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Stability analysis of time-averaged jet flows: fundamentals and application

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Abstract

We report on experimental and theoretical investigations of shear flow instabilities in jet flows. Linear stability analysis is applied to the time-averaged flow taken from experiments, contrasting the 'classic' stability approach that is based on a stationary base flow. To some extend, mean flow stability eigenmodes may deal as a model for instability waves at their nonlinearly saturated state, which is typically encountered in experiments. The capability of mean flow stability models is first demonstrated on laminar oscillating jets where the primary interaction takes place between the mean flow and the instability wave. We then focus on turbulent swirling jets where additional interactions occur between the fine-scale turbulence and the instability waves. Swirling flows are widely used in combustion applications where the associated high turbulence levels and internal recirculation zones (vortex breakdown bubble) are exploited for flame stabilization. We demonstrate the application of mean flow stability analysis on the flow field of a industry-relevant swirl-stabilized flame. We show that the flame response to acoutstic perturbations is closely linked to the flow receptivity predicted from linear stability analysis, which suggests that the adopted theoretical framework is very useful for thermoacoustic modeling.

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1. Theoretical concept of mean flow stability wave models

1.1. Governing equations

The stability equations of the mean (time-averaged) flow are derived from the classic triple decomposition. The instantaneous flow field vector $\mathbf{u}(\mathbf{x}, t)$ is decomposed into a time-averaged part $\overline{\mathbf{u}}(\mathbf{x})$, a periodic (coherent) part $\tilde{\mathbf{u}}(\mathbf{x}, t)$,

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and a randomly fluctuating (turbulent) part $\mathbf{u}'(\mathbf{x}, t)$, reading

$$\mathbf{u}(\mathbf{x}, t) = \overline{\mathbf{u}}(\mathbf{x}) + \tilde{\mathbf{u}}(\mathbf{x}, t) + \mathbf{u}'(\mathbf{x}, t). \tag{1}$$

The phase-average $\langle \mathbf{u}(\mathbf{x}, t) \rangle$ is used to separate the fine-scale turbulent fluctuations from the coherent motion such that $\tilde{\mathbf{u}}(\mathbf{x}, t) = \langle \mathbf{u}(\mathbf{x}, t) \rangle - \overline{\mathbf{u}}(\mathbf{x})$.

The triple decomposition is substituted into the incompressible Navier–Stokes equation and the continuity equation, and, after some manipulations, the governing equations for each of the three parts can be formulated[10]. The mean flow equations are

$$\overline{\mathbf{u}} \cdot \nabla \overline{\mathbf{u}} = -\nabla \overline{p} + \frac{1}{\mathrm{Re}} \nabla^2 \overline{\mathbf{u}} - \nabla \cdot \left(\overline{\mathbf{u}'\mathbf{u}'} + \overline{\tilde{\mathbf{u}}\tilde{\mathbf{u}}} \right) \tag{2a}$$

$$\nabla \cdot \overline{\mathbf{u}} = 0, \tag{2b}$$

indicating how the mean flow is modified through the generation of turbulent and coherent Reynolds stresses. The equations for the coherent motion are given as

$$\frac{\partial \tilde{\mathbf{u}}}{\partial t} + \tilde{\mathbf{u}} \cdot \nabla \overline{\mathbf{u}} + \overline{\mathbf{u}} \cdot \nabla \tilde{\mathbf{u}} = -\nabla \tilde{p} + \frac{1}{\mathrm{Re}} \nabla^2 \tilde{\mathbf{u}} - \nabla \cdot \left(\tau^N + \tilde{\tau} \right) \tag{3a}$$

$$\nabla \cdot \tilde{\mathbf{u}} = 0, \tag{3b}$$

where the nonlinear terms $\tau^N = \tilde{\mathbf{u}}\tilde{\mathbf{u}} - \overline{\tilde{\mathbf{u}}\tilde{\mathbf{u}}}$ are neglected in the following. The terms $\tilde{\tau} = \langle \mathbf{u}'\mathbf{u}' \rangle - \overline{\mathbf{u}'\mathbf{u}'} = \widetilde{\mathbf{u}'\mathbf{u}'}$ represent the modification of the turbulent field during the passage of a coherent structure. These turbulent-coherent interactions are unknown and must be modeled appropriately. The mean-coherent and mean-turbulent interactions are reflected in the actual mean flow shape and are implicitly accounted for in the perturbation equations (3). The turbulent-coherent interactions are modeled through a Newtonian eddy viscosity model

$$\tilde{\tau}_{ij} = -\widetilde{u_i'u_j'} = \nu_t \left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right), \tag{4}$$

where ν_t is the eddy viscosity of the undisturbed flow and the indices $i, j = 1, 2, 3$ indicate the three velocity components.

1.2. Solution for weakly nonparallel flows

Equations (3) are first solved for a parallel flow $\overline{\mathbf{u}}_0 = (f(r), 0, 0)^T$. In the presented studies, our interest lies in the *spatial* growth and decay of instabilities. Therefore, the perturbations have the form

$$\tilde{\mathbf{u}}(\mathbf{x}, t) = \hat{\mathbf{u}}_0(r) \mathrm{e}^{i(\alpha x + m\theta - \omega t)} + c.c., \tag{5}$$

with complex spatial wavenumber $\alpha = \alpha_r + i\alpha_i$, integer real azimuthal wavenumber m, and real temporal oscillation frequency ω. The conjugate complex of the perturbation is indicated by 'c.c.'. The imaginary part of α corresponds to the spatial growth rate of the parallel flow and determines whether a perturbation of a given m and ω grows $(-\alpha_i > 0)$ or decays $(-\alpha_i < 0)$ in the streamwise direction. Substituting the ansatz (5) and the equivalent for the pressure into (3) leads to the eigenvalue problem

$$\mathbf{D}(\omega)\psi_0 = \alpha \mathbf{E}(\omega)\psi_0, \tag{6}$$

with the eigenvalue α and the eigenfunction $\psi_0 = (\hat{u}_0, \hat{v}_0, \hat{w}_0, \hat{p}_0)^T$, and the matrices $\mathbf{D}$ and $\mathbf{E}$ containing the parallel flow profiles $\overline{\mathbf{u}}_0$.

The stability analysis is extended to weakly nonparallel flows by adopting the correction scheme developed by Crighton & Gaster[2]. To account for a slow streamwise jet divergence, we introduce a slow axial scale $X = \epsilon x$, where $\epsilon \ll 1$, and a radial component $\overline{v}_1 = \overline{v}/\epsilon$. The global perturbation field is given as

$$\tilde{\mathbf{u}}(X, r, \theta, t; \epsilon) = N(X)\hat{\mathbf{u}}_0(X, r; \epsilon) \exp\left(\frac{i}{\epsilon} \int^X \alpha(\xi) \mathrm{d}\xi + im\theta - i\omega t \right) + c.c., \tag{7}$$

with the amplitude factor $N(X)$ given as

$$\frac{\mathrm{d}N(X)}{\mathrm{d}X}G(X) + N(X)K(X) = 0. \tag{8}$$

The expressions G and K in (8) are derived from the solvability condition of the first order problem, and they contain the radial and streamwsie derivatives of eigenfunctions and their adjoints of the (zero order) parallel flow solution. The derivations of the multiple-scale approximation for swirling jets is given in Oberleithner et al.[7].

2. Results from recent laminar, turbulent, and reacting jet studies

In the following, we briefly describe the results from mean flow stability analysis of three different flow configurations. The first deals with a laminar oscillating jet. This study is aimed to asses the accuracy of the mean flow model in predicting the nonlinear fluctuations at strongly forced conditions. Details to this study are given in Oberleithner et al.[7]. The second configuration is a turbulent swirling jet at pre-breakdown conditions. In this study, we are interested in the helical modes that develop in the highly turbulent, konvectively unstable shear layer. Details to this study are given in Oberleithner et al.[6]. The third study combines the findings of the first and second investigation. We apply mean flow stability concepts to a turbulent swirling jet at post-breakdown conditions with an anchored V-shaped flame. This configuration is typically found in gas turbine combustion chambers. We compute the flow receptivity to acoustic perturbations at low and high amplitude acoustic forcing and show correlations to the respective flame response. Details to this study are given in Oberleithner et al.[8].

2.1. Laminar oscillating jets

The axisymmetric laminar jet was generated by a piston-cylinder-type arrangement and released into a large water tank. The jet's axisymmetric mode was excited by imposing a sinusoidal motion onto the piston's mean motion at relative amplitudes ranging from 0.1 % to 100 %. The flow field was obtained using high-resolution, low-speed, PIV. The oscillations corresponding to the fundamental and higher harmonics of the forcing frequency were extracted from the uncorrelated PIV snapshots using a POD-based phase reconstruction scheme. The experimental results are compared to the stability wave model, including amplitude and phase distribution, growth rates, and phase velocities.

Figure 1 shows the results for the jet forced sinusoidally at amplitudes of 5 % of the mean flow rate. The overall agreement between the stability wave model and the measured flow field fluctuations is excellent. The analysis of the flow forced at a wide range of amplitudes further reveals that the stability wave model captures well the growing phase and precisely predicts the neutral point and its upstream displacement with increasing forcing amplitudes. Nonlinear interactions, indicated by the higher harmonics seem not to affect the accuracy of the prediction. The decay phase of the stability wave is well predicted up to a streamwise location where the Reynolds stresses of the fundamental wave change their sign. The study suggests that the flow model fails once energy is transferred from the coherent fluctuations back to the mean flow.

The study gives credibility to mean flow stability analyses conducted recently, such as on the cylinder wake vortex shedding[1,4], on the vortex breakdown bubble[9], or on turbulent jets[3,6]. Inaccuracies of the model due to the interaction of the mean flow with the first harmonic, as observed for the cavity flow[11], have not been observed presently.

2.2. Turbulent swirling jets

The nearfield of a turbulent swirling jet is investigated at a Reynolds number of $\mathrm{Re}_D = 20000$. Four different swirl intensities were investigated, ranging from zero swirl to intensities that are just below the onset of vortex breakdown. The time-averaged flow field was measured via PIV and fed into the stability solver. The computed eigenmodes were then compared to hot-wire measurements that were phase-locked to low amplitude external forcing

Two excitation modes were considered. First the flow was excited a single frequency for different azimuthal modes. The phase-averaged velocities were then compared to the corresponding mean flow eigenmodes. This investigation reveals that the growth rate of the excited wave train is very well predicted of one accounts for the coherent-turbulent interactions (eddy viscosity model)[6].

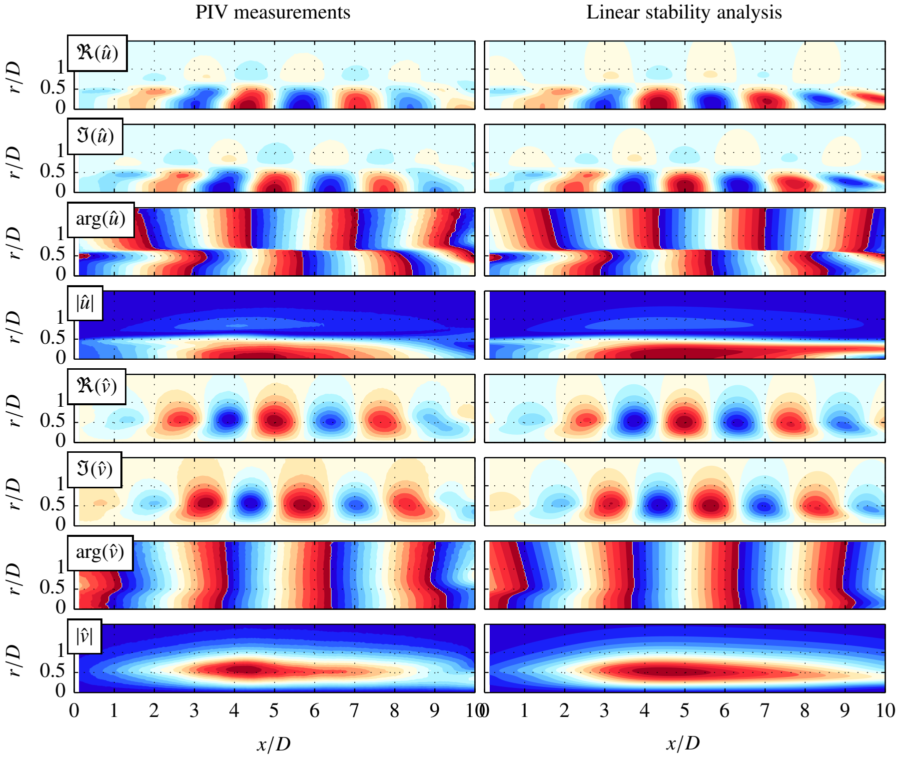


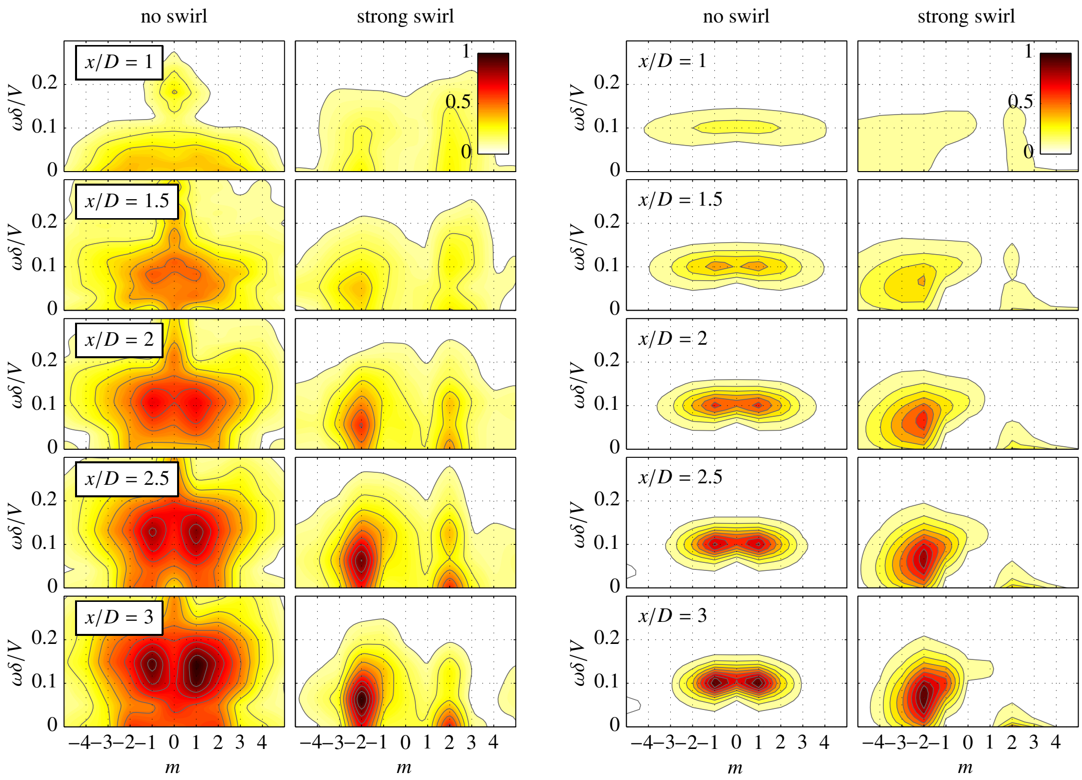
Fig. 1. Laminar axisymmetric jet forced at 5 % of the mean flow rate: Overall comparison of the flow oscillations measured via PIV (left) and the perturbation field obtained from the linear stability analysis (right). Displayed are the real and imaginary part, phase, and magnitude of the streamwise and radial component of the velocity fluctuations at the fundamental frequency. Nomenclature: streamwise coordinate x; radial coordinate r; nozzle diameter D. See Oberleithner et al.[7] for further information.

In a second experiment, the flow was excited by a time- and space-discrete pulse localized at the nozzle lip. The streamwise development of the wavepacket that was generated by this pulse was tracked by means of ensemble-averaged hot-wire measurements. Figure 2 shows results from these measurements. The modal content of the wavepacket measured at different streamwise positions is compared to the prediction from mean flow stability analysis. Results are shown for the non-swirling jet and the swirling jet. Evidently, the stability wave model is capably to predict the mode selection that occurs naturally in the flow for both, the swirling and non-swirling jet. The figure reveals that the non-swirling jet prefers the axisymmetric mode near the nozzle and the bending single-helical modes ($m = \pm 1$)further downstream. In contrast, the swirling jet prefers helical modes everywhere with strong amplification of weakly rotating and steady modes ($\omega = 0$) further downstream.

2.3. Reacting turbulent swirling jets

This study focuses on the formation of coherent flow structures in reacting combustor flows and its impact on the global heat release rate fluctuations. We consider a perfectly premixed swirl-stabilized flame that is acoustically forced axially mimicking the dynamics associated with thermoacoustic instability. Phase-averaged flow field and OH^*-chemiluminescence measurements show that heat release fluctuations are generated through the roll-up of the flame tip due to large-scale coherent flow structures imping the flame front. Details to the flame shape and dynamics are given in Oberleithner et al.[8].

The flame's response to the forcing is first characterized by a flame describing function (FDF) by essentially treating the flow field and the flame in a black box manner[5]. We obtain the global heat release rate fluctuations from a photomultiplier behind a OH^* filter. The left frame of figure 3 shows such a FDF measured in the laboratory at the TU Berlin.



(a) Hot-wire measurements (b) Linear stability analysis

Fig. 2. Modal amplitude distribution of a wavepacket traveling in the shear layer of a turbulent non-swirling and swirling jet. Swirl tends to break symmetry and destabilizes modes at low and zero frequencies (steady modes). Nomenclature: azimuthal wave number m; frequency ω; axial shear layer thickness δ; bulk velocity V; streamwise coordinate x; radial coordinate r; nozzle diameter D. See Oberleithner et al.[6] for further information.

The FDF is the main empirical input for thermoacoutic network models. However, the measurement of FDFs is very costly and time consuming and the demand for FDF prediction models is very high in the gas turbine industry. In order to develop prediction models we need to depart from this FDF-type black-box approach and develop specific flow and flame models. We suggest to employ hydrodynamic linear stability analysis based on the mean flow, which provides detailed information about the formation of the coherent flow structures excited by the inflow perturbations. In analogy to the FDF, the analytic approach provides a flow describing function, which characterizes the flow's response to the inflow perturbations.

The right frame in figure 3 shows such a flow transfer function derived from a local linear stability analysis. It reveals that the receptivity of the shear flow is highest for the unforced case with a significant dependence of the gain on the perturbation frequency. This dependence is also found in the FDF revealing that the maximum gain in the flame response is associated with maximum growth of the shear layer instability. With stronger forcing, the receptivity of the shear layer decreases and so does the gain in the corresponding FDF. At sufficiently strong forcing, the shear layer is saturated and the inflow perturbations are transported to the flame front without any amplification. A further discussion of the results is given in Oberleithner et al.[8]

3. Conclusion

The different studies mentioned here demonstrate the high accuracy and great potential of linear hydrodynamic stability analysis applied to time-averaged turbulent flows as encountered in many experimental and industrial envi-

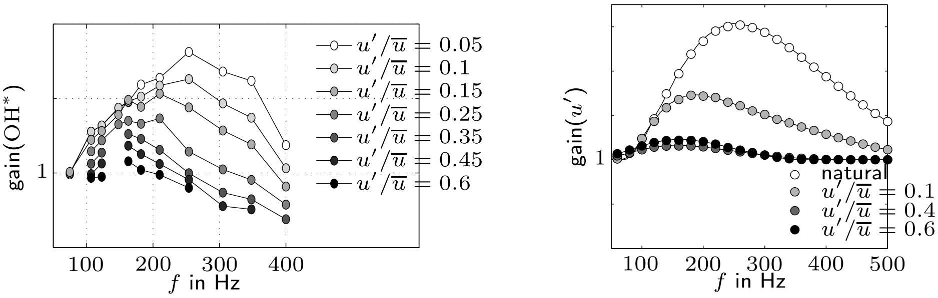


Fig. 3. left: Flame describing function (FDF) of a swirl-stabilized combustor. The graphs show the gain in the heat release rate fluctuations (OH^* emmision) versus the acoustic excitation frequency. Each graph corresponds to a different acoustic forcing amplitude. This figure discribes the saturation of the flame response at increasing excitation amplitude. right: Results from linear hydrodynamic stability analysis of the mean combustor flow. The graphs represent the gain of the axisymmetric Kelvin-Helmholtz mode that couples with the acoustic forcing. Apparently, the flow response shows the same qualitative behavior as the flame response suggesting that the flame's gain and saturation is directly linked to hydrodynamic instability.

ronments. The laminar jet experiments show that the type of interaction between the wave train and the mean flow is crucial for the linear model to hold. The turbulent jet experiments show that the interactions between the fine-scale structures and the modeled coherent structures must be accounted for accurate prediction of growth rates. The discussion of the swirling flame configuration shows an application of these flow models to a highly complex multi-shear-layered reacting flow, which demonstrates their robustness and ability to solve industry relevant problems.

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