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Estimating and forecasting international yield curves: a no-arbitrage VAR with macroeconomic and latent variables

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Abstract

The focus of this paper is to replicate and extend the estimation of a model for the prediction of interest rate affine term structures to G10 countries. The existing literature for the prediction of yield curves is vast, we will be focusing on the popular class of Gaussian affine term structure models. Our model uses measures of real activity and inflation as macroeconomic variables together with unobservable variables, through non-arbitrage VARs. Our purpose is to find how well this model fares at forecasting in different markets and if the model is good at predicting the correct shifts in the yield curve. This Work Project had the objective of delivering its results to BlackRock so they can trade based on forecasts from the model.

Key Words: *Interest Rate term structures, Affine Term Structure Models, Macroeconomic variables, Latent Variables, Minimum Chi-square Estimation*

Index

Introduction

- The present paper tries to set up a framework for the prediction of affine interest rates term structures, including macro factors as explanatory variables. This framework is based on the work done by **Hamilton and Wu (2012)** and also **Ang and Piazzesi (2003),** which elaborate on the already existing literature of **Gaussian Affine Term Structure models** developed by Vasicek (1977), Duffie and Kan (1996), Dai and Singleton (2002) and Dufee (2002).
- Other models that fit the yield curve such as the Nelson-Siegel model or the Svensson model do not do better than random walk models. Hence, one of our goals is to achieve a model than bares better forecasts than a random walk. The main framework for the model we are using includes both observable and unobservable factors (and their lags) when estimating affine interest rate term structure forecasts. The model also requires **non-arbitrage VARs** for the observable and unobservable factors. The estimation done by AP was a maximum likelihood estimation (MLE), however, we adopt the **minimum chi-square estimation (MCSE),** proven to be better by HW.
- In this work project our main intent is to expand the existing literature to other countries outside and inside the European Union and do an analysis of the quality of these obtained forecasts using RMSE (Root Mean Squared Error) and a measure of the corrected predicted yield movement.
- It was also important to ensure the model always takes in the most recent available data. This is important because one of the objectives is also that this can be an automatized predicting process that is always using the most recent and reliable data available.

Literature Review

Literature Review

Gaussian Affine Term Structure Models

- Oldrich Vasicek's paper in 1977 was one of the earliest literatures in modeling interest rate term structures, while incorporating a no-arbitrage assumption. This single-factor model is the basis for recent affine multi-factor models.

Vasicek, 1977

- *"An equilibrium characterization of the term structure".*
- *Journal of Financial Econometrics, volume 5, issue 2, November 1977, pgs. 177-188.*
- In this paper, Vasicek proposes to derive a general form of the term structure of interest rates taking into account the three following assumptions:
    1. The spot rate follows a continuous Markov process, meaning that the spot rate process is characterized by a single state variable which is its current value. Processes that are Markov and continuous are called diffusion processes which can be described by a stochastic differential equation.
    2. The price of a discount bond is determined by the assessment of the spot rate process over the term of the bond. The expectation hypothesis, the market segmentation hypothesis and the liquidity preference hypothesis all confirm this assumption.
    3. The market is assumed to be efficient, which means: no transaction costs, information is available to all investors simultaneously and they all act rationally.

Literature Review

Gaussian Affine Term Structure Models

- The movement of interest rates over time is arbitrary, so a random process is typically used to model it: continuous time process (also called diffusion process). In Vasicek the interest rate follows a Hornstein-Uhlenbeck process under the risk neutral measure Q as such: $dr_t = \kappa(\theta - r_t)dt + \sigma dW_t^Q$, where W(t) is a standard Brownian motion. The short-rate will, by construction, be mean-reverting because of $(\mu - r)$ and its implications on the expected interest rate change for the future.

- The pricing solution found by Vasicek for risk-free ZCBs is as such: $P_t(\tau) = \exp[a(\tau) + b(\tau)r_t]$

With $a(\tau) = \left(\theta - \frac{\lambda_0}{\kappa} - \frac{\sigma^2}{2\kappa^2}\right)\left[\frac{1}{\kappa}(1 - \exp(-\kappa\tau)) - \tau\right] - \frac{\sigma^2}{4\kappa^3}[1 - \exp(-\kappa\tau)]^2$

And $b(\tau) = -\frac{1}{\kappa}[1 - \exp(-\kappa\tau)]$

, where λ_0 is the risk premium, constant through time, and τ is the time of the bond until maturity.

- In sum, the paper delivers a single-factor interest rate model established on the idea that interest rates follow conditional and unconditional Gaussian distributions and are mean-reverting. The first equation is the core of more recent models, implying a Gaussian structure with constant volatility.

Literature Review

Gaussian Affine Term Structure Models

- John C. Cox, Jonathan E. Ingersoll, Jr., and Stephen A. Ross built a paper that, alongside Vasicek's, constituted the founding papers for affine term structure models. These models would remain as the workhorse models in the future literature.

Cox, Ingersoll and Ross, 1985

- *"A Theory of the Term Structure of Interest Rates".*
- *Econometrica, volume 53, issue 2, March, 1985*.
- Cox, Ingersoll and Ross (CIR) study the yield curve using an intertemporal general equilibrium asset pricing model. As the yield curve measures the relationship among the yields on default-free securities with different maturities, in order to define the determinants of this relationship, CIR take into account all of those factors that play a role in defining the price of bonds, such as: risk aversion, investment alternatives, anticipations, preferences and timing of consumptions, etc. As a result, their approach contains elements of all of the previous theories. They wanted to create a model which was able to predict how changes in a wide range of underlying variables would affect the yield curve. Their model is a single factor model of the term structure in which the dynamics of bond prices are given by a stochastic differential equation.

- They built a one factor model for the term structure but they also show how to incorporate multiple factors in this structure. Models to this time assumed that risk premia was constant through time.

Gaussian Affine Term Structure Models

Cox, Ingersoll and Ross, 1985

- The difference between Vasicek, 1977, and CIR, 1985, is the assumption of the process on the interest rates process throughout time. They use the square root process of the interest rate, like this $dr_t = \kappa(\theta - r_t)dt + \sigma\sqrt{r_t}dW_t^Q$.
- This makes the model more complex because now interest rates won't be normally distributed. However, this includes a very important restriction that in the future all interest rates will necessarily be non-negative. Which constitutes a very desirable feature in normal times for nominal interest rates because with interest rates negative then the non-arbitrage assumption falls to the ground, since bond prices exceed 100%.
- The final pricing specification for the risk-free ZCBs is $P_t(\tau) = \exp[a(\tau) + b(\tau)r_t]$

With $a(\tau) = \frac{2\kappa\theta}{\sigma^2}\log\left[\frac{2\gamma\exp(0.5\tau(\kappa+\lambda_1+\gamma))}{(\kappa+\lambda_1+\gamma)[\exp(\gamma\tau)-1]+2\gamma}\right]$

And $b(\tau) = \frac{-2[\exp(\gamma\tau)-1]}{(\kappa+\lambda_1+\gamma)[\exp(\gamma\tau)-1]+2\gamma}$ where $\gamma \equiv \sqrt{(\kappa+\lambda_1)^2 + 2\sigma^2}$

,where λ is the risk factor implied by a single agent's utility function. Notice that the risk premium consistent with the no-arbitrage assumption is $\lambda = \lambda_1 r_t$.

- In these models, prices of bonds of all maturities depend on a single random explanatory factor which is the spot interest rate.

Literature Review

Gaussian Affine Term Structure Models

- Ang and Piazzesi model is a special case of discrete-time versions of the affine class introduced by Duffie and Kan 1996, where bond prices are exponential affine functions of underlying state variables.

Duffie and Kan, 1996

- *"A yield-factor model for interest rates"*
- *Mathematical Finance, volume 6, issue 4, pgs. 379– 406*
- In this paper, Duffie and Kan present a consistent and arbitrage-free multi-variate model of the yield curve. The factors of their models are the yields of zero coupon bonds of different maturities which form a Markov process that can be seen as a multivariate version of the single factor model proposed by Cox, Ingersoll and Ross (1985).
- In other earlier models, the state-variable processes are treated as different kind of "shocks" which are not necessarily designed to be observable from the current yield curve. However, by solving the models for the term structure, the yield at any given maturity can be seen as to be a maturity-dependent affine function of the underlying state variables. Duffie and Kan, considering a set of maturities equal in number with the underlying factors, perform a change of basis under which the state variables are yields at these various fixed maturities.

Literature Review

Gaussian Affine Term Structure Models

- Duffie and Kan exploit the idea that yields at selected fixed maturities follow a parametric multi-variate Markov diffusion process with stochastic volatility factors that are a specified linear combination of yield factors. The yield model is ”affine” which means that there is, for each maturity, an affine function such that at any point in time the yield of any zero coupon bond is taken to be a maturity-dependent affine combination of the selected “basis” set of yields.

Duffie and Kan, 1996

- In particular, the short rate drift will follow $\mu(r_t) - \lambda(r_t) = \rho_0 + \rho_1 r_t$ and volatility will now have a function dependent on the interest rate at t $\sigma(r_t) = \sqrt{B_0 - B_1 r_t}$.
- For the pricing equation: $P_t(\tau) = \exp[a(\tau) + b(\tau) r_t]$, now a and b will not have closed form solutions like they had in the case of Vasicek and CIR.

- Empirical evidence showed that a multi factor specification was necessary in order to capture the dynamics of a risk-free asset term structure. Volatility dynamics did not seem to be completely captured neither with the constant solution nor with this conditional volatility.

Literature Review

Gaussian Affine Term Structure Models

- As a result of the extensive research on one-factor and two-factor models, by the 1990s, it was clear that at least three factors were necessary in order to model interest rate term structures.

Litterman and Scheinkman, 1991

- *“Common factors affecting bond returns”*
- *Journal of Fixed Income, volume 1, issue 1, pgs. 51-61.*
- It developed a three-factor model to figure out the common factors that have impacted returns on Treasury based securities in the past. These three factors are labeled as “level”, “slope” and “curvature” and they are built from the yields and returns themselves. Because they are dealing with unobservable factors, they assume zero mean, unit variance and zero covariance between any two factors. The conclusions yielded that these three factors could explain over 95 % of the variations on US Treasury Bond prices.

- The first step to building a multi factor affine term structure model is to establish a linear relationship between the yields and the factors included in the model: $\boldsymbol{r_t = \delta_0 + \delta' Y_t}$, where Y_t is a N vector of the factors with t realizations.
- Factor dynamics follow an affine process given by a Brownian motion under the measure *Q:* $dY_t = \chi(\theta - Y_t)dt + \Sigma\sqrt{S_t}dW_t^Q$.
- Then the pricing solution is $P_t(\tau) = \exp[A(\tau) + B(\tau)Y_t]$ with $A(\tau)\ and\ B(\tau)$ being solutions to the differential equations: $\frac{dA(\tau)}{d\tau} = -\theta\chi' B(\tau) + 0.5\sum_{i=1}^{N}[\Sigma' B(\tau)]_i^2 \alpha_i - \delta_0$ and $\frac{dB(\tau)}{d\tau} = -\chi'\beta(\tau) + 0.5\sum_{i=1}^{N}[\Sigma' B(\tau)]_i^2 \beta_i - \delta$.
- The pricing of the risk neutral asset in the economy is given by: $\Lambda = \sqrt{S_t}\lambda_0 + \sqrt{S_t^-}\lambda Y_t$ with S being an N-dimensional matrix where the diagonal elements take the form of $S_{t(ii)}^- = \begin{cases} \sqrt{\alpha_i + \beta_i' Y_t}\ if\ \inf(\alpha_i + \beta_i' Y_t) > 0 \\ 0\ if\ else \end{cases}$.

Gaussian Affine Term Structure Models

- In contrast to latent factor models, empirical studies try to directly model the relationship between bond yields and macro-variables by using VAR models.

Estrella and Mishkin, 1997

- *"The predictive power of the term structure of interest rates in Europe and the United States: Implications for the European Central Bank"*
- *European Economic Review, volume 41, issue 7, pgs. 1375-1401.*
- In this paper Estrella and Mishkin focus on the relationship between the yield curve and monetary policy instruments in both Europe and United States. In order to deal with the fact that both the term structure of interest rates and the central bank rate are endogenous variables, Estrella and Mishkin adopt a VAR formulation for the central bank (short and long term) rate. The strategy is to investigate the yield curve ability to predict future real activity and inflation throughout the estimation of a linear regression equation in which the contemporaneous value of the spread is used to forecast the change in real activity and inflation.

The result show that monetary policy is an important determinant of the term structure spread but it is not the only determinant. Indeed, there is a significant predictive power with regard to both inflation and real activity.

Therefore, the term structure of interest rate should be considered as an accurate and useful measure that, together with other information, can be used to guide monetary policy decisions.

Literature Review

Gaussian Affine Term Structure Models

- When using VAR models, the relationship between yields movements and shocks in macro variables can be inferred by IRs (impulse responses) and variance decomposition techniques.

Evans and Marshall, 1998

- *"Monetary policy and the term structure of nominal interest rates: evidence and theory"*
- *Carnegie-Rochester Conference Series on Public Policy, volume 49, issue 1, pgs. 53 – 111.*
- In this paper, Evans and Marshall investigate how exogenous impulses to monetary policy affect the term structure of interest rates and its shape as well as the impact of these shocks on term premia and ex-ante real rates. Bonds yields are measured as continuously compounded annualized returns on zero coupon bonds. In order to identify the monetary policy shocks, they use the Federal Funds rate as the monetary policy instrument and and the monetary policy rule is estimated as one equation within a restricted version of the VAR.

Evans and Marshall find that a contractionary monetary policy shocks cause the yield curve to flatten because they produce a large and significant (but short-lived and transitory) positive effect on short rates, with a decreasing effect on longer maturities. As a result, the short term effect of monetary policy takes the form of a liquidity effect. The IRs and variance decomposition suggest that the monetary policy shock resembles the "slope" factor identified in the finance literature.

With regard to term premia: monetary shocks could affect longer rates either through their effect on expected future short rates or by affecting term premia. According to Evans and Marshall, much of the response of longer-term rates can be explained by the expectation hypothesis rather than a positive response of term premia.

Literature Review

Gaussian Affine Term Structure Models

- A related asset-pricing literature tried to estimate VAR systems of yields under the null of the Expectations hypothesis (EH): Bekaert and Hodrick 2001. The term structure dynamics in our paper is consistent with deviations from the EHs.

Bekaert and Hodrick, 2001

- *"Expectations Hypotheses Tests"*
- *Journal of Finance, volume 56, issue 4, pgs. 1357-1394.*
- In this paper, Bekaert and Hodrick investigate the expectations hypothesis of the term structure of interest rates and exchange rates using VARs. The expectations hypothesis of the term structure of interest rates (EH-TS) states that information in current interest rates provides the conditional expectations of future changes in short-term interest rates. Most modern asset pricing theories imply that either expected future interest rates are related to current interest rates or to the addition of risk premia. EH holds when risk premia are constant with time.

There are three main potential reasons for the rejection of the EH: first of all, the EH is based on the assumption of rational expectations and unlimited arbitrage. Second, the presence of time-varying risk premia. Third, the tests themselves may lead to false rejections because of their poor properties in finite samples, which can be caused by highly persistent variables.

Bekaert and Hodrick consider the EHs in a VAR framework. They investigate: Wald tests, Lagrange multiplier (LM) and distance metric (DM) tests that require imposition of the null hypothesis in the estimation. What they find out is that: Wald tests grossly over-reject the null, LM tests slightly under-rejects the null and DM test over-reject.

- Fama and Bliss (1987) and Campbell and Shiller (1991), have presented strong evidence that risk premia are not constant through time. Hence, this was implying that the prices of ZCB incorporated time-varying risk premia.

Literature Review

Gaussian Affine Term Structure Models

- With the vast collection of affine models developed, Qiang Dai and Kenneth J. Singleton propose a characterization of these canonical affine models that would dominate the literature.

Dai and Singleton, 2000

- *"Specification Analysis of Affine Term Structure Models"*
- *The Journal of Finance, volume 55, issue 5, pgs. 1943-1978.*
- The authors of this paper explore the structural and goodness-of-fit differences among affine term structure models (ATSMs) in which the drifts and volatility coefficients of the state-variable processes are affine functions (Duffie and Kan, 1996). Dai and Singleton want to know what the most flexible specifications of ATSMs were existent at the time, what restrictions these specifications have in place, how those restrictions limit the yield curve dynamics and if they are over-identifying. The focal point was to determine whether these least restrictive specifications were sufficiently flexible to capture the joint dynamics of historical movements of short and long term bond yields.
- Two approaches have been used in the term structure literature to develop affine models: one assumes that the instantaneous short rate is a linear combination of an unobserved state vector which follows an affine diffusion model. According with this approach, it is convenient to decompose yield curve movements into changes in "level", "slope" and "curvature". The second approach defines the instantaneous short rate in terms of its own lag and other state variables. Whether the restriction imposed by the two approaches are *normalizations* or *over-identifying* restrictions depends on the information about the joint conditional distribution of bond yields used to identify the term structure model.

Literature Review

Gaussian Affine Term Structure Models

Dai and Singleton, 2000

- First of all, the authors point out that models based directly on the representation of the short rate are equivalent to models that define the short rate as a linear combination of unobserved state variables. Both approaches impose strong restrictions with regard to conditional correlations and variances of the state variables. This result makes easier the comparison and interpretation of the affine term structure models in the literature.
- Dai and Singleton want to figure out whether the over-identification restrictions imposed can be relaxed.
- By analyzing a three-factor affine model of the instantaneous short rate, they provide evidence that the over-identifying restrictions are strongly rejected by the data and the relaxation of the restrictions on the conditional second moments of the state variables is important for the explanation of movements in the short-end and long-end of the yield curve. Moreover, they point out that the drift of the instantaneous short rate is more complicated than simply the short rate mean reverting to a stochastic long-run mean.
- They show that many of these restrictions are not necessary to identify the term structure but needed in order to assure existence of solutions to the stochastic differential equations describing the state variables.
- Dai and Singleton shows that there is a trade-off between flexibility in modeling the conditional correlations and volatilities of the risk factors.

Literature Review

Gaussian Affine Term Structure Models

- Advancement in multi-dimensional models brings a shift in the research from fitting issues to forecasting and risk premia.
- Gregory R. Duffee defends, in 2002, that the previous class of ATSM, by Vasicek (1997) and Cox, Ingersoll and Ross (1985), which were extensively studied to the date, deemed as “completely affine” models, actually failed at forecasting.

Duffee, 2002

- *“Term Premia and Interest Rate Forecasts in Affine Models”.*
- *The Journal of Finance, volume 57, issue 1, pgs. 405-443.*
- In this paper, Duffee explained that previous affine term structure models were extremely poor at forecasting and defended that better forecasts were obtained by assuming that yields follow a random walk (RW). He unveiled the dilemma that the compensation for risk was a multiple of the variance of risk. This meant that there was an assumption dictating that compensation for risk could not vary independently from interest rate volatility. Hence, this paper proposed a new affine model that continued to retain the previous models manageability but allowed for an independent compensation for risk.

Duffee combines an ATSM with linear dynamics in the underlying state vector of the model to build an “essentially affine model”.

He concludes that only gaussian models in this class of “essentially affine” are flexible enough to produce trustworthy yield forecasts. Gaussian models are very tractable but can generate complicated yield dynamics.

In truth these models have fixed conditional variances, however, that does not pose a concern for this literature because its focus is on forecasting and not on producing conditional second moments

Literature Review

Gaussian Affine Term Structure Models

- From its results Duffee concludes that, for forecasting purposes, the class of completely affine models do worse than RW and states that purely Gaussian **essentially affine models do better that an RW.** However, in order to do this extension to essentially affine models, it is necessary to sacrifice flexibility in fitting interest rate volatility. Hence the difficulty of building a model that fares better at forecasting than a RW.
- Even though Duffee states that it is difficult for an affine model to do better than a RW, he did not consider a model with linear risk premia. In Ang and Piazzesi, 2003, they state that traditional affine models with linear risk premia show better results, nevertheless, they do not show results to corroborate this statement.

Duffee, 2002

- Basically Duffee took the class of affine models from Duffie and Kan, 1996, and combined it with linear dynamics of the state vector to build the classes of *completely affine* and *essentially affine.* These models differ in the way that they price the risk vector Λ_t.
- *Completely Affine:* $\Lambda_t = S_t\lambda_1$, with S_t being a diagonal matrix with the volatilities of the state factors represented by $S_{t(ii)} = \sqrt{\alpha_i + \beta_i{}'X_t}$. This class has the limitation that the excess returns are driven solely on yield volatility.
- *Essentially Affine:* the elements of the diagonal of S_t will be built to be bound away from zero, like so: $S^{-}_{t(ii)} = \begin{cases} \sqrt{\alpha_i + \beta_i{}'X_t} \; if \inf(\alpha_i + \beta_i' X_t) > 0 \\ 0 \; if \; else \end{cases}$ and $\Lambda_t = S_t\lambda_1 + S_t^{-}\lambda_2 X_t$. The completely affine can be seen as a class of essentially affine.

Literature Review

Gaussian Affine Term Structure Models

- Basically, an ATSM starts with the assumption of no arbitrage opportunities, which implies the existence of a stochastic process, a pricing kernel. Moreover, there are three important components when building a Gaussian ATSM:
    1. an equation for the vector of factors that are relevant for pricing bonds (seldom latent factors).
    2. an equation that describes the short-rate dynamics.
    3. a risk premium relationship associated to the shocks of the vector of factors.
- Chen and Scott, 1993, jointly estimated unobservable factors and coefficients by doing a maximum likelihood estimation.

ATSM

- Summing up, an affine term structure model can be characterized as follows:

$$y_{t,t+n} = -\frac{1}{n}(A_n + B_n' X_t) + e_{t,t+n} \text{ with } e_t \sim i.i.d.N(0, \sigma^2 I)$$

$$X_t = \mu + \phi X_{t-1} + v_t \text{ with } v_t \sim i.i.d.(0, \Omega)$$

With $b_{n+1} = \frac{1}{n+1}[\sum_{i=0}^{n} (\phi' - \lambda_1' \Omega)^i] b_1$ and $a_{n+1} = a_1 - \frac{1}{n+1} \sum_{i=0}^{n} B^i$, where $B^i = B_i'(\mu - \Omega\lambda_0) + \frac{1}{2} B_i' \Omega B_i$

Literature Review

ATSM presented by Ang and Piazzesi, 2003, JME

- As a consequence of these flexible models the focal point of analysis shifts once more, this time to econometric estimation and identification issues. Until today, lower bounds of the yields still pose a challenge.

Ang and Piazzesi (AP), 2003

- *"A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables".*
- *Journal of Monetary Economics, volume 50, issue 4, pgs. 745-787, 2003.*
- In this paper, Ang and Piazzesi propose including macroeconomic factors alongside latent factors in an ATSM, with the intent of proving that this specification is useful in predicting yields and that it out-preforms forecasting in a model that does not include macroeconomic factors. They examine the influences of inflation and real activity on the yield curve in an asset-pricing framework. As a result, bond yields are determined by three correlated unobservable factors together with inflation and real activity measures. They propose their model under a no-arbitrage assumption implying a pricing kernel, which is a stochastic discount factor that prices all bonds in the economy and is driven by shocks to both observable and unobservable factors.
- Moreover they do an analysis of the impact that each factor has on the yield curve using variances decompositions and impulse response functions. However, they have the constraint that the macro factors and latent factors (unobservable factors) follow independent VAR processes, which could be seen as a limitation.

Literature Review

ATSM presented by Ang and Piazzesi, 2003, JME

Ang and Piazzesi (AP), 2003

- Their term structure dynamics are given by a **Gaussian term structure model with time-varying risk premia**, consistent with deviations from the EHs, which will depend on the macro factors and latent factors under this no-arbitrage restriction. It is also assumed that the product and inflation factors are independent from the policy interest rates, the vector of observable factors is orthogonal to the vector of unobservable factors. As a result, their model can be seen as a special case of the affine class of these models in discrete time proposed to Duffie and Kan in 1996.
- AP methodology implies several advantages over existing empirical VAR approaches: first of all, in response to a macro shock, their model allows to define the behavior of the entire yield curve rather than just the yields included in the VAR. Second, by using AP model, it is possible to make a direct comparison between observable and unobservable factors in order to estimate the proportion of term structure movements attributable to macro variables and other latent factors. Finally, AP estimate a VAR subject to nonlinear no-arbitrage restrictions which means that their approach maintains the tractability of the VAR model.

- In sum, they reach the conclusion that a model including macroeconomic factors delivers better forecasts of yields, when compared to a "yields only" model and also that macroeconomic factors explain up to 85% of bond yield's variation, mostly in the short and middle term of the curve. Nevertheless, their conclusions do not reject the significance of the unobservable factors, especially in the long term part of the yield curve.

Literature Review

ATSM presented by Ang and Piazzesi, 2003, JME

- For the bond yields the sample period is from June 1952 to December 2000. For the macro variables the data range is from January 1952 to December 2000.

Data

Macro Variables

- Two principal components were created:
  1. "Real Activity": HELP, UE, EMPLOY and IP.
  2. "Inflation": CPI, CCI and PCOM.

➢ All series are normalized and the principal components inherit this trait.

➢ All growth rates are calculated as the log difference between this month's data and last year's month data, t and t-12.

Yields

- ZCB yields from the Fama CRSP files
- With maturities of 1m, 3m, 12m, 36m and 60m
- All bond yields are continuously compounded

➢ Yields of maturities 1m, 12m and 60m are measured without error, thus resulting in the three latent factors that the paper refers to. The 3m and 36m yields are measured with error, the main reason is that they have a 99% correlation with the 12m and 60m respectively.

Literature Review

ATSM presented by Ang and Piazzesi, 2003, JME

Methodology

- The vector that has the observable and unobservable factors follows a VAR(p) process.
- For the short rate dynamics, it is assumed that interest rates are an affine function of the state variables (the matrix that has the observable and unobservable contemporaneous variables and their lags - X), which can be seen as a different version of the conventional Taylor rule.
- They present two set ups for the model:
  1. Macro model - No Lags of the short-term dynamics.
  2. Macro Lag model - Short-term dynamics has lags of the macro variables.

Pricing Kernel

- With the assumption of no arbitrage there is the existence of a pricing kernel, which prices the assets in the economy according to a martingale measure.

Latent Factors

- Three latent factors are used, being consistent with the previously existing literature, representing “Level”, “Slope” and “Curvature”. For comparison purposes, they run a yields only model with strictly latent factors.

Literature Review

ATSM presented by Ang and Piazzesi, 2003, JME

Estimation

- Maximum Likelihood Estimation (MLE):
  - They had to do the estimation in steps due to the high computation capacity need:
    1. Estimate the VAR for "inflation" and "real activity" (observable variables).
    2. Estimate the short-term dynamics using an OLS, in this step the latent variables are seen as external shocks.
    3. Holding the first two estimates fixed, they run a maximum likelihood function for the yields and the variables.

- AP propose that the best fitting model would be one with a VAR(12) to capture the macroeconomic factor dynamics and a VAR(1) for the latent factors, using a normal Taylor rule for the short-rate dynamics (their called *Macro Model*).

<u>We will be discussing the results from this part further ahead in the methodology part, where the replication of Ang and Piazzesi is done. In the replication we get the same results as they did.</u>

Literature Review

Gaussian Affine Term Structure Models

Forecast Results from AP, 2003 – across models

Forecast comparisons

Yield (mths)	RW	Unconstrained VARs		VARs with cross-equation restrictions		
		VAR Yields Only	VAR with Macro	Yields Only	Macro model	Macro lag model
RMSE criteria						
1	0.3160	0.3905	0.3990	0.3012	**0.2889**	0.3906
3	**0.1523**	0.2495	0.2540	0.1860	0.2167	0.2876
12	0.1991	0.2776	0.2722	0.1914	**0.1851**	0.2274
36	0.2493	0.3730	0.3644	0.2489	**0.2092**	0.2665
60	0.2546	0.3793	0.3725	0.2497	**0.2333**	0.2530
MAD criteria						
1	0.2252	0.3076	0.3242	0.2155	**0.2039**	0.2981
3	**0.1159**	0.1987	0.2056	0.1442	0.1693	0.2344
12	0.1639	0.2176	0.2204	0.1616	**0.1559**	0.1870
36	0.1997	0.2991	0.2924	0.1974	**0.1604**	0.2111
60	0.2054	0.2957	0.2930	0.2017	**0.1883**	0.2064

- The table on the left shows results from an analysis of forecasts for the last 60 months of data, across all three models done by AP.
- The first one includes only the unobservable factors, the second one is the simple Taylor rule and the third one is the Taylor rule with the lags.
- According to the measures of RMSE and MAD, the *Macro model* is the best at forecasting OOS (out of sample).
- The *Macro lag model* could have suffered some overfitting due to the amount of irrelevant parameters, which could justify its poor performance OOS.

Literature Review

ATSM presented by Ang and Piazzesi, 2003, JME

Findings

- All yields are impacted positively by movements of "inflation", however, the impact of the shocks decrease with maturity. Defending that inflation impacts yields most in a short horizon.
- All yields are positively affected by movements in "real activity".
- Models that use macro factors have greater forecasting power than models that use latent factors exclusively.
- The three latent factors present in the AP model correspond to the conventional three latent factor models of the term structure. In their conclusions, the ''level'' factor survived almost intact when macro factors are incorporated, however, a significant fraction of the ''level'' and ''slope'' factors are attributed to macro factors, especially to inflation.
- Macro factors explain the short and middle part of the yield curve best.

- This was, at the time, a pioneer research in the field, bringing both observable macroeconomic factors and conventional latent factors together as a way to build a more robust factor model of the term structure. This extended previous studies by utilizing **t**ime-varying risk premia, together with no-arbitrage assumptions.

Literature Review

Identification and Estimation of ATSM, Hamilton and Wu, 2012, JE

- Hamilton and Wu (2012) (HW) propose an original way to identify and estimate Gaussian affine models. While previous papers emphasize on how ATSMs ought to be represented, Hamilton and Wu's focus is on how to estimate the model without bias and with certainty that the solved system is identified.

Hamilton and Wu, 2012

- *"Identification and estimation of Gaussian affine term Structure models".*
- *Journal of Econometrics, volume 168, pgs. 315-351.*
- In this paper, the authors focused on showing how previous models of the existing literature were unidentified solutions due to numerical challenges of the optimization process. Other than this previous contribution, the paper also investigates how minimum Chi-square estimation (MCSE) can produce more robust results for the model's parameters than a maximum likelihood estimation (MLE). Compared to MLE, MCSE is also stated to require easier computational methods. They also demonstrate that the maximum reached by AP was a local maximum an that in some cases MCSE can be useful to reach a global maximum with certainty.

- It's important to note that, in HW, the focus is on the popular class of ATSMs in which *N* yields are measured without error and *N* will be the number of unobservable factors included in the model. There is also another limitation, which is that the literature does not extend to non-Gaussian dynamics.

Literature Review

Identification and Estimation of ATSM, Hamilton and Wu, 2012, JE

Basic Framework for the model in discrete time

- The capture of the dynamics of F_t variables is done with a Gaussian vector autoregression: $F_{t+1} = c + \rho F_t + \Sigma u_{t+1}$.
- They produce the measure Q from a specific pricing kernel used to derive ATSMs. Under measure Q, risk-averse and risk-neutral investors value assets the same way, such that: $F_{t+1} = c^Q + \rho^Q F_t + \Sigma u^Q_{t+1}$. Being u^Q_{t+1} a vector of independent variables under measure Q.
- Their short rate dynamics is given by the following affine function of factors: $r_t = \delta_0 + \delta'_1 F_t$. (This is the model we are estimating so there is a full description of the equations for the short rate parameters further ahead)
- In the demonstration for the final steps they resort to AP (2003).
- They will use a number, N_l, of observed yields, measured without error, that correspond to the number of unobservable factors. Hence, N_e is the number of yields measured with error.

They then presented the final specification

$$\begin{bmatrix} Y_t^1 \\ (N_l \times 1) \\ Y_t^2 \\ (N_e \times 1) \end{bmatrix} = \begin{bmatrix} A_1 \\ (N_l \times 1) \\ A_2 \\ (N_e \times 1) \end{bmatrix} + \begin{bmatrix} B_1 \\ (N_l \times M) \\ B_2 \\ (N_e \times M) \end{bmatrix} F_t + \begin{bmatrix} 0 \\ (N_l \times N_e) \\ \Sigma_e \\ (N_e \times N_e) \end{bmatrix} \begin{matrix} u_t^e \\ (N_e \times 1) \end{matrix}$$

Literature Review

Identification and Estimation of ATSM, Hamilton and Wu, 2012, JE

- HW develop three models in their methodology: *Latent Factor Model* (so called "yields only model"), *Macro finance model with single lag* (MF1) and the *Macro finance model with 12 lags* (MF12). The model we will develop further ahead is the third one, for that reason we focus solely on the explanation of that one.

Macro finance model with 12 lags (MF12)

- This specification includes two observable macro variables: inflation and output gap, plus the three already mentioned latent factors.
- Similarly to the MF1, this model also includes the same observable macro factors ("inflation" and "Real Activity"), but with 12 monthly lags of the factors included. The latent factors still only have 1 lag (same as the *Macro Model* in AP):

$$f_t^m = \rho_1 f_{t-1}^m + \rho_2 f_{t-2}^m + \ldots + \rho_{12} f_{t-12}^m + \Sigma_{mm} u_t^m$$

$$f_t^l = c_l + \rho_{ll} f_{t-1}^l + \Sigma_{ll} u_t^l$$

- The short-rate dynamics is represented by:

$$r_t = \delta_0 + \delta_{1m}' f_t^m + \delta_{1l}' f_t^l$$

Literature Review

Identification and Estimation of ATSM, Hamilton and Wu, 2012, JE

Estimation

- HW utilized a minimum Chi-square estimation (MCSE). This involved a series of equations being solved, with equal number of unknown variables, so as to make sure that the system is identified.
- HW states that this is a more feasible method than preforming a MLE on all model parameters, which was the adopted method in the AP paper.

- After the base model that we mentioned in the previous slide, HW present a *reduced-form* model that they use in the initial steps of their estimation. This *reduced-form* model is a VAR(1) process that could be estimated through OLS thanks to the assumption that the latent variables are independent from the macro variables.
- The next steps were more computationally demanding and involved the Chi-square estimation.

Literature Review

Gaussian Affine Term Structure Models

MF 12 estimation results and comparison with MLE

	Global			Local1		
$\rho_{\ell\ell}$	0.9921	0	0	0.9918	0	0
	0	0.9462	0	0	0.9412	0
	0	−0.0034	0.9021	0	−0.0095	0.7712
$\delta_{1\ell}$	1.11E−04	4.27E−04	1.98E−04	1.09E−04	4.30E−04	1.92E−04
λ_ℓ	−0.0409	0	0	−0.0441	0	0
Λ_{mm}	2.8783	0.4303		−0.3430	0.1474	
	−6.1474	−0.8744		1.7675	−0.0607	
$\Lambda_{\ell\ell}$	−0.0048	0	0	−0.0045	0	0
	−0.0445	0	0.2910	−0.0474	0	0.2881
	−0.0322	0	0.3687	−0.0331	0	0.2110
χ^2	462.15			530.69		
LLF	20703			20668		
Frequency	14			84		

- In this table (extracted from HW, 2012) we can compare the results from the MLE in AP (Local 1) and the ones estimated through MCSE (Global). Their intent was to compare how MCSE fared in an over identified structure so they used all of the restrictions adopted in AP. It is evidenced that the chi-square is minimized (the lowest) and the corresponding log likelihood function (LLF) is also higher than the other case achieved through MLE. This proved that the result achieved in AP with their *Macro Model* was only a local maximum. The relevance of a correct estimation is highlighted by how the price of risk is substantially affected: signals for the price of inflation risk and the price of output risk are reversed across estimations.

Literature Review

Identification and Estimation of ATSM, Hamilton and Wu, 2012, JE

Findings

- HW managed to prove that their method is asymptotically equivalent to MLE. This approach of inferring structural parameters of the model through an unrestricted OLS for their use in the post Chi-square estimation was an innovation in the field, since Chi-square estimation had never been applied to such models. Moreover, the Chi-square had the advantage to be able to identify whether the result is a global maximum by means of calculating small-sample standard errors.
- They further managed to prove that other previous specifications of the model were unidentified and that AP found a local maximum with their MLE instead of a global one, due to unidentified regions of the parameter space.

- In light of these conclusions, we decide to focus on the estimation method applied in HW rather that AP, given the computational advantages mentioned and the possibility of ensuring a global maximum. Nevertheless, much of the data treatment and construction of the model is inspired in AP's *Macro lag model*.

Data Collection and Reasoning

Data Collection and Reasoning

Overview of Replication and Extension of the model

- Recapitulating, the objective of this project is to:
  - **replicate** the previous studies of AP and HW using the data of United States from 1952 to 2000.
  - **extend** the model to other developed countries.
- In order to do so, the first step was the collection of the macro data and yield data for each chosen country.

Data for the Replication

- In this section, the sample of data we consider is the same as the one used in AP, except for the bond yields for which we have a different starting month:
  - For the macro variables the data ranges from January 1952 to December 2000.
  - For the bond yields, which are from the Fama CRSP files, the sample period ranges from December 1952 (instead of June 1952) to December 2000.

Data for the Extension

- In this section, we extended the model to the following countries other than US:
  - United Kingdom, Germany, Japan and Canada.
- For all countries aforementioned we collected the most recent up-to-date data for both macroeconomic and yield variables. The sources for this data collection are detailed in the appendix, to collect the data we used a Bloomberg Terminal and Thomson Reuters Eikon.
- Besides these countries, we are also going to apply the model to the US, by extending the sample period to the most recent available data.

Data Collection and Reasoning

Structure and Reasoning behind Countries and Macroeconomic Factors

Choice of Countries

- For the analysis, the main goal was to compare the results between the US and some other countries also belonging to G10 group of countries. Hence, among these, we chose **Canada**, **Japan**, **Germany** and the **UK** because we thought these also showed some representative differences amongst themselves. We then end up with two representative examples of European countries.

Choice of Macro Variables

- We stuck closely to the framework used in AP. Therefore, the goal was to find good factors that could represent impacts on real activity and inflation, such that in a further step we could analyze these factors' impact on interest rate dynamics.
    - ➢ For capturing real activity dynamics we have the series: **HELP, EMPLOY, IP and UE.**
    - ➢ For capturing inflation dynamics we have the series: **CPI, PCOM and PPI.**
- Nevertheless, it is worth noticing that, across countries, some changes had to be made due to the data availability.

Choice of Yields Maturities across countries

- Our model takes in five different yields' maturities for representing the short, middle and long range of the yield curve. Across all countries we used the same maturities of: **1m, 3m, 12m, 36m and 60m**.

Data Collection and Reasoning

Implications of the Extension of the Model to other Countries

- Collecting data across different countries was difficult in terms of availability of the same series. As a consequence, for each country, we had to adjust the data set by finding the corresponding best proxies.

Macro Variables

- *CPI:* For UK, we used RPI as a proxy for CPI.
- *PCOM:* For UK, Germany and Japan we used CCI, adjusted with the exchange rate for each currency, as a proxy for PCOM. For Canada we used BCPI as a proxy for PCOM.
- *HELP:* For US, the HELP had been discontinued in Sep 2009. In order to make the series updatable, it has been combined with JOB VACANCIES, used as a proxy for HELP from September 2009 until today. For Canada, we could not a find a proxy for HELP, due to this, it was not included in the ”Real Activity” factor. For all the other countries we used JOB VACANCIES as a proxy for HELP, since they both reflect job market supply.

Yields

- As a proxy for the ZCB yields we used Interest Rate Swap (IRS) because it is a good approximation of the risk free rate the same as government ZCB.
- For shorter maturities, the 1m and 3m, however, we could not find the IRS. In these cases we used:
  - For Canada: Canada Bankers Acceptances.
  - For all the other countries: Ice Libor.
- Generally, there was a trade-off between consistent availability and length of the time series and quality in a sense of truly replicating zero coupon yields.

Appendix A - Data

Analysis and Methodology

Analysis and Methodology

Overview

- For the methodology, we thought it was reasonable to divide it in two critical sections: the replication of the model and the extension of the model.

Replication of the model

- This consisted of replicating the previous studies of AP and HW to the data of United States.
- This section's focus is mainly on illustrating our approach and how the model is built.
- The sample of data we consider is the same as the one used by AP, except for the bond yields, for which the sample period ranges from December 1952 (instead of June 1952) to December 2000

Extension of the model

- This consisted of the application of the same model used in the replication but now extending it to other countries.
- We are going to also apply the model to the US, by extending the sample period to the most recent available data.
- For all countries the code was meant to take in the most recent up-to-date data for both macroeconomic and yield variables.

- The following sections will be organized by the order of the process involved in building the model and in each section there will be a comparison analysis between the results obtained in the replication and extension parts of the model.

Analysis and Methodology

Data Treatment and Analysis – Replication of the Model

- For the replication part of the model, we stuck as much as possible to the structure used in AP, especially in the data treatment but differed in the optimization process, using the MCSE done by HW. Regarding the replication, it is expected that it will yield slightly different values from AP since we are using the most recent updated values for the data series.

Macro Variables

- Inflation measures: CPI, PPI, PCOM.
- Real activity measures: HELP, EMPLOY, IP, UE.
- The sample of data we consider is the same as the one used by AP
- This data ranged from January 1952 to December 2000.

Yields

- We use data on ZCB yields of maturities 1m, 3m, 12m, 36m and 60m from the Fama CRSP files.
- For the estimation of the model:
  - ➢ The 1m, 12m and 60 m yields are measured without error in order to represent the short, medium and long ends of the yield curve and to obtain the three latent factors.
  - ➢ The 3m and 36m yields are measured with error.
- This data ranged from December 1952 to December 2000.

Analysis and Methodology

Data Treatment and Analysis – Extension of the Model

- For each country's data set, we had to be careful with the length of the data series because there needs to be a match in the availability of months for yields and macro data. This is necessary for coding purposes.

Macro Variables

- For each country we collected the following macro data:
  - Inflation measures: CPI, PCOM, PPI
  - Real activity measures: HELP, EMPLOY, IP, UE
- There was special attention to collect the series with the most data available. Data series had to be cut according to the least data available, so that historically all series start at the same point, ending up with the same number of observations as well.
- Moreover, when dealing with updatable data, it might be that different series' newly released values are available in different days of the month. With the purpose of using the most recent values, we applied AR(1) to forecast the missing data of the updatable series, for the most recent period.

Yields

- The model always uses five different maturities for the yields: 1m, 3m, 12m, 36m and 60m, which remain constant across countries.
- Yields with maturities of 1m, 12m and 60m are observed without error. Recalling our model, this way we can obtain the three latent factors. The 3m and 36m yields are measured with error.
- The MatLab code needs the length of the yield data series to match the length of the macro data for the optimization. Consequently, we automatized the process so that it cuts the data by the lowest series' length of the macro and yields data series. Macro data is usually longer in the beginning to obtain better values for the PCA and only cut accordingly afterwards.

Analysis and Methodology

Data Treatment and Analysis – Central Moments and Autocorrelations

- We computed the growth rate of all of the inflation measures, employment and industrial production using log (X_t /X_{t-12}).
- Then, we calculated each series' central moments and autocorrelations up to 3 lags.

More model assumptions

- Distribution of yields is not a normal distribution.
- As AP did, we will assume a Gaussian yield curve in order to investigate the joint dynamics of yields and macro factors. So we are applying the Central Limit Theorem and assuming that they are normally distributed. This assumption is reasonable given the low amount of years of data available.

Analysis and Methodology

Data Treatment and Analysis – Central Moments and Autocorrelations

Summary statistics of data – Replication of the Model

- Autocorrelation of yields is **high** and **increasing** with maturity.
- Standard deviations of yields is generally **decreasing** with maturity.
- Excess kurtosis of yields is also **decreasing** with maturity.

		Central Moments			*Autocorrelations*		
	Mean	***Stdev***	***Skew***	***Kurt***	***Lag 1***	***Lag 2***	***Lag 3***
1m	5,1676	2,7311	1,0819	4,6825	0,9712	0,9428	0,9190
3m	5,5194	2,8453	1,0774	4,5939	0,9814	0,9584	0,9379
12m	5,9258	2,8306	0,8609	3,9275	0,9821	0,9602	0,9414
36m	6,2668	2,7465	0,7550	3,5501	0,9871	0,9714	0,9576
60m	6,4455	2,7059	0,6997	3,3092	0,9886	0,9757	0,9644
CPI	0,0386	0,0287	1,2748	4,3810	0,9936	0,9836	0,9710
PCOM	0,0093	0,1135	1,0111	5,8967	0,9639	0,9105	0,8535
PPI	0,0306	0,0363	1,4468	4,9371	0,9863	0,9689	0,9488
HELP	66,7483	22,0260	-0,1488	1,8730	0,9939	0,9877	0,9791
EMPLOY	0,0168	0,0153	-0,5108	3,2693	0,9373	0,8932	0,8372
IP	0,0344	0,0529	-0,5788	3,7912	0,9603	0,8876	0,7949
UE	5,7328	1,5664	0,4922	3,2462	0,9884	0,9738	0,9541

Analysis and Methodology

Data Treatment and Analysis – Central Moments and Autocorrelations

Extension of the model

- For all other countries, yields show high autocorrelation with their lags, as expected.
- Overall, the behavior is very similar across countries.
- US has the highest level of yields, although, it is lower than the replication sample results. Overall, Japan's yields are the lowest.
- For all countries, standard deviation of yields is decreasing with maturity, except for Japan's, which is increasing.

Analysis and Methodology

Data Treatment and Analysis – Central Moments and Autocorrelations

Summary statistics of data – Extension of the Model

US

	Central Moments				Autocorrelations		
	Mean	Stdev	Skew	Kurt	Lag 1	Lag 2	Lag 3
1m	3,3555	2,6690	0,3409	2,0617	0,9866	0,9706	0,9535
3m	3,4566	2,6359	0,3380	2,0707	0,9877	0,9707	0,9524
12m	3,8111	2,5248	0,3725	2,1546	0,9857	0,9655	0,9433
36m	4,2126	2,5458	0,1623	1,9781	0,9858	0,9676	0,9490
60m	4,6251	2,4210	0,1210	2,0066	0,9854	0,9672	0,9494
CPI	0,0341	0,0268	1,4837	5,4201	0,9904	0,9748	0,9571
PCOM	0,0169	0,1223	0,4803	4,6623	0,9559	0,8915	0,8225
PPI	0,0282	0,0354	1,1391	4,9579	0,9772	0,9462	0,9112
HELP	4041,090	1246,559	-0,0818	2,0381	0,9885	0,9805	0,9710
EMPLOY	0,0142	0,0158	-0,7159	4,2286	0,9499	0,9133	0,8661
IP	0,0270	0,0524	-0,6676	4,4720	0,9652	0,9024	0,8214
UE	5,8721	1,6236	0,6151	3,1063	0,9907	0,9786	0,9621

Analysis and Methodology

Data Treatment and Analysis – Central Moments and Autocorrelations

Summary statistics of data – Extension of the Model

UK

	Central Moments				Autocorrelations		
	Mean	Stdev	Skew	Kurt	Lag 1	Lag 2	Lag 3
1m	3,2391	2,5056	0,0568	1,4307	0,0566	0,0484	1,6259
3m	3,3547	2,4973	0,0576	1,4519	0,0310	0,1511	0,2495
12m	3,5011	2,4449	0,0759	1,4669	0,0495	0,0575	1,0963
36m	3,7481	2,2339	-0,0551	1,5014	0,0566	0,0484	1,6259
60m	3,9312	2,0582	-0,1387	1,6327	0,0310	0,1511	0,2495
RPI	0,0566	0,0484	1,6259	5,2693	0,9934	0,9813	0,9654
CCI	0,0310	0,1511	0,2495	3,6581	0,9377	0,8695	0,7901
PPI	0,0495	0,0575	1,0963	4,1575	0,9916	0,9763	0,9555
JOB VACANCIES	343,134	214,238	0,5771	1,8721	0,9905	0,9802	0,9717
EMPLOY	0,0053	0,0134	-0,8793	3,7166	0,9898	0,9698	0,9425
IP	0,0075	0,0403	-0,5667	5,3963	0,8703	0,7842	0,7036
UE	7,1119	2,3170	0,5156	2,0921	0,9964	0,9917	0,9856

Data Treatment and Analysis – Central Moments and Autocorrelations

Summary statistics of data – Extension of the Model

GERMANY

	Central Moments				Autocorrelations		
	Mean	Stdev	Skew	Kurt	Lag 1	Lag 2	Lag 3
1m	1,8460	1,6911	0,2495	1,7123	0,9893	0,9746	0,9574
3m	1,9680	1,6964	0,2585	1,8025	0,9902	0,9751	0,9572
12m	2,1435	1,7001	0,1953	1,8062	0,9891	0,9738	0,9552
36m	2,4493	1,7249	-0,0537	1,7294	0,9880	0,9729	0,9564
60m	2,7567	1,7097	-0,2025	1,7797	0,9880	0,9736	0,9589
CPI	0,0154	0,0082	0,3173	3,1806	0,9350	0,8728	0,8131
PCOM	0,0281	0,1525	-0,3854	3,0330	0,9420	0,8688	0,7756
PPI	0,0110	0,0266	-0,2325	3,6843	0,9720	0,9236	0,8553
JOB VACANCIES	387,541	125,994	0,7933	3,3039	0,9820	0,9618	0,9403
EMPLOY	0,0057	0,0091	-0,2475	2,7101	0,9741	0,9360	0,8914
IP	0,0136	0,0613	-1,7468	8,4147	0,9310	0,8789	0,8059
UE	7,4513	1,3550	0,1221	1,8467	0,9919	0,9815	0,9692

Analysis and Methodology

Data Treatment and Analysis – Central Moments and Autocorrelations

Summary statistics of data – Extension of the Model

JAPAN

	Central Moments				Autocorrelations		
	Mean	Stdev	Skew	Kurt	Lag 1	Lag 2	Lag 3
1m	0,2382	0,2571	1,0525	2,9269	0,9212	0,8732	0,8509
3m	0,2853	0,2756	0,9849	2,8517	0,9692	0,9339	0,9030
12m	0,3679	0,3039	0,7480	2,5392	0,9674	0,9417	0,9093
36m	0,5918	0,4776	0,9402	3,4983	0,9622	0,9237	0,8852
60m	0,8645	0,6411	0,9123	3,4775	0,9665	0,9306	0,8988
CPI	0,0239	0,0416	2,5254	10,4832	0,9900	0,9771	0,9621
CCI	0,0307	0,1512	0,2333	3,6216	0,9398	0,8712	0,7897
PPI	0,0129	0,0530	2,7140	12,4958	0,9878	0,9607	0,9234
JOB VACANCIES	567,02	174,45	0,4301	2,1692	0,9912	0,9818	0,9723
EMPLOY	0,0053	0,0095	-0,2316	2,8731	0,9103	0,8293	0,7988
IP	0,0158	0,0765	-1,5788	9,3267	0,9424	0,8751	0,7866
UE	3,1987	1,1633	0,3112	1,9394	0,9938	0,9885	0,9841

Analysis and Methodology

Data Treatment and Analysis – Central Moments and Autocorrelations

Summary statistics of data – Extension of the Model

CANADA

	Central Moments				*Autocorrelations*		
	Mean	*Stdev*	*Skew*	*Kurt*	*Lag 1*	*Lag 2*	*Lag 3*
1m	2,0656	1,2986	0,6768	2,2046	0,9788	0,9506	0,9171
3m	2,1109	1,2923	0,6603	2,1710	0,9798	0,9521	0,9180
12m	2,1893	1,2981	0,5680	1,9686	0,9777	0,9483	0,9127
36m	2,6348	1,3290	0,2975	1,6536	0,9762	0,9461	0,9127
60m	2,9934	1,3596	0,1479	1,6467	0,9764	0,9473	0,9175
CPI	0,0350	0,0289	1,3089	3,8893	0,9884	0,9755	0,9621
BCPI	0,0190	0,1772	-0,9435	4,9265	0,9230	0,8442	0,7492
PPI	0,0308	0,0392	0,7303	3,6360	0,9721	0,9292	0,8811
EMPLOY	0,0155	0,0159	-1,2153	5,3985	0,9784	0,9450	0,8961
IP	0,0214	0,0528	-0,2251	4,0667	0,9413	0,8833	0,8133
UE	8,3164	1,6439	0,8552	2,8350	0,9907	0,9795	0,9669

Analysis and Methodology

Principal Component Analysis

- First, we have separated the data in two groups, through a principal component method, making it possible to reduce the dimensionality of the system to two final macro factors: "Inflation" and "Real Activity"

PCA

- The first step involved normalizing (Z score) the series for all macroeconomic data, so that every series would have zero mean and unit variance. In order to do that, we have performed the normalization for each macro factor considering the complete time series. Only after the normalization we cut the series again according with the shortest data series available.
- We extract the principal component of each group of variables (inflation and real activity measures) separately. The objective is to define loadings of the first principal components for the inflation group and real activity group.
- According with AP, the first principal components explains more than 70% (50%) of the variance of the nominal variables for the inflation group (real activity group). Henceforth, we used only the factor loadings of the first principal component to build the new factors for both "Inflation" and "Real Activity".
- Then, we multiply the normalized series by -1. The main reason is that, negative shocks to the new factor: "Inflation", represent positive shocks to inflation, while positive shocks to the new factor: "Real Activity", represent positive shocks to economic growth. This way the loadings of the first principal components are negative for the inflation group. With regard to the real activity group they are also negative except for the UE component.
- Finally, we normalized the series again in order to find the two macro factors: "Inflation" and "Real Activity".

Analysis and Methodology

Principal Component Analysis

PCA – Replication of the Model

- The results of the PCA analysis for the replication is aligned with AP's findings.
- **More than 70%** of the variance of CPI, PCOM and PPI is explained by just the first principal components of the inflation group.
- **More than 50%** of the variance of HELP, EMPLOY, IP and UE is explained by just the first principal components of the real activity group.

	Principal Components: Inflation		
	1st	**2nd**	**3rd**
CPI	-0,6298	-0,3788	0,6782
PCOM	-0,4167	0,9016	0,1166
PPI	-0,6556	-0,2091	-0,7256
% variance explained	72,2128	97,7620	100

	Principal components: Real Activity			
	1st	**2nd**	**3rd**	**4th**
HELP	-0,3204	-0,7430	-0,5178	0,2776
UE	0,6287	-0,1707	0,2500	0,7163
EMPLOY	-0,6028	-0,1912	0,4403	-0,6373
IP	-0,3723	0,6182	0,6895	0,0612
% variance explained	52,6055	80,0930	95,2072	100

Analysis and Methodology

Principal Component Analysis

Extension of the model

- For all other cases in the extension, the first principal component always showed an **explanatory power above 50%.**
- Except for the first principal component of “Real Activity” for **UK (44%)** and the same for **Japan (49%).** However, since the explanatory power was not critically low we still decide to use only the first principle components for these two cases as well.
- This meant that the first principal component was enough in explaining the variability in the new “Inflation” and ”Real Activity” factors for all other countries, similarly to the replication sample.

Analysis and Methodology

Principal Component Analysis

PCA – Extension of the Model

US

	Principal Components: Inflation		
	1st	**2nd**	**3rd**
CPI	-0,6252	-0,4137	0,6618
PCOM	-0,4081	0,8961	0,1745
PPI	-0,6652	-0,1610	-0,7291
% variance explained	70,2452	97,0743	100

	Principal components: Real Activity			
	1st	**2nd**	**3rd**	**4th**
JOB VACANCIES	-0,3578	-0,8198	-0,3305	0,3010
UE	0,6252	-0,0944	0,2402	0,7365
EMPLOY	-0,5644	-0,2700	0,4924	-0,6051
IP	-0,4031	0,4960	0,7685	0,0279
% variance explained	54,4913	77,8308	95,4346	100

Analysis and Methodology

Principal Component Analysis

PCA – Extension of the Model

UK

	Principal Components: Inflation		
	1st	**2nd**	**3rd**
RPI	-0,6716	-0,2574	0,6947
CCI	-0,2766	0,9570	0,0872
PPI	-0,6873	-0,1336	-0,7140
% variance explained	66,2661	97,1014	100

	Principal components: Real Activity			
	1st	**2nd**	**3rd**	**4th**
JOB VACANCIES	-0,6268	-0,2501	-0,3399	0,6550
UE	0,5155	-0,4845	0,4523	0,5430
EMPLOY	-0,0951	-0,7861	0,4310	-0,4327
IP	-0,5765	0,2911	0,7029	0,2980
% variance explained	43,7514	76,4702	90,2831	100

Analysis and Methodology

Principal Component Analysis

PCA – Extension of the Model

GERMANY

	Principal Components: Inflation		
	1st	**2nd**	**3rd**
CPI	-0,5641	-0,6805	0,4677
CCI	-0,5566	0,7317	0,3934
PPI	-0,6099	-0,0384	-0,7915
% variance explained	63,9409	84,8136	100

	Principal components: Real Activity			
	1st	**2nd**	**3rd**	**4th**
JOB VACANCIES	-0,6317	-0,1468	-0,1089	0,7533
UE	0,5280	-0,3266	0,7350	0,2728
EMPLOY	-0,2109	-0,8048	0,5459	-0,0990
IP	-0,5270	0,4734	0,3873	0,5902
% variance explained	54,5769	83,9675	95,6890	100

Analysis and Methodology

Principal Component Analysis

PCA – Extension of the Model

JAPAN

	Principal Components: Inflation		
	1st	**2nd**	**3rd**
CPI	-0,6153	-0,4556	0,6434
CCI	-0,4177	0,8805	0,2241
PPI	-0,6686	-0,1309	-0,7320
% variance explained	68,1784	95,5409	100

	Principal components: Real Activity			
	1st	**2nd**	**3rd**	**4th**
JOB VACANCIES	-0,2581	-0,7773	-0,4534	0,3515
UE	0,6097	-0,2714	0,3789	0,6411
EMPLOY	-0,4328	-0,4888	0,7346	-0,1844
IP	-0,6118	0,2884	0,3334	0,6568
% variance explained	49,0135	78,4042	94,3431	100

Analysis and Methodology

Principal Component Analysis

PCA – Extension of the Model

CANADA

	Principal Components: Inflation		
	1st	**2nd**	**3rd**
CPI	-0,6034	-0,4938	0,6262
BCPI	-0,4396	0,8611	0,2554
PPI	-0,6653	-0,1212	-0,7366
% variance explained	62,9761	90,6970	100

	Principal components: Real Activity		
	1st	**2nd**	**3rd**
UE	0,7202	-0,0498	0,6920
EMPLOY	-0,6229	-0,4855	0,6134
IP	-0,3054	0,8728	0,3807
% variance explained	55,8555	91,0739	100

Analysis and Methodology

Correlation Analysis – Selected Correlation

- Once we had defined the “Inflation” and “Real Activity” factors, we calculated the following unconditional correlations:

Selected Correlation

- We calculated the correlation between:
  - the “Inflation” and “Real Activity” factors and their original time series: CPI, PCOM and PPI for “Inflation” and HELP, UE, EMPLOY and IP for “Real Activity”.
  - the “Inflation” and “Real Activity” factors and the 1m, 12m and 60m bond yields.
  - the “Inflation” and “Real Activity”.

- The unconditional correlation analysis gives us some intuition about the relationship of the two macro factors and the yield curve.

Analysis and Methodology

Correlation Analysis – Selected Correlation

Selected Correlation – Replication of the Model

- "Inflation" is mostly correlated with CPI and PPI. "Real Activity" is mostly correlated with EMPLOY and IP.
- "Inflation" is highly correlated with yields. This correlation is higher for short yields and smaller for long yields. "Real activity" is weakly correlated with yields of any maturity. However, this might not be representative for all the macro variables of real activity. Indeed, HELP is highly correlated with the 1m yields but "Real Activity" loads mostly on EMPLOY and IP, that way AP may underestimate the impact of real activity on the yield curve.
- The unconditional correlation between the two macro factors is weak.

	Inflation	CPI	PCOM	PPI
Inflation	1	0,9269	0,6133	0,9649
CPI	**0,9269**	1	0,3120	0,9221
PCOM	**0,6133**	0,3120	1	0,4416
PPI	**0,9649**	0,9221	0,4416	1

	Real Activity	HELP	EMPLOY	IP	UE
Real Activity	1	0,4648	0,9121	0,8745	-0,5400
HELP	**0,4648**	1	0,4470	0,1463	0,0415
EMPLOY	**0,9121**	0,4470	1	0,7407	-0,2807
IP	**0,8745**	0,1463	0,7407	1	-0,4111
UE	**-0,5400**	0,0415	-0,2807	-0,4111	1

	Inflation	Real Activity	1m	12m	60m
Inflation	1	-0,0014	0,6606	0,6410	0,5508
Real Activity	**-0,0014**	1	0,0575	0,0452	-0,0370
1m	**0,6606**	0,0575	1	0,9768	0,9179
12m	**0,6410**	0,0452	0,9768	1	0,9632
60m	**0,5508**	-0,0370	0,9179	0,9632	1

Analysis and Methodology

Correlation Analysis – Selected Correlation

Extension of the model

- For all other cases in the extension, the “Inflation” is **highly** correlated with CPI and PPI**.** Except for Germany. “Real Activity” is **mostly correlated** with EMPLOY and IP for US and Canada. For Japan and UK is **positive** correlated with, respectively, UE and HELP, and **negative** correlated with, respectively, EMPLOY and UE. For Germany it is **positive** correlated with HELP and EMPLOY, and **negative** correlated with UE.
- “Inflation” is **moderately correlated** with yields for US and Canada. However, this correlation is higher for short yields and smaller for long yields in all countries. “Real activity” is **moderately** and **positive** correlated with yields of any maturity for US and Canada. For the other countries is **weekly** and **negatively** correlated with all yields.

Analysis and Methodology

Correlation Analysis – Selected Correlation

Selected Correlation – Extension of the Model

US	Inflation	CPI	PCOM	PPI	
Inflation	1	0,9076	0,5925	0,9657	
CPI	0,9076	1	0,2495	0,8877	
PCOM	0,5925	0,2495	1	0,4449	
PPI	0,9657	0,8877	0,4449	1	

	Real Activity	HELP	EMPLOY	IP	UE
Real Activity	1	0,5283	0,9230	0,8333	-0,5952
HELP	0,5283	1	0,4635	0,1522	-0,1152
EMPLOY	0,9230	0,4635	1	0,7472	-0,3719
IP	0,8333	0,1522	0,7472	1	-0,3577
UE	-0,5952	-0,1152	-0,3719	-0,3577	1

	Inflation	Real Activity	1m	12m	60m
Inflation	1	0,2035	0,3070	0,2808	0,2588
Real Activity	0,2035	1	0,5315	0,5059	0,3702
1m	0,3070	0,5315	1	0,9887	0,9399
12m	0,2808	0,5059	0,9887	1	0,9528
60m	0,2588	0,3702	0,9399	0,9528	1

Analysis and Methodology

Correlation Analysis – Selected Correlation

Selected Correlation – Extension of the Model

UK	Inflation	CPI	PCOM	PPI	
Inflation	1	0,9470	0,3900	0,9691	
CPI	0,9470	1	0,1467	0,9064	
PCOM	0,3900	0,1467	1	0,2542	
PPI	0,9691	0,9064	0,2542	1	

	Real Activity	HELP	EMPLOY	IP	UE
Real Activity	1	0,8292	0,6820	0,1258	-0,7626
HELP	0,8292	1	0,3536	-0,1237	-0,5197
EMPLOY	0,6820	0,3536	1	0,3853	-0,2227
IP	0,1258	-0,1237	0,3853	1	0,0863
UE	-0,7626	-0,5197	-0,2227	0,0863	1

	Inflation	Real Activity	1m	12m	60m
Inflation	1	0,0833	0,0986	0,0886	0,0314
Real Activity	0,0833	1	0,0297	-0,0025	-0,1213
1m	0,0986	0,0297	1	0,9908	0,9473
12m	0,0886	-0,0025	0,9908	1	0,9681
60m	0,0314	-0,1213	0,9473	0,9681	1

Analysis and Methodology

Correlation Analysis – Selected Correlation

Selected Correlation – Extension of the Model

GERMANY	Inflation	CPI	PCOM	PPI
Inflation	1	0,7813	0,7708	0,8448
CPI	0,7813	1	0,3743	0,5077
PCOM	0,7708	0,3743	1	0,4917
PPI	0,8448	0,5077	0,4917	1

	Real Activity	HELP	EMPLOY	IP	UE
Real Activity	1	0,9334	0,7801	0,3116	-0,7786
HELP	0,9334	1	0,5988	0,1670	-0,7516
EMPLOY	0,7801	0,5988	1	0,3687	-0,3199
IP	0,3116	0,1670	0,3687	1	0,0960
UE	-0,7786	-0,7516	-0,3199	0,0960	1

	Inflation	Real Activity	1m	12m	60m
Inflation	1	-0,0203	0,4816	0,4849	0,3977
Real Activity	-0,0203	1	-0,3438	-0,3386	-0,4857
1m	0,4816	-0,3438	1	0,9851	0,9381
12m	0,4849	-0,3386	0,9851	1	0,9638
60m	0,3977	-0,4857	0,9381	0,9638	1

Analysis and Methodology

Correlation Analysis – Selected Correlation

Selected Correlation – Extension of the Model

JAPAN	Inflation	CPI	PCOM	PPI
Inflation	1	0,8799	0,5974	0,9562
CPI	0,8799	1	0,2156	0,8273
PCOM	0,5974	0,2156	1	0,4546
PPI	0,9562	0,8273	0,4546	1

	Real Activity	HELP	EMPLOY	IP	UE
Real Activity	1	0,3615	-0,8537	-0,6060	0,8566
HELP	0,3615	1	-0,0020	0,0300	0,4245
EMPLOY	-0,8537	-0,0020	1	0,4692	-0,6246
IP	-0,6060	0,0300	0,4692	1	-0,2246
UE	0,8566	0,4245	-0,6246	-0,2246	1

	Inflation	Real Activity	1m	12m	60m
Inflation	1	-0,4434	0,1962	0,2066	0,1344
Real Activity	-0,4434	1	-0,1707	-0,0914	-0,2024
1m	0,1962	-0,1707	1	0,9056	0,7366
12m	0,2066	-0,0914	0,9056	1	0,7747
60m	0,1344	-0,2024	0,7366	0,7747	1

Analysis and Methodology

Correlation Analysis – Selected Correlation

Selected Correlation – Extension of the Model

CANADA

	Inflation	CPI	PCOM	PPI
Inflation	1	0,8293	0,6043	0,9145
CPI	0,8293	1	0,1922	0,6795
PCOM	0,6043	0,1922	1	0,4133
PPI	0,9145	0,6795	0,4133	1

	Real Activity	EMPLOY	IP	UE
Real Activity	1	0,9323	0,8063	-0,3954
EMPLOY	0,9323	1	0,6125	-0,3440
IP	0,8063	0,6125	1	0,0664
UE	-0,3954	-0,3440	0,0664	1

	Inflation	Real Activity	1m	12m	60m
Inflation	1	0,6469	0,3440	0,3537	0,2227
Real Activity	0,6469	1	0,3399	0,3404	0,1207
1m	0,3440	0,3399	1	0,9744	0,8366
12m	0,3537	0,3404	0,9744	1	0,8951
60m	0,2227	0,1207	0,8366	0,8951	1

Analysis and Methodology

Correlation Analysis – Conditional Correlation

- We first performed a vector auto-regression model with 12 lags on the observable factors, VAR(12).
- Then we calculated the IRs from the VAR(12) on the observable factors.

Conditional Correlation

- **VAR(12)** for the two macro factors ("Inflation" and "Real Activity") with 12 lags:

$$f_t^o = \left(f_t^{o,1} f_t^{o,2}\right)'$$
$$f_t^o = \rho_1 f_{t-1}^o + \cdots + \rho_{12} f_{t-12}^o + \Omega u_t^o$$

- Where ρ_1 to ρ_{12} and Ω are 2×2 matrices with $u_t^o \sim$ IID N(0,I)
- Next step was the calculation of Impulse Responses (IRs) from the VAR(12) on macro factors:
  - ➢ Response of "Inflation" and "Real Activity" to shocks on "Inflation"
  - ➢ Response of "Inflation" and "Real Activity" to shocks on "Real Activity"

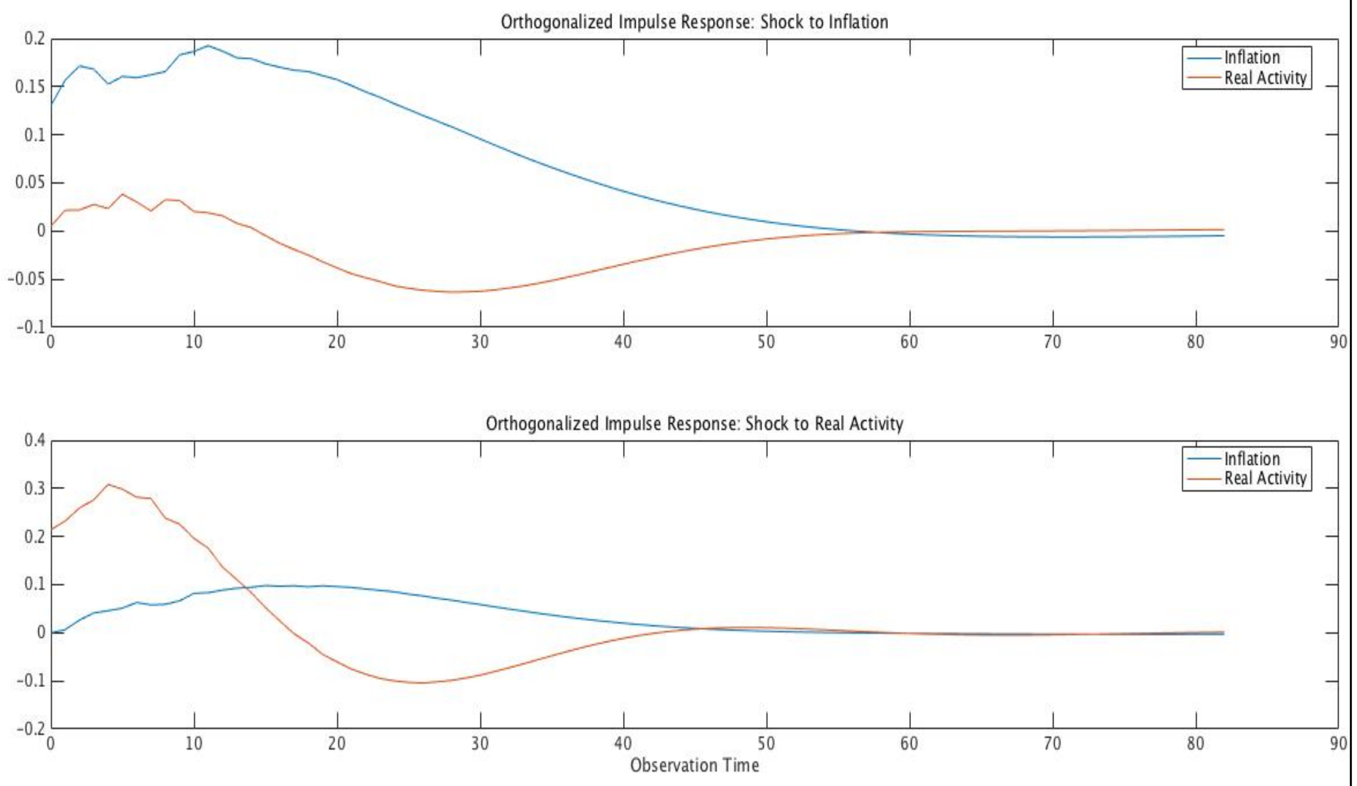
- The conditional correlation provides some more information about the relationship between the two macro factors.

Analysis and Methodology

Correlation Analysis – Conditional Correlation

IRs – Replication of the Model

- When calculating the Impulse Responses (IRs) from the VAR(12) on macro factors, we got the same result with regard to conditional correlation:
    - ➢ Response of inflation to shocks on real activity: positive and hump-shaped
    - ➢ Response of real activity to shocks on inflation: at the beginning it is weakly positive, then it becomes slightly negative before dying out



Observation Time is in months

Analysis and Methodology

Correlation Analysis – Conditional Correlation

Extension of the model

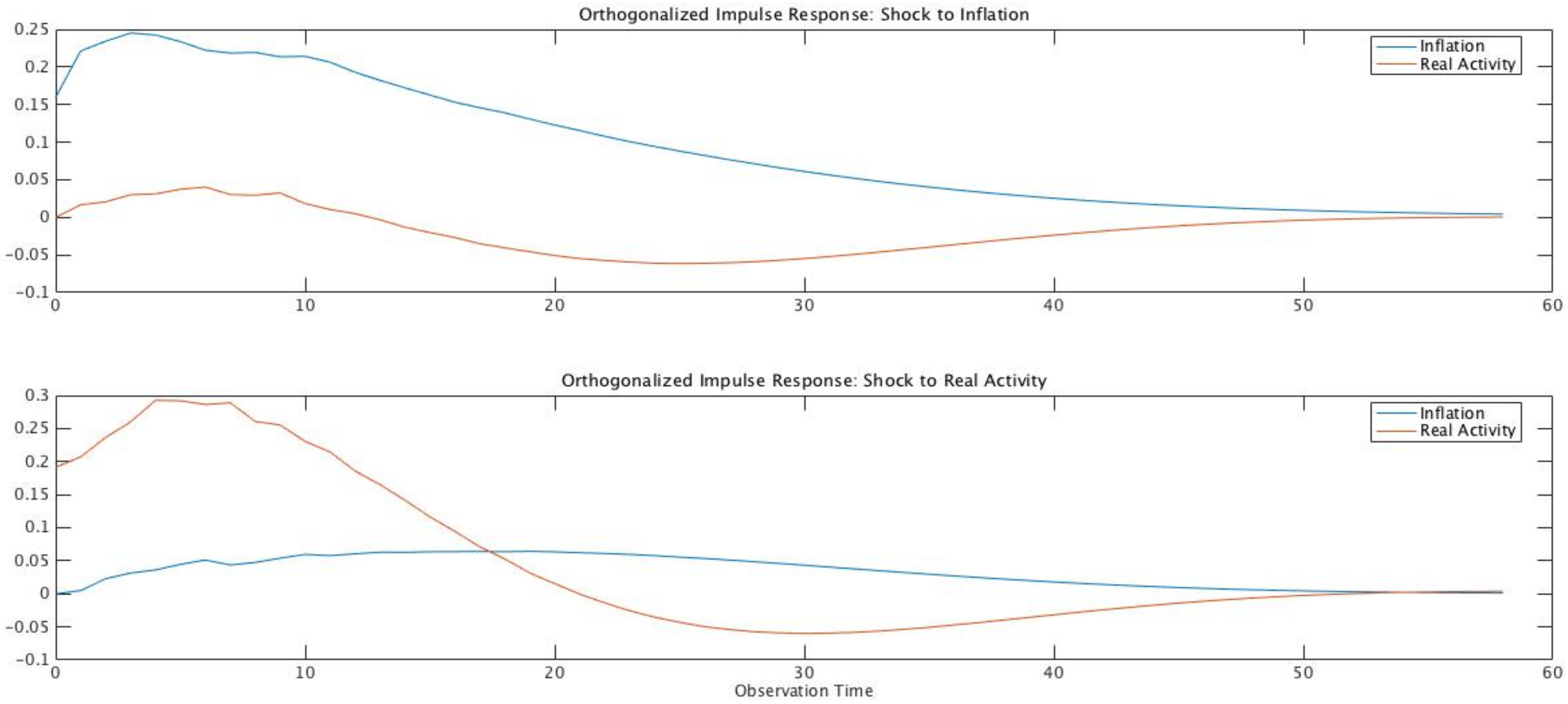
- The Impulse Responses (IRs) from the VAR(12) vary across different countries.
- For US, UK and Canada:
  - Response of inflation to shocks on real activity: **positive** and **hump-shaped**
  - Response of real activity to shocks on inflation: at the beginning it is **weakly positive**, then it becomes **slightly negative** before dying out
- For Germany:
  - Response of inflation to shocks on real activity: at the beginning it is **weakly positive**, then it becomes **negative** and return **positive**
  - Response of real activity to shocks on inflation: at the beginning it is **weakly negative**, then it becomes **slightly positive** before dying out
- For Japan:
  - Response of inflation to shocks on real activity: **negative** and **reverse hump-shaped**
  - Response of real activity to shocks on inflation: at the beginning it is **weakly negative**, then it becomes **strongly positive** before dying out

Analysis and Methodology

Correlation Analysis – Conditional Correlation

IRs – Extension of the Model

US



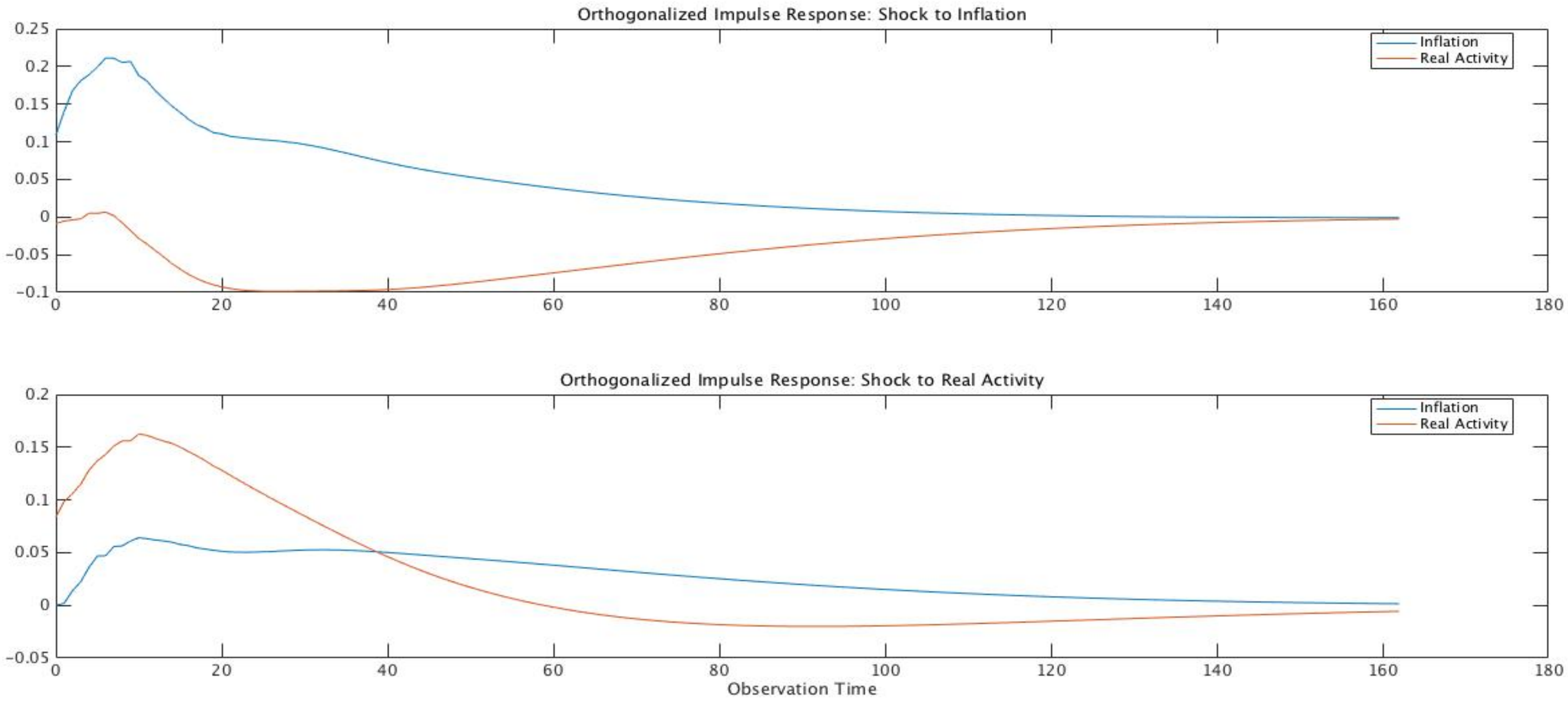
Observation Time is in months

Analysis and Methodology

Correlation Analysis – Conditional Correlation

IRs – Extension of the Model

UK



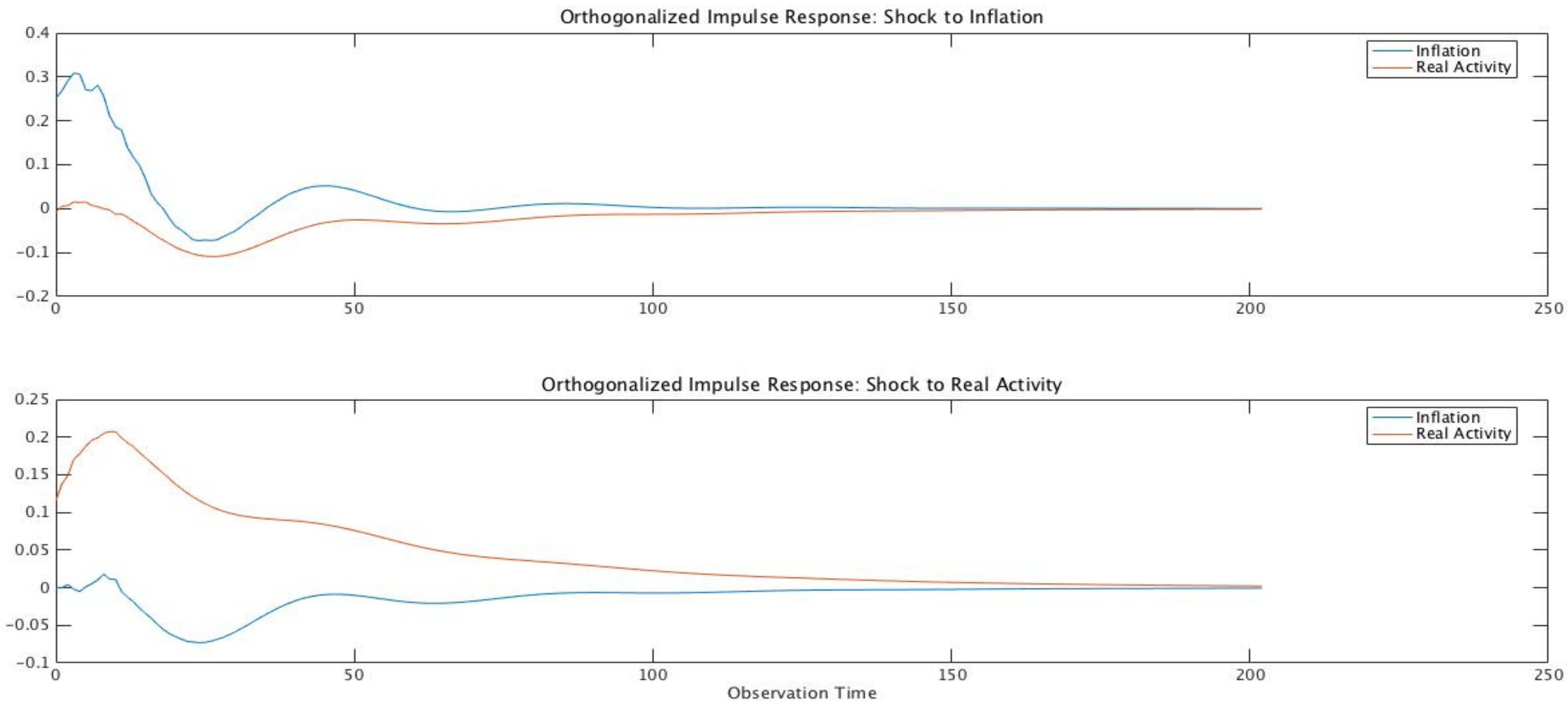
Observation Time is in months

Analysis and Methodology

Correlation Analysis – Conditional Correlation

IRs – Extension of the Model

GERMANY



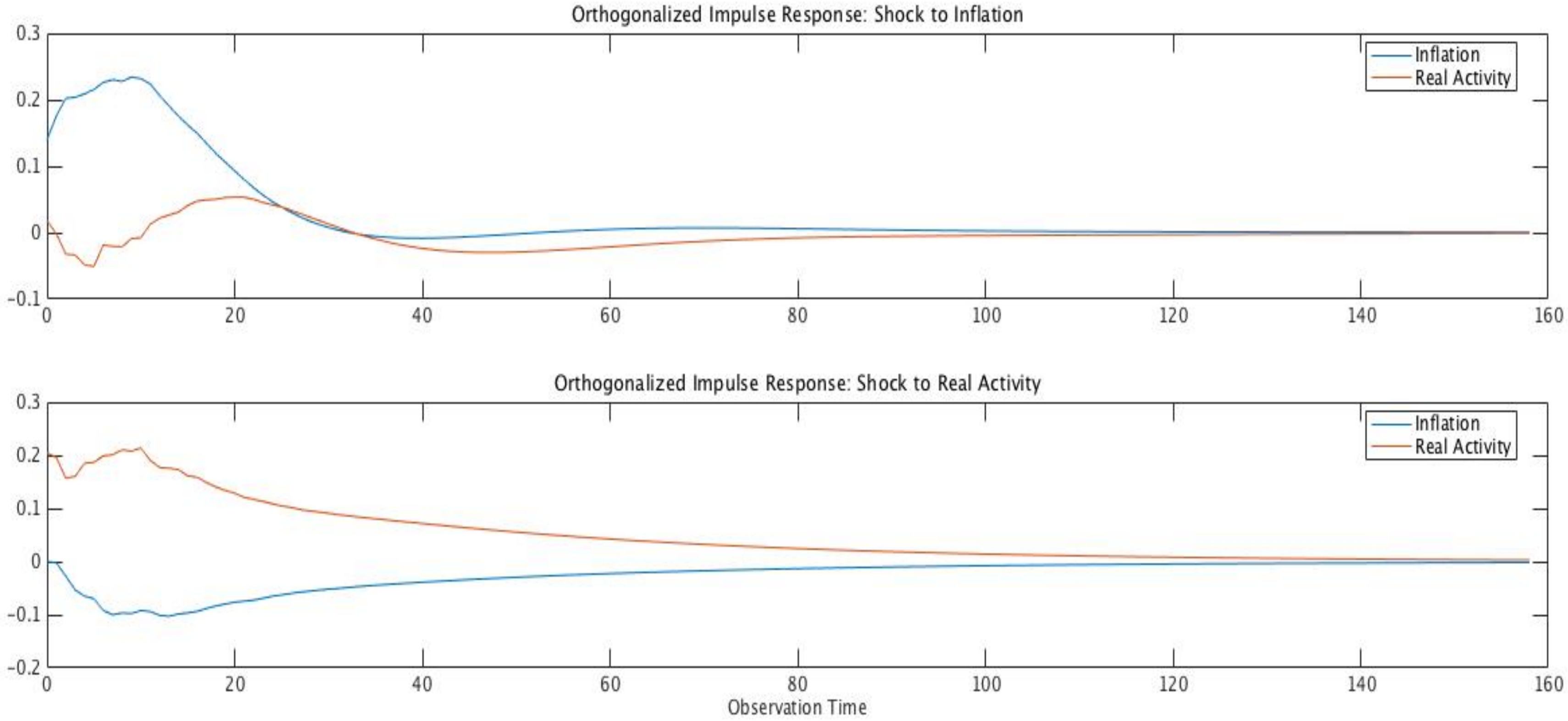
Observation Time is in months

Analysis and Methodology

Correlation Analysis – Conditional Correlation

IRs – Extension of the Model

JAPAN



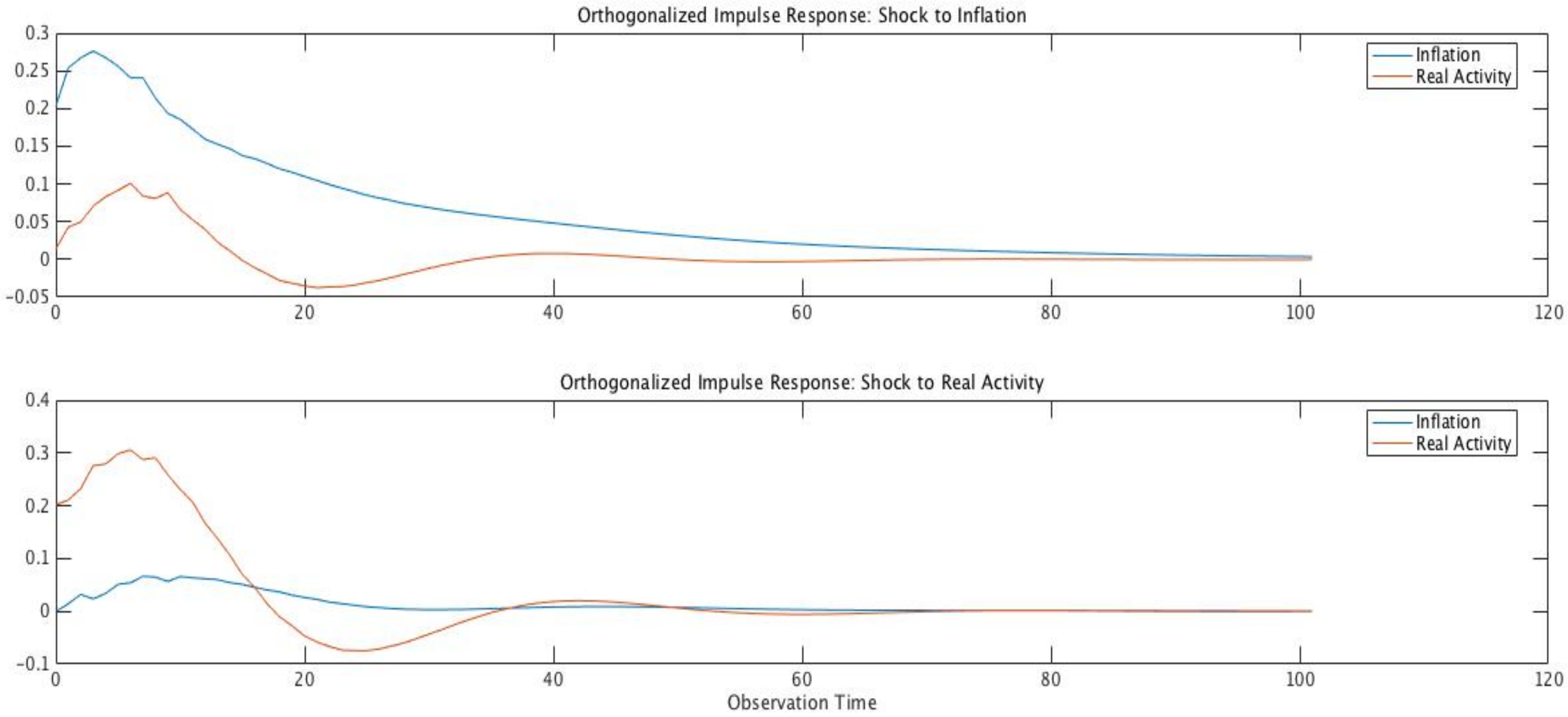
Observation Time is in months

Analysis and Methodology

Correlation Analysis – Conditional Correlation

IRs – Extension of the Model

CANADA



Observation Time is in months

Analysis and Methodology

Short Rate Dynamics – Policy Rules

- The short rate dynamics of the term structure used by AP can be seen as a version of the Taylor rule.

Taylor Rule (1993)

- The policy rule proposed by Taylor (1993) traces how the short rate r_t responds to simultaneous movements in two macro variables (inflation rate and output gap) as factors in f_t^0 and a component, orthogonal shock v_t, which is not explained by macro variables.

$$r_t = a_0 + a_1' f_t^0 + v_t$$

- The inflation rate is similar to AP inflation factor. With regard to the second variable (output gap): GDP data are quarterly while the AP real activity factor is built up considering only monthly series

Forward-looking Taylor Rule (2000)

- Proposed by Clarida (2000), the forward looking version implies that the central bank reacts to expected inflation and output gap by adding lagged macro variables (X_t^0) as arguments in the Taylor rule.
- The objective is to capture the information related with any variable that forecasts inflation or output

$$r_t = b_0 + b_1' X_t^o + v_t$$

$$X_t^o = (f_t^{o\prime}, f_{t-1}^{o\prime}, \dots, f_{t-p-1}^{o\prime})'$$

Where p is the lag length

- Duffie and Kan (1996) developed an affine term structure model based on the Taylor rule (1993) and an assumption on *risk premia* in which the short rate is an affine function of the underlying latent factors (X_t^u): $r_t = c_0 + c_1' X_t^u$. The latent factors follow affine processes and the VAR is a special Gaussian case of those.

Short Rate Dynamics – AP Specifications

- The short term dynamics of the term structure developed by AP can be seen as a version of the Taylor rule where the errors (v_t) are the latent factors.

Short Rate Dynamics (AP)

- AP combine the forward-looking Taylor rule together with the short rate dynamics in affine term structure developed by Duffie and Kan (1996). Indeed, both of them define the short rate as affine functions of factors:

$$r_t = \delta_0 + \delta'_{11}X_t^m + \delta'_{12}X_t^l$$

- They assume that X_t^0 and X_t^u are independent. Therefore, they can use ordinary least squares in order to estimate the coefficients on inflation and real activity in the aforementioned equation.
- Two regression are performed: the original Taylor rule and the its forward-looking version which contains lags of the macro variables.

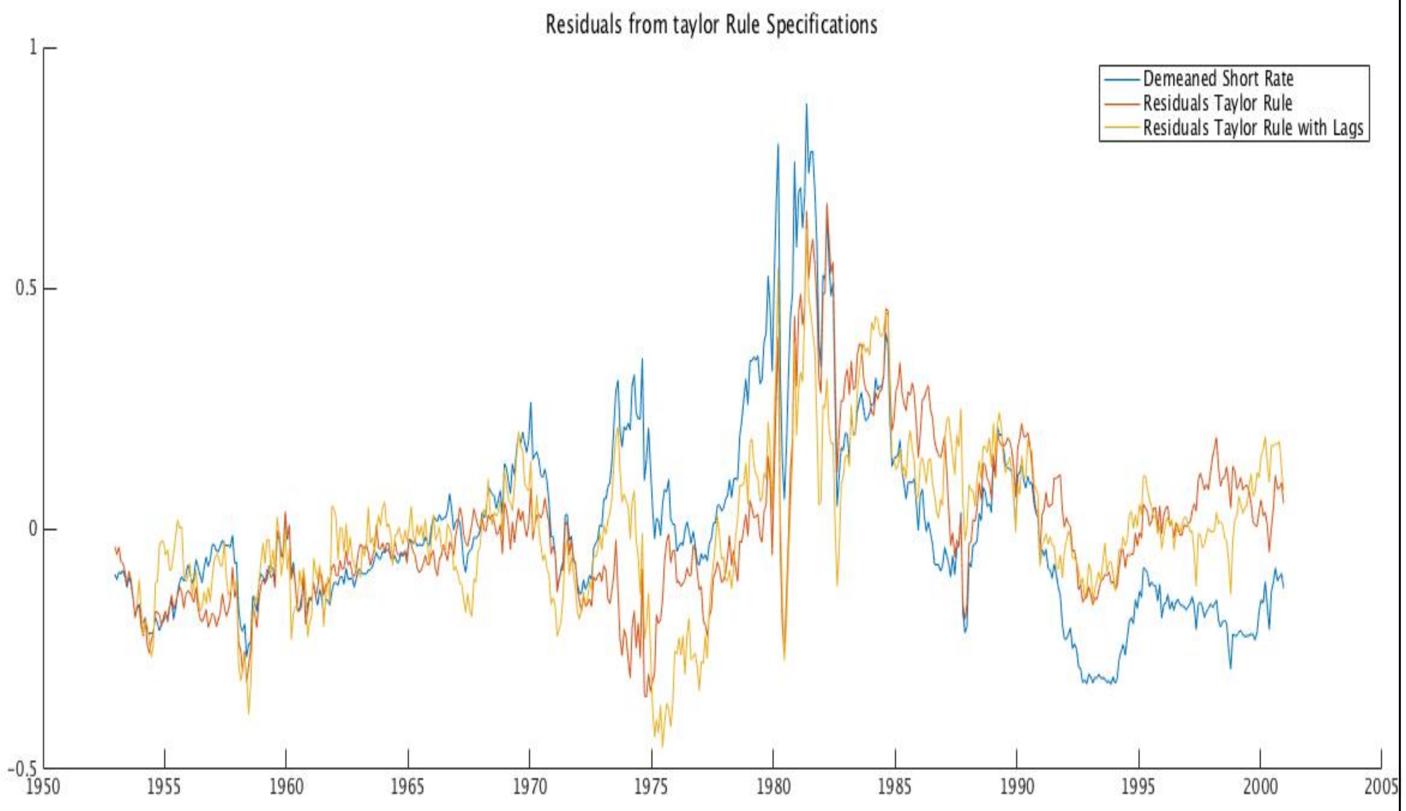
- The residuals from the Taylor rule regressions provide some intuition about the explanatory power of macro factors for shocks to the short rate (1m yield).

Analysis and Methodology

Short Rate Dynamics – Behaviour of the Residuals

Behaviour of the Residuals – Replication of the Model

- Looking at the plots, the demeaned short rate followed the behavior of the residuals for both the Taylor rule estimation and the forward Taylor rule estimation.
- This means that if a variable that mimics the behavior of the short rate itself is not set as an explanatory variable, then the residuals from the regression will follow the general pattern of the short rate.
- In conclusion, this is a point in favor of including unobservable factors, even though we are using macro variables.



Analysis and Methodology

Short Rate Dynamics – Behaviour of the Residuals

Extension of the model

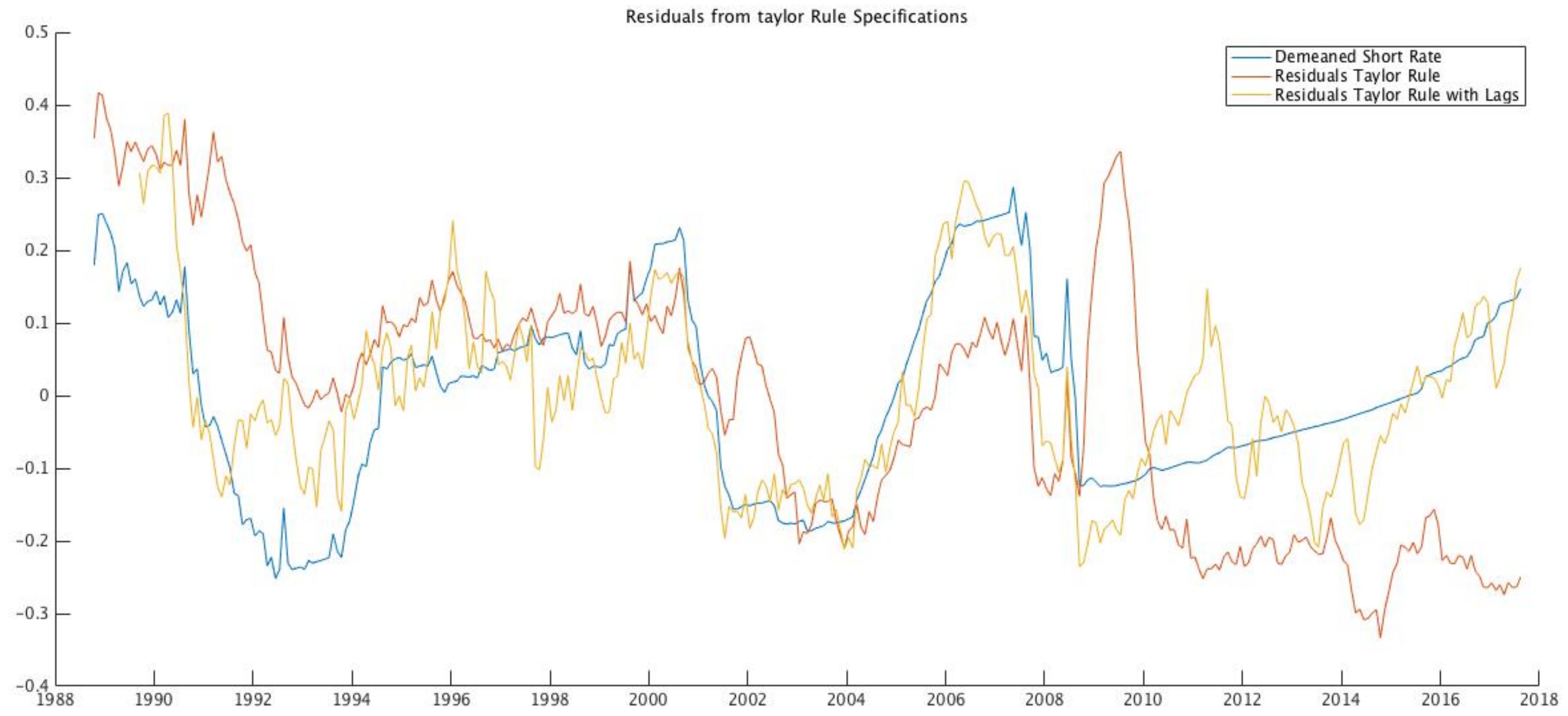
- Doing this analysis in the extension of the model was important to corroborate the importance of the unobservable factor “level” also to other countries.
- Overall, for all other countries we can see that the demeaned short-rate dynamics is mostly replicated by the movements of the residual in both the Taylor rule and the forward looking Taylor rule.
- The country that shows the best similarity is Japan whereas UK and Canada show the least, nevertheless, there is always evidence that the residuals can replicate the behavior of the short rate, hence, that the “Level” factor is relevant even with the inclusion of macro variables for the other countries.

Analysis and Methodology

Short Rate Dynamics – Behaviour of the Residuals

Behaviour of the Residuals – Extension of the Model

US

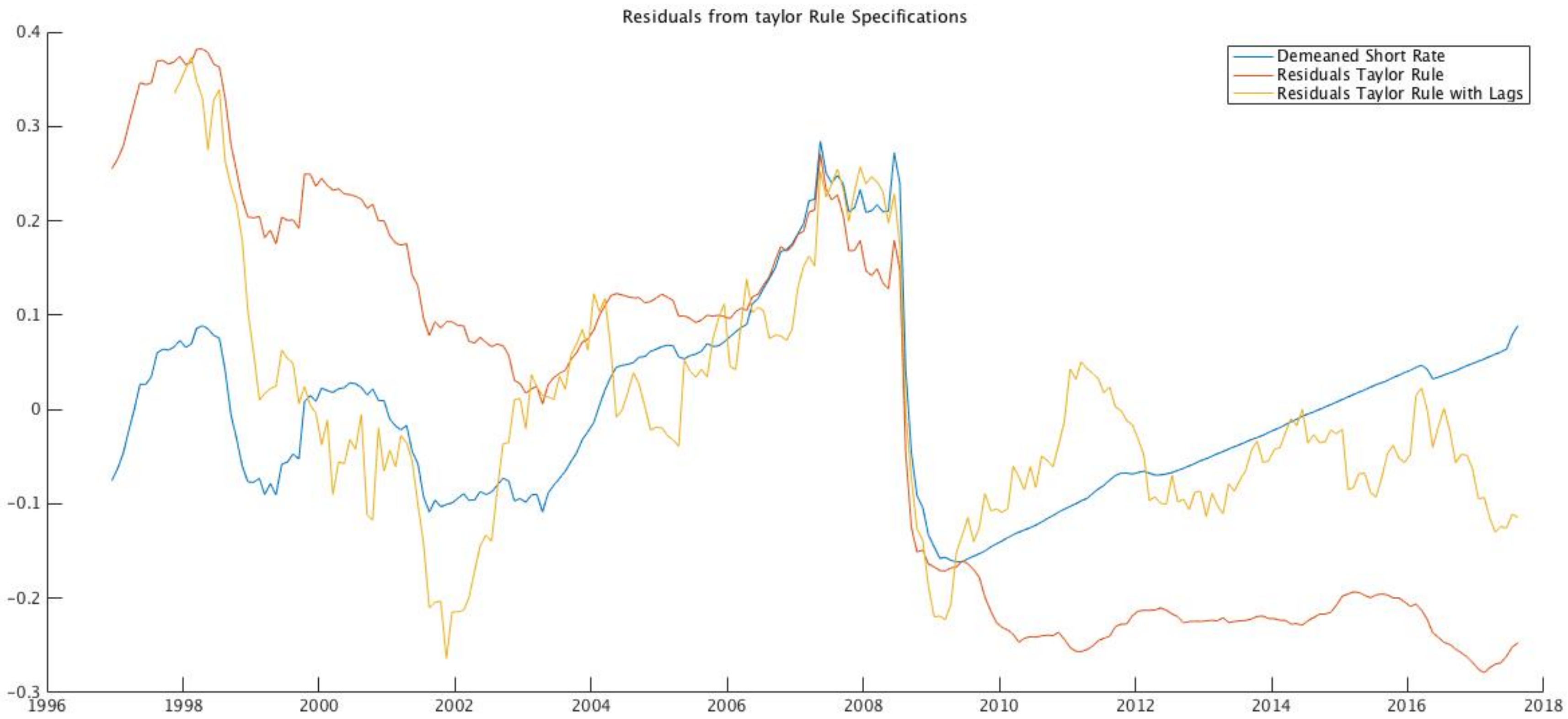


Analysis and Methodology

Short Rate Dynamics – Behaviour of the Residuals

Behaviour of the Residuals – Extension of the Model

UK

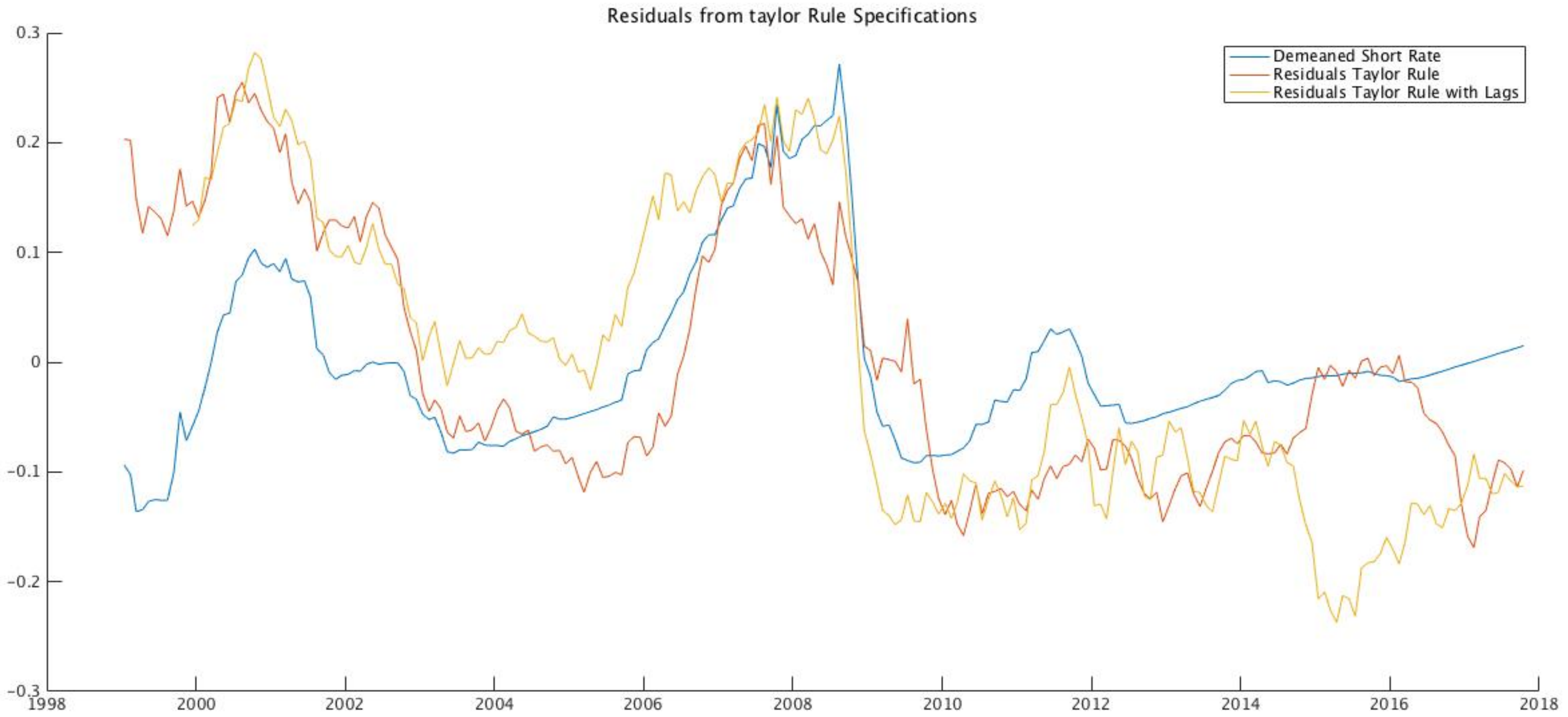


Analysis and Methodology

Short Rate Dynamics – Behaviour of the Residuals

Behaviour of the Residuals – Extension of the Model

GERMANY

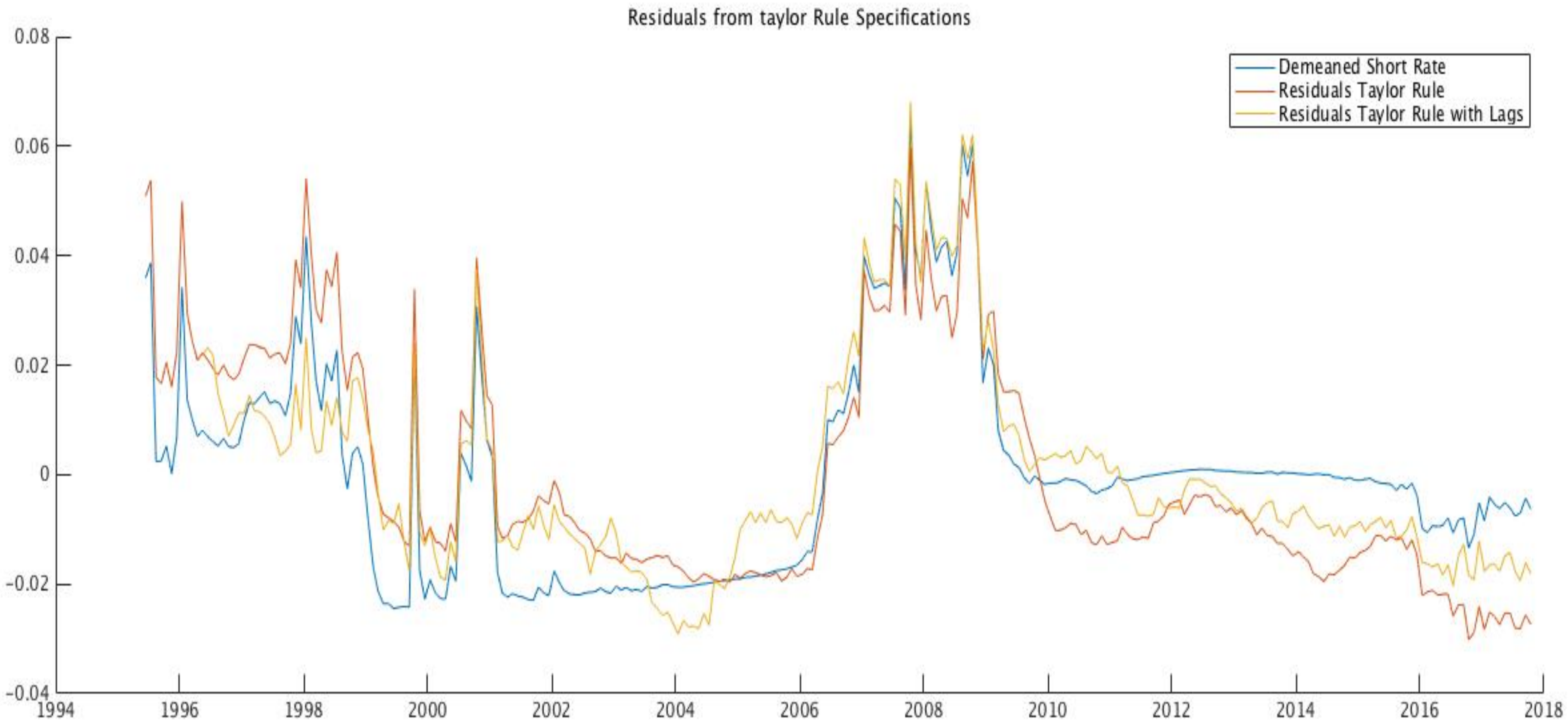


Analysis and Methodology

Short Rate Dynamics – Behaviour of the Residuals

Behaviour of the Residuals – Extension of the Model

JAPAN

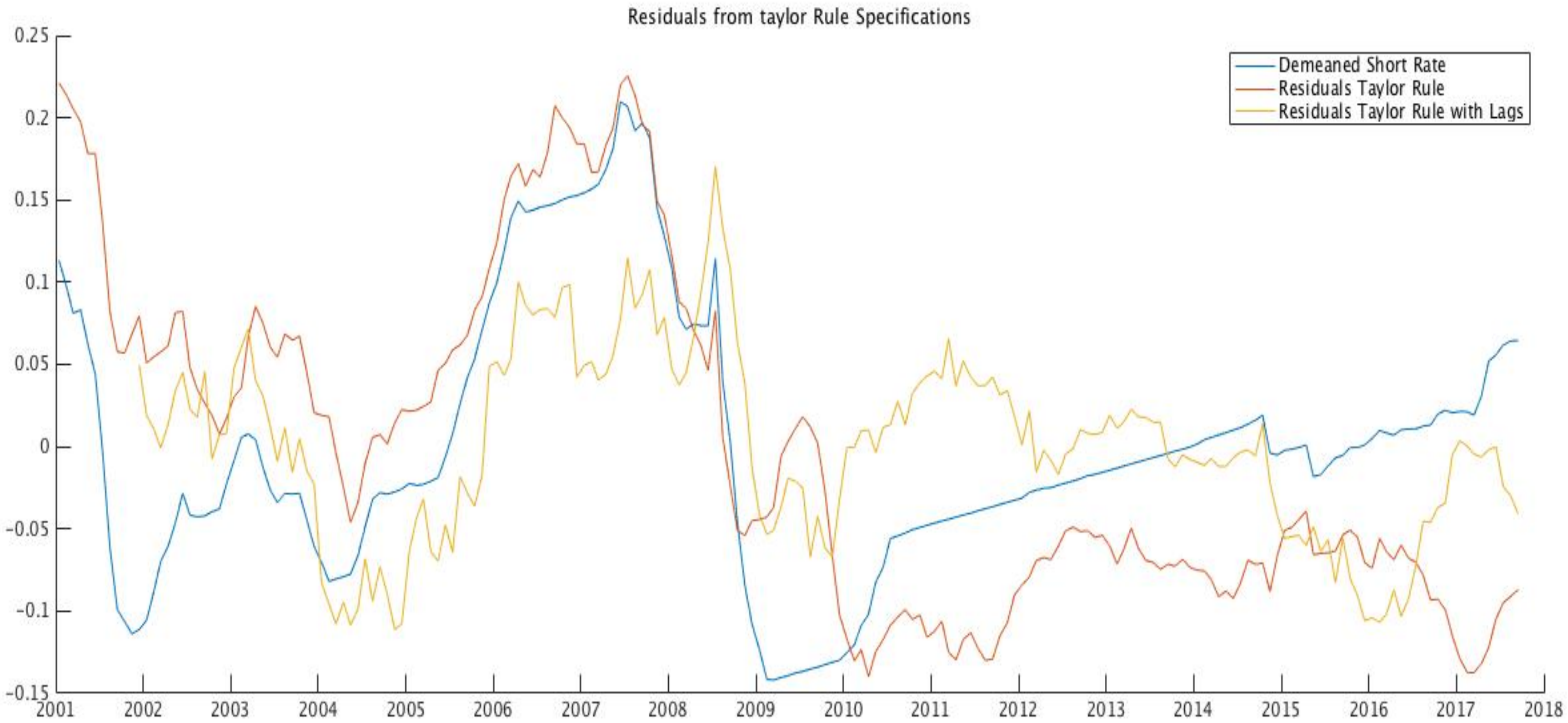


Analysis and Methodology

Short Rate Dynamics – Behaviour of the Residuals

Behaviour of the Residuals – Extension of the Model

CANADA



Analysis and Methodology

Short Rate Dynamics – Discussion of OLS Results

Short Rate Dynamics – Replication of the Model

- In the original Taylor Rule, the coefficients of "Inflation" and "Real Activity" are both significant and positive at, the 1% and 10% level, respectively.
- In the forward-looking version, most of the parameters are not significant, except for the 5th and 11th lag on real activity at 1% level. Moreover, the contemporaneous coefficients on inflation are also significant at a 5% level. This result suggests that using the forward-looking version of the Taylor Rule and including many lags may be the cause of over-parametrization and poorly behaviour of the system.

Taylor Rule	Constant	Inflation	Real Activity	Adjusted R
t	0,4271	0,1512	0,0132	0,4378
	*0,0071**	*0,0071**	*0,0071****	
Forward - Looking Taylor Rule	**Constant**	**Inflation**	**Real Activity**	**Adjusted R**
t	0,4335	0,1201	-0,0222	0,4784
	*0,0069**	*0,0540***	*0,0841*	
t - 1		-0,0326	0,0158	
		0,0844	*0,0848*	
t - 2		0,0316	-0,0135	
		0,0848	*0,0862*	
t - 3		-0,0312	0,0110	
		0,0860	*0,0851*	
t - 4		0,0144	-0,0778	
		0,0851	*0,0845*	
t - 5		0,0082	0,1439	
		0,0850	*0,0555**	
t - 6		0,0188	-0,0167	
		0,0320	*0,0472*	
t - 7		0,0009	0,0082	
		0,0473	*0,0470*	
t - 8		0,0085	0,0036	
		0,0467	*0,0470*	
t - 9		0,0251	-0,0439	
		0,0474	*0,0472*	
t - 10		-0,0248	-0,0078	
		0,0476	*0,0485*	
t - 11		-0,0162	0,1052	
		0,0486	*0,0326**	

Standard error significant at the 1% (*), 5% (**), and 10% (***).

Analysis and Methodology

Short Rate Dynamics – Discussion of OLS Results

Extension of the model

- Except for Germany, for all the other countries, the R^2 of the forward-looking Taylor Rule is higher than the R^2 of the original version. This result is aligned with the AP findings with regard to the explanatory power of macro factors to yield curve movements.
- However, for all the countries, in the forward-looking version, most of the parameters are not significant. This may be, the cause of over-parametrization and poorly behaviour of the system.

- These results provide some intuition about how much macro factors may explain with respect to the unobservable factors with regard to yield movements.

Analysis and Methodology

Short Rate Dynamics – Discussion of OLS Results

Short Rate Dynamics – Extension of the Model

US

Taylor Rule	Constant	Inflation	Real Activity	Adjusted R^2
t	0,3107	0,0646	0,1215	0,3199
	0,0104 *	*0,0141* *	*0,0112* *	
Forward - Looking Taylor Rule	**Constant**	**Inflation**	**Real Activity**	**Adjusted R^2**
t	0,2957	-0,1161	-0,0409	0,5973
	0,0074 *	*0,0590* **	*0,0915*	
t - 1		0,0419	0,0122	
		0,0920	*0,0927*	
t - 2		-0,0270	-0,0006	
		0,0920	*0,0926*	
t - 3		0,0520	-0,0011	
		0,0926	*0,0934*	
t - 4		-0,0050	-0,0076	
		0,0940	*0,0938*	
t - 5		0,0465	-0,0883	
		0,0939	*0,0622*	
t - 6		-0,0052	-0,0040	
		0,0310	*0,0445*	
t - 7		-0,0022	-0,0114	
		0,0447	*0,0442*	
t - 8		-0,0046	-0,0020	
		0,0439	*0,0440*	
t - 9		-0,0203	0,0327	
		0,0444	*0,0442*	
t - 10		0,0296	0,0119	
		0,0446	*0,0456*	
t - 11		0,0155	-0,1284	
		0,0459	*0,0310* *	

Standard error significant at the 1% (*), 5% (**), and 10% (***).

Analysis and Methodology

Short Rate Dynamics – Discussion of OLS Results

Short Rate Dynamics – Extension of the Model

UK

Taylor Rule	Constant	Inflation	Real Activity	Adjusted R^2
t	0,2858	0,0358	0,0072	0,0021
	*0,0254**	*0,0235*	*0,0212*	

Forward - Looking Taylor Rule	Constant	Inflation	Real Activity	Adjusted R^2
t	0,0933	0,0302	0,0211	0,5715
	*0,0154**	*0,0765*	*0,1123*	
t - 1		-0,0085	0,0164	
		0,1108	*0,1105*	
t - 2		0,0175	-0,0236	
		0,1098	*0,1101*	
t - 3		0,0054	0,0016	
		0,1108	*0,1111*	
t - 4		-0,0165	-0,0397	
		0,1115	*0,1112*	
t - 5		-0,0227	0,2099	
		0,1137	*0,0757**	
t - 6		-0,1796	0,0555	
		0,1093	*0,1771*	
t - 7		-0,0341	0,0769	
		0,1813	*0,1821*	
t - 8		-0,0234	0,0129	
		0,1816	*0,1821*	
t - 9		0,0093	-0,0173	
		0,1819	*0,1809*	
t - 10		0,0007	-0,0454	
		0,1808	*0,1789*	
t - 11		0,0163	0,0653	
		0,1773	*0,1134*	

Standard error significant at the 1% (*), 5% (**), and 10% (***).

Analysis and Methodology

Short Rate Dynamics – Discussion of OLS Results

Short Rate Dynamics – Extension of the Model

GERMANY

Taylor Rule	Constant	Inflation	Real Activity	Adjusted R^2
t	0,1688	0,0612	-0,0489	0,3377
	*0,0080**	*0,0070****	*0,0079*	
Forward - Looking Taylor Rule	**Constant**	**Inflation**	**Real Activity**	**Adjusted R^2**
t	0,1405	0,0000	-0,0012	-0,0338
	*0,0121**	*0,0421*	*0,0618*	
t - 1		0,0033	-0,0039	
		0,0612	*0,0608*	
t - 2		0,0031	-0,0115	
		0,0606	*0,0606*	
t - 3		0,0028	-0,0069	
		0,0605	*0,0603*	
t - 4		0,0074	-0,0025	
		0,0603	*0,0599*	
t - 5		0,0125	-0,0209	
		0,0589	*0,0404*	
t - 6		-0,0217	-0,0017	
		0,1038	*0,1668*	
t - 7		-0,0061	-0,0115	
		0,1710	*0,1714*	
t - 8		0,0142	0,0072	
		0,1733	*0,1732*	
t - 9		0,0264	0,0095	
		0,1740	*0,1739*	
t - 10		0,0312	0,0146	
		0,1726	*0,1728*	
t - 11		0,0596	-0,1625	
		0,1722	*0,1073*	

Standard error significant at the 1% (*), 5% (**), and 10% (***).

Short Rate Dynamics – Discussion of OLS Results

Short Rate Dynamics – Extension of the Model

JAPAN

Taylor Rule	Constant	Inflation	Real Activity	Adjusted R^2
t	0,0248	0,0066	-0,0032	0,0400
	*0,0020**	*0,0029***	*0,0020*	

Forward - Looking Taylor Rule	Constant	Inflation	Real Activity	Adjusted R^2
t	0,0156	-0,0043	-0,0004	0,0844
	*0,0032**	*0,0082*	*0,0132*	
t - 1		-0,0016	0,0041	
		0,0132	*0,0133*	
t - 2		0,0040	-0,0010	
		0,0134	*0,0133*	
t - 3		-0,0007	0,0017	
		0,0134	*0,0134*	
t - 4		0,0030	0,0042	
		0,0133	*0,0132*	
t - 5		0,0028	-0,0060	
		0,0133	*0,0084*	
t - 6		0,0015	-0,0019	
		0,0062	*0,0079*	
t - 7		0,0015	-0,0012	
		0,0080	*0,0083*	
t - 8		0,0014	0,0034	
		0,0082	*0,0082*	
t - 9		0,0004	-0,0017	
		0,0083	*0,0084*	
t - 10		-0,0022	-0,0036	
		0,0084	*0,0083*	
t - 11		-0,0025	0,0041	
		0,0082	*0,0062*	

Standard error significant at the 1% (*), 5% (**), and 10% (***).

Short Rate Dynamics – Discussion of OLS Results

Short Rate Dynamics – Extension of the Model

CANADA

Taylor Rule	Constant	Inflation	Real Activity	Adjusted R^2
t	0,1884	0,0291	0,0304	0,1334
	0,0087*	0,0118**	0,0130**	
Forward - Looking Taylor Rule	**Constant**	**Inflation**	**Real Activity**	**Adjusted R^2**
t	0,1036	0,0817	-0,0281	0,6704
	*0,0053**	*0,0397***	*0,0614*	
t - 1		0,0320	-0,0300	
		0,0603	*0,0614*	
t - 2		0,0319	-0,0202	
		0,0623	*0,0635*	
t - 3		0,0255	-0,0149	
		0,0650	*0,0652*	
t - 4		-0,0061	0,0096	
		0,0647	*0,0647*	
t - 5		-0,0196	0,0311	
		0,0640	*0,0404*	
t - 6		-0,0670	0,0128	
		*0,0213**	*0,0314*	
t - 7		-0,0024	0,0276	
		0,0307	*0,0311*	
t - 8		0,0095	0,0030	
		0,0316	*0,0314*	
t - 9		-0,0025	-0,0200	
		0,0308	*0,0306*	
t - 10		0,0098	-0,0177	
		0,0294	*0,0284*	
t - 11		0,0137	-0,0176	
		0,0302	*0,0209*	

Standard error significant at the 1% (*), 5% (**), and 10% (***).

Analysis and Methodology

Initial Model Framework

Discrete time Gaussian Model

- In statistics, a Gaussian model is a model that is build on a continuous frame.

Initial Set-up

- The pricing Kernel specified in our model is the same as the one used in HW and is the generally accepted pricing kernel used in the derivation of affine term structure models. According to this, we will be using a VAR measure for the factors that will be characterized by this measure Q:

1. $F_{t+1} = c + \rho F_t + \Sigma u_{t+1}$
2. $c^Q = c - \Sigma\lambda$ and $\rho^Q = \rho - \Sigma\Lambda$
3. $F_{t+1} = c^Q + \rho^Q F_t + \Sigma u^Q_{t+1}$

- With λ and Λ being the variables that represent **investors attitudes towards risk.**

- Under a VAR characterized by this measure Q, a risk-averse investor and a risk-neutral investor will value assets the same way.

Analysis and Methodology

Initial Model Framework

- According with the assumptions aforementioned, the yield on risk-free n-period pure-discount bond can be calculated by following the model developed by Hamilton and Wu.

Summarization of the model

- Following HW, the model can be represented by:

$$\boldsymbol{y_t^n = a_n + b_n' F_t}$$

- Where:
  - $b_n = \frac{1}{n}[I_M + \rho^{Q\prime} + \cdots + (\rho^{Q\prime})^{n-1}]\delta_1$
  - $a_n = \delta_0 + \frac{[b_n' + 2b_n' + \cdots + (n-1)b_{n-1}']c^Q}{n} - \frac{[b_1'\Sigma\Sigma' b_1 + 2^2 b_2'\Sigma\Sigma' b_2 + \cdots + (n-1)^2 b_{n-1}'\Sigma\Sigma' b_{n-1}]}{2n}$
- The idea is that by knowing $\boldsymbol{F_t, c^Q, \rho^Q, \delta_1, \delta_2}$ and $\boldsymbol{\Sigma}$ we can make a prediction of yields for different maturities, thus predicting the whole curve.
- With F_t being a matrix with a number of **observable** and **unobservable** factors for time t.
- By using the VAR process mentioned in the slide before, it is possible to get the values for the unobservable (latent) and observable (macroeconomic) variable values for t+1, thus predicting y_{t+1}^n.

Analysis and Methodology

Initial Model Framework

- Following the HW model, it should be possible to predict the value of one of the y_t^n as a linear function of the others. However, the empirical fit is never exact, so the model proposed assumes that some linear combinations will be measured with some error (N_e) whereas the other will be exact (N_l). We assume $N_d > N_l$ where N_d is a set of different maturities and N_l is the number of unobserved pricing factors.

Summarization of the model

- In the estimation proposed by AP, they assume that the model $\boldsymbol{y_t^n = a_n + b_n' F_t}$ holds for $\boldsymbol{N_l}$ linear combination of observed yields and the remaining $\boldsymbol{N_e = N_d - N_l}$ linear combinations differ from the predicted value by a small measurement error.
- The model specification is:

$$\begin{bmatrix} Y_t^1 \\ (N_l \times 1) \\ Y_t^2 \\ (N_e \times 1) \end{bmatrix} = \begin{bmatrix} A_1 \\ (N_l \times 1) \\ A_2 \\ (N_e \times 1) \end{bmatrix} + \begin{bmatrix} B_1 \\ (N_l \times M) \\ B_2 \\ (N_e \times M) \end{bmatrix} F_t + \begin{bmatrix} 0 \\ (N_l \times N_e) \\ \Sigma_e \\ (N_l \times N_e) \end{bmatrix} \underset{(N_e \times 1)}{u_t^e}$$

- Where, Y_t^1 represents the $(N_l \times 1)$ vector of linear combination of yields priced without error and Y_t^2 represents the $(N_e \times 1)$ vector of the remaining linear combinations priced with error.
- A_i and B_i are calculated according with the formulas in the previous slide; Σ_e represents the variance of the measurement error with $u_t^e \sim N\left(0, I_{N_e}\right)$.

Analysis and Methodology

Initial Model Framework – Latent factors

- In order to decide which are the yields priced without errors, HW followed AP and proposed 3 representative yields, which are those of maturities: 1m, 12m and 60m. The yields measured without error go into Y_t^1.

Summarization of the model

- In our model the 1m, 12m and 60m yields are priced without errors and will go into Y_t^1.
- The yields priced with errors are the 3m and 36m yields. Indeed, they are included in Y_t^2.
- We can define the full yields (measured with and without error) specification as:

$$\begin{bmatrix} y_t^1 \\ y_t^{12} \\ y_t^{60} \\ y_t^6 \\ y_t^{36} \end{bmatrix} = \begin{bmatrix} a_1 \\ a_{12} \\ a_{60} \\ a_6 \\ a_{36} \end{bmatrix} + \begin{bmatrix} b_1 \\ b_{12} \\ b_{60} \\ b_6 \\ b_{36} \end{bmatrix} F_t + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \Sigma_e \\ \Sigma_e \end{bmatrix} u_t^e$$

- It is clear, by looking at the errors matrix, that this would mean that, for optimization purposes, the yields of 1m, 12m and 60m are assumed not to have a pricing error, thus their respective variance covariance matrixes for the errors are zero.

Estimation Procedure – "Macro Finance Model with 12 lags" – MF12

- Given the results reached by AP we decided to use their model with 12 lags on the observable factors and only 1 lag on the latent factors, which was the one they deemed best for forecasts. We decided to use this model given that our purpose is not the estimation of a new model but the extension of their model to new markets.

Vector Autoregression

- Next step was to estimate a VAR for the two macro factors (“Inflation” and “Real Activity”) with 12 lags:

$$f_t^m = \left(f_t^{m,1} f_t^{m,2}\right)'$$

$$f_t^m = \rho_1 f_{t-1}^m + \cdots + \rho_{12} f_{t-12}^m + \Sigma_{mm} u_t^m$$

  - Where ρ_1 to ρ_{12} and Σ_{mm} are 2 × 2 matrices with $u_t^m \sim$ IID N(0,I)
  - The index 1 will correspond to the “Real Activity” ($f_t^{m,1}$), while the index 2 corresponds to “Inflation” ($f_t^{m,2}$).
  - AP suggest that only a 1-lag model is not sufficient to capture the whole dynamics for output and inflation.

- With regards to the latent variables, they will have only 1 lag:

$$f_t^l = c_l + \rho_{ll} f_{t-1}^l + \Sigma_{ll} u_t^l$$

  - According with AP, since the unobserved variables are independent from the observable ones the such terms as ρ_{ml} and ρ_{lm} are set to zero. Therefore, the two processes for the VARs can be and are independently estimated.

Analysis and Methodology

Estimation Procedure – "Macro Finance Model with 12 lags” – MF12

- In HW they focused on making sure that the model was identified. They concluded that the model by AP was undefined. To make sure, restrictions had to be applied to the procedure. Keeping in mind that our model has three yield measured without error and two measured with error, the following restrictions were put in place.

- Considering the model: $\boldsymbol{y_t^n} = \boldsymbol{a_n} + \boldsymbol{b'_n F_t}$; where $\boldsymbol{a_n}$ depends on $\boldsymbol{\delta_0, \delta_1, \rho^Q, c^Q}$ and $\boldsymbol{b'_n}$ depends on $\boldsymbol{\delta_1, \rho^Q}$**.** HW proposes the next restrictions so that an identified system is assured.

Identifying restrictions

- $\boldsymbol{\Sigma_{mm}}$ is lower triangular.
- $\boldsymbol{\Sigma_{ll}} = I_{N_l}$.
- $\boldsymbol{c_l} = 0$.
- $\boldsymbol{\rho_{ll}}$ is lower triangular and the elements in its diagonal are in descending order.

Analysis and Methodology

Estimation Procedure – "Reduced-Form" Model

- In HW they call their reduced-form equation the results that come from the calculation of VAR(1)s through OLS for the yields that are measured without error. This equation was obtained through an affine transformation.

Reduced form parameters

- First, thanks to the existence of the yields observed without error, it is possible to do an invariant affine transformation of the these yields because they have no pricing error and F_t becomes observable for them, which gets us the following regression that can be estimated through OLS:

$$\underset{(3\times1)}{\boldsymbol{Y_t^1}} = \boldsymbol{A_1^*} + \underset{(3\times24)}{\boldsymbol{\phi_{1o}^*}} \boldsymbol{F_{t-1}^o} + \underset{(3\times3)}{\boldsymbol{\phi_{11}^*}} \boldsymbol{Y_{t-1}^1} + \underset{(3\times2)}{\boldsymbol{\psi_{1o}^*}} \boldsymbol{f_t^o} + \boldsymbol{u_{1,t}^*};$$ where we have: $A_1^* = A_1 - B_1\rho B_1^{-1}A_1;\ \phi_{11}^* = B_1\rho B_1^{-1};\ u_{1,t}^* = B_1 u_t$ with $u_{1,t}^* \sim i.i.d.N(0,\ B_1 B_1')$.

- Then the **yields with errors** will take the form of the following equation that can also be estimated using OLS:

$$\underset{(2\times1)}{\boldsymbol{Y_t^2}} = \boldsymbol{A_2^*} + \underset{(2\times24)}{\boldsymbol{\phi_{2o}^*}} \boldsymbol{F_t^o} + \underset{(2\times3)}{\boldsymbol{\phi_{21}^*}} \boldsymbol{Y_t^1} + \boldsymbol{u_{2,t}^*}$$

- In HW they use these as the starting steps of the optimization process, that allows the calculation of $\boldsymbol{A_1^*}$, $\boldsymbol{A_2^*}$, $\boldsymbol{\phi_{11}^*}$, $\boldsymbol{\phi_{21}^*}$, $\boldsymbol{\phi_{2o}^*}$, $\boldsymbol{\phi_{1o}^*}$, $\boldsymbol{\psi_{1o}^*}$ and **the variance covariance matrix of $\boldsymbol{u_{1,t}^*}$ and $\boldsymbol{u_{2,t}^*}$**. Next, the focus is the calculation of the final parameters.

Analysis and Methodology

Estimation Procedure – "Reduced-Form" Model

Overview of the important parameters estimated by the OLS and their relation with the structural parameters.

Reduced form parameters

$$\underset{(2\times1)}{f_t^o} = \underset{(2\times24)}{\phi_{oo}^*} F_{t-1}^o + u_{o,t}^* \qquad \phi_{oo}^* = [\rho_1\ \rho_2 \ldots \rho_{12}]$$

$$A_1^* = A_1 - B_{1u}\rho_{uu}B_{1u}^{-1}A_1$$

$$\underset{(3\times24)}{\phi_{1o}^*} = \begin{bmatrix} \underset{(3\times22)}{B_{1o}^{(1)}} & \underset{(3\times2)}{0} \end{bmatrix} - B_{1u}\underset{(3\times3)}{\rho_{uu}}B_{1u}^{-1}\begin{bmatrix} \underset{(3\times2)}{B_{1o}^{(0)}} & \underset{(3\times22)}{B_{1o}^{(1)}} \end{bmatrix}$$

$$\phi_{11}^* = B_{1u}\rho_{uu}B_{1u}^{-1} \qquad \psi_{1o}^* = B_{1o}^{(0)} \qquad A_2^* = A_2 - B_{2u}B_{1u}^{-1}A_1 \qquad \phi_{2o}^* = B_{2o} - B_{2u}B_{1u}^{-1}B_{1o} \qquad \phi_{21}^* = B_{2u}B_{1u}^{-1}$$

$$Var\left(\begin{bmatrix} u_{o,t}^* \\ u_{1,t}^* \\ u_{2,t}^* \end{bmatrix}\right) = \begin{bmatrix} \Omega_o^* & 0 & 0 \\ 0 & \Omega_1^* & 0 \\ 0 & 0 & \Omega_2^* \end{bmatrix} = \begin{bmatrix} \Sigma_{oo}\Sigma_{oo}' & 0 & 0 \\ 0 & B_{1u}B_{1u}' & 0 \\ 0 & 0 & \Sigma_e\Sigma_e' \end{bmatrix}$$

Analysis and Methodology

Estimation Procedure – Minimum Chi-Square

- A Chi-square test statistics can measure how good the expected data fits to the observed data. In this sense, a very small Chi-square test statistic would mean that our expected values by the model fit the observed values extremely well. This is the optimization that is done by HW. Several iterations, with randomized starting parameters, in the code run for the log likelihood function and the one result that returns the minimum Chi-square measure will survive in the end.

Estimation Overview

- It is still necessary that the likelihood function is maximized.
- However, HW propose to maximize the likelihood function with respect to the reduced-form parameters, through a simple OLS, and then do a translation to the implications this would have on the structural parameters.
- With each estimation a new Chi-square is calculated, the lower the better. (Usually 30 iterations are enough, 100 to be sure).

- **Wald statistics**: This is the statistics that needs to be minimized in the minimum Chi-square estimation.
- The overall process consists of guessing $\boldsymbol{\delta_1, \rho^Q}$ and calculating B. Then guess $\boldsymbol{\delta_0, c^Q}$ and calculate A. Then, for each estimation, compute the Wald statistics and stop the process once it has reached the minimum.

Analysis and Methodology

Estimation Procedure – Minimum Chi-Square

- The **MCSE (Minimum Chi-square estimation)** was a method unused for estimating term structure models before its application in HW. In their paper they managed to prove that this was an asymptotically equal method to MLE but had further benefits.

- In *Appendix E* of the HW paper they prove that the MLE and MCSE are asymptotically equivalent and also state that MCSE has two main advantages over MLE:

    1. It is possible to be sure that a **global maximum** for the Likelihood function is achieved, when the Wald statistics they mention reaches a zero (its **global minimum**).

    2. Also, since the optimization process only requires OLS for the reduced-form parameters and a minimization of the Wald statistic, then this constitutes a far less demanding optimization procedure, when compared to the MLE of the structural parameters directly.

Estimation results

Estimation results

Impulse Response Analysis

Impulse Response Function

- An impulse response function will tell you what is the impact of an external shock to a certain variable and how that shock impacts the variable over time.

Factor weights across the yield curve

- Our replication results were in line with the results from AP.
- The effect of each macro factor on the yield curve is determined by the weights Bn that the term structure model assigns on each yield of maturity n. Moreover, these weights Bn also represent **the initial response** of yields to shocks from the various factors.
- The factor weights across the yield curve show the initial effect of shocks as a function of yield maturity.

From macro shocks

- To trace out the long-term responses of the yield curve from shocks to the macro variables after the yield curve's initial response, we examined the IRs from macro shocks.
- The yield on a n-period ZCB $\boldsymbol{y_t^n}$ is a linear combination of current and lagged values of $\boldsymbol{u_t}$:

$$\boldsymbol{y_t^n = a_n + \sum_{i=0}^{\infty} \psi_i^n \, u_{t-i}}$$

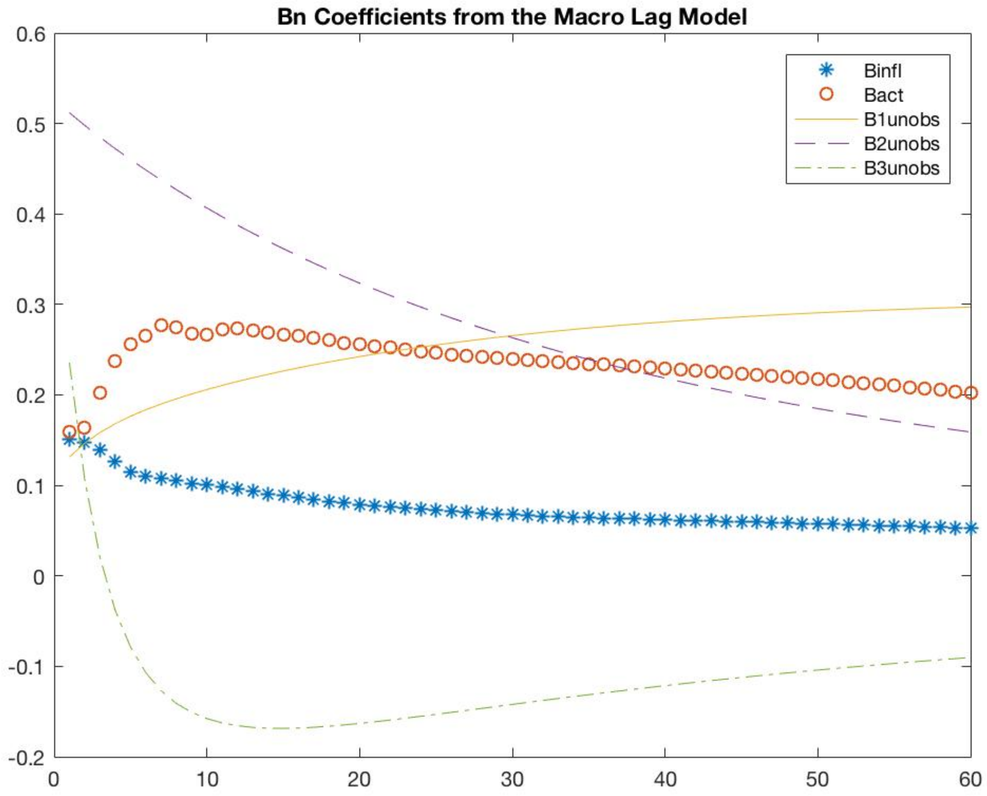
- where the row vectors $\boldsymbol{\psi_i^n}$ are functions of Bn.

Estimation results

Impulse Response Analysis – Factor weights across the yield curve

Impulse Response – Replication of the Model

- The weight on the most persistent factor (Unobs 1) is almost horizontal. Meaning that it affects yields of all maturities the same way, hence the name “level”.
- The coefficient of the second factor (Unobs 2) is downward sloping. It mainly moves the short end of the yield curve relative to the long end. Also called the “slope” factor.
- The coefficient on the least persistent factor (Unobs 3) is hump-shaped. Movement in this factor affects yields at short-end of the yield curve and middle and long-end of the yield curve with different signs. It has a twisting effect and it is called ”curvature”.
- Shocks to real activity impact the yield more than shocks to inflation. They mostly affect short yields and less so long yields.



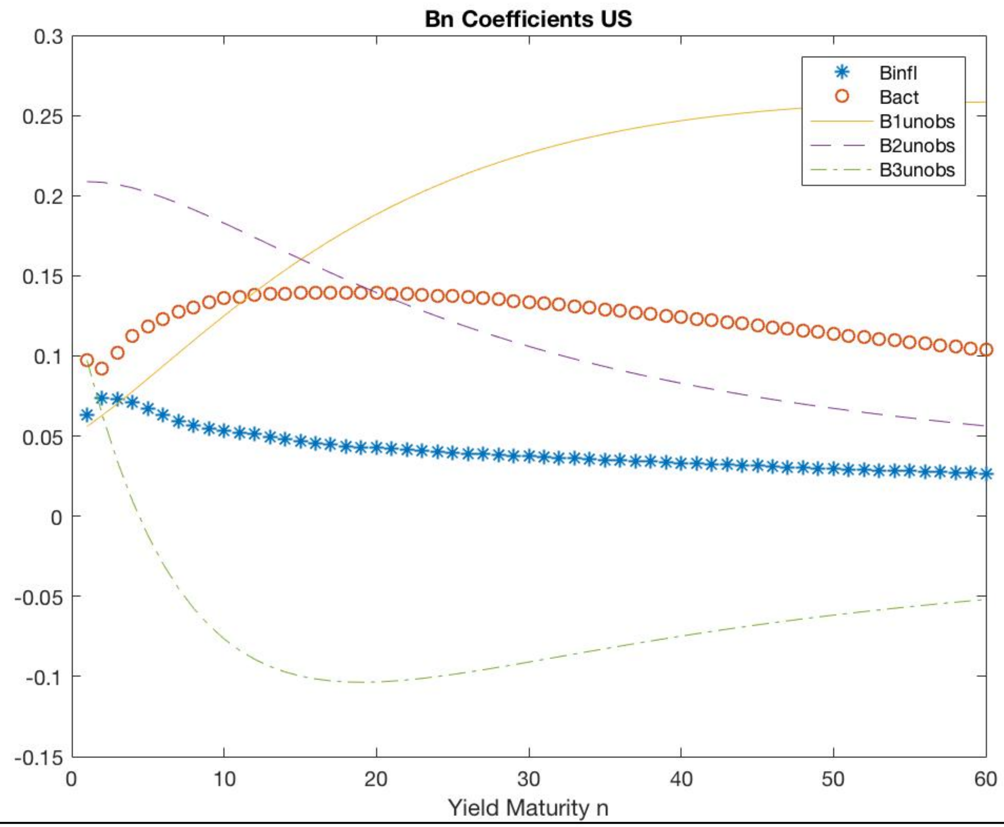
Estimation results

Impulse Response Analysis – Factor weights across the yield curve

Impulse Response – Extension of the Model

US

- The Unobs 1 and Unobs 2 factors are respectively upward and downward sloping.
- The Unobs 3 factor is hump-shaped and shocks to real activity impact the yield more than shocks to inflation. They mostly affect short yields and less so long yields. These factors act in almost the same way as the Replication model.
- The main reason of Bn coefficients is in the estimates of the Taylor rules in Short rate dynamics. Where the initial effect to Real Activity has a much stronger effect on yields than the Inflation.



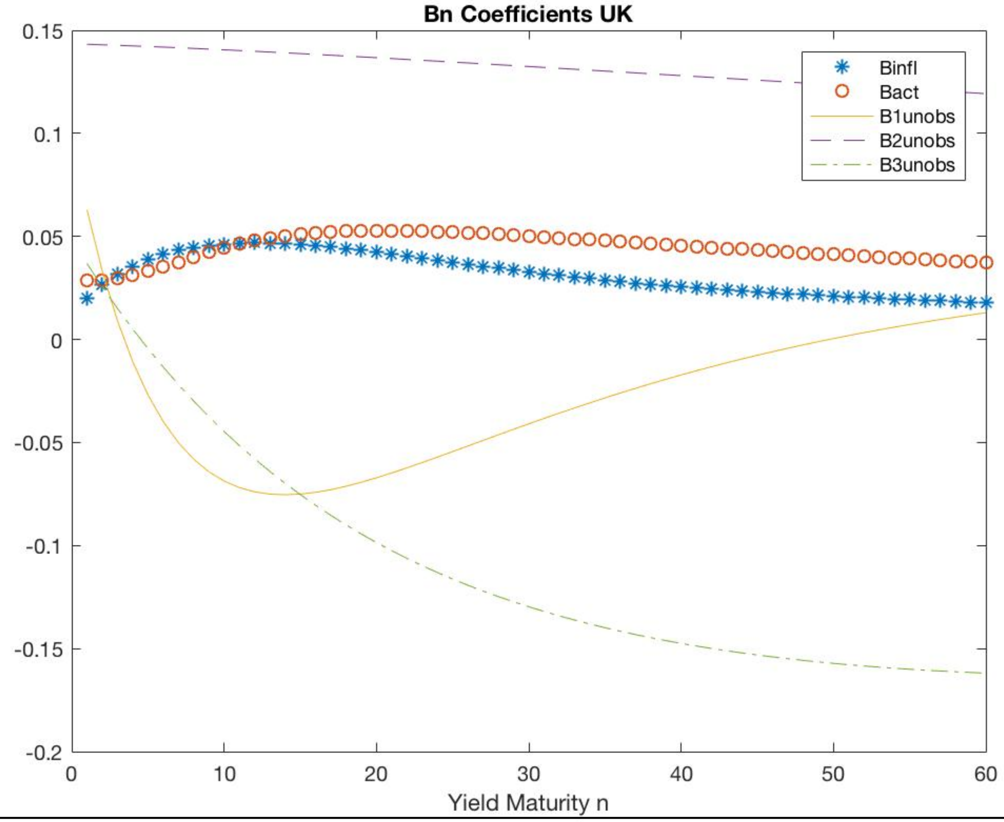
Estimation results

Impulse Response Analysis – Factor weights across the yield curve

Impulse Response – Extension of the Model

UK

- The Unobs 1 is hump-shaped.
- The Unobs 2 factor is flat.
- The Unobs 3 factor is downward sloping.
- Shocks to real activity and inflation impact the yield in the same way.
- According to the Taylor rules in Short rate dynamics, the initial effect to Inflation is slightly stronger across the yield curve than the Real Activity factor. Nevertheless, the Real Activity is more persistent than Inflation since the effect is higher up to the 11th lag.



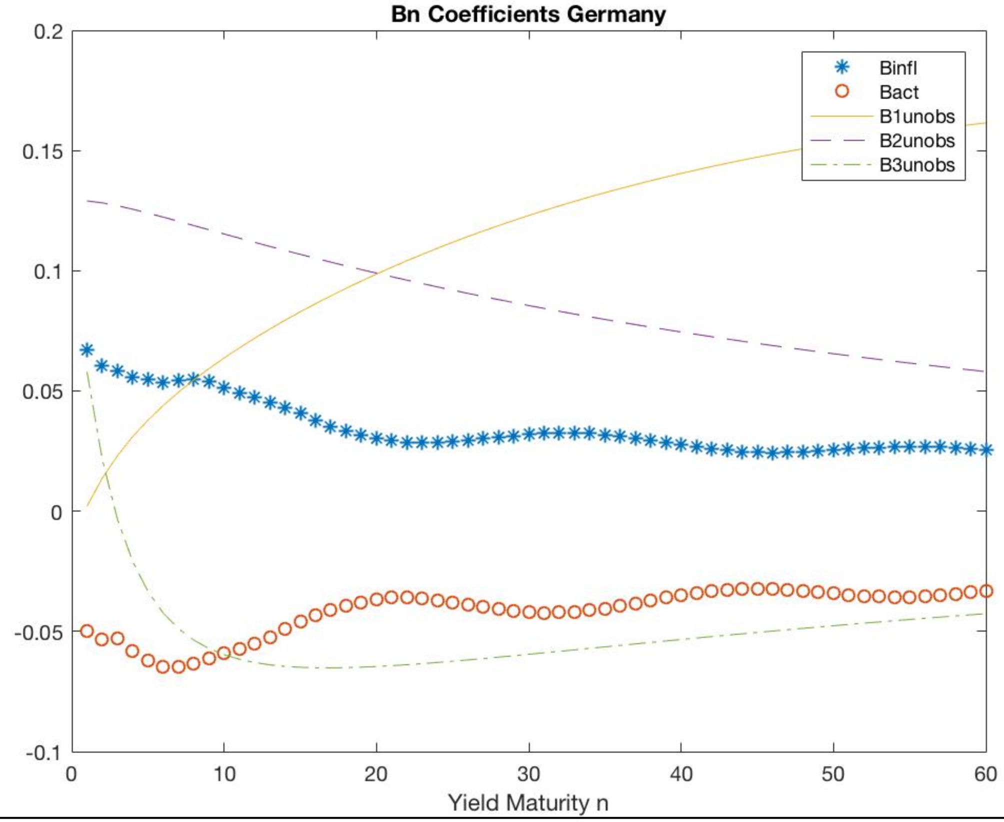
Estimation results

Impulse Response Analysis – Factor weights across the yield curve

Impulse Response – Extension of the Model

GERMANY

- The Unobs 1 is upward sloping.
- The Unobs 2 factor is downward sloping.
- The Unobs 3 factor is hump-shaped.
- Shocks to inflation impact the yield more than shocks to real activity.
- According to the Taylor rules in Short rate dynamics, the initial effect to Inflation is consistently much stronger across the yield curve than the Real Activity factor.



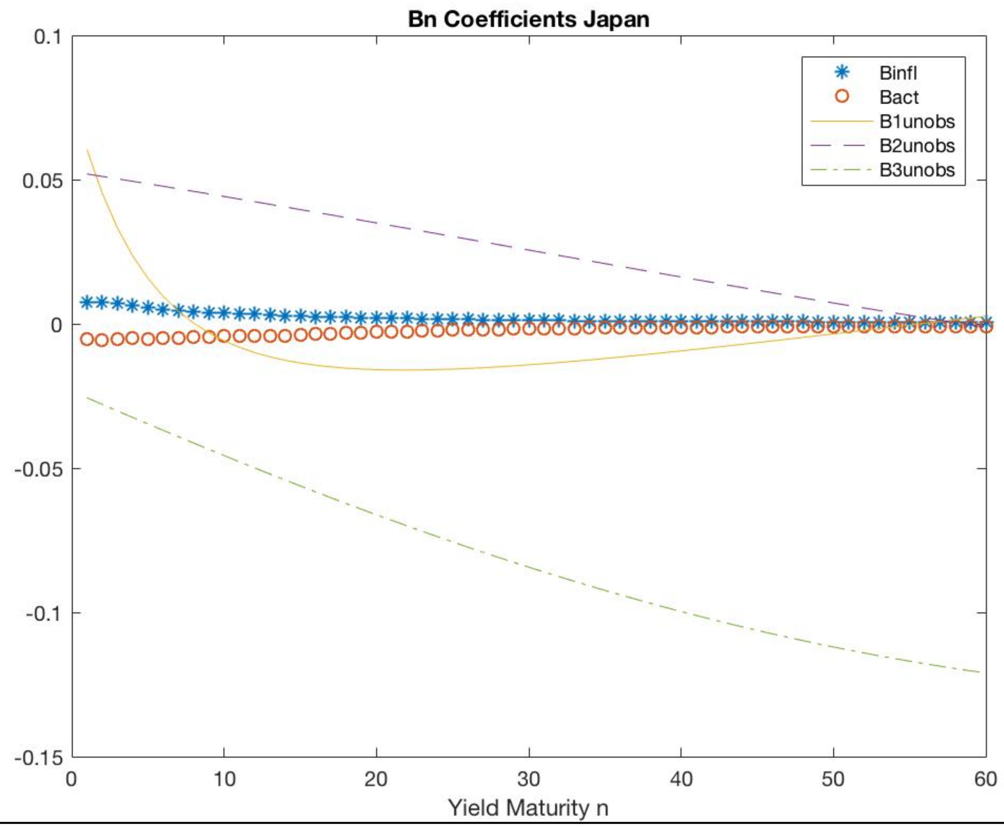
Estimation results

Impulse Response Analysis – Factor weights across the yield curve

Impulse Response – Extension of the Model

JAPAN

- The Unobs 1 is hump-shaped.
- The Unobs 2 and Unobs 3 factors are downward sloping.
- The initial shocks to inflation impact the yield more than shocks to real activity.
- According to the Taylor rules in Short rate dynamics, the effect to Inflation and Real Activity factors are very small. Moreover, the low effect persist up to the 11th lags.



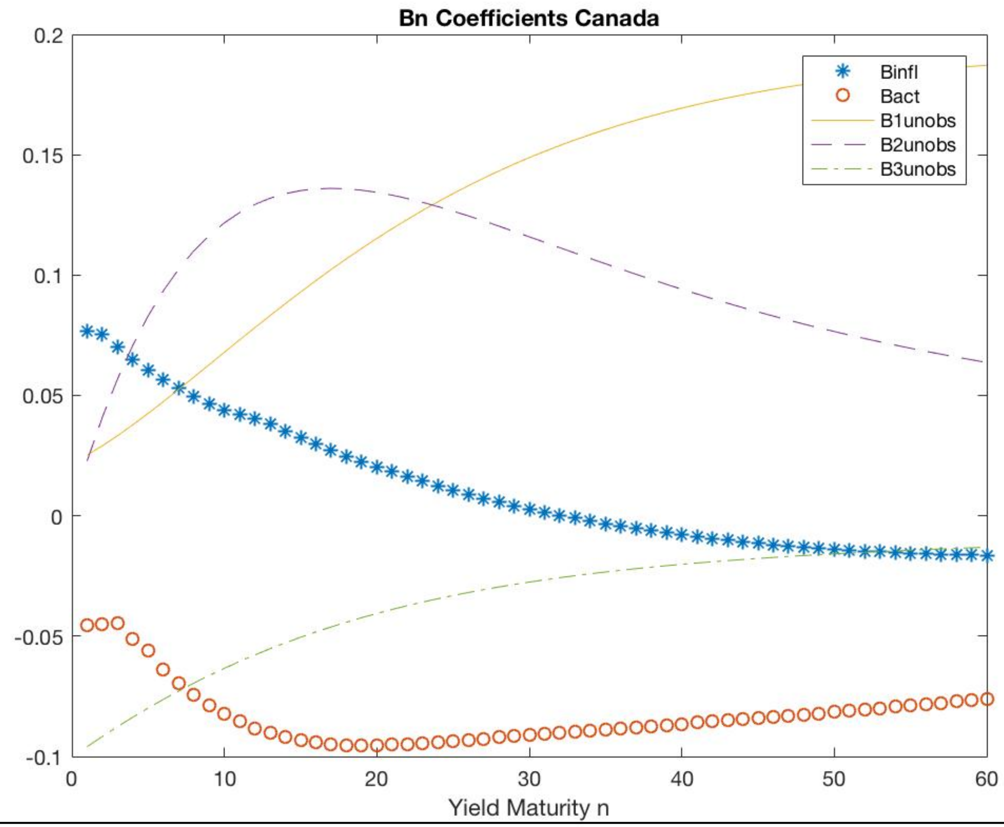
Estimation results

Impulse Response Analysis – Factor weights across the yield curve

Impulse Response – Extension of the Model

CANADA

- The Unobs 1 is upward sloping.
- The Unobs 2 factor is hump-shaped.
- The Unobs 3 factor is upward sloping.
- Shocks to inflation impact the yield more than shocks to real activity.
- According to the Taylor rules in Short rate dynamics, the initial effect to Inflation is consistently much stronger across the yield curve than the Real Activity factor.

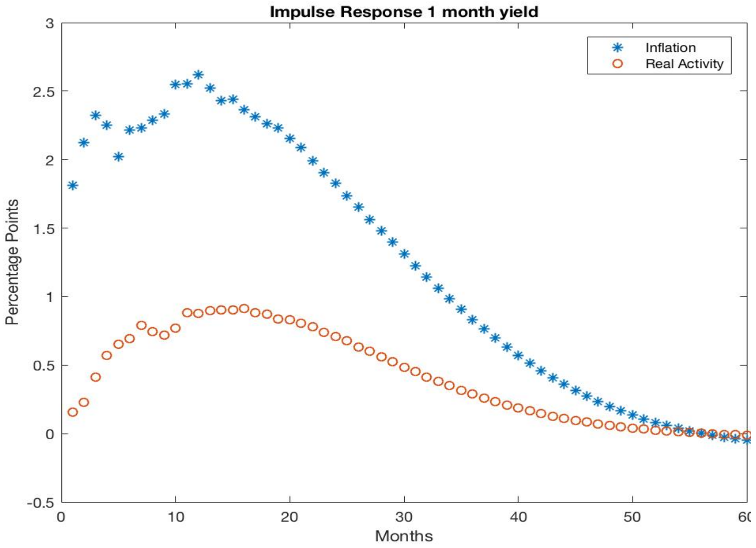


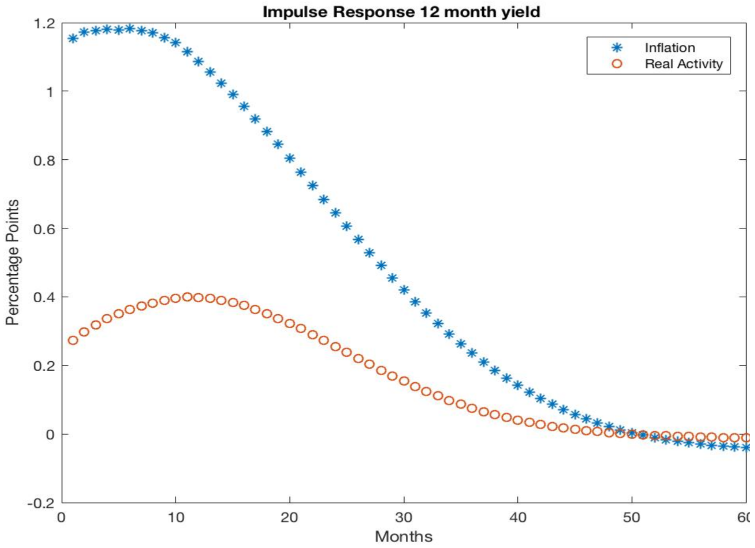
Estimation results

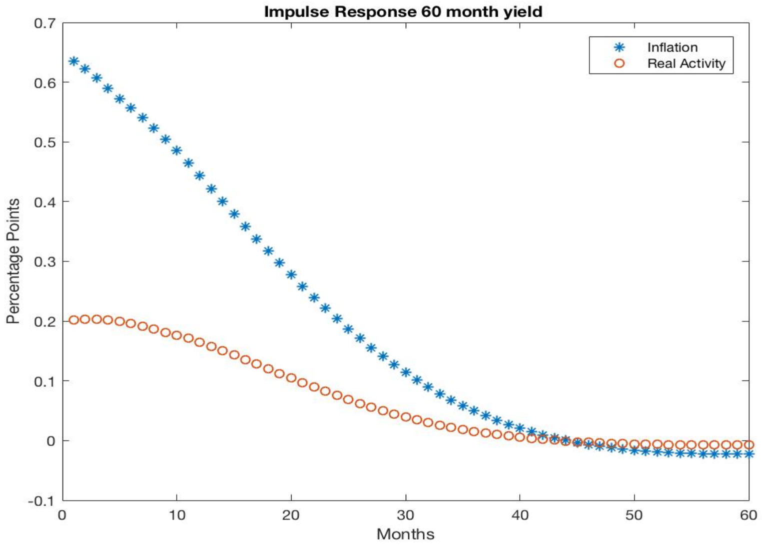
Impulse Response Analysis – From macro shocks

Impulse Response – Replication of the Model

- The effect of real activity is much smaller than the IRs from inflation shocks.
- This is due primarily to the small loading on real activity in the Taylor rule, compared to the much larger loading on inflation.







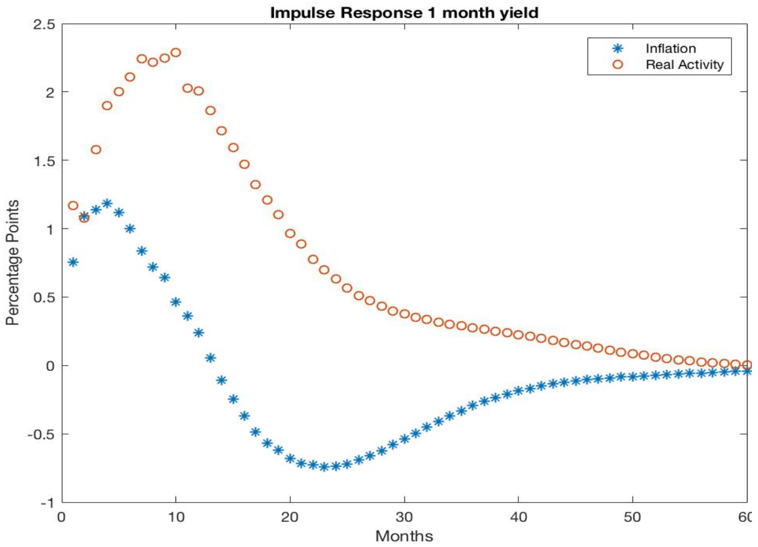
Estimation results

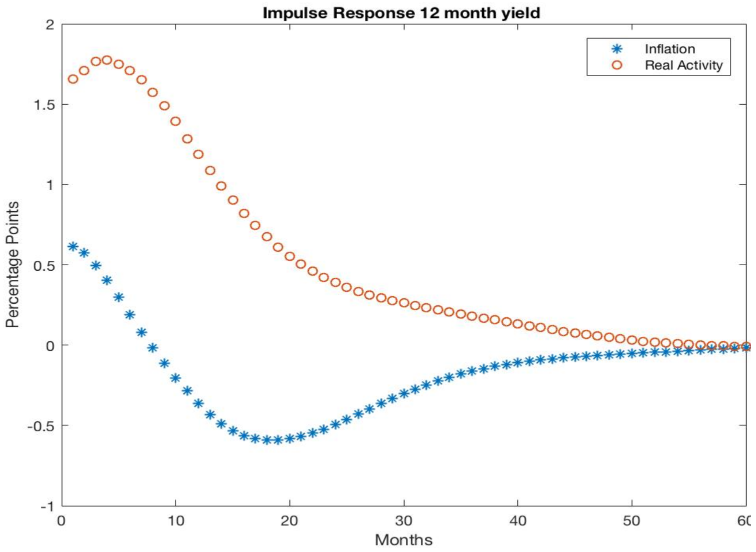
Impulse Response from macro shocks – Extension of the Model

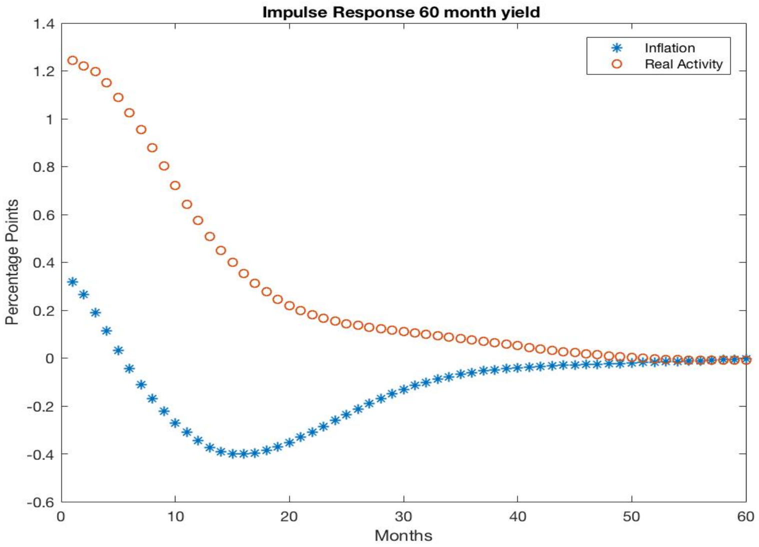
Impulse Response – Extension of the Model

US

- The effect of inflation is much smaller than the IRs from real activity shocks.
- This is due primarily to the small loading on inflation in the Taylor rule, compared to the much larger loading on real activity.







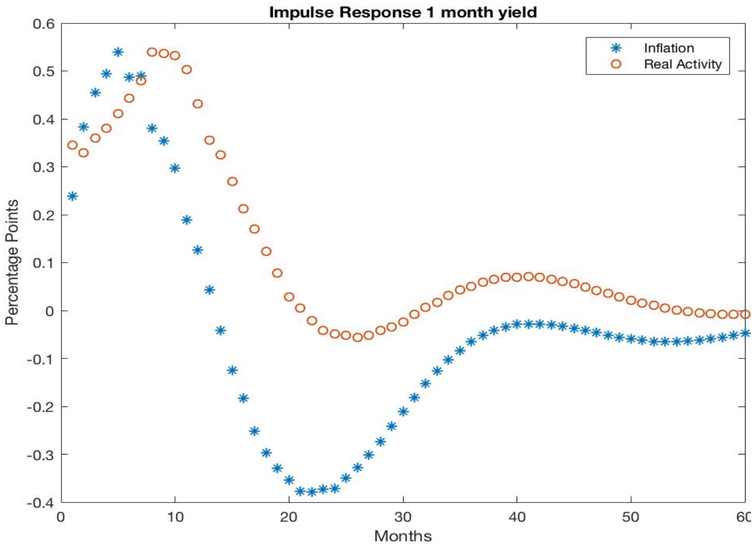
Estimation results

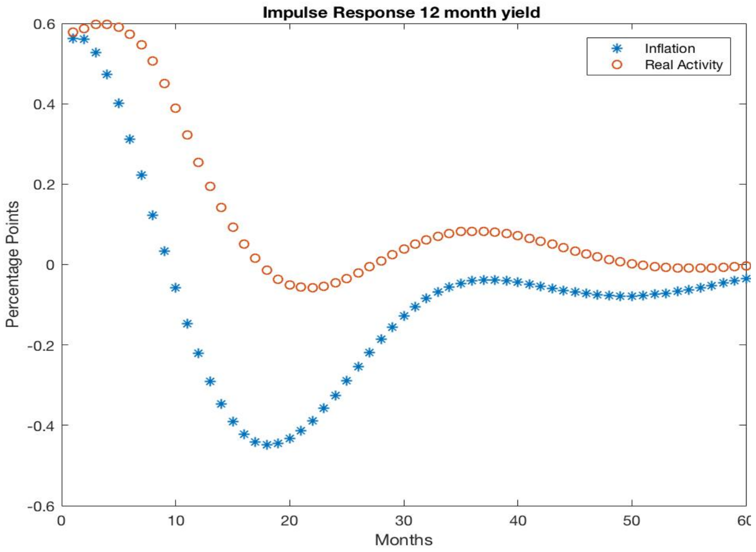
Impulse Response from macro shocks – Extension of the Model

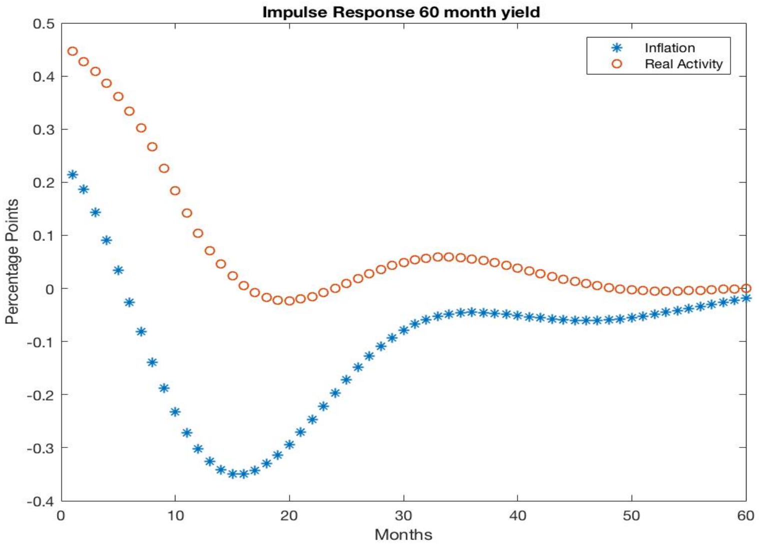
Impulse Response – Extension of the Model

UK

- The effect of inflation and real activity are almost the same order of magnitude for maturities up to 1 year. Afterwards, the inflation effect is much smaller than the IRs from real activity shocks.
- For real activity, there are much longer lagged effects. This is because the Taylor rule with lags has a significant weight on the 11th lag of real activity, which has its highest impact after 12 months.







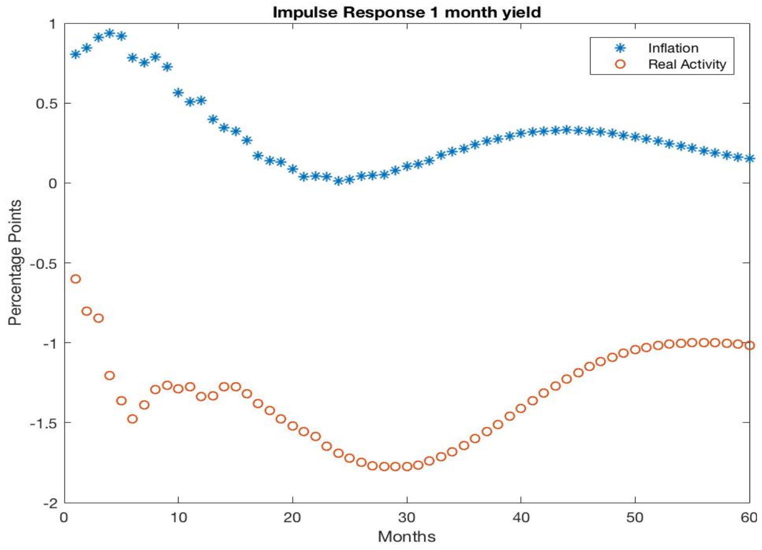
Estimation results

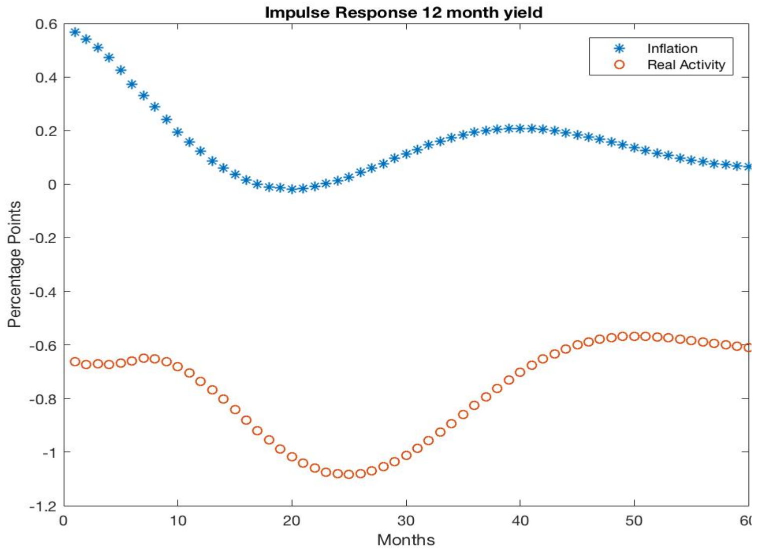
Impulse Response from macro shocks – Extension of the Model

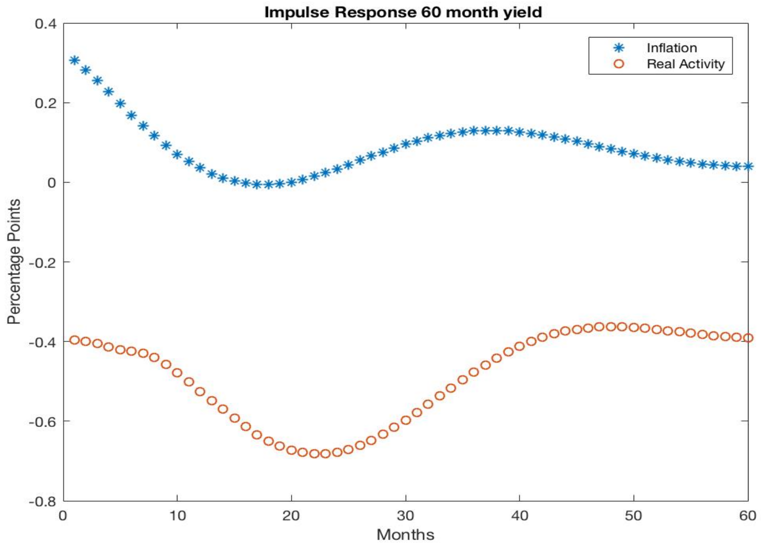
Impulse Response – Extension of the Model

GERMANY

- The effect of real activity is always smaller than the IRs from inflation shocks.
- This is due to the small loading on real activity in all the lags of the Taylor rule with lags, compared to the larger loading on inflation. The behaviour is quite similar between the three yields.







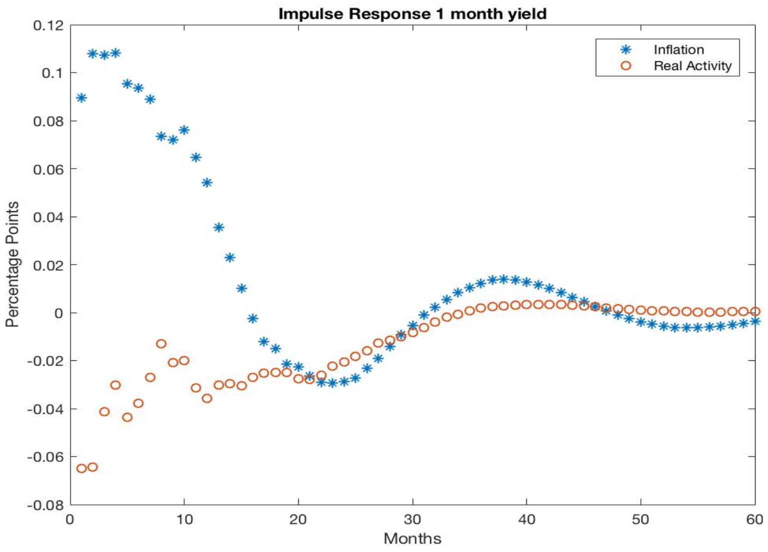
Estimation results

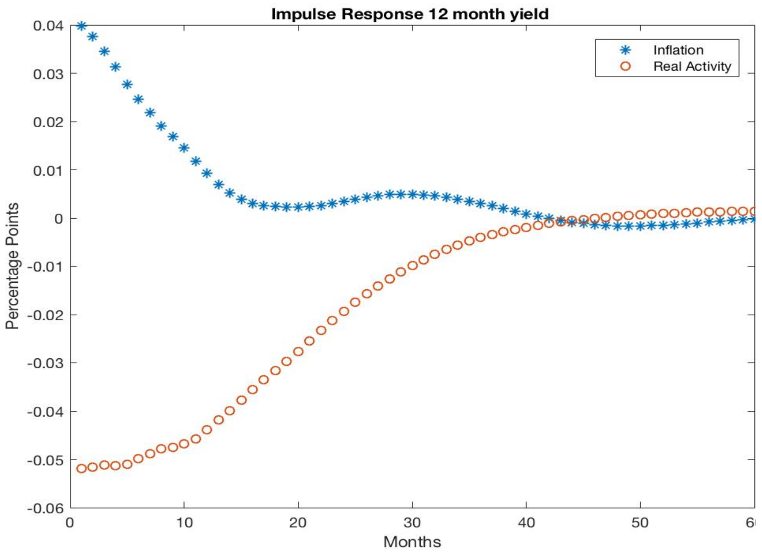
Impulse Response from macro shocks – Extension of the Model

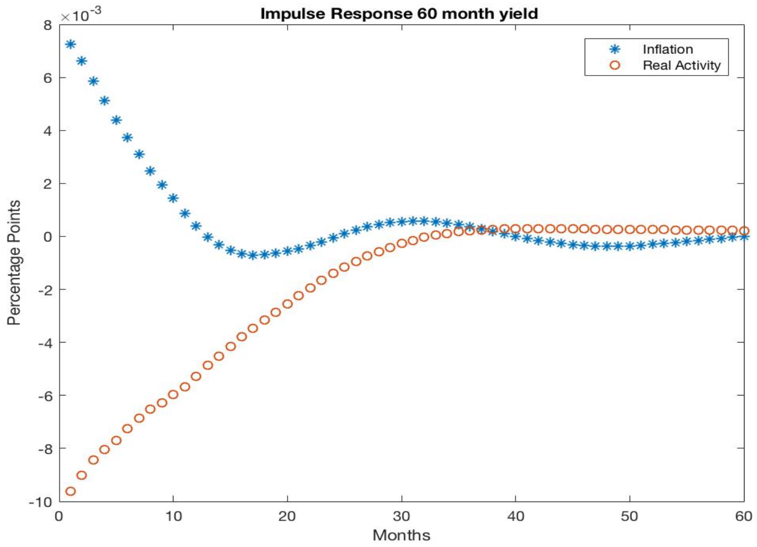
Impulse Response – Extension of the Model

JAPAN

- The effect of inflation is around 10 basis points bigger than the IRs from real activity shocks in the short end for all the yields. From the middle end the two factor have the same impact on the yield curve.
- This is due to the wide difference between the loading on real activity and on inflation.







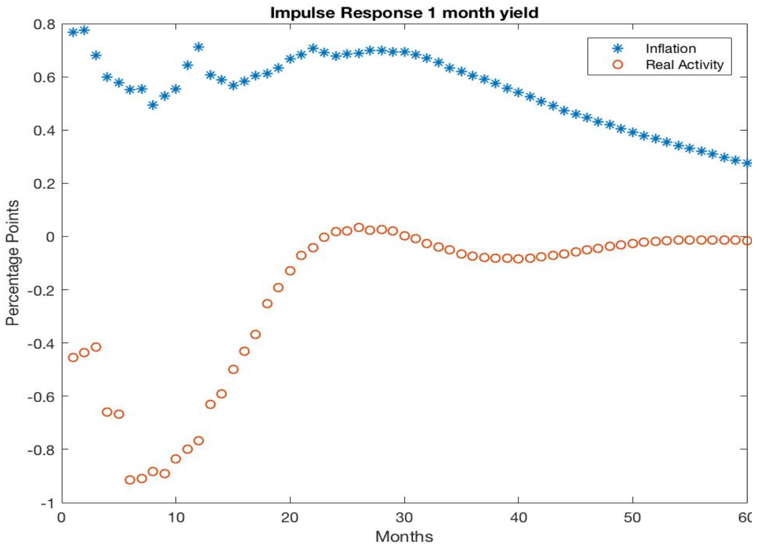
Estimation results

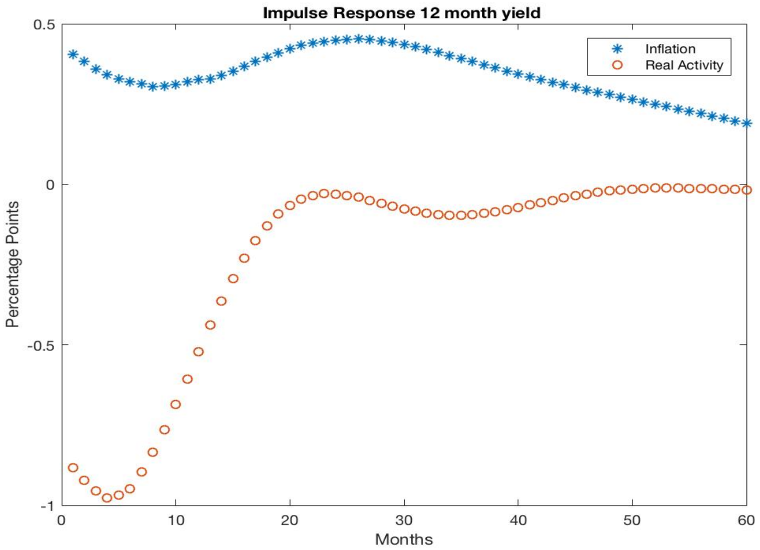
Impulse Response from macro shocks – Extension of the Model

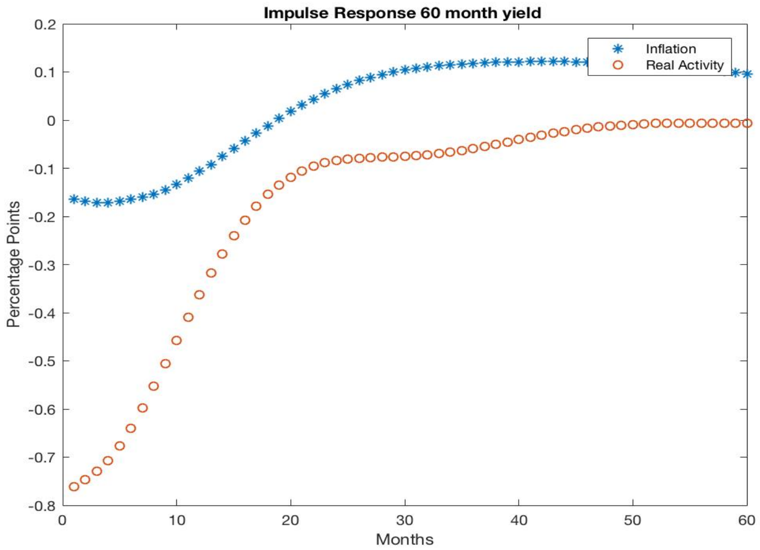
Impulse Response – Extension of the Model

CANADA

- The effect of real activity is much smaller than the IRs from inflation shocks.
- This is because the small loading on real activity in the Taylor rule with lags, compared to the much larger loading on inflation. The stronger effect of inflation persists up to the 11th lag.







Estimation results

Forecasts Analysis – RMSE

- In order to evaluate how well the model performs in terms of forecasting accuracy, a measure of Root Mean Squared Error (RMSE) was calculated.

Out-of-Sample Construction

- For the last 60 months, we did an out of sample forecasting evaluation.
- We have considered Data available until each point and run all necessary estimations including PCA and the Minimum-Chi-Square optimization for each point.
- We found that if we used previously estimated global minima parameters as starting values, consecutive estimations would likewise always converge to the global minimum. Thus in order to reduce computation time, we ran every 12 months the estimation 30 times and for the following months only once.

Period

- We only considered 1 step ahead forecasts, since the model forecasts 60 maturities at each point in time, returning a continuous yield curve.
- We also computed the percentage of how often the model predicted up or down moves in yields correctly and what is standard deviation terms the magnitude of the difference between forecasted and actual yields.

Estimation results

Forecasts Analysis – RMSE

- Out of Sample Forecasts for 60 months.
- MDF indicates the Chi-Square in the code, the lower this value is, indicates higher explanatory power of the model in an econometrical sense, but this does not necessarily affect the forecasting quality.
- In the following table, we first present the AP results for RMSE, followed by our own results for the replication and extension of the model compared with the random walk (rw) model.

Sample	Ang Piazzesi	Replication	Random Walk	US		UK		Germany		Japan		Canada	
				MLM	RW	MLM	RW	MLM	RW	MLM	RW	MLM	RW
Yield Sample	1951-2000	1951-2000	1951-2000	1988-2017		1997-2017		1999-2017		1995-2017		2001-2017	
Forecast Period				Last 60 months of the sample									
MDF	530.69	430.07	-	932.37	-	931.67	-	504.87	-	425.71	-	875.78	-
1M	0,3906	0,3254	0,3160	0,0966	0,0516	0,1190	0,0324	0,1599	0,0281	0,0233	0,0212	0,0959	0,0639
3M	0,2876	0,1937	0,1523	0,1376	0,0433	0,1145	0,0269	0,1570	0,0255	0,0254	0,0154	0,1130	0,0639
1Y	0,2274	0,1868	0,1991	0,0821	0,0578	0,1078	0,0598	0,0955	0,0497	0,0243	0,0230	0,1037	0,0999
3Y	0,2665	0,2448	0,2494	0,2206	0,1534	0,1753	0,1550	0,1294	0,0980	0,0476	0,0391	0,1689	0,1440
5Y	0,2530	0,2496	0,2543	0,1903	0,1887	0,1953	0,1926	0,1369	0,1288	0,0517	0,0501	0,1742	0,1718

- Our replication obtained significantly lower RMSE than what AP reported . This is mostly attributable to having obtained the global minimum instead of a local one.
- The overall lower values from the replicating countries can be explained that recent yields have been lower in general (particularly true for Japan), fact confirmed by the even lower random walk's RMSE.

Estimation results

Forecasts Analysis – Percentage of correctly predicted yield direction

- Although RMSE is a better measurement of the overall forecasting power of the model, we further inspected the predictions to analyze if the forecasted yields match the direction of the actual ones (if any predicted increase/decrease matches an actual increase/decrease in the yields).
- The values reported in the table below identifies the percentage of correctly predicted yield direction.

Sample	US	UK	Germany	Japan	Canada
Yield Sample	1988-2017	1997-2017	1999-2017	1995-2017	2001-2017
Forecast Period			Last 60 months of the sample		
MDF	932.37.00	931.67	504.87	425.71	875.78
1M	60.00%	50.00%	68.33%	31.67%	63.33%
3M	55.00%	45.00%	61.67%	45.00%	58.33%
1Y	56.67%	55.00%	45.00%	43.33%	58.33%
3Y	40.00%	55.00%	46.67%	50.00%	50.00%
5Y	53.33%	46.67%	56.67%	45.00%	53.33%

- The signals usually are between 40% and 60% depending on the yield and the country, thus not necessarily better than a coin-flip.
- Canada is the only country where the model predicted the signs correctly more than half of the time across all maturities.

Estimation results

Forecasts Analysis – Measuring magnitude of forecasted yields

- Focusing only on the forecasted yields whose direction has been correctly predicted, the accuracy of the prediction is here measured through the following formula : $(Yf_t$ - $Y_t)$ / σ_{12m} where Yf_t is the forecasted yield at time t, Y_t is the actual yield at time t and σ_{12m} is the standard deviation of the 12 previous months. The results obtained are then grouped in intervals that represents the difference in yields in standard deviation term.
- The analysis is performed over the forecasted yields in the last 60 months of the sample.

	US				UK				Germany				Japan				Canada			
	[-2,2]	[-1,1]	[-0.5,0.5]	[-0.25,0.25]	[-2,2]	[-1,1]	[-0.5,0.5]	[-0,25,0,25]	[-2,2]	[-1,1]	[-0.5,0.5]	[-0.25,0.25]	[-2,2]	[-1,1]	[-0.5,0.5]	[-0.25,0.25]	[-2,2]	[-1,1]	[-0.5,0.5]	[-0.25,0.25]
1M	73,0%	51,4%	35,1%	13,5%	20,7%	10,3%	6,9%	0,0%	58,3%	36,1%	22,2%	8,3%	90,9%	77,3%	59,1%	27,3%	45,2%	31,0%	19,0%	11,9%
3M	81,8%	63,6%	27,3%	21,2%	48,1%	18,5%	3,7%	3,7%	61,1%	44,4%	30,6%	19,4%	91,7%	50,0%	41,7%	25,0%	41,0%	35,9%	20,5%	15,4%
1Y	96,4%	82,1%	64,3%	50,0%	87,9%	60,6%	42,4%	21,2%	82,1%	64,3%	46,4%	42,9%	100,0%	100,0%	96,2%	73,1%	97,3%	91,9%	75,7%	43,2%
3Y	95,8%	79,2%	50,0%	29,2%	97,0%	90,9%	66,7%	42,4%	100,0%	76,5%	44,1%	26,5%	100,0%	93,9%	72,7%	42,4%	95,7%	87,0%	60,9%	56,5%
5Y	93,8%	87,5%	50,0%	34,4%	92,9%	85,7%	50,0%	25,0%	100,0%	87,9%	63,6%	42,4%	100,0%	81,5%	70,4%	51,9%	93,8%	84,4%	65,6%	46,9%

- The results show that the forecasted yields are more accurate for high maturities. 50% and more of the magnitude measures stands in fact in the |0.5σ| interval. Nonetheless the differences between forecasted and actual yields is quite high given the fact that, analysing in the last 60 months the difference between Y_t and Y_{t-1} in standard deviation terms, 50% of the magnitude measures stands in the |0.25σ| interval.

Estimation results

Different Periods Analysis

- Given the current environment for interest rates we decided to do an estimation of the model for other periods. This is mainly because since interest rates nowadays are close to zero, their variability is lower which might make them seem more predictable through these models. This could result in great forecast measures for the model while that is not due to the models intrinsic value and quality of predictions but thanks to this previously stated bias of predictability.

Different Periods Process

- For the analysis of different periods we estimated the model with an expanding window starting from the beginning of the data available for each country until December 2004.
- We then do a one step ahead forecast and save the RMSE for that forecast.
- Afterwards, we expand the previous window of estimation one month, redo the estimation, do again a new one step ahead forecast and save the RMSE from that estimation.
- In doing this analysis we end up with measures for one step ahead forecasts for monthly estimations of the model from the end of 2004 until November 2017.

Estimation results

Different Periods Analysis–RMSE Macro Lag Model(MLM) vs Random Walk(RW)

		US		UK		Germany		Japan		Canada	
		MLM	RW	MLM	RW	MLM	RW	MLM	RW	MLM	RW
1M	Total 2004-2017	0,35	0,2668	0,26	0,2485	0,23	0,1833	0,07	0,0705	0,18	0,1506
3M		0,31	0,2297	0,20	0,2283	0,17	0,1654	0,06	0,0419	0,16	0,1586
1Y		0,23	0,1960	0,24	0,2115	0,19	0,1670	0,05	0,0468	0,18	0,1843
3Y		0,35	0,2261	0,25	0,2186	0,21	0,1798	0,09	0,0716	0,22	0,2085
5Y		0,27	0,2436	0,24	0,2179	0,19	0,1784	0,09	0,0832	0,22	0,2198
1M	Pre crisis 2004-2007	0,21	0,1349	0,08	0,0921	0,13	0,1026	0,06	0,0672	0,28	0,1119
3M		0,20	0,1295	0,08	0,0952	0,10	0,0885	0,06	0,0567	0,19	0,1074
1Y		0,17	0,1576	0,13	0,1366	0,09	0,1225	0,08	0,0652	0,16	0,1605
3Y		0,21	0,2052	0,17	0,1520	0,16	0,1561	0,15	0,1071	0,16	0,1690
5Y		0,20	0,2095	0,16	0,1547	0,16	0,1593	0,14	0,1228	0,17	0,1656
1M	Recent 2014-2017	0,11	0,0722	0,12	0,0453	0,07	0,0198	0,03	0,0296	0,09	0,0716
3M		0,12	0,0603	0,10	0,0372	0,07	0,0172	0,03	0,0210	0,10	0,0713
1Y		0,06	0,0763	0,07	0,0682	0,07	0,0297	0,03	0,0312	0,09	0,0987
3Y		0,22	0,1613	0,17	0,1613	0,07	0,0653	0,06	0,0502	0,15	0,1226
5Y		0,18	0,1813	0,19	0,1915	0,11	0,1011	0,06	0,0550	0,14	0,1420

Estimation results

Different Periods Analysis – Percentage of correctly predicted yields' direction

	1M	3M	1Y	3Y	5Y
			Entire forecasted period 2004-2017		
US	55,5%	48,4%	41,9%	41,3%	48,4%
Germany	54,2%	64,5%	49,7%	49,0%	49,0%
UK	54,8%	57,4%	54,8%	52,3%	46,5%
Canada	69,7%	65,2%	65,2%	51,6%	53,5%
Japan	45,8%	49,0%	51,0%	51,6%	47,1%
			Pre crisis 2004-2007		
US	50,0%	43,3%	30,0%	50,0%	60,0%
Germany	73,3%	73,3%	80,0%	53,3%	46,7%
UK	76,7%	73,3%	70,0%	50,0%	53,3%
Canada	66,7%	60,0%	66,7%	73,3%	50,0%
Japan	60,0%	56,7%	43,3%	33,3%	40,0%
			Recent 2014-2017		
US	73,3%	73,3%	73,3%	36,7%	56,7%
Germany	76,7%	70,0%	36,7%	53,3%	50,0%
UK	43,3%	40,0%	53,3%	53,3%	50,0%
Canada	63,3%	60,0%	56,7%	36,7%	53,3%
Japan	56,7%	50,0%	56,7%	50,0%	53,3%

- In the entire forecasted period the corrected yields' directions range between 40% and 70%.
- Depending on the maturity and the country, the predictions become worse or better in the two other analysed periods, with relevant decreases in the prediction power in some cases.

Estimation results

Different Periods Analysis – Measuring magnitude forecasted yields

		Total forecasted period 2004-2017					Pre crisis 2004-2007					Recent 2014-2017				
		1M	**3M**	**1Y**	**3Y**	**5Y**	**1M**	**3M**	**1Y**	**3Y**	**5Y**	**1M**	**3M**	**1Y**	**3Y**	**5Y**
US	[-2,2]	67,4%	74,7%	90,8%	95,3%	96,0%	100,0%	100,0%	100,0%	100,0%	100,0%	90,9%	95,5%	100,0%	90,9%	100,0%
	[-1,1]	51,2%	61,3%	83,1%	81,3%	89,3%	80,0%	100,0%	100,0%	86,7%	83,3%	68,2%	72,7%	95,5%	72,7%	100,0%
	[-0.5,0.5]	37,2%	36,0%	66,2%	62,5%	60,0%	66,7%	61,5%	88,9%	73,3%	44,4%	45,5%	31,8%	77,3%	54,5%	58,8%
	[-0.25,0.25]	19,8%	29,3%	47,7%	32,8%	42,7%	40,0%	46,2%	66,7%	40,0%	33,3%	18,2%	22,7%	59,1%	36,4%	41,2%
UK	[-2,2]	51,8%	73,0%	85,9%	98,8%	97,2%	95,7%	100,0%	100,0%	100,0%	100,0%	30,8%	66,7%	87,5%	93,8%	86,7%
	[-1,1]	40,0%	55,1%	68,2%	91,4%	88,9%	78,3%	86,4%	100,0%	100,0%	93,8%	15,4%	33,3%	81,3%	87,5%	80,0%
	[-0.5,0.5]	31,8%	39,3%	52,9%	66,7%	55,6%	73,9%	63,6%	76,2%	73,3%	62,5%	7,7%	8,3%	81,3%	75,0%	46,7%
	[-0.25,0.25]	15,3%	25,8%	29,4%	37,0%	30,6%	34,8%	45,5%	42,9%	33,3%	31,3%	0,0%	8,3%	43,8%	43,8%	20,0%
DE	[-2,2]	77,4%	85,0%	90,9%	100,0%	98,7%	90,9%	95,5%	100,0%	100,0%	100,0%	60,9%	66,7%	63,6%	100,0%	100,0%
	[-1,1]	59,5%	75,0%	77,9%	85,5%	85,5%	77,3%	90,9%	91,7%	93,8%	85,7%	47,8%	47,6%	54,5%	68,8%	86,7%
	[-0.5,0.5]	41,7%	58,0%	63,6%	60,5%	60,5%	63,6%	77,3%	79,2%	81,3%	42,9%	30,4%	42,9%	54,5%	43,8%	60,0%
	[-0.25,0.25]	22,6%	38,0%	51,9%	38,2%	42,1%	45,5%	54,5%	62,5%	56,3%	35,7%	13,0%	28,6%	45,5%	25,0%	33,3%
JP	[-2,2]	74,6%	71,1%	97,5%	100,0%	98,6%	77,8%	76,5%	92,3%	100,0%	100,0%	88,2%	100,0%	100,0%	100,0%	100,0%
	[-1,1]	49,3%	46,1%	87,3%	92,5%	86,3%	50,0%	58,8%	53,8%	70,0%	75,0%	82,4%	66,7%	100,0%	86,7%	87,5%
	[-0.5,0.5]	32,4%	32,9%	73,4%	71,3%	64,4%	27,8%	41,2%	38,5%	60,0%	66,7%	58,8%	53,3%	94,1%	53,3%	75,0%
	[-0.25,0.25]	14,1%	25,0%	50,6%	40,0%	41,1%	16,7%	35,3%	23,1%	10,0%	25,0%	29,4%	33,3%	64,7%	33,3%	43,8%
CA	[-2,2]	66,7%	68,3%	99,0%	98,8%	97,6%	80,0%	88,9%	100,0%	100,0%	100,0%	89,5%	77,8%	94,1%	100,0%	93,8%
	[-1,1]	50,9%	62,4%	92,1%	88,8%	86,7%	70,0%	88,9%	90,0%	86,4%	86,7%	68,4%	72,2%	94,1%	90,9%	81,3%
	[-0.5,0.5]	38,9%	46,5%	73,3%	67,5%	67,5%	45,0%	44,4%	70,0%	72,7%	66,7%	42,1%	38,9%	76,5%	63,6%	62,5%
	[-0.25,0.25]	26,9%	30,7%	42,6%	47,5%	43,4%	20,0%	27,8%	35,0%	36,4%	20,0%	26,3%	27,8%	41,2%	63,6%	43,8%

Conclusions

Conclusions

- In conclusion, the general extension of the model to the other countries and the increased sample for the US yielded positive results. We managed to incorporate the initial code from AP with the Chi-square optimization from the HW paper. This yielded better RMSE values, which proved that the model fared better at forecasting. Comparing the replication RMSE with the AP's RMSE, we get lower values for every maturity and this is evidence that the optimization works well (since the data sample is very similar).
- Although AP reach the conclusion that the model is better at forecasting for certain parts of the yield curve, there is no universal conclusion that the model performs best for certain parts of the yield curve (short, mid or long term) across countries.
- There was an overall improvement of the code, making it easier to expand to newer countries and also to be incorporated in an active investment strategy.
- Nevertheless, we would like to be conservative in our conclusions and state that, although, our results were positive in comparison to previous studies, it is still too soon to trust such models in their entirety. This is a consequence of the lack of data available on the market and the amount of assumptions that are assumed and that in a real life situation do not hold. For example, it might be too optimistic to assume that the unobservable factors are independent from observable factors and that interest rates follow a normal distribution.
- Forecasts improve if we deliberately reduce the estimation period, since there was a structural break in the financial crisis. The period that returns the overall best RMSE measures is the one from 2014 to 2017.
- The yield curve predictions for December 2017 can be found in appendix B.

Appendix B - Prediction of monthly yield curve for December, 2017

Appendix

Appendix

Appendix A – Data

- US macro data

US

	Inflation			Real Activity			
	CPI	PCOM	PPI	HELP	EMPLOY	IP	UE
Time Frame	Monthly 01/1951 – 10/2017	Daily 01/1951 – 11/2017	Monthly 01/1951- 10/2017	Monthly 01/1951- 09/2009	Monthly 01/1951- 10/2017	Monthly 01/1951- 10/2017	Monthly 01/1951- 10/2017
Registration date	15th of the month	2nd of the month	15th of the month	15th of the month	15th of the month	15th of the month	15th of the month
Release dates	Last: 18/12/2017 Next: 13/01/2018	–	Last: 13/12/2017 Next: 12/01/2018	–	Last: 01/12/2017 Next: 06/01/2018	Last: 15/12/2017 Next: 17/01/2018	Last: 01/12/2017 Next: 06/01/2018
Adjustment	Price Index Seasonally Adjusted	Commodity Index	Price Index Seasonally Adjusted	Volume or constant price value index Seasonally Adjusted	Volumes Seasonally Adjusted	Volume or constant price value index Seasonally Adjusted	Seasonally Adjusted
Source	*Bureau of Labor statistics, U.S. department of Labor*	*Commodity Research Bureau*	*Bureau of Labor statistics, U.S. department of Labor*	*The Conference Board*	*Bureau of Labor statistics, U.S. department of Labor*	*Federal Reserve United States*	*Bureau of Labor statistics, U.S. department of Labor*
Ticker	USCONPRCE	CRBSPOT	USPFDOFGE	USBCINHAG	USEMPTOTO	USIPTOT.G	USUN%TOTQ

Appendix A – Data

- UK macro data

UK

	Inflation			Real Activity			
	RPI	CCI	PPI	JOB VACANCIES	EMPLOY	IP	UE
Time Frame	Monthly 01/1950 – 10/2017	Daily 09/1956 – 11/2017	Monthly 01/1957 – 10/2017	Monthly 07/1958 – 08/2017	Monthly 02/1971 – 08/2017	Monthly 01/1968 – 09/2017	Monthly 02/1971 – 08/2017
Registration date	15th of the month	28th of the month	15th of the month	15th of the month	Last day of the month	15th of the month	Last day of the month
Release dates	Last: 12/12/2017 Next: 17/01/2018	–	Last: 13/12/2017 Next: n/a	Last: 05/12/2017 Next: n/a	Last: 13/12/2017 Next: 18/01/2018	Last: 08/12/2017 Next: 10/01/2018	Last: 13/12/2017 Next: 18/01/2018
Adjustment	Price Index Not Seasonally Adjusted	Commodity Index	Price Index Not Seasonally Adjusted	Volumes Seasonally Adjusted	Volumes Seasonally Adjusted	Volume or constant price value index Seasonally Adjusted	Seasonally Adjusted
Source	*ONS Office for National Statistics, UK*	*Thomson Reuters*	*Main Economic Indicators, copyright OECD*	*Main Economic Indicators, copyright OECD*	*ONS Office for National Statistics, UK*	*ONS Office for National Statistics, UK*	*ONS Office for National Statistics, UK*
Ticker	UKCHAW..	NYFECRB	UKOPIMP2F	UKMLM004O	UKLF2G..O	UKIPTOT.G	UKUN%O16Q

Appendix

Appendix A – Data

- Germany macro data

GERMANY

	Inflation			Real Activity			
	CPI	CCI	PPI	JOB VACANCIES	EMPLOY	IP	UE
Time Frame	Monthly 01/1950 – 11/2017	Daily 09/1956 – 11/2017	Monthly 01/1950 – 10/2017	Monthly 01/1955 – 10/2017	Monthly 01/1992 – 10/2017	Monthly 01/1952 – 09/2017	Monthly 01/1950 – 11/2017
Registration date	15th of the month	28th of the month	15th of the month	15th of the month	15th of the month	15th of the month	15th of the month
Release dates	Last: 02/12/2017 Next: 30/12/2017	–	Last: 20/12/2017 Next: 20/01/2018	Last: 05/12/2017 Next: n/a	Last: 29/12/2017 Next: 30/01/2018	Last: 07/12/2017 Next: n/a	Last: 29/12/2017 Next: 02/01/2018
Adjustment	Price Index Seasonally Adjusted	Commodity Index	Price Index Seasonally Adjusted	Volumes Seasonally Adjusted	Volumes Seasonally Adjusted	Volume or constant price value index Seasonally Adjusted	Seasonally Adjusted
Source	*Deutsche Bundesbank*	*Thomson Reuters*	*Federal Statistical Office, Germany*	*Main Economic Indicators OECD*	*Bundesagentur fur Arbeit, Germany*	*Deutsche Bundesbank*	*Deutsche Bundesbank*
Ticker	BDCONPRCE	NYFECRB	BDCPPI..E	BDMLM004O	BDEMPTOTO	BDIPMMQLG	WGUN%TOTQ

Appendix

Appendix A – Data

- Japan macro data

JAPAN

	Inflation			Real Activity			
	CPI	CCI	PPI	JOB VACANCIES	EMPLOY	IP	UE
Time Frame	Monthly 01/1955 – 10/2017	Daily 09/1956 – 11/2017	Monthly 01/1960 – 10/2017	Monthly 01/1960 – 10/2017	Monthly 01/1953 – 10/2017	Monthly 01/1953 – 10/2017	Monthly 01/1953 – 10/2017
Registration date	15th of the month	28th of the month	15th of the month	15th of the month	15th of the month	15th of the month	15th of the month
Release dates	Last: 29/12/2017 Next: n/a	–	Last: 12/12/2017 Next: 11/01/2018	Last: 28/12/2017 Next: 30/01/2018	Last: 29/12/2017 Next: 30/01/2018	Last: 29/12/2017 Next: 16/01/2018	Last: 29/12/2017 Next: 30/01/2018
Adjustment	Price Index not Seasonally Adjusted	Commodity Index	Price Index Seasonally Adjusted	Volumes Seasonally Adjusted	Volumes Seasonally Adjusted	Volume or constant price value index Seasonally Adjusted	Seasonally Adjusted
Source	*Main Economic Indicators, OECD*	*Thomson Reuters*	*Ministry of Internal Affairs and Communication, Japan*	*Ministry of Health, Labour and Welfare*	*Ministry of Internal Affairs and Communication, Japan*	*Ministry of Economy, Trade and Industry, Japan (METI)*	*Ministry of Internal Affairs and Communication, Japan*
Ticker	JPOCP009F	NYFECRB	JPCPPI..E	JPVACTOTO	JPEMPTOTO	JPCIND..G	JPUN%TOTQ

Appendix

Appendix A – Data

- Canada macro data

CANADA

	Inflation			Real Activity			
	CPI	BCPI	PPI	HELP	EMPLOY	IP	UE
Time Frame	Monthly 01/1950 – 10/2017	Daily 01/1972 – 11/2017	Monthly 01/1956 – 10/2017	–	Monthly 01/1976 – 11/2017	Monthly 01/1961- 09/2017	Monthly 01/1966 – 11/2017
Registration date	15th of the month	12nd of the month	15th of the month	–	15th of the month	15th of the month	15th of the month
Release dates	Last: 22/12/2017 Next: 20/01/2018	–	Last: 29/12/2017 Next: 31/01/2018	–	Last: 08/12/2017 Next: 06/01/2018	Last: 29/12/2017 Next: n/a	Last: 08/12/2017 Next: 06/01/2018
Adjustment	Price Index not Seasonally Adjusted	Commodity Index	Price Index Seasonally Adjusted	–	Volumes Seasonally Adjusted	Volume or constant price value index Seasonally Adjusted	Seasonally Adjusted
Source	*CANSIM – Statistics Canada*	*Bank of Canada*	*CANSIM – Statistics Canada*	–	*CANSIM – Statistics Canada*	*CANSIM – Statistics Canada*	*CANSIM – Statistics Canada*
Ticker	CNCONPRCF	BCPITOT	CNCPPI..E	–	CNEMPTOTO	CNIP7500G	CNUN%TOTQ

Appendix A – Data

- Yields across countries

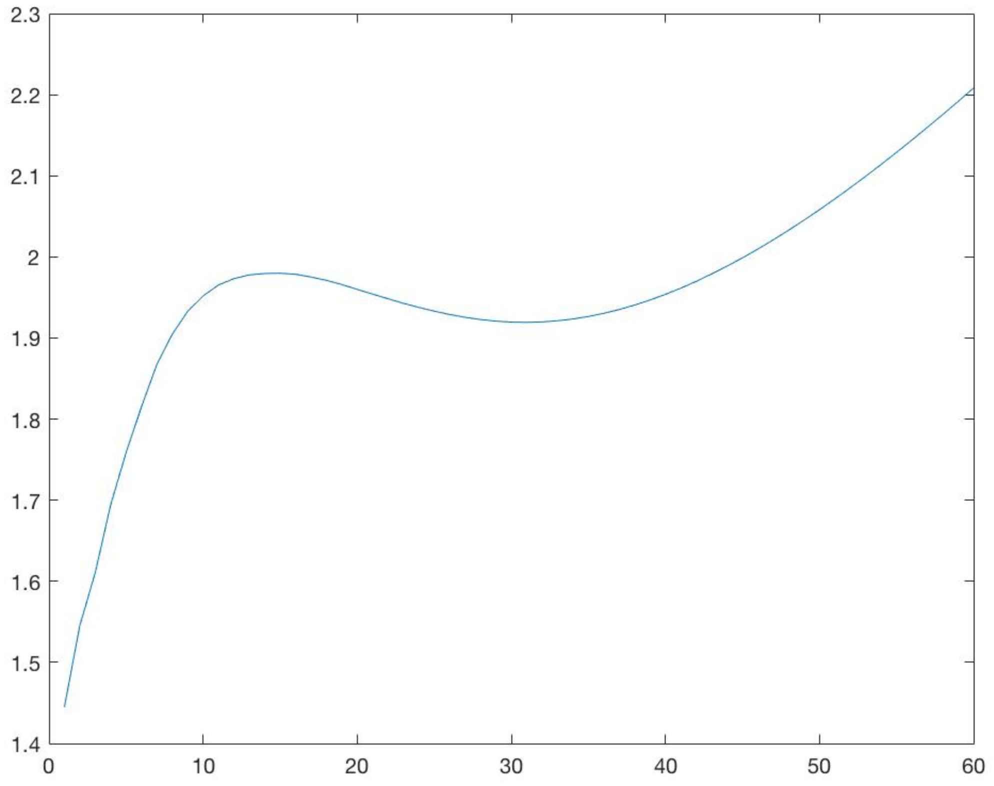
YIELDS

		US	UK	JAPAN	CANADA	GERMANY
1M	Ticker	US0001M	BP0001M	JY0001M	CDOR01	EUR001M
	Start Date	31/01/1989	31/03/97	31/07/95	30/03/2001	29/01/1999
3M	Ticker	US0003M	BP0003M	JY0003M	CDOR03	EUR003M
	Start Date	31/01/1989	31/03/97	31/07/95	30/03/2001	29/09/1999
1Y	Ticker	US0012M	BPSW1	JYSW1	CDSW1	EUSA1
	Start Date	31/01/1989	31/03/97	31/07/95	30/03/2001	31/01/90
3Y	Ticker	USSW3	BPSW3	JYSW3	CDSW3	EUSA3
	Start Date	31/01/1989	31/03/97	31/07/95	30/03/2001	31/01/90
5Y	Ticker	USSW5	BPSW5	JYSW5	CDSW5	EUSA5
	Start Date	31/01/1989	31/03/97	31/07/95	30/03/2001	31/01/90

Appendix

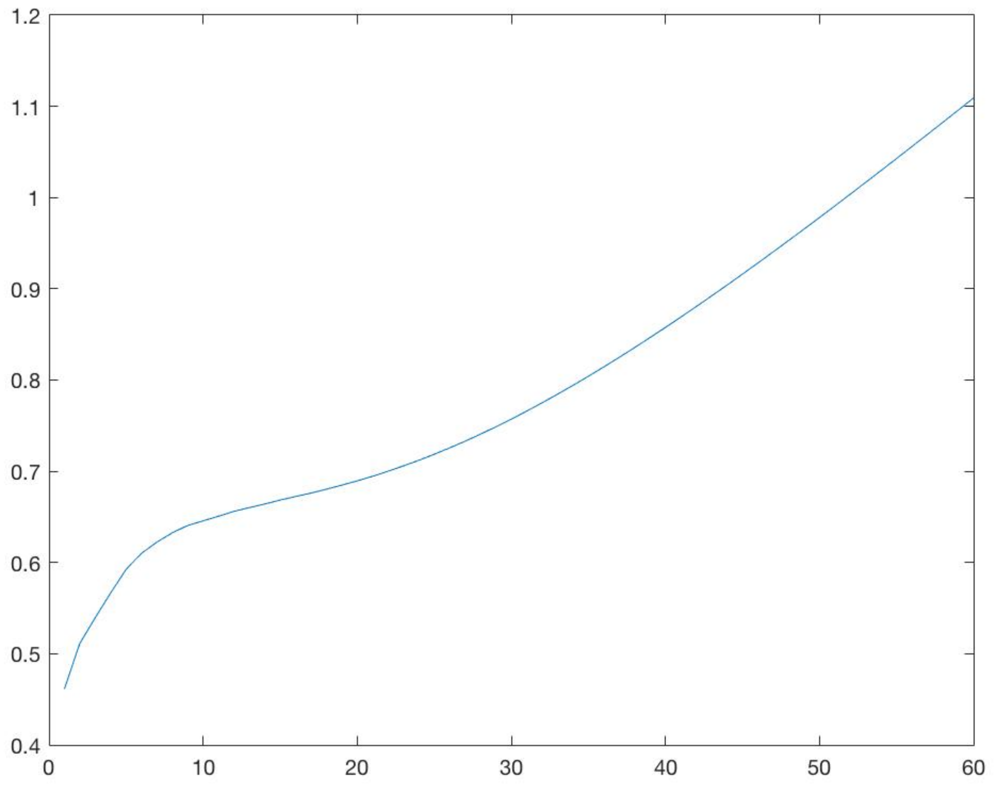
Appendix B – Prediction of monthly yield curve for December, 2017

1-step-ahead forecast for US



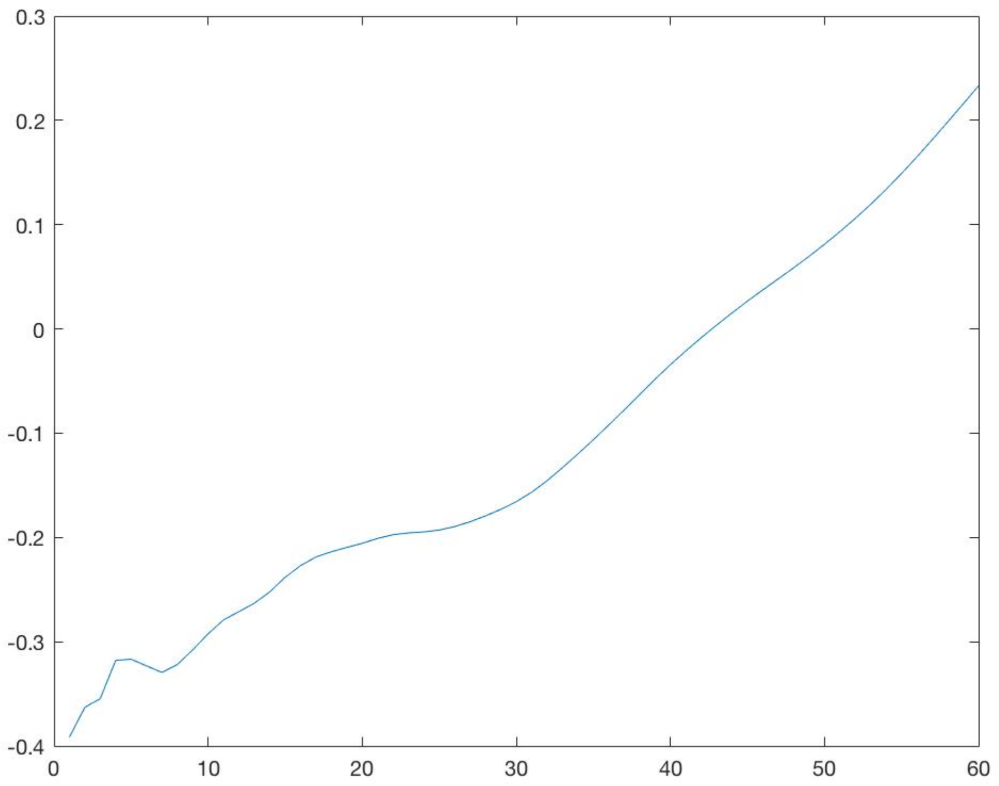
Appendix B – Prediction of monthly yield curve for December, 2017

1-step-ahead forecast for UK



Appendix B – Prediction of monthly yield curve for December, 2017

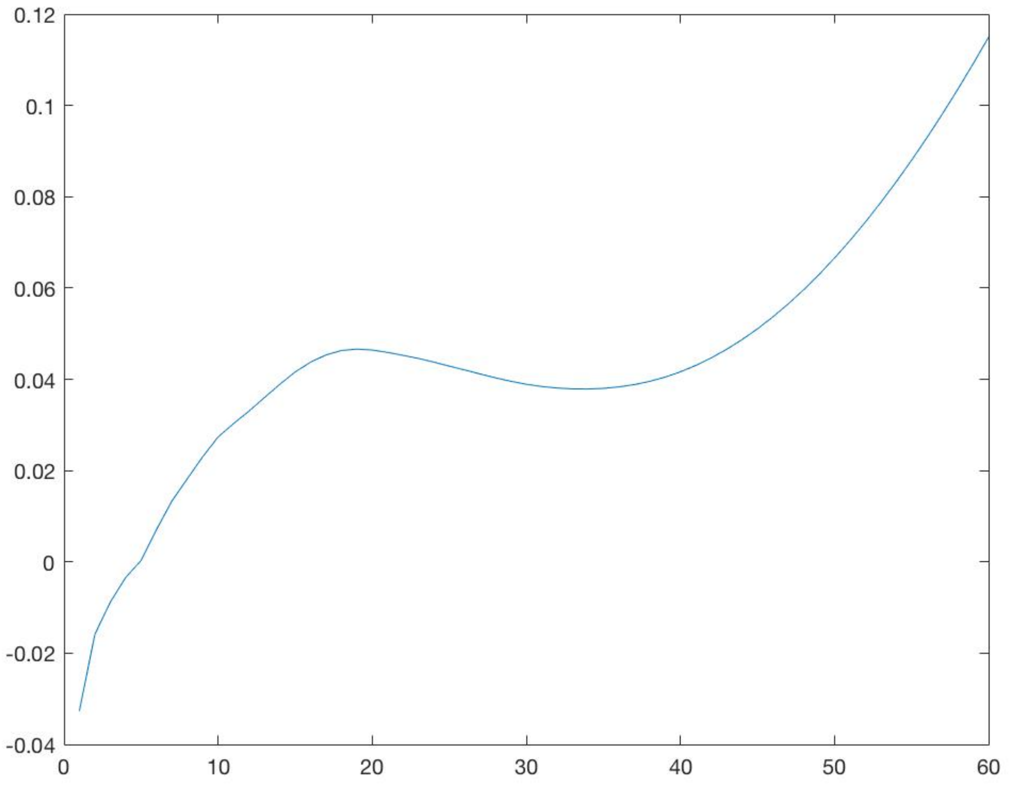
1-step-ahead forecast for Germany



Appendix

Appendix B – Prediction of monthly yield curve for December, 2017

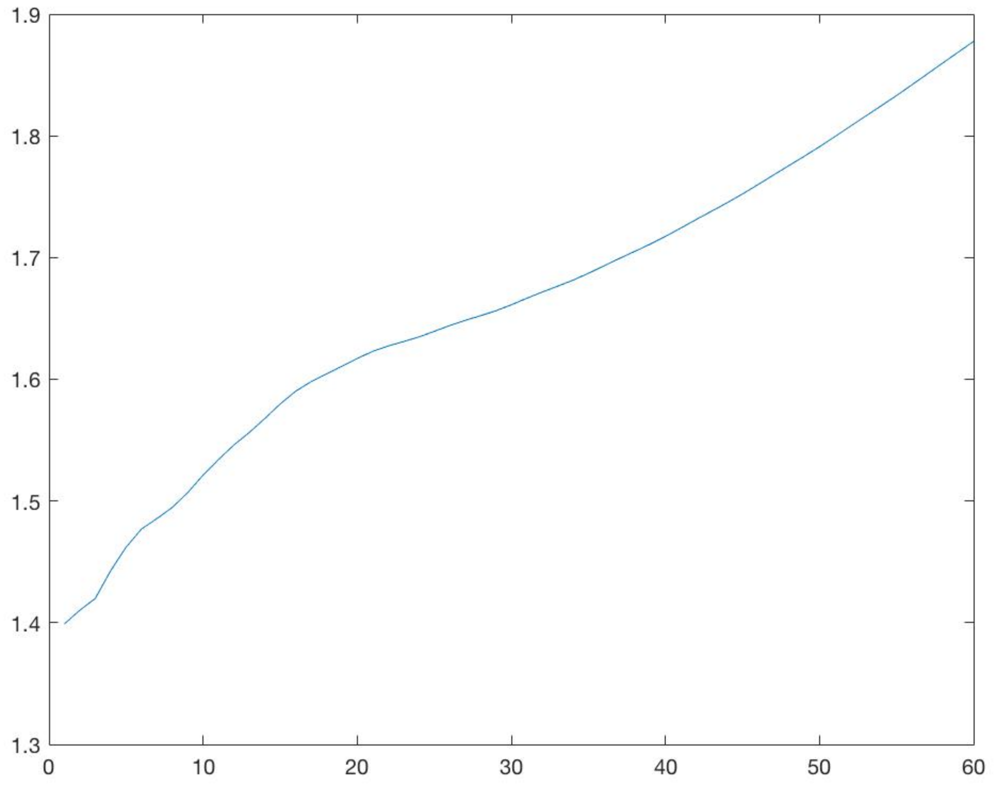
1-step-ahead forecast for Japan



Appendix

Appendix B – Prediction of monthly yield curve for December, 2017

1-step-ahead forecast for Canada



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