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Creep of Concrete: Influencing Factors and Prediction

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Effect of Creep and Shrinkage on the Behavior of Reinforced Concrete Members

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Properties and proportions of mix constituents, age of concrete at loading, storage conditions, stress-strength ratio, and other factors affecting creep are discussed. Various expressions for the creep-time relation are discussed, and prediction curves for creep of concrete of different properties and stored under different condition are presented.

Creep of Concrete: Influencing Factors and Prediction

By Adam M. Neville and Bernard L. Meyers

■ CREEP IS AN INCREASE with time in the strain of concrete subjected to stress; it is conveniently expressed at a constant stress. This definition is not adequate because concrete exhibits a change in strain with time when no external stress is acting—when drying (or swelling) takes place. This is of course drying shrinkage.

How are shrinkage and creep analyzed when they occur simultaneously? The common practice is to consider the two phenomena to be additive. The over-all increase in strain of a stressed and drying member is assumed to consist of shrinkage (equal in magnitude to that of a similar unstressed member) and of a change in strain due to stress (creep). This approach has the merit of simplicity but not of accuracy. Creep and shrinkage are not independent phenomena to which the principle of superposition can be applied, and in fact the effect of shrinkage on creep is to increase the magnitude of creep. In the case of many actual structures, however, creep and shrinkage occur simultaneously and the treatment of the two together is from the practical standpoint often convenient.

For this reason, and also because the great majority of the available data on creep were obtained on the assumption of the additive properties of creep and shrinkage, the discussion in this paper will, for the most part, consider creep as a deformation in excess of shrinkage. However, where a

more fundamental approach is warranted, distinction will be made between creep of concrete under conditions of no moisture movement to or from the ambient medium (true¹ or basic creep) and the additional creep caused by drying (drying creep).

The dependence of the stress-strain relation on time is shown not only by creep, as defined earlier, but also by relaxation. This is the change in stress in a member subjected to a constant strain. Creep and relaxation are closely inter-related but there is no simple way of translating creep values into relaxation values or vice versa.

For the sake of accuracy it should be noted that since the modulus of elasticity of concrete increases with time, the elastic strain decreases with time. Thus, strictly speaking, creep should be reckoned as strain in excess of the elastic strain at the time considered and not in excess of a fixed value of elastic strain.

The terms and definitions involved are illustrated in Fig. 1

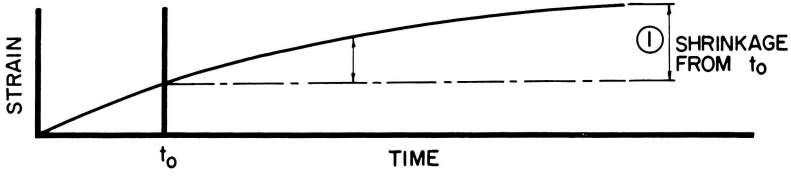
FACTORS INFLUENCING CREEP

Since the mix constituents are the main variables in concrete as a material, it is not surprising that nearly all the early studies on creep were concerned with the influence of the variation in the properties and quantities of the mix ingredients. Unfortunately, however, and this of course is well known, it is not possible to alter one constituent of concrete without altering at least one other. To give an example, a change in the water-cement ratio is accompanied by a change either in the content of the cement paste or in workability or in both.

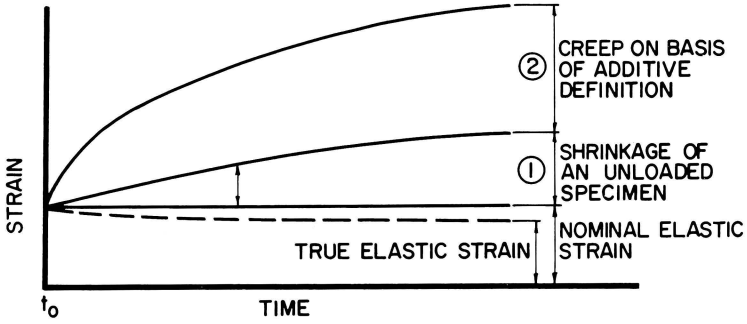
A great volume of test results on creep of concretes of different composition is available but the interpretation of the data requires considerable skill. It should not be surprising to find, in some cases, that the interpretation of test results in the light of the information now available may differ substantially from the investigator's original conclusions.

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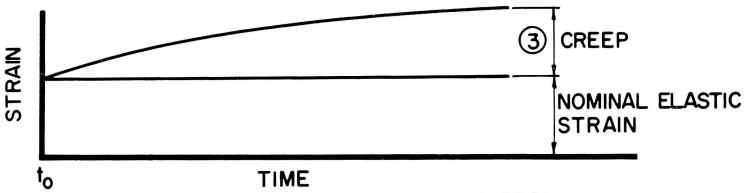
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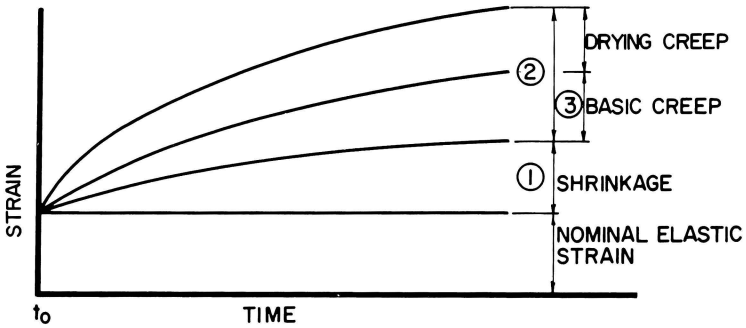
a.) SHRINKAGE OF AN UNLOADED SPECIMEN



b.) CHANGE IN STRAIN OF A LOADED AND DRYING SPECIMEN



c.) CREEP OF A LOADED SPECIMEN IN HYGRAL EQUILIBRIUM WITH THE AMBIENT MEDIUM



d.) CHANGE IN STRAIN OF A LOADED AND DRYING SPECIMEN

Fig. 1—Definitions of terms (t_0 is the time of application of load)

Aggregate

The usual normal weight aggregate used in concrete is not liable to creep to an appreciable extent,* so that it is reasonable to assume that the seat of creep is in the cement paste, but this does not mean that the aggregate does not influence the creep of concrete. The influence is in fact two-fold.

Aggregate content

Firstly, because the cement paste is subject to creep and the aggregate generally is not, the effect of the aggregate is to reduce the effective creep of concrete. Creep is thus a function of the volumetric content of cement paste in concrete, but the relation is not linear. Recent work⁶ indicates that creep of concrete, c , and the volumetric content of aggregate, g , are related by:

$$\log \frac{c_p}{c} = \alpha \log \frac{1}{1-g} \dots \dots \dots (1)$$

where c_p is creep of neat cement paste of the same quality as used in concrete, and

$$\alpha = \frac{3(1-\mu)}{1+\mu+2(1-2\mu_a)\frac{E}{E_a}} \dots \dots \dots (2)$$

Here, μ_a = Poisson's ratio of aggregate, μ = Poisson's ratio of surrounding material, E_a = modulus of elasticity of aggregate, and E = modulus of elasticity of the surrounding material.

Fig. 2 illustrates the relation between creep of concrete and its aggregate content. It may be noted that in the majority of the usual mixes, the variation in the aggregate content and therefore in creep is small.

The grading, maximum size, and shape of the aggregate have been suggested as factors in creep. It is now⁶ believed, however, that their main influence lies in the effect that these properties have directly or indirectly on the aggregate content. As shown earlier for a given water-cement ratio, the higher the content of aggregate, the lower the creep. The above statements and those to follow assume that in all cases the mix proportions are such that full consolidation can be obtained and that such consolidation has in fact been obtained.

Physical properties of aggregate

There are, however, certain physical properties of aggregate which influence the creep of concrete. The modulus of elasticity of aggregate is probably the most important factor. The higher the modulus, the greater

*Some aggregates are, however, liable to creep at stresses of no more than several hundred psi: McHenry³ demonstrated this for a volcanic agglomerate; the Bureau of Reclamation reported creep of the Glen Canyon sandstone⁴ and Taiwan greywacke.⁵

the restraint offered by the aggregate to the potential creep of the cement paste; this is evident from Eq. (2).

Porosity of aggregate has also been found to influence the creep of concrete but since aggregates with a higher porosity generally have a lower modulus of elasticity, it is possible that porosity is not an independent factor in creep. On the other hand, it can be visualized that the porosity of aggregate, and even more so its absorption (Fig. 3), plays a direct role in the transfer of moisture within concrete; this transfer may be associated with creep.

Because of the great variation in aggregate within any mineralogical or petrological type, it is not possible to make a general statement about the magnitude of creep of concrete made with aggregates of different types. However, as no more than an illustration, the findings of two investigators are quoted. In 1930 Davis and Davis⁷ found the aggregates in the order of increasing creep to be: limestone, quartz, granite, gravel, basalt, and sandstone. After 20 years' storage at a relative humidity of 50 percent, concrete made with sandstone aggregate exhibited creep more than twice as great as concrete made with limestone.⁸

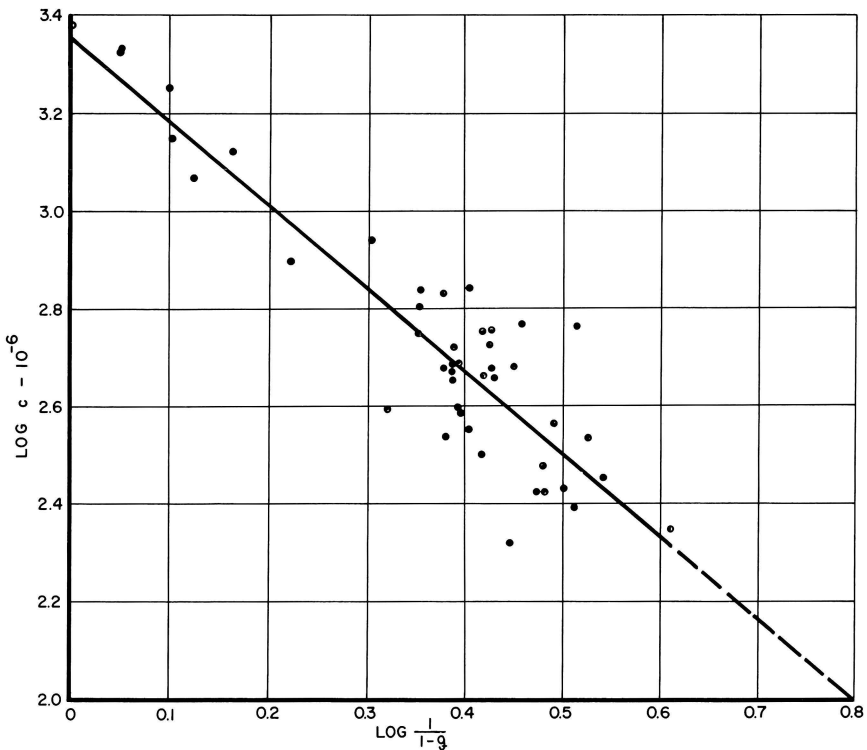


Fig. 2—Relation between creep c after 28 days under load and content of aggregate g for wet-stored specimens loaded at the age of 14 days to a stress-strength ratio of 50 percent

An even greater difference between creeps of concretes made with different aggregates was found by Rüsçh, Kordina, and Hilsdorf⁹ in 1962. After 18 months under load at a relative humidity of 65 percent, the maximum creep was five times the minimum value, the aggregates in the increasing order of creep being: basalt; quartz; gravel, marble and granite, and sandstone.

Rüsçh's⁹ tests indicate that the influence of the petrological type of aggregate on creep acts primarily through the modulus of elasticity of aggregate. The latter affects also the elastic deformation of concrete, and indeed a good correlation between creep and elastic deformation of concrete was obtained. At the same time, the absorption of aggregate and its modulus of elasticity were definitely related (Fig. 3) so that the pattern of the creep-elasticity-absorption relation, mentioned earlier, is confirmed.

Lightweight aggregate deserves a special mention because of the rather common belief that it leads to substantially higher creep than normal weight aggregate. Lightweight aggregate is increasingly used in structural concrete and the knowledge of its creep properties is important. Recent work both at the University of Missouri and elsewhere indicates that there is no fundamental difference between normal and lightweight aggregates as far as the creep properties are concerned, and the higher creep of concretes made with lightweight aggregates reflects only the lower modulus of elasticity of the aggregate. There is no inherent difference in the behavior of coated and uncoated aggregates or between those obtained by different

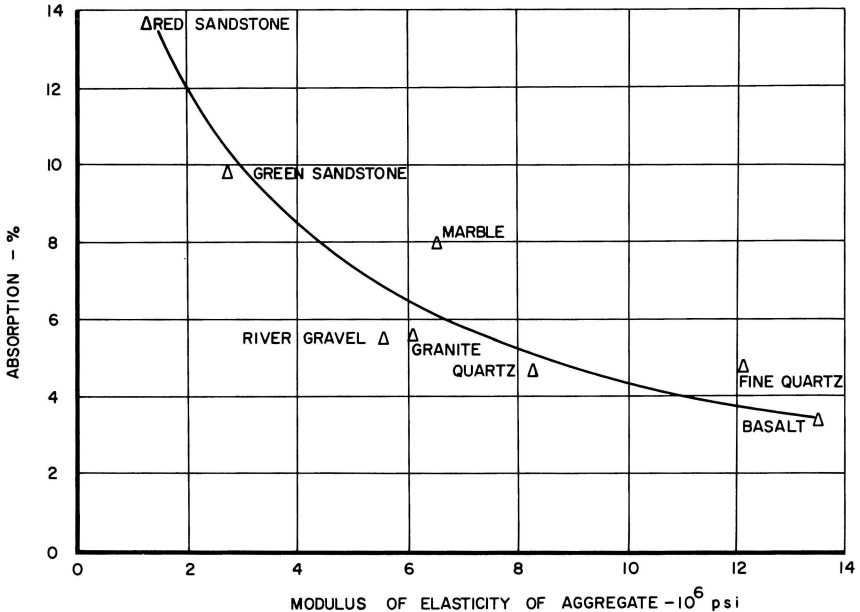


Fig. 3—Relation between absorption and modulus of elasticity of different aggregates

manufacturing processes, but this of course does not mean that all the aggregates lead to the same creep.

Cement

The usual portland cements differ from one another primarily in the rate of hydration but not in the ultimate strength. Any comparison of creep behavior must therefore take into account the degree of hydration at the time of application of the load. Unqualified statements such as "Type IV cement creeps more than Type I cement" are almost meaningless.

In conventional concrete design, the permissible stress forms a fixed proportion of the concrete strength at the time of application of the load or, more commonly, at some arbitrary age such as 28 days. For this reason, it is logical to compare concretes made with different cements under a load where the stress-strength ratio* is the same in all cases. Under these conditions, the type of cement (i.e., its composition or fineness) does not, in the first approximation, influence creep.^{10†} The composition is meant to include the major cement compounds, C_3S , C_2S , C_3A , C_4AF , and also the alkalis whose presence in cement tends to increase the creep and also to lower the gain in strength.¹²

The amount of gypsum in the cement may affect creep in a manner similar to the influence on shrinkage. This was suggested by Davis and Troxell¹³ and observed experimentally by Neville.¹⁰ The influence of gypsum is apparent when cements are reground in the laboratory without gypsum being added. With the vast majority of commercial cements, the gypsum content is near the optimum value for shrinkage and probably also for creep.

The statement that creep is not influenced by type of cement is believed to be of importance, but should be refined by considering the change in strength of concrete while under load. Therefore, concretes made with different cements and loaded at the same age at a constant stress-strength ratio should be considered. The increase in strength beyond this age will be different for different cements; being least for Type III cement and greatest for Type IV cement. It has been suggested that the decrease with time in the rate of creep is a function of the increase in strength, the decrease in the rate being greater, the greater the increase in strength.⁶ A quantitative verification of this relation is in progress (1964) at the University of Alberta, Calgary. In general terms it can be stated, with the qualifications made, that creep increases for cement Types IV, I, and III, respectively. There is no doubt, however, that for a constant applied stress at a fixed (early) age, creep increases in order for cement Types III, I, and IV. These two statements bring out clearly the need for a full qualification of statements about factors in creep.

*The ratio of the applied stress to the strength at the time of loading.

†This statement does not apply to cements other than Types I to V and white portland cement. For instance, portland blastfurnace cement results in a higher creep¹¹ than the cements mentioned above.

Entrained air

Sufficient information on the influence of entrained air on creep is not available but there are indications that the presence of air increases creep. It is logical to treat entrained air as aggregate with a modulus of elasticity of zero, and thus to consider air as materially contributing to creep. This statement is valid, however, only if we compare mixes which differ solely by the presence or absence of air. In practice, a mix designed to contain entrained air has properties such as workability and strength which are comparable with those of an air-free mix of different proportions. On this basis of comparison the influence of the air content on creep may not be significant.

Admixtures and pozzolans

No systematic tests on the influence of admixtures on creep have been made and no reliable information is available. This statement is hardly helpful, and all that can be recommended is that the behavior of untried admixtures be studied in the laboratory before field use.

Pozzolans probably do not directly affect creep. If they are used as a partial replacement of cement and the load is applied before the pozzolanic action has been fully developed, an increase in creep should be expected.

Mix proportions

The quality of the cement paste has a direct influence on creep, and this can be expressed approximately by saying that for a constant cement paste content, and the same applied stress, creep is inversely proportional to the strength of concrete. Thus strength is a convenient, but approximate, measure of the state of the cement paste, i.e., its composition and degree of hydration.

Creep, therefore, increases with an increase in the water-cement ratio* but the relation between creep and the water content of the mix is not basic. What happens depends on the influence of the water-cement ratio and aggregate-cement ratio, as these two factors control the water content of the mix. (The influence of the aggregate-cement ratio is the lesser of the two.)

Viewing creep as a function of water-cement ratio and aggregate-cement ratio gives a correct general picture of the influence of mix proportions on creep, and is worth emphasizing, for in the older literature there exist numerous misleading statements. For example: creep is proportional to the aggregate content of the mix (which can be explained by a concomitant increase in the water-cement ratio); or, creep is proportional to the water content of the concrete (which can be explained by the fact that for a constant workability an increase in water content must be accompanied by an increase in the aggregate-cement ratio, with a consequent

*Creep being approximately proportional to the square of the water-cement ratio, as suggested by Lorman.¹⁴

increase in the water-cement ratio; the effect of the latter on creep is greater than that of the aggregate-cement ratio for usual ranges of practical mixes). If both the aggregate content and water-cement ratio are varied, the net effect on creep would depend on the relative magnitude of the effects of variation in the paste content and its quality.

For these reasons, and also because the strength of structural concrete is a practical concern, relating creep to strength is thought to be both

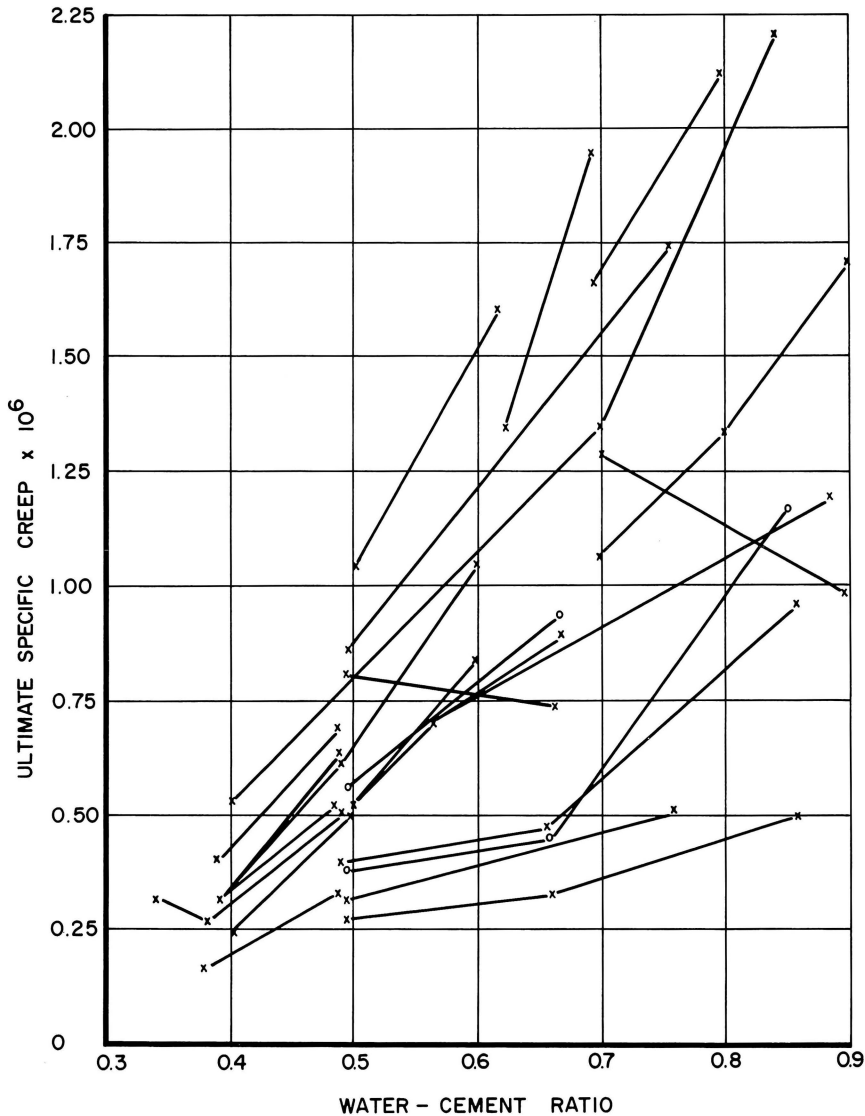


Fig. 4—Relation between specific creep and water-cement ratio in the tests of various investigators¹⁶

convenient and fairly reliable. As an example, Klieger's¹⁵ data on creep of concrete can be expressed in terms of strength, as shown in Table 1.

A good illustration of the general situation is given by Wagner's data.¹⁶ Fig. 4 shows the relation between specific creep (creep per psi) and water-cement ratio; there is no clear-cut pattern, and indeed the trend of some investigations is opposite to that of others. However, when a correction for the cement paste content has been made (by reducing the observed creep values to those which would exist if the content of cement paste were 20 percent by weight), the influence of the water-cement ratio becomes clear, as shown in Fig. 5. The ordinate of this figure represents the ratio of the actual creep to the creep of a mix with a water-cement ratio of 0.65. Such a relation exists both for long- and short-term creep.

It may be relevant to note that this mix with a water-cement ratio of 0.65 and a cement paste content of 20 percent by weight is used as a reference mix when the prediction of creep for mixes of different proportions by Wagner's method¹⁷ is considered later in the paper.

Mixing time and consolidation

No tests have been made on the influence of the mixing procedure and time on creep. These factors may affect the strength of concrete, and it

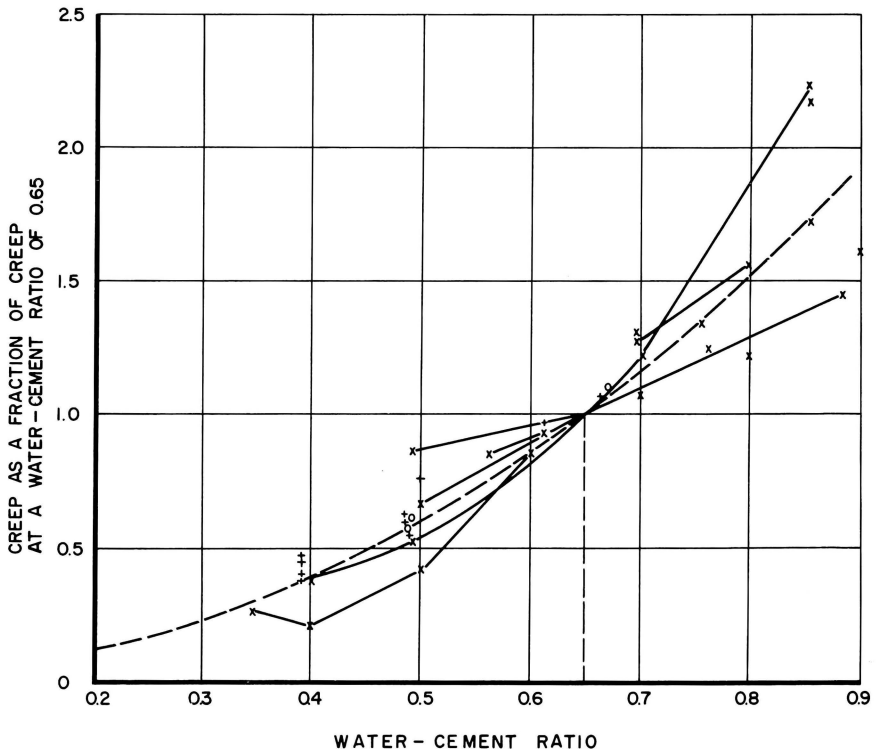


Fig. 5—Data of Fig. 4 adjusted for the content of cement paste (to a value of 20 percent) and expressed in terms of creep at a water-cement ratio of 0.65¹⁶

TABLE 1—CREEP OF CONCRETES OF DIFFERENT STRENGTHS

Compressive strength at time of application of load, psi	Ultimate specific creep, 10^{-6} per psi	Ultimate creep at a stress-strength ratio of 30 percent, 10^{-6}
2000	1.40	933
4000	0.80	1067
6000	0.55	1100
8000	0.40	1067

would therefore be logical to expect a corresponding influence on creep. In some cases, the influence on strength is transitory and may cease to be present at the age at which the sustained load is applied; no influence on creep would then be expected.

The influence of consolidation on creep is related to the strength which the concrete has developed. Voids due to incomplete consolidation affect creep in a manner similar to the capillary voids whose presence is reflected by the influence of the water-cement ratio on creep. It is, therefore, believed that experimental results relating the method of consolidation to creep should be interpreted in terms of the degree of consolidation achieved by the various means of consolidation. From the practical point of view, proper consolidation of concrete is of great importance.

Age of concrete

When the influence of the strength of concrete at the time of application of the load on creep was discussed, the variation in strength was considered to arise from the type of cement and mix proportions used. Age of concrete at loading acts in the same way, i.e., it is a factor in creep in so far as it influences the degree of hydration and development of strength. Thus creep has been found to correlate well with maturity.^{17,18}

Under conditions such that no sensible variation in the degree of hydration occurs, the age at loading ceases to influence creep. For instance, the influence of the age at which the load is applied is much smaller in the case of dry-cured concrete. Also, at later ages the rate of creep becomes independent of the age at loading.

Level of stress

In various experimental results there exists substantial evidence of the proportionality between creep and the applied stress, with a possible exception of specimens loaded at an early age. What is not certain is the upper limit of proportionality. (The lower limit is virtually at zero stress as creep is exhibited by concrete even at low stresses.) In terms of the stress-strength ratio, an upper limit between about 30 and 75 percent¹⁹ has been suggested.*

*Freudenthal and Roll²⁰ found the limit to be as low as 23 percent.

It is known that severe internal cracking takes place in a concrete compression specimen at a stress-strength ratio of 40 to 60 percent,²¹ and it is not surprising that, once the cracking has started, the creep behavior also changes. It is possible that the onset of cracking depends on the degree of heterogeneity of the concrete; for instance, mortars are less grossly heterogeneous than concrete containing a large-size aggregate and exhibit proportionality between creep and stress-strength ratio up to a higher limit, possibly 85 percent.¹⁰

It appears safe to conclude that within the range of working stresses the proportionality holds good and, with one exception,²² creep expressions and prediction diagrams (discussed later in the paper) assume this to be the case.

Above the limit of proportionality, creep increases with an increase in stress at an increasing rate, and there exists a stress level above which creep produces time failure. This level is in the region of 70-80 percent of the short-term static strength. Creep increases the total strain until this reaches a limiting value corresponding to the ultimate strain of the given concrete. This statement implies a limiting strain concept of failure of concrete, which the authors believe to be the case.¹²

Ambient humidity

Numerous tests have shown that creep increases with a decrease in the relative humidity of the surrounding medium. For instance, at a relative humidity of 50 percent creep may be 2 to 3 times greater than at 100 percent (Fig. 6). However, careful qualification is necessary because a statement that creep is higher the lower the relative humidity of the ambient medium may be misleading. Two points should be noted.

Firstly, the ambient relative humidity affects creep if drying takes place while the specimen is under load. But if the concrete has reached hygral equilibrium prior to loading, the magnitude of creep is independent of the relative humidity of the surrounding medium. This was reported by Neville,¹⁰ and confirmed by Kesler's tests on mature concretes. It appears thus that it is not the ambient humidity that is a factor in creep but the process of drying while the concrete is subject to creep. This is confirmed, for instance, by the fact that at later ages the rate of creep becomes sensibly independent of the ambient relative humidity (Fig. 6); by that time, shrinkage has virtually ceased.

The second point is really an extension of the first. Hansen²³ observed (1958) that alternating the ambient relative humidity between two limits results in a creep which is higher than that obtained at a constant humidity within the given limits. This was confirmed by L'Hermite²⁴ in 1960. The phenomenon was in fact first noticed as far back as 1942²⁵ but since no explanation was available it was simply ignored and forgotten. One effect of this behavior is that laboratory tests performed at a constant relative

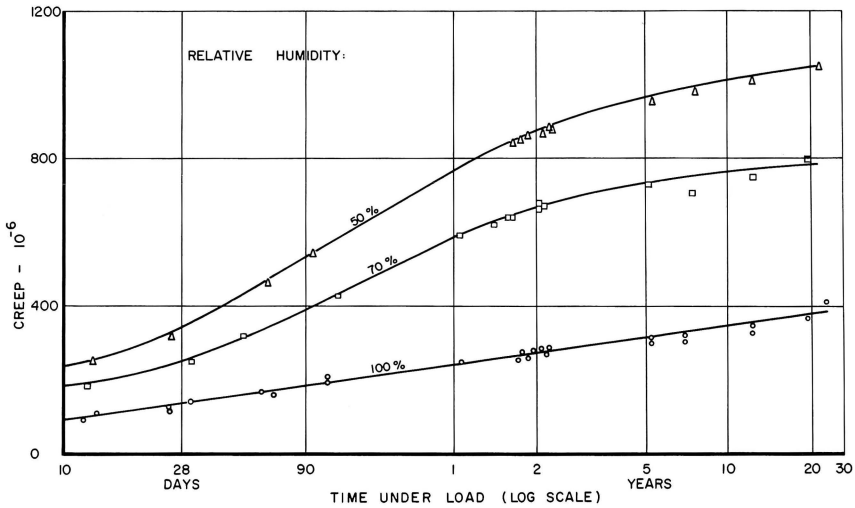


Fig. 6—Creep of concrete cured in fog for 28 days, then loaded and stored at different relative humidities⁸

humidity underestimate the creep under conditions of practical exposure in many countries.

It may be relevant at this stage to observe that the apparent influence of ambient humidity on creep does not act through the medium of an additional loss of water from the concrete. Numerous tests* have shown that an external load does not increase water loss to the surrounding air in excess of that which takes place under similar conditions without an external load acting. Alternating humidity, whether the concrete be under load or not, leads to moisture movement into and out of concrete. This appears to enhance creep²⁸ but creep does not enhance the moisture movement. An explanation of these apparently contradictory phenomena is offered in the second paper in this symposium.

We should note also that the ambient humidity affects the rate of hydration and of gain of strength. This influence is smaller in the case of Type III cement, and indeed the creep of concrete made with this cement is less influenced than when Type I cement is used. This is apparent, for instance, from Glanville's data.²⁹ The order of magnitude involved is given in Table 2.

Temperature

The influence of temperature on creep is of interest in connection with the use of concrete in the construction of prestressed concrete nuclear pressure vessels. The topic is thus rather specialized. Work of Ross and England³⁰ shows that creep increases with temperature, the increase being greater in the range 68 to 180F (say, fourfold after 80 days under load) than for higher temperatures up to 280F (when creep is about five times

*See References 26 and 27 for examples.

TABLE 2—INFLUENCE OF AMBIENT RELATIVE HUMIDITY ON CREEP

Relative humidity, percent	Creep of concrete as a fraction of creep at 70 percent relative humidity	
	Type I cement	Type III cement
100	0.50	0.70
85	0.77	0.87
70	1.00	1.00
55	1.22	1.10
45	1.34	1.17
35	—	1.22

the creep at 68F). This broad pattern was found both for concrete stored in air and for concrete under simulated conditions of mass curing. These results are in qualitative agreement with Theuer's findings³¹ but he observed also that the creep of desiccated concrete is independent of temperature. Experimental work being conducted by Nasser and Neville shows that the temperature effect is present also when concrete is stored in water.

Temperature influences creep even within the normal range (say, up to 100F) and this effect should not be neglected.

The difference between the temperature during the period while under stress and during the preceding period must be recognized; the latter influences the basic principles of concrete, its maturity, the structure of gel, etc., and should be viewed in that light.

Size of member

Several investigations have indicated an influence of the size of the specimen on creep. The measured creep decreases with an increase in the size of the specimen, but when the specimen thickness exceeds about 3 ft, the size effect is no longer noticeable. Some typical results are shown in Fig. 7.³²

This apparent influence of the size of the concrete member on creep complicates an evaluation of the test results of different investigators but the importance of the phenomenon lies in its causes. Why does size affect creep? The first observation which should be made is that the influence of size on creep is greatest during the initial period after the application of the load. Beyond, say, several weeks the rate of creep is the same in specimens of all sizes.

The original explanation of the size effect in terms of the loss of water to the ambient medium (which would be relatively greater in a smaller specimen) is not correct for it is now known that loss of water does not play a significant role in creep. However, in actual tests, creep and shrinkage operate simultaneously. As shown earlier these two phenomena are not independent and concurrent shrinkage enhances creep. Thus in a small specimen a greater part of the concrete is subjected to creep while drying

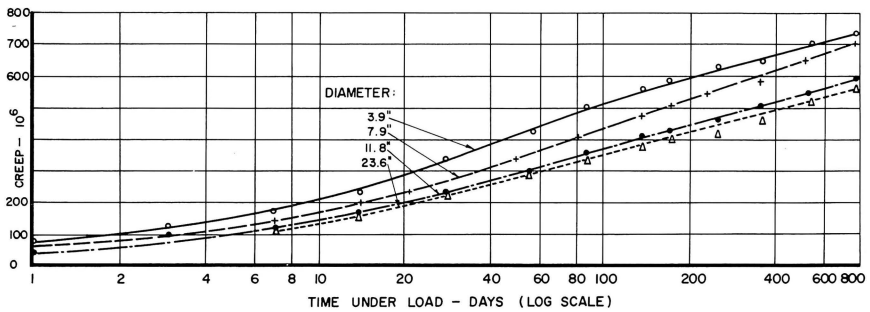


Fig. 7—Influence of specimen size on creep³²

takes place, and a greater creep is therefore recorded. The converse is true in a larger specimen, and even if with time the drying effect reaches the core, the concrete there will have changed substantially from the state which existed when load was first applied. A greater degree of hydration will have taken place and a higher strength will have been developed in the core so that the creep response to the creep-while-drying condition will be small. The size effect applies not to true creep or basic creep but to the increase in creep due to drying.

Recent work at the Portland Cement Association Laboratories indicates that both creep and shrinkage are functions of the surface-volume ratio of the member. Thus the size effect is an indirect one, involving the surface of the specimen, and it may be concluded that when a free surface is absent, creep is unaffected by the size of the member. In fact, in concrete cured under mass conditions, size effects do not appear to be present.

CREEP UNDER DIFFERENT STATES OF STRESS

The preceding discussion referred to uniaxial compression but creep also occurs in other loading situations and information about creep behavior under these situations is especially helpful in establishing the nature of creep. Unfortunately, the volume of experimental data is small and in many cases quantitative evaluation and comparison with the behavior in compression is not possible. For this reason no more than broad qualitative statements will be made.

Creep takes place under uniaxial tension; its magnitude is of the same order as creep in compression, and the creep-time curves for the two conditions are similar. Drying enhances creep in tension as well as creep in compression.

Creep occurs under torsional loading, and is affected by stress, water-cement ratio, and ambient relative humidity in qualitatively the same manner as creep in compression. The creep-time curve is also of the same shape.³³

Creep in a direction normal to the applied load can be expressed in terms of a creep Poisson's ratio. There are strong indications that this ratio is zero,³⁴ i.e., that axial creep produces no increase in the lateral

TABLE 3—AVERAGE INCREASE IN CREEP⁸

Creep after 1 year	1.00
Creep after 2 years	1.14
Creep after 5 years	1.20
Creep after 10 years	1.26
Creep after 20 years	1.33
Creep after 30 years	1.36

deformation, but this can be so only for loads which produce a stress-strength ratio not greater than, say, 40 to 50 percent because splitting may take place at higher stresses.²¹

Biaxial and triaxial stresses could thus be expected to lead to creep deformations which are sums of creeps caused by component uniaxial stresses. This appears to be the case but further work in this area is necessary.

END VALUE OF CREEP

The definition of creep refers to the increase in strain with time but says nothing about the presence or absence of a terminal value. All creep-time curves (with the exception of those for extremely high stress-strength ratios which lead to time failure) show a progressive decrease in the rate of creep with time. This is particularly noticeable in the early stages after the application of the load but it is not certain that the decrease continues throughout.

The problem is of fundamental importance for if this be the case creep approaches asymptotically a limiting value. If, on the other hand, the rate becomes stabilized at some value, creep increases indefinitely. There is no doubt, however, that even in the latter case the rate of increase would be so low that with a practical limit of duration of load, say 100 years, the long-term increase in creep would not bring about a large or dangerous defor-

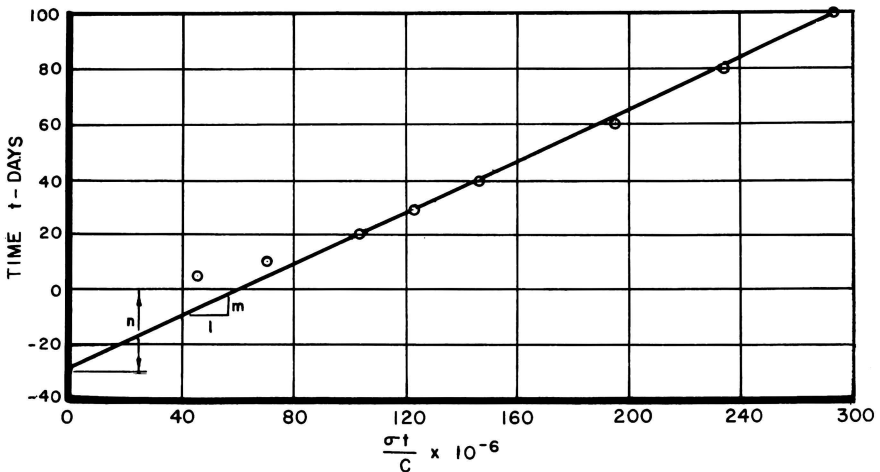


Fig. 8—Creep constants according to Lorman

mation. The practical significance of the problem is thus smaller. Table 3 shows average increase in creep⁸ beyond 1 year's duration of load.

The longest period for which creep data are available is less than 30 years⁸ and here a small but measurable rate of creep was observed. It is not possible to say whether this rate will vanish to zero. Flügge's interpretation³⁵ of various tests is that after about a year under load, creep continues at a constant rate so that no finite limit of creep can exist. These tests include the earlier tests of Davis and those of Glanville²⁹ who postulated a limiting value of creep. It seems that only time will resolve the problem.

CREEP EXPRESSIONS

It is convenient to express the creep-time relation in the form of an equation so that values of creep may be predicted without performing long-time tests. About a dozen equations have been suggested, most of these of a hyperbolic or an exponential type. In some cases creep is expressed by a "standard" curve, which is modified by a number of factors to allow for properties of a particular mix and storage conditions.

Among the equations which make creep tend toward a finite limit are those of Lorman, Ross, and Thomas. Lorman¹⁴ proposed a hyperbolic expression

$$c = \frac{mt}{n+t} \sigma \dots \dots \dots (3)$$

where, t = time since the application of load, σ = applied stress, and m and n are constants in appropriate units.

Eq. (3) can be written in the form

$$t = \frac{m\sigma t}{c} - n \dots \dots \dots (4)$$

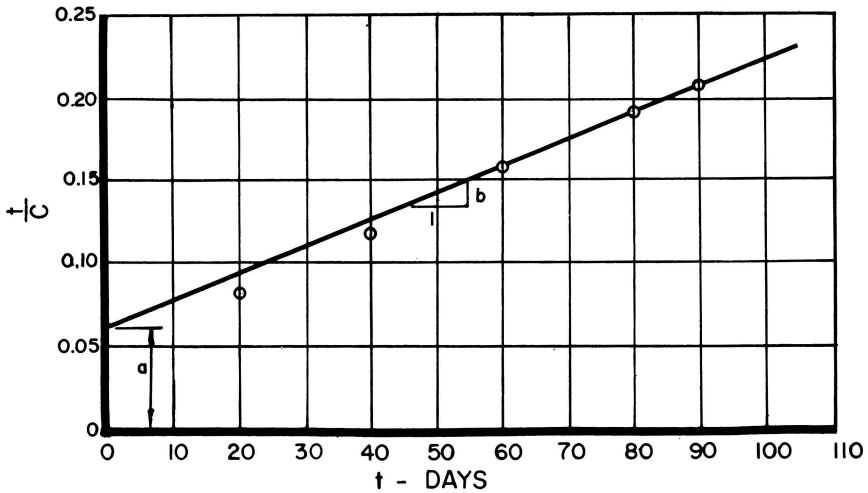


Fig. 9—Creep constants according to Ross

and hence, m and n can be obtained as the slope and intercept respectively of the plot of $\sigma t/c$ against time. This is shown in Fig. 8.

The ultimate creep, c_∞ , is creep when $t = \infty$. From Eq. (3)

$$c_\infty = m \sigma \dots \dots \dots (5)$$

It is interesting to note that when $t = n$, $c = \frac{1}{2} m \sigma$, i.e., one-half of the ultimate creep is realized at time $t = n$.

Ross³⁸ suggested a similar equation about 3 years before Lorman

$$c = \frac{t}{a+bt} \dots \dots \dots (6)$$

where a and b are constants.

A plot of t/c against t is a straight line, and the constants can be easily evaluated (Fig. 9). The ultimate creep is

$$c_\infty = \frac{1}{b} \dots \dots \dots (7)$$

and the ease with which this value can be obtained is an advantage of the hyperbolic expressions, Eq. (3) and (6).

It should be noted that in drawing the straight lines in Fig. 8 and 9 a greater weight is given to the points for larger values of t because this gives a better prediction of creep values for long periods under load and of the ultimate creep. This procedure is used throughout this paper.

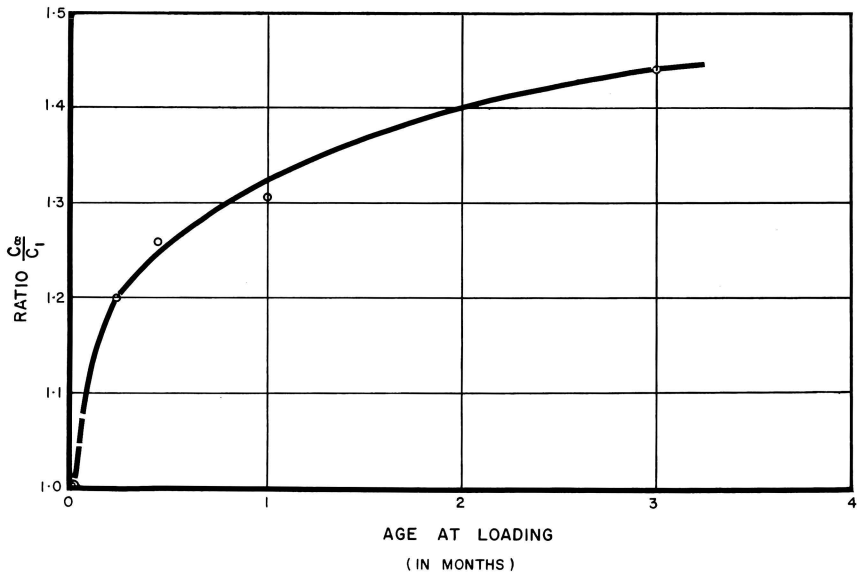


Fig. 10—The influence of the age at loading on the ratio of the ultimate creep, c_∞ , to creep after 1 year under load, c_1

The values of c_{∞} given by Eq. (6) agree closely with those given by Thomas³⁸ exponential expression

$$c = c_{\infty} \times \sigma \left(1 - e^{-A(t+d)^g - dg} \right) \dots \dots \dots (8)$$

where, A , d , and g are constants determined experimentally, and c_{∞} represents now the ultimate specific creep.

Thomas found that the ratio of the limiting creep to that occurring during the first year under load varies little and suggested that it does not exceed 4/3 for specimens loaded at the age of 28 days. For specimens loaded at later ages the creep after 1 year will be smaller and thus the ratio is an increasing function of the age at loading, as shown in Fig. 10. The figure is useful but in order to find the value of the limiting creep it is necessary to know the value of creep after the first year under load; such a period may not be convenient in practice. Furthermore, the value of 4/3 may be too low, as a value of 1.36 has been actually obtained (Table 3).

McHenry's³ exponential expression also assumes that creep is proportional to the amount of potential creep remaining. The specific creep is given by

$$c = (\alpha + \beta e^{pT}) (1 - e^{-rt}) \dots \dots \dots (9)$$

where, T = age at the time of application of load ($T > 5$ days), t = time since the application of load, and α , β , p , and r are constants determined experimentally.

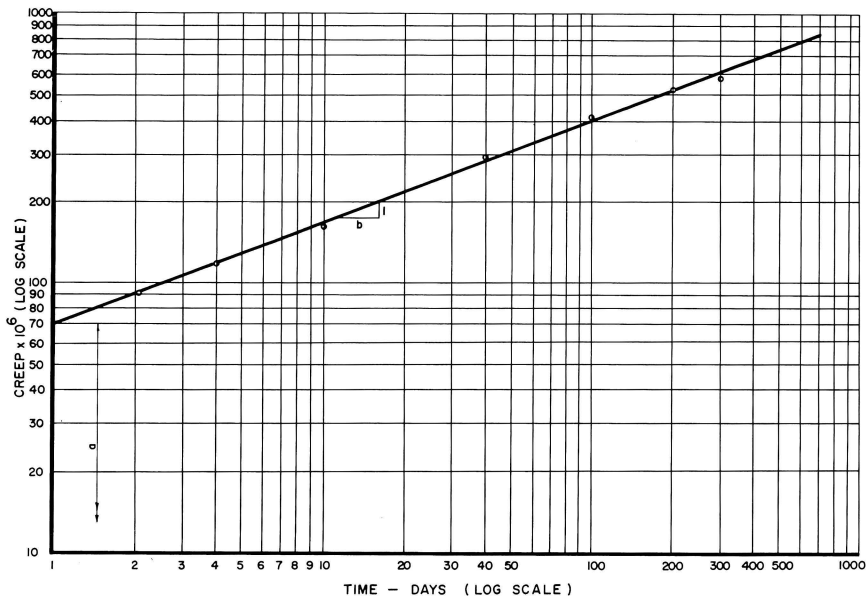


Fig. 11—Creep constants according to Shank

At an early stage, a transient creep may exist but this disappears at a rate proportional to itself. Thus,

$$c = \alpha(1 - e^{-rt}) + \beta e^{-pt} (1 - e^{-qt}) \dots \dots \dots (10)$$

where, q = an experimental constant. If $q = r$, Eq. 10 reduces to Eq. 9.

A simple exponential equation was suggested by Shank,³⁹ the specific creep is given by

$$c = at^{1/b} \dots \dots \dots (11)$$

where, a is a constant, and b is a coefficient depending on the properties of concrete.

The values of a and b can be obtained from a plot of $\log c$ against $\log t$ (Fig. 11) since the latter two plot as a straight line.

Eq. (11) can be used for estimating creep up to about 1 year under load only, as for longer periods under load creep increases at too great a rate. Furthermore, the expression postulates an indefinite increase in creep.

A similar approach is adopted by Saliger⁴⁰ who expresses creep approximately as

$$c = \alpha_t \times \sigma \dots \dots \dots (12)$$

where α_t is a creep coefficient of the form

$$\alpha_t = A \sqrt[3]{t} \dots \dots \dots (13)$$

and is plotted in Fig. 12.

Eq. (12) does not approach a finite limit, but Saliger assumes that maximum creep is reached at the age of 30 months. This would mean that concrete loaded at the age of 30 months will show no creep, which is not

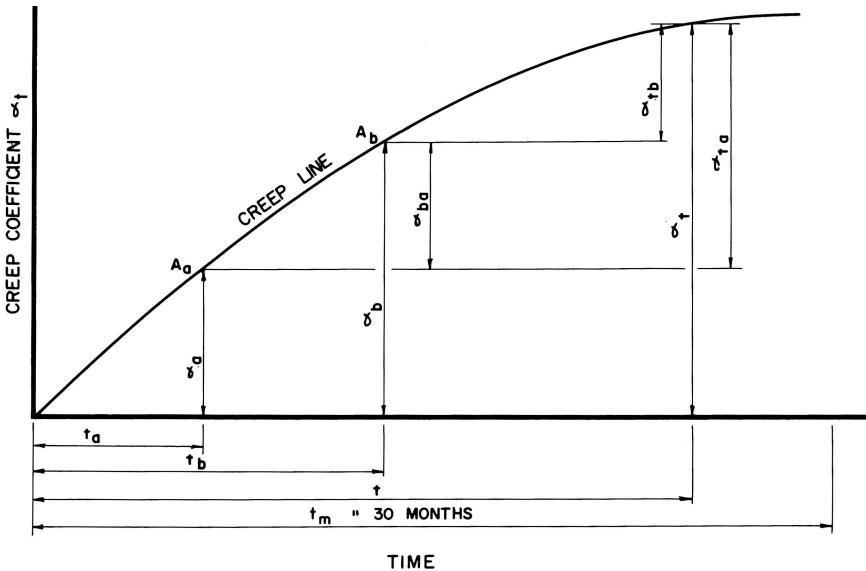


Fig. 12—Saliger's creep coefficient

the case, and for this reason Eq. (12) cannot be used for loading ages greater than a few months.

Saliger assumes that concrete subjected to a sustained load responds elastically to the action of live loads additional to the sustained load, i.e., that live loads produce only elastic deformations corresponding to the instantaneous modulus of elasticity at the time of application of the live load.

The creep coefficient, α_t , has been obtained empirically for a "standard" age at loading. If a sustained stress σ_a is imposed at time t_a greater than the standard age, creep should be calculated as if the curve showing the relation between the creep coefficient and time started from point A_a (Fig. 12). Now, if after time t_b an additional sustained stress σ_b begins to act, the additional creep is calculated as if the creep line started at A_b .

Saliger postulates that strains produced at any given time by a stress applied at the time t_o are independent of any stress applied either earlier or later than t_o . This principle of superposition was first postulated by McHenry,³ who applied it also to creep recovery by considering a relief of

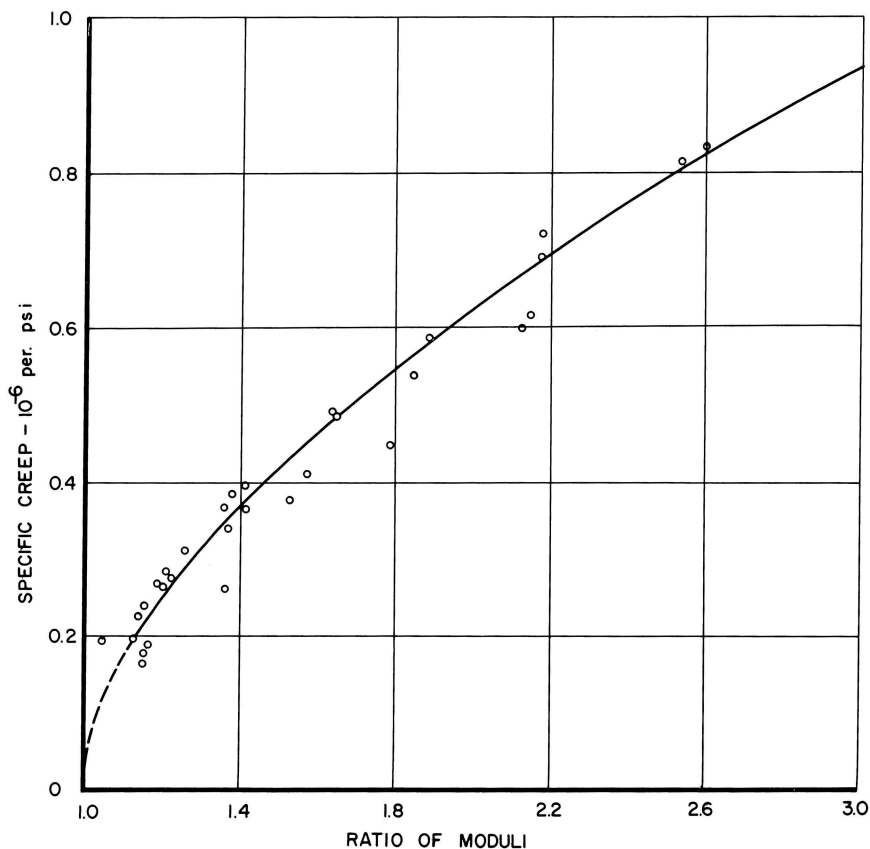


Fig. 13—Relation between specific creep at any time and the ratio of the modulus of elasticity at that time to the modulus at the time of application of load¹¹

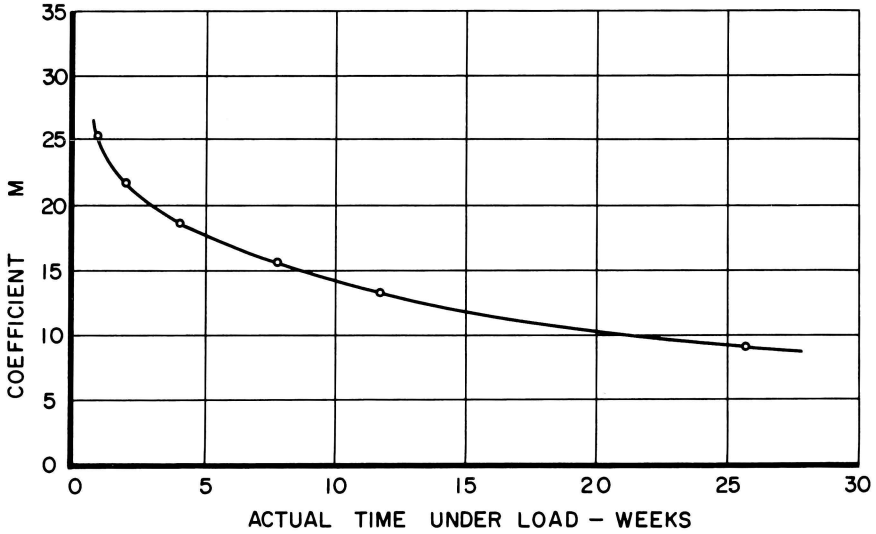


Fig. 14—Accuracy of predicting creep after 1 year under load from short-time tests

stress as a negative stress. In actual fact, the application of the principle of superposition introduces a fixed bias but is nevertheless convenient for most practical purposes.

The U. S. Bureau of Reclamation, which has made an extensive study of creep of concrete dams and of methods of predicting creep, has found that specific creep can be represented by the expression

$$C = F(T) \log (t+1) \dots \dots \dots (14)$$

where T is the age at which the load is applied, $F(T)$ is a function representing the rate of creep with time, and t is reckoned in days.

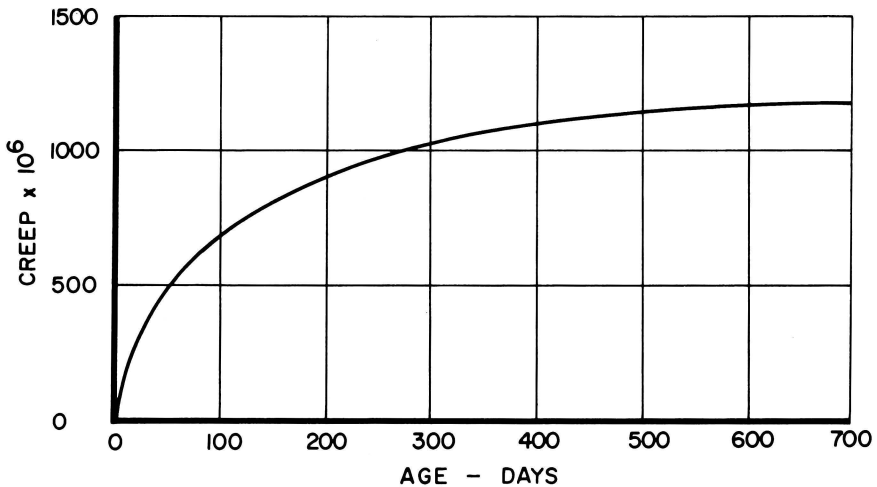


Fig. 15—"Standard" creep curve⁴²

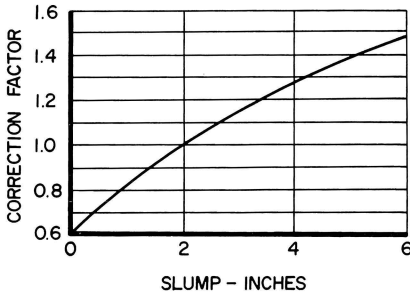


Fig. 16—Correction factor for slump

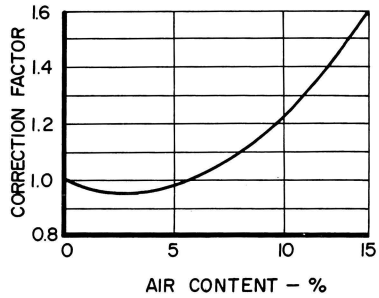


Fig. 17—Correction factor for air content

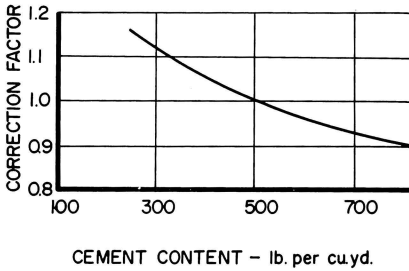


Fig. 18—Correction factor for cement content

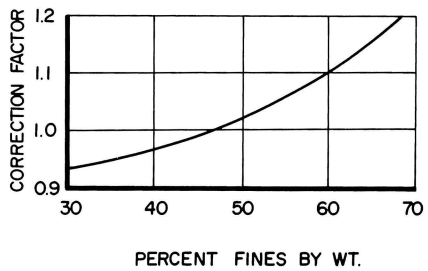


Fig. 19—Correction factor for percent fines

Creep plots thus as a linear function of the logarithm of the time under load (or, strictly speaking, $t + 1$), but values for short periods under load depart from the straight line (see discussion of Fig. 8 and 9).

In the Bureau studies,⁴¹ it was also shown that creep can be estimated from the change in the elastic properties of concrete. Fig. 13 shows the specific creep at any time plotted against the ratio of the modulus of elasticity at that time to the modulus at the time of application of the load; the data cover a wide range of ages at which the load was applied. Such a relation indicates that for a given mix, the modulus of elasticity of concrete (as a function of time) is a useful parameter in estimating creep. While the modulus is easier to determine than creep, the necessary information can be obtained only after a long period.

GENERAL PREDICTION DATA

In all the equations mentioned in the preceding section the constants have to be determined empirically, i.e., limited time creep tests must be made using the actual mix and storage conditions. The longer the time over which creep is actually measured the better the prediction. This can be seen from Fig. 14, which shows the error coefficient, M , after periods under load of 1 to 26 weeks for tests made at the University of Missouri. The coefficient is defined as

$$M = \frac{\sqrt{(c_t - c_1)^2/n}}{c_1} \dots \dots \dots (15)$$

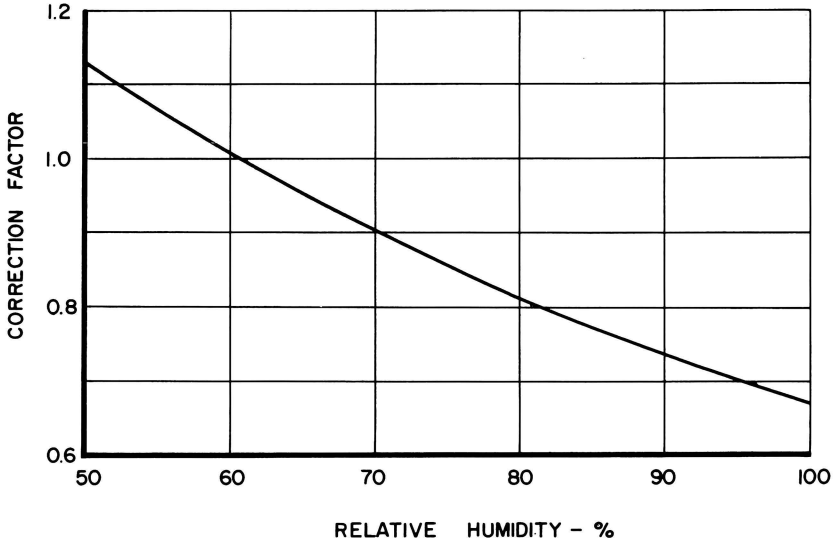


Fig. 20—Correction factor for relative humidity of storage

where c_t = creep after 1 year predicted from measured creep after time t weeks under load ($1 \leq t \leq 26$); c_1 = actual creep after 1 year under load; n = number of specimens for which creep was observed at time t . M is thus analogous to the coefficient of variation but deviation is measured from the “true” creep after 1 year. The difficulty of early prediction is apparent but if a coefficient M of about 15 percent is acceptable, creep tests need be run for about 60 days only.

There is a considerable amount of creep data available in technical literature and from this it might be possible to calculate coefficients and parameters for predicting creep of any concrete under any conditions. Two attempts have, in fact, been made.

Jones, Hirsch, and Stephenson⁴² use a “standard” creep curve shown in Fig. 15. This is for a concrete containing 540 lb of cement per cu yd, with

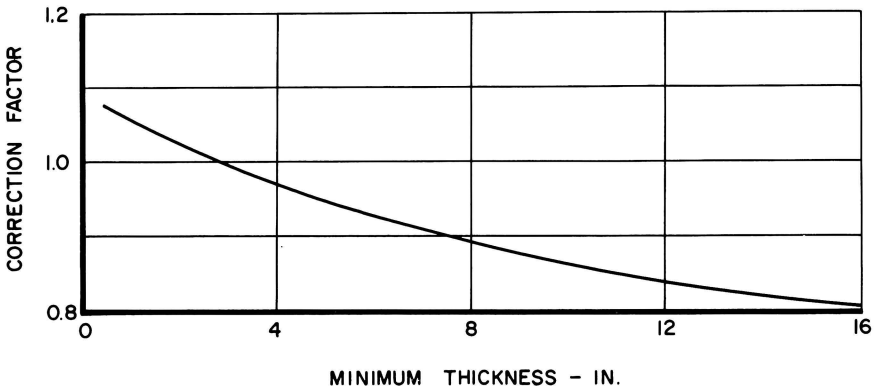
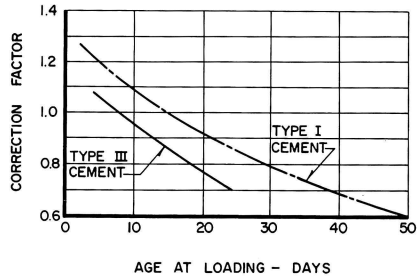


Fig. 21—Correction factor for minimum thickness of concrete

Fig. 22—Correction factor for age at loading



6 percent entrained air, and a 2-in. slump, subjected to a stress of 1420 psi at a relative humidity of 60 percent. The “standard” values of creep are then modified for the particular slump, air content, cement type and content, percent fines, relative humidity of storage, thickness of the member, and age at loading, using the correction factors shown in Fig. 16 to 22. This method was developed for lightweight concrete but appears to be valid for normal weight concrete as well. We may note that some of the factors listed (e.g., slump or fines content) are not basic factors in creep but are convenient for practical application.

A similar approach is used by Wagner.¹⁶ “Standard” values of ultimate specific creep are shown in Fig. 23; these values are then modified by coefficients given in Fig. 24 to 27. A comparison of the predicted ultimate creep using Wagner’s method with 4/3 of the measured 1-year creep is shown in Table 4. In most cases the agreement is poor and it is possible that this is at least in part due to the properties of aggregate which are not considered in Wagner’s curves.

Fig. 28 and 29 show creep curves derived from experimental data (for the conditions indicated) using some of the creep expressions of the preceding section. In calculating the constants creep values recorded during

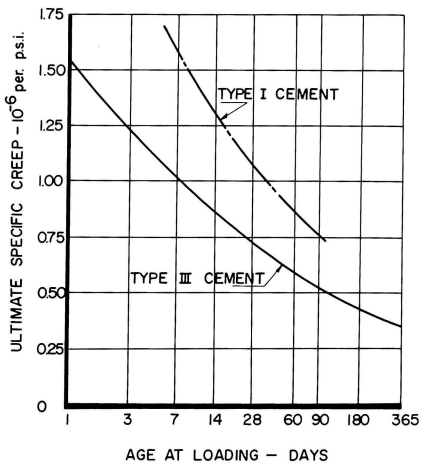


Fig. 23—“Standard” values of ultimate specific creep¹⁶

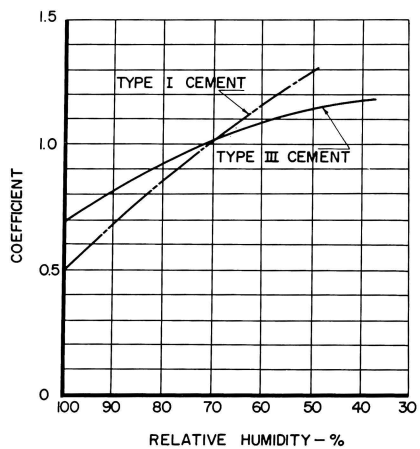


Fig. 24—Coefficient for relative humidity

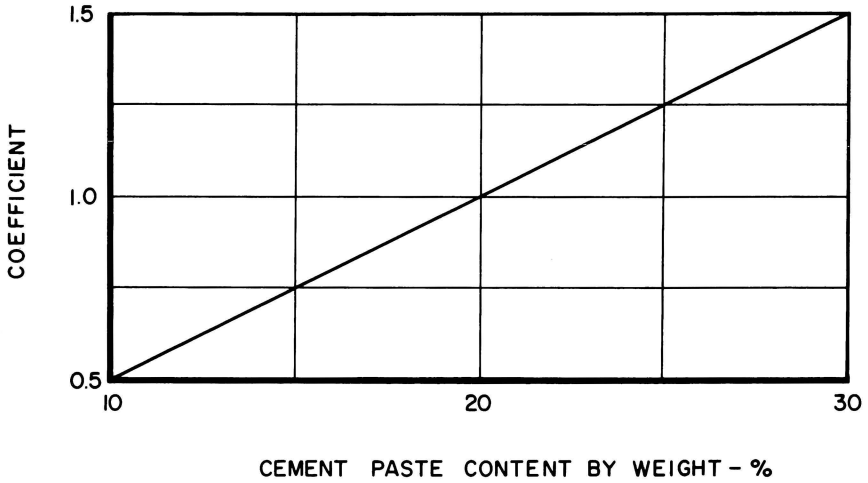


Fig. 25—Coefficient for cement content

the first 100 days under load were used. Observed creep values up to nearly 400 days under load are also plotted. The agreement between the observed and predicted values is fairly good but it must be remembered that prediction was based on measured creep up to 100 days under load, at which stage creep is equal to at least one-half of the total measured creep. All the equations were then tested using creep values recorded for periods of less than 100 days under load. For the most part the results were considered unsatisfactory (i.e., the coefficient *M* was more than 15 percent).

Summarizing, it seems that if it is possible to make at least a 60-day creep test, the equations of Lorman and Ross give satisfactory results. They are extremely easy to apply and have the advantage of quickly predicting the ultimate creep. If no tests can be made, the methods developed by Jones and by Wagner have to be used. These yield results which may not be sufficiently accurate when a structure is very sensitive to creep, but in the case of reinforced concrete members the predicted values are suffi-

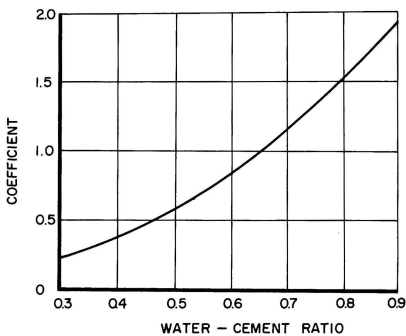


Fig. 26—Coefficient for water-cement ratio

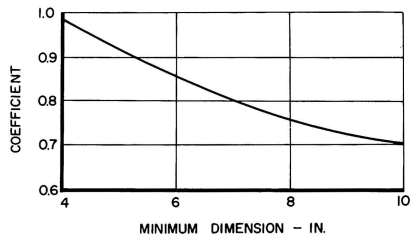


Fig. 27—Coefficient for specimen size

TABLE 4—COMPARISON OF PREDICTED AND OBSERVED CREEP

Age at loading, days	Predicted ultimate specific creep, 10^{-6} per psi (Fig. 23)	Sustained stress, psi	Predicted "standard" ultimate creep, 10^{-6}	Coefficient for relative humidity (Fig. 24)	W/C ratio	Coefficient for W/C (Fig. 25)	Cement paste content by weight	Coefficient for cement content (Fig. 26)	Predicted ultimate creep (Wagner)	Observed creep after 1 year	4/3 observed creep
31	1.05	1200	1260	1.29	0.624	0.85	0.206	1.00	1380	611	814
31	1.05	1200	1260	1.29	0.624	0.85	0.206	1.00	1380	626	835
31	1.05	1200	1260	1.29	0.624	0.85	0.206	1.00	1380	604	806
7	1.60	1200	1920	1.29	0.624	0.85	0.206	1.00	2110	1087	1450
3	2.00	1200	2400	1.29	0.624	0.85	0.206	1.00	2640	1150	1535
3	2.00	1200	2400	1.29	0.624	0.85	0.206	1.00	2640	1161	1550
2	2.30	1200	2700	1.29	0.624	0.85	0.206	1.00	3030	1080	1440

INFLUENCING FACTORS AND PREDICTION

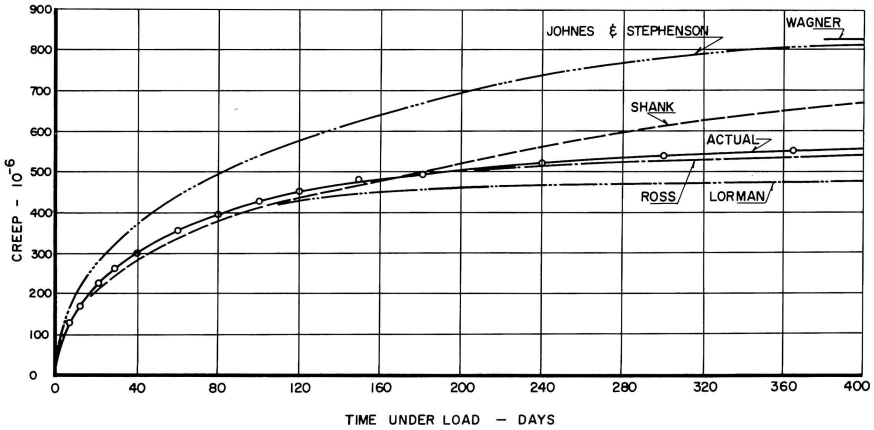


Fig. 28—Comparison of prediction methods for a 1:6.3 mix, $w/c = 0.625$, slump = 2 in., $f_c' = 5230$ psi, stress-strength ratio = 0.23, stored at 72 F, and a relative humidity of approximately 50 percent

cient for design purposes: the deflection of these members is not as sensitive to creep as has been thought in the past. This is clearly demonstrated by Pauw and Meyers in Paper No. 6 of this symposium.

CONCLUDING REMARKS

The paper has considered only the experimental-empirical approach to the prediction of creep. It must be mentioned, however, that rheological models can also describe the deformational response of concrete. They can use constants derived experimentally or from sonic properties of concrete. Kesler's³⁶ pioneer work on relating the natural frequency and logarithmic decrement to creep promises considerable success. Rheological models can be designed so that a full range of loading conditions can be studied on an analog computer. It would seem that the next logical step would be to develop models to simulate the actual behavior of a flexural member subjected to creep, and to investigate the stress redistribution under conditions of a non-linear stress-creep relation.

Note

ACI Committee 209 is currently preparing a bibliography on creep. The bibliography, which is annotated, starts with the earliest known papers on creep: I. H. Woolson's observations on "flow of concrete under pressure" in 1905 and W. K. Hatt's 1907 report on the increase in the deflection of reinforced concrete beams, and continues with more than 500 entries up to the present time. The bibliography will thus complement the symposium papers collected in the present volume. An earlier bibliography was prepared by Meyers.²

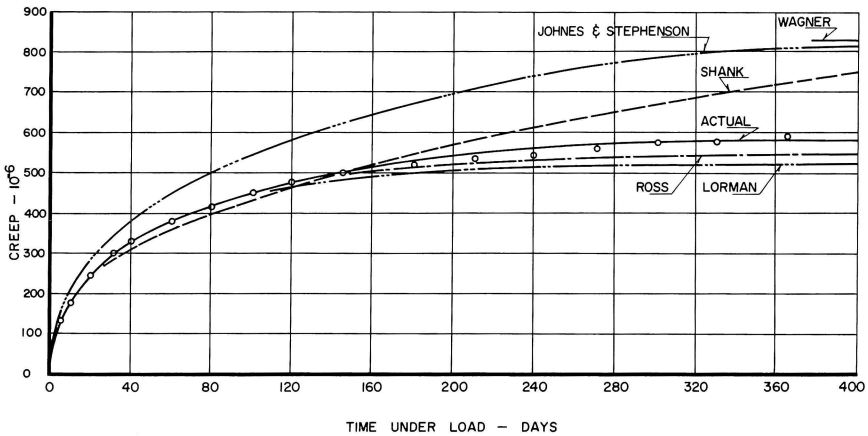


Fig. 29—Comparison of prediction methods for the same concrete as that of Fig. 28 but for different specimens

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DISCUSSION

By **C. H. Scholer***

In most studies of creep of concrete, it is assumed that concrete is a material composed of various constituents and that, under load, these constituents are forced to carry rather uniformly distributed stresses which in turn produce rather uniform deformations.

When a load is maintained uniformly for a prolonged period, it is found

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that the deformation usually increases with time, and this time-dependent deformation is considered to be creep. Shrinkage also will develop during this period and is recognized to be at least partly independent of creep.

As a matter of fact, concrete is not a material but a system of materials held together by a cement paste which bonds these particles together and causes the particular mass of concrete, called a specimen, to appear to deform much as though it were a uniform isotropic material having certain specific properties of elasticity, strength, creep, and shrinkage.

There is undoubtedly a wide variation in the way in which the discrete pieces of aggregate are loaded and in the amount and direction of yielding under these forces with time. The forces applied to the specimen are applied to the paste which has bonded to the surface of aggregate by contact. These surface forces will tend to deform these pieces of aggregate, to hold them together, and cause them to act much as though the whole mass was a homogenous, isotropic whole having certain elastic and strength characteristics.

The ability of the paste to cause the particles of matter to so perform will depend upon the strength of the paste (the water-cement ratio) and the relation between the surface which is grasped by the paste, and the volume and elastic properties of the particle. All forces must be transferred from the paste to the aggregate, and the relation between the surface area of the aggregate particle and the volume of the particle will be of great significance. The ratio of the surface area of a regular polyhedron to its volume is $6/D$, where D is the diameter of the inscribed sphere. This general relationship to the size of aggregate no doubt explains the significance of size of aggregate in relation to strength of concrete.

The elastic properties of the material in these discrete particles is one of the principal factors influencing the apparent elastic properties of the concrete. The apparent modulus of elasticity of concrete may vary from 50 to 100 percent, depending upon the aggregate and amount of aggregate in the concrete.

In a combination of paste and aggregate in a concrete specimen, the actual forces acting in the paste and mortar surrounding a large piece of aggregate are not by any means uniform. Creep which we measure in our specimen must be the result of a widely varying distribution of forces and deformations in the pieces of aggregate and layers of paste transmitting the different forces which are the result of the loading process. The natural creep under these varying stresses is an effort to redistribute the loads and deformations without causing failures. Looking at creep in this way, it is seen to be a beneficial and valuable property of concrete.

Many writers on the subject of creep have observed that the loads cause a loss of water to develop, and some have maintained that this loss is one of the major causes of creep. Usually, it is observed that this is again not strictly true, and water loss cannot be used as a definite criteria for such a purpose.

If we stop to consider that the forces acting upon the paste and mortar most certainly cannot be uniform in character at every section in a specimen, it is obvious that creep and the accompanying loss in water in the paste will not be uniform. Some of the water is merely redistributed and is not lost.

A careful elastic analysis of stress distribution in a simple specimen like a cylinder or a cube will show a wide variation in actual stress distribution.

In so far as the writer is aware, no studies have been made upon the effect of variations in the strength of concrete in which coarse aggregate is used having various known values in different specimens, and the resultant changes in creep.

Whether the forces to which concrete is subjected are caused by loads or temperature variations, and other natural external causes, it would not seem unreasonable to believe that, as these forces develop, the constituents of concrete will again attempt to creep in such a way as to prevent failure, if possible.

A study of natural rock formation, in various exposures, shows how extensive the creep has been to permit deformations to develop to prevent rupture and failure.

After all, failure in most materials is due to the deformations produced by the forces which a member is called upon to carry. If these forces develop slowly, the members of the structure try to yield without failure. Most certainly, these tendencies are present in concrete.

Can these known characteristics be utilized to design better concrete specimens which can, to a limited extent, adjust to the condition to which they are subjected?

Develops a procedure for including the effects of creep and shrinkage in design and analysis. The objective of the procedure is to permit the designer to automatically satisfy deflection criteria as well as allowable working stresses. Working equations for the general case are developed and equations are given for special cases, including beams with and without compressive reinforcement, and prestressed concrete beams.

Effect of Creep and Shrinkage on the Behavior of Reinforced Concrete Members

By **Adrian Pauw and B. L. Meyers**

■ IN CONVENTIONAL REINFORCED CONCRETE design, preliminary design dimensions are based on strength criteria regardless of the method of design used, whether working stress or ultimate strength. Such designs must of course be checked and modified to satisfy deflection criteria under service load conditions. The applicability of the conventional working stress design method is limited by the fact that creep and shrinkage causes a redistribution of internal stresses so that computed stresses do not reflect the actual stress distribution in the section. This difficulty is avoided in ultimate strength design by considering only the stress distribution at collapse load. At this load, the stress distribution is independent of the effects of creep and shrinkage.

Both design methods are deficient in that they are predicated on strength alone whereas the size of many structural elements may be governed by the deflection under service-load conditions, thus, requiring the designer to modify his preliminary design. The importance of deflection as a design criterion is recognized in the ACI Building Code (ACI 318-63).¹ Section 909 of this code (ACI 318-63) requires that "Reinforced concrete members subject to bending shall be designed to have adequate stiffness to prevent deflections or other deformations which may adversely affect the strength or serviceability of the structure." These requirements may be satisfied by using specified minimum thickness or depth ratios or

by computing the immediate elastic deflections under service loads. The effect of additional long-time deflections may be estimated by applying specified multiplication factors to the computed immediate deflections for the sustained-load component. Based on the analysis developed in this paper, and supported by a limited series of tests, it appears that under normal conditions the additional deflection due to creep is small compared to that due to shrinkage. Hence, computations based on an arbitrary multiplication factor may result in serious errors in predicting total deflection.

The analysis developed herein is based on the following premises:

1. Deflection criteria should be based on service-load conditions and section properties may be based on a cracked section.
2. Under service-load conditions and for the usual permissible deflection limits, the strain distribution at any section is linear both for instantaneous and sustained loads on the member.
3. The concrete stress in the compressive zone is proportional to the strain. The constant of proportionality is given by

$$E_{ct} = \frac{E_c}{C_{ct}} \dots \dots \dots (1)$$

where E_c is the modulus of elasticity of the concrete for initial load application, and C_{ct} is a creep coefficient assumed to be a constant for all stress levels in the working range and for a specified elapsed time after load application.

4. The stress in the reinforcement is proportional to the strain; the constant of proportionality being E_s the modulus of elasticity of the steel.

5. The potential shrinkage is assumed to be uniform across the section.

The assumption of a cracked section in computing the section properties is only slightly conservative since the increased section modulus in regions of low moment does not decrease the deflection materially. Moreover, this assumption gives more realistic relative values than does the usual assumption of basing the section modulus on the gross section. This is especially true in the case of beams reinforced for compression, T-beams, and beams using lightweight aggregate concrete. The justification for assuming a constant creep factor for all stress levels in the working range, i.e., a

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creep factor which is time-dependent only and is not stress-dependent, has been discussed in the first paper of this symposium. The assumption of a uniform shrinkage potential throughout the section is only approximately true for concretes that have been completely dried—a condition which is seldom encountered in practice. The assumption can only be justified as a convenience for determining an upper limit of the deflection component due to shrinkage.

The objective of the analysis which follows below is twofold: (1) to present a theoretical basis for explaining the effect of creep and shrinkage on the behavior of reinforced concrete members; and (2) to provide the designer with a convenient procedure for establishing the shallowest permissible depth which will satisfy specified deflection criteria.

Work previously reported² is extended by this analysis to include beams and slabs with compressive reinforcement, T-, I-, or box-beams, and includes the effect of creep and shrinkage on the deflection of flexural members. Charts are provided to clarify the effect of various design parameters and to simplify the calculations required.

NOTATION

The notation used is consistent with that used by the ACI code (ACI 318-63)¹ except for the symbols listed below.

Parameters affected by creep or shrinkage are designated by the following subscripts:

- i = subscript denoting initial value immediately following application of load.
- t = subscript denoting value at a given time after load application.
- T = subscript denoting ultimate time dependent value for sustained loads.
- sh = subscript denoting shrinkage.
- α = dimensionless coefficient dependent on the type of loading, degree of continuity, and distribution of relative stiffness along the beam with respect to stiffness at the critical section considered.
- γ = the ratio of the overhanging flange area to the web area in T-, I-, or box-beams.

$$\gamma = \frac{(b-b')t}{b'd} \dots \dots \dots (2)$$

Δ = the maximum deflection in same units as span L .

p = tensile steel ratio.

p' = compressive steel ratio.

Section parameters (Fig. 1) for rectangular beams, and for T-, I-, or box-beams where $kd < t$ are

$$p = \frac{A_s}{bd} \quad \text{and} \quad p' = \frac{A_s'}{bd}$$

Section parameters for T-, I-, or box-beams where $kd < t$

$$p = \frac{A_s}{b'd} \quad \text{and} \quad p' = \frac{A_s'}{b'd}$$

p_e' = equivalent effective area ratio for compression steel and/or the overhanging flange area in the case of T-, I-, or box-beams.*

Section parameters for rectangular beams, and for T-, I-, or box-beams where $kd < t$

$$p_e' = \frac{(n-1)}{n} p' \dots \dots \dots (3a)$$

Section parameters for T-, I-, or box-beams where $kd < t$

$$p_e' = \frac{(n-1)p' + \gamma}{n} \dots \dots \dots (3b)$$

p_r = effective reinforcement ratio.

$$p_r = \frac{p_e'}{p} \dots \dots \dots (4)$$

d_e' = depth to centroid of equivalent compressive steel area defined by p_e' .

$$d_e' = \frac{\gamma t + 2(n-1)p'd'}{2n p_e'} \dots \dots \dots (5)$$

d_r = effective steel depth ratio.

$$d_r = \frac{d_e'}{d} \dots \dots \dots (6)$$

S = ratio of tensile reinforcement in equilibrium with compression area $kb'd$ defined by Eq. (8) and (9).

K = creep effect ratio.

$$K_{f_s} = \frac{f_s T}{f_{ci}}; K_{f_c} = \frac{f_c T}{f_{si}}; K_{\Delta} = \frac{\Delta T}{\Delta i}$$

Q = deflection design parameter, product of three dimensionless ratios defined by Eq. (27).

SERVICE-LOAD STRESS ANALYSIS

Instantaneous load application

Essential to the analysis is the use of a realistic modular ratio. For computing the stress distribution and deflection immediately after application of the service loads this ratio is $n = E_s/E_c$. For deflection computations E_c is preferably determined by test but may be approximated by the relationship

$$E_c = w^{1.5} 33 \sqrt{f_c'}$$

Since for service loads, plane sections may be assumed to remain plane

*For working stress analysis the effect of the compression steel is increased to $(2n-1)p'/n$ in accordance with Section 1102c of ACI 318-63.¹

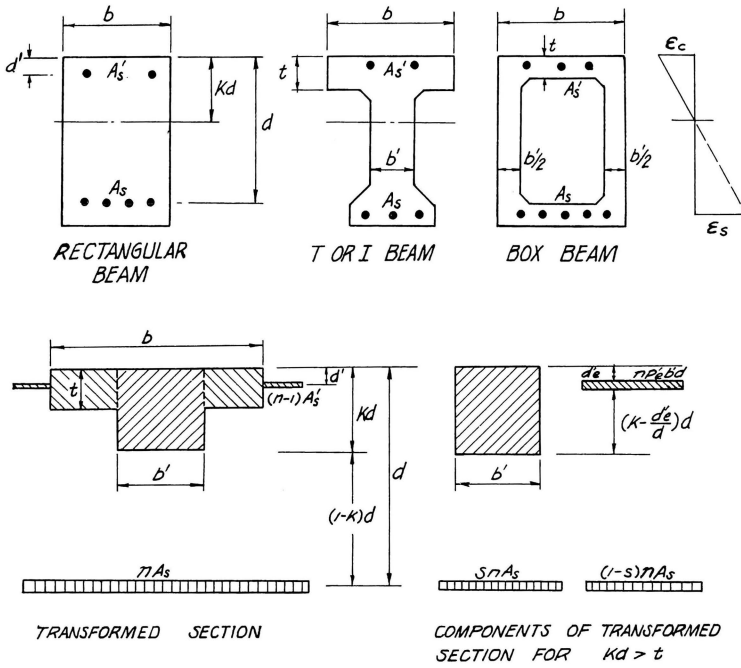


Fig. 1—Section parameters for beams

after loading, and stresses may be assumed to be proportional to strain, it follows that the strain ratio for any section is

$$\frac{\epsilon_c}{\epsilon_s} = \frac{k}{1 - k} = \frac{f_c}{f_s/n} \dots \dots \dots (7)$$

For purposes of considering equilibrium of the transformed section of a beam (Fig. 1) the tensile reinforcement may be split into two components, viz. SA_s in equilibrium with the compressive area in the stem, and $(1 - S)A_s$ in equilibrium with the compressive reinforcement. The two equations of equilibrium are therefore

$$npS = \frac{k^2}{2(1 - k)} \dots \dots \dots (8)$$

$$p_r = \frac{p_e'}{p} = \frac{(1 - S)(1 - k)}{(k - d_r)} = \frac{(1 - S)}{k_r} \dots \dots \dots (9)$$

where

$$k_r = \frac{(k - d_r)}{(1 - k)} \dots \dots \dots (10)$$

For a given design the above equations are readily solved for S and k by

a simple trial-and-error procedure using the graphs in Fig. 2. With S set equal to unity an initial trial value for k is determined from the graph for npS . With this trial value the value of k_r is obtained for the appropriate d_r ratio. With the value k_r and the appropriate ratio for p_r , the value of S is determined and npS is computed to establish a revised value of k . The process is repeated until the values of S and k are in balance. This normally requires less than three cycles. For T-, I-, and box-beams an initial value of kd can best be estimated by first assuming that $kd \leq t$ and calculating n/r and p_e'/n on the basis of the over-all width b of the section. The procedure will be further illustrated in the examples.

The moment of inertia of the transformed section can be expressed in terms of S and k by substituting the results of Eq. (8) and (9), and may be shown to be

$$I = np \frac{(1-k)}{3} [3(1-d_r) + (3d_r - k)S] b'd^3 \dots \dots \dots (11)^*$$

For singly-reinforced rectangular beams and slabs Eq. (11) reduces to

$$I = np \frac{(1-k)(3-k)}{3} bd^3 \dots \dots \dots (12)$$

*For $kd < t$, b' is replaced by b .

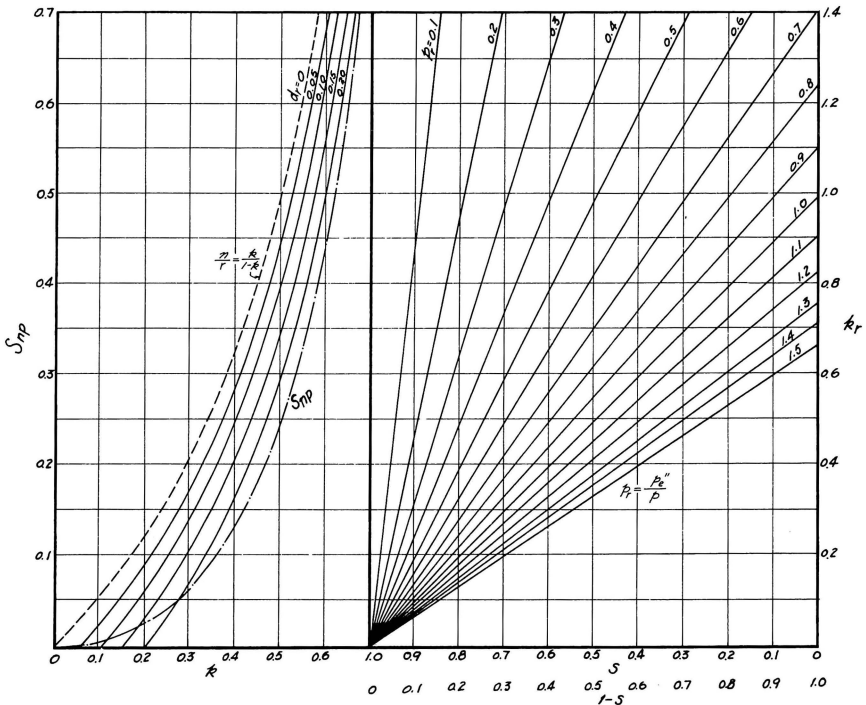


Fig. 2—Charts for determining S and k for doubly-reinforced beams

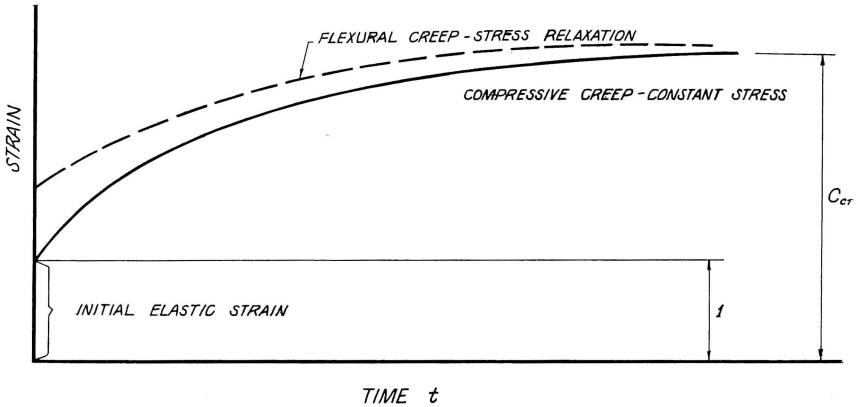


Fig. 3—Basic configuration of creep

The stress in the tensile steel is given by

$$\begin{aligned}
 f_s &= n \frac{M(1-k)d}{I} = \frac{3M}{p[3(1-d_r)+(3d_r-k)S]b'd^2} \\
 &= \frac{6Sn(1-k)M}{k^2 [3(1-d_r)+(3d_r-k)S]b'd^2} \dots \dots \dots (13)
 \end{aligned}$$

Eq. (13) reduces to

$$f_s = \frac{3M}{p(3-k)bd^2} = \frac{6n(1-k)M}{k^2 (3-k)bd^2} \dots \dots \dots (14)$$

for singly-reinforced rectangular beams.

The compressive stress in the extreme fiber can be obtained by solving Eq. (7) for f_c and substituting in the above equations, thus for the general case

$$f_c = \frac{6SM}{k[3(1-d_r)+(3d_r-k)S]b'd^2} \dots \dots \dots (15)$$

and for the special case of a singly-reinforced rectangular beam

$$f_c = \frac{6}{k(3-k)} \frac{M}{bd^2} \dots \dots \dots (16)$$

Effect of sustained loads on stress distribution

Under a sustained load, flexural stresses are redistributed as a result of creep deformations in the compressive zone. This redistribution is a gradual process roughly paralleling the basic creep curve, but it is modified by a relaxation of the concrete stresses in the extreme compressive fibers. As shown in Fig. 3, the maximum flexural creep strains, as a result of this stress relaxation, tend to stabilize somewhat more rapidly than do compressive creep strains under constant stress.

For ultimate strains, i.e., at large values of t , the total strain, and hence the creep coefficient C_{ct} is primarily dependent on the final stress level. It has been shown that for large values of t , C_{cT} for a given concrete approaches a constant value C_{ct} which is relatively independent of stress. The ultimate stress distribution for sustained service loads may therefore be determined with reasonable accuracy by replacing E_c in the previous analysis by a modified elastic modulus shown by Eq. (1). Eq. (3) and (4), and (7) through (16) therefore remain valid by replacing the modular ratio n by

$$n_T = C_{cT} n$$

and by designating the n -dependent parameters p_r , d_r , k , k_r , S , and r by subscripts T .

The values of $p_{r,T}$ and $d_{r,T}$ can be computed directly by substituting n_T for n in Eq. (3) and (5) respectively. The values of k_T and S_T may be obtained by the use of the charts in Fig. 2, utilizing the procedure described in the previous section.

The effect of creep on the ultimate redistribution of stress may be determined by comparing the value of k_T with the value of k for instantaneous load application. Three classes of flexural members should be considered.

1. Singly-reinforced rectangular beams and slabs. S is equal to unity and hence k_T may be determined directly from Fig. 2 by entering with $C_{cT} np$ on the $Sn p$ curve. k_T is therefore greater than k , indicating an increase in the depth of the compression zone. This increase results in a small decrease of the internal moment arm, $1 - (k_T/3)$ and hence a corresponding small increase in the tensile-steel stress. The increase in the depth of the compression zone results in a marked decrease of the concrete stress in the extreme fiber as shown in Fig. 4.

2. Doubly-reinforced beams and slabs. The increase in k due to creep is moderated by the stiffening effect of the compression steel. This becomes immediately clear by considering the curves in Fig. 2. An increase in k is reflected by a corresponding increase in k_r . However, p_r is relatively unaffected by a change in the modular ratio and hence an increase in k_r results in a corresponding decrease in S . Thus, the ratio $S_T n_T p / Sn p$ is less than C_{cT} and the ratio k_T / k is therefore smaller than for the previous case.

3. Singly-reinforced T-beams. For this case

$$p_r = \frac{\gamma}{np}$$

and hence $p_{r,T}/p_r$ is inversely proportional to C_{cT} . Hence S_T is greater than S and the ratio $S_T n_T p / Sn p$ is greater than C_{cT} . For such beams the ratio k_T / k is greater than for singly-reinforced beams with a corresponding initial compressive depth ratio.

The increased tensile steel stress can be computed conveniently by using

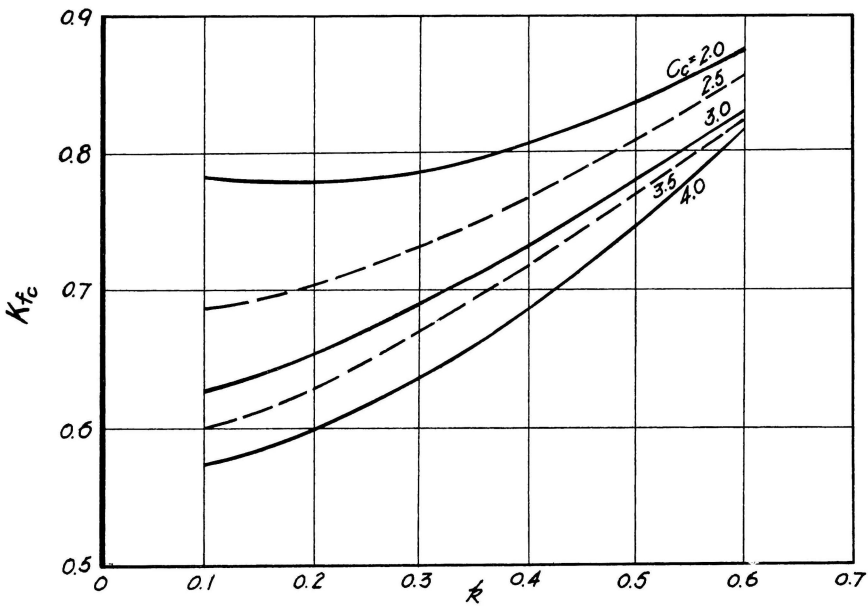
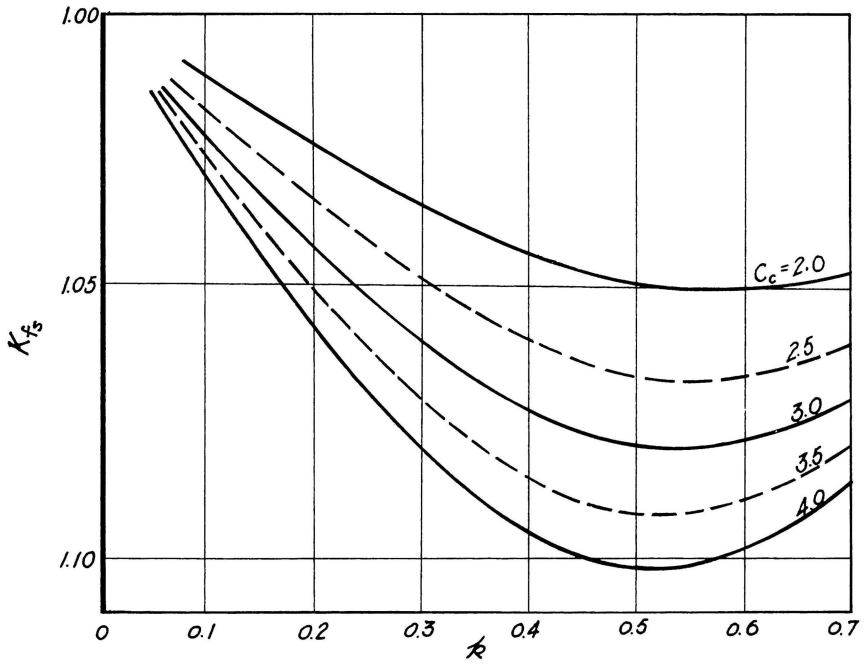


Fig. 4—Creep stress coefficients for singly-reinforced rectangular beams

the ratio K_{f_s} of the stress after creep to the immediate stress. Since M , p , b' , b and d are invariant quantities, we have from Eq. (13)

$$K_{f_s} = \frac{3(1-d_r) + (3d_r - k)S}{3(1-d_{rT}) + (3d_{rT} - k_T)S_T} \dots \dots \dots (17)$$

For singly-reinforced rectangular beams $d_r = d_{rT} = 0$, $S = S_T = 1$, and hence Eq. (17) reduces to

$$K_{f_s} = \frac{3 - k}{3 - k_T} \dots \dots \dots (18)$$

For this case the ratio k_T can be expressed in terms of k and C_{cT} and hence K_{f_s} can also be expressed as a function of C_c and k . This relationship is shown graphically in Fig. 4.

The ratio of the compressive stresses in the extreme concrete fiber can be obtained in a similar manner from Eq. (15). Thus,

$$K_{f_c} = \frac{S_T k [3(1-d_r) + (3d_r - k)S]}{S k_T [3(1-d_{rT}) + (3d_{rT} - k_T)S_T]} = \frac{S_T k}{S k_T} K_{f_s} \dots \dots \dots (19)$$

For rectangular beams without compressive reinforcement, Eq. (19) reduces to

$$K_{f_c} = \frac{k(3-k)}{k_T(3-k_T)} = \frac{k}{k_T} K_{f_s} \dots \dots \dots (20)$$

From the graphical representation of Eq. (20) in Fig. 4 it may be seen that creep has a beneficial effect in that the resulting stress redistribution markedly reduces the concrete compressive stress due to sustained loads.

From Eq. (17) and (19) it may also be deduced that

$$\frac{f_{sT}}{f_{cT}} = \frac{K_{f_s} f_s}{K_{f_c} f_c} = \frac{S k_T}{S_T k} r \dots \dots \dots (21)$$

where $r = f_s / f_c$.

DEFLECTION CALCULATIONS

Instantaneous load deflection

The maximum deflection Δ in a beam of span L subjected to any loading may be expressed by the relationship

$$\Delta = \alpha \frac{ML^2}{EI} \dots \dots \dots (22)$$

where M is the maximum moment and α is a constant which is a function of the type of loading, the type of end restraints, and the relative moment

of inertia with respect to the moment of inertia, I , at the point of the maximum moment M . Typical values for α for simple beams of constant moment of inertia may be shown to be 0.0833 for a concentrated load at the center and 0.1042 for a uniformly distributed load.² Values of α for continuous beams are readily obtained by the conjugate-beam method or other suitable structural analysis methods, e.g., by applying the slope-deflection equations.

Eq. (13) is valid for all concrete beams, both with or without compressive reinforcement. Solving Eq. (13) for M/I and substituting into Eq. (22), the deflection ratio Δ/L is found to be

$$\frac{\Delta}{L} = \frac{\alpha}{(1-k)} \cdot \frac{f_s}{E_s} \cdot \frac{L}{d} \dots \dots \dots (23)$$

From Eq. (23) it is immediately clear that for a given tensile-steel stress level, the deflection ratio Δ/L is a function of k and of the length-to-depth ratio L/d and is independent of the moment of inertia of the section.

Deflections due to creep

The increase in deflection due to creep is readily determined from Eq. (23) by replacing k by k_T and f_s by $K_{f_s} f_s$. The deflection ratio is therefore

$$K_{\Delta} = \frac{\Delta_T}{\Delta_i} = \frac{1-k}{1-k_T} K_{f_s} \dots \dots \dots (24)$$

and hence the deflection ratio for a sustained load after creep can be computed directly from the immediate deflection, since

$$\left(\frac{\Delta}{L} \right)_i = K_{\Delta} \left(\frac{\Delta}{L} \right) \dots \dots \dots (25)$$

For singly-reinforced rectangular beams K_{Δ} reduces to

$$K_{\Delta} = \frac{(1-k)(3-k)}{(1-k_T)(3-k_T)} \dots \dots \dots (26)$$

which can be evaluated in terms of k and C_{cT} . This relationship is shown graphically in Fig. 5.

When the desired deflection ratios Δ/L and the corresponding tensile steel stress levels can be predetermined (as is usually the case in design problems) it is convenient to define a parameter.

$$Q = \left\{ \frac{L}{\Delta} \right\} \Sigma \alpha \left\{ \frac{f_s}{E_s} \right\} K_{\Delta} \dots \dots \dots (27)$$

The summation in Eq. (27) should consider the relative deflections for the various loads applied together with a proper multiplication factor K_{Δ} for sustained loads and a reduction in the permissible Δ/L ratio to allow for

deflection due to shrinkage.* Eq. (23) can then be written in the form²

$$\frac{L}{\bar{d}} = \frac{1-k}{Q} \dots \dots \dots (28)$$

Eq. (28) is general and applies to all concrete beams. Since *k*, *Q*, and *L/d* are related linearly it is a simple matter to determine one value when the other two are known or specified. With *Q* estimated by means of Eq. (27), either *L/d* or *k* may be determined when the other parameter is specified. For design analysis, Eq. (23) to (25) are more convenient and reliable than the use of the parameter *Q*.

Shrinkage deflections

For the purpose of this analysis it is assumed that each element of the concrete in the entire cross section is subjected to a uniform shrinkage potential ϵ_{sh} which develops gradually. Therefore the resulting stresses are treated as though they are due to a sustained load. Since the reinforcement restrains the development of shrinkage the effect of shrinkage can be estimated by considering a fictitious force as shown in Fig. 6

$$F_{sh} = - \epsilon_{sh} E_s (A_s + A_s') \dots \dots \dots (29)$$

applied at the centroid of the total steel area. With this applied force the concrete develops its full unrestrained shrinkage potential. Since the shrinkage strains then are uniform across the entire cross section of the beam, such shrinkage strains do not produce beam deflections.

The deflection of the beam due to shrinkage can then be computed by considering the effect, on the transformed section, of an externally-applied load, equal and opposite to the fictitious force F_{sh} , required to restore force equilibrium. This restoring force subjects the beam to a uniform moment

$$M_{sh} = - F_{sh} e$$

where

$$e = \frac{A_s d + A_s' d'}{A_s + A_s'} - k_T d \dots \dots \dots (30)$$

Substituting $p = A_s/b'd$ and $p' = A_s'/b'd$ the uniform shrinkage moment may be shown to be

$$M_{sh} = pd' d^2 \left[1 - \frac{p'(k_T - d'/d)}{p(1 - k_T)} \right] (1 - k_T) \epsilon_{sh} E_s \dots \dots (31)$$

Substituting this moment into Eq. (22), and since from Eq. (11)

$$I_T = n_T p \frac{(1 - k_T)}{3} [3(1 - d_r) + (3d_r - k_T) S_T] b' d^3$$

*A method for estimating shrinkage is described in the section on the next page.

the following general expression for the shrinkage deflection-length ratio is obtained

$$\frac{\Delta_{sh}}{L} = 3\alpha \frac{1 - \frac{p'(k_T - d'/d)}{p(1 - k_T)}}{3(1 - d_r) + (3d_r - k_T)S} \epsilon_{sh} \frac{L}{d} \dots \dots \dots (32)$$

For a simple-span, singly-reinforced rectangular beam of uniform section properties, $\alpha = 1/8$, $d_r = 0$, and $S_T = 1$. Hence Eq. (32) reduces to the relatively simple form

$$\frac{\Delta_{sh}}{L} = \frac{3}{8} \frac{\epsilon_{sh}}{(3 - k_T)} \frac{L}{d} \dots \dots \dots (33)$$

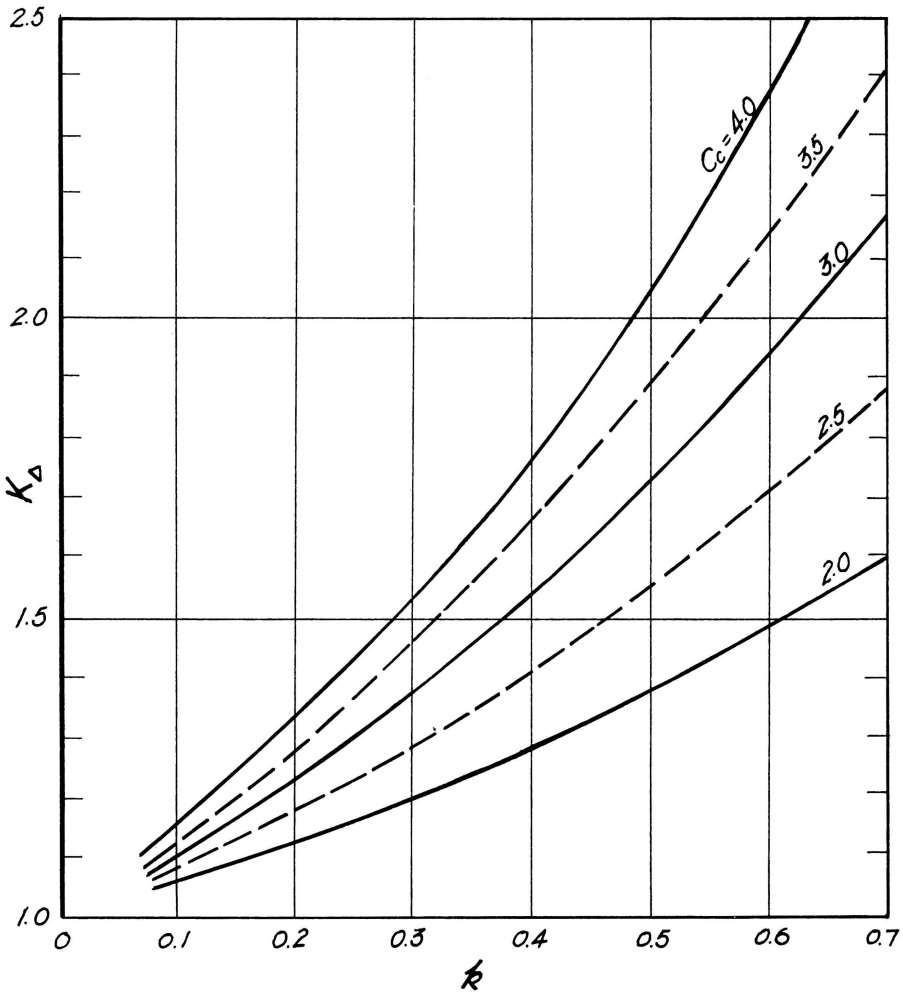


Fig. 5—Creep-deflection coefficients for singly-reinforced rectangular beams

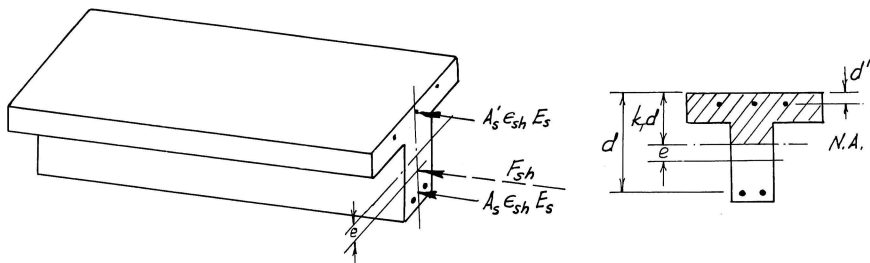


Fig. 6—Notation for shrinkage analysis

A graphical representation of Eq. (33) is given in Fig. 7. For a doubly-reinforced beam

$$p_{e'} = \frac{n_T - 1}{n_T} p' = \frac{C_{eT} n - 1}{C_{eT} n} p' \approx p'$$

From Eq. (9)

$$\frac{p'}{p} = \frac{1 - S_T}{k_{rT}}$$

and from Eq. (10)

$$k_r = \frac{k_T - d'/d}{1 - k_T}$$

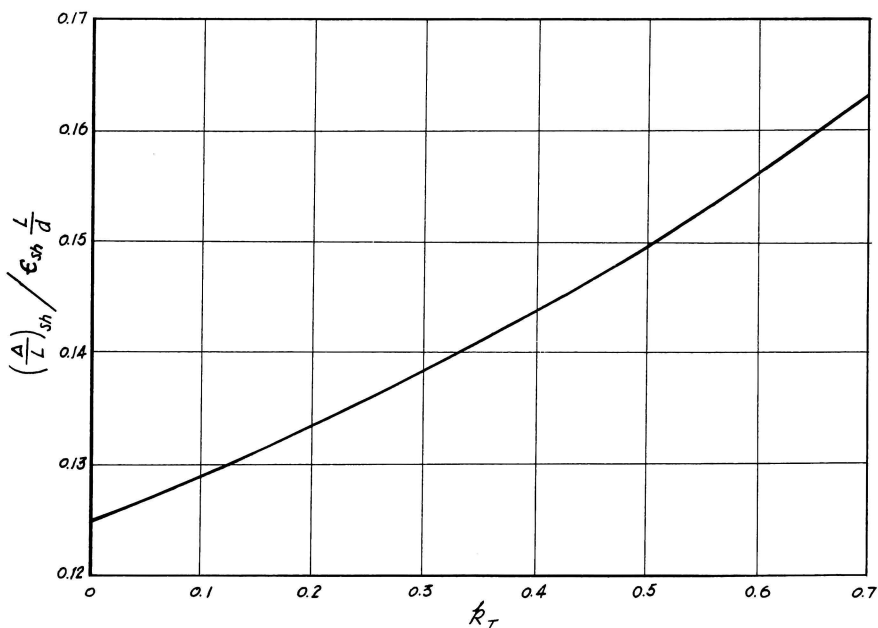


Fig. 7—Shrinkage deflection coefficients for singly-reinforced rectangular beams

TABLE 1—DIMENSIONS AND PROPERTIES OF TEST BEAMS

Beam	A_s , sq. in.	b_c^* in.	c_c^* in.	d_c^* in.	P , lb	E_c psi $\times 10^6$	E_s , psi $\times 10^6$	f_c , psi
R1	0.614	7.0	2.70	6.5	2117	4.75	29.0	4900
R2	0.884	7.0	2.84	6.5	3312	4.75	29.0	4870
R3	0.884	7.0	3.91	6.5	2844	2.36	29.0	5650
R4	1.202	7.0	3.96	6.5	4308	2.36	29.0	5620

* b_c = width of compression face; c_c = distance from extreme compression fiber to neutral axis (experimental value); d_c = distance from extreme compression fiber to centroid of the tension reinforcement.

since $d_r = d'/d$. Hence for a simple-span, doubly-reinforced beam the deflection-length ratio given by Eq. (32) reduces to

$$\frac{\Delta_{sh}}{L} = \frac{3}{8} \frac{S_T \epsilon_{sh}}{3(1-d'/d) + (3d'/d - k_T)S_T} \frac{L}{d} \dots \dots \dots (34)$$

Note that Eq. (34) reduces to Eq. (33) when $p' = 0$ since for that case $S_T = 1$ and $d'/d = 0$.

DEFLECTION OF PRESTRESSED BEAMS

The general method of analysis may be extended to apply to prestressed concrete members and is consistent with the report of Subcommittee 5, ACI Committee 435, Deflection of Concrete Building Structures.⁴ In one respect, the analysis is simpler than for ordinary reinforced beams. If the tensile cracking strains are not exceeded under service-load conditions, and the prestressing steel ratio is small, the moment of inertia and the section modulus may be assumed to remain constant under creep. This assumption is not valid for partial prestressing or when supplemental unstressed reinforcement is employed. The effect of the prestress can be conveniently analyzed by resolving the prestressing force into a centroidal force and an equivalent sustained external load.⁵ The centroidal force does not contribute to the deflection, it merely causes an axial shortening of the member. Similarly, if the steel ratio is small, the eccentricity of the shrinkage restraining force, F_{sh} , applied at the centroid of the steel reinforcement is relatively small and shrinkage does not contribute appreciably to the deflection. On the other hand, the analysis is more complex because the equivalent external load is not constant but is reduced by the effects of creep, tendon relaxation, and shrinkage. While the reduction ratio due to tendon relaxation and shrinkage can be assumed constant, the reduction due to creep varies with the position of the tendon and is

TABLE 2—CREEP AND SHRINKAGE COEFFICIENTS

Time, days	Beam R1		Beam R2		Beam R3		Beam R4	
	C_{ct}	$\epsilon_{sh}^* \times 10^6$	C_{ct}	$\epsilon_{sh}^* \times 10^6$	C_{ct}	$\epsilon_{sh}^* \times 10^6$	C_{ct}	$\epsilon_{sh}^* \times 10^6$
0	1.00	000	1.00	000	1.00	000	1.00	000
7	1.60	55	1.48	47.5	1.34	130	1.35	98
14	1.79	105	1.66	109	1.46	200	1.47	129
28	1.99	155	1.92	154	1.59	284	1.58	210
60	2.30	230	2.22	202	1.74	321	1.73	296
90	2.51	230	2.38	227	1.84	321	1.82	315
120	2.69	250	2.46	253	1.89	321	1.86	315
150	2.78	250	2.50	253

*Shrinkage coefficients are given in microin. per in.

a function of the creep strain at the tendon level. The change in deflection due to the external load reduction resulting from creep must therefore be superimposed as a final correction. Finally the analysis is predicted on the assumption that creep strains developed between the time of prestressing and the application of the sustained load are fully recoverable.

Since the change in the top fiber stress is given by

$$\Delta f_c = \frac{Mkd}{I}$$

the corresponding strain increment is

$$\Delta \epsilon_c = \frac{Mkd}{EI}$$

Solving for M and substituting in Eq. (22), it can be seen that the deflection ratio is given by the equation

$$\frac{\Delta}{L} = \frac{\sum_j \alpha_j \Delta \epsilon_{ij}}{k} \frac{L}{d} \dots \dots \dots (35)$$

Hence the designer can select the L/d ratio required for a given Δ/L limitation from suitable strain increment levels for critical combinations of instantaneous, sustained, and equivalent prestress loads, modified by appropriate α -coefficients.

EXPERIMENTAL VERIFICATION

The analysis presented herein has been verified experimentally for rectangular beams reinforced for tension only, by a limited test series consisting of four 8-ft beams. The beams were loaded at the third-points of a 7.5 ft simple span, center-to-center bearings, and the load was sustained for a maximum period of 150 days. The section properties of the beams, the load P at each point of load application, and measured values of elastic modulus E_c and E_s of the concrete and reinforcement, respectively, are shown in Table 1. The creep coefficients, C_{ct} , shown in Table 2 were obtained from companion prism specimens subjected to a sustained constant stress of 1200 psi. Shrinkage coefficients were obtained from unstressed companion specimens.

The theoretical deflection curves shown in Fig. 8 were calculated on the basis of the creep coefficients, C_{ct} , and the shrinkage strains ϵ_{sh} obtained from the companion specimens without correction for the effect of stress relaxation in the concrete. A typical deflection calculation for Beam R1, 150 days after application of the sustained load is given below.

The data required for this calculation is: $L = 90$ in.; $b = 7$ in.; $d = 6.5$ in.; $A_s = 0.614$ in.²; $w = 61.9$ lb per ft; $P = 2117$ lb; $E_c = 4.75$ ksi;

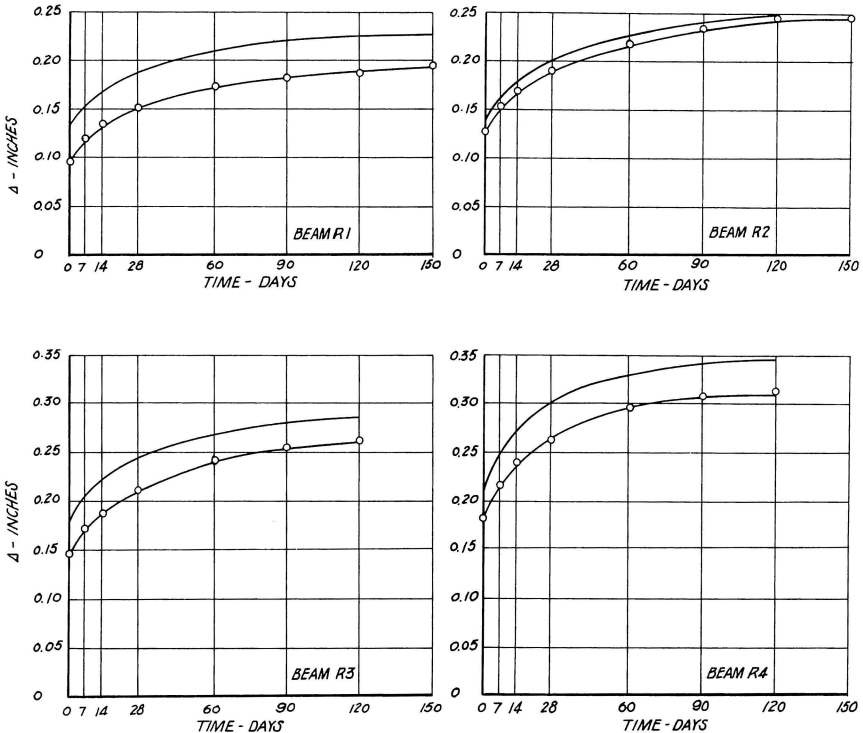


Fig. 8—Theoretical and experimental deflection curves

$E_s = 29,000$ ksi; $C_{c(150)} = 2.78$; and $\epsilon_{sh(150)} = 0.00025$. The parameters n , p and k are determined

$$n = \frac{E_s}{E_c} = \frac{29}{4.75} = 6.1$$

and

$$p = \frac{A_s}{bd} = \frac{0.614}{(7)(6.5)} = 0.0135$$

Since the beam is reinforced for tension only, $S=1$ and k may be read directly from the Snp curve in Fig. 2 by entering with $Snp = (1)(6.1)(0.0135) = 0.0824$. Thus $k = 0.332$ and $j = 1 - k/3 = 0.889$.

The dead-load moment is

$$M_D = \frac{wL^2}{8} = \frac{61.9(7.5)^2}{8} = 435.5 \text{ ft-lb}$$

and the sustained live-load moment is

$$M_L = \frac{PL}{3} = (2117)(2.5) = 5293 \text{ ft-lb}$$

The corresponding reinforcement stresses are

$$f_{sD} = \frac{M_D}{A_s j d} = \frac{(435.5)(12)}{(0.614)(0.889)(6.5)} = 1,473 \text{ psi}$$

and

$$f_{sL} = \frac{M_L}{A_s j d} = \frac{(5293)(12)}{(0.614)(0.889)(6.5)} = 17,900 \text{ psi}$$

The α -coefficients for a simple span may be shown to be $5/48$ for uniform load and $23/216$ for third-point loading.² From Eq. (23) the instantaneous load deflection is found to be

$$\frac{\Delta_i}{L} = \frac{\alpha}{1-k} \cdot \frac{f_s}{E_s} \cdot \frac{L}{d} = \frac{\left(\frac{5}{48}\right)(1473) + \left(\frac{23}{216}\right)(1790)}{(0.668)(29,000,000)} \cdot \frac{90}{6.5} = 1.47 \times 10^{-3}$$

Hence, $\Delta_i = (0.001,47)(90) = 0.132$ in. For $C_{ct} = 2.78$, $n_t p S_t = (2.78)(0.0824) = 0.229$. From Fig. 2 it may be seen that $k_t = 0.486$. The deflection ratio K_Δ may be computed from Eq. (26)

$$K_\Delta = \frac{(1-k)(3-k)}{(1-k_t)(3-k_t)} = \frac{(0.668)(2.668)}{(0.514)(2.514)} = 1.38$$

or read directly from Fig. 5.

The total elastic-plus-creep deflection is therefore $\Delta_{ct} = (0.132)(1.38) = 0.181$ in. To this, the estimated shrinkage deflection must be added. The shrinkage deflection ratio is computed from Eq. (33)

$$\frac{\Delta_{sh}}{L} = \frac{3}{8(3 - k_t)} \epsilon_{sh} \frac{L}{d} = \frac{(0.375)(0.00025)}{2.514} \frac{90}{6.5} = 0.000516$$

or it may be determined from Fig. 7. Hence, $\Delta_{sh} = (0.000516)(90) = 0.046$ and the total deflection is $\Delta = \Delta_{ct} + \Delta_{sh} = 0.227$ in.

This computed deflection should be compared to the measured deflection of Beam R1 of 0.191 in. at 150 days as is shown in Fig. 8. For this beam the analysis gave a conservative result, the predicted value being approximately 19 percent greater than the measured value.

The analysis was based on a shrinkage potential determined by means of a 3 x 4 x 16-in. prism (Table 2). Tests by others have shown that the shrinkage rate is affected by the size of the specimen.⁶ Based on these test results, the shrinkage potential should have been reduced approximately 50 percent, considering the age of the concrete and the size of the beam relative to the companion specimen. This correction would have reduced the predicted total deflection to about 0.204 in. and this result would then have been in excellent agreement with the observed deflection.

This correction was not made for the predicted deflection curves shown in Fig. 8 because, in general, the designer is only interested in the end value of the deflection due to all causes including creep and shrinkage. The terminal shrinkage potential is primarily a function of the quality of the concrete rather than of the size of the specimen. Computed deflections

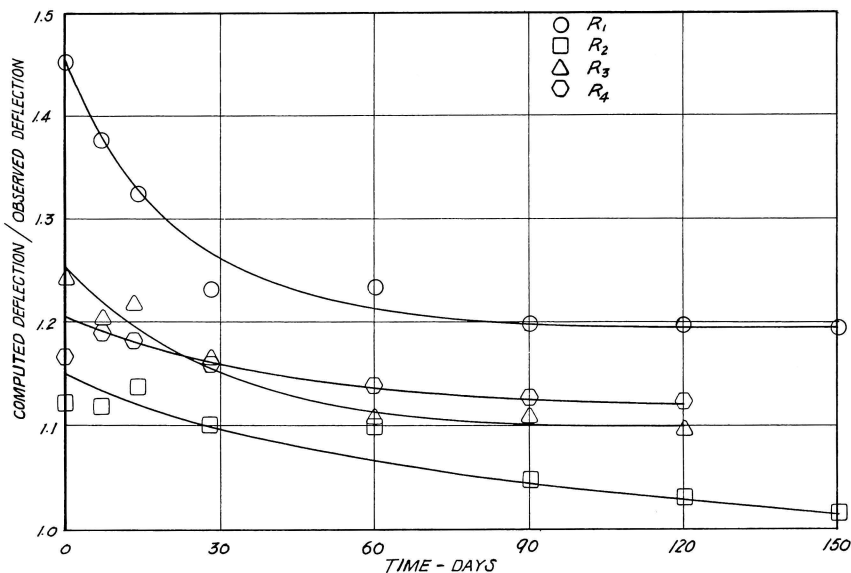


Fig. 9—Comparison of computed deflection to measured deflection of experimental verification

based on the shrinkage potential for relatively small specimens will tend to approach the measured deflection asymptotically with increased age. This trend is especially noticeable if the ratio of computed to observed deflection is plotted as shown in Fig. 9.

EXAMPLES

To illustrate the design procedure as well as the analysis for predicting total deflection, detailed calculations are shown in the three examples below. In the first example a simple singly-reinforced beam is designed for specified loads and deflection-length ratio. The design is checked to show that both the load capacity and the deflection-length ratio limitation is satisfied. In the second example the depth is reduced, requiring an increase in the tensile-reinforcement ratio, as well as compression steel to satisfy the specified deflection-length ratio. In the third example a T-beam of comparable span and load requirements is analyzed to illustrate the effect of increased width of the compression zone on the deflection-span ratio.

Example 1

A concrete beam is to be designed for the 30-ft simple span shown in Fig. 10. The beam is to support its own weight, two sustained 1-kip loads at the third points, and a uniformly distributed live load of 400 lb per lineal ft. The total center line deflection is not to exceed 1/180 of the span length. The design is based on the following design data:

Compressive strength of concrete	$f'_c = 4000$ psi
Yield strength of reinforcement	$f_y = 40,000$ psi
Modulus of elasticity of concrete	$E_c = 3625$ ksi
Modulus of elasticity of steel	$E_s = 29000$ ksi
Weight of concrete	$w = 150$ pcf
Ultimate creep coefficient	$C_{cT} = 4$
Shrinkage potential	$\epsilon_{shT} = 0.0004$
Modular ratio (initial)	$n = E_s/E_c = 8$

Preliminary design.—To estimate the minimum depth d , the following additional assumptions are made:

Dead load of concrete beam	$g = 200$ lb per ft
Compression-zone depth ratio	$k = 0.4$
Length-depth ratio	$\frac{L}{d} = 25$

These assumptions must, of course, be checked and the design modified if necessary.

From Fig. 2, for k equal to 0.4, Snp is found to be 0.13. Since S is equal to unity for a singly-reinforced beam, $Snp = SnC_{cT}p = 0.52$. Hence, from

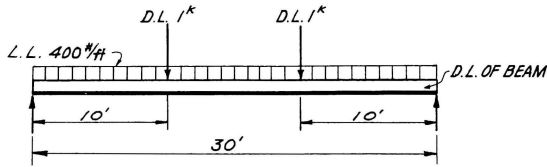


Fig. 10—Loading of beam for Example 1

Fig. 2, $k_T = 0.625$ and from Fig. 8, for $k_T = 0.625$

$$\frac{\Delta/L}{\epsilon_{sh}(L/d)} = 0.158$$

Hence the deflection-length ratio for shrinkage is approximately

$$\left(\frac{\Delta}{L}\right)_{sh} = 0.158 \epsilon_{sh} \frac{L}{d} = (0.158)(0.0004)(25) = 0.00158$$

The permissible Δ/L ratio for instantaneous and sustained loads is therefore

$$\frac{\Delta}{L} = \frac{1}{180} - 0.00158 = 0.00556 - 0.00158 = 0.00398 = \frac{1}{252}$$

Deflection design parameter.—In order to compute the parameter Q , Eq. (27), it is necessary to estimate the service-load stress levels in the reinforcement due to instantaneous and sustained loads. The ultimate strength design moments are as follows:

Sustained load:

$$\text{Self-weight} \quad M = \frac{gL^2}{8} = \frac{0.2 \times 900}{8} \times 12 = 270 \text{ in.-kips;}$$

$$M_U = 1.5M = 405 \text{ in.-kips.}$$

$$\text{Concentrated loads} \quad M = \frac{PL}{3} = 1.0 \times 10 \times 12 = 120 \text{ in.-kips;}$$

$$M_U = 1.5M = 180 \text{ in.-kips.}$$

Total

$$M_{SD} = 390 \text{ in.-kips;}$$

$$M_{UD} = 585 \text{ in.-kips.}$$

Live load:

$$M_{SL} = \frac{qL^2}{8} = \frac{0.4 \times 900}{8} \times 12 = 540 \text{ in.-kips;}$$

$$M_{UL} = 1.8$$

$$M_{SL} = 972 \text{ in.-kips.}$$

Total moment:

$$M_S = 930 \text{ in.-kips;}$$

$$M_U = 1557 \text{ in.-kips.}$$

Stress level:

Since the ultimate-moment-capacity reduction factor ϕ is equal to 0.9 (Section 1504 of ACI 318-63), the approximate service-load stress levels are:

$$\text{Self-weight stress } f_s = \frac{270}{1557} (0.9)(40) = 6.25 \text{ kips per sq in.}$$

$$\text{Concentrated load stress } f_s = \frac{120}{1557} (0.9)(40) = 2.78 \text{ ksi}$$

$$\text{Uniform live load stress } f_s = \frac{540}{1557} (0.9)(40) = 12.50 \text{ ksi}$$

$$\text{Total service-load working stress } f_s = 6.25 + 2.78 + 12.50 = 21.53 \text{ ksi}$$

The deflection ratio K_Δ for sustained loads may be computed from Eq. (26) or read directly from Fig. 5 for $k = 0.4$ and found to be $K_\Delta = 1.75$. From Reference 2 it may be found that for a simply supported beam $\alpha = 5/48$ for a uniformly distributed load and $\alpha = 23/216$ for third-point loading. Thus substituting in Eq. (27)

$$Q = \frac{L}{\Delta} \sum \alpha \left\{ \frac{f_s}{E_s} \right\} K_\Delta$$

$$Q = \frac{252 \left[\frac{5}{48} (6.25)(1.75) + \frac{23}{216} (2.78)(1.75) + \frac{5}{48} (12.5) \right]}{29000} = 0.0255$$

Length-depth ratio.—The maximum length-depth ratio may be computed by solving Eq. (28) for L/d , thus

$$\frac{L}{d} = \frac{1-k}{Q} = \frac{0.6}{0.0255} = 23.4$$

This value is slightly less than that assumed in the shrinkage deflection computation and should therefore give a conservative design. The minimum design depth is therefore

$$d = \frac{L}{23.5} = \frac{360}{23.5} = 15.4 \text{ in.}$$

A design depth of 16 in. should therefore satisfy the deflection-length ratio criterion and is selected. The approximate amount of reinforcement required is most readily determined from the service-load moment. For an assumed value of $k = 0.40$

$$jd = \left(1 - \frac{0.4}{3}\right)16 = 13.85 \text{ in.}$$

The required steel reinforcement area is therefore

$$A_s = \frac{M_s}{f_s jd} = \frac{930}{(21.53)(13.85)} = 3.12 \text{ sq in.}$$

Four # 8 bars may be selected to give

$$A_s = 3.16 \text{ sq in.}$$

For $k = 0.4$, from Fig. 2,

$$Snp = 0.13$$

hence

$$p = \frac{A_s}{bd} = \frac{0.13}{8} = 0.01625$$

Solving for b ,

$$b = \frac{A_s}{pd} = \frac{3.16}{(0.01625)(16)} = 12.1 \text{ in.}$$

Four # 8 bars require a minimum b of 11 in. hence a b of 12 in. is selected. The design section is therefore as shown in Table 3.

Ultimate resisting moment.—The dead load of the 12 × 18-in. beam is 225 lb per ft instead of 200 lb per ft as assumed for the preliminary design. The required ultimate moment is 1607 in.-kips. The ultimate resisting moment is 1640 in.-kips as determined by Eq. (16 - 1) of ACI 318-63 and is therefore slightly greater than the required design moment, hence the design is satisfactory for strength.

Check of deflection-length ratio.—To check the deflection-length ratio the contributions due to sustained load, live load, and shrinkage must be computed separately and added. The section was analyzed in the manner described in the previous section. The results obtained are given in Table 3.

Example 2

The beam in the first example is modified by reducing the depth by 2 in. To satisfy the strength requirement the tensile-reinforcement area must be increased. Compressive reinforcement must also be added to avoid excessive deflection resulting from the increased L/d ratio. The dead load for the beam is 200 lb per ft which corresponds to the initial assumption made in the first example. Assuming that the k ratio is to be held at approximately 0.4, the tensile steel area is inversely proportional to the internal moment arm jd . Hence

$$A_s \approx 3.12 \frac{13.85}{11.85} = 3.64 \text{ sq in.}$$

Selecting two # 8 and two # 9 bars,

$$A_s = 3.58 \text{ sq in.}$$

For this area,

$$p = \frac{A_s}{bd} = \frac{3.58}{(12)(14)} = 0.0213$$

To assess the effect of compression steel on the deflection characteristics the trial section shown in Table 3 was analyzed. The compression steel was assumed to consist of two # 5 bars at a depth $d' = 1.4$ in. The results of the analysis of this design are also summarized in Table 3.

Example 3

The T-section shown in Table 3 is to be analyzed for ultimate-load resistance and service-load deflection. The section is used for a 30-ft simple

span. The span is subjected to a sustained load equal to the dead load of the section itself and a live load of 400 lb per ft.

Service-load and ultimate-load design moments.—The weight of the section is 350 lb per ft.

$$M_{SD} = \frac{gL^2}{8} = \frac{(350)(900)}{8} (12) = 472 \text{ in.-kips}$$

$$M_{UD} = (1.5)(472) = 708 \text{ in.-kips}$$

$$M_{SL} = \frac{qL^2}{8} = \frac{(400)(900)}{8} (12) = 540 \text{ in.-kips}$$

$$M_{UL} = (1.8)(540) = 972 \text{ in.-kips}$$

$$M_U = 708 + 972 = 1680 \text{ in.-kips}$$

Ultimate resisting moment.—The moment may be determined by Eq. (16-1) of ACI 318-63.

$$t = 3 > 1.18 \frac{qd}{k_1} = \frac{1.18 A_s f_y}{k_1 b f'_c} = \frac{(1.18)(3.58)(40)}{(0.85)(60)(4)} = 0.83 \text{ in.}$$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(3.58)(40)}{(0.85)(4)(60)} = 0.7 \text{ in.}$$

and

$$M_U = \phi[A_s f_y (d - \frac{a}{2})] = 0.9[(3.58)(40)(13.65)] = 1760 \text{ in.-kips}$$

The resisting moment is therefore about 5 percent greater than the design moment required and the design is adequate for strength.

Deflection-length ratio.—For T-beams, the initial trial values for k and k_T can most readily be estimated by considering the overall width. In this example

$$S_{np} \approx \frac{(1)(8)(3.58)}{(60)(14)} = 0.034 \text{ for which } k = 0.23$$

and

$$S_{C_{eT} np} \approx \frac{(32)(3.58)}{(60)(14)} = 0.136 \text{ for which } k_T = 0.40$$

More exact values can be obtained by the use of Eq. (8) to (10) or by the use of Fig. 2. The following ratios must first be calculated:

$$p = \frac{A_s}{b'd} = \frac{3.58}{(12)(14)} = 0.0213$$

$$\gamma = \frac{(b - b')t}{b'd} = \frac{(48)(3)}{(12)(14)} = 0.587$$

$$\dot{p}_{eT}' = \frac{\gamma}{n} = \frac{0.857}{8} = 0.107 \text{ and } p_{eT}' = \frac{\gamma}{C_{eT} n} = \frac{0.857}{32} = 0.026$$

Hence,

$$p_{ri} = \frac{p_e}{p} = \frac{0.107}{0.0213} = 5.03 \text{ and } p_{rT} = \frac{0.0268}{0.0213} = 1.26$$

Also

$$d_r = \frac{t/2}{d} = \frac{1.5}{14} = 0.107$$

Using the above ratios and Fig. 2, the following parameters may be obtained

$$\begin{aligned} k_r &= 0.16 & k_{rT} &= 0.59 \\ S &= 0.20 & S_T &= 0.25 \\ k &= 0.23 & k_T &= 0.44 \end{aligned}$$

The service-load stress levels are readily computed from the moments. Since $A_s j d = (3.58)(1-0.766)(14) = 46.3$ cu in.; the stress levels are $f_s = 472/46.3 = 10.20$ kips per sq in. for the sustained dead load and $f_s = 540/46.3 = 11.68$ kips per sq in. for the live load. From Eq. (17), $K_{f_s} = 1.018$ and from Eq. (24), $K_\Delta = 1.40$. Substituting these results into Eq. (23), the sustained load Δ/L ratio is found to be

$$\begin{aligned} \left(\frac{\Delta}{L}\right)_D &= \frac{\alpha}{(1-k)} \frac{f_s}{E_s} \frac{L}{d} K_\Delta \\ &= \frac{1}{0.77} \frac{(10.2)(25.7)}{29000} \frac{5}{48} (1.40) = 0.00171 \end{aligned}$$

and the live-load ratio is

$$\begin{aligned} \left(\frac{\Delta}{L}\right)_L &= \frac{\alpha}{(1-k)} \frac{f_s}{E_s} \frac{L}{d} \\ &= \frac{1}{0.77} \frac{11.68(25.7)}{29000} \frac{5}{48} = 0.00140 \end{aligned}$$

The contribution due to shrinkage may be computed from Eq. (32). Since $\alpha = 1/8$, $p' = 0$, and $d_r = t/2d$; Eq. (32) reduces to

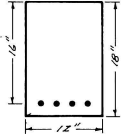
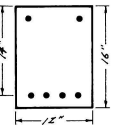
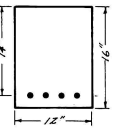
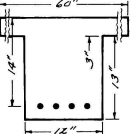
$$\begin{aligned} \frac{\Delta_{sh}}{L} &= \frac{3}{8} \frac{\epsilon_{sh}}{3(1-t/2d) + [(3t/2d) - k_T] S_T} \cdot \frac{L}{d} \\ &= \frac{3}{8} \frac{0.0004}{3(1-0.107) + (0.321 - 0.44) 0.25} 25.7 = 0.00145 \end{aligned}$$

The total deflection-length ratio is therefore $\Delta/L = 0.00171 + 0.00140 + 0.00145 = 0.00456 = 1/219$. This Δ/L ratio is approximately 20 percent less than the maximum specified.

SUMMARY AND CONCLUSIONS

The results of the computations for the three examples are summarized in Table 3 below. In addition, the deflections for the beam in Example 2 without compressive reinforcement are also given for purposes of comparison. It should be noted that the shrinkage deflection ranges from about one-fifth to about one-third of the total deflection, and that this ratio is largest for the T-beam and smallest for the doubly reinforced beam. It

TABLE 3—SUMMARY OF RESULTS; EXAMPLES 1-3

Item	Example 1	Example 2		Example 3
	$A_s = 3.16$ sq. in.	$A'_s = 0.62$ sq. in. $A_s = 3.58$ sq. in.	$A_s = 3.58$ sq. in.	$A_s = 3.58$ sq. in.
				
M_u (design)	1607 in.-kips	1157 in.-kips	1157 in.-kips	1680 in.-kips
M_u (resisting)	1640 in.-kips	1620 in.-kips	1580 in.-kips	1760 in.-kips
M_D	424 in.-kips (44%)	390 in.-kips (42%)	390 in.-kips (42%)	472 in.-kips (47%)
M_L	540 in.-kips (56%)	540 in.-kips (58%)	540 in.-kips (58%)	540 in.-kips (53%)
K_Δ	1.77	1.62	1.87	1.40
$(\Delta/L)_D$	0.00232 (42.8%)	0.00236 (42.5%)	0.00281 (43.0%)	0.00171 (37.5%)
$(\Delta/L)_L$	0.00166 (30.6%)	0.00200 (36.0%)	0.00207 (31.7%)	0.00140 (30.7%)
$(\Delta/L)_{sh}$	0.00144 (26.6%)	0.00119 (21.5%)	0.00165 (25.3%)	0.00145 (31.8%)
Δ/L	0.00542 (100%)	0.00555 (100%)	0.00653 (100%)	0.00456 (100%)
$(\Delta/L)_{creep}$	0.00101 (18.6%)	0.00090 (16.2%)	0.00131 (20.0%)	0.00049 (10.8%)

should also be noted that the incremental deflection due to creep in all cases is less than that due to shrinkage. This increase for the T-beam is offset by a marked reduction in the total deflection due to the increase in stiffness resulting from the flange. The results for Example 2 are of special interest in that they illustrate the effectiveness of a relatively small amount of compression reinforcement in controlling deflections due to creep and shrinkage. Without compression reinforcement, the 2-in. reduction in depth in Example 2 would result in a 20 percent increase in deflection as may be seen by comparing the deflection with that of Example 1. This increase may be avoided by the use of a compression steel ratio equal to 17 percent of the tensile steel ratio. This compression steel reduces the sustained load deflection by about 16 percent, the instantaneous live-load deflection by about 3 percent and the shrinkage deflection by 28 percent.

Eq. (23) and (33) are of special significance in that they give a qualitative appraisal of the design parameters which tend to increase deflection. Recent design trends such as longer spans, high-strength reinforcement, and the use of ultimate-strength design which results in higher allowable steel ratios and hence greater k values all tend to increase the deflection-span ratio. Moreover, the effect of these trends to increase deflection is not merely algebraic but is geometric. This is immediately evident from Eq. (23) and (33) which show that the Δ/L ratios are proportional to the tensile steel stress level, the span-to-depth ratio, and the shrinkage coefficient, and inversely proportional to the depth of the compression zone. The creep deflection coefficient may also be shown to be proportional to the k -value.

The present emphasis on strength as the principal if not sole design criterion could easily result in serious difficulties if these design trends continue to develop. The designer therefore requires a reasonably simple but reliable method of analysis for predicting the behavior of the structure he designs under service-load conditions and for comparing the relative merits of alternate design solutions. The authors believe that the procedures developed in this paper can be helpful in satisfying this need.

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DISCUSSION

By M. V. Pregnoff*

The paper is timely because it treats the subject of time-dependent behavior of concrete beams. Lately many engineers have begun to realize that creep, shrinkage and temperature affect the eventual stresses in concrete members to such an extent that the designer's computations of stresses lose a great deal of the exactness usually attached to them. The results of computations should be tempered by an understanding of the time-dependent behavior of the concrete as a structural material. While the phenomena of creep and shrinkage are surrounded by multitudes of variables, one should not conclude that the problem is hopeless. As the authors of this paper do, one should endeavor to solve the problem in some simple way.

Using the methods of this paper with appropriate creep (C_{ct}) and shrinkage (ϵ_{sh}) coefficients, the engineer can compute the deflection of concrete beam during construction and control the time of installation of nonstructural fragile elements under or over the beam. For instance, a certain concrete beam may deflect 0.25 in. immediately after the shores are removed. Then it will creep and shrink; eventually adding another 0.5 in. About one-half of these deflections will take place during the first 3 months after removal of the shores. Thus only 0.25 in. of eventual total deflection will be left to be considered, provided that the nonstructural fragile elements are installed 3 months after removal of the shores. It is advisable to camber the beam by the amount of deflection during construction. If we treat the problem this way we do not put the concrete beam at a disadvantage in comparison with a steel beam which does not creep and shrink.

Those engineers and researchers who work with concrete and understand it, know that creep, or plastic flow, is not necessarily a detrimental quality of concrete. "Mainly because of plastic flow, a concrete structure tries with admirable docility to adapt itself to our calculations—which do not always represent the most logical and spontaneous answer to the request of the forces at play—and even tries to correct our deficiencies and errors. Sections and regions too highly stressed yield and channel some of their loads to other sections or regions, which accept this additional task with commendable spirit of collaboration, within the limits of their own

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strength. (If something like this happened in human relations, misery and injustice would disappear from our troubled lives).” Thus wrote Pier Luigi Nervi, the famous Italian engineer.*

While the paper treats cracked sections, the basic method can be used to calculate deflections of a beam using gross sections if desired.

The question is often raised whether cracked or uncracked section should be used in computing deflections. Because of cracking, an ordinary reinforced concrete beam is a member whose various crosssections along the span are subjected to distinctly different stress configurations, thus in effect giving variable flexural rigidities, EI . At points of small bending moments the concrete works both in tension and compression. At points of greater moments the concrete fails in tension and minute cracks are formed. However, the concrete between cracks still resists some tension and contributes to the rigidity of the beam. Under long duration of loading the influence of tensile concrete upon the rigidity of the beam is diminished due to shrinkage stresses. The use of cracked or uncracked sections in computing deflection depends upon the make-up of a member. Shallow, heavily reinforced members act differently from deep, husky, lightly reinforced members. Concrete, while not reliable in tension to carry life and limb, is capable of resisting an ultimate direct tension of $4\sqrt{f'_c}$ to $6.5\sqrt{f'_c}$, where f'_c is an ultimate 28-day compressive cylinder strength. The above ultimate tension corresponds to a modulus of rupture of about 1.9 times the above values.† Reasonably deep continuous beams at actual service loads are working on uncracked section with steel transformed to concrete. Aggravated by shrinkage, concrete cracks and the steel carries the burden of the stresses across these minute gaps. The presence of steel decreases the unfavorable influence of the heterogeneous structure of concrete and softens the effect of stress risers in the tension zone. The lower moment of inertia at the cracks often does not affect greatly the whole deflection of a deep beam. This is the reason why the Portland Cement Association recommended in the past the use of uncracked (gross) concrete sections in computing deflections of concrete beams.

To get realistic deflections the extent of cracked sections along the span needs to be considered in some important typical cases.

Consideration of deflections often dictates deeper beams. While the steel stresses in these beams are at the maximum allowable, the concrete tensile and compressive stresses are rather low under actual service loads. In many cases of continuous beams the length of the cracked region is $1/20$ of the span at supports and $1/8$ of the span near mid-length.

Under long duration loading the end moments of continuous beams are reduced by about 15 to 20 percent, thus increasing the mid-span moment and deflection. It is advisable, therefore, to use only 85 or 80 percent of

*Nervi, Pier Luigi, *Structures*, F. W. Dodge Corp., New York, 1956, p. 44.

†Kessler, Clyde E., "Statistical Relation between Cylinder Modified Cube and Beam Strength of Plain Concrete, *Proceedings*, ASTM, 1954, p. 1178.

computed elastic end moments as available while calculating deflections of continuous beams.

AUTHORS' CLOSURE

The authors would like to thank Mr. Pregnoff for his interest and for his valuable comments. They appreciate the fact that many beams do not crack at all and that most beams only crack in the region where the ultimate direct tensile strength of the concrete is exceeded. It is true that the basic method proposed can be modified to be applied to gross sections. However, it should also be pointed out that, with the stresses permitted by current building codes, the assumption of a cracked section throughout the tensile zone does not overestimate the deflection as much as might be expected.

In the first place, assuming that concrete has an ultimate tensile strength f'_T of $6.5\sqrt{f'_c}$, that the modulus of rupture is $1.9 f'_c$ and that the modulus of elasticity E_c is approximately $60,000\sqrt{f'_c}$ for normal-weight concrete, cracks in the tensile zone of a beam can be expected to develop when the steel stress exceeds

$$f_s = E_s \epsilon_t = E_s \frac{1.9f'_c}{E_c} \doteq 6200 \text{ psi}$$

Hence in most designs, tensile cracks can develop throughout most of the tensile zone, even for dead load alone, and these cracks will propagate at least up to the level of the main reinforcement.

In the second place, the effect of increased rigidity, at the supports in a simple beam and in the neighborhood of the points of inflection in a continuous beam, on the deflection is not proportional to the ratio of the uncracked zone length to the span length. This conclusion can be demonstrated by considering the effect on the elastic load in the conjugate-beam analysis and noting that the elastic load is modified only in the neighborhood of the points of zero moment. The authors therefore believe that for most cases this reduction can be neglected or, if need be, corrected by incorporating a small reduction factor in the α -coefficient computed on the basis of the cracked section.

It was not the authors' intent to suggest that the proposed method of analysis be considered an exact method for computing deflections under all conditions of loading, creep, and shrinkage. Rather, it was their objective to provide the designer with a relatively simple design procedure for selecting minimum depth and reinforcement requirements without the need for an elaborate deflection analysis to assure compliance with a pre-determined deflection criterion.

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