Rabin vs. Streett Automata*

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─────── **Abstract** ───────

The Rabin and Streett acceptance conditions are dual. Accordingly, deterministic Rabin and Streett automata are dual. Yet, when adding nondeterminsim, the picture changes dramatically. In fact, the state blowup involved in translations between Rabin and Streett automata is a longstanding open problem, having an exponential gap between the known lower and upper bounds.

We resolve the problem, showing that the translation of Streett to Rabin automata involves a state blowup in $\Theta(n^2)$, whereas in the other direction, the translations of both deterministic and nondeterministic Rabin automata to nondeterministic Streett automata involve a state blowup in $2^{\Theta(n)}$.

Analyzing this substantial difference between the two directions, we get to the conclusion that when studying translations between automata, one should not only consider the state blowup, but also the *size* blowup, where the latter takes into account all of the automaton elements. More precisely, the size of an automaton is defined to be the maximum of the alphabet length, the number of states, the number of transitions, and the acceptance condition length (index).

Indeed, size-wise, the results are opposite. That is, the translation of Rabin to Streett involves a size blowup in $\Theta(n^2)$ and of Streett to Rabin in $2^{\Theta(n)}$. The core difference between state blowup and size blowup stems from the tradeoff between the index and the number of states. (Recall that the index of Rabin and Streett automata might be exponential in the number of states.)

We continue with resolving the open problem of translating deterministic Rabin and Streett automata to the weaker types of deterministic co-Büchi and Büchi automata, respectively. We show that the state blowup involved in these translations, when possible, is in $2^{\Theta(n)}$, whereas the size blowup is in $\Theta(n^2)$.

1 Introduction

Automata on infinite words were introduced in the 60's, in the course of solving fundamental decision problems in mathematics and logic [4, 19, 15, 21]. Today, they are widely used in formal verification and synthesis of nonterminating systems, where their size and the complexity of performing operations on them play a key role. Unlike automata on finite words, there are several types of automata on infinite words, differing in their acceptance conditions, most notably Büchi [4], Muller [19], Rabin [21], Streett [27], and parity [18]. Each of the types has its own advantages, for which reason there is an extensive research on the state blowup involved in the translations between them [22, 23, 12, 20, 28, 9, 13, 25, 2, 26, 1].

For most translations, there are satisfactory solutions, in the sense that the upper bound on the state blowup involved in the translation algorithm is close to the theoretical lower bound on the inevitable blowup. For some stubborn cases, however, there is still an exponential gap between the lower and upper bounds. This situation is especially frustrating, as it implies that not only is something missing in our understanding of automata on infinite words, but also that we may be using algorithms that can be significantly improved.

Most of these stubborn cases concern Rabin and Streett automata. In particular, the best known algorithm for translating deterministic and nondeterministic Streett automata to nondeterministic Rabin automata involves a $2^{O(n)}$ state blowup [8, 22], while the current lower bound is only $\Omega(n)$. As for the other direction, one can deduce an exponential state blowup in the translation of nondeterministic Rabin to Streett automata, due to the doubly exponential blowup in determinizing Rabin automata [7] and the singly exponential blowup in determinizing Streett automata [5]. Yet, for the translation of deterministic Rabin to nondeterministic Streett automata, there is currently also an $\Omega(n)$ lower bound and a $2^{O(n)}$ upper bound [8].

We resolve these problems, providing tight bounds for both directions. Interestingly, we show that the translation of Streett to Rabin automata involves a state blowup in $\Theta(n^2)$, whereas the translations of both deterministic and nondeterministic Rabin automata to nondeterministic Streett automata involve a state blowup in $2^{\Theta(n)}$.

For the translation of Streett to Rabin automata, we provide in Section 3 a new algorithm. Given a Streett automaton with n states, the constructed Rabin automaton has up to $2n^2$ states. We couple it with a lower bound proof, showing that a quadratic state blowup is optimal.

The challenge in translating Streett to Rabin comes from the conjunctive nature of the former and the disjunctive nature of the latter. That is, by the Streett acceptance condition, one can require, for example, to visit each of n states infinitely often, whereas the Rabin condition allows to ask for infinitely many visits in at least one of the n states.

The standard solution to require with a Rabin condition a visit in each of n different states is to have n copies of the original automaton and allow a move from the i-th copy to the next one only upon visiting the i-th state [8, 22]. A Streett acceptance condition on an automaton with n states might induce a choice between $2^{O(n)}$ different requirements to visit each of $O(n)$ different states, implying that the resulting Rabin automaton has up to $n2^{O(n)} = 2^{O(n)}$ states.

Our construction allows to use the same n copies for all of the $2^{O(n)}$ requirements. The idea is to add "bridges" between each two such copies, and provide a Rabin acceptance pair $\langle B, G \rangle$ for each requirement, such that the "bad" set B of the Rabin condition forces a transition through the bridge only when the relevant state is visited.

For the other direction, we provide in Section 4 a $2^{\Omega(n)}$ lower bound on the state blowup involved in the translation of deterministic Rabin to nondeterministic Streett automata. The lower bound builds on the property of the Streett condition, according to which the union of two accepting cycles is accepting. (By a "union of cycles" we mean a cycle whose states are the union of the states of the two cycles.) We describe a family $\{\mathcal{D}_n\}_{n \geq 1}$ of deterministic Rabin automata, and for each automaton $\mathcal{D}_n$, a set of $2^{\Omega(n)}$ words, such that $\mathcal{D}_n$ accepts each of the words, but none of their combinations. We then show that each such word can be associated with a unique state of a Streett automaton equivalent to $\mathcal{D}_n$.

Upfront, the bold asymmetry of the state blowup involved in the two directions is very surprising. Yet, a close look on the lower and upper bound results reveals the reason—there is a tradeoff between the number of states and the acceptance condition length (index). In

the translation of Rabin to Streett there is an exponential state blowup and no index blowup, whereas in the translation of Streett to Rabin there is a quadratic state blowup and an exponential index blowup.

We thus argue that when studying translations between automata, one should not only consider the state blowup, but also the *size* blowup, where the latter takes into account all of the automaton elements. More precisely, the size of an automaton is defined to be the maximum of the alphabet length, the number of states, the number of transitions, and the index. There are literature results that take the index blowup into account, for example [24], but it is often not the case.

Out of the four elements that constitute the automaton size, the number of states and the index are the dominant ones. Considering the alphabet, the common practice is to provide the upper bounds for arbitrary alphabets and to seek lower bounds with fixed alphabets. For example, [14] strengthen the lower bound of [16] by moving to a fixed alphabet, and [28] starts with automata over a rich alphabet and then moves to a fixed alphabet. As for the number of transitions, they are bounded by the size of the alphabet times quadratically the number of states, and the transition blowup tends to go hand in hand with the state blowup.

The state and size blowups involved in the translations between Rabin and Streett automata are summarized in Table 1. The differences between the results that concern the state blowup and those that concern the size blowup stem from the fact that the index of Rabin and Streett automata might be exponential in the number of states.

Next, we look into the translations of deterministic Rabin and Streett automata to the weaker types of deterministic co-Büchi and Büchi automata, respectively. It is known that a deterministic Rabin automaton that has an equivalent deterministic Büchi automaton has one on its own structure, namely an equivalent Büchi automaton exists over the same states and transitions [11]. Yet, for the translation of deterministic Rabin to deterministic co-Büchi automata, the upper bound on both the state and size blowups is $2^{O(n)}$ [2] with only an $\Omega(n)$ lower bound. We show that the state blowup of this translation is in $2^{\Theta(n)}$ and that the size blowup is in $\Theta(n^2)$. (The same holds for the dual Streett to Büchi case.)

To this end, we provide a new algorithm for translating, when possible, a deterministic Rabin automaton with n states and index k to a deterministic co-Büchi automaton with nk states. The translation goes through an intermediate nondeterministic co-Büchi automaton, as per the constriction in [2]. We analyze the intermediate automaton to be of a special form, having nk states. We then use its special form for determinizing it over the same structure.

In all of our results, whenever possible, we also consider the translations of automata with the more descriptive Muller condition.

2   Preliminaries

Given a finite alphabet Σ, a *word* over Σ is a (possibly infinite) sequence $w = w(0) \cdot w(1) \cdots$ of letters in Σ.

An *automaton* is a tuple $\mathcal{A} = \langle \Sigma, Q, \delta, Q_0, \alpha \rangle$, where Σ is the input alphabet, Q is a finite set of states, $\delta : Q \times \Sigma \to 2^Q$ is a transition function, $Q_0 \subseteq Q$ is a set of initial states, and α is an acceptance condition. The first four elements, namely $\langle \Sigma, Q, \delta, Q_0 \rangle$, are the automaton's *structure*. The automaton $\mathcal{A}$ may have several initial states and the transition function may specify many possible transitions for each state and letter, and hence we say that $\mathcal{A}$ is *nondeterministic*. In the case where $|Q_0| = 1$ and for every $q \in Q$ and $\sigma \in \Sigma$, we have $|\delta(q, \sigma)| \leq 1$, we say that $\mathcal{A}$ is *deterministic*. We then use q_0 instead of Q_0 to denote the single initial state. For a state q of $\mathcal{A}$, we denote by $\mathcal{A}^q$ the automaton that is derived

Table 1 The blowup involved in the translations between Rabin and Streett automata.

State Blowup

From \ To	Rabin		Streett	From \ To	Streett		
	Det.	Non-Det.			Det.	Non-Det.
Det. Streett	$\Theta(2^{n\log n})$ [10, 14]	$\Theta(n^2)$ Thm. 1, 2, Cor. 3		Det. Rabin	$\Theta(2^{n\log n})$ [10, 14]	$2^{\Theta(n)}$ Thm. 4, Cor. 5
Non-Det. Streett	$2^{\Theta(n^2\log n)}$ [6, 5]			Non-Det. Rabin	$2^{2^{\Theta(n)}}$ [22, 7]	

Size Blowup

From \ To	Rabin		Streett	From \ To	Streett		
	Det.	Non-Det.			Det.	Non-Det.
Det. Streett	$\Theta(2^{n\log n})$ [10, 14]	$2^{\Theta(n)}$ [8, 24]		Det. Rabin	$\Theta(2^{n\log n})$ [10, 14]	$O(n^2)$ [8]
Non-Det. Streett	$2^{\Theta(n^2\log n)}$ [6, 5]			Non-Det. Rabin	$2^{\Theta(n^2\log n)}$ [22, 7]	

from $\mathcal{A}$ by changing the set of initial states to $\{q\}$.

A *run*, or a *path*, $r = r(0), r(1), \cdots$ of $\mathcal{A}$ on $w = w(0) \cdot w(1) \cdots \in \Sigma^\omega$ is an infinite sequence of states such that $r(0) \in Q_0$, and for every $i \geq 0$, we have $r(i+1) \in \delta(r(i), w(i))$.

Acceptance is defined with respect to the set $inf(r)$ of states that the run r visits infinitely often. Formally, $inf(r) = \{q \in Q \mid \text{for infinitely many } i \in \mathbb{N}, \text{ we have } r(i) = q\}$. As Q is finite, it is guaranteed that $inf(r) \neq \emptyset$.

Several acceptance conditions are studied in the literature; the main ones are:

- *Büchi*, where $\alpha \subseteq Q$, and r is accepting iff $inf(r) \cap \alpha \neq \emptyset$. (The states of α are *accepting*.)
- *co-Büchi*, where $\alpha \subseteq Q$, and r is accepting iff $inf(r) \cap \alpha = \emptyset$. (The states of α are *rejecting*.)
- *weak* is a special case of the Büchi condition, where every strongly connected component of the automaton is either contained in α or disjoint to α; that is, no strongly connected component has a state in α and some other state not in α.
- *parity*, where $\alpha = \{S_1, S_2, \ldots, S_{2k}\}$ with $S_1 \subset S_2 \subset \cdots \subset S_{2k} = Q$, and r is accepting if the minimal index i for which $inf(r) \cap S_i \neq \emptyset$ is even.
- *Rabin*, where $\alpha = \{\langle B_1, G_1 \rangle, \langle B_2, G_2 \rangle, \ldots, \langle B_k, G_k \rangle\}$, with $B_i, G_i \subseteq Q$ and r is accepting iff for some $i \in [1..k]$, we have $inf(r) \cap B_i = \emptyset$ and $inf(r) \cap G_i \neq \emptyset$.
- *Streett*, where $\alpha = \{\langle B_1, G_1 \rangle, \langle B_2, G_2 \rangle, \ldots, \langle B_k, G_k \rangle\}$, with $B_i, G_i \subseteq Q$ and r is accepting iff for all $i \in [1..k]$, we have $inf(r) \cap B_i = \emptyset$ or $inf(r) \cap G_i \neq \emptyset$.
- *Muller*, where $\alpha = \{\alpha_1, \alpha_2, \ldots, \alpha_k\}$, with $\alpha_i \subseteq Q$ and r is accepting iff for some $i \in [1..k]$, we have $inf(r) = \alpha_i$.

A run that is not accepting is *rejecting*. Notice that Büchi and co-Büchi are special cases of the parity condition, which is in turn a special case of both the Rabin and Streett conditions. In the latter conditions, we refer to the B_i and G_i sets as the "bad" and "good" sets, respectively.

The number of sets in the parity and Muller acceptance conditions or pairs in the Rabin and Streett acceptance conditions is called the *index* of the automaton. For weak, co-Büchi, and Büchi automata, the index is 1.

The *size* of an automaton is the maximum size of its elements; more precisely, it is the maximum of the alphabet length, the number of states, the number of transitions, and the index.

An automaton accepts a word if it has an accepting run on it. The language of an automaton $\mathcal{A}$, denoted by $L(\mathcal{A})$, is the set of words that $\mathcal{A}$ accepts. We also say that $\mathcal{A}$ *recognizes* the language $L(\mathcal{A})$. Two automata, $\mathcal{A}$ and $\mathcal{A}'$, are *equivalent* iff $L(\mathcal{A}) = L(\mathcal{A}')$.

For a finite path $C = q_1 q_2 \cdots q_n$, we say that C is accepting (resp., rejecting) if the infinite path C^ω is accepting (resp., rejecting). Notice that the union of two Rabin-rejecting (finite) paths is Rabin-rejecting, and of two Streett-accepting (finite) paths is Streett-accepting.

The *class* of an automaton characterizes its transition mode (deterministic or nondeterministic) and its acceptance condition. In the more technical paragraphs, we shall denote the different classes of automata by three letter acronyms in $\{D, N\} \times \{W, B, C, P, R, S, M\} \times \{W\}$. The first letter stands for the transition mode of the automaton (deterministic or nondeterministic); the second for the acceptance-condition (weak, Büchi, co-Büchi, parity, Rabin, Streett, or Muller); and the third indicates that the automaton runs on words. For example, DBW stands for deterministic Büchi automata on words.

Büchi, parity, Rabin, Streett, and Muller automata have the same expressive power, recognizing all ω-regular languages. Weak and co-Büchi automata, as well as deterministic Büchi automata, are less expressive. When an automaton $\mathcal{A}$ of type γ has an equivalent automaton of type γ', we say that $\mathcal{A}$ is γ'-*recognizable*, for example NCW-recognizable.

3 Streett to Rabin

In this section we consider the translation of Streett to Rabin automata. The best known algorithm involves a $2^{O(n)}$ state blowup [8, 22], while the current lower bound is only $\Omega(n)$. We provide a new translation algorithm that involves a $2n^2$ state blowup, and show that a quadratic blowup is optimal. We then analyze the size blowup, and show that very differently from the state blowup it is exponential.

State blowup

We show that every Muller automaton can be translated to a Rabin automaton with only a quadratic state blowup, implying the result for translating Streett to Rabin. We start with an informal explanation of the construction, followed by an illustrated example and a formal proof.

Consider an automaton structure A with n states, and a Muller acceptance set S. A run r is accepting according to S if it visits infinitely often all the states in S and only finitely often the states out of S. Let us look first what can be done with a Rabin automaton that is defined over A. We can easily define a Rabin acceptance pair $\langle B, G \rangle$ that partially corresponds to the Muller acceptance set S—We define B to include all the states out of S, ensuring that they are visited only finitely often. The problem is that the set G cannot force visits in all states of S; It can only force a visit in some states of S.

In order to force a visit in every state of a set $S = \{q_1, q_2, \ldots, q_{|S|}\}$, one can take $|S|$ copies of A (which we call "components"), move from the i-th component to the next one (modulo $|S|$) upon reaching the state q_i, and setting, say, q_1 in the first component to be the only accepting state. This is, for example, the idea in translating a generalized Büchi
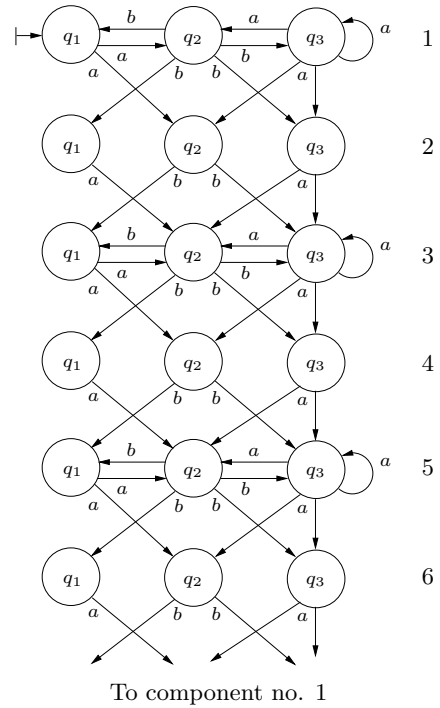
A Muller automaton $\mathcal{A}$: An equivalent Rabin autmaton $\mathcal{A}'$:
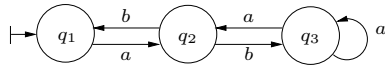
The Muller acceptance sets:

i) $\{q_1, q_3\}$

ii) $\{q_2, q_3\}$

The corresponding Rabin acceptance pairs:

i) $\langle B, G \rangle$, where

$G = \{\langle q_1, 2 \rangle\}$

$B = B_1 \cup B_2$, with

$B_1 = \{\langle q_2, 1 \rangle, \langle q_2, 2 \rangle, \langle q_2, 3 \rangle, \langle q_2, 4 \rangle, \langle q_2, 5 \rangle, \langle q_2, 6 \rangle\}$

$B_2 = \{\langle q_2, 2 \rangle, \langle q_3, 2 \rangle, \langle q_1, 6 \rangle, \langle q_2, 6 \rangle\}$

ii) $\langle B, G \rangle$, where

$G = \{\langle q_2, 4 \rangle\}$

$B = B_1 \cup B_2$, with

$B_1 = \{\langle q_1, 1 \rangle, \langle q_1, 2 \rangle, \langle q_1, 3 \rangle, \langle q_1, 4 \rangle, \langle q_1, 5 \rangle, \langle q_1, 6 \rangle\}$

$B_2 = \{\langle q_1, 4 \rangle, \langle q_3, 4 \rangle, \langle q_1, 6 \rangle, \langle q_2, 6 \rangle\}$

To component no. 1

Figure 1 An example of the translation of a Muller automaton to an equivalent Rabin automaton with an $O(n^2)$ state blowup, following the construction in the proof of Theorem 1.

automaton to a Büchi automaton. (The generalized-Büchi acceptance condition allows for several sets of states, and a run is accepting if it visits infinitely often each of these sets.)

The problem with the above approach is that we need $|S|$ copies of A for every Muller acceptance set S. As there might be 2^n Muller acceptance sets, we get an exponential state blowup. This blowup is indeed inevitable when translating to a Büchi automaton [24], yet it is not clear if and how the Rabin acceptance condition can help. We show that it certainly can, allowing all the Muller sets S to be handled over the same set of components.

We extend the above approach of having copies of A, by adding a "bridge" between each two copies. A bridge is a limited copy of A, in which all states can only move to the next component. Then, for every Muller acceptance set S, we define a Rabin acceptance pair $\langle B, G \rangle$ that forces for every state $q_i \in S$, a visit to the state q_i of the i-th bridge—We add to B all the states of the i-th bridge, except for q_i. In bridges of a component j, such that $q_j \notin S$, the run can visit any state of S.

An example of a translation of a Muller to Rabin automaton along this construction is illustrated in Figure 1.

▶ **Theorem 1.** *For every NMW with n states, m transitions, and index k, there is an equivalent NRW with $2n^2$ states, $3nm$ transitions, and index k.*

Proof.

Construction. Consider an NMW $\mathcal{A} = \langle \Sigma, Q = \{q_1, q_2, \ldots, q_n\}, Q_0, \delta, \alpha \rangle$ with n states, m transitions, and index k. We define the NRW $\mathcal{A}' = \langle \Sigma, Q', Q'_0, \delta', \alpha' \rangle$, which we claim to be equivalent to $\mathcal{A}$, as follows.

- $Q' = Q \times [1..2n]$. (We shall call each instance of Q a "component" of $\mathcal{A}'$.)
- $Q'_0 = Q_0 \times \{1\}$.
- For every state $\langle q, j \rangle \in Q'$ and $\sigma \in \Sigma$, the transition function is defined as follows.
    - If j is odd, then $\delta'(\langle q, j \rangle, \sigma) = \{\langle \hat{q}, \hat{j} \rangle \mid \hat{q} \in \delta(q, \sigma)$ and $\hat{j} \in \{j, j+1\}\}$.
    - If j is even, then $\delta'(\langle q, j \rangle, \sigma) = \{\langle \hat{q}, (j+1) \bmod 2n \rangle \mid \hat{q} \in \delta(q, \sigma)\}$.
- For every Muller accepting set $S \in \alpha$, we have in α' the Rabin acceptance pair $\langle B, G \rangle$, where B and G are defined as follows. Let $x \in [1..n]$ be the minimal index i of a state $q_i \in S$.
    - $G = \{\langle q_x, 2x \rangle\}$ consists of the single state q_x in the $2x$ component.
    - B is the union of two sets B_1 and B_2. The first includes all the states that are not in S, along all the components. The second handles the transitions through the even components (the bridges), adding every state $q_i \in S$ that appears in a component j, such that $j \neq i$ and $q_j \in S$. Formally, $B_1 = \{\langle q, j \rangle \mid q \notin S$ and $j \in [1..2n]\}$, and $B_2 = \{\langle q_i, 2j \rangle \mid i \neq j \in [1..n]$ and $q_j \in S\}$.

Correctness. Consider a word $w \in L(\mathcal{A})$. Then there is a run r of $\mathcal{A}$ and a set $S \in \alpha$, such that r visits infinitely often exactly the states in S. We will describe a run r' of $\mathcal{A}'$ that satisfies the Rabin acceptance pair $\langle B, G \rangle$ that corresponds to S. The left-projection of r', namely the q_i element of the $\langle q_i, j \rangle$ states that r' visits, is identical to r. We will describe the right-projection of r', namely the series of components that r' traverses along the run.

Notice that when r' is in an even component it must move to the next component, and when it is in an odd component it has the choice of whether to stay there or move to the next one. We explain next how r' behaves in the odd components. Let t be the first position of r after which it only visits states in S. The run r' remains in the first component until position t. After position t, when r' is in component $2j - 1$, it remains there until one of the following happens: i) the next state of r is q_j and $q_j \in S$; or ii) the next state of r is q_x and $q_j \notin S$. Note that one of the above events must indeed eventually happen: r must eventually visit q_x, because $q_x \in S$, and in the case that $q_j \in S$, r must also eventually visit q_j.

Observe that r' satisfies the Rabin acceptance pair $\langle B, G \rangle$: Considering the "good" set $G = \{\langle q_x, 2x \rangle\}$, r' visits all the components infinitely often, and when in component $2x$, it visits the state q_x. Considering the "bad" set $B = B_1 \cup B_2$, the states of B_1 are visited only finitely often, since the left-projection of r' is identical to r. As for B_2, its states are visited only finitely often, since the only case in which r' visits after position t a state $\langle q_i, 2j \rangle$ such that $i \neq j$ is when $q_j \notin S$.

As for the other direction, consider a word $w \in L(\mathcal{A}')$. Then there is a run r' of $\mathcal{A}'$ that satisfies some Rabin acceptance pair $\langle B, G \rangle$ of $\mathcal{A}'$. By the construction of $\mathcal{A}'$, the pair $\langle B, G \rangle$ corresponds to some Muller set $S \in \alpha$. We claim that the left-projection of r' is a run r of $\mathcal{A}$ that visits infinitely often exactly the states in S.

First observe that due to the subset B_1 of B, the run r visits states out of S only finitely often. Next, observe that r' must visit infinitely often all components—it visits $\{\langle q_x, 2x \rangle\}$ infinitely often, and going from $\{\langle q_x, 2x \rangle\}$ back to itself enforces a visit in all components. Now, by the subset B_2 of B, when r' is in component $2j$ and $q_j \in S$, it can visit finitely often only a state different from $\langle q_j, 2j \rangle$. Hence, r' visits infinitely often $\langle q_j, 2j \rangle$, for every $q_j \in S$, and therefore r visits infinitely often all states in S. ◄

Next, we provide a matching lower bound. As opposed to the upper bound construction, a lower bound on the state blowup involved in the translation of Muller to Rabin automata does not hold for the translation of Streett automata, as the Streett condition is less descriptive

Deterministic Streett automata $\mathcal{S}_n$
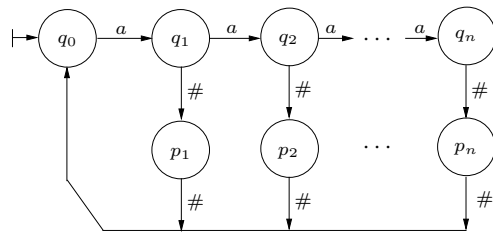
The acceptance condition:
  The pair $\langle \{q_0\}, \{q_n\} \rangle$
  For every $i \in [1..n-1]$, the pair $\langle \{q_i\}, \{p_i\} \rangle$

Figure 2 Deterministic Streett automata $\mathcal{S}_n$ with $O(n)$ states, for which equivalent nondeterministic Rabin automata have at least $n^2/2$ states.

than the Muller one. Yet, it turns out that the family of languages used in [1] for the former, can also serve us for the latter (and even for translating generalized Büchi automata).

▶ **Theorem 2.** *For every $n \in \mathbb{N}$, there is a DSW over a two-letter alphabet with $2n+1$ states, $3n$ transitions, and index n, for which equivalent NRWs have at least $n^2/2$ states.*

Proof. Consider the DSWs $\mathcal{S}_n$ depicted in Figure 2. Observe that a run of $\mathcal{S}_n$ is accepting iff it visits all of $\mathcal{S}_n$'s states infinitely often. Indeed, every run must visit q_0 infinitely often, and by the first acceptance pair, it must also visit q_n infinitely often. Every visit to q_n entails a visit to p_n and to q_i, for all $i \in [1..n-1]$, which in turn entail, by the rest of the acceptance pairs, a visit to p_i, for all $i \in [1..n-1]$.

Hence, $\mathcal{S}_n$ is equivalent to a Muller automaton $\mathcal{M}_n$ over the same structure with a single acceptance set that includes all of $\mathcal{M}_n$'s states. It is shown in [1, Proof of Theorem 9] that every NRW equivalent to $\mathcal{M}_n$ has at least $n^2/2$ states.   ◀

We conclude with the corresponding tight bounds.

▶ **Corollary 3.** *The state blowup in the translations of deterministic and nondeterministic Streett and Muller automata to nondeterministic Rabin automata is in $\Theta(n^2)$.*

Proof. The upper bounds follow from Theorem 1. The lower bound for Streett from Theorem 2 and for Muller from [1, Proof of Theorem 9] .   ◀

Size blowup

The size blowup involved in translations of Streett to Rabin automata is very different from the state blowup, as it is exponential, even when considering a deterministic Streett automaton: In [22, Lemma 2.3], there is a family of languages L_n, for $n > 0$, over a fixed alphabet, such that L_n is recognized by a DSW with $O(n)$ states, transitions, and index, while an equivalent NBW requires $\Omega(2^n)$ states. As every NRW with n states and index k can be translated to an equivalent NBW with nk states [8], it follows that the size blowup in translating DSW to NRW is in $2^{\Omega(n)}$, and together with known constructions [24], it is in $2^{\Theta(n)}$.

4   Rabin to Streett

In this section we consider the translation of Rabin to Streett automata. It turns out that the state and size blowups in this case "switch roles" with the corresponding blowups in the translation of Streett to Rabin—the size blowup is known to be quadratic [8], while

Deterministic Rabin automata $\mathcal{D}_n$
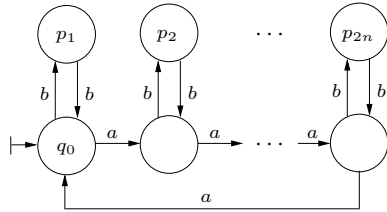
The acceptance condition:

Let Q be the set of all states and $P = \{p_1, p_2, \ldots, p_{2n}\}$.

The acceptance pairs are $\{\langle B, Q \rangle \mid B \subseteq P$ and $|B| = n\}$.

■ **Figure 3** Deterministic Rabin automata with $O(n)$ states, for which equivalent nondeterministic Streett automata need at least 2^n states.

we provide an exponential lower bound on the state blowup involved in the translation of deterministic Rabin to nondeterministic Streett automata.

State blowup

Our lower bound proof for the state blowup involved in the translation of Rabin to Streett automata builds on the property of the Streett condition, according to which the union of two accepting cycles is accepting. The challenge is to come up with a family $\{\mathcal{D}_n\}_{n \geq 1}$ of deterministic Rabin automata, and for each such automaton a set of $2^{\Omega(n)}$ words, such that $\mathcal{D}_n$ accepts each of the words, but none of their combinations.

We describe such a family in Figure 3. The automaton $\mathcal{D}_n$ accepts words on which it visits finitely often at least n out of the $2n$ states of the set P. We then define a set of $\binom{2n}{n}$ periodic words on which $\mathcal{D}_n$ visits finitely often *exactly* n states of P. Each word in the set corresponds to a choice of n out of the $2n$ states of P. (Recall that $\binom{2n}{n} > 2^n$.)

The repeated finite word in each such infinite word corresponds to a cycle of $\mathcal{D}_n$ from q_0 back to itself, avoiding the relevant n states of P and visiting the other n states of P. As a result, an infinite word that combines two such different finite words is rejected by $\mathcal{D}_n$, as the run of $\mathcal{D}_n$ on it visits finitely often less than n states of P. Accordingly, we can show that for an equivalent Streett automaton, accepting runs on different such words cannot share the same state in positions that start the repeated finite word. Hence, the Streett automaton has at least $\binom{2n}{n}$ different states.

▶ **Theorem 4.** *For every $n \in \mathbb{N}$, there is a DRW over a two-letter alphabet with $4n$ states and $6n$ transitions, for which equivalent NSWs have at least 2^n states.*

Proof. Consider the family $\{\mathcal{D}_n\}_{n \geq 1}$ of DRWs depicted in Figure 3, and let $\mathcal{A}$ be an NSW equivalent to $\mathcal{D}_n$. Observe that $\mathcal{D}_n$ has an index $k = \binom{2n}{n} > 2^n$. We show that $\mathcal{A}$ has a unique state for every acceptance pair of $\mathcal{D}_n$, implying that it has more than 2^n states.

For every $i \in [1..k]$, let B_i be the "bad" (left) set in the i-th acceptance pair of $\mathcal{D}_n$, and let u_i be the minimal finite word that takes $\mathcal{D}_n$ from q_0 back to q_0, while avoiding the states in B_i and visiting (once) every state in $P \setminus B_i$. For example, for $n = 3$ and $B_i = \{2, 5, 6\}$, we have $u_i = bbaabbabbaaa$.

For every $i \in [1..k]$, define $w_i = u_i^\omega$, and notice that $\mathcal{D}_n$ accepts w_i due to its i-th acceptance pair. We shall call the positions of w_i in which $\mathcal{D}_n$ reaches q_0 "big positions". (These are the positions of w_i after every full instance of u_i.) Let r_i be an accepting run of $\mathcal{A}$ on w_i. We now show that for every $i \neq j \in [1..k]$, the states that r_i and r_j visit infinitely often in big positions are disjoint.

Assume toward contradiction that for some $i \neq j \in [1..k]$, both r_i and r_j visit the same state s infinitely often in big positions. Let t and t' be big positions of w_i in which r_i visits s, and between which r_i visits exactly the states that it visits infinitely often. Let u be the subword of w_i between positions t and t'. Now, let w be the word that is derived from w_j by adding u in every big position in which r_j visits s.

Consider the run r of $\mathcal{A}$ on w that follows r_j, while making extra cycles from s back to itself in every big position that u was added to w. In these extra cycles, r follows the cycle that r_i does between positions t and t'. Notice that the states that r visits infinitely often are the union of the states that r_i and r_j visit infinitely often. Hence, due to the property of the Streett condition that the union of two accepting cycles is accepting, we have that r is accepting.

On the other hand, when $\mathcal{D}_n$ runs on w, it reads infinitely often both u_i and u_j from the state q_0, implying that it visits infinitely often both $P \setminus B_i$ and $P \setminus B_j$. Thus, it visits finitely often less than n states in P, and therefore rejects w. Contradiction.   ◀

As the Rabin condition is less detailed than the Muller condition (namely, every Rabin automaton as an equivalent Muller automaton over the same structure), and the Streett condition is more detailed than the parity and Büchi conditions, the above results can be generalized as follows.

▶ **Corollary 5.** *The translations of deterministic Rabin and Muller automata to nondeterministic Büchi, parity, and Streett automata involve a state blowup in $2^{\Theta(n)}$.*

Proof. The lower bounds follow from Theorem 4. The upper bounds for translating Rabin automata are given in [8], and for Muller automata are folklore (see [1] for details).   ◀

Size blowup

Every Rabin automaton with n states and index k can be translated to an equivalent Büchi automaton with nk states [8], which can also be viewed as a Streett automaton, providing a quadratic size blowup.

5 Rabin to Co-Büchi

In this section we resolve the open problems of translating deterministic Rabin and Streett automata to the weaker types of deterministic co-Büchi and Büchi automata, respectively. For both the state and size blowups, the known upper bound is in $2^{O(n)}$ [2] with only an $\Omega(n)$ lower bound. We show that the state blowup of these translations is in $2^{\Theta(n)}$ and that the size blowup is in $\Theta(n^2)$.

State blowup

Upfront, the lower bound for the translation of Rabin to Streett automata (Theorem 4) does not follow to the case of translating to a deterministic co-Büchi automaton, as the latter does not recognize all ω-regular languages. Yet, our family $\mathcal{D}_n$ of NRWs, as depicted in Figure 3, is NCW-recognizable, providing a lower bound also for the co-Büchi case.

▶ **Theorem 6.** *For every $n \in \mathbb{N}$, there is an NCW-recognizable DRW over a two-letter alphabet with $4n$ states and $6n$ transitions, for which equivalent NCWs have at least 2^n states.*

Proof. Observe that in the DRW $\mathcal{D}_n$ of Figure 3, each Rabin acceptance pair is actually a co-Büchi condition. Hence, $\mathcal{D}_n$ is the union of DCW's, and is therefore NCW-recognizable. By Theorem 4, an NSW equivalent to $\mathcal{D}_n$ has at least 2^n states, and therefore also an equivalent NCW.   ◀

Combining Theorem 6 with known results on the expressive power of the different automata types, we get the following generalization.

▶ **Corollary 7.** *The translations of deterministic Rabin and Muller automata to nondeterministic weak automata, as well as to deterministic and nondeterministic co-Büchi automata involve a state blowup in $2^{\Theta(n)}$.*

Proof. The lower bounds follow from Theorem 6. The upper bounds are given in [2].   ◀

Another consequence of Theorem 6 concerns the translation of deterministic Streett to deterministic Büchi automata. It is known that there is an exponential state blowup in the translation of deterministic Streett to nondeterministic Büchi automata [24]. Yet, the languages used in [24] are not DBW-recognizable, leaving open the translation of DSWs to DBWs. By the duality of DSWs and DRWs and the duality of DBWs and DCWs, we get from Corollary 7 a corresponding answer.

▶ **Corollary 8.** *The translations of deterministic Streett and Muller automata to deterministic Büchi automata involve a state blowup in $2^{\Theta(n)}$.*

Size blowup

We now move from the negative results on the exponential state blowup to positive results on the quadratic size blowup. More precisely, given a DRW with n states and index k that is NCW-recognizable, we construct an equivalent DCW with nk states.

Our construction starts with translating a given DRW with n states and index k to an equivalent NCW, as per the translation of an NRW to an NCW given in [2]. In general, the constructed NCW might have $kn2^n$ states. However, we analyze the special case in which the translated NRW is a DRW, and show that the constructed NCW has up to kn states.

The next step is to determinize the constructed NCW. In general, co-Büchi determinization is done via the breakpoint (Miyano-Hayashi) construction, and might result in an exponential state blowup [17, 3]. Yet, we analyze the constructed NCW to be of a special form, a union of DCWs over the same structure, for which we provide a different determinization construction that introduces no state blowup.

We start with a definition from [2], which provides the central building block in the translations to co-Büchi automata.

▶ **Definition 9** (Augmented subset construction [2]). Let $\mathcal{A} = \langle \Sigma, A, \delta_{\mathcal{A}}, A_0 \rangle$ be an automaton structure. We define its *augmented subset construction* $\mathcal{A}'$ as the product of $\mathcal{A}$ with its subset construction. Formally, $\mathcal{A}' = \langle \Sigma, A', \delta_{\mathcal{A}'}, A_0' \rangle$, where

- $A' = A \times 2^A$. That is, the states of $\mathcal{A}'$ are all the pairs $\langle a, E \rangle$ where $a \in A$ and $E \subseteq A$.
- For all $\langle a, E \rangle \in A'$ and $\sigma \in \Sigma$, we have $\delta_{\mathcal{A}'}(\langle a, E \rangle, \sigma) = \delta_{\mathcal{A}}(a, \sigma) \times \{\delta_{\mathcal{A}}(E, \sigma)\}$. That is, $\mathcal{A}'$ nondeterministically follows $\mathcal{A}$ on its A-component and deterministically follows the subset construction of $\mathcal{A}$ on its 2^A-component.
- $A_0' = A_0 \times \{A_0\}$.

We continue with three lemmas from [2], which will serve us in analyzing the constructed NCW.

▶ **Lemma 10** ([2, Lemma 5.3]). *Every NRW and NMW $\mathcal{A}$ with index k is equivalent to the union of k NSWs over the same structure of $\mathcal{A}$.*

▶ **Lemma 11** ([2, Lemma 5.4]). *Consider k NSWs, $\mathcal{S}_1, \ldots, \mathcal{S}_k$, over the same structure. There is an NSW $\mathcal{S}$ over the disjoint union of their structures, such that $L(\mathcal{S}) = \bigcup_{i=1}^{k} L(\mathcal{S}_i)$.*

▶ **Lemma 12** ([2, Proof of Theorem 5.1]). *For every NCW-recognizable NSW $\mathcal{S}$, there is an equivalent NCW $\mathcal{C}$ over the augmented subset construction of $\mathcal{S}$.*

Next, we provide an alternative determinization procedure for an NCW that is a union of DCWs over the same structure. The constructed DCW is generated on the same structure of the given NCW.

The idea is as follows. Consider an NCW $\mathcal{A}$ that is a union of several DCWs over the same structure. A run of $\mathcal{A}$ is accepting if one of the DCWs accepts it. Instead of guessing which DCW will accept it, as the NCW does, a global DCW $\mathcal{D}$ that is equivalent to $\mathcal{A}$ can move between the local DCWs whenever it is in a rejecting state, and remain in the local DCW if it is in an accepting state. Its rejecting states are the union of the rejecting states of the local DCWs. Since the local DCWs share the same structure, there is no harm in moving between them, and if one of them is accepting, the global DCW will eventually remain there forever.

▶ **Lemma 13.** *Consider k DCWs $\mathcal{D}_1, \mathcal{D}_2, \ldots, \mathcal{D}_k$ over the same structure of n states and m transitions. Then there is a DCW $\mathcal{D}$ with nk states and mk transitions that is equivalent to their union, namely $L(\mathcal{D}) = \cup_{i=1}^{k} L(\mathcal{D}_i)$.*

Proof.
Construction. For every $i \in [1..k]$, let $\mathcal{D}_i = \langle \Sigma, Q, q_0, \delta, \alpha_i \rangle$, where the alphabet Σ, the set Q of n states, the initial state q_0, and the transition function δ are common to all the DCWs, while the set $\alpha_i \subseteq Q$ of rejecting states is possibly different in each of them.

We define the DCW $\mathcal{D}' = \langle \Sigma, Q', q_0', \delta', \alpha' \rangle$, which we claim to recognize $\cup_{i=1}^{k} L(\mathcal{D}_i)$, as follows.

- $Q' = Q \times [1..k]$.
- $q_0' = \langle q_0, 1 \rangle$.
- For every state $\langle q, i \rangle \in Q'$ and $\sigma \in \Sigma$, the transition function is defined as follows.
    - If $q \in \alpha_i$, then $\delta'(\langle q, i \rangle, \sigma) = \langle \delta(q, \sigma), (i{+}1) \bmod k \rangle$.
    - If $q \notin \alpha_i$, then $\delta'(\langle q, i \rangle, \sigma) = \langle \delta(q, \sigma), i \rangle$.
- For every $q \in Q$ and $i \in [1..k]$, $\langle q, i \rangle \in \alpha'$ iff $q_i \in \alpha_i$;

Correctness. Observe that by the definition of $\mathcal{D}'$, for every position t of a word w, the runs of all $\mathcal{D}_i$'s are at the same state q, while $\mathcal{D}'$ is in a state $\langle q, i \rangle$ for some $i \in [1..k]$. Thus, by the definition of δ', we have:
$w \in L(\mathcal{D}')$ iff
there exists $i \in [1..k]$, such that from some position of w, $\mathcal{D}'$ remains in $(Q_i \setminus \alpha_i) \times \{i\}$ iff
there exists $i \in [1..k]$, such that w is accepted by $\mathcal{D}_i$ iff
$w \in \cup_{i=1}^{k} L(\mathcal{D}_i)$. ◀

We are now in position to provide the upper bound proof. We give it for the translations of both deterministic Rabin and Muller automata.

▶ **Theorem 14.** *For every NCW-recognizable DRW and DMW with n states, m transitions, and index k, there is an equivalent DCW $\mathcal{C}$ with nk states and mk transitions.*

Proof. Consider a DRW or a DMW $\mathcal{A} = \langle \Sigma, A, A_0, \delta, \alpha \rangle$ with n states, m transitions, and index k. By Lemmas 10 and 11, there is an NSW $\mathcal{S}$ equivalent to $\mathcal{A}$ whose structure consists of k copies of the structure of $\mathcal{A}$. That is, $\mathcal{S} = \langle \Sigma, A \times [1..k], A_0 \times [1..k], \delta_{\mathcal{S}}, \alpha_{\mathcal{S}} \rangle$, where for all $a \in A, i \in [1..k]$, and $\sigma \in \Sigma$, we have $\delta_{\mathcal{S}}(\langle a, i \rangle, \sigma) = \langle \delta(a, \sigma), i \rangle$.

Let $\mathcal{C}$ be the NCW equivalent to $\mathcal{S}$, defined over the augmented subset construction of $\mathcal{S}$, as per Lemma 12. Note that $\mathcal{S}$ has nk states, suggesting that an application of the augmented subset construction on it might result in an NCW with up to $nk2^{nk}$ states. Yet, we show below that due to the special structure of $\mathcal{S}$, the resulting NCW $\mathcal{C}$ presents no state blowup, and is defined over a structure that is isomorphic to the structure of $\mathcal{S}$.

Indeed, applying the augmented subset construction on $\mathcal{S}$, we get the product of $\mathcal{S}$ and its subset construction, where the latter has a state for every reachable subset of S. As S consists of k disjoint copies of the same deterministic structure of $\mathcal{A}$, each reachable subset of S is of the form $\{\langle a, 1 \rangle, \langle a, 2 \rangle, \ldots, \langle a, k \rangle\}$, for some $a \in A$. Thus, the subset construction of $\mathcal{S}$ results in a structure that is isomorphic to the structure of $\mathcal{A}$.

Now, as the structure of $\mathcal{C}$ is the product of S and its subset construction, a state of $\mathcal{C}$ is of the form $\langle \langle a, i \rangle, a \rangle$, for some $a \in A$ and $i \in [1..k]$. Hence, the state space of $\mathcal{C}$ is isomorphic to that of $\mathcal{S}$. It remains to see that the transition function $\delta_{\mathcal{C}}$ of $\mathcal{C}$ follows the isomorphism between the states of $\mathcal{S}$ and $\mathcal{C}$. Indeed, for every $a \in A$ and $i \in [1..k]$, $\delta_{\mathcal{C}}(\langle \langle a, i \rangle, a \rangle) = \langle \delta_{\mathcal{S}}(\langle a, i \rangle), \delta(a) \rangle = \langle \langle \delta(a), i \rangle, \delta(a) \rangle$.

Next, we need to determinize the NCW $\mathcal{C}$ into an equivalent DCW. The standard co-Büchi determinization might result in an exponential state blowup [17, 3]. Yet, since the structure of $\mathcal{C}$ is isomorphic to that of $\mathcal{S}$, it follows that $\mathcal{C}$ is the disjoint union of k DCWs over the same structure. Hence, we can determinize it as per Lemma 13, getting a DCW with nk states and mk transitions. ◄

For providing a corresponding lower bound, we look on the dual translation of a DSW to a DBW. Observe that the family of DSWs depicted in Figure 2 are DBW-recognizable. Hence, we get from Theorem 2 the desired bound.

▶ **Theorem 15.** *For every $n \in \mathbb{N}$, there is a DBW-recognizable DSW over a two-letter alphabet with $2n+1$ states, $3n$ transitions, and index n, for which equivalent DBWs have at least $n^2/2$ states.*

Proof. Consider the DSWs $\mathcal{S}_n$ depicted in Figure 2. As explained in the proof of Theorem 2, a run of $\mathcal{S}_n$ is accepting iff it visits all of $\mathcal{S}_n$'s states infinitely often. Hence, $\mathcal{S}_n$ is equivalent to the intersection of n DBWs that are defined over its structure, where each of them has a different single accepting state. As the set of DBW-recognizable languages is closed under intersection, we have that $\mathcal{S}_n$ is DBW-recognizable.

By Theorem 2, every NRW equivalent to $\mathcal{S}_n$ has at least $n^2/2$ states, and therefore also every such DBW. ◄

We conclude with the tight bound.

▶ **Corollary 16.** *The translations of deterministic Rabin automata to deterministic co-Büchi automata and of deterministic Streett automata to deterministic Büchi automata involve a size blowup in $\Theta(n^2)$.*

6 Conclusions

The duality between the Rabin and Streett acceptance conditions, when combined with automata nondeterminism, turns out to result in a duality between the number of states

of an automaton and the length of its acceptance condition (index): We resolve the open problems of the blowup involved in the translations between Rabin and Streett automata, showing that from Rabin to Streett there might be an exponential state blowup, while having no index blowup, whereas from Streett to Rabin there can be only a quadratic state blowup, yet having an exponential index blowup.

Moreover, the state blowup and index blowup are interconnected—The translation from Streett to Rabin can be done not only with a quadratic state blowup and an exponential index blowup, but also with an exponential state blowup and no index blowup; Yet, every algorithm that translates Streett to Rabin must involve either an exponential state blowup or an exponential index blowup.

Following these results, we argue that when studying translations between automata, one should also consider the *size* blowup, where the size of an automaton is the maximum of its elements, namely the alphabet length, the number of states, the number of transitions, and the index. Out of the four elements, the number of states and the index are the dominant ones.

The substantial difference between state blowup and size blowup takes place also in the translations of deterministic Rabin and Streett automata to the weaker types of deterministic co-Büchi and Büchi automata, respectively. We resolve the open problems of the blowup involved in these translations, when possible, showing that the state blowup is exponential and that the size blowup is quadratic.

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