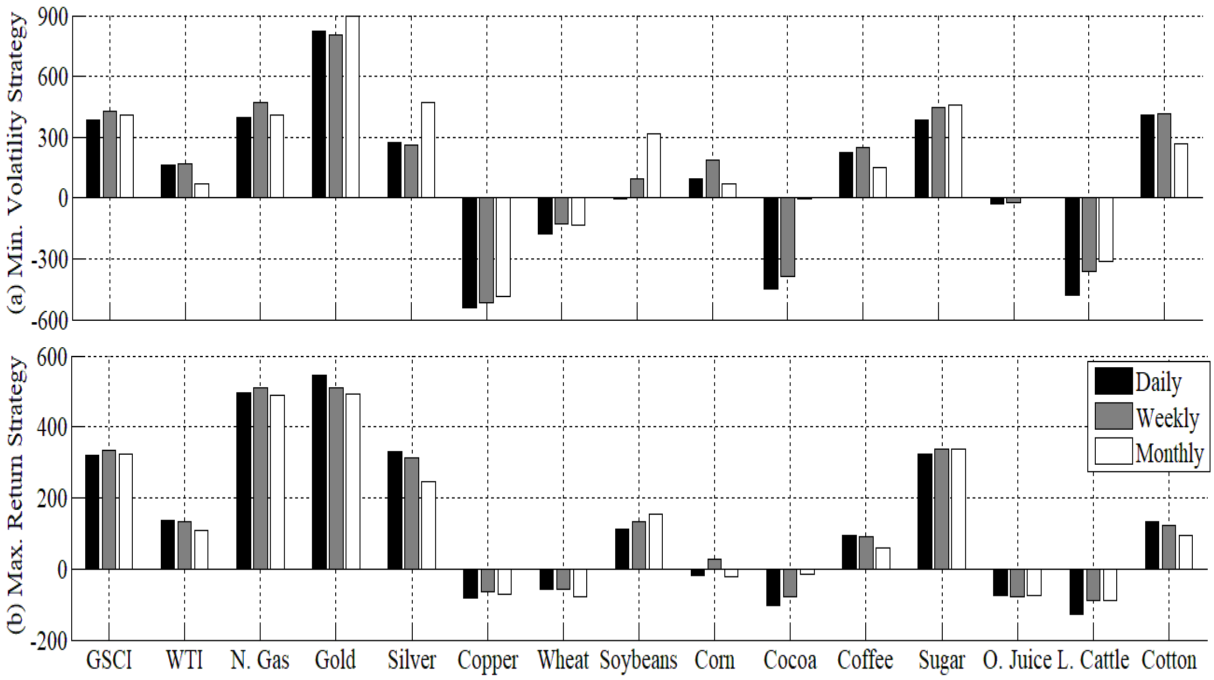

Figure 6: Average Out-of-Sample Risk-adjusted Abnormal Returns. This figure illustrates the evolution of the $M2$ measure (Eq. 18) measure, in annual bps from 2005 to 2011. $M2$ (Modigliani and Modigliani, 1997) quantifies the abnormal return a portfolio comprising stock, bond, commodity and cash would have earned if it had the same risk as the benchmark stock, bond and cash only portfolio. Results of daily portfolio optimizations use optimum weights that either (a) minimize volatility while setting a target expected return of 10 percent or (b) maximize return subject to a target volatility level of 12 percent. The three columns correspond to three different rebalancing frequencies, i.e., daily (black), weekly (grey) and monthly (white). Each column corresponds to the average across dynamic strategies abnormal returns during the out-of-sample period.

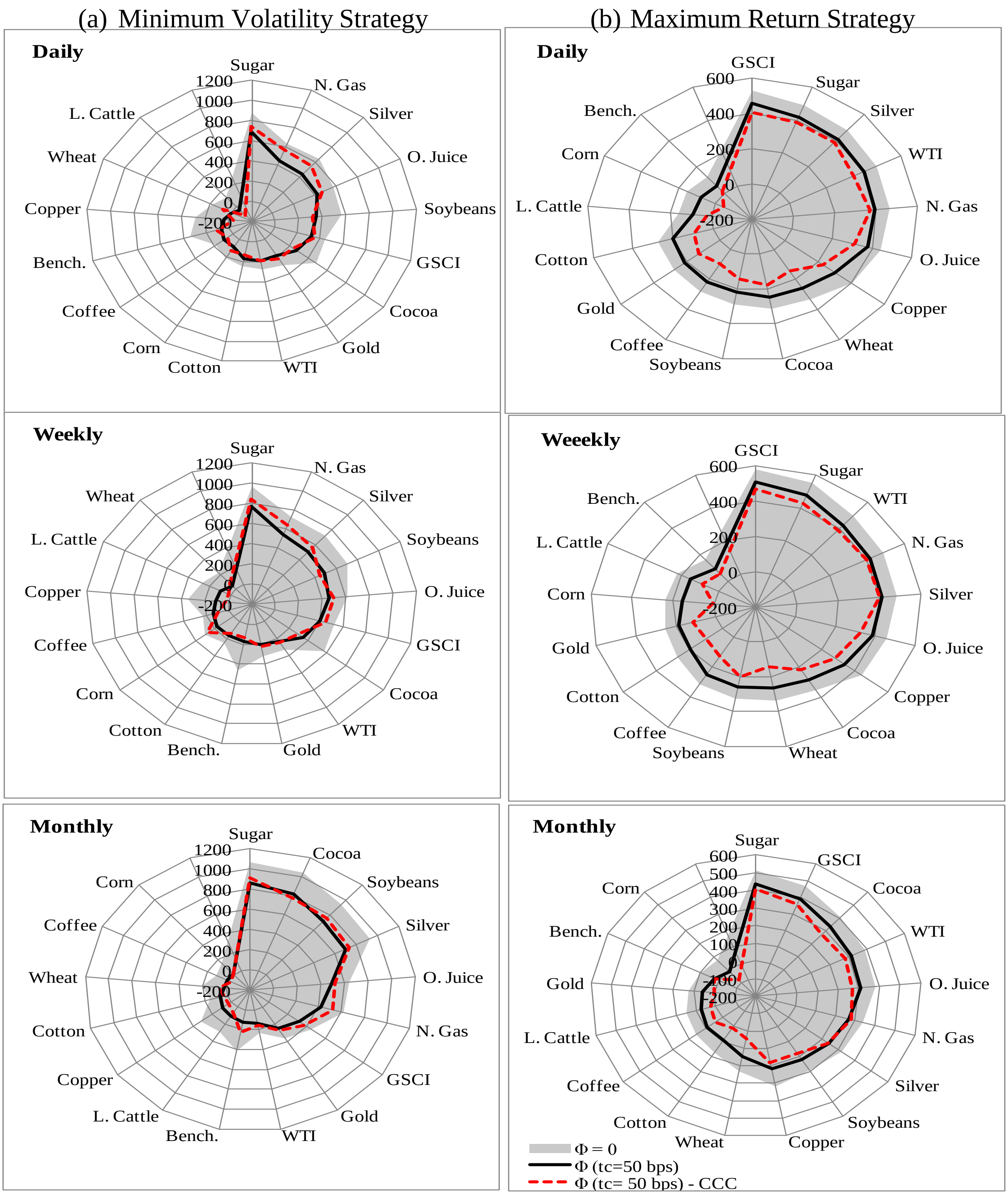

Figure 7: Out-of-Sample Performance fees. This figure illustrates the performance fees in annual bps during the out-of-sample period from January 2005 to January 2012, for (a) minimum volatility and (b) maximum return, mean-variance efficient portfolios. Bench is the benchmark portfolio that contains stock, bond and cash. The shadowed area shows the annual fees an investor is willing to pay for switching from a static allocation strategy to a volatility and correlation timing strategy (maximum of DCC, MDC, RSC). The black line represents the corresponding fees when proportional 50 bps transaction costs are assumed. Red dotted line portrays the fees, net of transaction costs generated by CCC. Each row in the plot corresponds to the associated rebalancing frequency, i.e., daily, weekly and monthly. Portfolios are ranked clockwise according to the net performance fee they generate.