Classes of Ordinary Differential Equations Obtained for the Probability Functions of Kumaraswamy Inverse Rayleigh Distribution

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***Abstract*— In this paper, differential calculus was used to obtain the ordinary differential equations (ODE) of the probability density function (PDF), Quantile function (QF), survival function (SF), inverse survival function (ISF), hazard function (HF) and reversed hazard function (RHF) of Kumaraswamy inverse Rayleigh distribution. The parameters and support that define the distribution inevitably determine the nature, existence, uniqueness and solution of the ODEs. The method can be extended to other probability distributions, functions and can serve an alternative to estimation and approximation. Computer codes and programs can be used for the implementation.**

***Index Terms*— Differentiation, quantile function, survival function, approximation, hazard function, Rayleigh.**

I. Introduction

CALCULUS is a very key tool in the determination of mode of a given probability distribution and in estimation of parameters of probability distributions, amongst other uses. The method of maximum likelihood is an example.

Differential equations often arise from the understanding and modeling of real life problems or some observed physical phenomena. Approximations of probability functions are one of the major areas of application of calculus and ordinary differential equations in mathematical statistics. The approximations are helpful in the recovery of the probability functions of complex distributions [1-6].

Apart from mode estimation, parameter estimation and approximation, probability density function (PDF) of probability distributions can be expressed as ODE whose solution is the PDF. Some of which are available. They include: beta distribution [7], Lomax distribution [8], beta prime distribution [9], Laplace distribution [10] and raised cosine distribution [11].

The aim of this research is to develop homogenous ordinary differential equations for the probability density function (PDF), Quantile function (QF), survival function (SF), inverse survival function (ISF), hazard function (HF) and reversed hazard function (RHF) of Kumaraswamy inverse Rayleigh distribution. This will also help to provide the answers as to whether there are discrepancies between the support of the distribution and the necessary conditions for the existence of the ODEs. Similar results for other distributions have been proposed, see [12-24] for details.

Kumaraswamy inverse Rayleigh distribution is a submodel of the Kumaraswamy-inverse Weibull distribution proposed by [25]. The distribution was later proposed explicitly by Roges [26]. The Estimation of the parameters of the distribution under certain conditions was done by [27] and [28]. The boundary conditions of the supports of the distribution are similar to the Kumaraswamy distribution [29]. The distribution has been extended to Kumaraswamy exponentiated inverse Rayleigh distribution by [30].

The ordinary differential calculus was used to obtain the results.

II. Probability Density Function

The probability density function (PDF) of the Kumaraswamy inverse Rayleigh distribution is given by;

$$f(x)=\frac{2bc^2}{x^3}\mathrm{e}^{-\left(\frac{c}{x}\right)^2}(1-\mathrm{e}^{-\left(\frac{c}{x}\right)^2})^{b-1} \qquad (1)$$

When b =1, the PDF reduces to the PDF of the inverse Rayleigh distribution.

Differentiate equation (2), to obtain;

$$f'(x)=\left\{-\frac{3x^{-4}}{x^{-3}}+\frac{\frac{2c^2}{x^3}\mathrm{e}^{-\left(\frac{c}{x}\right)^2}}{\mathrm{e}^{-\left(\frac{c}{x}\right)^2}}-\frac{\frac{2c^2(b-1)}{x^3}\mathrm{e}^{-\left(\frac{c}{x}\right)^2}(1-\mathrm{e}^{-\left(\frac{c}{x}\right)^2})^{b-2}}{(1-\mathrm{e}^{-\left(\frac{c}{x}\right)^2})^{b-1}}\right\}f(x) \qquad (2)$$

The condition necessary for the existence of the equation is $x,b,c>0$.

$$f'(x)=\left\{-\frac{3}{x}+\frac{2c^2}{x^3}-\frac{2c^2(b-1)\mathrm{e}^{-\left(\frac{c}{x}\right)^2}}{x^3(1-\mathrm{e}^{-\left(\frac{c}{x}\right)^2})}\right\}f(x) \qquad (3)$$

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The second derivative is obtained

$$f''(x)=\left\{-\frac{3}{x}+\frac{2c^2}{x^3}-\frac{2c^2(b-1)\mathrm{e}^{-\left(\frac{c}{x}\right)^2}}{x^3(1-\mathrm{e}^{-\left(\frac{c}{x}\right)^2})}\right\}f'(x)$$

$$+\left(\frac{3}{x^2}-\frac{6c^2}{x^4}\right)f(x)-\left\{\begin{matrix}-\frac{4c^4(b-1)(\mathrm{e}^{-\left(\frac{c}{x}\right)^2})^2}{x^6(1-\mathrm{e}^{-\left(\frac{c}{x}\right)^2})^2}\\ -\frac{6c^2(b-1)\mathrm{e}^{-\left(\frac{c}{x}\right)^2}}{x^4(1-\mathrm{e}^{-\left(\frac{c}{x}\right)^2})}\\ -\frac{4c^4(b-1)\mathrm{e}^{-\left(\frac{c}{x}\right)^2}}{x^6(1-\mathrm{e}^{-\left(\frac{c}{x}\right)^2})}\end{matrix}\right\}f(x) \quad (4)$$

The condition necessary for the existence of the equation is $x,b,c>0.$

The following equations obtained from equation (3) are required to simplify equation (4);

$$\left\{-\frac{3}{x}+\frac{2c^2}{x^3}-\frac{2c^2(b-1)\mathrm{e}^{-\left(\frac{c}{x}\right)^2}}{x^3(1-\mathrm{e}^{-\left(\frac{c}{x}\right)^2})}\right\}=\frac{f'(x)}{f(x)} \quad (5)$$

$$\frac{2c^2(b-1)\mathrm{e}^{-\left(\frac{c}{x}\right)^2}}{x^3(1-\mathrm{e}^{-\left(\frac{c}{x}\right)^2})}=\frac{2c^2}{x^3}-\frac{3}{x}-\frac{f'(x)}{f(x)} \quad (6)$$

$$\left(\frac{2c^2(b-1)\mathrm{e}^{-\left(\frac{c}{x}\right)^2}}{x^3(1-\mathrm{e}^{-\left(\frac{c}{x}\right)^2})}\right)^2=\left(\frac{2c^2}{x^3}-\frac{3}{x}-\frac{f'(x)}{f(x)}\right)^2 \quad (7)$$

$$\frac{4c^4(b-1)(\mathrm{e}^{-\left(\frac{c}{x}\right)^2})^2}{x^6(1-\mathrm{e}^{-\left(\frac{c}{x}\right)^2})^2}=\frac{1}{b-1}\left(\frac{2c^2}{x^3}-\frac{3}{x}-\frac{f'(x)}{f(x)}\right)^2 \quad (8)$$

$$\frac{6c^2(b-1)\mathrm{e}^{-\left(\frac{c}{x}\right)^2}}{x^4(1-\mathrm{e}^{-\left(\frac{c}{x}\right)^2})}=\frac{3}{x}\left(\frac{2c^2}{x^3}-\frac{3}{x}-\frac{f'(x)}{f(x)}\right) \quad (9)$$

$$\frac{4c^4(b-1)\mathrm{e}^{-\left(\frac{c}{x}\right)^2}}{x^3(1-\mathrm{e}^{-\left(\frac{c}{x}\right)^2})}=2c^2\left(\frac{2c^2}{x^3}-\frac{3}{x}-\frac{f'(x)}{f(x)}\right) \quad (10)$$

$$\frac{4c^4(b-1)\mathrm{e}^{-\left(\frac{c}{x}\right)^2}}{x^3(1-\mathrm{e}^{-\left(\frac{c}{x}\right)^2})}=\frac{2c^2}{x^3}\left(\frac{2c^2}{x^3}-\frac{3}{x}-\frac{f'(x)}{f(x)}\right) \quad (11)$$

Substitute equations (5), (6), (7) and (11) into equation (4);

$$f''(x)=\frac{f'^2(x)}{f(x)}+\left(\begin{matrix}\frac{3}{x^2}-\frac{6c^2}{x^4}\\ +\frac{1}{b-1}\left(\frac{2c^2}{x^3}-\frac{3}{x}-\frac{f'(x)}{f(x)}\right)^2\end{matrix}\right)f(x)$$

$$+\left\{\begin{matrix}\frac{3}{x}\left(\frac{2c^2}{x^3}-\frac{3}{x}-\frac{f'(x)}{f(x)}\right)+\\ \frac{2c^2}{x^3}\left(\frac{2c^2}{x^3}-\frac{3}{x}-\frac{f'(x)}{f(x)}\right)\end{matrix}\right\}f(x) \quad (12)$$

The required differential equations are computed based on the given parameters.

III. Quantile Function

The Quantile function (QF) of the Kumaraswamy inverse Rayleigh distribution is given by;

$$Q(p)=\frac{c}{\sqrt{\ln\left(\frac{1}{1-(1-p)^{\frac{1}{b}}}\right)}} \quad (13)$$

Differentiate equation (13), to obtain;

$$Q'(p)=-\frac{c(1-p)^{\frac{1}{b}-1}}{2b(1-(1-p)^{\frac{1}{b}})\left(\sqrt{\ln\left(\frac{1}{1-(1-p)^{\frac{1}{b}}}\right)}\right)^3} \quad (14)$$

The condition necessary for the existence of the equation is $b,c>0, 0<p<1.$

Equation (14) is simplified as follows;

$$\sqrt{\ln\left(\frac{1}{1-(1-p)^{\frac{1}{b}}}\right)}=\frac{c}{Q(p)} \quad (15)$$

$$\left(\sqrt{\ln\left(\frac{1}{1-(1-p)^{\frac{1}{b}}}\right)}\right)^3=\frac{c^3}{Q^3(p)} \quad (16)$$

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Substitute equation (16) into equation (14) to obtain;

$$Q'(p) = -\frac{(1-p)^{\frac{1}{b}-1}Q^3(p)}{2bc^2(1-(1-p)^{\frac{1}{b}})}$$

(17)

$$2bc^2(1-(1-p)^{\frac{1}{b}})Q'(p) + (1-p)^{\frac{1}{b}-1}Q^3(p) = 0 \quad (18)$$

The ordinary differential equations can only be obtained for the particular values of the parameters. Some cases considered are shown in **Table 1**.

Table 1: Classes of differential equations obtained for the quantile function of Kumaraswamy inverse Rayleigh distribution for different parameters

b	c	ordinary differential equation
1	1	$2pQ'(p)+Q^3(p)=0$
1	2	$8pQ'(p)+Q^3(p)=0$
1	3	$18pQ'(p)+Q^3(p)=0$
2	1	$4(\sqrt{1-p})(1-\sqrt{1-p})Q'(p)+Q^3(p)=0$
2	2	$16(\sqrt{1-p})(1-\sqrt{1-p})Q'(p)+Q^3(p)=0$
2	3	$36(\sqrt{1-p})(1-\sqrt{1-p})Q'(p)+Q^3(p)=0$

IV. SURVIVAL FUNCTION

The survival function (SF) of the Kumaraswamy inverse Rayleigh distribution is given by;

$$S(t) = (1-\mathrm{e}^{-\left(\frac{c}{t}\right)^2})^b \quad (19)$$

Differentiate equation (19), to obtain;

$$S'(t) = \frac{2bc^2}{t^3}\mathrm{e}^{-\left(\frac{c}{t}\right)^2}(1-\mathrm{e}^{-\left(\frac{c}{t}\right)^2})^{b-1} \quad (20)$$

The condition necessary for the existence of the equation is $t, b, c > 0$.

Substitute equation (19) into equation (20);

$$S'(t) = \frac{2bc^2\,\mathrm{e}^{-\left(\frac{c}{t}\right)^2}S(t)}{t^3(1-\mathrm{e}^{-\left(\frac{c}{t}\right)^2})} \quad (21)$$

Equation (19) is simplified as follows;

$$1-\mathrm{e}^{-\left(\frac{c}{t}\right)^2} = S^{\frac{1}{b}}(t) \quad (22)$$

$$\mathrm{e}^{-\left(\frac{c}{t}\right)^2} = 1 - S^{\frac{1}{b}}(t) \quad (23)$$

Substitute equations (22) and (23) into equation (21);

$$S'(t) = \frac{2bc^2(1-S^{\frac{1}{b}}(t))S(t)}{t^3S^{\frac{1}{b}}(t)} \quad (24)$$

$$S'(t) = \frac{2bc^2(S^{1-\frac{1}{b}}(t) - S(t))}{t^3} \quad (25)$$

$$t^3S'(t) - 2bc^2(S^{1-\frac{1}{b}}(t) - S(t)) = 0 \quad (26)$$

$$S(1) = (1-\mathrm{e}^{-c^2})^b \quad (27)$$

The first order ordinary differential equation of the survival function of the Kumaraswamy inverse Rayleigh distribution is given can be obtained for the particular values of the parameters b and c.

V. INVERSE SURVIVAL FUNCTION

The inverse survival function (ISF) of the Kumaraswamy inverse Rayleigh distribution is given by;

$$Q(p) = \frac{c}{\sqrt{\ln\left(\frac{1}{1-p^{\frac{1}{b}}}\right)}} \quad (28)$$

Differentiate equation (28), to obtain;

$$Q'(p) = \frac{cp^{\frac{1}{b}-1}}{2b(1-p^{\frac{1}{b}})\left(\sqrt{\ln\left(\frac{1}{1-p^{\frac{1}{b}}}\right)}\right)^3} \quad (29)$$

The condition necessary for the existence of the equation is $b, c > 0, 0 < p < 1$.

Equation (28) is simplified as follows;

$$\sqrt{\ln\left(\frac{1}{1-p^{\frac{1}{b}}}\right)} = \frac{c}{Q(p)} \quad (30)$$

$$\left(\sqrt{\ln\left(\frac{1}{1-p^{\frac{1}{b}}}\right)}\right)^3 = \frac{c^3}{Q^3(p)} \quad (31)$$

Substitute equation (31) into equation (29) to obtain;

$$Q'(p) = \frac{p^{\frac{1}{b}-1}Q^3(p)}{2bc^2(1-p^{\frac{1}{b}})} \quad (32)$$

$$2bc^2(1-p^{\frac{1}{b}})Q'(p) - p^{\frac{1}{b}-1}Q^3(p) = 0 \quad (33)$$

The ordinary differential equations can only be obtained for the particular values of the parameters. Some cases considered are shown in **Table 2.**

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Table 2: Classes of differential equations obtained for the inverse survival function of Kumaraswamy inverse Rayleigh distribution for different parameters

b	c	ordinary differential equation
1	1	$2(1-p)Q'(p)-Q^3(p)=0$
1	2	$8(1-p)Q'(p)-Q^3(p)=0$
2	1	$4(\sqrt{p}-p)Q'(p)-Q^3(p)=0$
2	2	$16(\sqrt{p}-p)Q'(p)-Q^3(p)=0$

VI. Hazard Function

The hazard function (HF) of the Kumaraswamy inverse Rayleigh distribution is given by;

$$h(t)=\frac{2bc^2\,e^{-\left(\frac{c}{t}\right)^2}}{t^3(1-e^{-\left(\frac{c}{t}\right)^2})} \quad (34)$$

Differentiate equation (34), to obtain;

$$h'(t)=\left\{-\frac{\frac{2c^2}{t^3}e^{-\left(\frac{c}{t}\right)^2}}{e^{-\left(\frac{c}{t}\right)^2}}-\frac{3t^{-4}}{t^{-3}}-\frac{\frac{2c^2}{t^3}e^{-\left(\frac{c}{t}\right)^2}(1-e^{-\left(\frac{c}{t}\right)^2})^{-2}}{(1-e^{-\left(\frac{c}{t}\right)^2})^{-1}}\right\}h(t) \quad (35)$$

$$h'(t)=-\left\{\frac{2c^2}{t^3}+\frac{3}{t}+\frac{2c^2\,e^{-\left(\frac{c}{t}\right)^2}}{t^3(1-e^{-\left(\frac{c}{t}\right)^2})}\right\}h(t) \quad (36)$$

The condition necessary for the existence of the equation is $t,b,c>0$.

$$h'(t)=-\left\{\frac{2c^2}{t^3}+\frac{3}{t}+\frac{h(t)}{b}\right\}h(t) \quad (37)$$

The first order ordinary differential equation of the hazard function of the Kumaraswamy inverse Rayleigh distribution is given as;

$$bt^3h'(t)+b(2c^2+3t^2)h(t)+t^3h(t)=0 \quad (38)$$

$$h(1)=\frac{2bc^2\,e^{-c^2}}{t^3(1-e^{-c^2})}=\frac{2bc^2}{t^3(e^{c^2}-1)} \quad (39)$$

VII. Reversed Hazard Function

The reversed hazard function (RHF) of the Kumaraswamy inverse Rayleigh distribution is given by;

$$j(t)=\frac{\frac{2bc^2}{t^3}e^{-\left(\frac{c}{t}\right)^2}(1-e^{-\left(\frac{c}{t}\right)^2})^{b-1}}{1-(1-e^{-\left(\frac{c}{t}\right)^2})^b}$$

(40) Differentiate equation (40), to obtain;

$$j'(t)=\left\{-\frac{3t^{-4}}{t^{-3}}+\frac{\frac{2c^2}{t^3}e^{-\left(\frac{c}{t}\right)^2}}{e^{-\left(\frac{c}{t}\right)^2}}-\frac{\frac{2c^2(b-1)}{t^3}e^{-\left(\frac{c}{t}\right)^2}(1-e^{-\left(\frac{c}{t}\right)^2})^{b-2}}{(1-e^{-\left(\frac{c}{t}\right)^2})^{b-1}}-\frac{\frac{2bc^2}{t^3}e^{-\left(\frac{c}{t}\right)^2}(1-e^{-\left(\frac{c}{t}\right)^2})^{b-1}(1-(1-e^{-\left(\frac{c}{t}\right)^2})^b)^{-2}}{(1-(1-e^{-\left(\frac{c}{t}\right)^2})^b)^{-1}}\right\}j(t)$$

(41)

$$j'(t)=\left\{-\frac{3}{t}+\frac{2c^2}{t^3}-\frac{\frac{2c^2(b-1)}{t^3}e^{-\left(\frac{c}{t}\right)^2}}{(1-e^{-\left(\frac{c}{t}\right)^2})}-\frac{\frac{2bc^2}{t^3}e^{-\left(\frac{c}{t}\right)^2}(1-e^{-\left(\frac{c}{t}\right)^2})^{b-1}}{(1-(1-e^{-\left(\frac{c}{t}\right)^2})^b)}\right\}j(t)$$

(42)

The condition necessary for the existence of the equation is $t,b,c>0$.

$$j'(t)=\left\{-\frac{3}{t}+\frac{2c^2}{t^3}-\frac{\frac{2c^2(b-1)}{t^3}e^{-\left(\frac{c}{t}\right)^2}}{(1-e^{-\left(\frac{c}{t}\right)^2})}-j(t)\right\}j(t)$$

(43)

The second derivative is obtained

$$j''(t)=\left\{-\frac{3}{t}+\frac{2c^2}{t^3}-\frac{2c^2(b-1)\mathrm{e}^{-\left(\frac{c}{t}\right)^2}}{t^3(1-\mathrm{e}^{-\left(\frac{c}{t}\right)^2})}-j(t)\right\}j'(t)$$
$$+\left(\frac{3}{t^2}-\frac{6c^2}{t^4}-j'(t)\right)j(t)$$
$$-\left\{\begin{array}{l}-\frac{4c^4(b-1)(\mathrm{e}^{-\left(\frac{c}{t}\right)^2})^2}{t^6(1-\mathrm{e}^{-\left(\frac{c}{t}\right)^2})^2}\\ -\frac{6c^2(b-1)\mathrm{e}^{-\left(\frac{c}{t}\right)^2}}{t^4(1-\mathrm{e}^{-\left(\frac{c}{t}\right)^2})}-\frac{4c^4(b-1)\mathrm{e}^{-\left(\frac{c}{t}\right)^2}}{t^6(1-\mathrm{e}^{-\left(\frac{c}{t}\right)^2})}\end{array}\right\}j(t) \quad (44)$$

The condition necessary for the existence of the equation is $t,b,c>0$.

The following equations obtained from equation (43) are required to simplify equation (44);

$$\left\{-\frac{3}{t}+\frac{2c^2}{t^3}-\frac{2c^2(b-1)\mathrm{e}^{-\left(\frac{c}{t}\right)^2}}{t^3(1-\mathrm{e}^{-\left(\frac{c}{t}\right)^2})}-j(t)\right\}=\frac{j'(t)}{j(t)} \quad (45)$$

$$\frac{2c^2(b-1)\mathrm{e}^{-\left(\frac{c}{t}\right)^2}}{t^3(1-\mathrm{e}^{-\left(\frac{c}{t}\right)^2})}=\frac{2c^2}{t^3}-\frac{3}{t}-\frac{j'(t)}{j(t)}+j(t) \quad (46)$$

$$\left(\frac{2c^2(b-1)\mathrm{e}^{-\left(\frac{c}{t}\right)^2}}{t^3(1-\mathrm{e}^{-\left(\frac{c}{t}\right)^2})}\right)^2=\left(\frac{2c^2}{t^3}-\frac{3}{t}-\frac{j'(t)}{j(t)}+j(t)\right)^2 \quad (47)$$

$$\frac{4c^4(b-1)(\mathrm{e}^{-\left(\frac{c}{t}\right)^2})^2}{t^6(1-\mathrm{e}^{-\left(\frac{c}{t}\right)^2})^2}=\frac{1}{b-1}\left(\frac{2c^2}{t^3}-\frac{3}{t}-\frac{j'(t)}{j(t)}+j(t)\right)^2 \quad (48)$$

$$\frac{6c^2(b-1)\mathrm{e}^{-\left(\frac{c}{t}\right)^2}}{t^4(1-\mathrm{e}^{-\left(\frac{c}{t}\right)^2})}=\frac{3}{t}\left(\frac{2c^2}{t^3}-\frac{3}{t}-\frac{j'(t)}{j(t)}+j(t)\right) \quad (49)$$

$$\frac{4c^4(b-1)\mathrm{e}^{-\left(\frac{c}{t}\right)^2}}{t^3(1-\mathrm{e}^{-\left(\frac{c}{t}\right)^2})}=2c^2\left(\frac{2c^2}{t^3}-\frac{3}{t}-\frac{j'(t)}{j(t)}+j(t)\right) \quad (50)$$

$$\frac{4c^4(b-1)\mathrm{e}^{-\left(\frac{c}{t}\right)^2}}{t^3(1-\mathrm{e}^{-\left(\frac{c}{t}\right)^2})}=\frac{2c^2}{t^3}\left(\frac{2c^2}{t^3}-\frac{3}{t}-\frac{j'(t)}{j(t)}+j(t)\right) \quad (51)$$

Substitute equations (45), (48), (49) and (51) into equation (44);

$$j''(t)=\frac{j'^2(t)}{j(t)}+\left(\begin{array}{l}\frac{3}{t^2}-\frac{6c^2}{t^4}-j'(t)\\ +\frac{1}{b-1}\left(\frac{2c^2}{t^3}-\frac{3}{t}-\frac{j'(t)}{j(t)}+j(t)\right)^2\end{array}\right)j(t)$$
$$+\left\{\begin{array}{l}\frac{3}{t}\left(\frac{2c^2}{t^3}-\frac{3}{t}-\frac{j'(t)}{j(t)}+j(t)\right)\\ +\frac{2c^2}{t^3}\left(\frac{2c^2}{t^3}-\frac{3}{t}-\frac{j'(t)}{j(t)}+j(t)\right)\end{array}\right\}j(t) \quad (52)$$

The condition necessary for the existence of the equation is $t,c>0,b>1$.

The required differential equations are computed based on the given parameters.

VIII. Concluding Remarks

Ordinary differential equations (ODEs) has been obtained for the probability density function (PDF), Quantile function (QF), survival function (SF), inverse survival function (ISF), hazard function (HF) and reversed hazard function (RHF) of Kumaraswamy inverse Rayleigh distribution. This differential calculus and efficient algebraic simplifications were used to derive the various classes of the ODEs. The parameter and the supports that characterize the Kumaraswamy inverse Rayleigh distribution determine the nature, existence, orientation and uniqueness of the ODEs. The results are in agreement with those available in scientific literature. Furthermore several methods can be used to obtain desirable solutions to the ODEs [31-41]. This method of characterizing distributions cannot be applied to distributions whose PDF or CDF are either not differentiable or the domain of the support of the distribution contains singular points.

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