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Mathematics, Writing, and Rhetoric: Deep Thinking in First-Year Learning Communities

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Abstract

Through the process of combining two seemingly unlikely bedfellows, mathematics and composition, two instructors explain how rhetoric connects the art of writing and the art of doing mathematics in an inquiry-based learning community. Combining these two courses in a learning community enables students and instructors to practice the deep thinking valued by each instructor and by a traditional liberal arts education while challenging both our and our students' individual, disciplinary, and rhetorical conventions and beliefs. Using student writing from our course, our assignments from mathematics and composition, and survey evaluation results, we demonstrate how engaging in inquiry-based education provides unconventional (and conventional) learning opportunities for both students and instructors. Furthermore, through our discussions of the four iterations of our Learning Community, we examine some ways interdisciplinary learning challenges structural, individual, and disciplinary expectations, conventions, and learning.

Keywords: *learning community, writing, English, composition, mathematical explorations, peer review, inquiry-based learning, meta-goals, Discovering the Art of Mathematics, Mathematics for Liberal Arts*

“Mathematics is the art of explanation.”

–Paul Lockhart

A note on structure:

We have organized this article by dividing it into sections, using the same sorts of questions we encourage our students to embrace in some of the assignments we give—“Who? Where? When? What? Why?” and “How?” Our intended audience is both high school instructors (ELA and Mathematics teachers) and college-level composition and mathematics instructors/professionals who have ever wondered whether fruitful connections can be forged between composition and mathematics. We seek to examine both pragmatic and larger discursive ideas and strategies we used while referencing student voices and feedback. The reader familiar with and convinced of the ideas behind inquiry-based learning and learning communities in general can skip the “Why?” section and instead learn in the section “How?” about our specific implementation and tools.

WHO?

Prof. Jen DiGrazia loves to teach writing and ask questions. Learning communities like the one described below help her to stay “fresh” in her field, to see writing as an endeavor that spans all disciplines and contexts. They encourage her to take the sorts of intellectual risks she hopes students will take in their own writing and thinking. She also has an abiding interest in queer theory and disability studies. She has a Master’s Degree in English from Boise State University, and she earned her Ph.D. from University of Massachusetts’s composition program. She currently coordinates the composition and first-year read programs at Westfield State University, and she continues to recover from her self-proclaimed mathematics phobia.

Prof. Christine von Renesse loves teaching at all levels—from elementary school through college. She uses open inquiry techniques in all her classes, believing that this is the most effective and enjoyable way of learning and teaching. Her students learn to take responsibility, to think independently and to enjoy the endeavor of challenging questions with growing confidence. Prof. von Renesse has a Master’s Degree in Elementary Education, a Minor

in Music and a Master's Degree in Mathematics from the Technical University Berlin, Germany. After receiving her Ph.D. in Mathematics at the University of Massachusetts, Amherst, she is now professor at Westfield State University. She is also an integral part of the project "[Discovering the Art of Mathematics](#)", and has been facilitating workshops for faculty and teachers to promote and implement inquiry-based learning.

WHERE?

Westfield State University (WSU) is a residential public university in Massachusetts with about 4500 students, most of whom are undergraduates. It is traditionally a teaching college; many students are first generation, and the student population is predominantly white.

WHAT?

Between 2012 and 2016, we have taught four fully integrated **learning communities** (LCs) that combine two of our existing courses, *Honors Composition* (English 105), and an honors section of *Mathematical Explorations* (Math 110), with a cohort of honors students who elect to take the course. The course sessions are co-taught, with both professors in the classroom and run for a total of five hours a week. While *Mathematical Explorations* and *Honors Composition* both satisfy our university's core course requirements, neither counts for credit toward a mathematics or an English major or minor. We usually have a few mathematics and English majors in the LC anyway, but most students come from other majors such as psychology, music, criminal justice, history, theater, education, biology, etc.

Both authors use an **inquiry-based approach** to teaching in their discipline, and that teaching philosophy influences why we persist in bringing these two disciplines together in our LC. In mathematics, we use the term inquiry-based learning (IBL) as described in Sandra Laursen's study [21], which defines IBL courses as those characterized by:

- Learning goals focused on problem-solving and communication,
- A curriculum driven by a carefully constructed sequence of problems or proofs,

- Driving toward a small number of big ideas,
- Course pace set by students' progress through this sequence,
- Class time used for a mix of active and collaborative problem-solving tasks,
- Instructors who guide student work instead of delivering information.

This approach to teaching challenges the traditional idea of lecturing in which the instructor transmits knowledge to the students who listen, memorize and apply the knowledge. Instead, students develop definitions, theorems, and proofs themselves (what we call sense-making in [12]), while the instructor facilitates the students' learning process through active listening, guided questions, and by providing emotional support as needed. The larger mathematical problems are usually posed by the facilitator but can be extended through students' further questions and conjectures, see [29].



Figure 1: Prof. von Renesse supporting group work. Photo by the authors from Fall 2013.

The term inquiry-based learning is used in composition to describe student learning and writing that stems from and is driven by students' questions. Examples of texts that address this way of teaching composition include Bruce Ballenger's multiple editions of *The Curious Researcher* [2] written to help students identify issues and topics that stem from their own curiosity and to search for ways to understand the topic/issue and write their assignment. In these texts, Ballenger encourages students to pose their own questions, to explore ideas and problems that are "interesting" to them. His texts are focused on helping students navigate their own investment in the issue with the resources they find through interviews, observations, academic and popular texts, trade texts, documentaries, etc.

Similarly, in "Looking for Trouble: Finding Your Way into Writing Assignments" [31], published in *Writing Spaces*, Catherine Savini explains that a hallmark of a good writing assignment is its capacity to invite the student to pose questions that students—with instructor guidance—work through to identify a "project" that is possible for a student to complete within the frame of the course or the length of the assignment. She suggests, "There are four steps toward finding problems and developing meaningful projects of your own." They include "Noticing", "Articulating a problem and its details", "Posing fruitful questions", and "Identifying what is at stake" [31].

Student-based inquiry-driven approaches to composition and mathematics require students to be invested in seeing and articulating the problem or issue; they invite students to explore others' perspectives, and to ultimately situate their evolving perspective within the framework of what others have said. Such approaches run counter to assignments that pre-establish the issues or problems students are supposed to write about or "solve", and then typically require them to navigate/use course texts to justify their answer. The audience for the texts produced in many less inquiry-based composition and mathematics courses is usually the instructor, while inquiry-based writing assignments and student work on proofs will often ask students to explore, understand, and negotiate more complex rhetorical contexts, including thinking about a potential audience for their written work that extends beyond the walls of the classroom. Those of us who use such an approach value the ways writing and producing proofs happens in specific rhetorical contexts, and we purposefully draw students' attention to those contexts in the writing we ask students to produce and read.



Figure 2: Prof. DiGrazia in a rope exploration. Photo by the authors from Fall 2013.

Learning Communities

Published in PRIMUS (*Problems, Resources, and Issues in Mathematics Undergraduate Studies*) in 2016, authors Victor Piercey and Roxanne Cullen of “Teaching Inquiry with Linked Classes and Learning Communities” [24] argue that a learning community combining *Quantitative Reasoning for Business* and an *English Composition* class they recently taught had similar goals and benefits to ours. They suggest that it led to “greater student metacognition” and enable “deeper learning and recognition of the role of logic in problem-solving” (pages 1–2). We also witnessed these benefits and seek to add to their work by offering more examples and by show-casing different iterations—and the resultant thinking and knowing—different populations of students experienced in our Learning Community.

Historically, the benefits of learning communities are well-documented by scholars from a range of disciplines. Emerging from the conversations about Writing Across the Curriculum (WAC) scholarship and the importance of interdisciplinary learning, learning communities can take many forms, from partially-integrated or linked (two or more courses share a similar assignment, reading or concept) to fully integrated courses like those that we taught (both instructors are present in both classrooms; share grading efforts and create combined standards and classroom norms). Barbara Leigh Smith and Jean MacGregor [33], Jodi Levine Laufgraben and Nancy Shapiro [19, 32], Juan Carlos Huerta and Jennifer Bray [15], and Gerald Graff [14] all suggest that LCs have the potential to reform undergraduate education. The most important benefits include: creating connections among students, faculty and disciplines, improving retention among first-year students [32], increased participation on campus communities and an increased willingness and desire to participate in campus activities and initiatives on the part of students and instructors.

In a Joint Report from the National Center for Public Policy and Higher Education and The National Center for Higher Education Management Systems [4], Patrick M. Callan, Peter T. Ewell, Joni E. Finney and John Jones claim that LCs are good practice for improving productivity in the educational pipeline, particularly for 18-24-year olds. Shapiro and Levine also explain the documented benefits of LCs for first-year students: “Several studies illustrate that participation in learning communities has a positive impact on student achievement and retention” [32, page 171]. Based on the extent to which many of our former LC student demonstrate leadership on campus and their willingness to collaborate with us outside of class, we would agree. We think that the LC experience fosters students’ intellectual curiosity and willingness to explore opportunities and options that arise within the span of their time at WSU.

Combining Mathematics and Composition

In the United States, people are used to seeing mathematics and English / composition as separate brain activities. They are taught with little connection in our public school system—and in many college contexts—and this separation extends to the ways they are represented in most social, academic, and cultural contexts. In fact, often, people claim not be a “math person”,

or they claim not to have “inherited the writing gene”. People tend to suggest that they are “good at math, but not English” or vice versa. We discuss how we had to continually challenge and resist this perspective in ourselves while engaging in our learning community and encourage students to become aware of and challenge this tendency toward separation of math and English in themselves. We were surprised at the degree to which our collaboration challenged our own limited understandings of the other’s discipline. For both the instructors and the students, our LCs have required a significant departure from previous assumptions about mathematics and writing.

One student’s comments from Fall 2015 help to illustrate that change. In her final reflection of the semester, English major Anna notes that “Forbidding me to say that I dislike math has actually been very beneficial. I have censored myself from expressing hatred of math even outside of the hearing range of Professor Von. I do not actually hate math—something I have always known—I simply hate not being able to understand it [yet].” Anna was in fact able to change her mindset about mathematics and was, at the end of the semester, not only interested in making sense of mathematics, but also confident enough to go to the board during class to explore her ideas, create mathematical conjectures and explain her proof ideas to the rest of the class. Another student reflects in his journal: “Before this class I thought writing in math was crazy. I thought that there was no way I could transform all the mathematics I do on paper into a written assignment. Now, however, I can’t imagine doing mathematics without incorporating writing. It has become just as important to my learning process in mathematics as anything else. Throughout our time investigating the maypoles, I had a general understanding of the mathematics, but it wasn’t until I wrote my final proof that I began to see how everything was connected.”

We suspected, and this belief has become stronger throughout our four semesters of teaching our LC, that as a traditional liberal art, rhetoric can serve as connective tissue between our disciplines. Our collaboration was further strengthened by our combined interests in inquiry-based learning as a method of thinking, facilitating the classroom interactions, and approaching both composition and mathematics assignments. Each semester, the population of first-year honors students who takes the course is intrigued by the “unusual” combination of courses and persuaded by testimony provided by the authors during orientation or by former students’ enthusiasm for the course. While the courses are different each year, one feature remains consistent:

the resulting courses require both students and instructors to acknowledge and come to terms with their existing beliefs about mathematics and writing—and check them at the door of the classroom in order to position themselves as learners.

As education becomes more professionalized, colleges and universities today struggle to define themselves as supporting either liberal arts or the trades—and our school is no exception. While both mathematics and composition tend to be valued for their applications, for the ways they can help students “succeed” in “the real world”, we have found that bringing the two disciplines together in an inquiry-based learning community actually reinforces both disciplines’ roots in the liberal arts, valuing deep thinking and problem-based learning.

Meta-Goals instead of Content-Goals

As a Composition Coordinator who has revamped composition outcomes and course descriptions for *Composition* I and II, and as a member of the innovative *Discovering the Art of Mathematics* team working to change the ways mathematics is taught in the schools and at the college level, both authors helped to develop and were intimately familiar with the course outcomes in their respective disciplines.

Despite common assumptions both within academe and outside of it, neither of these courses is content-driven. They are not about learning the basics of proper sentence structure, nor are they about memorizing and applying equations or theorems, activities which often drive perceptions of these courses. Instead, they are process-based, and the point of both courses (individually and together) is to develop critical thinking, to encourage students to think and learn in ways that might be unfamiliar to them.

We want students to become more confident, curious, and creative thinkers, to know what it is like to be a writer or a mathematician, and to think, feel, and behave like active members of these respective disciplines. We also want students to become better communicators of mathematical ideas, both orally and in writing. Central for both courses is the ability to persevere and collaborate on difficult problems by working in groups.

It might be surprising, especially in mathematics, that there are no specific content goals like memorizing specific equations or mathematical ideas,

or practicing methods to solve problems. While we recognize the value of such courses for specific majors, e.g. from STEM fields, our mathematical meta-goals can be achieved by working on interesting, open-ended problems from any content area of mathematics. Lower-stakes writing activities and peer review reinforce the process of exploration and illustrate the ways real audiences of students and instructors influence one another's thinking and learning. When taught using an inquiry-driven process-based rhetorical approach, composition is flexible and can lend itself to almost any discipline or subject.

Student Voices

The following three examples from students' voices demonstrate how they took risks in the LC and the resulting changes in their beliefs about themselves and their attitudes toward mathematics and writing.

Shauna (Fall 2015):

In her final reflection, Shauna notes a significant change in her thinking: "I did not consider myself a math person. Due to this, I felt as if my ideas did not have a lot of significance. I felt as if other people did not really care about what I had to say during the class. However, this was a very wrong assumption. Even when I am wrong about a proof, sharing the ideas in the groups helps to see each other's different thought processes. The realization that other people in the class actually did value my work, and I valued theirs is what ultimately helped me "broaden my horizons." Instead of shutting down when I get a wrong answer on a proof, I now find it easier to accept the fact and try again."

Shauna realized that all ideas are valuable and important whether they are correct or not because they are part of the process of learning. She learned that mathematics is not just about "getting the right answer" and that you need to persevere to get to the desired connections. While Shauna tended to be more comfortable with writing than with mathematics, mathematics provided her with important information: not knowing is valuable, and she was similarly able to take more risks in her writing and thinking as the course progressed.

In her book [3], Jo Boaler draws upon Carol Dweck’s growth mindset research to argue that we need to re-think the ways we teach mathematics. She provides opportunities for creating new connections that help students to bridge the gap between how we actually learn mathematics and how mathematics is traditionally taught. Like all “growth mindset” advocates, she suggests that mistakes are useful and generative opportunities for growth. When we focus too much on the final “product”, whether it is in mathematics or composition, we deny students the opportunity to grow and instead reward what they have always done. We argue that our focus on process and on allowing students (and ourselves) to make and reflect on “mistakes” is one of the best features of our LC. See also [29].

Shauna also became more confident and started to enjoy doing mathematics: “From the very first moment we started the Stone Game, I was able to understand it and was actually excited to discover more ways in which a player could win. After the class ended, I remember I went back to my dorm room, tore up a piece of paper into little squares, and began to search for more ways that a player could win. After I was finished working through some of my ideas, I looked around at all the little scraps of paper that were covering my desk, and I was amazed with myself. That had been the first time I had ever gotten so excited about working on a math problem outside of the classroom.”

Jane (Fall 2015):

In Jane’s first project, which asked students to explore the ways they had been taught and come to understand the role of mathematics, writing and god in their lives (reflecting that year’s first-year read, *The Bonobo and The Atheist* [5] by Franz de Waal), she explained that she found comfort in the rote learning and memorization “plug and chug” mathematics classes she had taken. She found them comfortable because “Math and science just seemed to have all the answers”. In contrast, she explains, “I dislike writing, and I find it hard to believe in god for similar reasons. There are no real concrete answers in either writing or religion, so I tend to be more apt to like things that have the answers, like mathematics.” While Jane was a strong mathematical thinker, she did not have the confidence or interest to create conjectures or try out new ideas—both of which are central tenets of doing real mathematics.

She used self-deprecating jokes to distance herself from any risk she could have taken and preferred to follow more outspoken students in their reasoning.

However, in her third assignment of the semester, she wrote about the institution of education, inquiring whether she wanted to pursue a career as a teacher when much of the curriculum (at least as she had experienced it) discouraged students from critical inquiry. In her third writing assignment of the semester, titled, “If the U.S. Education System is so Flawed, How can we Power Through?”, she reflects on her own experiences as a student and asked how she, as a potential teacher could “do school” differently if she chose to pursue a career as a teacher. To establish the “problem” or to identify the issue, she describes her own experiences and notes, “the more research that I do about the field of education, the more discouraged I feel . . . I have also realized that it is a deeply flawed establishment.” Referencing the practice of “teaching to the test” [25], Jane explored the ways her own exposure to this method of learning took the joy out of subjects like mathematics and composition. While identifying the ways high-stakes testing rewards students and teachers for this practice, she ultimately asks if it is possible for teachers to do school differently.

In her final course reflection, Jane references her experiences in our course, which purposefully required her to unsettle the assumptions and thinking—about both mathematics and writing—with which she had entered the course. Of her experiences in our joint class, she explains that, “true learning involves struggling . . . ideas I was pretty uncomfortable with, but through these classes, I was forced to face my fears.” Collectively, Jane’s papers and progress throughout the course show a student who is revising her understanding of the purpose and function of education.

Jim (Fall 2015):

In his reflection of the semester, Jim, a very creative thinker who surprised himself and the class continually with his beautiful conjectures and explanations, states, “As we progressed further throughout the math portion of the class, I always was learning new things. Professor Von and myself noted my improvement and confidence growth during the semester. At the beginning, I was unsure and hesitant about what I was doing. When the class was asked how confident we were with our proofs, the responses were not great.

Throughout the past few weeks, I have begun to quickly understand the theories, to some degree, fairly quickly. I have learned new ways of thinking such as deductively and inductively, and problem solving skills. I have even gained a liking for talking in front of people and I am more comfortable with standing in front of a class. I have even gained enough confidence in my answers to present them to the class and explain them while answering questions. This is a stage I never saw myself at, during the beginning of the class. I find myself excited when I figure out why the theory mathematically works and I make the connection and solve it. I look forward to my math class every day I attend; this is quite surprising compared with my mindset from high school. If in the past, I saw where I was at today, I would be quite surprised that I could ever be excited for a math class.” Through the process of metacognitive thinking about both math and writing, Jim’s confidence grew over the course of just one semester; perhaps more importantly, he recognized that growth.

Evaluation - Survey Results

Jane’s, Anna’s, and Jim’s stories above document what they see as significant changes in their thinking and learning. Combining the mathematics and the composition elements of our LC provides numerous opportunities for these sorts of reflections and growth. Additionally, matched pre- and post-survey evaluations in three of our learning communities (43 matched responses) support these students’ reflections.¹

For example, students report that they know more about what it means to do mathematics, who mathematicians are, and what they might feel. Students find mathematics more interesting and enjoy working on challenging mathematical problems and making discoveries. They find mathematics more beautiful and perceive an increased ability to understand and critique written or spoken mathematical arguments. Students also report that their ability to communicate and reason effectively has increased, they feel more empowered as learners, their curiosity about the world around them has increased, and their confidence in taking responsibility for their own learning has increased.

¹ The survey we used is available at <https://www.artofmathematics.org/sites/default/files/daomsurvey2013-2016.pdf>, last accessed on January 17, 2018.

Almost all survey responses moved in the desired direction on the 5-point scale. These are statistically significant shifts, many with high effect sizes. Compared to the other results of the *Discovering the Art of Mathematics* project (see [10]), and the larger IBL community (see [20]), our changes are statistically significant and the effect sizes are higher. While this could be the result of teaching a smaller class of honors students, we believe that the learning community itself had a large impact on these results.

Instructor Voices

The following personal reflections of both authors show the personal growth and professional development that teaching a learning community can have.

Prof. von Renesse's Reflection

I always thought of myself as a “bad writer”, and co-teaching the LC has increased my confidence. As a non-native speaker who failed English in high school, it took me a while to realize that I have the creative and critical thinking that is needed for writing, and everything else I am missing are techniques I can learn. I now know that just as mathematics is not mostly about performing arithmetic, so is writing not mostly about spelling and grammar.

Besides strengthening my writing skills, the LC allows me to think in-depth about topics that usually don't cross my professional paths. I tend to read most often about the teaching and learning of mathematics, or about mathematical topics. The learning community encourages me to read the “first year read”, which is usually a book completely outside of mathematics. Part of why I chose mathematics as my field of study is my discomfort with questions or problems that have at best a “grey” answer. I tend to feel safe with topics that are “right or wrong” and where problems have (eventually) a definite answer. Our course has given me the opportunity to think about and discuss topics like oppression of women, the problems of the Western medical system, racism, etc., and this has changed how I position myself inside and outside the university context.

I enjoy co-teaching and learning with Prof. DiGrazia so much that I am willing to teach an additional class for no compensation for the 4th time—and that should say enough about its benefits for me.

Prof. DiGrazia's Reflection

I love to do learning communities, and, prior to doing this LC with Professor Von, I participated in LCs combining composition and courses from movement science, women's studies and criminal justice. While they were/are all interesting, the material in these disciplines was also accessible to me. I like theories about how and why crime happens. The philosophical origins of physical education and the mind/body connection are interesting to me, and women's studies has consistently informed how I teach composition. However, entering into an LC with a mathematics professor scared me because I avoided mathematics at all costs. Like many of our students claim at the beginning of the semester, I thought I was "bad" at mathematics and "not a mathematics person".

Probably the most significant lesson I have learned is to see mathematics differently. While I continue to struggle with my facility with mathematics and mathematical reasoning, I, like Professor Von, truly enjoy approaching it from an inquiry-based, problem-driven perspective. Participating in this LC also continuously reminds me what it feels like to struggle academically, to fight my own assumptions about myself, to recognize and wrestle with my own limitations and to consistently challenge myself to try again, to see and to consider material and ways of thinking a-new. While I don't know that I am "cured" of my mathematics anxiety, I don't feel the need to loudly proclaim my inability anytime something related to mathematics or mathematical reasoning arises in a conversation. I've also learned to see and to continuously disrupt the bias we hold against mathematics and to challenge assumptions about what counts as mathematical thinking and reason. Finally, I think exposure to mathematical reasoning helps me to think better and more flexibly about writing and teaching—that, more than an ability to "relate to" or "speak the language of" the criminal justice or movement science or the few women's studies students in my composition classroom, benefits all students in my first-year writing classes.

HOW?**Course Texts and First Year Read**

At Westfield, all composition courses draw upon a first-year read for at least one (and usually more) major assignment. The first-year read books take on different topics/themes each year, and they are always chosen by committee

using the following criteria: they are non-fiction, they model academic inquiry, they build upon or reference the work of other speakers and scholars, and they somehow address (or lend themselves to addressing) issues of social justice. Additionally, the first-year read often allows us to find connections between and address social issues related to writing and mathematics. For example, with our 2015 first-year read, Sheryl WuDunn and Nicholas Kristoff's *Half the Sky* [16], we discussed and asked students to consider the value of micro-loans as a means to support entrepreneurs in developing countries while examining the larger social inequities that create problems associated with human trafficking and gender inequality.

In addition to the first-year read described above, we assign *A Mathematician's Lament: How School Cheats Us of our Most Fascinating Art Form* [22] by Paul Lockhart. Lockhart explains in his lament how flawed the education of mathematics in K-12 classes in the US currently is. His informal opinionated writing style and the challenges he poses to the education system give great fodder for discussion and personal reflection.

“Shitty First Drafts”, an excerpt from Anne Lamott's *Bird by Bird: Some Instructions on Writing and Life* [17] (also see [18]), does some similar work and is a foundational text in our LC. Lamott argues that all writers need to learn to trust the process of writing. Her claim that the best way to start writing and avoid writer's block and procrastination by writing a shitty first draft is often reassuring to students. It relieves them of the assumption that good writers, published writers, write beautifully initially, and she frees them up to begin. Once we have something on paper, we can develop, revise and even abandon the ideas in favor of new ones.

Through course texts—the first-year read, Lamott, and Lockhart—our learning community automatically inherits a direction of thought and is given the opportunity to examine how three other writers negotiate rhetorical contexts. Furthermore, the texts help us to consider the problems these writers identify and the larger implications these problems pose for society. All three texts are texts students return to, draw upon and analyze in different ways throughout the rest of the course.

While we use the texts to prompt thinking, we also use them in a range of ways: to provide models of different types of writing, to use as sources in students' writing, to introduce and test out ideas and perspectives, and to

examine the role of inductive and deductive reasoning in both the discipline of mathematics and writing. Once students have drafts of their work for any given mathematics or writing project, our students' writing and mathematical assignments become the central "texts" that drive the course. We use those texts and classroom-inspired examples and moments to illustrate how revision can work, to raise awareness of and challenge disciplinary conventions, to discuss audience expectations, and to demonstrate and foster a range of problems and lines of inquiry.

Guiding Questions

Metacognitively, as is true of many instructors invested in rhetoric and the liberal arts, the type of thinking we encourage students to do and the process of learning is very similar for both authors. Part of what we think makes this integrated course interesting and worthwhile is this similarity, and, each semester, our ability to come at the meta-goals from different disciplinary perspectives determines how we will accomplish that. To focus our efforts of uniting our courses, we developed these guiding questions:

- What does a mathematical proof have to do with writing?
- How are both writing and proving forms of persuasion?
- How can rhetorical tools and exposure to forms of mathematical logic enhance your ability to argue effectively?
- How does perseverance benefit us as thinkers in both English and mathematics?

These questions guide the structure of the writing assignments and the types of mathematical reasoning we ask students to practice each semester. For example, our first major composition assignment of the semester asks students to compose a narrative with analysis where students explore their beliefs about mathematics and writing, and the first assignment in mathematics is to produce a proof of a problem introduced in class, see [9]. For both assignments, we introduce the process of peer review (see below), emphasizing to students that while some of the criteria for writing

might differ depending upon audience, context, and discipline, the act of collaboration when writing and learning is vital for everyone's development. The two assignments allow us to illustrate early on the ways audience, context, and genre are shaped by rhetorical and disciplinary conventions, and reinforce the idea that good thinking and learning require students to take risks, make mistakes, and reflect. All writing is persuasive, the process of working through mathematical proofs strengthens students' inductive and deductive reasoning skills, and the success of projects like the ones we give depend largely upon students' abilities to persevere. Revision based on audience feedback is required for both assignments.

Taking Risks to Learn Vulnerability

“Transformational learning occurs when learners reject deeply held ideas, reorganize what they know, and restructure and question their basic assumptions and frameworks for learning”

Susan Loucks-Horsley *et al.*

When we ask our students to reconcile contradictions in mathematical reasoning or in their own interpretations or experiences, we model that process with students in ways that arise naturally through the discussion around assignments and thinking. That includes making visible for students our own vulnerabilities and uncertainties both in and out of the composition and mathematics classroom. We believe that transformational learning requires students and instructors to exhibit vulnerability, which, for us, arises from exposing our “differences”—personal, academic and methodological—to students. Prof. von Renesse's acknowledgement of her German ancestry and the historical and cultural baggage it carries for her and her discussion of her anxieties about writing is one example, and Prof DiGrazia's exposure of herself as “not straight” or “lesbian” or “queer”, and her revelation about her deep anxiety about mathematics is another. Our point is that these aspects of ourselves influence how we “read” anything (people, texts, world events), and we are asking students to come to an understanding of how their own identities influence their reading and thinking as well. In sharing these aspects of ourselves, we are inviting students to find the ways they “see” the world and to examine how and why.

We believe that having two instructors who are willing to be students in the other's subject area, which includes making mistakes and not feeling confident is a great model for vulnerability. We ask our students to reflect deeply on themselves in every paper they write; often, they start changing their points of view—about mathematics, writing, and issues related to social justice—during the semester. Every mathematical conjecture they create carries the risk of “being wrong” and “feeling stupid”, and every piece of writing—whether a first or final draft—carries the risk of being ignored or dismissed as unworthy. To an extent, our own vulnerability invites theirs.

While the previous examples of our disclosures illustrate some of our vulnerability, it also happens most notably when we are learning something about the other's discipline. For example, during class time, while deep in discussion about a proof focusing on why $0.9999 \dots = 1$, Prof. DiGrazia might say, “Wait. I don't get it. Can we back up? Here is where I got lost.” She is asking members of the learning community for help untangling her thinking, demonstrating that it is okay—even for an instructor—not to know, or not to be able to follow the thinking as quickly as others seem to be processing it. In a recent interview, Anna (discussed previously) said that when she got lost in class, she would often look at Prof. DiGrazia and be reassured because it was obvious from the confusion and frustration on her face that she was struggling as well. Through such modeling, we help students better understand how to be vulnerable, how to “not know”, and we model how we ask for help and take responsibility for our own learning. We try to establish it as the most valued and valuable part of our course.

Similarly, Prof. von Renesse might look confused (or even annoyed) when some discussion around the first-year read does not have a clear “outcome” and instead, the conversations keep circling around connected ideas. She finds it uncomfortable that there are no proofs for most claims outside of mathematics and that all you can do is try to persuade others of your opinion. By expressing her frustration while staying part of the conversation, she models to the students how she is confronting her discomfort with “the gray areas” of life. We examine these moments of confusion for inspiration because they model the very real impact that the process of learning something new might have.

Peer Review

We regularly use peer review for both composition and mathematics assignments. Prof. von Renesse first learned from Prof. DiGrazia how to successfully implement peer review in composition. DiGrazia's prompts focus on composition theorist Peter Elbow's feedback strategies of "Sayback" "Pointing" and "Questions". She later added "Assignment Criteria" to the list. We lay important groundwork when we first introduce students to the practice of peer review during a composition assignment and model the process as a full class sitting in a circle. A volunteer reads their paper and then students comment/ask questions using the protocol. Afterwards the students meet in small groups to continue the peer review so all students can get feedback.



Figure 3: Composition peer review. Photo by the authors from Fall 2013.

It took some time to find a version of peer review that worked for mathematics assignments. One of the differences between offering peer review for composition versus peer review for mathematics papers was that students had more difficulty deciding if the mathematical work of other students was correct or not. They also had few ideas about how mathematical writing could be improved or how to change their mathematical writing for a specific audience.

Using the language of proofs and the practices of peer review from composition, we adapted the prompts students used to respond to one another's work as follows:

Audience: Who seems to be the intended audience? What suggests that to you? What is this audience's interest in the proof? Why might they care? Do you have ideas about how the writer might shift language and structure (and ideas?) to more effectively address this audience?

Introduction: Does the writer offer an overview that outlines the problem? Does the writer explain what s/he will do, and why? Does the writer give some context for the problem and the proof? Do you have ideas about what the intro might do that it doesn't, or what the writer might do differently?

Statement/Conjecture: Restate the conjecture that you hear or that you almost hear. Are there ways you might suggest that the writer re-shape and re-state his/her conjecture? Why?

Specificity/Conciseness: Choose 1-2 sentences that seem "bulky" or less clear. As a group, re-work the sentences, paying particular attention to: use of pronouns (is the antecedent clear?), double negatives, active verbs (watch for "ing" verbs and "would" verbs) and clear connections between the agent of the sentence and the action of the sentence.

Transitions: Are you able to follow the logic of the proof? What connections is the writer making between ideas? Would repetition of the previous concept/term help to clarify? How about a transitional phrase or term?

Complete Proof: Would you call this proof "complete"? Why/why not? What would you like to see developed?

Holes: Do you see any gaps in the writer's reasoning or ideas? Are there concepts or ideas or connections that you think could be clarified or strengthened?

Images to help the reader if applicable: How does the writer use graphics or images? What are these graphics or images being used to do? Is it clear how the graphics/images work with the text?

Would you like more/less graphics/images? Why? Do you have ideas about how the writer might tighten the connection between the images/graphics and the written text?

Process: Did you get a sense of how the writer arrived at his/her proof? What process did s/he follow? Do you want a better sense of that process? Where and how might the writer incorporate that?

Curiosity and Questions: Was there anything that you found particularly interesting or intriguing? Do you have questions?

Prior to providing peer review of a student's mathematical proof, the facilitators choose the homework sample of a student and, with the student's permission, share copies with the class. Students then work in groups on the chosen piece of writing using the above prompts. We interject during whole class discussions to compare ideas throughout the process of providing peer review and decide on the changes we liked best. All students (including the author of the work discussed) can then redo their homework using the insights they gain from reviewing each other's work.

The continuity of peer review from composition to mathematics and vice versa develops and strengthens throughout the course. As a student from our 2014 course noted in her reflection, "In math class, we did a peer review exercise that we typically do in English, but in this case, it was applied to a proof instead of an essay. Personally, this exercise was extraordinarily helpful, as it forced us to view our proofs from a different perspective and gave us a way to "get out of our own heads" so to speak. This was definitely something with which I struggled at the beginning of the year, especially when I was trying to write up my proof for the $3a + 5b$ problem. In my head, I was confident that I understood what was going on, but when it came to writing that down in an explanation, that confidence fell short. Suggestions and constructive criticism such as "I know you understand it, but you need to work on your wording and explanations" and "be more precise here" were among the types of feedback that I received for this proof. Had we done the peer review exercise sooner, I may have been able to more clearly recognize my shortcomings and use that peer review as a way to go over my work from the perspective of an outsider who can't read my mind or know anything about my brain activity."

In both composition and mathematics, the process of peer review helps students realize that all writers benefit from feedback, from discussion, and from having the space to “try out” new ideas and perspectives. The writing we do throughout the course makes it much more organic to discuss the different audience expectations of a “mathematics reader” who has a strong need for certainty and proof (and someone unfamiliar with what counts as “evidence”, like quotes, summaries, and examples in a composition class) and an “English reader” used to living in the “grey” areas (and unfamiliar with the discourses and conventions of mathematics).

Conferences

This awareness of audience is developed through the feedback that students give each other through peer review and when we give students feedback on their writing-in-process during one-on-one 20-minute individual conferences between the student and the instructor(s). For each of the four larger composition projects, students meet with DiGrazia or with DiGrazia and von Renesse (depending upon time constraints and teaching schedules) to get feedback on their progress with the assignment. We ask questions to challenge the student to “go deeper”, and often make suggestions about what to revise in their work and what to develop further. Often, our (the professors’) collaboration during conferences illustrates for students the different understandings people might have of their work. Our ability to talk through differences with one another and with the student or to support one another’s interpretation of a student’s text further models the process of collaborative learning and reinforces the power of audience in writing.

Sample Assignments

The second assignment given during our 2016 LC was a composition assignment that required students to think about proof and truth in the context of the first year read: Will Allen’s book *The Good Food Revolution* [1]. This was one of four substantial composition projects that the students would produce during the semester. Notice how we were seeking to draw from both disciplines, and to address specific requirements while working to give students freedom of choice in purpose, audience, and genre.

Project Two: Adaptability and Perseverance: Good Food and Proofs

“I can listen no longer in silence. I must speak to you by such means as are within my reach.”

Jane Austen, *Persuasion*.

Persuading yourself and others about persuasion. We will be getting meta with this project in that you will be examining how you interpret information to determine what’s true. What does it take to convince you? Do our criteria change, depending upon the situation? How so? Are there different standards we purposefully or unconsciously apply to different contexts? What counts as “proof” or evidence in math or in composition? How do those standards differ, and which do we rely upon in our daily lives? This assignment asks you to use the work you have done in Math 110, a section of Allen’s text (you choose it), and one supplementary text to consider different standards of proof. How are mathematicians convinced about truth? How are people in mainstream culture (say, readers of a text like Will Allen’s) convinced of an idea? When you look at the different tools and techniques used in each of these contexts, what do you notice? What does that suggest to you?

Your **purpose** is to use the data you gather from both your own work and from an analysis of a section of Allen’s text to write a **text** (a letter, an essay, a FAQ, or an article) to **an audience you identify** encouraging the audience to think more critically about the ways we consume and use information. Are some standards better than others? Does it depend entirely upon context?

To gather one set of data for your text, think about (and examine carefully) the steps you use when writing a proof. To gather another set of data, choose a section of Allen’s text and examine the ways he works to persuade an audience of his point. What evidence is used in each context? Compare and contrast these sets of data. What is different or similar about these acts of persuasion? What sense do you make of those similarities and differences,

and what do they suggest to you about persuasion in more daily contexts? What works? What doesn't? Are there standards or types of information you think we might want to reconsider or look at differently? Why?

As you examine the different intellectual and rhetorical moves made in each context, consider the many variables that come into play when we seek to convince or persuade our audiences. What is interesting or important about these insights? How can you communicate these insights to your chosen audience?

Steps to Take When Drafting and Due Dates (these may shift in response to our actual progress in class, but they should give you a sense of the trajectory of the project):

1. Explore: Gather data. Brainstorm in and out of class: Explore Allen's text and other texts (film and speakers' events and supplemental texts). Examine the ways you go about writing a proof. Look for differences and/or similarities between these types of texts and think about what those differences/similarities suggest to you.
2. Draft: Using the data you have gathered and the insights you are developing, write a Shitty first draft (SFD). As your writing takes shape, you will start to consider what you might want a potential audience to understand or know—and why it matters. This is a good time to begin thinking about an overall idea or message about persuasion and information you might want to communicate. Bring your SFD to class for peer review on Tuesday, October 11 (it acts as a Monday schedule).
3. Revise: After peer review, use the feedback you received from your peers to revise your work, developing your main idea, fleshing out ideas and details, considering different questions and interpretations of your ideas.
4. Analyze possible publication venues and potential audiences: At this point, you can also start to identify and clarify your target audience and the possible genres that would allow you to reach this audience. Examine possibilities for com-

municating with an audience. What sorts of genres are appropriate? Why? What are the expectations of a particular audience and/or genre? Do a genre and audience analysis.

5. Conference: Sign up for a conference date/time between Oct. 14-17. When you come to conference, bring two drafts of your paper and your genre/audience analysis, and we'll work to both clarify the purpose of your paper, the development it needs, discuss genre conventions and audience expectations.
6. Revise: Using the feedback you receive from me and/or Professor Von, revise again. Bring a revised draft to class on 10/19 for a final workshop. Incorporate all three texts (Allen's, the supplementary text, and your analysis of your proofs), and work to meet the genre conventions and to communicate with your target audience.
7. Final: Your final draft and all accompanying work (SFD, peer review draft, conference draft, genre analysis, post-conference draft, final draft) is due on October 24, depending upon our progress.

Requirements for Final Draft

1. Your final text should have a clear purpose, be written for a specific audience and in a specific genre, all of which you choose.
2. Your text should be persuasive, speak to/identify the overall main point and address some larger "So What?" questions associated with persuasion.
3. Your text should meet the expectations of your target audience and fulfill the conventions of your chosen genre. Length, style, type of documentation depend upon the genre and will be determined for each student in class and during conferences.
4. Introduce and give your audience a context for understanding the texts you use if the audience is unfamiliar with them.
5. Meet all due-dates and course requirements.

A second paper assignment example is provided in Appendix B.

The following assignment offers an example of a weekly mathematical writing assignment. While the assignment is much shorter, there is a longer rubric explaining the expectations for writing a “homework story”, see [27]. Appendix A contains a sample homework story from Fall 2016.

Recall that in class we were clapping our first names, using regular claps for consonants and slaps on the legs for vowels. We then played our name-rhythms in a loop, emphasizing the first letter to keep track of the beginning. When you performed your name-rhythm simultaneously with another student, we noticed that eventually the first letters lined up again. We were wondering if we could predict when this would happen...

Please write up your first draft of a proof for the clapping names conjectures. Why is the least common multiple $+1$ the number where the first letter claps meet again? If the word “proof” scares you, think of writing a convincing argument or “making sense” of your reasoning. You are not expected to write a formal proof.

(You might also need to prove how the greatest common factor and the least common multiple relate or why you can “reduce” number pairs by taking out common factors.)

Additionally, student work from our maypole dancing explorations is provided online at [27].

Grading and Assessment

We grade our classes separately instead of giving one grade for the whole learning community. The grades have to be reported separately anyway, and each of our courses satisfies different types of core requirements. Also some students are stronger in one of the subject areas.

For the final grade in composition, the students have to submit a portfolio containing at least two of the larger writing projects, each of which includes all drafts, and some of the mathematics assignments. The bulk of the portfolio grade comes from students’ accompanying reflective metacognitive letter or essay, which is meant to introduce their work. In the mathematics portion,

students are graded on participation during class, weekly homework-stories [26] and (sometimes) a final exam. Courses share a final portfolio in which students choose work from both courses to demonstrate their learning gains.

Preparation and Planning

To think about the larger themes of the LC, choose the mathematical activities, and design the writing assignments, we meet several times during the summer. During the semester, we try to meet once a week to prepare the big ideas of the next classes. Additionally, we reflect on past classes during phone conversations. It is very helpful to have two facilitators in the classroom at all times and to reflect and receive feedback on a regular basis. But it is also very time consuming. Our class sessions are still labeled “Mathematics” or “Composition”, and only one professor is responsible for planning the details of the respective class period. Similarly, the mathematical homework stories and composition projects are graded by the respective professors.

We are both present for all class periods, and, as noted, we conference with students together as often as possible. In our perfect world, both of us would complete all the course assignments from the other course ourselves, but in reality, we do not have the time. We hope that there will be more support for LCs that will allow us to dive even deeper into each others’ disciplines.

Classroom Tools and Resources

Just to reiterate: during class time, there is no lecture. Instead, we do mathematics, free write, facilitate peer review, conference with students, revise writing and mathematics projects, and discuss students’ questions, struggles, and ideas. Generally, we see our role as that of supporting deep student conversations instead of responding to students’ questions and struggles with our own professional opinions. Detailed descriptions of some of the tools for the mathematical investigations and discussions can be found at <https://www.artofmathematics.org/>.

Institutional Constraints and Organization

Despite our efforts at collaboration, the grading structure we use (separately reporting grades) demonstrates ways that our institutional structure makes

the sort of learning we advocate difficult. Despite our success as documented by our own and students' testimonies and texts, our school has an inconsistent record of support for LCs. In the past, LCs have received administrative funding through a series of internal grants, but they have been deemed "unsupportable" due to the high level of investment they require of faculty. Essentially, because our contract requires us to teach a 4-4 course load, when we undertake a fully integrated LC like ours, we take on a fifth class [6]. Despite many faculty members' clear investment in LCs, our administration has not been willing (or is unable) to provide sustained support for LCs.

Additionally, different class sizes amongst disciplines and courses create challenges because it is hard to find classes where the numbers of students are easily matched. We are lucky that the honors program at WSU continues to be very supportive of our learning community, and since all honors classes are kept at 15 students, we solved our problem of class caps (traditional *Composition* courses are capped at 19 students, and *Mathematical Exploration* courses at 30 students) by offering our course through the honors program. While working with honors students is highly rewarding, as instructors who hope to reach and provide all students with a range of opportunities, we are also interested in seeing this course offered to all students at WSU.

For those interested in the scholarship and specifics associated with LCs, we suggest consulting the Washington Center resources, including a journal and an email list (<http://wacenter.evergreen.edu/>).

Areas to Think About and Work On

We know from reading our students' assignments and final papers that in each installment of the course almost all of them successfully work toward the meta-goals we mentioned above. The few students who really struggle (at most 2 out of 15 per semester) have explained in their survey responses that they were not ready to be challenged by this way of thinking. Some realized that being in the honors program was too challenging for them, and some struggled with the transition from high school to college. Our learning community is fun for both us and the students—but it is by no means easy. We also wonder if the type of thinking we are encouraging students to do in this LC privileges some students' ways of knowing over others. For example, some students with learning disabilities may struggle with the theoretical and metacognitive nature of the type of thinking we encourage.

We continue to think about ways to support our students so that all of them can be successful. Maybe discussing equity, access, learning styles, and students' understandings of metacognition as it functions as a goal in our course more explicitly will offer more options for students who disengage and/or struggle.

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A. A Sample Homework Story

The following homework story, about a mathematical activity inspired by Karl Schaffer (<http://www.mathdance.org>), does not show average student work but provides an idea of how unusual and informal the writing of a “homework story” can be. We encourage this personal writing style.

Since you’ve told us we can write just about anything here and are encouraged to ramble on about our whole process, I’ll give it my best try. Warning, this might be pretty long. I was driving home for the weekend and explaining the homework I had to do to my mom, a fourth grade teacher, when the book, *A Mathematician’s Lament*, really began to make sense to me. She instantly understood from the clapping skit we did that this was a Least Common Multiple problem, and didn’t really get why such a basic concept, nor why we would have to prove it. To her, knowing the definition of what LCM is would be all you needed as this was clearly a LCM problem. I, myself, was a little surprised when Professor von Renesse (I know you said we could call you Christine, though it feels weird calling a teacher by their first name) immediately asked the class to try and disprove me, rather than saying I was right and explaining why. But then, thinking of the book, I understood what was going on. *A Mathematician’s Lament* talks about how teachers sometimes make math cute for those who dislike it, though they still teach though lecture afterward. My mom in her class would have done the same. But here, after the skit and instead of a lecture, once the correct answer was given, the class was assigned to go about disproving me. In this process, they of course had to go through my logic and see it for themselves and therefore truly begin to understand it. And still after the end of class, it was not explained to us. Instead, since my “conjecture” had not been disproven, we were tasked with proving it, assuring each and every one of us must come to the LCM conclusion ourselves. I truly enjoyed the book, as well as the idea behind this assignment. Now, having praised the book and your approach to this class enough, on with the actual proof. I was first pretty discouraged when my two subsequent attempts to prove the LCM conjecture were also labeled as conjectures

and not proofs of conjecture 1. But I understand why you did so. Firstly, had anything I said been considered a genuine proof, then we would not have been able to do this assignment. It would also have allowed for anyone in the class who did not yet understand how the clapping skit tied into Least Common Multiples to either remain ignorant, or memorize the correct answer without truly understanding. Also, it's not truly a proof. I explained how the numbered pairs can be changed so long as they stay in the same ratio (2,3-4,6-8,12) and still have the same number of repetitions until the cycle begins anew (in this examples case, no matter what you do to these numbers, multiply or divide, as long as they stay in a 2:3 ratio with one another, the first number will always be repeated 3 times and the second number repeated twice before they start up at the same time together). Greatest Common Factor is the inverse of LCM. It's the greatest whole number you can take out of two values via division and still be left with two whole numbers. Taking out the GCF from two values keeps them in the same ratio. Taking out any common factor or multiplying by any number (as long as that number's the same for both sides) keeps the two values in the same ratio with each other. Infinite values can be in a 2:3 ratio with one another. Same with a 1:4 ratio. Or a 3:5 ratio. Or anything that can't be simplified down by taking out a GCF, as there no longer is a GCF. Well technically there is, the GCF is 1, but by the laws of multiplication and division, the number 1 is actually useless. This explanation provides an infinite number of examples that work. It shows how anything in the 2:3 ratio will always repeat twice and 3 times before starting again at the same time. I had also explained how in each of these infinite examples it will always be the Least Common Multiple of the two numbers (6 for 2 and 3, 12 for 4 and 6 and so on) that will be the total number of claps before the cycle begins anew. This doesn't necessarily prove that the LCM will always be the first time the cycles of clapping start together again, it simply provides an infinite number of examples where that is true. Testing an infinite number of examples separately is not a proof, there needs to be some mathematical law to explain it. In high school math classes, a proof like this would require simply citing the definition of Least Common Multiple, as we are lucky enough to be

alive long after LCM for first discovered and we only need know it exist rather than prove it ourselves. Someone else did it for us. However that thinking of someone else did it for us directly contradicts the point of why we did this and the ideology seen in *A Mathematician's Lament*. Thus, I have to go deeper down the mathematical rabbit hole. A common multiple between two numbers is a number both of them can be multiplied to. Multiplication is repeated adding. If I add the same number to itself many times, each time I do I hit another multiple of that number. A common multiple between two numbers is when both of those numbers can be added to themselves repeatedly, and eventually both will hit the same value. A common multiple can always be found when you multiply two numbers together. Clearly that number is a multiple of both of them, since, well, they both just got there through multiplication. Being able to get to a number though multiplication by another whole number is what a multiple is. However, the Least Common Multiple is the lowest number that both of the values you're working with have as a Common Multiple. Sometimes it is the number you get when you multiply them together (those times being when the numbers cannot be simplified to any smaller a fraction). Other times, they can be simplified. And when that happens, the Least Common Multiple between the numbers is a lower number than those numbers multiplied together. At this point I've sort of devolved into stating facts I've been told in math classes, which is exactly the thing we're not supposed to do occurring to the book, though I assure you I do genuinely understand the meaning behind what I'm saying. Contrary to Paul Lockhart's beliefs, some of us can learn in a normal school mathematical environment. But you should get the picture. No matter what numbers you're clapping with, it will be the Least Common Multiple between the two numbers where they finish their cycle of being desynced from one another. This is because the total number of claps you and your partner are now on is the same, and you have both finished a cycle. You both must have finished a cycle of clapping at that exact moment because if the number of claps you are on is a multiple of the number you're repeating (adding over and over us multiplying so clapping the same number of claps over and over is multiplying

that number of claps by how many cycles you've gone through), then by definition of multiple you both finish at the same time. The next clap you do will be the start of another round. A round which will be finished the next time you hit a common multiple between the two numbers you're clapping in sequence with. The class couldn't prove me wrong because I wasn't. This is a Least Common Multiple problem plain and simple. If an infinite number of examples is not a proof, then I'm not too sure this is much better, but for those out there who respond better to words than examples with numbers, I hope this suffices.

B. A Sample Composition Assignment from our 2016 LC

Project 3: Writing about Mathematics

“Mathematics is about problems and problems must be made the focus of a student's mathematical life.”

Paul Lockhart, *A Mathematician's Lament*

Lockhart goes on to explain that frustration and the ability to persist through that frustration and adapt to circumstances (something emphasized in the Fermat video as well) is a central component to studying mathematics. Your next project should be driven by your already existing or emerging interests, curiosity and passion for mathematics. Your ability to adapt to availability of resources and to persist in the face of frustration (from taking on tasks that seem hard!) should sustain you throughout this project.

Your **purpose** for this project is to discover and to demonstrate (to prove!) the relevance and/or beauty of mathematics as a method of understanding our world using a range of sources, including: academic publications, texts from mainstream publications magazines and newspapers, documentaries or films, websites and pamphlets. You will write for an **audience** of people who would not initially recognize or understand your interests and points, for those who don't see mathematics as inquiry, and/or those who might assume mathematics either isn't relevant beyond

the two math classes required in college or only useful as a skill or in the service of work you do “in the real world”. You might also address the value of persistence and adaptability in such endeavors by demonstrating your own or referencing other types of texts. You will **choose the genre** depending upon your purpose and target audience. Some possible options include: traditional research paper written for an audience in a chosen discipline or a multi-modal article or research paper written for an audience in a mainstream publication. You should also feel free to propose a genre if neither of these options works for you. The project will be comprised of three parts:

1. A proposal to Prof. von Renesse; you need her approval to proceed.
2. The text of the project: this can be written in almost any genre, depending upon your purpose, audience and interests.
3. A presentation of your project to be delivered to both your classmates and your professors.

Steps to Take and Due-Dates:

- I. Part One: Proposal. This is due on October 28 and should have four parts: It should outline the problem you are pursuing, articulate some of the questions that are motivating your exploration/your interest in the problem, discuss possible sources that might help you to address or explore this problem, name the genre you think this project will take, and explain why you think that genre is appropriate.
- II. Part Two: Drafting the Text. In addition to doing exploratory and reflective writing in and out of class, following class guidelines, you will bring a shitty first draft (SFD) to class for peer review on 11/7-9. Conferences will happen on 11/14-16. Bring a revised draft to class on 11/18. Discuss possibilities for presentation at that time. Sign up for presentations. Hard copy of final draft and all accompanying work due to Prof DiGrazia for grading on 11/21.
- III. Part Three: Presentations 11/28-12/9. Each person will present their project at some point right after fall break.

Presentations should be about 5-10 minutes, engage the audience, convey your overall argument/point and show the beauty/relevance of your project. Multi-modal methods of presentation are appreciated.

Part II Final Requirements:

1. Use at least three sources. Cite them correctly according to your intended audience and chosen genre.
2. Organize your text according to the genre's conventions, your audience and your purpose.
3. Make clear to the audience how mathematics is involved and/or relevant. This includes the original problem you posed or questions, you pursued as well as the
4. Make sure you address the "So What?" question, or explain why and how this issue is interesting or important.
5. Incorporate mathematics to the extent you are able (students will determine this in conjunction with Professor von Renesse).
6. Meet all due-dates and deadlines.