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Bidirectional dc/dc power converters with current limitation based on nonlinear control design

George C. Konstantopoulos and Antonio T. Alexandridis

Abstract— A new nonlinear controller for bidirectional dc/dc power converters that guarantees output voltage regulation with an inherent current limitation is proposed in this paper. In contrast to traditional single or cascaded PI controllers with a saturation unit that can lead to integrator windup and instability, the proposed controller is based on a rigorous nonlinear mathematical analysis and, using Lyapunov stability theory, it is proven that the current of the converter is always limited without the need of additional saturation units or limiters. The proposed concept introduces a virtual resistance at the input of the converter and a controllable voltage that can take both positive and negative values leading to bidirectional power flow capability. The dynamics of this voltage are proven to remain bounded and with a suitable choice of the voltage bound and the virtual resistance, the upper limit for the converter current is guaranteed at all times, even during transients. Simulation results for a bidirectional converter equipped with the proposed controller are presented to verify the current-limiting capability and the desired voltage regulation.

I. INTRODUCTION

DC/DC power converters are known to convert a dc input voltage to a higher or lower level at their output and are widely used in renewable energy systems [1], [2], energy storage systems [3] and micro-grids [4], [5], [6]. Although for the cases of photovoltaic systems or dc loads, a unidirectional power flow is required, there are applications such as battery energy storage systems and electric vehicles where a bidirectional dc/dc converter is essential. Hence, the control design of these bidirectional converter applications has drawn a lot of attention recently due to the rising integration of storage systems to the power grid in order to guarantee a stable and reliable operation.

The operation of bidirectional dc/dc converters is based mainly on pulse-width modulation and considering a high switching frequency, the nonlinear average model of the converter can be obtained in order to design a suitable control method. The most widely used technique for regulating the voltage or current of a bidirectional dc/dc converter is using traditional single or cascaded PI controllers [7]. Based on linearization and the small-signal model of the converter, traditional PI controllers can be designed to guarantee local stability of the desired equilibrium point. However, the nonlinear dynamics of the converter dictate a need to design advanced control methods that can be applied directly to

the nonlinear model of the system, such as sliding control [8], [9], [10], [11] or passivity-based control [12], [13], [14], [15]. These advanced nonlinear control methods can guarantee nonlinear closed-loop system stability based on a strong mathematical theory but in most of the cases require the information of the system or load parameters that may vary depending on the application.

Although several nonlinear controllers have been recently designed for dc/dc converters to guarantee closed-loop system stability and convergence to a desired equilibrium based on Lyapunov theory [16], [17], [18], [19], an upper limit for the converter current cannot be guaranteed at all times, i.e. under changes of the load, input voltage or reference signal. Such a high current can be damaging for the converter in real applications. To overcome this crucial issue, traditional control methods introduce an inner current control loop with a saturation unit to limit the reference value of the current obtained by the outer loop [20]. However, as pointed out in [21], these saturated controllers introduce two main drawbacks: i) although the reference value of the converter current is limited, the actual value can violate the limit during transients and ii) saturated integrators can lead to integrator windup and instability. To this end, a current-limiting control method for different types of dc/dc converters was recently introduced in [21] to achieve accurate regulation and current limitation without any saturation units. The main disadvantage of this method is that it can be applied only to unidirectional dc/dc converters; hence it is not applicable to energy storage applications.

In this paper, a new nonlinear current-limiting controller is designed for bidirectional dc/dc converters without requiring knowledge of the load or converter parameters. The proposed controller introduces a constant virtual resistance and a varying voltage based on nonlinear dynamics that aim to accomplish the desired output voltage regulation. Inspired by the enhanced version of the bounded integral controller [22], the voltage dynamics are formulated to satisfy given bounds and can take both positive and negative values enabling the bidirectional power flow, required by the converter application. Based on nonlinear input-to-state stability (ISS) theory, the converter current is rigorously proven to remain limited below a given maximum value that can be provided by the technical requirements of the converter. To verify the theory and validate the effectiveness of the proposed controller, a bidirectional dc/dc converter equipped with the proposed current-limiting controller is tested by simulations in Matlab/Simulink using the Simpower Systems toolbox. It is shown that under several changes of the load, the output

voltage regulation is accomplished under an inherent current protection of the converter, introduced by the proposed control design.

The rest of the paper is organized as follows. In Section II, the nonlinear dynamic model of the bidirectional dc/dc converter is presented and the desired problem is formulated. In Section III, the new current-limiting nonlinear control scheme is provided and the dynamics of the controller an analyzed. In addition, it is shown that under a suitable choice of the controller parameters, the current-limiting property of the converter is guaranteed independently from the direction of the power flow. In Section IV, simulation results illustrate the proposed controller performance and in Section V the conclusions of this work are drawn.

II. BIDIRECTIONAL DC/DC POWER CONVERTER MODEL

The bidirectional dc/dc converter shown in Fig. 1 consists of two switching elements, an inductor L at the input and a capacitor C at the output. The dc input voltage of the converter is represented as V_{in}, while the output current is denoted as i_L, which can be positive or negative to represent a load or a source at the output of the converter, respectively. The converter is operated using pulse-width-modulation with a high switching frequency and the two switches are controller in a complementary manner, i.e. when u is closed then $\bar{u}$ is open and vice versa. Under the common assumption of a high operating switching frequency and continuous conduction mode, using average theory, the continuous-time dynamic model of the bidirectional dc/dc converter can be obtained as shown below [12]:

$$L\frac{di}{dt} = -(1-u)v + V_{in} \qquad (1)$$

$$C\frac{dv}{dt} = (1-u)i - i_L \qquad (2)$$

where i and v are the inductor current and capacitor voltage, respectively, and represent the system states while u is the control input representing the duty-ratio of the converter and is limited in the range $[0,1]$. Note that the dynamics of the bidirectional converter are nonlinear due to the terms uv and ui, which make the control design and stability analysis a challenging task. Furthermore, when $u = 1$, it is clear from (1) that the inductor current will continuously increase and will reach high values that can cause damage to the converter. Hence, maintaining the inductor current i limited below a given maximum value is crucial in converter applications.

Although traditional control techniques introduce a cascaded control structure (outer voltage controller and inner current controller) with saturation units, the current is not maintained limited during transients and saturation units can lead to instability. To this end, very recently, a current-limiting control for dc/dc converter was introduced without suffering from windup and instability to overcome these issues [21]. Nevertheless, this controller can be only applied to unidirectional dc/dc converters and does not allow bidirectional power flow; hence cannot be applied in energy storage systems and electric vehicles.
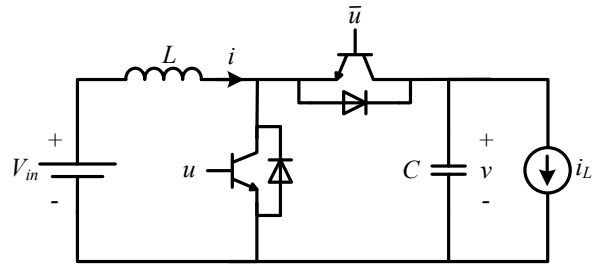

Fig. 1. Bidirectional dc/dc power converter

III. PROPOSED CONTROL DESIGN AND ANALYSIS

A. The proposed controller

The main goal of this work is to design a controller that can be applied to bidirectional dc/dc converters to achieve output voltage regulation and a current-limiting capability without saturation units that can lead to instability. The proposed concept is based on the idea of partially decoupling the inductor current dynamics, introducing a constant virtual resistance and a bounded controllable voltage. This bounded voltage will guarantee the desired upper limit for the converter current independently from the direction of the power flow (positive or negative current). Hence, the proposed controller takes the form:

$$u = 1 - \frac{r_v i + V_{in} - E}{v}, \qquad (3)$$

where $r_v > 0$ represents a constant virtual resistance and E a controllable voltage which introduces the following nonlinear dynamics:

$$\dot{E} = -k\left(\frac{E^2}{E_m^2} + E_q^{2l} - 1\right)E + cE_q^{2l}(v_{ref} - v) \qquad (4)$$

$$\dot{E}_q = -k\left(\frac{E^2}{E_m^2} + E_q^{2l} - 1\right)E_q - \frac{cEE_q(v_{ref} - v)}{E_m^2}, \qquad (5)$$

where E_q is an additional control state, c, k, E_m, are positive constants, $l \geq 1 \in \mathcal{N}$ and v_{ref} represents the desired converter output voltage. To further understand the choice of the controller dynamics (4)-(5), let the following Lyapunov function candidate

$$W = \frac{E^2}{E_m^2} + \frac{E_q^{2l}}{l}. \qquad (6)$$

Taking the time derivative of W and incorporating the control system (4)-(5), then

$$\begin{aligned}
\dot{W} &= \frac{2E\dot{E}}{E_m^2} + 2E_q^{2l-1}\dot{E}_q \\
&= -2k\left(\frac{E^2}{E_m^2} + E_q^{2l} - 1\right)\frac{E^2}{E_m^2} + \frac{2cEE_q^{2l}}{E_m^2}(v_{ref} - v) \\
&\quad -2k\left(\frac{E^2}{E_m^2} + E_q^{2l} - 1\right)E_q^{2l} - \frac{2cEE_q^{2l}}{E_m^2}(v_{ref} - v) \\
&= -2k\left(\frac{E^2}{E_m^2} + E_q^{2l} - 1\right)\left(\frac{E^2}{E_m^2} + E_q^{2l}\right). \qquad (7)
\end{aligned}$$

It is clear from (7) that $\dot{W}$ is negative outside the curve

$$W_0 = \left\{ E, E_q \in R : \frac{E^2}{E_m^2} + E_q^{2l} = 1 \right\} \qquad (8)$$

and positive inside except from the origin, where $\dot{W} = 0$. Therefore, for any initial conditions E_0 and E_{q0} except from the origin, the controller state trajectory will be attracted on the curve W_0. From (7), one can realize that the larger the gain k, the faster the attraction on the curve W_0. For $l = 1$, W_0 represents an ellipse on the $E - E_q$ plane while for $l > 1$, W_0 takes the form as depicted in Fig. 2.

Based on the Lyapunov analysis, since $\dot{W}$ is negative outside W_0, then considering the set $S = \{W(E, E_q) \leq 1\}$, i.e.

$$S = \left\{ E, E_q \in R : \frac{E^2}{E_m^2} + \frac{E_q^{2l}}{l} \leq 1 \right\},$$

then $\dot{W} < 0$ outside of S since the curve W_0 is contained inside and on the boundaries of S. This can be easily proven if one takes into account that $l \geq 1$. Hence, every trajectory starting inside the set S will remain in the set for all future time. As a result, for any initial conditions E_0 and E_{q0} inside S, the controller states will satisfy

$$\frac{E^2(t)}{E_m^2} + \frac{E_q^{2l}(t)}{l} \leq 1, \forall t \geq 0,$$

yielding

$$|E(t)| \leq E_m, \forall t \geq 0,$$

which results in a bounded voltage E below a given value E_m that can be selected by the controller operator. Therefore, $E \in [-E_m, E_m]$ for all $t \geq 0$.

For an arbitrary large constant k, the controller states E and E_q are quickly attracted and remain close to W_0 resulting in the dynamics

$$\dot{E} \approx c E_q^{2l} (v_{ref} - v) \qquad (9)$$

$$\dot{E}_q \approx -\frac{c E E_q (v_{ref} - v)}{E_m^2}. \qquad (10)$$

As it can be seen from Fig. 2, for a large l, the controller state E_q will be closer to the value 1 for the entire range of $E \in [-E_m, E_m]$. Hence, the dynamics of E_q are faster since $\dot{E}_q$ takes larger values in (5) compared to a case with a lower l. Then, the voltage dynamics (4) approximate the dynamics of the integrator

$$\dot{E} \approx c(v_{ref} - v),$$

where c represents the integral gain (for more details the reader is referred to [22]). Hence, (4)-(5) represent bounded integral control dynamics without the need of a saturation unit that may lead to instability.

Considering $(E_0, E_{q0}) \neq (0,0)$, the possible equilibrium points of the controller dynamics are any points on the curve W_0 that satisfy: i) $v = v_{ref}$, which corresponds the the desired output voltage regulation or ii) $(E_e, E_{qe}) = (\pm E_m, 0)$, corresponding to the case of current limitation as explained in the sequel.
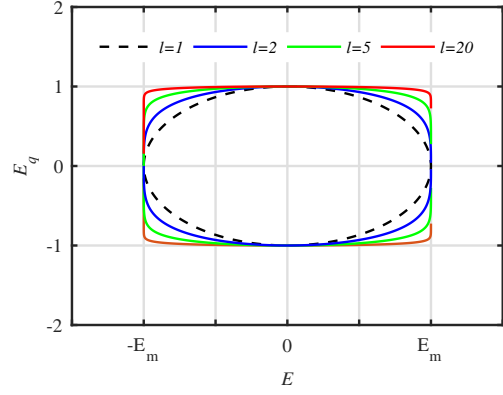

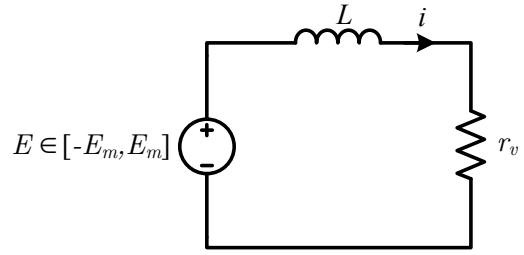Fig. 2. W_0 for different values of l

Fig. 3. Equivalent circuit of the closed-loop system

B. Current limitation

By applying the proposed controller expression (3) into the converter dynamics (1)-(2), the closed-loop system equation for the inductor current i becomes

$$L\frac{di}{dt} = -r_v i + E, \qquad (11)$$

and it is obvious that r_v represents a virtual resistance in series with the converter inductor L. The equivalent circuit diagram of the closed-loop system is shown in Fig. 3.

In order to investigate how the selection of the virtual resistance and the bounded controller dynamics of E are related to the desired current-limiting property, let the Lyapunov function candidate

$$V = \frac{1}{2} L i^2$$

for closed-loop current dynamics (11). The time derivative of V yields

$$\begin{aligned} \dot{V} &= L i \frac{di}{dt} = -r_v i^2 + E i \\ &\leq -r_v i^2 + |E| \, |i| \leq -r_v i^2 + E_m \, |i| \, , \end{aligned}$$

given the bounded voltage $E \in [-E_m, E_m]$, which implies that

$$\dot{V} < 0, \forall |i| > \frac{E_m}{r_v}.$$

Hence, if initially $|i(0)| \leq \frac{E_m}{r_v}$, then it holds that

$$|i(t)| \leq \frac{E_m}{r_v}, \forall t \geq 0, \qquad (12)$$

Parameters	Values	Parameters	Values
L	2 mH	l	50
C	50 μF	k	1000
V_{in}	100 V	c	10
R_L	150 Ω	E_m	10 V
i_{max}	5 A	r_v	2 Ω

due to the invariant set property. Based on the desired current-limiting property, it should hold true that

$$|i(t)| \leq i_{max}, \ \forall t \geq 0, \tag{13}$$

for a given maximum value i_{max} of the inductor current. Then by combining (12) and (13), one can suitably select the parameters E_m and r_v in the proposed controller in order to satisfy

$$E_m = r_v i_{max}. \tag{14}$$

Hence, any selection of the constant and positive parameters E_m and r_v that satisfy (14) results in the desired current-limiting property (13) of the converter inductor current independently from the load or system parameters (e.g. converter inductance, capacitance).

Note from the closed-loop dynamics (11) together with the controller dynamics (4)-(5) that when at the steady state there is $v = v_e = v_{ref}$, then $E = E_e$ on the curve W_0 and the value of the inductor current becomes $i_e = \frac{E_e}{r_v}$. Since $E_e \in [-E_m, E_m]$, then the inductor current can be both positive and negative satisfying the bidirectional operation of the converter. When $E_e = E_m$ then $i_e = \frac{E_m}{r_v} = i_{max}$ and when $E_e = -E_m$ then $i_e = -\frac{E_m}{r_v} = -i_{max}$ corresponding to the current limitation of the converter in both directions of the current.

It should be underlined that opposed to the existing traditional current-limiting controllers, the proposed design maintains the current limited during transients, as proven by the nonlinear ISS property and does not require any external limiters or saturation units that can lead to instability, which highlights the superiority of the proposed controller.
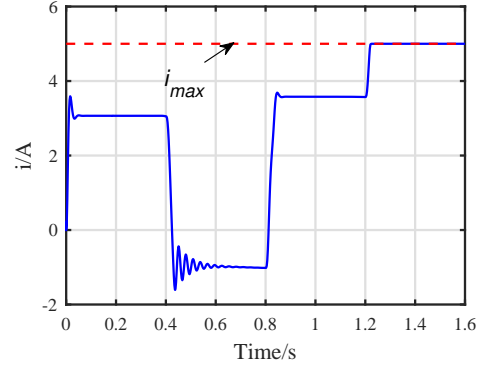
IV. SIMULATION RESULTS

In order to validate the proposed controller performance and the current-limiting property of the converter, a bidirectional dc/dc power converter equipped with the proposed control scheme is considered and connected to a resistive load R_L in parallel with a current source i_L. The current source can change from positive to negative values to enable the bidirectional operation of the converter. The system is simulated using the Simpower Systems toolbox of Matlab/Simulink based on the parameters given in Table I. Note that since the maximum current of the converter is $i_{max} = 5$ A, the controller parameters E_m and r_v are selected as $E_m = 10$ V and $r_v = 2\,\Omega$ to satisfy (14).
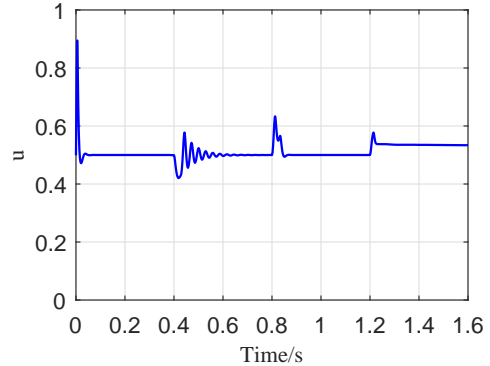
The desired task is to regulate the converter output voltage v to $v_{ref} = 200$ V independently from the variations of the
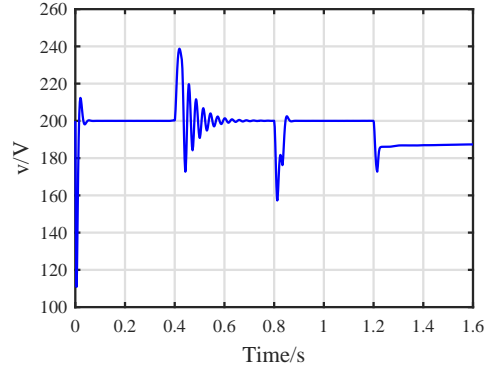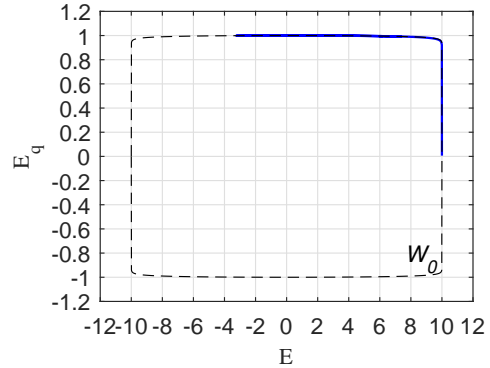

(a) output voltage

(b) inductor current

(c) duty ratio

(d) $E - E_q$ plane

Fig. 4. Simulation results of the bidirectional d/dc converter equipped with the proposed current-limiting controller

current source. Initially $i_L = 0.2\,\text{A}$ and as it can be seen from the simulations results in Fig. 4(a), the output voltage is quickly regulated at the desired level. Since the current of the load is positive, then the converter feeds the load and the inductor current is positive as well (Fig. 4(b)). In order to test the bidirectional operation of the converter under the proposed controller, at $t = 0.4\,\text{s}$, the load current i_L changes to $-1.8\,\text{A}$. In this case, the proposed controller regulates again the output voltage at v_{ref} after a short transient but as it is observed in Fig. 4(b), the inductor current is regulated at a negative value. At the time instant $t = 0.8\,\text{s}$, the load current i_L becomes $0.5\,\text{A}$ and the controller leads the converter output voltage at the desired level once again with a positive inductor current. In order to verify the current-limiting property of the controller, at $t = 1.2\,\text{s}$, the load current i_L changes to $1.5\,\text{A}$. In this case, as shown in Fig. 4(b), the converter current reaches the upper limit ($i_{max} = 5\,\text{A}$) and remains limited as analytically proven in this paper. The output voltage v automatically drops to a lower value to maintain the current-limiting property of the converter and hence protect the device from high load demands that request power levels that exceed the technical limits of the converter, as shown in 4(a). The transient response of the duty-ratio control input of the converter is illustrated in 4(c). To verify the theory presented in this paper, the trajectory of the controller states E and E_q is plotted on the $E - E_q$ plane where it becomes clear that it remains very close to the curve W_0 at all times, proving the desired bounded dynamics of the controller states.

V. Conclusions

A nonlinear dynamic control scheme was designed for bidirectional dc/dc converters to guarantee output voltage regulation with a current-limiting capability. By introducing a virtual resistance and bounded voltage dynamics, it was analytically proven that the inductor current of the converter will never violate a given maximum value. This current limitation is proven without any additional saturation units or limiters; hence overcoming integrator windup and instability problems that often occur with traditional current-limiting controllers. The effectiveness of the proposed design and its current-limiting property were verified by simulating a bidirectional converter under several changes of the load current.

Future work will focus on proving the asymptotic stability of the desired equilibrium of the bidirectional dc/dc converter and extend the proposed controller application to dc micro-grid systems with energy storage components.

References

[1] G. C. Konstantopoulos and A. T. Alexandridis, "Non-linear voltage regulator design for DC/DC boost converters used in photovoltaic applications: analysis and experimental results," *IET Renewable Power Generation*, vol. 7, no. 3, pp. 296–308, 2013.

[2] S. K. Kim, J. H. Jeon, C. H. Cho, J. B. Ahn, and S. H. Kwon, "Dynamic modeling and control of a grid-connected hybrid generation system with versatile power transfer," *IEEE Transa. on Ind. Electron.*, vol. 55, no. 4, pp. 1677–1688, April 2008.

[3] Y. X. Wang, F. F. Qin, and Y. B. Kim, "Bidirectional DC-DC converter design and implementation for lithium-ion battery application," in *2014 IEEE PES Asia-Pacific Power and Energy Engineering Conference (APPEEC)*, Dec 2014, pp. 1–5.

[4] M. Kabalan, P. Singh, and D. Niebur, "Large signal Lyapunov-based stability studies in microgrids: A review," *IEEE Transactions on Smart Grid*, to appear, early Access.

[5] J. A. Solsona, S. G. Jorge, and C. A. Busada, "Nonlinear control of a buck converter which feeds a constant power load," *IEEE Trans. on Power Electron.*, vol. 30, no. 12, pp. 7193–7201, 2015.

[6] M. Car, M. Vasak, and V. Lesic, "Control of a buck-boost DC-DC power converter for microgrid energy storage," in *19th International Conference on Electrical Drives and Power Electronics (EDPE)*, Oct 2017, pp. 122–127.

[7] J.-H. Su, J.-J. Chen, and D.-S. Wu, "Learning feedback controller design of switching converters via matlab/simulink," *IEEE Transactions on Education*, vol. 45, no. 4, pp. 307–315, 2002.

[8] S. Singh, D. Fulwani, and V. Kumar, "Robust sliding-mode control of dc/dc boost converter feeding a constant power load," *IET Power Electronics*, vol. 8, no. 7, pp. 1230–1237, 2015.

[9] Z. Chen, W. Gao, J. Hu, and X. Ye, "Closed-loop analysis and cascade control of a nonminimum phase boost converter," *IEEE Trans. Power Electron.*, vol. 26, no. 4, pp. 1237–1252, 2011.

[10] J. A. Barrado, A. E. Aroudi, H. Valderrama-Blavi, J. Calvente, and L. Martinez-Salamero, "Analysis of a self-oscillating bidirectional DC-DC converter in battery energy storage applications," *IEEE Trans. Power Del.*, vol. 27, no. 3, pp. 1292–1300, July 2012.

[11] L. Albiol-Tendillo, E. Vidal-Idiarte, J. Maixe-Altes, S. Mendez-Prince, and L. Martinez-Salamero, "Seamless sliding-mode control for bidirectional boost converter with output filter for electric vehicles applications," *IET Power Electronics*, vol. 8, no. 9, pp. 1808–1816, 2015.

[12] R. Ortega, A. Loria, P. J. Nicklasson, and H. Sira-Ramirez, *Passivity-based Control of Euler-Lagrange Systems, Mechanical, Electrical and Electromechanical Applications*. Springer-Verlag. Great Britain, 1998.

[13] R. Sira-Ramirez, H.and Silva-Ortigoza, *Control Design Techniques in Power Electronics Devices*. Springer, London, 2006.

[14] H. Rodriguez, R. Ortega, G. Escobar, and N. Barabanov, "A robustly stable output feedback saturated controller for the boost dc-to-dc converter," *Systems & Control Letters*, vol. 40, no. 1, pp. 1–8, 2000.

[15] Y. I. Son and I. H. Kim, "Complementary PID controller to passivity-based nonlinear control of boost converters with inductor resistance," *IEEE Trans. Control Syst. Technol.*, vol. 20, no. 3, pp. 826–834, May 2012.

[16] Z. Wu and C.-H. Liu, "Nonlinear current-mode controller for DC/dc boost power converters," *Electronics Letters*, vol. 47, no. 3, pp. 209–211, 2011.

[17] K. F. Krommydas and A. T. Alexandridis, "Modular control design and stability analysis of isolated PV-source/battery-storage distributed generation systems," *IEEE Journal on Emerging and Selected Topics in Circuits and Systems*, vol. 5, no. 3, pp. 372–382, 2015.

[18] G. C. Konstantopoulos and A. T. Alexandridis, "Generalized nonlinear stabilizing controllers for Hamiltonian-passive systems with switching devices," *IEEE Trans. Control Syst. Technol.*, vol. 21, no. 4, pp. 1479–1488, 2013.

[19] G. C. Konstantopoulos, Q.-C. Zhong, B. Ren, and M. Krstic, "Bounded integral control of input-to-state practically stable nonlinear systems to guarantee closed-loop stability," *IEEE Trans. Autom. Control*, vol. 61, no. 12, pp. 4196–4202, Dec 2016.

[20] K. Saichand and J. Vinod, "Virtual resistance based control for ultracapacitor based bidirectional dc-dc backup system," *Department of Electrical Engineering Indian Institute of Science*, 2015.

[21] G. C. Konstantopoulos and Q. C. Zhong, "Current-limiting DC/DC power converters," *IEEE Trans. Contr. Syst. Technol.*, vol. PP, no. 99, pp. 1–9, 2018.

[22] G. C. Konstantopoulos, "Enhanced bounded integral control of input-to-state stable nonlinear systems," *IFAC-PapersOnLine*, vol. 50, no. 1, pp. 8151 – 8156, 2017, 20th IFAC World Congress.