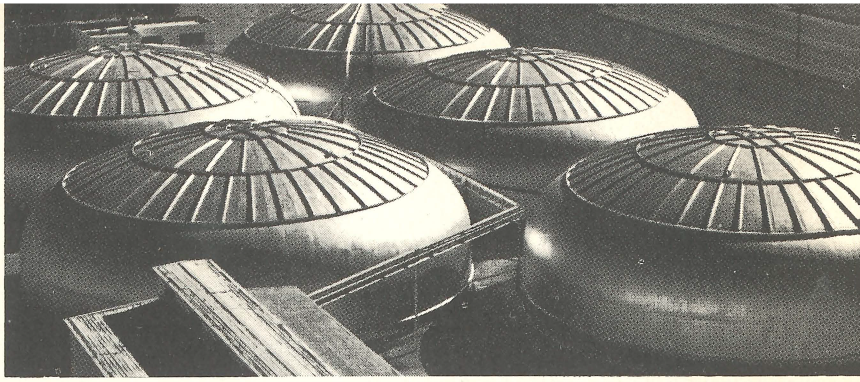
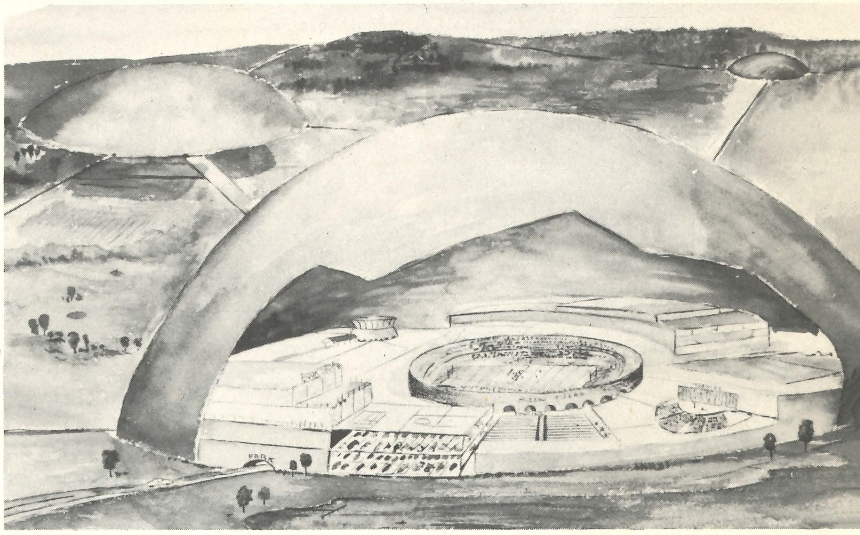
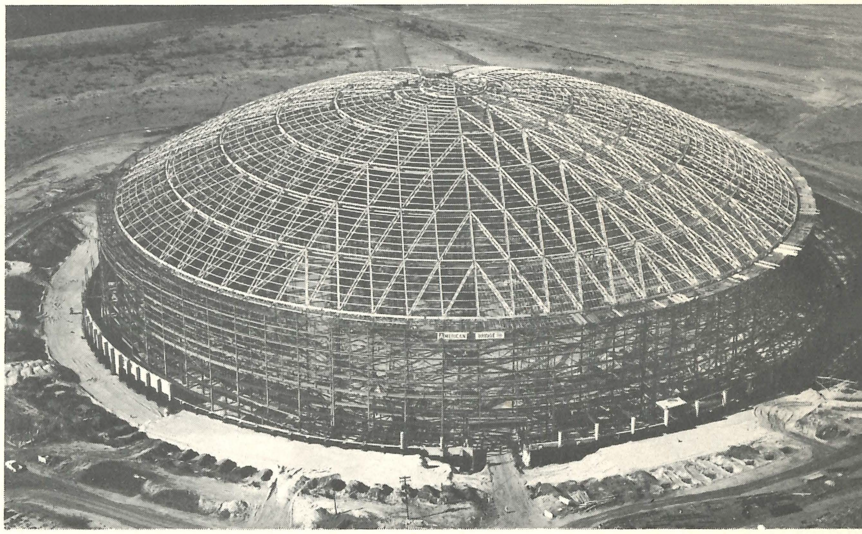
# EFFECT OF EDGE CONDITIONS ON BUCKLING OF STIFFENED AND FRAMED SHELLS

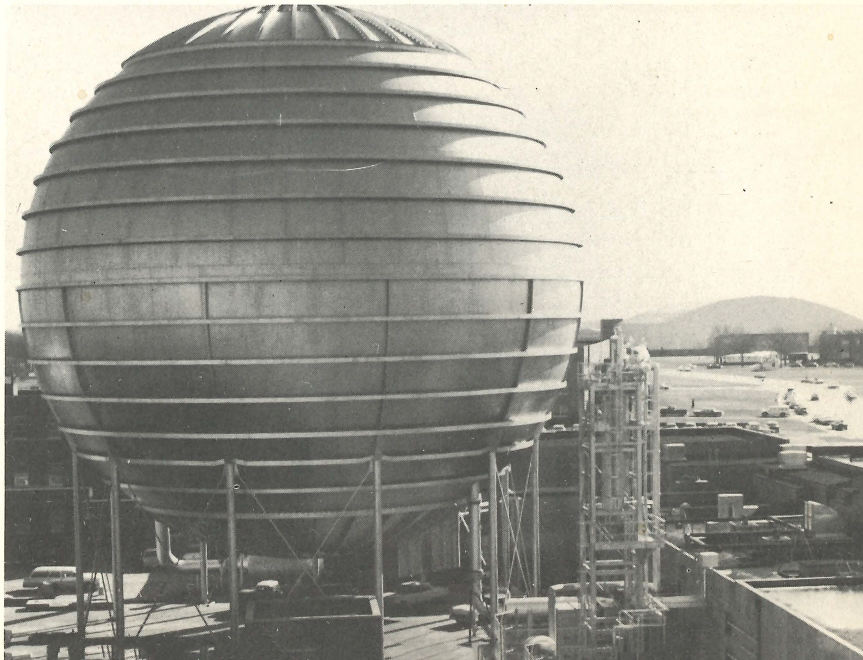
by

Kenneth P. Buchert









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# EFFECT OF EDGE CONDITIONS ON BUCKLING OF STIFFENED AND FRAMED SHELLS

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## SUMMARY

A series of stiffened shells made of plastic were tested to verify the theoretical equations for the effect of edge conditions on the buckling of stiffened and framed shells. The theory was developed by calculating the deflections during loading and prior to buckling, and by using a large deflection stability approach. The agreement between the test results and the theory was good. Tests also confirmed that the edges could be stiffened and relatively high buckling loads could be obtained by increasing the meridional curvature near the edge of the shell.

## INTRODUCTION

The use of shell and shell-like structures has increased significantly in recent years. The trend in shell construction has been toward larger and relatively thinner structures. As these structures become larger and thinner, buckling becomes one of the most important design considerations.

It was demonstrated[1,2,3,4,5,6,7][*] that the general buckling load of a dome type structure can be increased considerably by increasing the bending stiffness of the structure. This can be done with a minimum amount of material. The cost of this type of structure, particularly the erection cost, has been reduced considerably by using a minimum amount of shoring, and in some cases using no shoring at all. Erecting the structure by the cantilever method is made possible by taking advantage of the double curvature of the structure[1,2].

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[*] Superscripts refer to entries in the Bibliography.

Numerous methods have been used to increase the bending stiffness of the shell-like structure while basically maintaining membrane shell action. Full scale structures have been built by;

1) using a relatively thin shell and adding stiffeners to the shell.[2,3]
2) eliminating the shell entirely and using stiffeners only (such as pipes and structural shapes[5]).
3) using various combinations of materials as well as stiffening (such as structural shapes or pipes as stiffeners and concrete as the shell).

It has been demonstrated theoretically and experimentally, (both by the writer[1,2,8] and later[9]) that shell edge conditions play an important role in the capability of a shell-like structure to resist buckling. Poor edge conditions result in very low buckling loads. It has been demonstrated in the Shell Structures Laboratory of the University of Missouri that the buckling load can be near zero if very poor edge conditions are provided. On the other hand, with the proper design of the edges, the edge effect can practically be eliminated and the buckling can be made to occur away from the edge at a relatively high load[2].

Other factors that influence the buckling load are the yield strength of the material[10,11] and local stability[1,2,7,12,13].

Several studies have been made on the optimum design of these types of structures[3,14,16,17,18].

The purpose of the tests and theory presented herein was to develop quantitative criteria for the edge effect on stiffened and framed domes.

The funds provided by the University of Missouri, the American Iron and Steel Institute, and the National Aeronautics and Space Administration to carry out the study are gratefully acknowledged. The tests reported were conducted in the Shell Structures Laboratory at the University of Missouri.

## TEST PROGRAM

Boltaron plastic was used for the specimens for this study. Plastic was selected primarily for its low cost to fabricate and for its high yield strain. In the case of plastic shells, yield strain effects are minimized. The use of plastic allowed the shell portion of the specimen to be tested prior to fabrication of the stiffeners. In this way the stiffener effect could be better evaluated. In most cases the same specimen could be tested several times and the results showed that repeatable values of the buckling pressure could be obtained.

The specimens were formed on a vacuum former (see Photograph 1). A female forming die was fabricated by using a standard 24 inch spherical segment rupture disc made of stainless steel. The rupture disc was attached to a 1/4 inch steel cylinder,and hydrostone was poured behind the disc for reinforcing. A flat sheet of plastic was then vacuum formed to the die.

Six unstiffened shells were tested to verify the repeatability of the results. All specimens had a measured radius of curvature of 16.83 in. The details of the edge support are shown in Figure 1,and the results are given in Table I.

The modulus of elasticity of the material was determined in accordance with the ASTM "Standard Method of Test for Flexural Properties of Plastics" D790-66. The average modulus of elasticity of the 0.055 in. thick shell material was 204,000 psi, and the modulus of the 0.115 in. thick shell material was 243,000 psi.

The stiffened shells were fabricated by cutting 0.115 inch material into stiffeners, or by vacuum forming a 0.115 in. thick shell and then cutting away portions of the shell so that only the stiffeners and desired edge ring remained. The stiffeners were attached to the shell with epoxy cement. In all stiffened shell tests the clear distance between stiffeners was 1.5 in. and the width of the stiffeners was 1.0 in. The distance between stiffeners was selected such that buckling between stiffeners would not occur prior to general buckling.

Various edge ring configurations were tested. The conditions varied from no edge ring to several edge rings. The same basic specimen was used in all tests. After each buckle test the edge stiffening was increased. Table II gives the results. Photograph 2 shows the typical buckle. The dark line indicates the buckle configuration for specimen 8. The buckle shown is a result of the test of specimen 9. Note that the buckle of specimen 9 is farther away from the edge than the buckle of specimen 8. This result is typical. That is, as the buckle is forced away from the edge, the buckling load is higher.

After the tests on specimens 1 thru 9 the female die was modified by pouring a 5 in. wide ring of plastic at the edge of the die. The plastic was then machined such that the meridional curvature at the edge was 8 in. or approximately one-half of the spherical shell radius (see Photograph 3). This curvature raised the buckling pressure to 13.4 psi even though a single edge ring was used (see Table 10).

# THEORY

The effects of deflections during loading and prior to buckling have been presented by the writer[1,8]. The results are of the form

$$\frac{\sigma_{cr} R}{E t_m} = 0.41 \left(\frac{t_B}{t_m}\right)^{3/2} - 0.81 \frac{\Delta}{t_m} \qquad \text{----(1)}$$

for $$\frac{\Delta}{t_m} << 1 \qquad \text{----(2)}$$

where $\sigma_{cr}$ = the critical buckling stress

$R$ = the radius of curvature

$E$ = the modulus of elasticity

$t_m$ = the effective membrane thickness

$t_B$ = the effective bending thickness

$\Delta$ = the deflection during loading and prior to buckling

The buckling pressure is given by

$$p_{cr} = \frac{2 t_m \sigma_{cr}}{R} \qquad \text{----(3)}$$

For a stiffened shell

$$t_m = t + \frac{A}{d} \qquad \text{----(4)}$$

and

$$\frac{t_B^3}{12} = \frac{t^3}{12} + \frac{I}{d} \qquad \text{----(5)}$$

where $t$ = the shell thickness

$A$ = the area of the stiffener

$d$ = the distance between stiffeners

$I$ = the effective moment of inertia of the stiffener.

The effective membrane and bending thicknesses for framed, reticulated, and other structures are given in the Bibliography references, 2, 3, 4, and 5.

Equation (1) applies only when equation (2) holds. If $\Delta/t_m$ is not much less than 1 the following equation holds.

$$\frac{\sigma_{cr} R}{E\, t_m} = -0.54 \frac{\Delta}{t_m} - 0.145 \left[9.9 \left(\frac{\Delta}{t_m}\right)^2 + 3.08 \left(\frac{t_B}{t_m}\right)^3 \right]^{1/2}$$

$$+ \left\{ 1.09\left(\frac{\Delta}{t_m}\right)^2 - 0.03 \frac{\Delta}{t_m} \left[9.9 \left(\frac{\Delta}{t_m}\right)^2 + 3.08 \left(\frac{t_B}{t_m}\right)^3 \right]^{1/2} + 0.359 \left(\frac{t_B}{t_m}\right)^3 \right\}^{1/2} \qquad \text{--- (6)}$$

For most practical applications and for the tests reported herein, equation (6) is the one most applicable. Equation (6) agrees quite well with the experimental curve given in reference 19 for unstiffened shells.

In order to apply equations (1) or (6) the value of $\Delta$ must be determined or must be known by test. The following paragraphs give an approximate method for calculating $\Delta$.

The basic differential equation for secondary edge effects as given by Haas[20] is (see Figure 2).

$$\frac{d^4\psi}{d\phi^4} + 4\kappa\psi = 0 \qquad \text{--- (7)}$$

Where $\psi$ is the slope.

For a stiffened or framed shell

$$\kappa = \left(\frac{3R^2 t_m}{t_B{}^3}\right)^{1/4} \qquad \text{--- (8)}$$

The slope is

$$\psi = Ce^{-\kappa\alpha} \sin(\kappa\alpha + \frac{\pi}{4}) \qquad \text{----(9)}$$

If equation (9) is solved and the results are added to the primary effects the results may be used as described below.

Solve for

$$CE = \frac{-\left[\dfrac{r\, N_{\phi s} \cos \phi_s}{A_b} + \dfrac{N_{\theta s}}{t_m}\right]}{\left[\dfrac{\sqrt{2}}{2\kappa} + \dfrac{\sqrt{2}\; t_m r}{4\, \kappa^2 A_b \sin \phi_s}\right]} \qquad \text{----(10)}$$

$$N_\theta'' = \frac{-\,Et_B^3}{6} \;\frac{\kappa^2}{R^2}\; Ce^{-3\frac{\pi}{4}} \;\cot(\phi_s - \frac{3\,\pi}{2\,\kappa}) \qquad \text{----(11)}$$

$$N_\phi'' \doteq \frac{-\sqrt{2}\;ECt_m}{4\kappa^2}\; e^{-\frac{\pi}{2}} \;\cot(\phi_s - \frac{\pi}{2\,\kappa}) \qquad \text{----(12)}$$

$$\sigma_{b_{max}} = \frac{-t_B \kappa e^{-\frac{\pi}{4}}\, CE}{2R} \qquad \text{----(13)}$$

$$\delta_{max} = \frac{-RC \sin(\phi_s - \frac{3\,\pi}{4\,\kappa})\, e^{-\frac{3\,\pi}{4}}}{2\kappa} \qquad \text{----(14)}$$

$$\Delta \approx \frac{Ct_B^3}{3\sqrt{2}\; t_m}\; \frac{\kappa^3}{R}\; \sin^2\phi_s - \frac{N_{\theta s}R}{t_m E} \qquad \text{----(15)}$$

where $C$ = constant of integration

$N_{\phi s}$ = primary membrane meridional stress

$N_{\theta s}$ = primary membrane circumferential stress

$N''_{\phi}$ = secondary membrane meridional stress

$N''_{\theta}$ = secondary membrane circumferential stress

$r$ = base radius

$A_b$ = area of edge ring

$\sigma_b$ = bending stress

$\delta$ = secondary deflection

The above equations were derived based upon small torsional and bending stiffnesses of the edge ring. In practical shells this assumption is usually satisfactory because of the relatively large base of the shell compared to the torsional and bending stiffnesses of the ring.

## COMPARISONS OF THEORY AND TESTS

The test results were compared with the theory in the previous section for specimens 1 thru 10. The result is shown in Table III. The ratio of the test pressure to the theoretical pressure for the stiffened shells vary from 0.85 to 1.04. This accuracy is well within the known accuracy of loads usually encountered in this type of structure.

The unstiffened shells buckled from 6% to 45% higher than the predicted values. This is partially attributed to the addition edge extensional fixity provided to the very thin shells by the test fixture , which is not accounted for in the theoretical equations.

The theory for variable curvature with different edge conditions is not available at this time. A basic approach to this problem could be formulated by extending the techniques developed in reference 2.

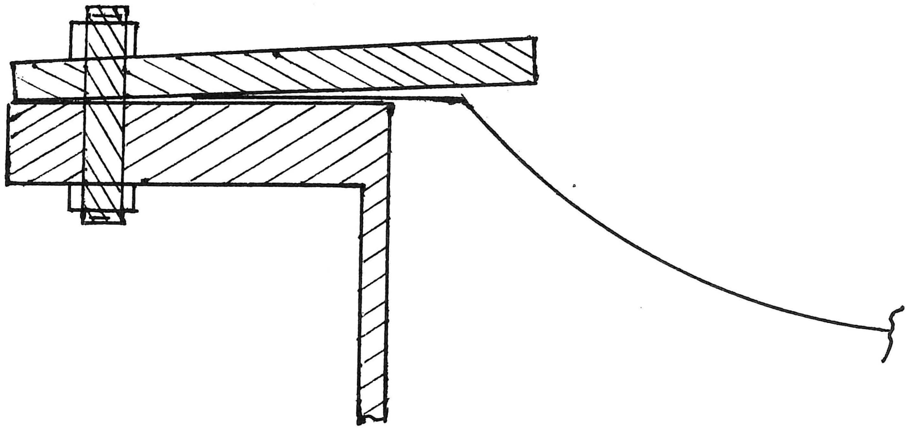


Figure 1 - EDGE DETAIL OF SHELLS

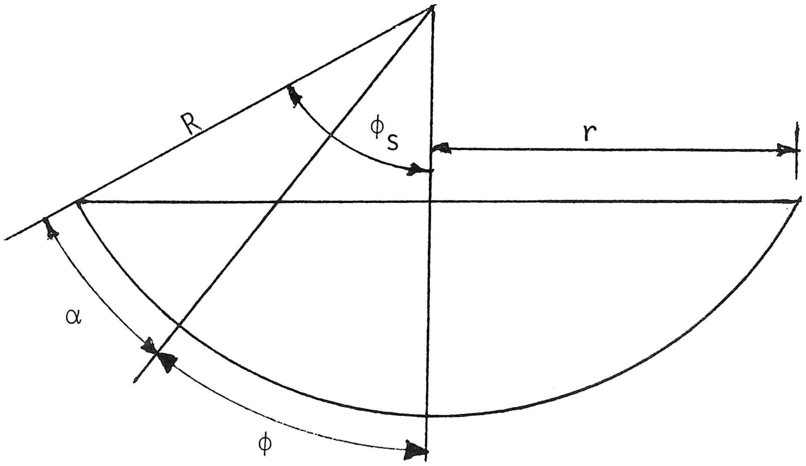
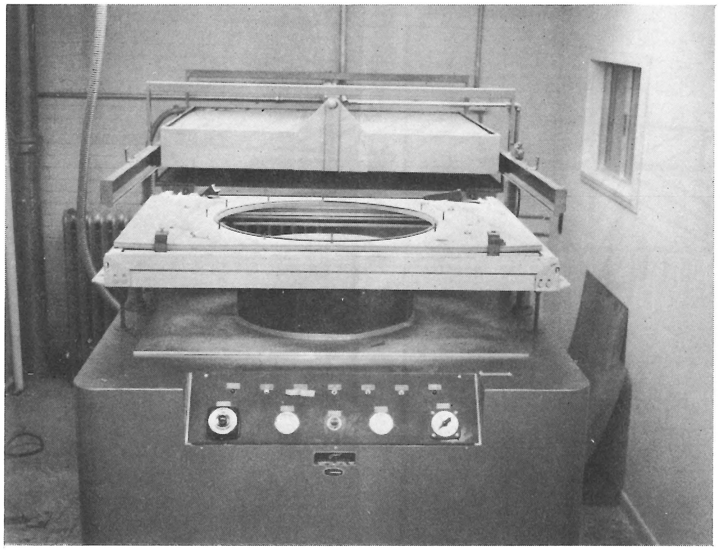
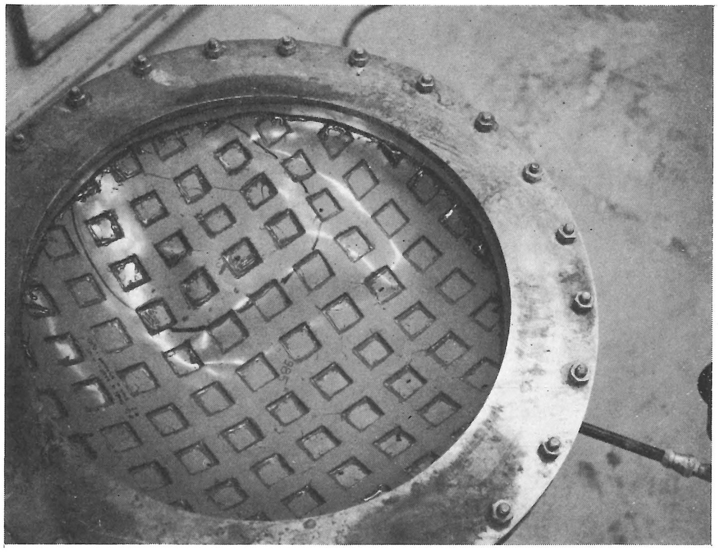


Figure 2 - GEOMETRIC NOTATION



Photograph 1 - VACUUM FORMER



Photograph 2 - BUCKLED STIFFENED SHELL

Photograph 3 - MACHINING VARIABLE

CURVATURE SHELL

## TABLE I - UNSTIFFENED SHELLS

| Specimen | Thickness | Buckling Pressure-psi |
|---|---|---|
| 1 | 0.55 | 1.30 |
| | | 1.26 |
| 2 | 0.55 | 0.97 |
| | | 1.00 |
| 3 | 0.55 | 1.17 |
| | | 1.27 |
| 4 | 0.55 | 1.16 |
| | | 1.12 |
| 5 | 0.115 | 6.21 |
| | | 6.21 |
| 6 | 0.115 | 6.21 |
| | | 6.21 |

For 0.55" Shells
Average pressure = 1.15 psi
Deviation + 13%, - 15%

## TABLE II - STIFFENED SHELLS

| Specimen No. | Buckling Pressure-psi | Edge Ring |
|---|---|---|
| 7 | 6.5 | 0.060" x 3" ring |
| 8 | 8.3 | 0.060" x 3" ring plus<br>0.115" x 2.5" ring |
| 9 | 9.6 | 0.060" x 3" ring plus<br>2-0.115" x 2.5" rings |
| 10 | 13.4 | 1-0.060" x 3" ring with 8 in. radius of curvature at edge |

## TABLE III - COMPARISON OF THEORY AND EXPERIMENT

| Specimen | Test Pressure-psi | Theoretical Pressure-psi | Ratio of Test to Theoretical Pressure |
|---|---|---|---|
| 1 | 1.30 | 0.90 | 1.45 |
| | 1.26 | 0.90 | 1.40 |
| 2 | 0.97 | 0.90 | 1.08 |
| | 1.00 | 0.90 | 1.11 |
| 3 | 1.17 | 0.90 | 1.30 |
| | 1.27 | 0.90 | 1.41 |
| 4 | 1.16 | 0.90 | 1.29 |
| | 1.12 | 0.90 | 1.25 |
| 5 | 6.21 | 5.90 | 1.06 |
| | 6.21 | 5.90 | 1.06 |
| 6 | 6.21 | 5.90 | 1.06 |
| | 6.21 | 5.90 | 1.06 |
| 7 | 6.5 | 7.7 | 0.85 |
| 8 | 8.3 | 8.7 | 0.95 |
| 9 | 9.6 | 9.3 | 1.04 |

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Note - For a more complete bibliography see items 2 and 8 below.

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