Classes of Ordinary Differential Equations Obtained for the Probability Functions of inverse Rayleigh Distribution

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***Abstract*— Differential calculus was used to obtain the ordinary differential equations (ODE) of the probability density function (PDF), Quantile function (QF), survival function (SF), inverse survival function (ISF), hazard function (HF) and reversed hazard function (RHF) of inverse Rayleigh distribution. The parameters and support that define the distribution inevitably determine the nature, existence, uniqueness and solution of the ODEs. The method can be extended to other probability distributions, functions and can serve an alternative to estimation and approximation. Computer codes and programs can be used for the implementation.**

***Index Terms*— Differentiation, quantile function, survival function, approximation, hazard function, Rayleigh.**

I. Introduction

CALCULUS is a very key tool in the determination of mode of a given probability distribution and in estimation of parameters of probability distributions, amongst other uses. The method of maximum likelihood is an example.

Differential equations often arise from the understanding and modeling of real life problems or some observed physical phenomena. Approximations of probability functions are one of the major areas of application of calculus and ordinary differential equations in mathematical statistics. The approximations are helpful in the recovery of the probability functions of complex distributions [1-10].

Apart from mode estimation, parameter estimation and approximation, probability density function (PDF) of probability distributions can be expressed as ODE whose solution is the PDF. Some of which are available. They include: beta distribution [11], Lomax distribution [12], beta prime distribution [13], Laplace distribution [14] and raised cosine distribution [15].

The aim of this research is to develop homogenous ordinary differential equations for the probability density function (PDF), Quantile function (QF), survival function (SF), inverse survival function (ISF), hazard function (HF) and reversed hazard function (RHF) of inverse Rayleigh distribution. This will also help to provide the answers as to whether there are discrepancies between the support of the distribution and the necessary conditions for the existence of the ODEs. Similar results for other distributions have been proposed, see [16-28] for details.

Inverse or inverted Raylegh distribution was earlier studied by [29]. Estimation is one of the areas of the distribution that have been studied extensively. The details can be seen in the research outputs of [30-36]. In particular, emphasis was placed on comparison of the efficiency of different estimators [37-39]. Estimation under censoring features prominently in the works of [40-42]. Prakash [43] work was based strictly on Bayes estimation of the parameters of the distribution. Other areas already explored are acceptance sampling based on the distribution [44-45]. Rosaiah et al. [46] applied the distribution to economic reliability analysis. Recently a new approach of correctly estimating the probability density function (PDF) and cumulative distribution function (CDF) of the distribution was proposed by [47]. Process capability and system availability analysis of the distribution was described by [48]. Generalizations, compounding and modifications include: Bivariate inverse Rayleigh distribution [49], beta inverse Rayleigh distribution [50], discrete inverse Rayleigh distribution [51], transmuted inverse Rayleigh distribution [52], transmuted modified inverse Rayleigh distribution [53], modified inverse Rayleigh distribution [54], Kumaraswamy inverse Rayleigh distribution [55], mixture of inverse Rayleigh distribution [56] and others.

The ordinary differential calculus was used to obtain the results.

II. Probability Density Function

The probability density function (PDF) of the inverse Rayleigh distribution is given by;

$$f(x) = \frac{2\theta}{x^3} e^{-\frac{\theta}{x^2}} \qquad (1)$$

Differentiate equation (1), to obtain;

$$f'(x) = \left\{ -\frac{3x^{-4}}{x^{-3}} - \frac{2\theta e^{-\frac{\theta}{x^2}}}{x^3 e^{-\frac{\theta}{x^2}}} \right\} f(x) \qquad (2)$$

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$$f'(x) = -\left(\frac{3}{x} + \frac{2\theta}{x^3}\right) f(x) \quad (3)$$

The condition necessary for the existence of the equation is $x, \theta > 0.$

The first order ordinary differential equation of the probability density function of the inverse Rayleigh distribution is given as;

$$x^3 f'(x) + (3x^2 + 2\theta) f(x) = 0 \quad (4)$$

$$f(1) = 2\theta e^{-\theta} \quad (5)$$

III. Quantile Function

The Quantile function (QF) of the inverse Rayleigh distribution is given by;

$$Q(p) = \sqrt{\frac{\theta}{\ln\left(\frac{1}{p}\right)}} \quad (6)$$

Differentiate equation (6), to obtain;

$$Q'(p) = \frac{\theta}{2p\left(\ln\left(\frac{1}{p}\right)\right)^2 \sqrt{\frac{\theta}{\ln\left(\frac{1}{p}\right)}}} \quad (7)$$

The condition necessary for the existence of the equation is $\theta > 0, 0 < p < 1.$

Substitute equation (6) into (7);

$$Q'(p) = \frac{\theta}{2p\left(\ln\left(\frac{1}{p}\right)\right)^2 Q(p)} \quad (8)$$

Equation (6) can also be written as;

$$Q^2(p) = \frac{\theta}{\ln\left(\frac{1}{p}\right)} \Rightarrow \ln\left(\frac{1}{p}\right) = \frac{\theta}{Q^2(p)} \quad (9)$$

$$\left(\ln\left(\frac{1}{p}\right)\right)^2 = \frac{\theta^2}{Q^4(p)} \quad (10)$$

Substitute equation (10) into (8);

$$Q'(p) = \frac{Q^3(p)}{2\theta p} \quad (11)$$

The first order ordinary differential equation of the Quantile function of the inverse Rayleigh distribution is given as;

$$2\theta p Q'(p) - Q^3(p) = 0 \quad (12)$$

$$Q(0.1) = \sqrt{\frac{\theta}{\ln 10}} = 0.659\sqrt{\theta} \quad (13)$$

IV. Survival Function

The Survival function (SF) of the inverse Rayleigh distribution is given by;

$$S(t) = 1 - \mathrm{e}^{-\frac{\theta}{t^2}} \quad (14)$$

Differentiate equation (14), to obtain;

$$S'(t) = \frac{2\theta}{t^3} \mathrm{e}^{-\frac{\theta}{t^2}} \quad (15)$$

The condition necessary for the existence of the equation is $t, \theta > 0.$

Substitute equation (14) into (15);

$$S'(t) = \frac{2\theta}{t^3}(1 - S(t)) \quad (16)$$

The first order ordinary differential equation of the survival function of the inverse Rayleigh distribution is given as;

$$t^3 S'(t) + 2\theta S(t) - 2\theta = 0 \quad (17)$$

$$S(1) = 1 - \mathrm{e}^{-\theta} \quad (18)$$

V. Inverse Survival Function

The inverse survival function (ISF) of the inverse Rayleigh distribution is given by;

$$Q(p) = \sqrt{\frac{\theta}{\ln\left(\frac{1}{1-p}\right)}} \quad (19)$$

Differentiate equation (19), to obtain;

$$Q'(p) = -\frac{\theta}{2(1-p)} \sqrt{\frac{\theta}{\left(\ln\left(\frac{1}{1-p}\right)\right)^3}} \quad (20)$$

The condition necessary for the existence of the equation is $\theta > 0, 0 < p < 1.$

Equation (19) can also be written as;

$$Q^2(p) = \frac{\theta}{\ln\left(\frac{1}{1-p}\right)} \Rightarrow \ln\left(\frac{1}{1-p}\right) = \frac{\theta}{Q^2(p)} \quad (21)$$

$$\left(\ln\left(\frac{1}{1-p}\right)\right)^3 = \frac{\theta^3}{Q^6(p)} \quad (22)$$

Substitute equation (22) into (20);

$$Q'(p) = -\frac{Q^3(p)}{2\theta(1-p)} \quad (23)$$

The first order ordinary differential equation of the inverse survival function of the inverse Rayleigh distribution is given as;

$$2\theta(1-p)Q'(p) + Q^3(p) = 0 \quad (24)$$

$$Q(0.11) = \sqrt{\frac{\theta}{0.116533}} \quad (25)$$

VI. Hazard Function

The hazard function (HF) of the inverse Rayleigh distribution is given by;

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$$h(t)=\frac{2\theta\,\mathrm{e}^{-\frac{\theta}{t^2}}}{t^3(1-\mathrm{e}^{-\frac{\theta}{t^2}})} \quad (26)$$

Differentiate equation (26), to obtain;

$$h'(t)=\left\{-\frac{2\theta\,\mathrm{e}^{-\frac{\theta}{t^2}}}{t^3\,\mathrm{e}^{-\frac{\theta}{t^2}}}-\frac{3t^{-4}}{t^{-3}}-\frac{2\theta\,\mathrm{e}^{-\frac{\theta}{t^2}}(1-\mathrm{e}^{-\frac{\theta}{t^2}})^{-2}}{t^3(1-\mathrm{e}^{-\frac{\theta}{t^2}})^{-1}}\right\}h(t) \quad (27)$$

$$h'(t)=-\left\{\frac{2\theta}{t^3}+\frac{3}{t}+\frac{2\theta\,\mathrm{e}^{-\frac{\theta}{t^2}}}{t^3(1-\mathrm{e}^{-\frac{\theta}{t^2}})}\right\}h(t) \quad (28)$$

The condition necessary for the existence of the equation is $t,\theta>0.$

$$h'(t)=-\left\{\frac{2\theta}{t^3}+\frac{3}{t}+h(t)\right\}h(t) \quad (29)$$

The first order ordinary differential equation of the hazard function of the inverse Rayleigh distribution is given as;

$$t^3h'(t)+t^3h^2(t)+(3t^2+2\theta)h(t)=0 \quad (30)$$

$$h(1)=\frac{2\theta\,\mathrm{e}^{-\theta}}{t^3(1-\mathrm{e}^{-\theta})}=\frac{2\theta}{t^3(\mathrm{e}^{\theta}-1)} \quad (31)$$

VII. Reversed Hazard Function

The reversed hazard function (RHF) of the inverse Rayleigh distribution is given by;

$$j(t)=\frac{2\theta}{t^3} \quad (32)$$

Differentiate equation (32), to obtain;

$$j'(t)=-\frac{6\theta}{t^4} \quad (33)$$

The condition necessary for the existence of the equation is $t,\theta>0.$

Substitute equation (32) into equation (33);

$$j'(t)=-\frac{3j(t)}{t} \quad (34)$$

The first order ordinary differential equation of the reversed hazard function of the inverse Rayleigh distribution is given as;

$$tj'(t)+3j(t)=0 \quad (35)$$

$$j(1)=2\theta \quad (36)$$

VIII. Concluding Remarks

Ordinary differential equations (ODEs) has been obtained for the probability density function (PDF), Quantile function (QF), survival function (SF), inverse survival function (ISF), hazard function (HF) and reversed hazard function (RHF) of inverse Rayleigh distribution. This differential calculus and efficient algebraic simplifications were used to derive the various classes of the ODEs. The parameter and the supports that characterize the inverse Rayleigh distribution determine the nature, existence, orientation and uniqueness of the ODEs. The results are in agreement with those available in scientific literature. Furthermore several methods can be used to obtain desirable solutions to the ODEs [57-62]. This method of characterizing distributions cannot be applied to distributions whose PDF or CDF are either not differentiable or the domain of the support of the distribution contains singular points.

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References

[1] W.T. Shaw, T. Luu and N. Brickman, “Quantile mechanics II: changes of variables in Monte Carlo methods and GPU-optimised normal quantiles,” Euro. J. Appl. Math., vol. 25, no. 2, pp. 177-212, 2014.
[2] G. Derflinger, W. Hörmann and J. Leydold, “Random variate generation by numerical inversion when only the density is known,” ACM Transac.Model. Comp. Simul., vol. 20, no. 4, Article 18, 2010.
[3] J. Leydold and W. Hörmann, “ Generating generalized inverse Gaussian random variates by fast inversion,” Comput. Stat. Data Analy., vol. 55, no. 1, pp. 213-217, 2011.
[4] G. Steinbrecher, G. and W.T. Shaw, “Quantile mechanics” Euro. J. Appl. Math., vol. 19, no. 2, pp. 87-112, 2008.
[5] H.I. Okagbue, M.O. Adamu and T.A. Anake “Quantile Approximation of the Chi-square Distribution using the Quantile Mechanics,” In Lecture Notes in Engineering and Computer Science: Proceedings of The World Congress on Engineering and Computer Science 2017, 25-27 October, 2017, San Francisco, U.S.A., pp 477-483.
[6] H.I. Okagbue, M.O. Adamu and T.A. Anake “Solutions of Chi-square Quantile Differential Equation,” In Lecture Notes in Engineering and Computer Science: Proceedings of The World Congress on Engineering and Computer Science 2017, 25-27 October, 2017, San Francisco, U.S.A., pp 813-818.
[7] Y. Kabalci, “On the Nakagami-m Inverse Cumulative Distribution Function: Closed-Form Expression and Its Optimization by Backtracking Search Optimization Algorithm”, Wireless Pers. Comm. vol. 91, no. 1, pp. 1-8, 2016.
[8] C. Yu and D. Zelterman, “A general approximation to quantiles”, Comm. Stat. Theo. Meth., vol. 46, no. 19, pp. 9834-9841, 2017.
[9] I.R.C. de Oliveira and D.F. Ferreira, “Computing the noncentral gamma distribution, its inverse and the noncentrality parameter”, Comput. Stat., vol. 28, no. 4, pp. 1663-1680, 2013.
[10] W. Hörmann and J. Leydold, “Continuous random variate generation by fast numerical inversion,” ACM Transac.Model. Comp. Simul., vol. 13, no. 4, pp. 347-362, 2003.
[11] W.P. Elderton, Frequency curves and correlation, Charles and Edwin Layton. London, 1906.
[12] N. Balakrishnan and C.D. Lai, Continuous bivariate distributions, 2nd edition, Springer New York, London, 2009.
[13] N.L. Johnson, S. Kotz and N. Balakrishnan, Continuous Univariate Distributions, Volume 2. 2nd edition, Wiley, 1995.
[14] N.L. Johnson, S. Kotz and N. Balakrishnan, Continuous univariate distributions, Wiley New York. ISBN: 0-471-58495-9, 1994.
[15] H. Rinne, Location scale distributions, linear estimation and probability plotting using MATLAB, 2010.
[16] H.I. Okagbue, P.E. Oguntunde, A.A. Opanuga, E.A. Owoloko “Classes of Ordinary Differential Equations Obtained for the Probability Functions of Fréchet Distribution,” In Lecture Notes in Engineering and Computer Science: Proceedings of The World Congress on Engineering and Computer Science 2017, 25-27 October, 2017, San Francisco, U.S.A., pp 186-191.

[17] H.I. Okagbue, P.E. Oguntunde, P.O. Ugwoke, A.A. Opanuga "Classes of Ordinary Differential Equations Obtained for the Probability Functions of Exponentiated Generalized Exponential Distribution," In Lecture Notes in Engineering and Computer Science: Proceedings of The World Congress on Engineering and Computer Science 2017, 25-27 October, 2017, San Francisco, U.S.A., pp 192-197.

[18] H.I. Okagbue, A.A. Opanuga, E.A. Owoloko, M.O. Adamu "Classes of Ordinary Differential Equations Obtained for the Probability Functions of Cauchy, Standard Cauchy and Log-Cauchy Distributions," In Lecture Notes in Engineering and Computer Science: Proceedings of The World Congress on Engineering and Computer Science 2017, 25-27 October, 2017, San Francisco, U.S.A., pp 198-204.

[19] H.I. Okagbue, S.A. Bishop, A.A. Opanuga, M.O. Adamu "Classes of Ordinary Differential Equations Obtained for the Probability Functions of Burr XII and Pareto Distributions," In Lecture Notes in Engineering and Computer Science: Proceedings of The World Congress on Engineering and Computer Science 2017, 25-27 October, 2017, San Francisco, U.S.A., pp 399-404.

[20] H.I. Okagbue, M.O. Adamu, E.A. Owoloko and A.A. Opanuga "Classes of Ordinary Differential Equations Obtained for the Probability Functions of Gompertz and Gamma Gompertz Distributions," In Lecture Notes in Engineering and Computer Science: Proceedings of The World Congress on Engineering and Computer Science 2017, 25-27 October, 2017, San Francisco, U.S.A., pp 405-411.

[21] H.I. Okagbue, M.O. Adamu, A.A. Opanuga and J.G. Oghonyon "Classes of Ordinary Differential Equations Obtained for the Probability Functions of 3-Parameter Weibull Distribution," In Lecture Notes in Engineering and Computer Science: Proceedings of The World Congress on Engineering and Computer Science 2017, 25-27 October, 2017, San Francisco, U.S.A., pp 539-545.

[22] H.I. Okagbue, A.A. Opanuga, E.A. Owoloko and M.O. Adamu "Classes of Ordinary Differential Equations Obtained for the Probability Functions of Exponentiated Fréchet Distribution," In Lecture Notes in Engineering and Computer Science: Proceedings of The World Congress on Engineering and Computer Science 2017, 25-27 October, 2017, San Francisco, U.S.A., pp 546-551.

[23] H.I. Okagbue, M.O. Adamu, E.A. Owoloko and S.A. Bishop "Classes of Ordinary Differential Equations Obtained for the Probability Functions of Half-Cauchy and Power Cauchy Distributions," In Lecture Notes in Engineering and Computer Science: Proceedings of The World Congress on Engineering and Computer Science 2017, 25-27 October, 2017, San Francisco, U.S.A., pp 552-558.

[24] H.I. Okagbue, P.E. Oguntunde, A.A. Opanuga and E.A. Owoloko "Classes of Ordinary Differential Equations Obtained for the Probability Functions of Exponential and Truncated Exponential Distributions," In Lecture Notes in Engineering and Computer Science: Proceedings of The World Congress on Engineering and Computer Science 2017, 25-27 October, 2017, San Francisco, U.S.A., pp 858-864.

[25] H.I. Okagbue, O.O. Agboola, P.O. Ugwoke and A.A. Opanuga "Classes of Ordinary Differential Equations Obtained for the Probability Functions of Exponentiated Pareto Distribution," In Lecture Notes in Engineering and Computer Science: Proceedings of The World Congress on Engineering and Computer Science 2017, 25-27 October, 2017, San Francisco, U.S.A., pp 865-870.

[26] H.I. Okagbue, O.O. Agboola, A.A. Opanuga and J.G. Oghonyon "Classes of Ordinary Differential Equations Obtained for the Probability Functions of Gumbel Distribution," In Lecture Notes in Engineering and Computer Science: Proceedings of The World Congress on Engineering and Computer Science 2017, 25-27 October, 2017, San Francisco, U.S.A., pp 871-875.

[27] H.I. Okagbue, O.A. Odetunmibi, A.A. Opanuga and P.E. Oguntunde "Classes of Ordinary Differential Equations Obtained for the Probability Functions of Half-Normal Distribution," In Lecture Notes in Engineering and Computer Science: Proceedings of The World Congress on Engineering and Computer Science 2017, 25-27 October, 2017, San Francisco, U.S.A., pp 876-882.

[28] H.I. Okagbue, M.O. Adamu, E.A. Owoloko and E.A. Suleiman "Classes of Ordinary Differential Equations Obtained for the Probability Functions of Harris Extended Exponential Distribution," In Lecture Notes in Engineering and Computer Science: Proceedings of The World Congress on Engineering and Computer Science 2017, 25-27 October, 2017, San Francisco, U.S.A., pp 883-888.

[29] R. Voda, "On the inverse Rayleigh variable", Rep. Stat. Apph. Res. Juse, vol. 19, no. 4, pp. 15-21, 1972.

[30] A.A. Abdel-Monem, "Estimation and Prediction for the Inverse Rayliegh life distribution", M.Sc. Thesis. Faculty of Education, Ain Shames University, 2003.

[31] S. Dey, "Bayesian estimation of the parameter and reliability function of an inverse Rayleigh distribution", Malay. J. Math. Sci., vol. 6, no. 1, pp. 113-124, 2012.

[32] A. Soliman, E.A. Amin and A.A. Abd-El Aziz, "Estimation and prediction from inverse Rayleigh distribution based on lower record values", Appl. Math. Sci., 4, no. 62, pp. 3057-3066, 2010.

[33] M.K. Gharraph, "Comparison of estimators of location measures of an inverse Rayleigh distribution", Egypt. Stat. J., vol. 37, 295-309, 1993.

[34] G. Fan, "Bayes estimation for inverse Rayleigh model under different loss functions", Res. J. Appl. Sci. Engine. Tech., vol. 9, no. 12, pp. 1115-1118, 2015.

[35] D. Kumar, "On estimation of inverse Rayleigh progressive type-II censored data with binomial removals", Int. J. Agric. Stat. Sci., vol. 13, no. 1, pp. 197-203, 2017.

[36] S. Ali and M. Riaz, "Cumulative quantity control chart for the mixture of inverse Rayleigh process", Comp. Indust. Engine., vol. 73, pp. 11-20, 2014.

[37] N. Feroze and M. Aslam, "On posterior analysis of inverse Rayleigh distribution under singly and doubly type II censored data", Int. J. Prob. Stat., vol. 1, no. 5, pp. 145-152, 2012.

[38] S. Manzoor and A.Z. Memon, "Analysis of inverse rayleigh distribution based on lower record values", Inter. J. Adv. Sci. Technol., vol. 102, pp. 35-48, 2017.

[39] R. Kishan and D. Kumar, "On Estimation Of The Scale Parameter Of Inverse Rayleigh Distribution With Progressive Type-Ii Censoring Under Different Loss Functions", Int. J. Agric. Stat. Sci., vol. 12, no. 2, pp. 477-482, 2016.

[40] A.A. El-Helbawy and Abd-El-Monem, "Bayesian Estimation and Prediction for the Inverse Rayleigh Lifetimd Distribution". Proceeding of the 40st annual conference of Statistics , Computer sciences and Operation Research, Cairo University, pp. 45-59, 2005.

[41] T.N. Sindhu, M. Aslam and N. Feroze, "Bayes estimation of the parameters of the inverse Rayleigh distribution for left censored data", Prob. Stat. Forum, vol. 6, pp. 42-59, 2013.

[42] B. Tarvirdizade and H. Kazemzadeh Garehchobogh, "Interval Estimation of Stress-Strength Reliability Based on Lower Record Values from Inverse Rayleigh Distribution", J. Qual. Relia. Engine., Article ID 192072, 2014.

[43] G. Prakash, "Bayes estimation in the inverse Rayleigh model", Elect. J. Appl. Stat. Analy., vol. 6, no. 1, pp. 67-83, 2013.

[44] K. Rosaiah and R.R.L. Kantam, "Acceptance sampling based on the inverse Rayleigh distribution", Econ. Qual. Control, vol. 20, no. 2, pp. 277-286, 2005.

[45] G.V. Sriramachandran and M. Palanivel, "Acceptance Sampling Plan from Truncated Life Tests Based on Exponentiated Inverse Rayleigh Distribution", Amer. J. Math. Magt. Sci., vol. 33, no. 1, pp. 20-35, 2014.

[46] K. Rosaiah, R.R.L. Kantam and J.P. Reddy, J. P. (2008). "Economic reliability test plan with inverse Rayleigh variate", Pak. J. Stat.-All Series-, vol. 24, no. 1, 57, 2008.

[47] F. Maleki Jebely, K. Zare and E. Deiri, "Efficient estimation of the PDF and the CDF of the inverse Rayleigh distribution", J. Stat. Comput. Simul., vol. 88, no. 1, pp. 75-88, 2018

[48] S. Ali, M. Aslam, N. Abbas, S.M.A. Kazmi and T. Hasan, "On process capability and system availability analysis of the inverse Rayleigh distribution", Pak. J. Stat. Oper. Res., vol. 11, no. 1, pp. 53-66, 2015.

[49] S.P. Mukherjee and L.K. Saran, "Bivariate inverse Rayleigh distributions in reliability studies", J. Indian Stat. Assoc., 22, 23-31, 1984.

[50] J. Leao, H. Saulo, M. Bourguignon, R. Cintra, L. Rêgo and G.M. Cordeiro, "On some properties of the beta inverse Rayleigh distribution", Chilean J. Stat., vol. 4, no. 2, pp. 111-131, 2013.

[51] Hussain and M. Ahmad, "Discrete Inverse Rayleigh Distribution", Pak. J. Stat., vol. 30, no. 2, pp. 203-222, 2014.

[52] A. Ahmad, S.P. Ahmad and A. Ahmed, "Transmuted Inverse Rayleigh distribution: a generalization of the Inverse Rayleigh distribution", Math. Theo. Model., vol. 4, no. 7, pp. 90-98, 2014.

[53] M.S. Khan and R. King, "Transmuted modified inverse Rayleigh distribution", Austrian J. Stat., vol. 44, no. 3, pp. 17-29, 2015.

[54] M.S. Khan, "Modified inverse Rayleigh distribution", Int. J. Comput. Appl., vol. 87, no. 13, pp. 28-33, 2014.

[55] D.L. Roges, "The Kumaraswamy inverse Rayleigh distribution", J. Stat. Comput. Simul., vol. 84, pp. 39-290, 2014.
[56] S. Ali, "Mixture of the inverse Rayleigh distribution: Properties and estimation in a Bayesian framework", Appl. Math. Model., vol. 39, no. 2, pp. 515-530, 2015.
[57] A.A. Opanuga, E.A. Owoloko and H.I. Okagbue, "Comparison Homotopy Perturbation and Adomian Decomposition Techniques for Parabolic Equations," Lecture Notes in Engineering and Computer Science: Proceedings of The World Congress on Engineering 2017, 5-7 July, 2017, London, U.K., pp. 24-27.
[58] A. A. Opanuga, E.A. Owoloko, H. I. Okagbue and O.O. Agboola, "Finite Difference Method and Laplace Transform for Boundary Value Problems," Lecture Notes in Engineering and Computer Science: Proceedings of The World Congress on Engineering 2017, 5-7 July, 2017, London, U.K., pp. 65-69.
[59] A.A. Opanuga, H.I. Okagbue and O.O. Agboola "Application of Semi-Analytical Technique for Solving Thirteenth Order Boundary Value Problem," Lecture Notes in Engineering and Computer Science: Proceedings of The World Congress on Engineering and Computer Science 2017, 25-27 October, 2017, San Francisco, U.S.A., pp 145-148.
[60] A.A. Opanuga, E.A. Owoloko, O.O. Agboola and H.I. Okagbue, "Application of Homotopy Perturbation and Modified Adomian Decomposition Methods for Higher Order Boundary Value Problems," Lecture Notes in Engineering and Computer Science: Proceedings of The World Congress on Engineering 2017, 5-7 July, 2017, London, U.K., pp. 130-134.
[61] S.O. Edeki , A.A. Opanuga, H.I. Okagbue , G.O. Akinlabi, S.A. Adeosun and A.S. Osheku, "A Numerical-computational technique for solving transformed Cauchy-Euler equidimensional equations of homogenous type", Advanced Studies Theor. Physics, vol. 9, no. 2, pp. 85-92, 2015.
[62] H.I. Okagbue, M.O. Adamu, T.A. Anake (2018) Ordinary Differential Equations of the Probability Functions of Weibull Distribution and their application in Ecology, Int. J. Engine. Future Tech., vol. 15, no. 4, pp. 57-78, 2018.