Investigating the Social Efficiency of Merchant Transmission Planning through a Non-Cooperative Game-Theoretic Framework

Antonio De Paola, *Member, IEEE,* Dimitrios Papadaskalopoulos, *Member, IEEE,* David Angeli, *Fellow, IEEE*
Goran Strbac, *Member, IEEE*

***Abstract*—Merchant transmission planning is considered as a further step towards the full liberalization of the electricity industry. However, previous modeling approaches do not comprehensively explore its social efficiency as they cannot effectively deal with a large number of merchant companies. This paper addresses this fundamental challenge by adopting a novel non-cooperative game-theoretic approach. Specifically, the number of merchant companies is assumed sufficiently large to be approximated as a continuum. This allows the derivation of mathematical conditions for the existence of a Nash Equilibrium solution of the merchant planning game. By analytically and numerically comparing this solution against the one obtained through the traditional centralized planning approach, the paper demonstrates that merchant planning can maximize social welfare only when the following conditions are satisfied: a) fixed investment costs are neglected and b) the network is radial and does not include any loops. Given that these conditions do not generally hold in reality, these findings suggest that even a fully competitive merchant transmission planning framework, involving the participation of a very large number of competing merchant companies, is not generally capable of maximizing social welfare.**

***Index Terms*—Game theory, merchant transmission investors, Nash equilibrium, transmission planning.**

NOMENCLATURE

A. Indices

$t \in T$ Index and set of time periods
$m \in M$ Index and set of network branches
$n \in N$ Index and set of network nodes
$l \in L$ Index and set of network loops
$i \in I$ Index and set of merchant transmission companies

B. Parameters

w_t Weighting factor of period t
T_m^F Fixed investment cost of branch m (£/h)
T_m^V Variable investment cost of branch m (£/MWh)
a_n^G Quadratic cost coefficient of generation at node n (£/MW^{2}h)
b_n^G Linear cost coefficient of generation at node n (£/MWh)
G_n^{max} Maximum generation limit at node n (MW)
a_n^D Quadratic benefit coefficient of demand at node n (£/MW^{2}h)
b_n^D Linear benefit coefficient of demand at node n (£/MWh)
F_m^0 Existing capacity of branch m (MW)
$\mathbf{\Phi}$ Matrix of sensitivities $\phi_{n,m}$ for outflow from node n with respect to power flow on branch m
$\mathbf{\Psi}$ Matrix of sensitivities $\psi_{l,m}$ for voltage drop of loop l with respect to power flow on branch m
n_m^s Reference sending node of branch m
n_m^r Reference receiving node of branch m

C. Variables

$\boldsymbol{u}$ Vector of binary variables u_m expressing whether new capacity is added on branch m ($u_m = 1$ if it is, $u_m = 0$ otherwise)
$\boldsymbol{F}$ Vector of total capacity additions F_m on branch m (MW)
$\boldsymbol{F(i)}$ Vector of individual capacity additions $F_m(i)$ by company i on branch m (MW)
$G_{n,t}$ Power generation at node n and period t (MW)
$D_{n,t}$ Power demand at node n and period t (MW)
$f_{m,t}$ Power flow on branch m at period t (MW)
$P_{n,t}$ Net power outflow from node n at period t (MW)
$\lambda_{n,t}$ Locational marginal price at node n and period t (£/MWh)

D. Functions

$T_m(\cdot)$ Transmission investment cost for branch m (£/h)
$C_{n,t}(\cdot)$ Cost of generation at node n and period t (£/h)
$B_{n,t}(\cdot)$ Benefit of demand at node n and period t (£/h)
$J(i, \cdot)$ Profit of company i (£/h)

I. INTRODUCTION

A. Motivation

During the last two decades, deregulation of the electricity industry has been observed worldwide, involving the unbundling of vertically integrated monopoly utilities, the introduction of competition in the generation and supply sectors, and the open access to the electricity networks. In this deregulated environment, two general approaches are adopted for transmission network planning [1]–[4]. Under the first approach, planning is centrally carried out by a regulated transmission company, which realizes under regulatory supervision the optimal transmission expansion plan that maximizes the social welfare while ensuring security of supply. The required capital cost plus a suitable rate of return for the transmission company is recovered from the network users. In this context, research efforts have focused on the solution of the centralized

A. De Paola, D. Papadaskalopoulos, D. Angeli and G. Strbac are with the Department of Electrical and Electronic Engineering, Imperial College, London. (e-mail: antonio.de-paola09@ic.ac.uk; d.papadaskalopoulos08@ic.ac.uk; d.angeli@ic.ac.uk; g.strbac@ic.ac.uk).

optimal transmission planning problem [5]–[10], as well as the allocation of transmission costs among the users [11]–[16].

Under the second approach, known as merchant transmission investment [2], transmission planning relies on competitive market forces and profit-driven decisions of self-interested investors, known as merchant transmission companies. These merchant companies are rewarded on the basis of the collected congestion revenues created by their network investments. This paradigm is gaining continuously ground as the participation of multiple merchant companies and the resulting competition arising in transmission planning are advocated as a further step towards the deregulation and liberalization of the electricity industry [2]–[4], [17]. The first instances of merchant transmission planning can be found in the USA, Australia, Argentina and Brazil, although the adopted frameworks constitute a mix of centralized and merchant planning [2], [3], [18]–[22].

However, a critical question towards the widespread application of this merchant transmission planning paradigm is whether this approach can achieve the same (maximum) social welfare as the centralized planning approach.

B. Relevant Work

A few recent papers have developed quantitative models of this merchant investment framework in order to answer this question. Through Lagrangian Relaxation (LR) principles, authors in [17] demonstrate that the merchant paradigm leads to the same planning solution as the one obtained by the centralized paradigm, leading to the conclusion that introduction of competition in network planning is plausible. However, this outcome is subject to two simplifying assumptions. First of all, the fixed costs of network assets are neglected despite the undeniable economies of scale associated with transmission investment [1], [2]; this assumption is made as LR is generally unable to produce the centralized solution in the presence of non-convexities [23]. More importantly, merchant investors are assumed competitive, price-taking entities, considering the locational marginal prices (LMP) as exogenous signals that cannot be influenced by their individual actions. In reality however, in a similar fashion with strategic behavior observed in energy markets, merchant investors will attempt to exercise market power and manipulate the LMP to increase their profits beyond the centralized planning levels, through strategic network investments [2], [3], [24]–[27].

In [25], [26] and [2], [3], this competitive behavior assumption is removed. Authors in [25], [26] show that under certain conditions (neglect of fixed costs of network assets, congestion rights satisfying certain feasibility constraints, no imperfections in the energy market), merchant investments are socially efficient. In the seminal work [2], the authors demonstrate through theoretical discussion and illustrative examples that this conclusion does not hold when the above simplifying conditions are relaxed. However, these papers investigate the social efficiency of investments by a single merchant company, neglecting that the very essence of the merchant planning paradigm lies in the introduction of competition in transmission planning, through the participation of multiple merchant companies. More specifically, authors in [2] recognize through a simple 3-node example that gaming interactions between multiple merchant companies are likely in reality (Section 9.3, pages 54-55), but they do not provide a comprehensive modeling framework capturing these interactions.

Authors in [3] make the first attempt to consider a setting with multiple merchant companies and analytically derive the relation between the procured transmission capacity under the centralized and the merchant transmission paradigm. The results indicate that the merchant paradigm leads to under-investment with respect to the centralized approach but such under-investment is reduced as the number of merchant companies is increased. Specifically, in a simple 2-node system example, the results demonstrate that the deviation between the merchant and the centralized planning solution is less than 10% when 4 merchant companies are considered, less than 1% when 7 merchant companies are considered, and less than 0.1% when 10 merchant companies are considered. Extending the analysis to the theoretical case where the number of merchant companies approaches infinity, the authors in [3] demonstrate that the differences between the planning solutions of the two paradigms in this case tend to zero. Although this theoretical scenario with an infinite number of merchant companies does not correspond to a realistic setting, this result is of great significance as it implies that under a “sufficiently large” number of competing merchant companies, the socially optimal transmission planning solution can be approached. However, this paper [3] carries out simplifying assumptions that limit the generality of this result; transmission branches are presumed congested at the optimal solution and fixed costs of network assets are neglected. More importantly, the multiple merchant companies are assumed to make investment decisions sequentially, accounting for past but not possible future investments by competitors. In other words, the adopted approach does not comprehensively model the decision-making interactions between multiple merchant companies.

As discussed in [27], a non-cooperative game-theoretic modeling framework is required to capture the strategic behavior and interactions of multiple merchant investors. Such a framework has been investigated in [28], where an equilibrium programming approach has been employed to search for Nash Equilibria (NE) of the merchant planning game. Case studies in a simple 2-node system indicate that as the number of competing merchant companies increases, the merchant planning solution approaches the centralized one. Specifically, the deviation between the two solutions in terms of the procured capacity is shown to be less than 20% when 4 merchant companies are considered, and less than 10% when 9 merchant companies are considered. However, the adopted approach cannot guarantee convergence to existing NE, especially as the number of players and the size of the network increase; as a result, the case studies are limited to a 2-node system with up to 10 merchant companies. In other words, although this approach captures the decision making interactions between competing merchant companies and accounts for fixed costs of transmission assets (aspects which are not captured by the modeling framework of [3]), it cannot establish whether the important finding of [3] (i.e. that merchant planning yields the same solution as centralised planning under the participation of a “sufficiently large” number of competing investors) is valid or not, as it cannot deal with a large number of merchant companies, especially in large networks.

C. Contributions

This paper fills the knowledge gap that exists between the previous papers [3] and [28]. As in [28], the strategic interactions between multiple merchant companies are captured by adopting a non-cooperative game-theoretic modeling framework. However, this framework is adapted to deal with a large number of merchant companies, in order to validate the important finding of [3]. To achieve this, the set of merchant companies is approximated as a continuum [29]. Similar approaches have been previously considered in other economic [30], [31] and smart grid [32], [33] applications. The proposed approximation makes the impact of each infinitesimal player’s decisions on system quantities negligible, allowing us to derive mathematical conditions for the existence of the merchant planning solution, characterized as a Nash equilibrium.

Based on this approach, this paper investigates the validity of the finding of [3], through an analytical and numerical comparison of this merchant transmission planning solution against the one obtained through a traditional centralized approach. This comparison demonstrates that merchant planning can achieve the same (maximum) social welfare as the centralized planning approach only when the following conditions are satisfied: a) fixed investment costs are neglected and b) the network is radial and does not include any loops. As the mentioned conditions do not generally hold in reality, our findings suggest that even a fully competitive merchant transmission planning framework, involving the participation of a very large number of competing merchant companies, is not generally capable of maximizing social welfare, as implied by the previous work [3]. Numerical simulations supporting these findings are carried out on a 2-node, a 3-node, a 6-node and a 24-node system, while the largest case study examined in the previous relevant papers discussed in Section I-B corresponds to a 6-node system.

It should be mentioned that the transmission planning model investigated in this paper - as well as the models in the relevant merchant transmission planning literature [3] and [28] - assumes a fixed generation mix and therefore ignores the strategic interactions between transmission and generation expansion decisions, which have been explored in papers [34]–[39]. As discussed in Section VI though, these interactions constitute the subject of future research which aims at developing an integrated transmission and generation planning framework in order to compare the impacts of centralized and merchant transmission planning on generation expansion decisions.

D. Paper Structure

The rest of this paper is organized as follows. Section II outlines a basic model of centralized planning. Section III details the proposed game-theoretic model of merchant planning. Section IV theoretically proves the equivalence of the centralized and merchant planning solutions under a set of assumed conditions. Section V presents numerical results of case studies in four different systems. Finally, Section VI discusses conclusions and further extensions of this work.

II. Centralized Transmission Planning Model

Under the centralized transmission planning paradigm, the regulated transmission company determines the capacity to be added in the existing network, so as to maximize the long-term social welfare or, equivalently, minimize the long-term system cost. The latter is given by the sum of two terms: the difference between generation cost and demand benefit, plus the investment cost required for delivering the new capacity.

Definition 1: The centralized solution (CS) of the transmission planning problem is determined through the following optimization problem:

$$\min_{\substack{u_m, F_m, f_{m,t}, \\ G_{n,t}, D_{n,t}, \lambda_{n,t}}} \ \forall m, \forall n, \forall t \ \sum_t \sum_n w_t \left[C_{n,t}(G_{n,t}) - B_{n,t}(D_{n,t}) \right] + \sum_m T_m(u_m, F_m) \tag{1}$$

Where:

$$T_m(u_m, F_m) = u_m \left(T_m^F + T_m^V F_m \right) \tag{2}$$

subject to:

$$0 \leq F_m \quad \forall m \tag{3}$$
$$D_{n,t} + P_{n,t} - G_{n,t} = 0 \ : \ \lambda_{n,t} \quad \forall n, \forall t \tag{4}$$
$$-\left(F_m^0 + u_m F_m \right) \leq f_{m,t} \leq F_m^0 + u_m F_m \quad \forall m, \forall t \tag{5}$$
$$P_{n,t} = \textstyle\sum_m \phi_{n,m} f_{m,t} \quad \forall n, \forall t \tag{6}$$
$$\textstyle\sum_m \psi_{l,m} f_{m,t} = 0 \quad \forall l, \forall t \tag{7}$$
$$0 \leq G_{n,t} \leq G_n^{max} \quad \forall n, \forall t. \tag{8}$$

Following the realistic economic properties of network investments, the investment cost T_m for branch m in (2) includes i) a fixed component, which does not depend on the procured capacity but only on the binary decision (u_m) of whether new capacity will be added on branch m or not and ii) a variable component, which is incurred when this binary decision is positive ($u_m = 1$), and is proportional to the procured capacity F_m. DC load flow constraints are expressed by (4)-(7); the Lagrangian multipliers $\lambda_{n,t}$ associated with the nodal demand-supply balance constraints (4) express the LMP at the respective node n and period t. Generation limits are enforced by (8).

III. Modeling Merchant Transmission Planning as a Game with Very Large Number of Players

A. Game Setting

In the merchant planning paradigm, each rational merchant transmission company i determines its network expansion proposals $F_m(i)$ on each branch m so as to maximize its profit function $J(i, \cdot)$ in (9). This is defined as the difference between the congestion revenue (second term) and the investment cost (first term) associated with the network capacity procured by company i on each network branch. The share of total fixed investment cost paid by company i is equal to the share of the total capacity addition it procures, as expressed by the ratio in the first term of (9). Likewise, out of the total congestion revenue associated with branch m, the share belonging to company i is equal to the share of the total capacity of branch m it owns, as expressed by the ratio in the second term of (9).

$$\begin{aligned} J(i,\boldsymbol{F(i)},\boldsymbol{u},\boldsymbol{F}) = -\sum_m u_m \left(T_m^F \tfrac{F_m(i)}{F_m} + T_m^V F_m(i) \right) \\ + \sum_m u_m \left[\sum_t w_t \left(\lambda_{n_m^r,t} - \lambda_{n_m^s,t} \right) f_{m,t} \tfrac{F_m(i)}{F_m + F_m^0} \right]. \end{aligned} \tag{9}$$

If planning involves multiple companies, their expansion proposals are inter-dependent: the profit $J(i,\cdot)$ of company i depends not only on the vector $\boldsymbol{F(i)}$ of its strategic decisions but also on the aggregate decisions $\boldsymbol{F}$ of all companies, which impact the final capacity $F_m + F_m^0$ of each branch m and consequently the power flows and the LMP affecting the companies' profits. It follows that each merchant company needs to account for the decisions of its competitors. These interactions can be described through a non-cooperative game, modeling the merchant companies as competing players.

As discussed in Section II, previous game-theoretic approaches for modeling merchant planning cannot effectively deal with a large number of merchant companies. Therefore, they cannot accurately determine whether merchant planning leads to the same solution as centralized planning under the participation of many competing investors. In order to address this fundamental challenge, this paper adopts a novel approach, in which the number of merchant companies is assumed sufficiently large to be approximated as a continuum [29]. In other words, the set of merchant companies is not described as a finite collection $I = \{1, 2, \ldots, |I|\}$ but as a closed interval $I \subset \mathbb{R}$. With this approximation, system quantities such as investment decisions u_m and F_m are not impacted by each infinitesimal player's strategies but only depend on the aggregation of all players' strategies. In this context, the total capacity addition on branch m is expressed as:

$$F_m = \int_I F_m(i)\, di \tag{10}$$

and the binary investment decision u_m corresponds to:

$$u_m = \begin{cases} 0 & \text{if} \quad F_m = 0 \\ 1 & \text{if} \quad F_m > 0 \end{cases} \tag{11}$$

B. Determining Nash Equilibria of the Game

In the proposed non-cooperative game setting, the interest of regulators and policy makers lies in determining the likely outcome of the strategic interaction between multiple merchant companies, i.e. the Nash equilibrium (NE) of the game. NE expresses a condition where none of the players can increase its profits by unilaterally modifying its decisions [40], as formalized by the following definition:

Definition 2: Consider a feasible vector of individual network capacity additions $\boldsymbol{F^*(i)}$ and the corresponding total capacity additions $\boldsymbol{F^*}$ and binary investment decisions $\boldsymbol{u^*}$. These quantities constitute a NE for the game of Section III-A if, for any feasible vector of individual network capacity additions $\boldsymbol{F(i)}$, the following holds:

$$J(i,\boldsymbol{F^*(i)},\boldsymbol{u^*},\boldsymbol{F^*}) \geq J(i,\boldsymbol{F(i)},\boldsymbol{u^*},\boldsymbol{F^*}). \tag{12}$$

As mentioned in Section III-A, a single player i cannot impact the total capacity additions $\boldsymbol{F}$ and the binary investment decisions $\boldsymbol{u}$ but it can only modify the individual capacity additions $\boldsymbol{F(i)}$. It is thus critical to analyze which values of the vector $\boldsymbol{F(i)}$ maximize the profit function (9) for fixed values of the vectors $\boldsymbol{F}$ and $\boldsymbol{u}$. Note that $J(i,\cdot)$ is linear with respect to each individual capacity addition $F_m(i)$ and can alternatively be written as:

$$J(i,\boldsymbol{F(i)},\boldsymbol{u},\boldsymbol{F}) = \sum_m \Lambda(u_m, F_m) \cdot F_m(i) \tag{13}$$

where the term $\Lambda(u_m, F_m)$ is expressed as:

$$\Lambda(u_m, F_m) = u_m \frac{\sum_t w_t \left(\lambda_{n_m^r,t} - \lambda_{n_m^s,t} \right) f_{m,t}}{F_m + F_m^0} - u_m \left(\frac{T_m^F}{F_m} + T_m^V \right). \tag{14}$$

Three different conditions need to be examined for each term of the sum in (13):

- $\Lambda(u_m, F_m) > 0$: the function J is monotonically increasing with respect to $F_m(i)$. It follows that the profit of player i can always be improved by selecting a higher value of $F_m(i)$ and therefore a NE can never be reached.
- $\Lambda(u_m, F_m) < 0$: the function J is monotonically decreasing with respect to $F_m(i)$. Therefore, the profit of player i can always be increased by choosing a lower value of $F_m(i)$. As a result, a NE could potentially be reached if and only if $F_m(i) = 0$, $\forall i$. This is never the case, as the mentioned conditions would lead to $u_m = 0$ and $\Lambda(u_m, F_m) = 0$, contradicting the initial hypothesis.
- $\Lambda(u_m, F_m) = 0$: the function J does not depend on $F_m(i)$. If this is true for all $m \in M$, (12) holds as equality and a NE is reached. In this case, the marginal value and the marginal cost of an additional unit of network capacity investment by player i are equal.

Based on the three conditions examined above, the following result can be deduced:

Theorem 1: The individual network capacity additions $\boldsymbol{F^*(i)}$ and the corresponding vector of total capacity additions $\boldsymbol{F^*}$ and binary investment decisions $\boldsymbol{u^*}$ constitute a NE for the game of Section III-A if and only if:

$$\begin{aligned} u_m^* F_m^* \sum_t w_t \left(\lambda_{n_m^r,t} - \lambda_{n_m^s,t} \right) f_{m,t} \\ = u_m^* (F_m^* + F_m^0)(T_m^F + T_m^V F_m^*). \end{aligned} \quad \forall m \tag{15}$$

Proof: The three above conditions for $\Lambda(u_m, F_m)$ are considered. When $\Lambda(u_m, F_m) > 0$, we have established that a NE does not exist. This is consistent with the theorem statement, as (15) does not hold in this case. In fact, since $\Lambda(u_m, F_m) > 0$, the term in the left-hand side of (15) is strictly larger than the term in the right-hand side of (15). A similar procedure can be followed for the case $\Lambda(u_m, F_m) < 0$: having established that a NE is never reached, it is sufficient to note that the left-hand side of (15) is strictly smaller than its right-hand side. When $\Lambda(u_m, F_m) = 0$, it has been shown that a NE is reached and (15) always holds, thus concluding the proof. ■

Theorem 1 provides the necessary and sufficient conditions (15) for existence of a NE in the proposed non-cooperative game. According to game-theory literature, uniqueness of NE solutions is generally not guaranteed [40]. Therefore, it is possible that multiple different investment solutions fulfill (15). Since the focus of this paper is not on identifying all possible NE of the merchant planning game but rather on investigating whether merchant planning can yield the same social welfare maximizing solution as centralized planning, we will seek for the NE solution yielding the largest social welfare.

Definition 3: The merchant solution (MS) of the transmission planning problem is determined through the following optimization problem:

$$\min_{\substack{u_m, F_m, f_{m,t}, \\ G_{n,t}, D_{n,t}, \lambda_{n,t}}} \ _{\forall m, \forall n, \forall t} \sum_t \sum_n w_t \left[C_{n,t}(G_{n,t}) - B_{n,t}(D_{n,t}) \right] + \sum_m T_m(u_m, F_m) \quad (16)$$

Subject to (3)-(8),

$$u_m F_m \sum_t w_t \left(\lambda_{n^r_m,t} - \lambda_{n^s_m,t} \right) f_{m,t} = u_m (F_m + F^0_m)(T^F_m + T^V_m F_m). \quad \forall m \quad (17)$$

This problem is similar to the one solved under centralized planning, but it also considers the NE condition (17) of Theorem 1, to be verified on each network branch.

IV. Social Efficiency of Merchant Transmission Planning

This paper investigates under which conditions the merchant planning paradigm yields the social welfare maximizing solution or, in other words, under which conditions the centralized planning approach of Section II and the merchant planning approach of Section III yield the same solution. We claim that this equivalence holds if the following conditions hold:

A_1) Fixed investment costs are neglected, i.e. $T^F_m = 0$, $\forall m$.

A_2) The network is radial and does not include any loops.

A. Theoretical Analysis

The sufficiency of the aforementioned conditions A_1 and A_2 is theoretically proved through Theorem 2 below. This theorem claims that if A_1 and A_2 hold, then the centralized and merchant planning solutions coincide. In order to simplify the theoretical analysis, two additional conditions are introduced:

B_1) The operational timescale of the planning problem includes a single period, i.e. $|T| = 1$.

B_2) The existing capacity of every branch is zero, i.e. $F^0_m = 0$, $\forall m$.

Theorem 2: The centralized solution CS of Definition 1 and the merchant solution MS of Definition 3 coincide if conditions A_1-A_2 and B_1-B_2 hold.

Proof: Without loss of generality, it is assumed that the total capacity addition of the CS is positive for all branches, i.e. $F_m > 0$, $u_m = 1$, $\forall m$. If this is not the case for some branches, the analysis provided next can be performed on the subset of branches $\tilde{M} \subset M$ for which this assumption holds, with $\tilde{M} = \{m \in M : F_m > 0,\ u_m = 1\}$. If this is not the case for any branch (the CS does not involve any capacity addition, i.e. $F_m = 0$, $u_m = 0$, $\forall m$), it can be shown that the CS and MS coincide as both sides of the NE conditions (17) are zero. Given condition B_1, the subscript t is omitted in the remainder of this proof. Under the current assumptions, a simplified expression can be derived for the optimization problem (1)-(8) returning CS. From condition A_1, the investment cost term (2) in the objective function (1) can be rewritten as:

$$T_m = T^V_m F_m \quad \forall m. \quad (18)$$

Regarding the constraints, equations (7) are omitted as a result of A_2. Assuming without loss of generality a "positive" power flow on each branch (i.e. power flows from the reference sending node to the reference receiving node), we implicitly account for constraints (5) by imposing:

$$f_m = F_m \quad \forall m. \quad (19)$$

These equations hold since i) $f_m > F_m$ violates (5) given that $F^0_m = 0$ from B_2 and ii) $f_m < F_m$ is suboptimal as the unused capacity $F_m - f_m$ increases the objective function (1). As a result of the above, by combining (4) and (6) and by rewriting (8) as two separate constraints, the optimization problem returning the CS can be formulated as:

$$\min_{\substack{F_m, \forall m \\ G_n, D_n, \forall n}} \sum_m T^V_m F_m + \sum_n \left[C_n(G_n) - B_n(D_n) \right] \quad (20)$$

subject to:

$$D_n + \sum_m \phi_{n,m} F_m - G_n = 0 : \lambda_n \quad \forall n. \quad (21)$$

$$-G_n \leq 0 : \mu^-_n \quad \forall n. \quad (22a)$$

$$G_n - G^{max}_n \leq 0 : \mu^+_n \quad \forall n. \quad (22b)$$

The Lagrangian function associated with this optimization problem is expressed as:

$$L = \sum_m T^V_m F_m + \sum_n \left[C_n(G_n) - B_n(D_n) \right] + \sum_n \lambda_n (D_n + \sum_m \phi_{n,m} F_m - G_n) - \sum_n \mu^-_n G_n + \sum_n \mu^+_n (G_n - G^{max}_n). \quad (23)$$

Derivation of the Lagrangian with respect to F_m yields the following set of necessary conditions for optimality:

$$\frac{\partial L}{\partial F_m} = T^V_m + \sum_n \phi_{n,m} \lambda_n = 0 \quad \forall m. \quad (24)$$

The term $\phi_{n,m}$ in (24) denotes the element in the n-th row and m-th column of the sensitivity matrix $\mathbf{\Phi}$, describing the network topology. For each column m of $\mathbf{\Phi}$ we have $\phi_{n^s_m,m} = 1$ and $\phi_{n^r_m,m} = -1$, while $\phi_{n,m} = 0$ for all nodes n not connected to branch m. Therefore, (24) can be rewritten as:

$$T^V_m + \lambda_{n^s_m} - \lambda_{n^r_m} = 0 \quad \forall m. \quad (25)$$

Regarding the MS, as a result of conditions A_1, B_1, B_2 and equation (19), the necessary and sufficient conditions (17) for achieving NE can be rewritten as:

$$\lambda_{n_m^r} - \lambda_{n_m^s} = T_m^V \quad \forall m. \tag{26}$$

Note that the optimality conditions (25) of the CS are equivalent to the NE conditions (26) of the MS. This implies that the CS and the MS coincide, concluding the proof. ■

Although the above theoretical analysis considers the simplifying hypotheses B_1-B_2, the case studies presented in Section V will numerically demonstrate that B_1-B_2 are not necessary. In other words, it will be shown that CS and MS coincide even when B_1-B_2 do not hold. On the other hand, these case studies indicate that A_1-A_2 are not only sufficient but also necessary: the CS and MS are in principle different when one of the conditions A_1 or A_2 does not hold.

B. Discussion

This section aims at discussing the physical significance behind these two sufficient and necessary conditions:

Condition A1: Under the CS, the total congestion surplus in the network covers exactly the variable component of the total investment cost, but it does not cover fixed costs [1]. On the other hand, the NE condition (15) of the MS requires that the total congestion surplus covers exactly the total investment cost (both variable and fixed components), as the rational merchant investors do not accept economic losses. Therefore, as demonstrated by the case studies of Section V, when fixed costs are accounted for, the total network capacity procured under the MS is lower than the respective capacity procured under the CS, in order to increase the collected congestion surplus and thus cover the fixed costs.

Condition A2: Under the CS, although the total congestion surplus in the network is equal to the variable component of the total investment cost, this equality does not necessarily hold for each individual network branch when the network is meshed; in such cases, some branches may generate higher congestion surplus than their variable investment cost, while other branches may generate lower congestion surplus [1]. On the other hand, the NE existence condition (15) of the MS requires that this equality holds on an individual branch basis, as demonstrated in the case studies of Section V. This requirement makes sense since the impact of each infinitesimal investor's decisions on system conditions is negligible. As a result, each of these investors assesses its decision for each branch individually, ignoring the impact of this decision on the congestion surplus associated with other branches; it will strive to increase its procured capacity on a branch m if the obtainable congestion surplus from m is higher than the required investment cost and decrease it if the obtainable congestion surplus from m is lower than the required investment cost. Therefore, as demonstrated in the case studies of Section V, the CS and MS do not coincide when the network is meshed.

V. Case Studies

A. Scope and Implementation

The objective of the examined case studies is to determine, in a simulative context, under which conditions the CS of Section II and the MS of Section III yield the same solution. In order to achieve this, four systems of different sizes and topologies are examined under different sets of assumptions. Both planning models have been implemented in MATLAB on a computer with a 4-core 2.40 GHz Intel(R) Xeon(R) E5620 processor and 12 GB of RAM.

The optimization problems corresponding to the centralized and merchant planning approaches include the binary decision variables u_m, $\forall m$. For simplicity and accuracy, these variables are accounted in the resolution process through a complete enumeration approach. In other words, the optimization problems have been solved for each different combination of 2 alternative values of u_m for each of the $|M|$ branches of the network, i.e. $2^{|M|}$ alternative solutions are determined and the (feasible) solution yielding the lowest objective function is selected. Given this simplification (and the assumption that generation costs and demand benefits are quadratic functions), the optimization problem (1)-(8) determining the CS is a quadratic programming problem with linear constraints. Therefore, it has been solved using the *quadprog* MATLAB optimization routine. On the other hand, the optimization problem determining the MS has been solved using the *fmincon* optimization routine, given that constraints (17) are non-linear. Despite the employment of a complete enumeration approach for dealing with the binary variables u_m, $\forall m$, the proposed algorithm requires low computational times. For the largest examined system (24-node system of Section V-E), the CS and MS have been calculated in 51s and 896s, respectively.

B. 2-Node System

The relevant generation and demand data [1] of the 2-node system considered in the first case study are illustrated in Fig. 1. It is assumed that the existing capacity of the single branch is zero and that the operational timescale of the planning problem includes a single time period. Generation costs are assumed to be quadratic functions of the respective power productions. The demands in the two nodes are assumed inelastic and equal to constant values, i.e. their benefit functions are constant and can thus be omitted from the two optimization problems. Two different cases have been examined, with the respective CS and MS presented in Table I.

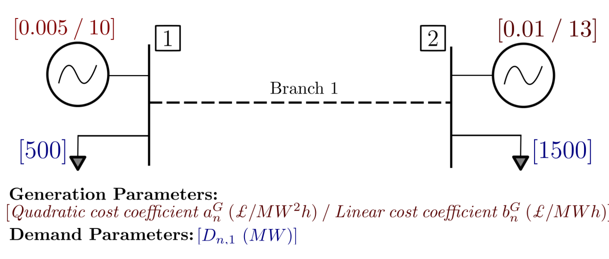

Fig. 1. Topology and parameters of the 2-node system.

Case 1.1): The investment cost includes only a variable component $T_1^V = 4£/MWh$, while fixed costs are neglected ($T_1^F = 0$). In this case, the centralized and the merchant planning model yield the same solution, involving investment on a line of $800MW$ (Table I). This result is expected from

Theorem 2, as conditions A_1-A_2 and B_1-B_2 hold and therefore the CS and MS must coincide.

Case 1.2): The investment cost includes both a variable component $T_1^V = 4£/MWh$ and a fixed component $T_1^F = 2283£/h$. The capacity procured under the CS does not change with respect to Case 1.1 and, as discussed in Section IV-B, the congestion surplus does not cover the full investment cost, due to the existence of fixed costs. On the other hand, the capacity procured under the MS is now reduced to 690MW, to ensure that the congestion surplus covers the full investment cost (Section IV-B). This result suggests that condition A_1 (zero fixed investment costs) is a necessary condition for the CS and MS to coincide.

TABLE I
CENTRALIZED (CS) AND MERCHANT PLANNING (MS) SOLUTIONS IN 2-NODE SYSTEM

	Case 1.1		Case 1.2	
	CS	MS	CS	MS
F_1 (MW)	800	800	800	690
Congestion surplus (£/h)	$3,200$	$3,200$	$3,200$	$5,042$
Investment cost (£/h)	$3,200$	$3,200$	$5,483$	$5,042$

C. 3-Node System

The employed 3-node system along with its relevant generation and demand data [1] is shown in Fig. 2. It is assumed that the existing capacity of the three branches is zero, their investment costs are equal and their reactances after any capacity addition are equal. The operational timescale of the planning problem includes two time periods with weighting factors $w_1 = 0.25$ and $w_2 = 0.75$. Generation costs are assumed to be quadratic functions of the respective power productions and demands are assumed inelastic. Four different cases have been examined, presenting in Table II the respective CS and MS.

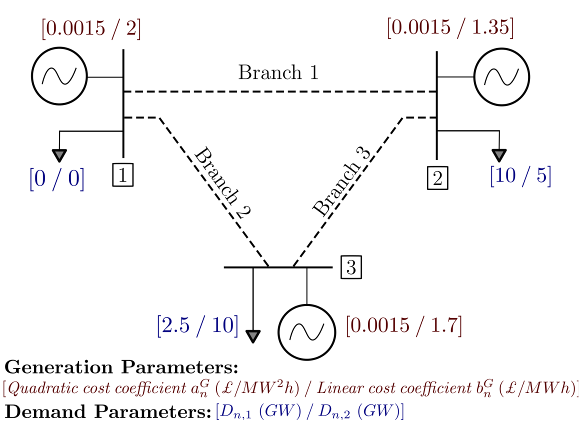

Fig. 2. Topology and parameters of the 3-node system.

Case 2.1: The investment cost includes only a variable component $T_m^V = 3.42£/MWh, \forall m$, while fixed costs are neglected. In contrast to Case 1.1, the centralized and the merchant planning model do not yield the same solution (Table II). As discussed in Section IV-B, while the equality between congestion surplus and investment cost holds for each individual network branch under the MS, the same does not hold under the CS. This result suggests that condition A_2 (no network loops) and/or condition B_1 (single period in the operational timescale) is/are necessary condition(s) for the CS and MS to coincide.

Case 2.2: In order to investigate which of the conditions A_2 and B_1 is critical, we consider a case where capacity can be added only on branches 1 and 2, imposing $F_3 = 0$ in the two optimization problems. All the other parameters remain the same as in Case 2.1. In this scenario, the centralized and the merchant planning model yield the same solution (Table II). This suggests that A_2 is a necessary condition for the CS and the MS to coincide, since in this scenario the network is radial and does not include loops. On the other hand, it also demonstrates that condition B_1 is not necessary for the CS and MS to coincide.

Case 2.3: In order to further explore this interesting result, we consider a theoretical scenario where capacity can be added on all three branches but the Kirchhoff's voltage law (KVL), expressed through (7), is neglected in both optimization problems. All the other parameters remain the same as in Case 2.1. In this theoretical scenario, the centralized and the merchant planning model again yield the same solution (Table II). This result suggests that the physical reason behind the necessity of condition A_2 lies in the unavoidable consideration of the KVL in meshed networks. As already noted for Case 2.2, it seems that condition B_1 is not necessary for the equivalence between the CS and the MS.

Case 2.4: The KVL is neglected as in Case 2.3 but the investment cost also includes a fixed component $T_m^F = 2283£/h, \forall m$. In contrast to Case 2.3, the CS and the MS do not coincide. As discussed in Section IV-B, while the total congestion surplus covers the total investment cost under the MS, the same does not hold under the CS due to the existence of fixed costs. Like in Case 1.2, this result suggests that A_1 is necessary for the CS and MS to coincide.

D. Garver's 6-Node System

Garver's 6-node system, along with its relevant network, generation and demand data [3], is illustrated in Fig. 3. The solid lines 1-6 represent existing branches (with positive existing capacity) that can be expanded while the dashed lines 7-8 represent new branches (with zero existing capacity) that can be built. The operational timescale of the planning problem includes a single time period. Generation costs and demand benefits are assumed to be quadratic functions of the power productions and consumptions, respectively. In contrast with the previous case studies, the generation limits (8) are taken into account, with the considered values of G_n^{max} reported in Fig. 3.

Three different cases have been examined, presenting in Table III the corresponding CS and MS. Note that the capacity additions for branches 1, 2, 4 and 5 are zero in all three cases (under both CS and MS), and are therefore omitted from Table III.

Case 3.1: The investment cost includes only a variable component (presented in Fig. 3 for each branch), while fixed

TABLE II
CENTRALIZED (CS) AND MERCHANT PLANNING (MS) SOLUTIONS IN 3-NODE SYSTEM.

	Case 2.1		Case 2.2		Case 2.3		Case 2.4	
	CS	MS	CS	MS	CS	MS	CS	MS
F_1 (MW)	$1,963$	$2,193$	$2,678$	$2,678$	$2,044$	$2,044$	$2,044$	$1,686$
F_2 (MW)	$2,887$	$2,808$	$3,991$	$3,991$	$2,089$	$2,089$	$2,089$	$2,211$
F_3 (MW)	$1,387$	$1,609$	0	0	$2,156$	$2,156$	$2,156$	$1,811$
Congestion surplus - Branch 1 (£/h)	$6,690$	$7,511$	$9,172$	$9,172$	$7,002$	$7,002$	$7,002$	$8,057$
Congestion surplus - Branch 2 (£/h)	$8,337$	$9,615$	$13,667$	$13,667$	$7,154$	$7,154$	$7,154$	$9,855$
Congestion surplus - Branch 3 (£/h)	$6,333$	$5,510$	0	0	$7,382$	$7,382$	$7,382$	$8,485$
Congestion surplus - Total (£/h)	$21,360$	$22,636$	$22,839$	$22,839$	$21,538$	$21,538$	$21,538$	$26,397$
Investment cost - Branch 1 (£/h)	$6,723$	$7,511$	$9,172$	$9,172$	$7,002$	$7,002$	$9,285$	$8,057$
Investment cost - Branch 2 (£/h)	$9,887$	$9,615$	$13,667$	$13,667$	$7,154$	$7,154$	$9,437$	$9,855$
Investment cost - Branch 3 (£/h)	$4,750$	$5,510$	0	0	$7,382$	$7,382$	$9,665$	$8,485$
Investment cost - Total (£/h)	$21,360$	$22,636$	$22,839$	$22,839$	$21,538$	$21,538$	$28,387$	$26,397$

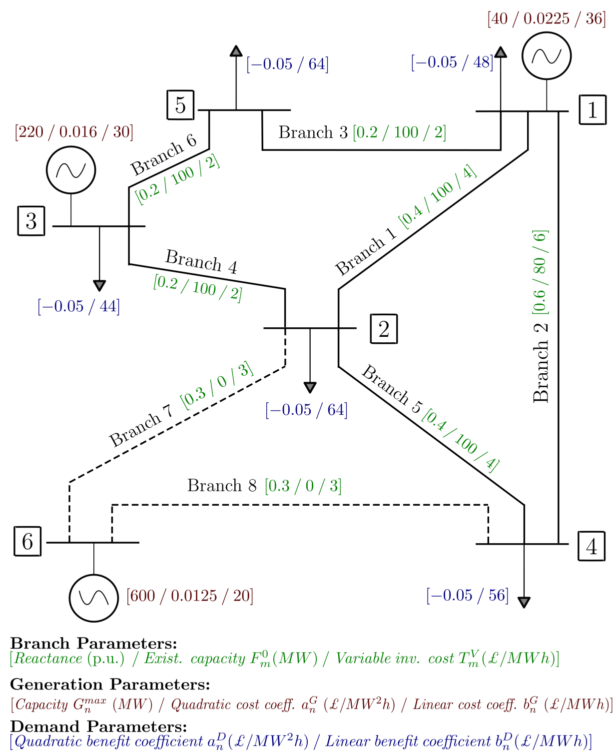

Fig. 3. Diagram of the 6-node Garver's system.

costs are neglected. As in Case 2.1, the centralized and the merchant planning model do not yield the same solution (Table III). This suggests that A_2 (no network loops) and/or B_2 (zero existing capacity on every branch) is/are necessary condition(s) for the CS and MS to coincide.

Case 3.2: In order to investigate which of the conditions A_2 and B_2 is critical, following the rationale of Case 2.3, we consider a theoretical scenario where the KVL is neglected. As in Case 2.3, the centralized and the merchant planning model yield the same solution (Table III). This result again supports the idea that condition A_2 is necessary for the CS and MS to coincide. On the other hand, this result demonstrates that condition B_2 is not required to obtain correspondence between the two planning paradigms.

TABLE III
CENTRALIZED (CS) AND MERCHANT PLANNING (MS) SOLUTIONS IN GARVER'S SYSTEM.

	Case 3.1		Case 3.2		Case 3.3	
	CS	MS	CS	MS	CS	MS
F_3 (MW)	0	0	17.41	17.41	0	0
F_6 (MW)	77.93	75.61	24.77	24.77	0	0
F_7 (MW)	319.13	317.44	229.12	229.12	208.48	184.18
F_8 (MW)	276.52	275.89	362.16	362.16	366.30	366.80
Cong. surplus (£/h)	$1,903$	$1,931$	$1,858$	$1,858$	$1,724$	$2,053$
Inv. cost (£/h)	$1,943$	$1,931$	$1,858$	$1,858$	$2,124$	$2,053$

Case 3.3: The KVL is neglected as in Case 3.2 but the investment cost also includes a fixed component $T_m^F = 200£/h, \forall m$. In contrast to Case 3.2, the CS and the MS are different. As in Cases 1.2 and 2.4, this suggests that condition A_1 is necessary for the CS and MS to coincide.

It should be noted that the maximum generation limits at nodes 1 and 3 are active in all examined cases. However, the same trends regarding the correspondence between the centralized and the merchant planning solution are observed, suggesting that physical generation limits do not impact the conditions ensuring social efficiency of the MS.

E. IEEE 24-Node System

The IEEE 24-node system [41], along with its relevant network, generation and demand data, is illustrated in Fig. 4. All lines represent existing branches that can be expanded. The operational timescale of the planning problem includes a single time period. Generation costs and demand benefits are assumed to be quadratic functions of the power productions and consumptions, respectively. Three different cases have been examined, analogous to the ones examined in Section V-D, presenting in Table IV the corresponding CS and MS. For compactness, branches with zero capacity additions in all three cases have been omitted from Table IV.

Case 4.1: The investment cost includes only a variable component (presented in Fig. 4 for each branch), while fixed costs are neglected. As in Case 3.1, the centralized and the merchant planning model do not yield the same solution (Table IV), suggesting that A_2 (no network loops) and/or B_2 (zero existing capacity on every branch) is/are necessary condition(s) for the CS and MS to coincide.

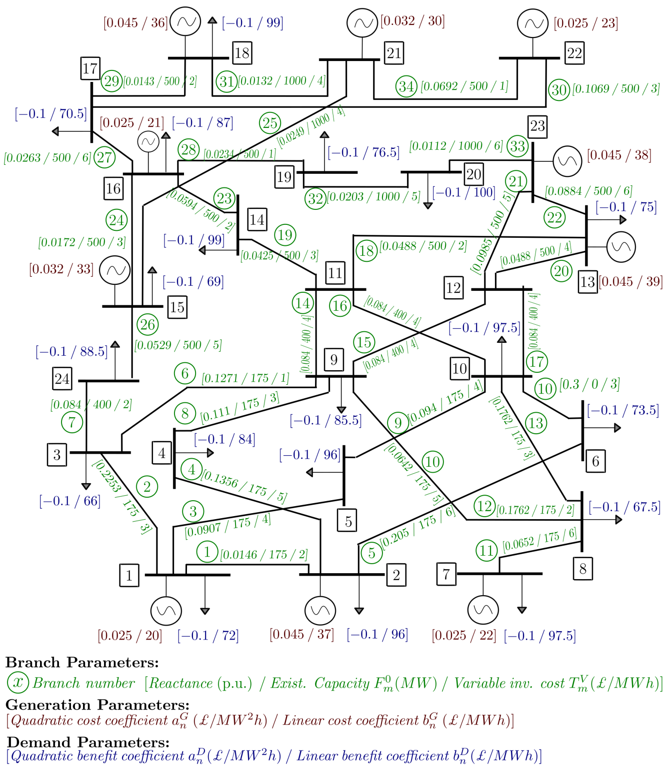

Fig. 4. Diagram of the IEEE 24-node system.

TABLE IV
CENTRALIZED (CS) AND MERCHANT PLANNING (MS) SOLUTIONS IN IEEE 24-BUS SYSTEM.

	Case 4.1		Case 4.2		Case 4.3	
	CS	MS	CS	MS	CS	MS
F_1 (MW)	322.02	323.03	241.53	241.53	241.53	246.39
F_3 (MW)	185.06	179.81	93.91	93.91	93.91	88.39
F_{11} (MW)	140.21	144.91	146.45	146.45	146.45	142.36
F_{23} (MW)	311.42	310.99	117.54	117.54	117.54	82.12
F_{26} (MW)	209.62	202.36	0	0	0	0
F_{28} (MW)	234.70	240.27	548.91	548.91	548.91	567.43
F_{34} (MW)	4.33	0	0	0	0	0
Cong. surplus (£/h)	3,928	4,108	2,521	2,521	2,521	2,682
Inv. cost (£/h)	4,135	4,108	2,521	2,521	2,771	2,682

Case 4.2: In order to investigate which of the conditions A_2 and B_2 is critical, following the rationale of Cases 2.3 and 3.2, we consider a theoretical scenario where the KVL is neglected. As in these cases, the centralized and the merchant planning model yield the same solution (Table IV). This result again supports the idea that condition A_2 is necessary for the CS and MS to coincide, while condition B_2 is not required to obtain correspondence between the two planning paradigms.

Case 4.3: The KVL is neglected as in Case 4.2 but the investment cost also includes a fixed component $T_m^F = 50£/h, \forall m$. In contrast to Case 4.2, the CS and the MS are different. As in Case 3.3, this suggests that condition A_1 is necessary for the CS and MS to coincide.

VI. CONCLUSIONS AND FUTURE WORK

The investigation of the merchant transmission planning paradigm requires a non-cooperative game-theoretic framework that is able to capture the strategic behavior and interactions of multiple merchant investors. However, previous relevant modeling approaches cannot deal with a large number of merchant companies, due to the difficulties in determining NE solutions emerging from their interaction. As a result, they cannot accurately determine whether merchant planning can yield the same solution as centralized planning under the participation of a large number of merchant companies. In order to address this challenge, this paper has proposed a novel non-cooperative game-theoretic approach, in which the number of merchant companies is assumed sufficiently large so that they can be approximated as a continuum. This approximation allows the derivation of mathematical conditions for the existence of a NE solution.

By analytically and numerically comparing this solution against the one obtained through traditional centralized planning, the paper demonstrates that the merchant planning paradigm can maximize social welfare only when the following conditions are satisfied: a) fixed investment costs are neglected and b) the network is radial and does not include any loops. When these two conditions do not hold, the results of the case studies show that the centralized and the merchant planning solutions do not coincide, although their differences in certain cases (especially in the 6-node and the 24-node system) are relatively small. As the above conditions do not generally hold in reality, our findings suggest that even a fully competitive merchant transmission planning framework, involving the participation of a very large number of competing merchant companies, is not generally capable of maximizing social welfare, as implied by the previous work [3].

It should be noted that the merchant investors examined in this paper are assumed not to be involved in the electricity generation of supply business. However, the presented modeling framework can be extended in a straightforward fashion to cases where generation or demand players are allowed to carry out merchant network investments. Analysis of such cases carried out by the authors demonstrated that the above conclusions regarding the conditions for social optimality of merchant transmission planning are not altered.

In order to simplify the presented analysis, the developed model - as well as the models in the relevant literature - does not consider a) the "lumpiness" of actual transmission investment practices which often include a certain discrete set of standardized capacity options and b) the complex operational constraints of the generation side (e.g. ramp rates and minimum up and down times) and of the demand side (e.g. energy conservation constraints of flexible demand technologies). Future work aims at incorporating an accurate representation of these elements in the centralized and merchant planning models and exploring their impact on the equivalence between the two solutions. Furthermore, the developed model - as well as the models in the relevant literature - assumes a fixed generation mix and does not consider generation expansion decisions. In reality however, transmission and generation expansion decisions are interdependent. In this context, future work aims at developing an integrated transmission and generation planning framework and comparing the impacts of centralized and merchant transmission planning on generation expansion decisions.

Finally, on the policy front, our results suggest that some sort of regulatory interventions will be required to align the outcome of merchant planning with the socially optimal solution. However, these interventions need to remain at a minimum level, in line with the deregulation vision. The design of such regulatory measures constitutes a significant challenge for future research.

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