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Designing evacuation routes with GeoGebra

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ABSTRACT: We expose in this paper how the command *ShortestDistance* can be used in GeoGebra to design evacuation routes in buildings in a dynamic and interactive way. For those cases in which a legal regulation or procedure compels specific requirements in the design, we also indicate how to make use of JavaScript in order to implement a more general version of Dijkstra's algorithm that makes possible to deal with such specifications.

KEYWORDS: Evacuation route, Graph Theory, Dijkstra's algorithm.

1 Introduction

Nowadays, the importance of designing optimal evacuation routes in public and private buildings is unquestionable due to the fact that they guarantee, with a very high probability, the safe evacuation of all their occupants in case of any kind of emergency. In the case of buildings like schools, hospitals or hotels, all these routes have to be clearly visible to everybody and, because of

that, it is usual or even mandatory to post evacuation maps in all their rooms and corridors. The problem of these maps is that they are usually fixed and static and they could point exactly at the cause of the emergency, like the seat of a fire, for instance. In order to avoid these cases, it is interesting to have dynamic screens or LED lamps that could show in real time which is exactly the evacuation route that must be follow the person who is watching them. Keeping this idea in mind, we expose in this paper how to use GeoGebra in order to design a dynamic evacuation map that make possible a fast update of the evacuation routes of a building in case of being necessary.

The paper is organized as follows. In Section 2 we introduce the shortest path problem in Graph theory and expose how the command *ShortestDistance*, which is implemented by defect in GeoGebra, can be used to solve this problem. We focus specifically on the creation of a template in GeoGebra that makes possible to design a dynamic and interactive evacuation map of a building whose plan has been previously inserted in the worksheet. In Section 3 we make use of JavaScript in order to implement in GeoGebra the Dijkstra's algorithm, which is commonly used to solve the shortest path problem for any connected and weighted graphs with non-negative weights. This implementation makes possible to deal with legal regulations or procedures that compel specific requirements in the design of evacuation routes.

2 The shortest path problem

We start with some basic results on Graph theory that are used throughout the paper. For more details on this topic we refer to the monograph of Gross and Yellen [GY04].

A *graph* $G = (V, E)$ is a pair formed by a set V of points called *vertices* and a set E of segments connecting them called *edges* (see Figure 1). It is said to be *directed* if its edges has a direction from one of its vertices to the other one. Otherwise, the graph is said to be *undirected*. A *path* between two given vertices v and v' in E is a series of vertices $v_0 = v, v_1, \ldots, v_n = v'$ such that $v_i v_{i+1}$ determines an edge in E, for all $i \in \{0, \ldots, n - 1\}$. The graph G is said to be *connected* if there exists a path for any pair of its vertices. Further, if the graph G is *weighted*, that is, if every edge e in E has associated a numerical label or *weight* W_e, then the *shortest path problem* consists of finding a path between two given vertices that minimizes the sum of weights of its edges. This minimum sum of weights constitutes the *length* of the minimal path or the *graph distance* between the two given vertices.

This problem can be solved in GeoGebra for weighted graphs whose weights are either constant or are determined by the Euclidean distance between connected vertices, that is, by the length of the edges. In the first case, the shortest path between two given points of the graph is a path with the minimum possible number of edges between both points. Once a weighted undirected graph *G* has been drawn in a worksheet of GeoGebra as a set of points (vertices) and segments (edges) and the initial and final points of the path, *A* and *B*, have been identified, it is enough to create a list *L* containing all the segments of the graph. Then, we enter in the input bar the command

ShortestDistance[L, A, B, <Boolean Weighted>],

where the Boolean value can be *true* or *false* depending, respectively, on whether we use the Euclidean distance or the number of edges between two given points. Figure 1 shows an example in this regard. The red edges determine the shortest path between the points *B* and *E* according to the Euclidean distance whereas the green ones determine the path with the minimum number of edges between both points.

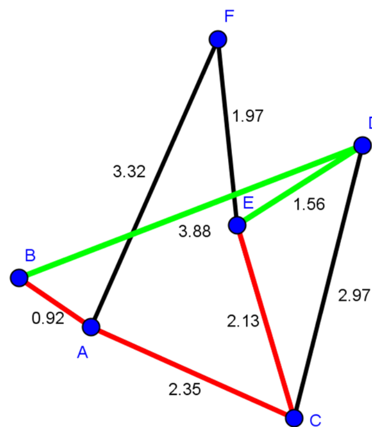

Figure 1: Shortest path problem in GeoGebra.

The command *ShortestDistance* together with the dynamical structure of the graphics view of Geogebra constitutes an especially useful tool to design dynamic and interactive evacuation maps. In this regard, let us suppose that we are interested in designing an evacuation map of the building whose plan is shown in Figure 2. It represents the ground floor of the School of Building Engineering at the University of Seville, in which we can observe the existence of three emergency exit doors. This map has conveniently been inserted in the graphic view of GeoGebra in such a way that the image respects the real scale. In this case, the real size of the plan is 720 x 740 m^2.

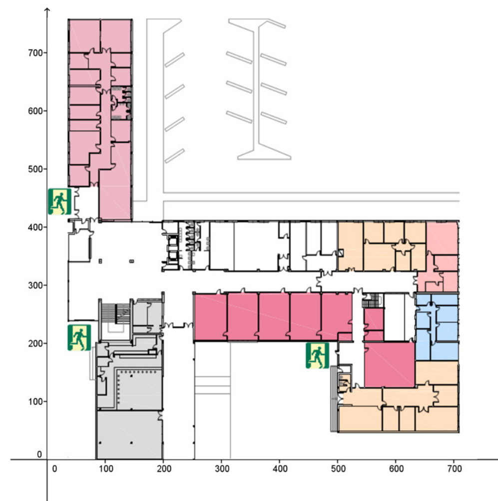

Figure 2: Plan of the first ground of a school.

In order to design our evacuation map in GeoGebra, it is required to determine a graph that represents the distribution of rooms, corridors and stairs. To this end we draw a series of points and edges by following the next sequential order:

- We draw a point for each door or stair in the plan (see Figure 3.a). The points related to the emergency exit doors are called *Exit1*, *Exit2* and *Exit3*.
- We draw an edge for each pair of such points that are related to distinct doors in a same room (see Figure 3.b).
- In front of each door and stair, in the middle of corridors, we draw a point connected by an edge with the point of the corresponding door or stair. Adjacent points in the corridor are also connected with an edge (see Figure 3.c).

The resulting graph (see Figure 3.d and Figure 4) is called the *base graph* of the evacuation map. Once this graph is constructed, we click on the tool *Create List* in the Toolbar of GeoGebra and we create the list *L* of edges that will be used as argument of the command *ShortestDistance*. After that, we define in the input bar the text *Origin* = *"A"*, which we hide immediately after from the graphics view and for which we create a related input box (see Figure 5). In the *On Update* tab of the *Scripting* tab of the input box, we write the next code in GeoGebra script:

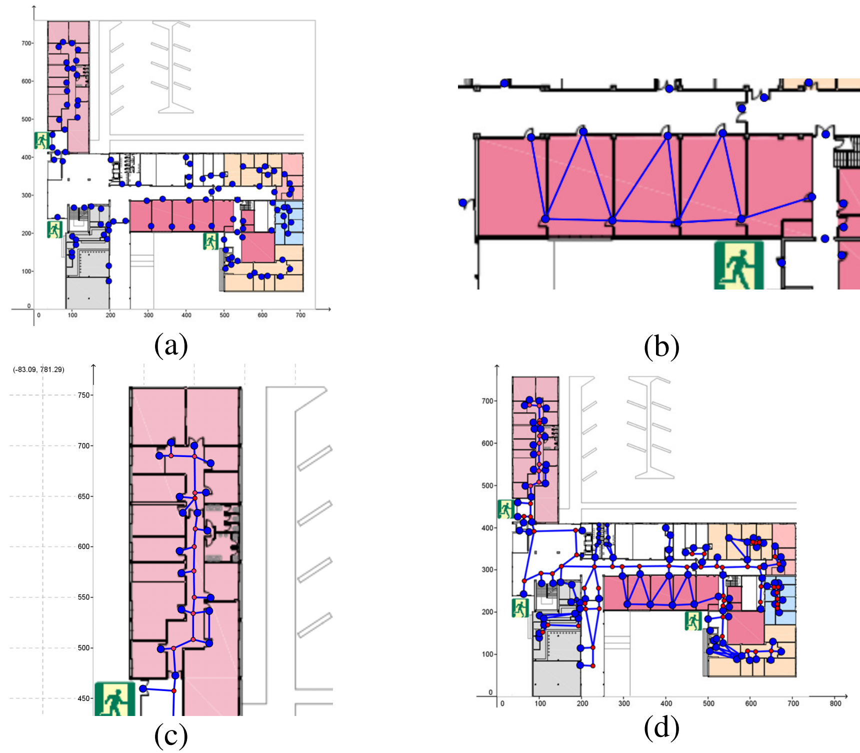

Figure 3: Construction of the base graph of an evacuation map.

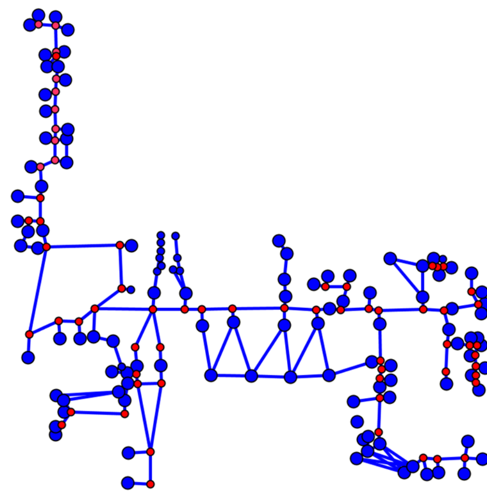

Figure 4: Base graph of an evacuation map.

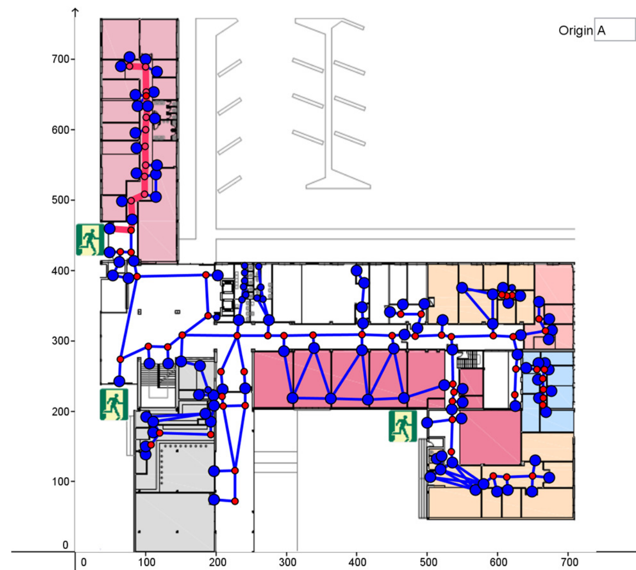

Figure 5: Evacuation route in GeoGebra.

Execute[{"path1=ShortestDistance[L,"+Origin+", Exit1, true]",
"path2=ShortestDistance[L, "+Origin+", Exit2, true]",
"path3=ShortestDistance[L, "+Origin+", Exit3, true]"}]
path={Length[path1], Length[path2], Length[path3]}

This code involves the computation of the three possible evacuation routes from the given initial point to the three possible emergency exit doors and of their respective lengths. Since we are only interested in the evacuation route of minimum length, we have to make use of some *condition to show object* in the advanced properties of each one of the three paths. Specifically, we impose in *path1*, *path2* and *path3* the respective conditions:

Min[RemoveUndefined[path]] ≟ Length[path1]
Min[RemoveUndefined[path]] ≟ Length[path2]
Min[RemoveUndefined[path]] ≟ Length[path3]

After that, our template constitutes a dynamical and interactive evacuation map of our building where it is enough to define in the created input box the initial point from where the shortest evacuation route is required. Moreover, the use of the command *RemoveUndefined* in the just exposed three conditions makes possible to delete a series of edges in the base graph of the evacuation map that, in case of an emergency, are temporally

blocked, due to fire propagation or gas leak, for example. These edges are directly eliminated from the list L of edges and, after that, new alternative evacuation routes are automatically obtained (see Figure 6, where there are indicated a pair of alternative routes of the shortest path shown in Figure 5, in case of being blocked one or two of the closer emergency exit doors) .

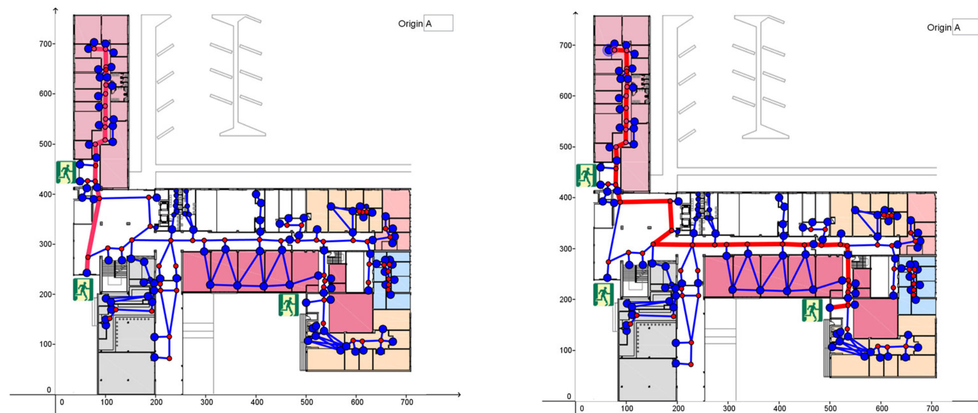

Figure 6: Alternative routes when some emergency exit doors are blocked.

Nevertheless, even if our evacuation route is automatically selected and exposed by GeoGebra as the shortest path between the initial point in question and the three possible emergency exit doors, two important remarks have to be done to our construction:

1) Real evacuation routes must facilitate paths through wide corridors and avoid, whenever is possible, to enter from one corridor to a small room as a crossing point of our path. It is due to the fact that this type of rooms could constitute bottle necks in case of an emergency. Nevertheless, since our base graph is undirected and our shortest path is obtained by taking into account the Euclidean distance, GeoGebra include these small rooms as possible crossing points of the final evacuation route (see Figure 7).
2) Depending on the type of building, there can exist a series of legal regulations or procedures that compel specific requirements in the design of an evacuation route like the minimum width or height of doors and stairs, the lengths of corridors or the maximum number of people (*capacity*) that can use a certain route, amongst others. Nevertheless, none of these factors can be taken into account in our previously designed evacuation route.

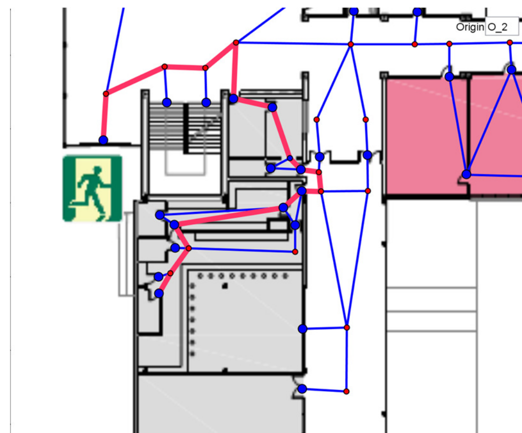

Figure 7: Evacuation route proposed by GeoGebra.

It is therefore necessary to generalize our construction in order to deal with more realistic evacuation routes. To this end, we propose in the next section to make use of Javascript in order to implement the Dijkstra's algorithm in GeoGebra.

3 The Dijkstra's algorithm

In 1959, Edsger W. Dijkstra [Dij59] established an algorithm that solves the shortest path problem for connected and weighted graphs with non-negative weights. These graphs can be directed or undirected. In the course of the algorithm, given a weighted graph $G = (V, E)$ and the initial and final vertices, v and v', of the path, Dijkstra subdivides the set of vertices V into three sets:

- The set V_1 of vertices w for which the path of minimum length from v is known.
- The set V_2 of vertices in $V - V_1$ that are connected by one edge to at least one vertex of V_1. If the graph is directed, then this edge must be oriented from the vertex in V_1 to the vertex in V_2.
- The set V_3 formed by the rest of vertices.

Dijkstra also subdivides the set of edges E in other three sets:

- The set E_1 of edges in the minimal paths from v to the vertices of V_1.
- The set E_2 of edges in $E - E_1$ that connect vertices of V_1 and V_2.

- The set E_3 formed by the rest of edges.

At the beginning of the algorithm, all the vertices are in V_3 and all the edges are in E_3. The vertex v is then the first vertex to be included in V_1. At each step of the algorithm, it is considered all the edges e that connect the last vertex w included in V_1 with vertices w' in $V_2 \cup V_3$.

- If the vertex w' is in V_3, then it is added to V_2 and the edge e is added to E_2. The path of minimum length l_w between v and w together with the edge e of weight W_e determines a provisional path between v and w' of length $l_{w'} = l_w + W_e$.
- If the vertex w' is already in V_2, then there exists exactly one edge e' in E_2 that connects w' with a vertex w'' in V_1 so that the path of minimum length $l_{w''}$ between v and w'' together with the edge e' of weight $W_{e'}$ determines a provisional path of minimum length $l_{w'} = l_{w''} + W_{e'}$. If $l_{w'} \leq l_w + W_e$, then the edge e is rejected. Otherwise, the edge e replaces the edge e' in V_2.

After that, the vertex in V_2 with the shortest provisional path from v and its related edge in E_2 are respectively transferred to V_1 and E_1, and the procedure is then repeated for this new vertex in V_1. The algorithm finishes when the final vertex v' is transferred to V_1.

In practice, the n vertices of the graph are initially ordered and labeled as $v_1, \ldots, v_n$. An auxiliary ordered list L of cardinality n, initialized as $\{0,\ldots,0\}$, can then be defined in the course of the algorithm in such a way that its ith element is the immediately previous vertex through which any shortest path starting in v has to pass to get the vertex v_i. It coincides with the second vertex related to the edge that is transferred to E_1 at the same time that v_i is transferred to V_1. Once the algorithm finishes, this list determines all the previous vertices through which the shortest path from v to v' has to pass.

Further, since the algorithm can be repeated for any pair of vertices, it is also possible to determine the *distance matrix* $D = (d_{ij})$ of the graph, where d_{ij} is the length of the shortest path between the vertices v_i and v_j of the graph.

As an example of implementation of the Dijkstra's algorithm, we show in Figure 8 the steps that are sequentially required to determine the shortest path between two vertices, A and I, of a given undirected graph. Orange edges determine the vertices and edges that are transferred in each step to the sets V_1 and E_1, respectively. Below each image we indicate the vertices w that are in the set V_2 after each step, which are indicated as pairs (w, l_w), where l_w is the weight of the provisional minimum path to w from the vertex A by only taking into consideration the edges in E_2. The ordered auxiliary list L is also

indicated in each step. We have used the lexicographical order. Let us examine the procedure in detail.

- **Step 0**: The initial vertex A is transferred to V_1.
- **Step 1**: We transfer to V_2 the vertices in V_3 connected to A, that is, B and C. The edges AB and AC are transferred to E_2. Since both edges have weight 2, we can transfer any of the vertices B or C to the set V_1. We have selected B. The edge AB is transferred to E_1.
- **Step 2**: We transfer to V_2 those vertices in V_3 that are connected to B, which are D and H, and to E_2 the edges BH and BD. The distances to A of these two new vertices according to the edges of E_2 are, respectively, $l_D = 6$ and $l_H = 7$. Hence, the vertex in V_2 with minimum distance to A by using the edges of E_2 is C ($l_C = 2$), which is transferred to V_1. The edge AC is transferred to E_1.
- **Step 3**: We transfer to V_2 the vertex E, which is the only vertex in V_3 connected to C. The edges CD and CE are transferred to E_2 and hence, the distance from A to D by using E_2 decreases from 6 to 5. The vertex with minimum distance to A is then E ($l_E = 4.5$), which is transferred to V_1. The edge CE is transferred to E_1.
- **Step 4**: We transfer to V_2 the vertex F, which is the only vertex in V_3 connected to E. The edge EF is transferred to E_2. The distance from A to F by using E_2 is $l_F = 6.5$. The vertex with minimum distance to A in V_2 is D ($l_D = 5$). It is transferred to V_1 and the edge CD to E_1.
- **Step 5**: We transfer to V_2 the vertex I, which is the only vertex in V_3 connected to D. The edges DI and DF are transferred to E_2. The distance from A to I by using E_2 is $l_I = 7$. The vertex with minimum distance to A in V_2 is F ($l_F = 6.5$). It is transferred to V_1 and the edge DF to E_1.
- **Step 6**: The vertex G is transferred to V_2 and the edge FG to E_2. It is $l_G = 9.5$. The minimum distance to A in V_2 is $l_H = l_I = 7$. We transfer to V_1 the final vertex I and the edge DI to E_1. The algorithm finishes here and the shortest path is formed by the edges AC, CD and DI, with total length 7.
- **Solution**: Once the final vertex I is reached, the shortest path is uniquely determined by the auxiliary list L, which is lexicographically ordered. In our case, it can be observed in this list that I is reached from D, D from C and C from A.

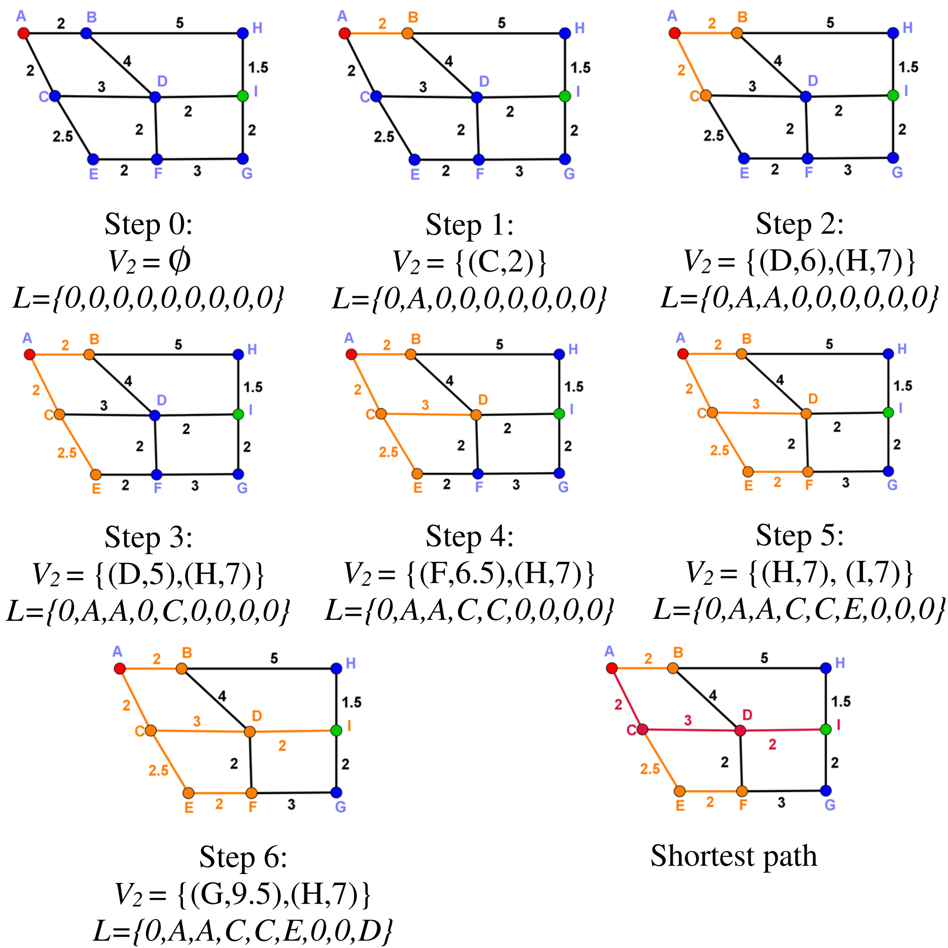

Figure 8: Dijkstra's algorithm.

The Dijkstra's algorithm can be implemented in GeoGebra by making use of Javascript. Previously, the vertices of our weighted graph G have to be ordered and its *adjacency matrix* defined, that is, the $n \times n$ binary matrix $M = (m_{ij})$ such that, if there exists an edge between the vertices i and j (from the vertex i to the vertex j if the graph is directed), then m_{ij} coincides with the weight of such an edge. Otherwise, it is $m_{ij} = 0$. The adjacency matrix of a directed graph is, therefore, symmetric, but it is not true in general if the graph is undirected.

The specific construction in GeoGebra is as follows: Let us suppose that we are interested in a directed graph of six vertices and adjacency matrix

$$M = \begin{pmatrix} 0 & 1 & 2 & 0 & 4 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 2 & 1 & 0 \\ 0 & 2 & 0 & 0 & 1 & 3 \\ 1 & 0 & 0 & 2 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

In order to draw this graph in GeoGebra, we create six points, *A*, *B*, *C*, *D*, *E* and *F*, in the graphics view of GeoGebra and we enter in the input bar the next six sentences:

V = {A, B, C, D, E, F}
n=Length[V]
M={0,1,2,0,4,0,0,0,0,1,1,1,1,0,0,2,1,0,0,2,0,0,1,3,1,0,0,2,0,1,1,0,1,0,1,0}
Edges=Sequence[Sequence[If[Element[M, (i - 1) n+ j] > 0, Vector[Element[V, i], Element[V, j]]], j, 1, n], i, 1, n]
Initial = 1
Final = 1

The last two sentences will refer to the position in the list *V* of the initial and final vertices of our path. In order to design an interactive template, we create two input boxes in the graphics view that are respective related to these two vertices (see Figure 9).

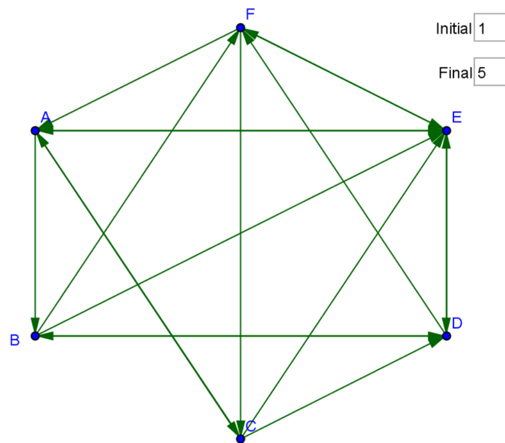

Figure 9: Directed graph in GeoGebra.

The next step consists of implementing the Dijkstra's algorithm in GeoGebra. To this end, we enter in the *JavaScript Global* tab, the next code in JavaScript:

```
function distance(){
var i, d, m, v, a, a0;
var s=0;
var n = ggbApplet.getValue("n");
var D = new Array();
var NM = new Array();
var P = new Array();
for(i =0; i < n*n; i++){
    D[i] = 100;}
for(a0 =1; a0 < n+1; a0++){
    for(i =0;i<n;i++){
        NM[i]=0;
        P[i]=-1;
        D[n*n+(a0-1)*n+i]=-1;}
    a=a0;
    D[(a0-1)*n+a-1]=0;
    NM[a-1]=1;
    while(a!=0){
        for(i =0;i<n;i++){
            m=ggbApplet.getListValue("M",(a-1)*n +i+1);
            if (NM[i]==0 & m!=0 & D[(a0-1)*n+i]>D[(a0-1)*n+a-1]+m){
                D[(a0-1)*n+i]=D[(a0-1)*n+a-1]+m;
                P[i]=a;
                D[n*n+(a0-1)*n+i]=a;}}
        d=100;
        for(i=0;i<n;i++){
            if (NM[i]==0){
                if (s==0){
                    s=i;
                    d=D[(a0-1)*n+i];}
                else{
                    if (D[(a0-1)*n+i]<d){
                        d=D[(a0-1)*n+i];
                        s=i;}}}}
        if (d==100){
            v=0;}
        else{
            v=s;}
        if (v==0){
            a=0;}
        else{
            NM[v]=1;
            a=v+1;}}}
      return(D);}
```

The output of this function is an array D of cardinality $2n^2$, such that its n^2 first elements determine the distances among the vertices of the graph. These first elements are initialized to *100*, which would correspond to the maximum possible weight of the graph in question. Depending on the graph, this initial value can conveniently be increased. Further, the n^2 last elements of the array D determine the list of previous vertices in any shortest path from a given initial vertex.

Since D is not yet an explicit object in GeoGebra, we enter the next code in JavaScript in the *On Update* tab of the *Scripting* properties of the numbers *Initial* and *Final*:

ggbApplet.evalCommand("d={"+distance()+"}");

We can then press *F9* or to introduce a number in any of the two input boxes of our template to define automatically an auxiliary list d with all the $2n^2$ elements of the array D. In particular, the distance matrix of our graph can be explicitly defined by entering in the input bar the sentence

DM=Sequence[Sequence[If[Element[d,(i-1) n+j]==100, ∞, Element[d,(i-1) n+j]], j, 1, n], i, 1, n]

In our case, this distance matrix is

$$DM = \begin{pmatrix} 0 & 1 & 2 & 2 & 2 & 2 \\ 2 & 0 & 2 & 1 & 1 & 1 \\ 1 & 4 & 0 & 2 & 1 & 2 \\ 2 & 2 & 3 & 0 & 1 & 2 \\ 1 & 4 & 2 & 2 & 0 & 1 \\ 1 & 5 & 1 & 3 & 1 & 0 \end{pmatrix}$$

The next step is to define the auxiliary list of previous vertices in a shortest path with initial vertex that one that has been inserted in the corresponding input box of our template. To this end, we enter the next sequence in the input bar:

L=Sequence[If[Element[d, n² + (Initial - 1) n + i] ≟ -1, 0, Element[d, n² + (Initial - 1) n + i]], i, 1, n]

This list can be used to determine the shortest path from the initial to the final vertices introduced in the corresponding input boxes. This path is got by adding the next code in Javascript in the *Global JavaScript* tab:

```
function path(){
    var i,j,m;
    var ip = ggbApplet.getValue("Initial");
    var fp = ggbApplet.getValue("Final");
    var n = ggbApplet.getValue("n");
    var A = new Array();
    var C = new Array();
    for(i =0;i<n;i++){
```

```
        m=ggbApplet.getListValue("L",i+1);
        A[i]=m;}
    C[0]=A[fp-1];
    j=0;
    for(i =0;i<n-1;i++){
        if (C[i]==0){
            C[i+1]=0;}
        else{
            if (C[i]!=ip){
                C[i+1]=A[C[i]-1];}
            else{
                C[i+1]=0;}}}
    return(C);}
```

The output of this function is a list of numbers that determine the position in the list *V* of all the vertices through which our shortest path has to pass before reaching the final vertex. In order to have this list as an explicit object in GeoGebra, we add the next code in JavaScript in the *On Update* tab of the *Scripting* properties of the numbers *Initial* and *Final*:

```
ggbApplet.evalCommand("path= {"+path()+"}");
```

Finally, the shortest path is drawn in GeoGebra (see Figure 10) by entering the next sequence in the input bar:

```
Shortest=Union[Sequence[If[Element[path, i] Element[path, i + 1] > 0,
Vector[Element[V, Element[path, i + 1]], Element[V, Element[path, i]]]], i, 1, n],
{If[Element[path, 1] > 0, Vector[Element[V, Element[path, 1]], Element[V,
Final]]]}]
```

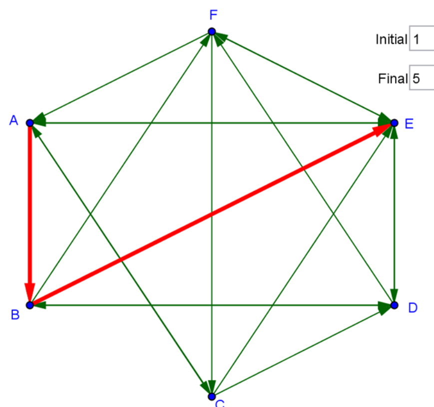

Figure 10: Shortest path in GeoGebra by using the Dijkstra’s algorithm.

The template that we have just constructed can be considered as the base worksheet from which we can elaborate any dynamic and interactive evacuation map in a similar way to the construction that we have done in Section 2. It is due to the fact that this template is completely parametric, not only with respect to the initial and final points of our path, but also with respect to the initial graph. Specifically, if we change the list V of vertices of the new graph and the list M that corresponds to the entries of its adjacency matrix, all the constructed objects are conveniently redefined. It is particularly useful in case of being required to eliminate some vertex or edge of the base graph of the evacuation map because of an emergency.

The advantage of this template with respect to that one constructed in Section 2 is that the new one facilitates the use of directed edges in the base graph of the evacuation map and also the use of any type of weight, not only constant weights or weights based on the Euclidean distance among vertices. It makes possible the implementation of our model in the design of evacuation routes of any type of building, with independence of the particular requirements compelled by the corresponding legal regulation.

4 Conclusions

In this paper we have exposed how to use GeoGebra in order to design a dynamic and interactive evacuation map that can be used in real time to optimize the evacuation plan of public and private buildings like schools, hospitals or hotels, amongst others. The use of the command *ShortestDistance*, which is implemented by defect in GeoGebra, is a good alternative to this end. Nevertheless, this command can only be used for weighted graphs labeled with constant weights or with weights based on the Euclidean distance among vertices. To deal with more general cases, we have made use of JavaScript to implement in GeoGebra the Dijkstra's algorithm. A more comprehensive study for possible implementation of our model in real buildings has to be further developed.

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