A work Project, presented as a part of the requirements for the Award of a Master Degree in Finance from the NOVA-School of Business and Economics.

CUSTOMIZED INVESTMENT SOLUTIONS

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Customized Investment Solutions

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- Portuguese investment decisions
- Reasons for low investment in securities
- Problems of Portuguese investment decisions
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- Goals based investing
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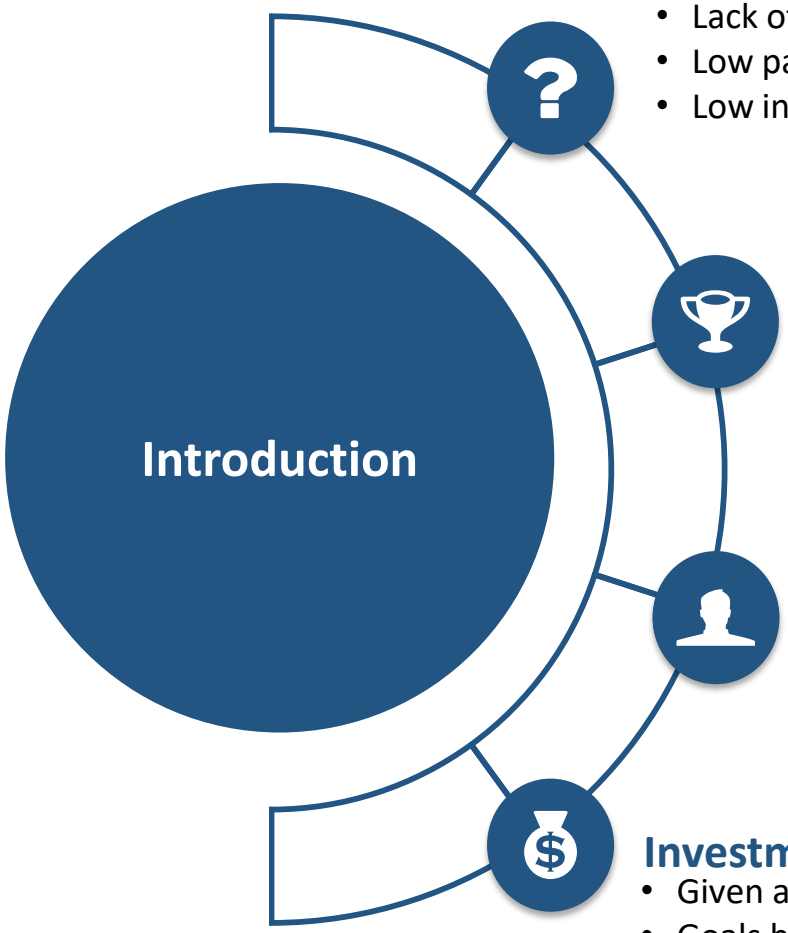
Introduction

CUSTOMIZED INVESTMENT SOLUTIONS



Main problems

- Lack of confidence in the banking sector;
- Low participation in equity markets from Portuguese investors;
- Low interest rates and uncertain future projections of these

Goal

- Develop a model of financial portfolio adequacy to lifecycle (dynamic model – Matlab - vs. static model - Excel)

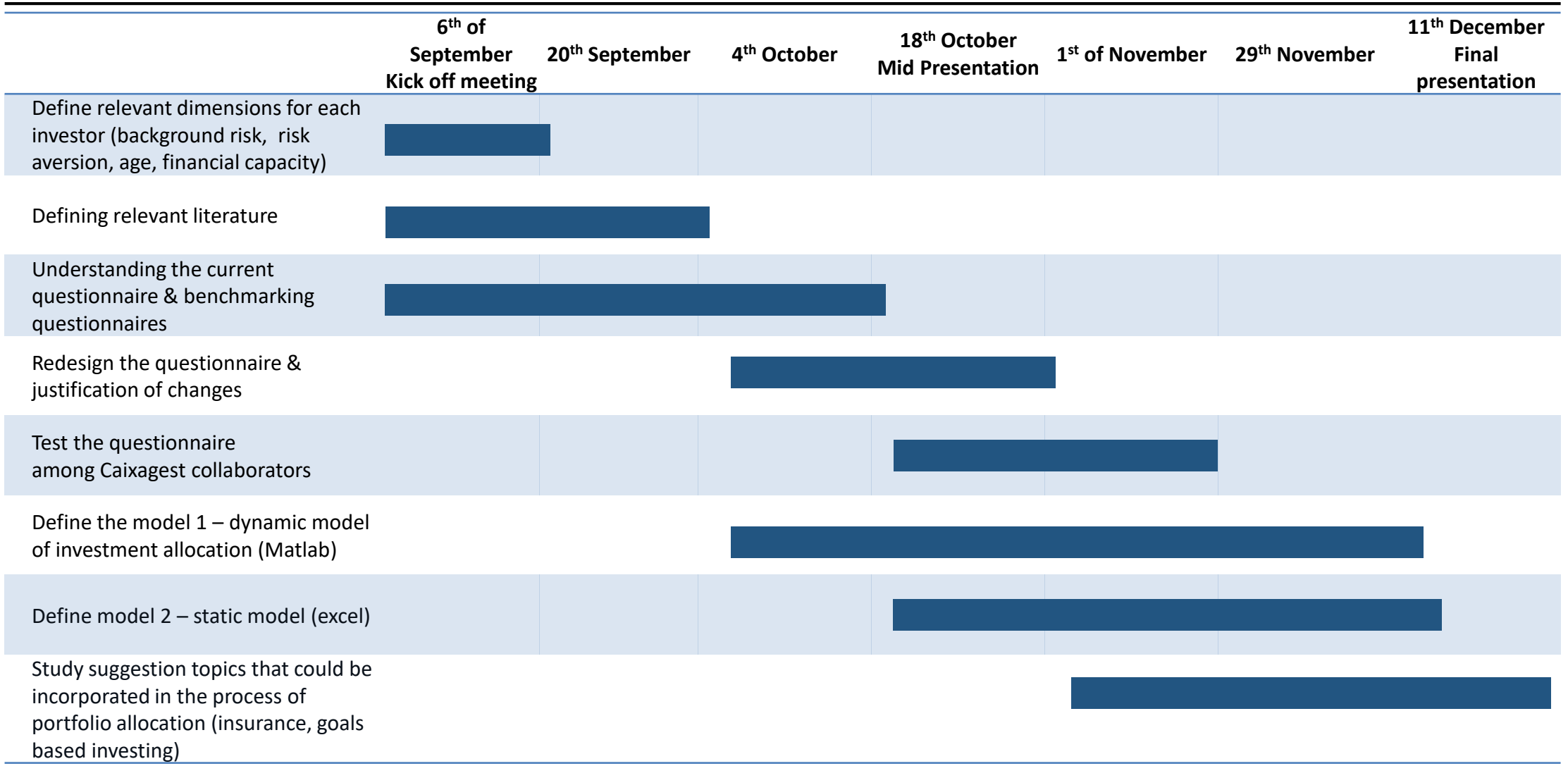
Find each investor risk profile

- Restructure the risk profile questionnaire
- Extract each investors level of risk aversion
- Extract some other important factors (income, wealth, others)

Investment allocation

- Given an investor risk profile, which is the portfolio that maximizes lifetime utility?
- Goals based investing (a suggestion)

GROUPS' SCHEDULE



Portugal Overview

To begin our work project we found it was important to explain how Portugal ranks in terms of financial literacy, Portuguese investment decisions and its problems and why people should start investing.

BANK OF PORTUGAL USED A SURVEY TO MEASURE FINANCIAL LITERACY IN PORTUGAL BASED ON THE FOLLOWING METRICS

1. **Financial inclusion** was measured through indicators of access to the banking system, such as the holding of one or more deposit accounts and the frequency of their movement, the holding of insurance and investment products.
2. **Family budget planning and saving habits** were measured through attitudes and behaviours related to the frequency of family budgeting planning, the regularity and purpose of savings, the ability to deal with unexpected expenses and planning for retirement.
3. **Financial products and the selection criteria** were measured through the identification of the factors considered in the choice of the financial products, the sources of information considered, the habits of reading the pre-contractual and contractual information.
4. **Characterization of sources of information** was measured through the type of financial news followed and the resource used in case of disagreement with the financial institution or difficulty in paying the loans.
5. **Assessment of financial knowledge** was performed by evaluating simple numerical operations resolution and with questions about key concepts associated with some financial products.

64% OF THE RESPONDENTS DO NOT HAVE ANY KNOWLEDGE OF FINANCIAL PRODUCTS OR MARKETS

The characteristics of the respondents intentionally reflect the proportion of each class in the Portuguese population aged 16 or over, according to the data published by the Instituto Nacional de Estatística (INE) in the 2011 Census.

- People interviewed: 1100.
- 47% of the respondents were female.
- 53% of the respondents were male.
- 44% of the respondents were active in their labour situation.

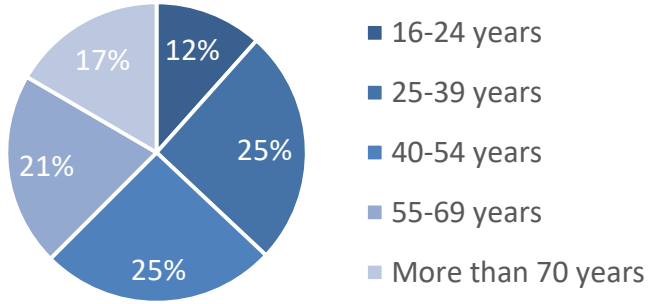


Figure I – Dispersion of age among sample individuals.
Source: *Relatório do Inquérito à Literacia Financeira da População Portuguesa, 2015*

Financial Products' Knowledge

- Only 35,7% of the respondents have knowledge of financial products or markets;
- 51,7% of the respondents follow the financial news;
- 31% of the respondents follow the interest rates' behaviour;
- 19% of the respondents know the financial products' regulations;
- 49,4% of the respondents already heard about commercial paper.
- 36,9% of the respondents are aware of the existence of complex financial products.

Financial Knowledge Sources

- 58,7% of the respondents identified their account manager as a source;
- 45,3% of the respondents identified their family and colleagues as a source;
- 22,2% of the respondents use the TV and the radio to keep informed about the financial news;
- 11,2% of the respondents choose the internet to keep informed.

Interest Rates

- 75% of the respondents know the interest rates applied to their savings and loans;
- 15% of the respondents do not know at all the interest rates applied to their savings and loans;
- Only 20% of the respondents refer to know the exact value of the interest rates applied to their savings and loans.

PORTUGAL RANKS ABOVE AVERAGE IN TERMS OF FINANCIAL LITERACY AND BELOW AVERAGE IN MANAGEMENT OF SAVINGS IN ANOTHER STUDY BY OECD

Positives

- Portugal occupies the 10th place in a list of 29 countries[1] in terms of knowledge, attitudes and behaviours when managing money.
- Where the Portuguese score relatively well is in the knowledge of concepts, by responding aptly above the OECD average in six of the eight questions regarding inflation, interest calculation or risk / return ratio, for example.
- Portugal shows a habit of drawing up a family budget (72%, compared to an average of 57% in the OECD)
- Portugal uses an account manager less times than the average OECD countries to manage their personal finance (52% versus 68%).
- Portugal scores relatively well in terms of timely payment of bills and in the absence of recourse to credit to cover the usual monthly expenses.

Negatives

- One of the most negative aspects concerns passivity in the management of savings: only 35% of the respondents are active, below the 60% average in the OECD and far from the over 80% of affirmative answers obtained among the Norwegians.

[1] France, Finland, Norway, Canada, Hong Kong, New Zealand, Korea, Belgium, Austria, Portugal, Lithuania, Netherlands, Estonia, Latvia, UK, British Virgin Islands, Thailand, Albania, Jordan, Czech Republic, Turkey, Hungary, Georgia, Malaysia, Russia Federation, Brazil, Croatia, Belarus, Poland

PORTUGUESE ARE CONSERVATIVE AND HAVE LOW DIVERSIFICATION IN THEIR PORTFOLIOS

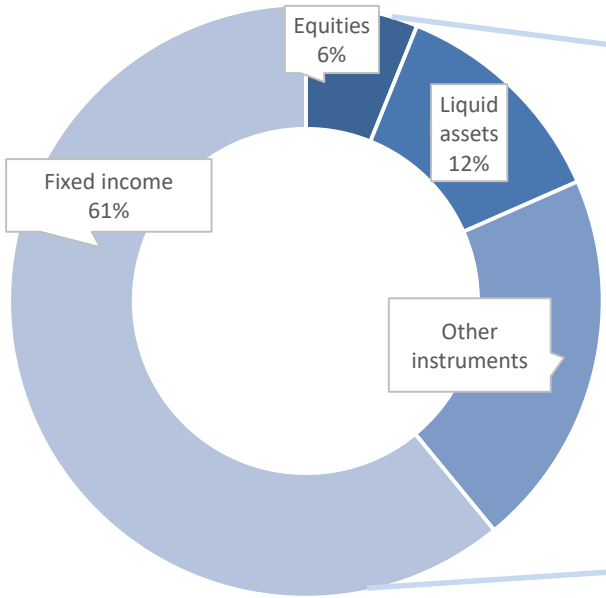


Figure II - Investment mix in Portugal.
Source: CMVM, Relatório inv. de gestão de ativos 2.º trimestre de 2017

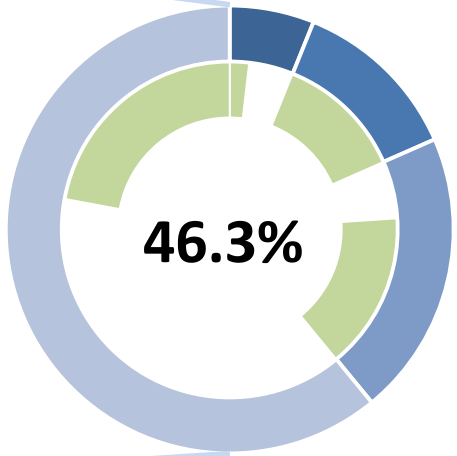


Figure III – International component of the Portuguese investment mix in green.
Source: CMVM, Relatório inv. de gestão de ativos 2.º trimestre de 2017.

The current investment mix in Portugal

- Large component of fixed income (61%), composed of corporate and Portuguese government bonds.
- Some liquid assets (12%), including foreign government bonds.
- Small share of equities (6%), both Portuguese and foreign.
- The small share of equities suggests that portfolios are missing out on diversification gains. Higher returns might be obtained by including more exposure to equities.
- Some other instruments, including participation units and others.
- Bias towards national investments: 53.7% of all investments are in Portuguese assets; the average portfolio is heavily dependent on the Portuguese economy.

THE MAIN REASON TO INVEST IN SECURITIES IS THEIR HIGHER RETURN COMPARED TO BANK DEPOSITS

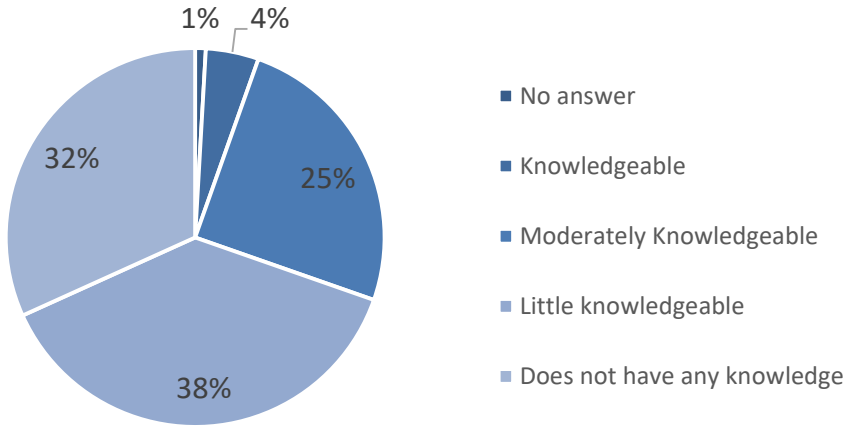


Figure IV - Investment products and securities market knowledge.
Source: Relatório do Inquérito à Literacia Financeira da População Portuguesa, 2015

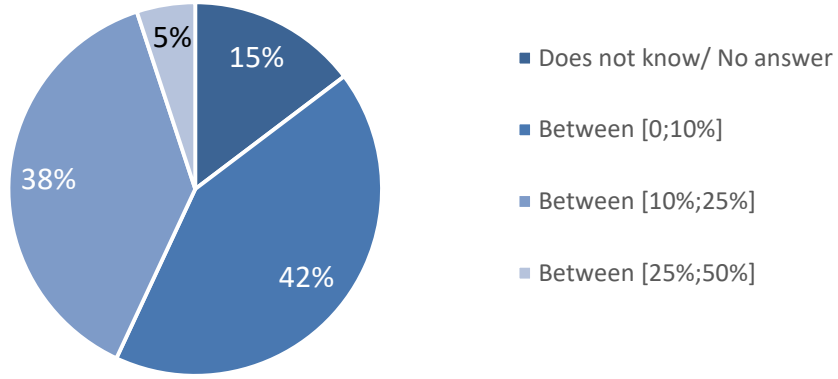


Figure V - Assets invested in securities.
Source: Relatório do Inquérito à Literacia Financeira da População Portuguesa, 2015

Financial Products

- 92,5% of the respondents say they have a deposit account;
- 73,1% of the respondents have at least one insurance;
- In addition to these products respondents have: term deposits (38,7%), credit cards (31,6%), mortgage credit (21,9%), other credits such as the automobile (15,5%) and retirement savings plans (15%). However, 6,3% do not have any financial product;
- 79,1% use their own savings to invest in securities.

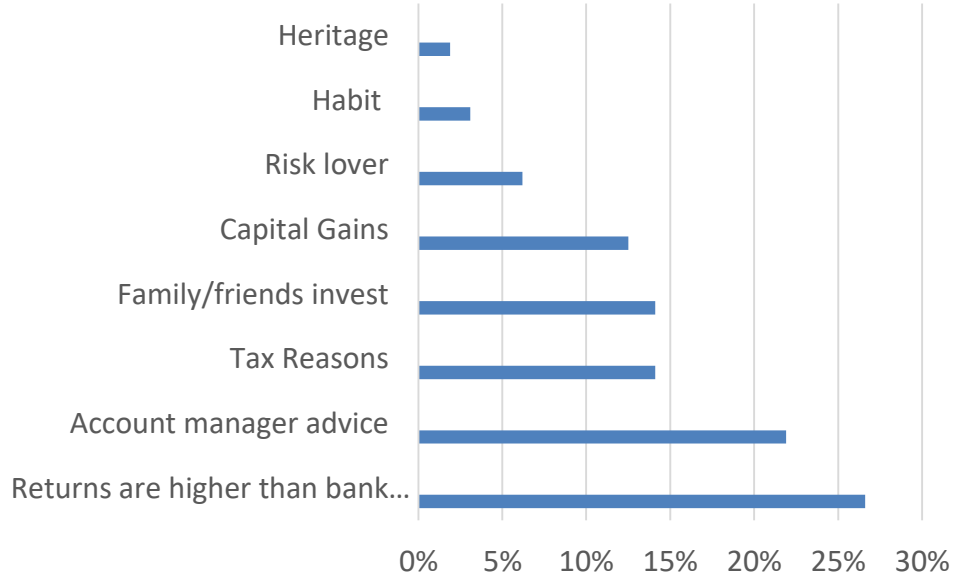


Figure VI - Reasons to invest in securities.
Source: Relatório do Inquérito à Literacia Financeira da População Portuguesa, 2015

ALMOST HALF OF THE INTERVIEWEES DO NOT INVEST IN SECURITIES DUE TO THE LACK OF INCOME

Reasons for low investing

- If there was a greater stability of the economy that would influence Portuguese future investments.
- For those who never invested in securities, the lack of money is the main reason appointed (47,1%), while 7% said they do not have enough knowledge to invest in financial markets (figure VII).
- **Equity Premium Puzzle** - Describes the higher historical real returns of stocks over government bonds. **The equity premium, which is defined as equity returns minus bond returns, has been about 6% on average for the past century.** It is supposed to reflect the relative risk of stocks compared to "risk-free" government bonds, but the puzzle appears because even though the premium exists there is a **strange high level of risk aversion among investors**.

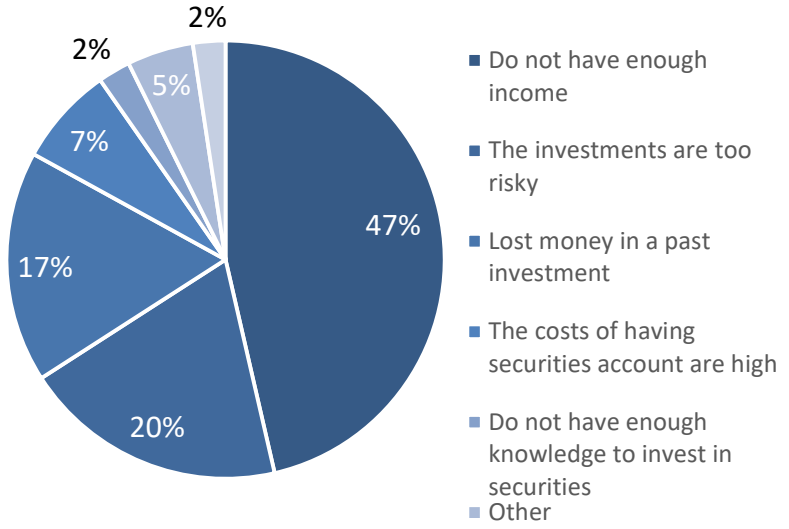


Figure VII - Reasons why people are not investing in securities.
Source: Relatório do Inquérito à Literacia Financeira da População Portuguesa, 2015

Retirement Plan

- Most respondents (82,2%) say they will fund their retirement with social security or another mandatory contributory scheme, and only 11,9% indicate that they have a private retirement savings plan.
- Of those who have a retirement savings plan, 84,2% say they have guaranteed capital and 77,6% indicate that they have guaranteed income (coming from interests). Most (73,3%) claim that their retirement savings plan has both guaranteed capital and income.

Investing Behaviour

- 58% of the respondents would not sell their position in the event of a sharp drop in the price.
- 1/3 stopped investing because of liquidity needs.
- 24,4% stopped investing because they lost money.
- 19,5% stopped investing because they considered the risk was too high.

PORTUGUESE INVESTORS HAVE NOT BEEN INVESTING AS WELL AS THEY SHOULD

Having in mind the world average, Portugal is considered a high income country where wealth is more evenly distributed (figure VIII). Therefore a significant proportion of the Portuguese people could be investors. The uncertain future of the social security system calls for an investment in more diversified financial instruments. Over the last decade, investors have turned to cash (figure IX), which is in fact a problem due to the (1) probability of collapse of the social security system and (2) inflation. Moreover, only about 4% of the Portuguese hold non-deposit or non-insurance investments (figure X) and, as exposed before, they are overly exposed to the national economy – only 46.3% is invested abroad (figure XI).

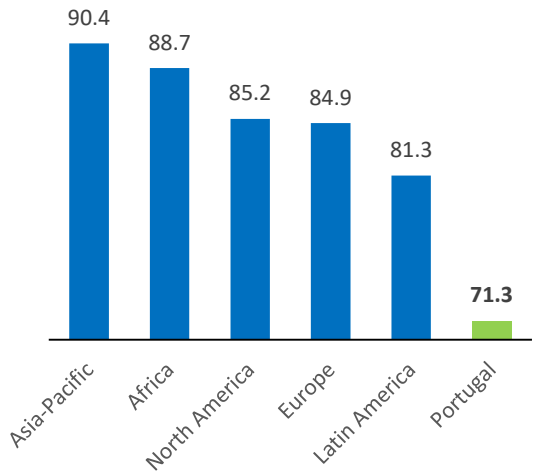


Figure VIII - Concentration of wealth: Gini Coefficient.
Source: Crédit Suisse Research Institute, Global Wealth Databook, 2016

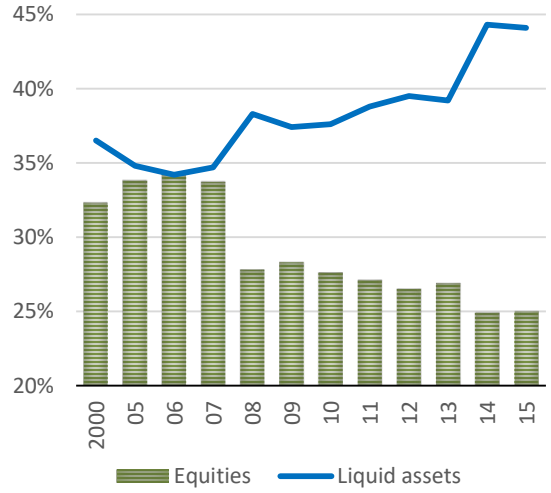


Figure IX - Evolution of the investment mix in Portugal, 2000-2015
Source: CMVM, Relatório de gestão de ativos 2.º trimestre de 2017

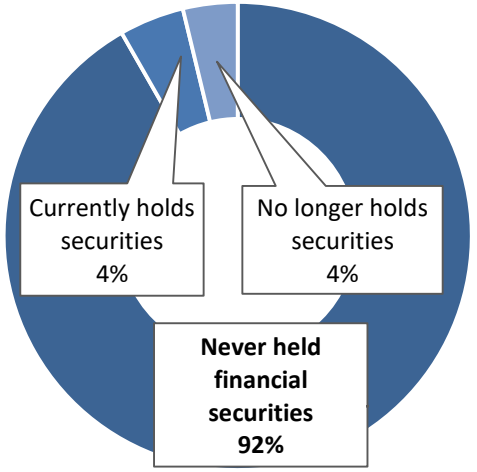


Figure X - Proportion of security-holding Portuguese investors
Source: Banco de Portugal, Relatório do Inquérito à Literacia Financeira na População Portuguesa 2017

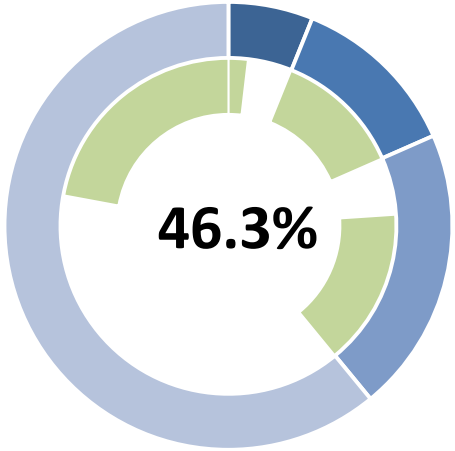


Figure XI - International component of the Portuguese investment mix in green
Source: Relatório de Sustentabilidade Financeira da Segurança Social, 2017

ONE OF THE REASONS PEOPLE SHOULD START SAVING AND INVESTING FOR THEIR RETIREMENT IS SOCIAL SECURITY INSTABILITY

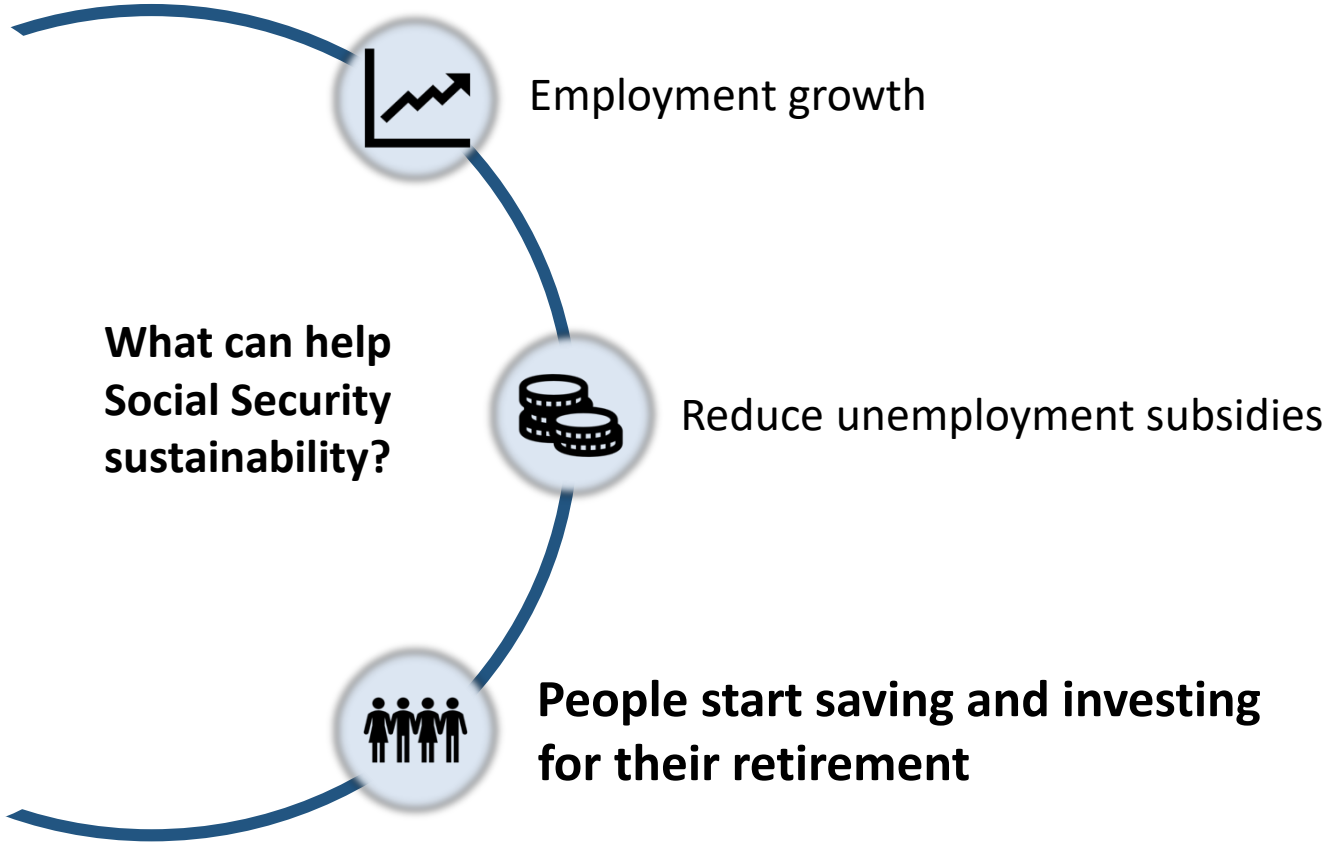
Social Security Financial Sustainability Report states that even if negative balances of the social security system are projected in the mid-2020s, at that time, the annual FEFSS (Social Security Financial Stabilization Fund) simulation is going to be used to cope with these deficits. However, by the early 2040s, it is projected that the current social security system would be unsustainable.

Current Situation

- The Stabilization Fund has about 14,1 billion euros, which would pay more than a year of pensions in case the government is unable to cover the deficit.

Situation in 2040

- As deficits are forecast to remain until 2020, the FEFSS will reach 2040 with only 6,5 billion euros. Moreover, since the population is getting older that will increase the expenditure in pensions (expected to reach 20,5 billion euros, or 8% of GDP), the Fund will sustain four months of pensions (or one third of their value).



BANK OF PORTUGAL PROJECTIONS INDICATE THAT IT IS A GOOD TIME FOR CAIXAGEST TO MARKET ITS PRODUCTS AND FOR PORTUGUESE TO INVEST

Main Projections

- Due to a projected growth in internal and external demand, there are strong expectations of an increased confidence on all productive sectors of the economy, thus **projecting a recovery on the main sectors, where industry, tourism, construction and automotive sectors stand out**.
- **Private consumption is estimated to grow at a smaller rate than the GDP**, reflecting a modest growth in salaries, due to small gains in productivity, and the continuous need for families to reduce their level of indebtedness.
- Nominal salaries are expected to grow gradually between 2017 and 2019. However, in average annual terms, this growth will be only slightly above that projected for the inflation rate. The latter shall be of 1,4% in 2017 and keep rising in the next two years.

	2016	2017	2018	2019
Employment	1,6	2,4	1,3	1,3
Unemployment rate	11,1	9,4	8,2	7

Reflecting the evolution of employment in the private sector, since no major changes on employment are expected in the public sector.

Figure XII - Growth rate (%) of employment and unemployment.
Source: 12th October 2016, OECD

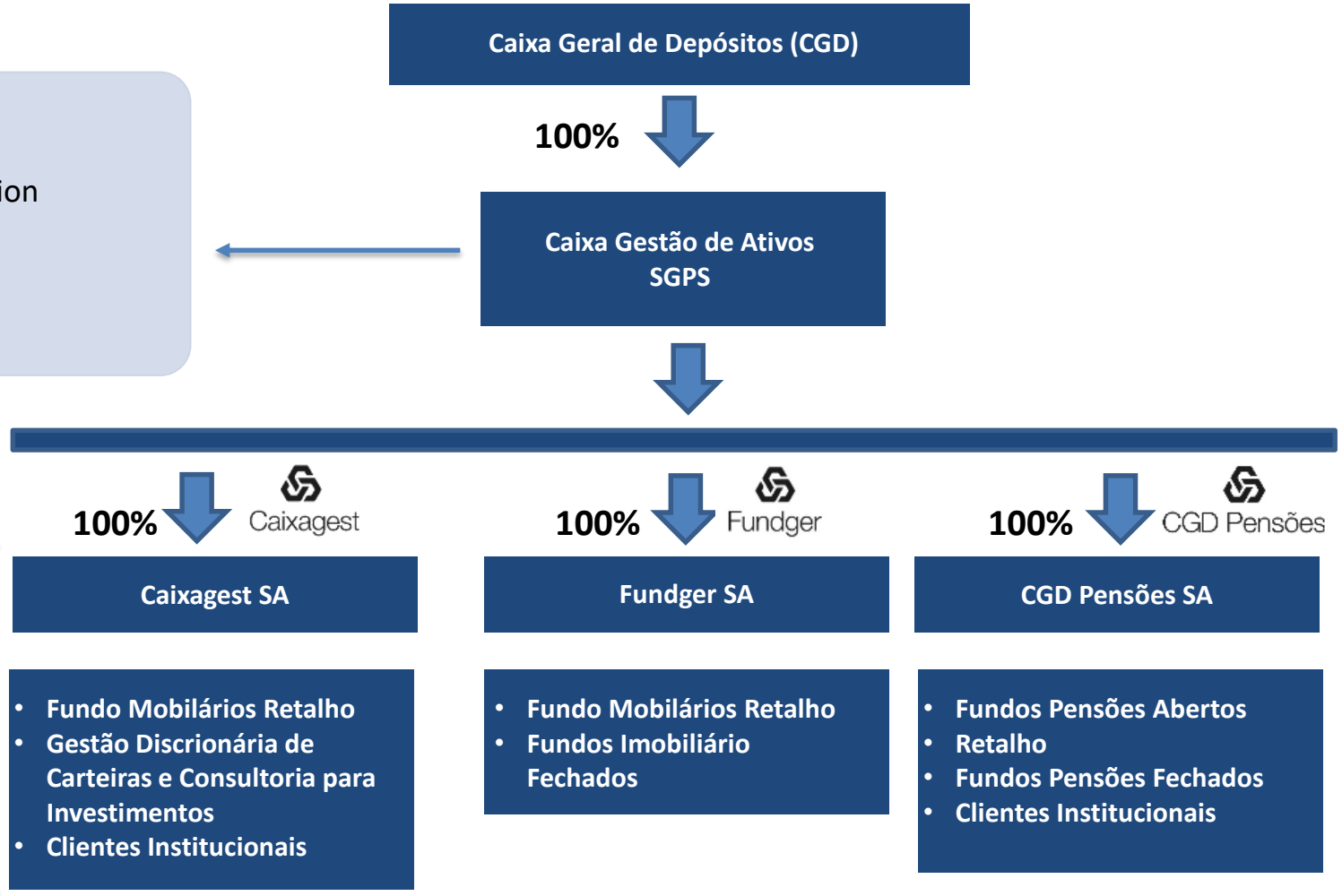
Caixagest

In this section we will focus mainly on Caixagest's activity in wealth management and how the process is, since the investor asks for advice until the optimal investment allocation is defined.

CAIXAGEST, FUNDGER AND CGD PENSÕES BELONG TO CAIXA GESTÃO DE ATIVOS

Distinguishing Features

1. Agency Business Model
2. Limited Risk balance and capital consumption
3. Protection of clients' assets
4. Compensation based on an annual fee
5. Regulation and risk principals
6. Transparency in the assets valuation



Specialized in the **management** of investment funds and in **advising** private clients in portfolio management decisions, benefiting from the experience, know-how and management culture acquired during 125 years of tradition of Caixa Geral de Depósitos in the financial area.

Figure XIII – Caixa Geral de Depósitos structure.
Source: Caixagest internal documents

CAIXAGEST RECEIVED SEVERAL AWARDS DURING THE PAST YEAR

+38%
Debt Funds

+29%
CXG Global Leader Equities

-5%
Other Equities Funds

+10%
Multi Equities

Caixagest

2017 Awards

- Morningstar: Best national fund of euro bonds (Long-Term Caixagest Bonds).
- APFIPP & Jornal de Negócios: Best fund of Other International Assets Funds.
- Institutional Investor: Best Alternative Investment Team in Portugal.
- Extel Europe: One of the employees was distinguished as the number one in management among other national and European professionals.

EXTEL IDENTIFYING EXCELLENCE

Institutional Investor

Prémios Melhores Fundos - 2017 JORNAL DE NEGÓCIOS/APFIPP

MORNINGSTAR

+3,6%
Penetration of individuals clients

+269.523
Clients with securities

133.501
Clients with funds

Opportunities to Caixagest

Improving some potential strategies of growth

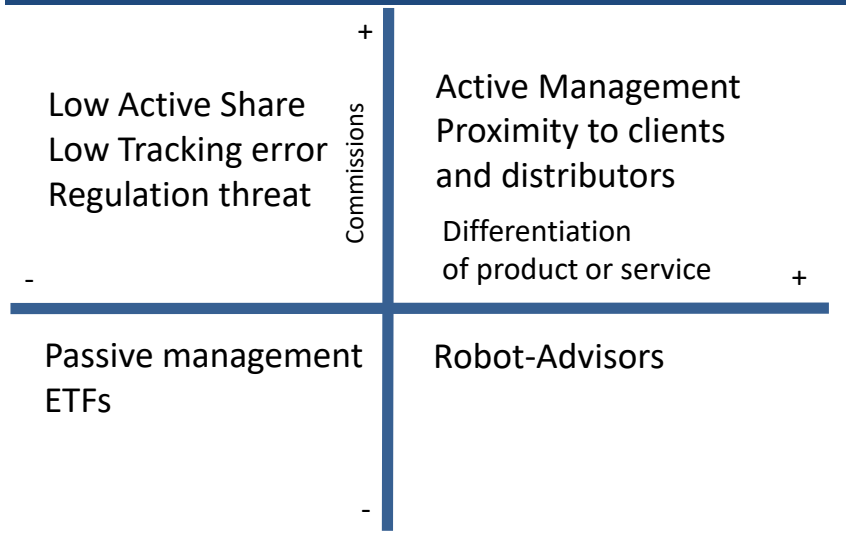
- Funds of Penetration versus balance sheet funds.
- Current average commission (0,59%, while BPI Bank presents 0,68%, Millennium BCP Bank 0,83%, Santander Totta 1,13% and Novo Banco and Best Bank 1,18%).

CAIXAGEST USES DIFFERENT TOOLS TO COMMUNICATE AND KEEP THE CLIENTS INFORMED

Structure

7	+500	+2000	1416
Consultant sessions	Agencies	Commercial network Collaborators	Training Sessions

Industry trends

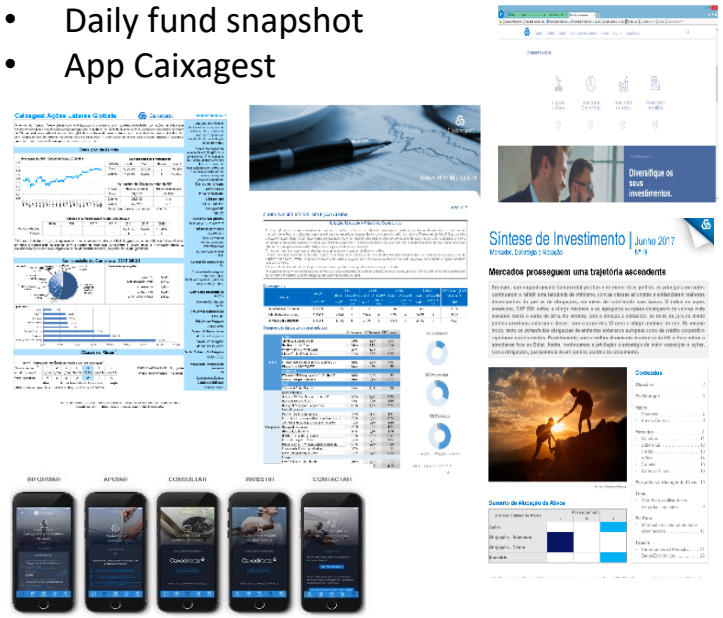


Opportunities to local managers

- Improving management to increase the active share.
- **Advisory**: Promotion of financial literacy.
- **Distribution**: Privileged access to local distributors.

Tools

- On-line Investment Funds
- Multi Equities News
- Monthly Financial Markets Snapshot
- Daily fund snapshot
- App Caixagest



CAIXAGEST USES 4 PROFILES TO MATCH THE CLIENTS 'CHARACTERISTICS TO THE INVESTMENT ALLOCATION

1

Individual Questionnaire which is mandatory under Artº 314 of Código dos Valores Mobiliários
A confidential questionnaire is performed to gauge the investor knowledge of financial markets, financial capacities and its risk profile.
The investor must obey two rules:

1. Have knowledge regarding financial markets and its risks.
2. Have sufficient wealth to support possible losses inherent to the financial products.

2

Investor Risk Profile

Profiles	Knowledge of Financial Markets	Temporal Horizon	Financial situation vs responsibilities	Return on investment
Prudente	Little	Less than 1y	Inferior	Need to obtain the certain amount previously discussed
Equilibrado	Some	1y-3y	Similar	Returns similar to interest rates without risk
Dinâmico	Reasonable	3y-5y	Equal	Returns similar to interest rates without risk. Small capacity to capital losses
Arrojado	Reasonable and with some experience	Higher than 3y	Equal or Superior	Returns higher than interest rates without risk. Higher capacity to capital losses

Figure XIV – Summary of investment profiles and its characteristics.
Source: Caixagest internal documents

CAIXAGEST MAKES SOME ADJUSTMENTS AFTER DECIDING WHICH INVESTOR RISK PROFILE CORRESPONDS TO THE CLIENT

2

Investor Risk Profile

According to the regulation CMVM 2/2015, the profiles are characterized in the following way:

Profiles	Volatility Range	Weight on M
Prudente	0%-2%	0%
Equilibrado	2%-5%	35%
Dinâmico	5%-10%	70%
Arrojado	>10%	100%

Figure XV – Summary of investment profiles tolerance to volatility and respective weight on M.
Source: Caixagest internal documents

3

Final adjustment

For each risk profile, the optimal portfolio will combine the portfolio offered by Caixagest (which we will call portfolio M from now on and that we will describe after) and one riskless asset (which is a low volatility portfolio). Afterwards, the portfolio is tested, so as to guarantee it is subordinated in what concerns to the volatility range historical data.

Optimal Portfolio return $= weight\ on\ portfolio\ M\ * Portfolio\ M\ return + (1 - weight\ on\ M) * Riskless\ Portfolio\ return$

The portfolio M is composed by 58% of equities. This is low when considering an arrojado investor that desires to invest 60% on equities. To avoid short selling, a re-composition was made, increasing the weigh on some sub-equities classes.

Benchmarking Questionnaires

In order to improve the questionnaire developed by Caixagest we decided to analyse questionnaires used by 8 other banks on some topics: the most common type of questions asked, the extension and the extraction of the level of risk aversion for each individual. We will also present some drawbacks of using questionnaires.

QUESTIONS REGARDING TOLERANCE TO VOLATILITY AND THE REACTION TO LOSS WERE FOUND IN MOST QUESTIONNAIRES

DAVIDSON FINANCIAL GROUP PTY LTD

Goldman Sachs

Fidelity INVESTMENTS

RBC

RBS

Principal Financial Group

Morgan Stanley

MAIN CONCLUSIONS

- 10 questions on average.
- Mostly multiple choice and no open ended questions[1] .
- Points are attributed to each option to find the risk profile.
- Between 3-5 risk profiles.

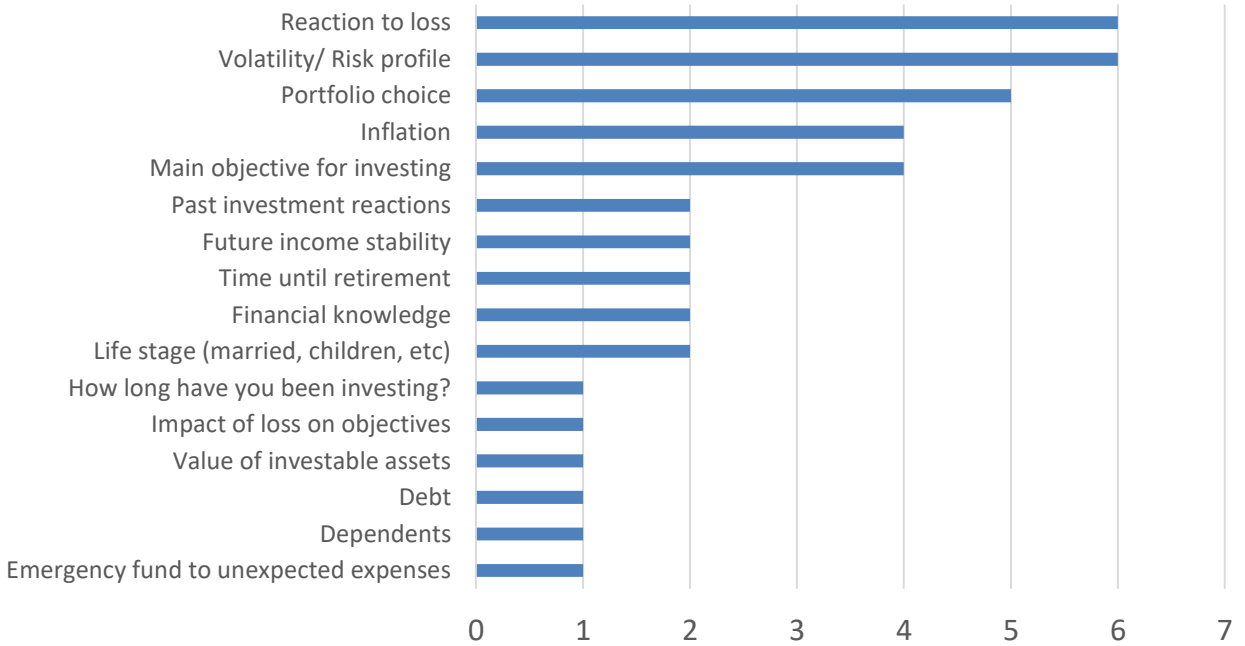


Figure XVI – Amount of benchmarking questionnaires in which each type of question appears
Source: authors' own calculations

[1] Except for some questions in the Goldman Sachs questionnaire because it is focused on retirement planning.

THERE ARE SOME PROBLEMS WITH MEASURING RISK PROFILE USING QUESTIONNAIRES

Because survey questions ***are not incentive compatible****, economists are sceptical about whether self-reported personal attitudes and traits are behaviourally meaningful. Various factors, including self-serving biases, inattention, and strategic motives could cause respondents to distort their reported risk attitudes (for a discussion, see Camerer and Hogarth 1999). Experimental studies, which measure risk-taking behaviour with real money at stake, on the other hand, offer an incentive-compatible measure of risk attitudes. However, a drawback of this technique is that it is costly and difficult to perform with a large, representative sample, preventing large-scale studies.* ---- Individual Risk Attitudes: Measurement, Determinants, and Behavioral Consequences

Quantitative measures of risk aversion have drawbacks too. First, ***when asked about willingness to pay, individuals tend to underreport, which overestimates their true risk aversion*** *(Kachelmeir and Shehata, 1992). Second, answers may be affected by how questions are framed. Third, the validity of this methodology rests on the assumption that respondents know how they would behave in a hypothetical settings and that they are willing to reveal their choices truthfully (Kahneman and Tversky, 1979)*
Household Finance: An Emerging Field

To solve this problems of incentive compatibility and the behavioural bias of investors in hypothetical settings, some suggestions will be made in the individual part of one of the group members.

Questionnaire

Proposed, Suggestions & Questions Excluded

The following section will present the final version of the questionnaire proposed by the group to Caixagest, as well as some suggestions of topics that could be further explored. The questionnaire will be presented in Portuguese, because it is to be used in that language. The structure will be as follow: 1) slide with question and justification for its inclusion; 2) slide with suggestion (if existent). After some consideration and the first test of the questionnaire (among relatives of the group members), some questions were excluded from the initial versions. The reasons are mainly due to correlations between questions/answers, lack of variability in answers and difficulty in reaching conclusions.

PRE - QUESTION

THIS QUESTION COULD BE FOLLOWED BY A VIDEO PRESENTATION TO CLARIFY THE RESPONDENTS WHICH HAVE LESS KNOWLEDGE OF FINANCIAL MARKETS

Quão familiarizado está com mercados financeiros/investimento?
Não tenho conhecimento.
Conheço algumas classes de ativos e tipos de investimentos, mas não sei mais do que isso.
Conheço algumas classes de ativos e tipos de investimentos e tenho experiência suficiente para perceber a importância da diversificação.
Conheço a maioria das classes de ativos e tipos de investimentos e percebo que os mercados flutuam e que diferentes sectores oferecem diferentes retornos, crescimento e têm diferentes impostos associados.
Conheço perfeitamente as classes de ativos e tipos de investimentos e percebo os vários fatores que influenciam a performance dos mercados financeiros.

We decided it was important to ask this question before entering in the actual questionnaire even though it will not be directly used in the models developed after. The answer to this question could be followed by a short video presentation to clarify the respondents regarding financial markets specificities (as explained in the next slide). Such video could also be used as a Marketing tool for Caixagest.

VIDEO

A VIDEO WOULD BE USEFUL TO CLARIFY INVESTORS REGARDING THE IMPORTANCE OF INVESTING AND THE ROLE OF THE WEALTH MANAGER

1. Impact of Financial Literacy on decisions

- Briefly explanation of assets classes, the mechanics of financial markets and consequences of not understanding those concepts

2. Little return on traditional investments

- Lower rate of return on deposits and an incentive to invest on alternative investments that maintain the required return with safety

3. Uncontrolled and unsubstantiated emotions of an investor given the lack of financial knowledge

- Panic regarding price fluctuations, impulsive decisions and lack of confidence on the markets
- Role of "Gestor de Riqueza" and credibility of financial institutions

Aiming to…
Provide a better understanding of financial markets
Gain enough confidence to invest beyond traditional methods

To…
Investors that lack financial knowledge

PERSONAL QUESTIONS

AGE AND EDUCATIONAL BACKGROUND COULD HAVE AN IMPACT IN THE RISK AVERSION PARAMETER

1. Nome

2. Género

3. Idade

- *(…) risk averse parameters are also positively correlated with age which explains patterns of portfolio choice in the life-cycle.* (Dohmen et al. 2011, Barsky et al.1997, Guiso and Paiella 2008)

4. Habilitações académicas

- Ensino básico (do 1º ao 9º ano de escolaridade)
- Ensino secundário (10º, 11º, 12º ano de escolaridade)
- Curso médio/politécnico/bachalerato
- Licenciatura, pós-graduação, mestrado ou doutoramento – especializado economia/gestão
- Licenciatura, pós-graduação, mestrado ou doutoramento – outras especializações

- *Using US data, Polkovnichenko (2005) (….) shows that, among the respondents in the SCF survey that hold stocks directly, those with higher education invest a lower proportion of financial wealth in directly held stocks and in risky assets. Guiso and Jappelli (2008) are able to study the effect of financial literacy on portfolio diversification directly.*

PERSONAL QUESTIONS

TO FIND THE INVESTMENT ALLOCATION WE WILL NEED WAGE AVERAGE GROWTH AND VOLATILITY WHICH IS BASED ON SECTOR AND QUALIFICATION LEVEL

5. Profissão

5.1 Sector

1) Educação
2) Construção
3) Água e Pesca
4) Indústria Extractiva
5) Indústria Transformadora
6) Alojamento e Restauração
7) Transporte e Armazenamento
8) Electricidade, Gás e Água
9) Atividades Financeiras e Seguros
10) Saúde e Apoio Social
11) Administração pública e Defesa
12) Comércio Grosso e Retalho

5.2 Nível de qualificação

1) Quadros superiores
2) Quadros médios
3) Profissionais qualificados
4) Profissionais semiqualificados
5) Profissionais não qualificados
6) Praticantes e aprendizes

The usefulness of asking this questions will be understood in the models section of the work project.

FOR THE EFFECTIVENESS AND UTILITY OF THE QUESTIONNAIRE IS IMPORTANT THAT RESPONDENTS ANSWER CORRECTLY

The questionnaire must be done with a glossary, presented below, with the definitions of the most technical terms. As it was explained before, the questionnaire was structured in Portuguese and so was the glossary. The first set of definitions is to be used in question 5.2 regarding the qualification level.

Question 5.2	
Quadros superiores	Quadros e técnicos da área administrativa, comercial ou de produção da empresa com funções de coordenação nessas áreas de acordo com planificação estabelecida superiormente, bem como funções de responsabilidade, ambas requerendo conhecimentos técnico-científicos de nível superior.
Quadros médios	Quadros e técnicos das áreas administrativas, comercial ou de produção com funções de organização e adaptação da planificação estabelecida superiormente, as quais requerem conhecimentos técnicos de nível médio.
Profissionais qualificados	Trabalhadores com funções de carácter executivo, complexas ou delicadas e normalmente não rotineiras, enquadradas em directivas gerais bem definidas, exigindo o conhecimento do seu plano e execução.
Profissionais semiqualificados	Trabalhadores com funções de execução totalmente planificadas e definidas, de carácter predominantemente mecânico ou manual, pouco complexas, normalmente rotineiras e, por vezes repetitivas.
Profissionais não qualificados	Trabalhadores que executam tarefas simples, diversas e normalmente não especificadas, totalmente determinadas.
Praticantes e aprendizes	Trabalhadores que, sob orientação de trabalhadores especializados, adquirem conhecimentos técnico-profissionais que lhes possam permitir desempenhar uma função administrativa, de produção ou outra.

Figure XVI – Set of definitions regarding qualification levels for question 5.2 of the questionnaire
Source: Pordata

THIS TYPE OF QUESTIONS ALLOWS TO MEASURE FINANCIAL CAPACITY OF THE INVESTOR

6. Qual é o seu rendimento líquido anual?

- *High income or wealth levels may increase the willingness to take risks because they cushion the impact of bad realizations.*
  Dohmem et al. 2011, "Individual Risk Attitudes: Measurement, Determinants and Behavioral Consequences"

7. Assinale as fontes de que é proveniente o seu rendimento? Indique cada uma em % do rendimento e se é considerada uma fonte estável ou instável.

	%	Estável / Instável
Trabalho por conta de outrem		
Trabalho por conta própria		
Rendimentos de capital		
Rendimentos prediais		
Pensões		
Outra categoria de rendimentos		

- *Investors are endowed with their own individual risk. (…) To the extent that their endowment risk is correlated with financial securities, investors should tilt their portfolios away from the market in order to reduce their exposure to those assets that are correlated with their own endowment risk (Duffie et al., 1997; Davis and Willen, 2000; Calvet, Gonzales-Eiras and Sodini, 2004; Cochrane, 2008).*

GLOSSARY

FOR THE EFFECTIVENESS AND UTILITY OF THE QUESTIONNAIRE IS IMPORTANT THAT RESPONDENTS ANSWER CORRECTLY

The questionnaire must be done with a glossary, presented below, with the definitions of the most technical terms. As it was explained before, the questionnaire was structured in Portuguese and so was the glossary. The set of definitions is to be used in question 7 regarding the income sources and definitions are in accordance with social security information.

Question 7	
Trabalho por conta de outrém	Pessoas que exercem uma atividade remunerada ao serviço de uma entidade empregadora.
Trabalho por conta própria	Indivíduo que exerce uma atividade independente, com associados ou não, obtendo uma remuneração que está diretamente dependente dos lucros (realizados ou potenciais) provenientes de bens ou serviços produzidos e que, habitualmente, não contrata trabalhador(es) por conta de outrem para com ele trabalhar(em).
Rendimentos de capital	O valor dos rendimentos de capitais (juros de depósitos bancários, dividendos de ações ou rendimentos de outros ativos financeiros); 5% do valor total do património mobiliário (créditos depositados em contas bancárias, ações, certificados de aforro ou outros ativos financeiros).
Rendimentos prediais	Habitação permanente, Restantes imóveis, excluindo a habitação permanente: O valor das rendas auferidas; 5% do valor patrimonial de todos os imóveis (excluindo habitação permanente).

Figure XVII – Set of definitions regarding income sources for question 7 of the questionnaire.
Source: Portuguese social security

SAVINGS REPRESENT THE AMOUNT THAT IS NOT USED BY THE INDIVIDUAL TO CONSUME AND THUS THAT COULD BE INVESTED

8. Qual é a sua poupança acumulada?

- *For the typical household, human capital is the largest form of wealth early in life, when little savings have been accumulated. It progressively loses importance until retirement age when most households stop accumulating assets. As a consequence, background risk is particularly relevant for the young who have very little buffer savings and have still a long horizon over which earnings can be affected by persistent labour income shocks.* ----- Household Finance: An Emerging Field

9. A sua poupança financeira acumulada permite-lhe fazer face às suas despesas correntes de quanto tempo?

- Até 3 meses
- Até 6 meses
- Até 1 ano
- Entre 1 e 2 anos
- Entre 2 e 3 anos
- Superior a 3 anos

- *For the typical household, human capital is the largest form of wealth early in life, when little savings have been accumulated. It progressively loses importance until retirement age when most households stop accumulating assets. As a consequence, background risk is particularly relevant for the young who have very little buffer savings and have still a long horizon over which earnings can be affected by persistent labour income shocks.*
  Household Finance: An Emerging Field

10. Qual é, em média, a sua taxa de poupança anual em percentagem dos seu rendimento?

- This question is to be used in the excel model which will be explained after.

THIS QUESTION WILL DISTINGUISH WHICH MODEL TO FOLLOW AFTER: MAXIMIZE UTILITY VS. GOALS BASED INVESTING

11. Investimento futuro

11.1 Qual é o objetivo central do seu investimento?

Maximização de capital	Reforma	Vários objetivos
	11.1.1 Quantos anos lhe faltam para a reforma? 11.1.2 Qual o rendimento que gostaria de ter disponível, mensalmente, após reforma?	11.1.1 Indique-os consoante a sua prioridade, sendo 1 o mais importante de atingir.
	11.1.1 *Retirement income is modelled as a constant fraction of permanent labour income in the last working-year.* Cocco, J.; Gomes, F., "Consumption and Portfolio Choice over the Life Cycle"	11.1.1 (…) *listing the clients' various goals by priority and time horizon.(…)* "Goals-Based Investing: Aligning Life and Wealth"

	Objetivo	Horizonte	Valor
1			
2			
3			
(..)			

THE MAIN MODELS WE DEVELOPED FOCUS ON UTILITY MAXIMIZATION ALTHOUGH WE HAVE PROPOSED ANOTHER BASED ON GOALS ACCOMPLISHMENT

Maximização de capital

Maximize utility function
Matlab Model
or
Excel Model
In both models we will assume that the investor wants to maximize capital until death which we assumed to be at the age of 82, as it is average life expectancy in Portugal (value of 2015).

Reforma/ Vários objetivos

In the suggestions' section we will present an example in which we tested how much the investor should allocate to the portfolio (in this case the one offered by the bank) when he/she has different objectives.

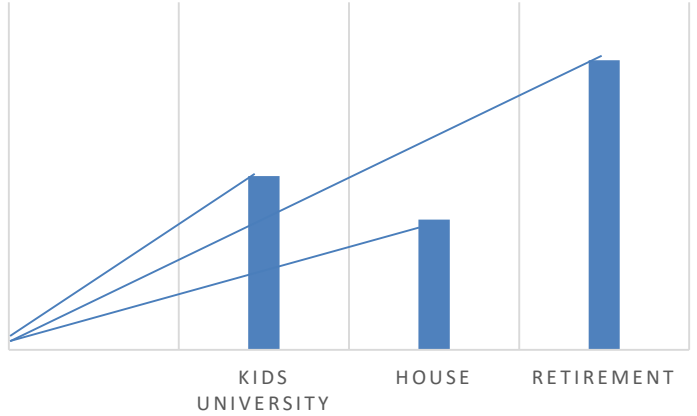


Figure XVIII – Illustration of goals based investing
Source: authors' own figure

THESE QUESTIONS CONTRIBUTE TO CALIBRATE THE LEVEL OF RISK AVERSION

12. Qual das carteiras seguintes lhe parece mais adequada aos seus objetivos, considerando as rendibilidades anualizadas num período de 3 anos e no período de um ano (na melhor e na pior situação obtida)?

	Rendibilidades anualizadas - período de 3 anos rolante*		Rendibilidades anuais período de 1 ano rolante*	
	Pior retorno obtido	Melhor retorno obtido	Pior retorno obtido	Melhor retorno obtido
Carteira A	-0,4%	7,0%	-6,0%	16,0%
Carteira B	-4,0%	10,0%	-17,0%	26,0%
Carteira C	-10,0%	15,0%	-31,0%	43,0%

Outra Carteira: Sem perda de capital.

Figure XIX – Different portfolios worst and best return
Source: Caixagest actual questionnaire

13. Se a meio do seu horizonte temporal tivesse uma queda de 20%, 30%, 40%, no valor do seu investimento em qual das situações liquidaria a sua posição?

- 12 was already asked in the original questionnaire by Caixagest and 14 is asked in most of our benchmark of risk profile questionnaires (75%).

14. Tem mais de 36% de prestações de crédito relativamente ao seu rendimento mensal?

- Based on rule 28/36: a rule of thumb stating that a household should not spend more than 28% of its gross monthly income on total housing expenses and no more than 36% on total debt service, including housing and other debt such as car loans.
  Note: This question may be subject to change, depending on what CMVM believes to be the most appropriate level of indebtedness for the investor. The Comissão de Mercado de Valores Imobiliário (CMVM) sets rules that Caixagest needs to follow to construct the questionnaire and the final output results from a set of discussions between CMVM and the bank, however, our group did not participate in those.

THIS QUESTION WAS PROPOSED IN ORDER TO INTRODUCE A MORE DYNAMIC WAY OF MEASURING RISK AVERSION RATHER THAN THE TRADITIONAL ONES

15. Assuma uma lotaria em que tem 50% de probabilidade de ganhar 300€ e igualmente 50% de não ganhar nada (0€). Selecione apenas o patamar a partir do qual preferia receber o montante certo.

(Por exemplo, se prefere 130€ certos em vez de jogar na lotaria acima mencionada selecione essa opção.)

Montante certo	Escolher montante certo
0 €	
10 €	
20 €	
30 €	
40 €	
50 €	
60 €	
70 €	
80 €	
90 €	
100 €	
110 €	
120 €	
130 €	
140 €	
150 €	
160 €	
170 €	
180 €	
190 €	

- *In the lottery they could win either €300 or €0 with 50% probability (…). In each row the lottery was exactly the same but the safe option increased from row to row. In the first row the safe option was 0, in the second it was 10, and so on up to 190 in row 20.*
  Once a respondent preferred the safe option to playing the lottery, the interviewer asked whether the respondent would also prefer the even higher safe payments to playing the lottery, and all subjects responded in the affirmative. The switching point is informative about a subject's risk attitude. Since the expected value of the lottery is 150, weakly risk-averse subjects should start to prefer the safe option over the lottery for save payments that are less than 150. They should also prefer larger safe payments to the lottery. Only risk-loving subjects should opt for the lottery when the offered safe option is 160, 170, 180, or 190. (…) As very few individuals are typically extremely risk loving, a maximum of 190 was chosen. As discussed in what follows, the fact that the table is bounded at 190 is inconsequential for our results.
  Dohem, T. et al, "Individual Risk Attitudes"

THIS WOULD BE A DYNAMIC WAY OF MEASURING RISK AVERSION, HOWEVER WE WERE NOT ABLE TO TEST IT, BECAUSE IT REQUIRES PROGRAMMING SKILLS

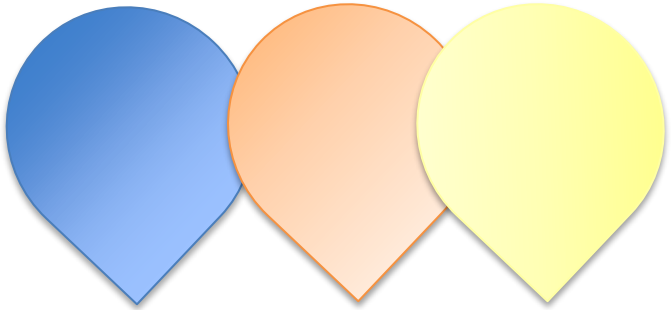
The Balloon Analogue Risk Task – The original game

Example: https://www.brainturk.com/bart

3 balloons of different colours: blue, orange and yellow are presented one at a time. For each pump, the balloon grows in size and the individual earns money that is deposited into a temporary reserve. The value of the reserve is never revealed to the participant.
The bigger the balloon the higher the probability of popping. If it pops the participant loses all the money in the temporary reserve. At any given time, the participant can either pump the balloon or collect what he/she has earned so far.
Since each successive pump carried an increased risk of causing the balloon to pop, **the authors took the average number of pumps, excluding balloons that exploded, to be the adjusted value corresponding to the individual's risk preference**.
Charness, G.; "Experimental methods: Eliciting risk preferences"

Sure amount

Pump



Probability of popping	Blue	Orange	Yellow
After first pump	1/128	1/8	1/32
After second	1/127	…	…
…	….	…	….
Guaranteed explosion	After 128 pumps	After 8 pumps	After 32 pumps

CAIXAGEST FOUND IT WAS IMPORTANT TO KEEP THIS QUESTION IN ORDER TO ALIGN CLIENT EXPECTATIONS WITH PORTFOLIO ALLOCATION

16. Investimentos muito conservadores geram, normalmente, retornos inferiores à taxa de inflação, o que pode levar a uma redução do poder de compra do investidor. Considerando os seus objetivos de investimento, qual das seguintes frases se adequa mais?

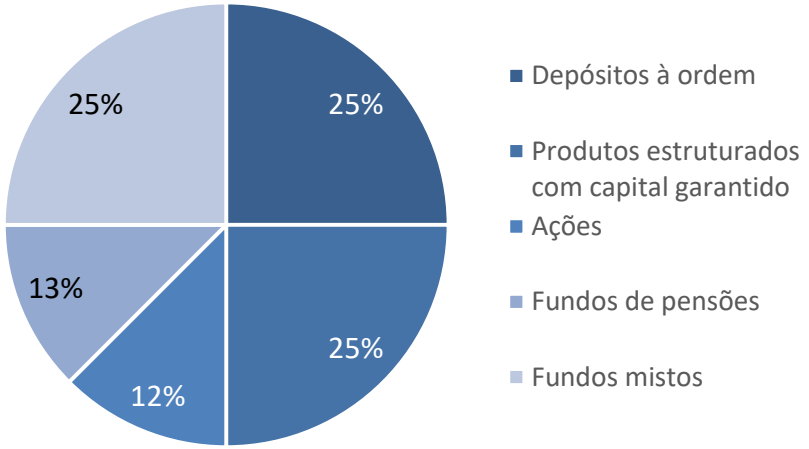
É preferível que os meus investimentos sejam seguros, mesmo que isso signifique que estes não seguem o mesmo ritmo da taxa de inflação.
Desde que os meus investimentos sigam, aproximadamente, o mesmo crescimento que a taxa de inflação, estou disposto a arriscar pequenas perdas no valor do meu investimento.
Procuro que os meus investimentos cresçam a uma taxa superior à da inflação e para este fim estaria disposto a algum risco adicional.
Procuro que os meus investimentos cresçam a uma taxa bastante superior à da inflação, para aumentar consideravelmente o meu poder de compra, estando disposto a tomar um risco bastante superior.

- Asked in 50% of our benchmark of risk profile questionnaires.
- After the mid presentation and some discussion with Caixagest, they found it was important to ask this question, even though it is not used in the model. The reason to include this question was that the bank needs to be aware of what the clients expectations are in terms of risk-return trade off to make sure the final allocation proposed will meet these requirements.

THIS QUESTION COULD BE ADDED TO THE QUESTIONNAIRE TO UNDERSTAND WHICH INVESTMENT CATEGORIES INVESTORS ARE MORE RETICENT WITH

Investimento passado

Produtos	Conhece?	Já utilizou?	Consideraria investir de novo?
Depósitos à Ordem e/ou a Prazo			
Produtos Estruturados com Capital Garantido			
Obrigações			
Ações ou Fundos de Ações			
Seguros			
Fundos de pensões			
Fundos mistos			
Outro tipo de investimento			



The graph illustrated to the left presents the investors, from all those interviewed, that after using a specific product do not wish to use it again. This type of information could be used by Caixagest to understand investors' fears, clarify them and adjust their investment allocation.

Figure XX – Products that investors have used once and do not want to use again
Source: authors' own calculations

SOME QUESTIONS WERE EXCLUDED BECAUSE THE GROUP DID NOT CONSIDER THEM RELEVANT

Investimento passado

Teve uma perda significativa nos seus últimos investimentos? Se sim, qual foi a sua reação? Se não, qual seria?

Teve algum momento de incerteza? Se sim, como reagiu? Se não como reagiria?

Em algum dos momentos anteriores (11.1 e 11.2) se sentiu ansioso ou stressado?

14.1, 14.2, 14.3. Although the distinction between loss aversion and risk aversion is an important field of study, we did not find it relevant when determining the optimal investment portfolio.

Through a superficial approach we detected a certain correlation between the lack of knowledge of financial markets and the negative emotions (anxiety, nervousness) upon changes in the value of a portfolio. We then ran a regression to see if this lack of knowledge would be enough to predict the answer of the client on the emotions question. With such, we proved (through the p-value approach) that the coefficient on this regression is not statistically different from zero, at a 5% confidence level (no statistical evidence to reject the null hypothesis of b=0). Thus, the question presented was deleted from the questionnaire.

Figure XXI – Knowledge in financial markets
Source: authors' own calculations

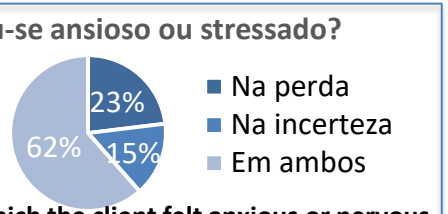


Figure XXII– Situations in which the client felt anxious or nervous
Source: authors' own calculations

Preferia manter o seu salário atual para sempre ou passar a ter um trabalho com salário instável?

This question did not show much variability in the answers so it was also excluded.

Planeia com antecedência as suas viagens?

Tem carta de condução de moto?

Pratica desporto?

É fumador?

Even though these indicators can help to extract some risk attitudes of our clients, it is not clear whether they are optimal to reflect risk attitudes towards financial investments.
Individual Risk Attitudes: Measurement, Determinants, and Behavioral Consequences

Answers

After some discussion regarding the content of the questionnaire, it was tested, anonymously, among Caixagest collaborators and some masters' teachers for 1 week. In this section we will present the results collected and some of the drawbacks of its application, suggesting further improvement.

OUR SAMPLE CHOOSES MOSTLY LOW RISK PORTFOLIOS BUT SEEKS RETURNS MUCH HIGHER THAN THE INFLATION RATE, WHICH IS QUITE CONTRADICTORY

24
answers

14
answers that can be used in the model

40
years (average of age)[1]

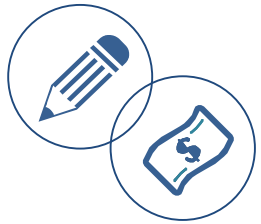
9
respondents with specialization in economics/management[1]

54.950€
average accumulated savings[1]

33.114€
average annual income[1]

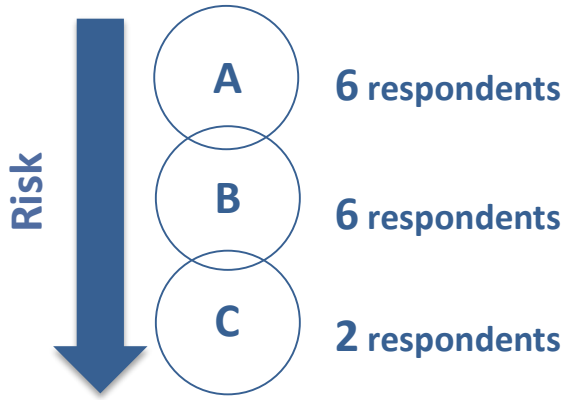
Sector[1]

1
education



13
financial services

Investment portfolio chosen[1]



Job title[1]

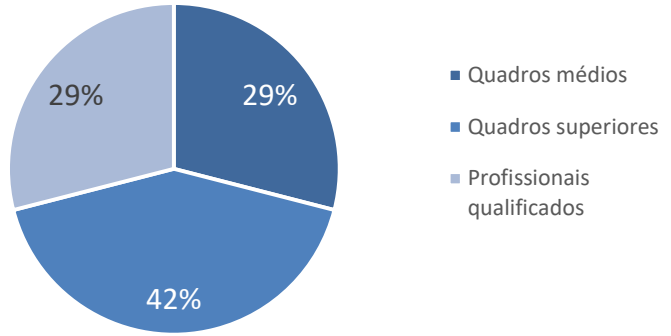


Figure XXIII– Job title
Source: authors' own calculation[1]

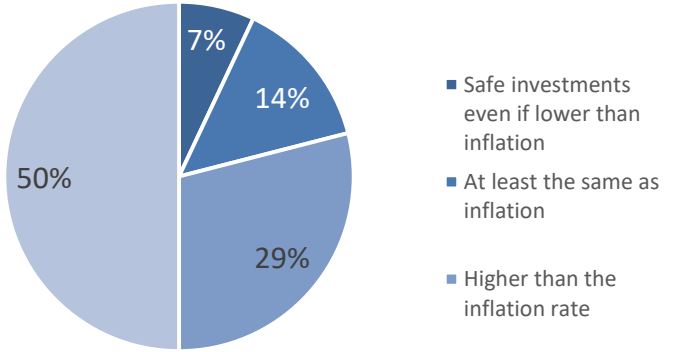


Figure XXIV – Expected return
Source: authors' own calculation[1]

[1] Based on the 14 answers that could be used in the models presented after.

THE LOW RATE OF PARTICIPATION IS ONE OF THE MAIN PROBLEMS WE HAD WITH OUR TEST

- **Low rate of participation** and the respondents were not actual clients interested in investing with the bank.
- **Low diversification in educational background** (which did not allow to test in the models the impact of educational background in optimal investment allocation, at least with real answers, because we performed some tests with hypothetical answers as it will be seen after).
- **Difficult to calibrate the level of risk aversion** (even after reading numerous studies regarding this matter).
  - Janecek, K. (2004) suggests that the risk aversion parameter may be between **30 – 300**.
  - Cecchetti, S. et al (1990) suggest a risk aversion parameter between **0 - 77**.
  - Mehra & Prescott (1985) suggest a parameter between **0 - 10.**

Amplify and diversify the sample

Provide some clarification or glossary before the application (check suggestions section)

Make the questionnaire more appealing

MatLab Model

In this section, we will present the first model that was developed, which is a dynamic model that allows the finding of an optimal allocation to the portfolio M (Caixagest portfolio) versus the risk free asset (by maximizing utility subject to consumption), based on portfolio M and investors' inputs. The inputs are: the risk free rate; portfolio M expected return and volatility; the investors' age; the level of risk aversion (gamma parameter, γ); cash on hand (X_0); average growth and volatility of wage.

IN SEARCH OF A MODEL: OPTIMAL INVESTING STARTS BY SOLVING THE INTERTEMPORAL CONSUMER PROBLEM

What is the intertemporal consumer problem?

- Maximizing a consumer's utility subject to his/her consumption preferences across a time interval.
- The consumer will allocate his/her income to consumption so that his overall utility is conditional not only on consumption today – but also in the fore coming periods.

Why the intertemporal consumer problem?

- The ultimate objective of investing is to generate utility
- Utility can be measured through production of cash flows in a monotonic function: more cash flow is more useful than less.
- Consumption can be seen as cash withdrawals, which derive utility for the investor.
- Even though in the actual model, income is seen not only as wages, for the sake of simplicity in our model is assumed to be only wages.
- In order to produce a yield on investment, the investor must invest his(her) savings on a portfolio.
- In order to attain maximum utility, the investor must optimally allocate his investments between the tangency ("M") portfolio and a riskless asset.

References

- We are currently following a simplified version of the approach proposed by Cocco and Gomes (2005), from The Review of Financial Studies Vol. 18, No. 2.
- The article shows how to calibrate a real life cycle model of consumption and portfolio choices, whereas portfolio choices is our primary variable of interest.
- The authors suggest a mathematical utility function, conditional on consumption and risk-aversion.

THE PROBLEMATIZATION FROM COCCO AND GOMES (2005) ASSUMES UTILITY IS DERIVED FROM CONSUMPTION

- The authors propose measuring utility at a given point in time, t, by a constant relative risk aversion (CRRA) function:

$$U_t = \frac{C_t^{1-\gamma}}{1-\gamma} + \beta U_{t+1} \qquad \text{(I)}$$

β is the discount factor
C_t is the consumption at t.

- Utility is derived from consumption.
- Risk-aversion γ implies that the more risk-averse the investor is, the less he values consumption at a particular time t; he wants to be sure he invests enough to be able to consume at $t+i$.
- CRRA function exhibits constant relative risk aversion, meaning that decisions won't be affected by scale. This is useful for the model to reach a wider range of investors.
- Overall the Bellman equation for this problem is:

$$V_t(X_t) = \max\left[U\left(C_t\right) + \beta E(V_{t+1})\right] \equiv \max\left[U(c_t) + \cdots\right] \qquad \text{(II)}$$

- Moreover,

$$X_{t+1} = Y_{t+1} + (X_t - C_t)\left(\alpha_t R_{M,t+1} + (1-\alpha_t)R_f\right) \qquad \text{(III)}$$

- Where Y_{t+1} is the expected wage, X_{t+1} is the initial wealth in period $t+1$, α_t is the proportion invested in the risky portfolio "M" and R_i are the returns on M and the risk-free asset, respectively.
- At the last period T, the investor consumes everything :

$$X_T = C_T = Y_T + (X_{T-1} - C_{T-1})\left(\alpha_{T-1} R_M + (1-\alpha_{T-1})R_f\right) \qquad \text{(IV)}$$

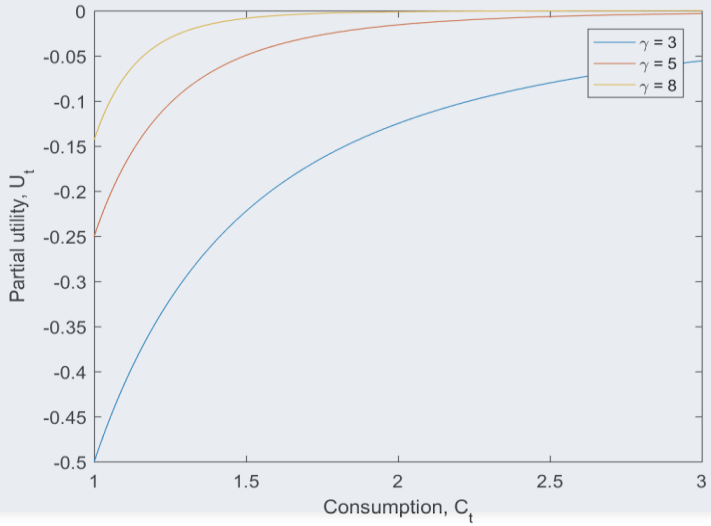


Figure XXV - Reaction of the CRRA utility function to risk aversion (1)
A risk averse investor prefers consumption today
Source: Own calculations.

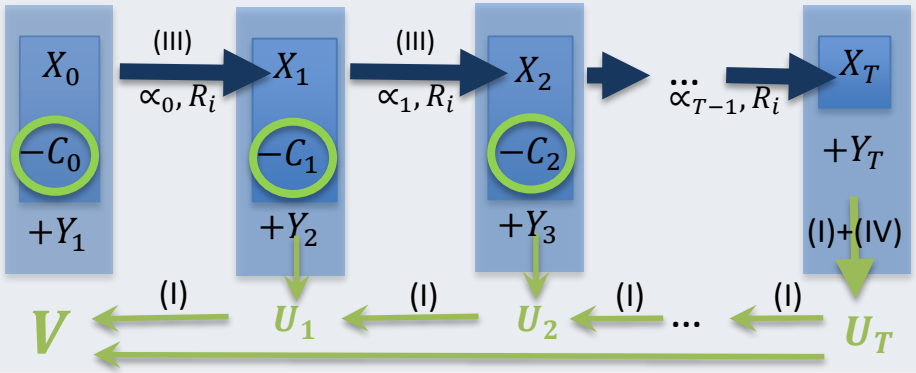


Figure XXVI - Representation of the utility generation flow Source: Own Calculations
Equations (I) through (IV)

OPTIMIZATION IN PRACTICE: THE BINOMIAL MODEL HELPS SIMPLIFYING THE PROBLEM

The binomial model uses discrete time approximations of the varying values of the variables of interest. This framework can be drawn as a tree, also known as binomial lattice. Binomial lattices are typically used for pricing derivative instruments.

Why a binomial lattice?

- A binomial lattice can be used to estimate the possible paths for the investor's wage, and the performance of the risky portfolio, "M".
- This relies on the assumption for coefficients of growth (u_i) or decrease (d_i) in either wages or M.
- This approach is intuitive and keeps our analysis simple.
- We can manipulate the binomial lattice to consider the cross states of wages and M (when both go up, both go down, when wage goes up and market goes down and vice versa).

Assumptions of the binomial lattice

- The coefficients are calculated such that

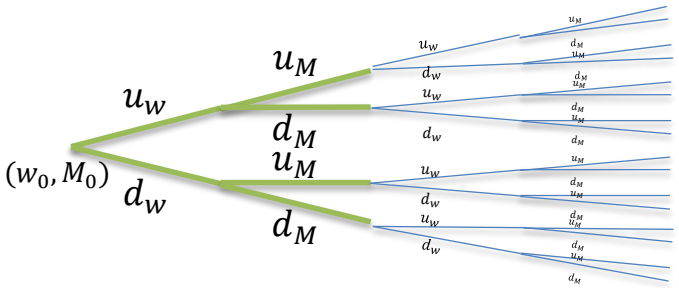
$$\text{Growth multiplier}_i = u_i = 1 + \mu_i + \sigma_i \quad \text{(V)}$$

$$\text{Decrease multiplier}_i = d_i = 1 + \mu_i - \sigma_i \quad \text{(VI)}$$

- The size of each time step is assumed to be one year for simplicity.
- μ_i and σ_i. are based on historical return series of wages and of portfolio M (the calculation of those is presented in the inputs section).

Figure XXVII - An illustration of the three-period lattice
Source: authors' own figure

The first period is represented by the green arrows and the second by the blue. There are 4^{T-1} final nodes. Pictured here is the 3-step case.



Why *not* a binomial lattice only

- Using a binomial lattice only runs into computational trouble.
- For every year, each node originates 4 nodes. E.g., if $T = 15$, the matrix will have $4^{14} = 268{,}435{,}456$ rows!

An approximation to facilitate computation

- We depart from the fact that we can approximate the shape of the CRRA function by $V_t \cong -\exp(a + b \ln X_t)$, where a and b are the estimators from running OLS $\ln V_{T,i} = a + b \ln X_{t,i} + \varepsilon_i$, and $i = 1,2, \dots,$ grid_size. grid_size reflects the hypothetical range of values of X_T, standardized by the initial wage.
- This way, we can easily estimate future utilities based on the binomial framework's expected utility for the next period.
- We use backward induction in order to achieve the optimal allocation, which is given by the set $(\propto_0, c_0)$.
- All variables are standardized around wages.

Excel Model

In order to have a simpler model we developed a model in Excel, which is not dynamic and only has in consideration the utility of the last period. The inputs are the same with the exception that savings are now given by the investor (in question 10 of the proposed questionnaire) and not computed by the optimization problem.

EXCEL MODEL

UNLIKE THE OTHER THIS MODEL IS STATIC AND REQUIRES A NEW INPUT WHICH IS THE SAVINGS RATE

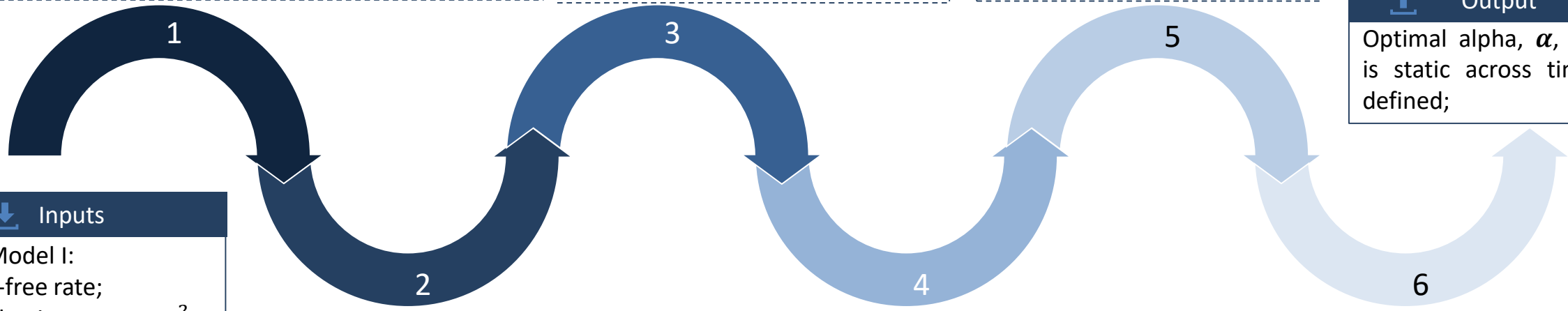
Simulate 6000x#tsteps different shocks for wage and market returns:
$\varepsilon_{j,i}$, where j = W,M and i = 1, 2, …., 6000x#tsteps;

Compute wage simulations:
$\boldsymbol{W_1 = W_0 * (1 + r_{w,1})}$ **(VIII)**
$\boldsymbol{W_{i+1} = W_i * (1 + r_{w,i+1})}$; **(IX)**

Compute utility, at T, through CRRA utility function:
$\mathbf{U_T = -(X_T^{1-\gamma})}$ **(XII)**

Output

Optimal alpha, $\boldsymbol{\alpha}$, which is static across time, T, defined;



Inputs

As in Model I:
- Risk-free rate;
- Market inputs: μ_M, σ_M^2;
- Wage inputs: μ_W,σ_W^2, W_0;
- Wealth input, W_0;
- Time horizon, T;
- Risk aversion coefficient, γ;

New input:
- Savings rate, s;

Compute market and wage returns through Monte Carlo simulation:
$\boldsymbol{r_{j,i} = \mu_j * \Delta t + \sigma_j * \sqrt{\Delta t} * \varepsilon_{j,i}}$; **(VII)**

Compute wealth values:
$\boldsymbol{X_{i+1} = [X_i + W_i * s] * (1 + r_{p,i+1})}$ **(X)**
Where $\boldsymbol{r_{p,i+1} = \alpha * r_{M,i+1} + (1 - \alpha) * r_f}$ **(XI)**

Compute the average utility, at T, of the 6000 simulations and maximize it, using Solver, by changing $\boldsymbol{\alpha}$.

If $W_0 = 0 \rightarrow \alpha = \frac{RP}{\gamma \sigma_M^2}$, where $RP = \mu_M - r_f$

Inputs

In this section we will present the inputs that we need to run our models, which are: the risk free rate, portfolio's (M) average return and volatility, wage average growth rate and volatility and risk aversion parameter (gamma).

THE RISK FREE RATE IS DIFFICULT TO DEFINE, USUALLY THE EURIBOR IS USED, BUT SINCE THEY ARE ALL NEGATIVE, WE ASSUMED 2% FOR OUR MODELS

Traditional portfolio theory

- In basic portfolio theory, a risky asset and a riskless asset are assumed to exist.
- Caixagest supplies the risky "M" asset, which is composed of several diversified indices.

"There ain't no such thing as a free lunch"

- There is no absolutely riskless asset: there is always credit risk or liquidity issues.
- Yields on 10-year bonds of the Bundesbank, the Fed and the BoE have displayed significant historical variability over the last decades.
- No unique ("correct") way of calculating risk-free rates.
- For valuation purposes, the ECB (2014) suggests using futures of overnight interest rate swaps (EONIA).

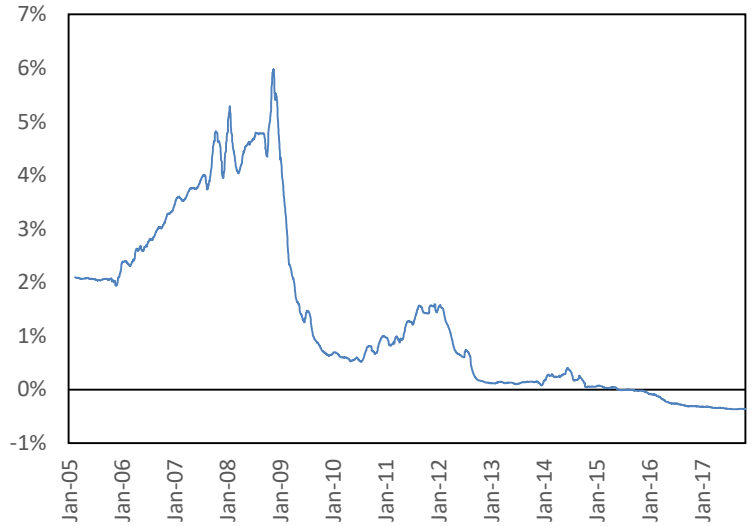
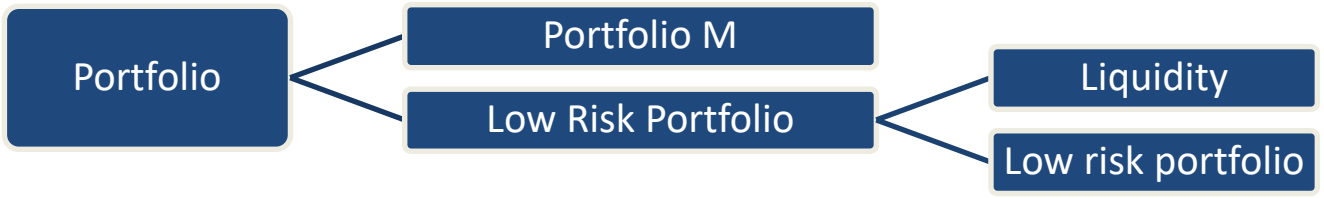


Figure XXVIII - 3-month EURIBOR (annualized): 1-month daily moving average.
Source: authors' own figure

For simplification…

- We present the model with the risk-free rate set at a hypothetical 2% for simplification.
- EURIBOR was widely used as a risk-free proxy before the financial crisis of 2007 (ECB, 2014).
- Spot rates on EURIBOR would be appropriate if they weren't all negative rates – and no small investor would procure such a product.
- For implementation purposes, this rate should be replaced by an extremely low risk portfolio to be designated by Caixagest, as we will explain after.
- Volatility should be low enough not to violate the model's assumption that the riskless asset has zero volatility.

THE LOW RISK PORTFOLIO IS COMPOSED BY TWO COMPONENTS: LIQUIDITY AND THE REST OF THE LOW RISK PORTFOLIO



Low Risk Portfolio

- The liquidity component is the Euribor rate which is used as a proxy of the risk free rate as explained before.
- The low risk portfolio is composed by bonds, more precisely Corporate debt Investment grade which is debt with relatively low risk of default and public debt with low maturity.
- For the corporate debt Caixagest uses a Barclays index. This euro aggregate bond index is a benchmark that measures the investment grade, euro-denominated, floating rate bond market, including treasuries, government related, corporate and securitized issues. Caixagest also uses another index that measures the non-securitizes component of the US Aggregate Index. It includes investment grade, US dollar denominated, fixed rate treasuries, government-related and corporate securities.
- For the public debt Caixagest uses a JP Morgan Index that represent government bonds for one and three years.

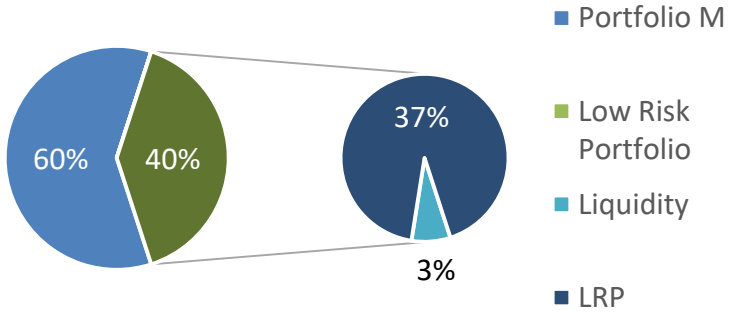


Figure XXIX - In this example, a client allocates 60% of his/her wealth to the risky asset and 40% to the low risk portfolio, this allocation is divided into bonds and the risk free asset.
Source: Caixagest internal documents

Weights

- The liquidity component has a fixed weight in the composition of the portfolio, which is 3%.
- The bonds component's weight depends on the weight of the risky portfolio (portfolio M) . Since the liquidity component is fixed, the weight of the low risk portfolio is (100% - 3% - ω_M)

IT WAS SUGGESTED BY CAIXAGEST THAT WE COULD COMPUTE OUR OWN SERIES OF RETURNS AND THUS WE APPLIED THE FOLLOWING PROCEDURE

1. After downloading the prices for the indexes included in the portfolio M from Bloomberg between 1/01/2005 until 31/10/2017
2. We computed the log return =$\ln(\frac{p_{t+1}}{P_t})$ (XIII).
3. After we computed the daily log portfolio return multiplying the log returns of each index by their specific weight in the M portfolio;
   - The formula used was $Portfolio\ \log return = \sum_{t=1}^{T} w_i * \log return_i$ (XIV)
4. The total log return of the portfolio was obtained summing all log returns per day of the portfolio
   - After the exponential log return of the portfolio was computed $e^{Total\ portfolio\ \log return} - 1$ (XV)
5. After the arithmetic annualized return was computed using the following formula $Arithmetic\ annualized\ return = Exponential\ portfolio\ \log return * \frac{number\ of\ base\ days}{number\ of\ observations}$ (XVI)
6. The annualized volatility of the portfolio was computed by multiplying the daily standard deviation of the returns by the square root of the number of base days (250 trading days).

Arithmetic Annualized Return	8,81%
Annualized Standard Deviation	8,86%

PORTFOLIO M IS HIGHLY DIVERSIFIED, HOWEVER IT HAS SOME EXCHANGE RISK

Portfolio M

Daily data from Bloomberg, from Jan/2005 to October/2017, incl.;

μ_M = 8.81%, annualized arithmetic return for 250 trading days a year;

σ_M= 8.86%, annualized

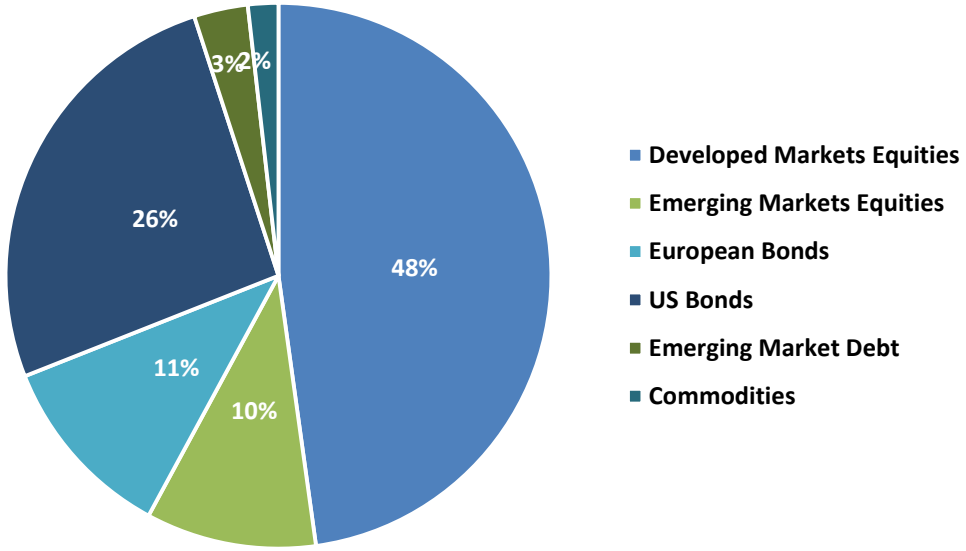


Figure XXX – Portfolio M composition.
Source: Caixagest internal documents

Details

- Portfolio comprises different indexes, of different asset classes, countries and currencies;
- There are four indexes European bonds and five American ones. Regarding emerging market currencies, there are three while in commodities are used just one index;
- Some of these indexes are composed by assets in different currencies, e.g. LP01TREU Index's currency breakdown: 82.4% Euro, 16.6% Sterling, 1% Krona;
- Portfolio was constructed, assuming an index of currencies, i.e. each index in its currency.
- Thus, 12.7% of the portfolio has exchange risk, as the currency of the indexes is not, entirely or partially, the same as the currency in which the index is denominated[1];

$$r_{p,i} = \sum_{j=1}^{22} w_j * r_{j,i} \S \qquad \text{(XVII)}$$

where I = 1,2,…,3346 and $r_{j,t+1} = \ln(\frac{S_{j,t+1}}{S_{j,t}})$

[1]This approach to calculate exchange rate risk was suggested by Caixagest

DEVELOPED MARKETS EQUITIES ARE INCORPORATED IN PORTFOLIO M

Figure XXXI - MSCI Daily TR Gross North America USD.
Source: Caixagest internal documents

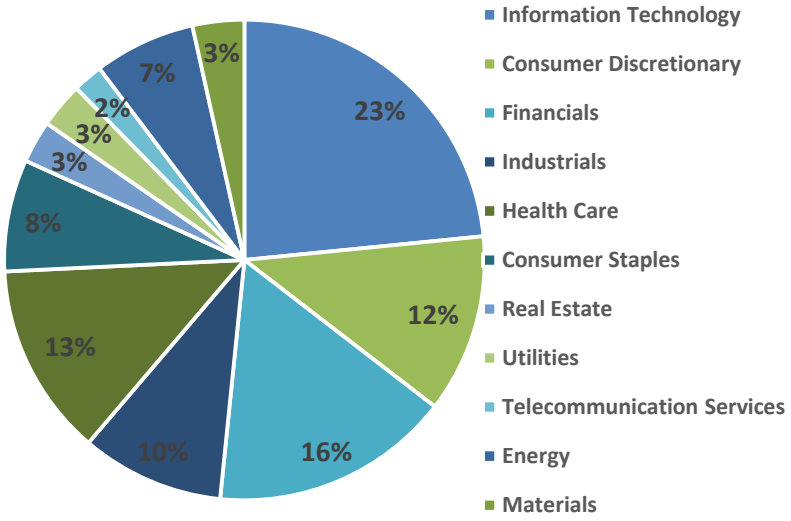
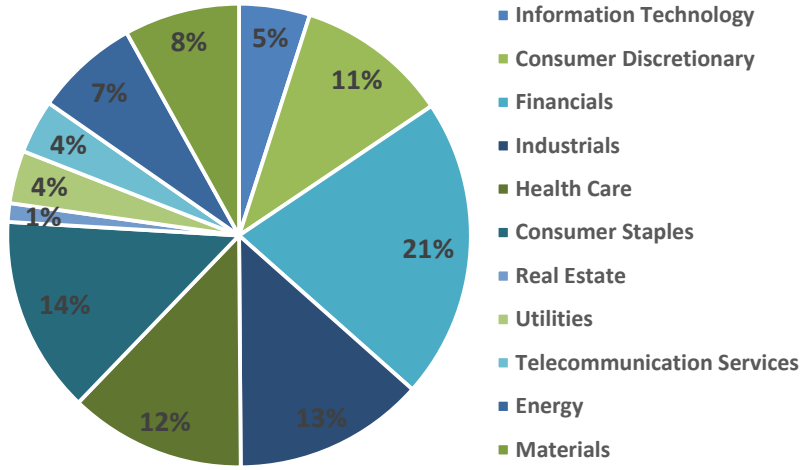


Figure XXXII - MSCI Daily TR Euro Local
Source: Caixagest internal documents

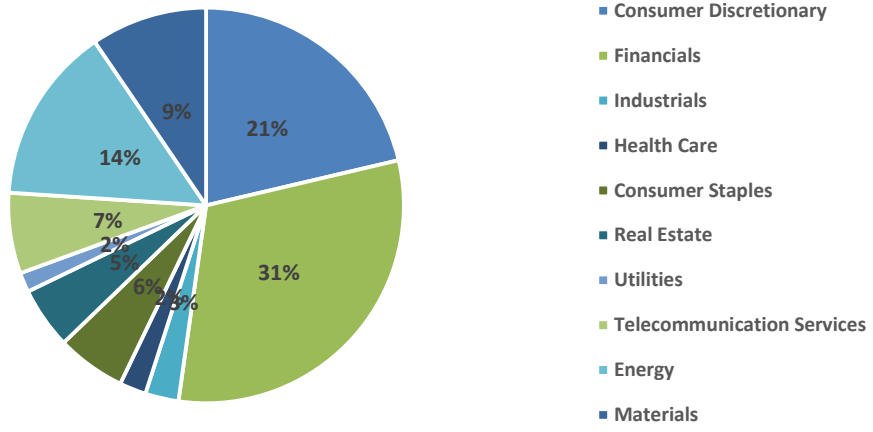


Portfolio M is exposed to developed markets such as North America (MSCI Daily TR Gross North America USD), Euro area (MSCI Daily TR Euro Local), Japan (MSCI Daily TR Gross Japan Local), United Kingdom (MSCI Daily TR Gross UK Local), Hong Kong (MSCI Daily TR Gross Hong Kong Local), Australia (MSCI Daily TR Gross Australia Local) and Switzerland (MSCI Daily TR Gross Switzerland Local). In several sectors, such as technology, financials, industrials, health care, real estate, energy and minerals etc. Above two of these indexes are presented.

EMERGING MARKETS EQUITIES ARE ALSO INCORPORATED IN PORTFOLIO M

Figure XXXIII - MSCI Daily TR EM Europe & Middle East & Africa USD

Source: Caixagest internal documents



Portfolio M is also exposed to emerging markets such as Asia (MSCI Daily TR Gross EM Asia USD), Latin America (MSCI Daily TR Emerging Markets Latin America USD) and emerging markets in Europe, Middle East and Africa (MSCI Daily TR Emerging Markets Europe & Middle East & Africa USD). In several sectors, such as technology, financials, industrials, health care, real estate, energy and minerals , among others.

Figure XXXIV - MSCI Daily TR EM Latin America USD

Source: Caixagest internal documents

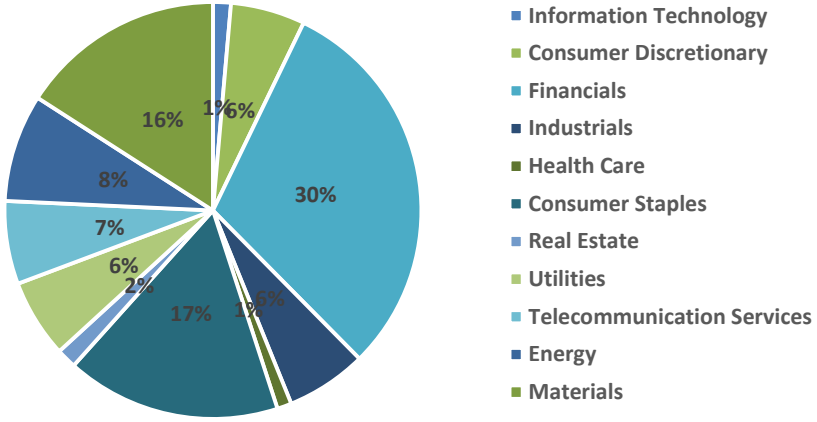
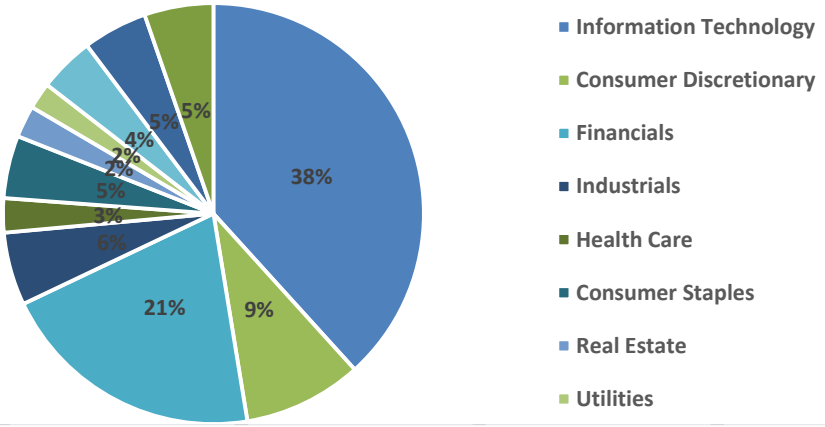


Figure XXXV - MSCI Daily TR Gross EM Asia USD

Source: Caixagest internal documents



WAGE AVERAGE GROWTH IS AROUND 1.7% FOR ALL SECTORS AND QUALIFICATION LEVELS, HOWEVER VOLATILITY IS AROUND 3%

			Níveis de Qualificação						
			Total	Quadros superiores	Quadros médios	Profissionais qualificados	Profissionais semiqualificados	Profissionais não qualificados	Praticantes e aprendizes
Sectores	Educação	Avg (10y)	1,42%	1,08%	-0,46%	1,64%	1,54%	2,14%	2,25%
		St Deviation (10y)	3,42%	2,43%	4,34%	2,50%	4,92%	4,03%	6,73%
	Construção	Avg (10y)	2,22%	1,01%	0,96%	2,07%	1,89%	2,81%	3,31%
		St Deviation (10y)	2,19%	5,32%	4,04%	1,54%	1,98%	1,81%	1,67%
	Água e Pesca	Avg (10y)	2,37%	0,79%	-1,23%	3,63%	1,88%	2,30%	2,36%
		St Deviation (10y)	3,33%	10,15%	7,07%	6,07%	1,87%	2,49%	4,14%
	Indústria Extractiva	Avg (10y)	2,33%	2,80%	2,54%	2,16%	1,61%	2,01%	3,65%
		St Deviation (10y)	1,26%	7,83%	6,70%	1,20%	1,75%	2,68%	4,79%
	Indústria Transformadora	Avg (10y)	1,90%	1,45%	0,53%	1,81%	1,73%	2,05%	2,58%
		St Deviation (10y)	1,63%	4,11%	2,26%	1,04%	1,00%	1,17%	1,91%
	Alojamento e Restauração	Avg (10y)	1,60%	2,23%	1,44%	2,05%	1,63%	1,76%	2,48%
		St Deviation (10y)	1,78%	4,48%	2,18%	1,55%	1,89%	1,64%	1,87%
	Transporte e Armazenamento	Avg (10y)	0,34%	1,73%	1,67%	0,71%	-0,16%	1,25%	2,33%
		St Deviation (10y)	3,27%	5,63%	7,51%	2,13%	5,95%	4,97%	1,93%
	Electricidade, Gás e Água	Avg (10y)	4,59%	3,51%	2,73%	3,14%	1,09%	-0,25%	10,53%
		St Deviation (10y)	7,33%	8,50%	4,67%	8,38%	14,40%	18,96%	30,02%
	Atividades financeiras e Seguros	Avg (10y)	2,13%	1,01%	1,71%	1,39%	0,50%	1,94%	1,22%
		St Deviation (10y)	2,33%	5,19%	1,76%	1,38%	5,45%	6,11%	4,36%
	Saúde e Apoio Social	Avg (10y)	2,30%	1,73%	0,15%	0,99%	1,04%	1,98%	1,99%
		St Deviation (10y)	2,90%	4,90%	2,69%	1,38%	1,34%	1,74%	4,55%
	Administração pública e Defesa	Avg (10y)	-2,54%	1,74%	0,67%	0,30%	-0,44%	3,30%	0,34%
		St Deviation (10y)	7,25%	9,07%	12,47%	2,85%	4,36%	7,38%	18,22%
	Comércio e Retalho	Avg (10y)	1,68%	1,45%	0,78%	1,42%	1,64%	1,92%	2,49%
		St Deviation (10y)	1,52%	3,58%	2,40%	1,35%	1,30%	1,27%	1,69%

Figure XXXVI – 10 year average growth and volatility of men's wage by sector and qualification.

Source: Pordata

♂

In order to compute the average wage increase and its volatility, the group used data from Pordata. The data collected presented the evolution of wages in Portugal for the last 20 or 30 years in different sectors. The group decided that it would be more accurate and representative to select the last 10 years of data to analyse the average wage increase and its volatility.

WAGE AVERAGE GROWTH AND VOLATILITY ARE A LITTLE BIT HIGHER FOR WOMEN, 2% AND 3.2%, RESPECTIVELY

			Níveis de Qualificação						
			Total	Quadros superiores	Quadros médios	Profissionais qualificados	Profissionais semiqualificados	Profissionais não qualificados	Praticantes e aprendizes
Sectores	Educação	Avg (10y)	1,83%	0,57%	0,78%	1,17%	1,39%	1,67%	-
		St Deviation (10y)	3,85%	1,18%	2,62%	1,52%	1,22%	1,28%	1,39%
	Construção	Avg (10y)	2,57%	0,81%	2,31%	2,00%	1,91%	3,17%	3,14%
		St Deviation (10y)	2,31%	2,52%	8,76%	1,76%	2,04%	2,90%	2,37%
	Água e Pesca	Avg (10y)	2,55%	0,38%	1,28%	1,96%	1,87%	2,49%	2,28%
		St Deviation (10y)	1,71%	5,77%	10,25%	4,09%	1,47%	1,72%	3,92%
	Indústria Extractiva	Avg (10y)	2,61%	1,27%	3,48%	1,69%	2,32%	2,54%	3,32%
		St Deviation (10y)	2,03%	4,47%	10,87%	3,26%	5,74%	1,91%	6,91%
	Indústria Transformadora	Avg (10y)	2,50%	1,71%	0,77%	1,98%	2,16%	2,39%	2,77%
		St Deviation (10y)	1,70%	4,28%	2,65%	1,70%	1,06%	1,43%	1,57%
	Alojamento e Restauração	Avg (10y)	2,14%	2,17%	1,60%	2,17%	1,77%	2,39%	2,53%
		St Deviation (10y)	1,35%	3,82%	2,32%	1,32%	1,23%	1,45%	1,78%
	Transporte e Armazenamento	Avg (10y)	1,30%	0,57%	1,89%	0,52%	-0,23%	1,52%	3,14%
		St Deviation (10y)	3,42%	2,54%	6,04%	5,03%	4,14%	2,29%	2,34%
	Electricidade, Gás e Água	Avg (10y)	4,49%	3,98%	2,89%	1,76%	3,51%	2,18%	-1,44%
		St Deviation (10y)	8,13%	8,85%	10,59%	6,21%	15,26%	20,61%	10,45%
	Atividades Financeiras e Seguros	Avg (10y)	1,82%	0,43%	1,87%	1,19%	0,45%	1,31%	1,27%
		St Deviation (10y)	1,53%	6,57%	1,79%	1,06%	7,08%	7,44%	3,64%
	Saúde e Apoio Social	Avg (10y)	2,47%	1,69%	0,97%	1,23%	1,48%	2,14%	1,99%
		St Deviation (10y)	2,02%	3,73%	1,68%	1,21%	1,06%	1,38%	2,16%
	Administração pública e Defesa	Avg (10y)	-1,54%	3,50%	1,15%	-0,49%	-1,31%	2,42%	-2,29%
		St Deviation (10y)	9,42%	9,84%	8,99%	6,58%	7,00%	4,17%	18,71%
	Comércio e Retalho	Avg (10y)	2,30%	1,20%	1,20%	1,86%	1,93%	2,28%	2,56%
		St Deviation (10y)	1,38%	3,59%	2,75%	1,46%	1,21%	1,45%	1,64%

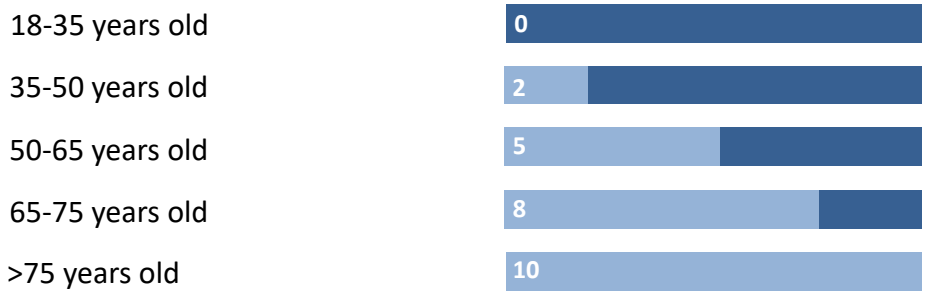
Figure XXXVII– 10 year average growth and volatility of women's wage by sector and qualification.

Source: Pordata

♀ The group repeated the same procedure for the case of women.

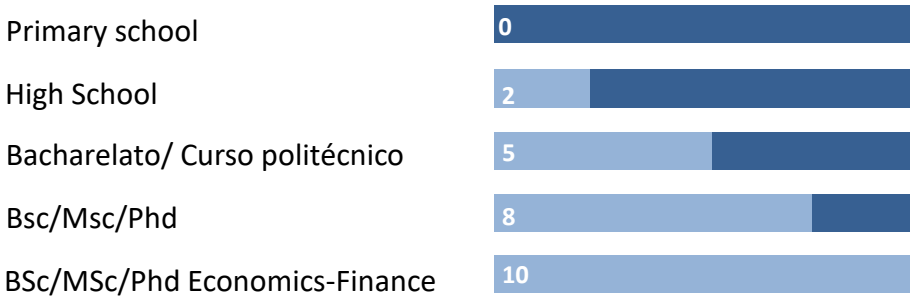
THE QUESTIONS CHOSEN FOR GAMMA CALIBRATION WERE THE FOLLOWING AND POINTS WERE GIVEN SUCH THAT HIGHER POINTS SUGGEST HIGHER RISK AVERSION

AGE



Points given as Caixagest

ACADEMIC QUALIFICATIONS

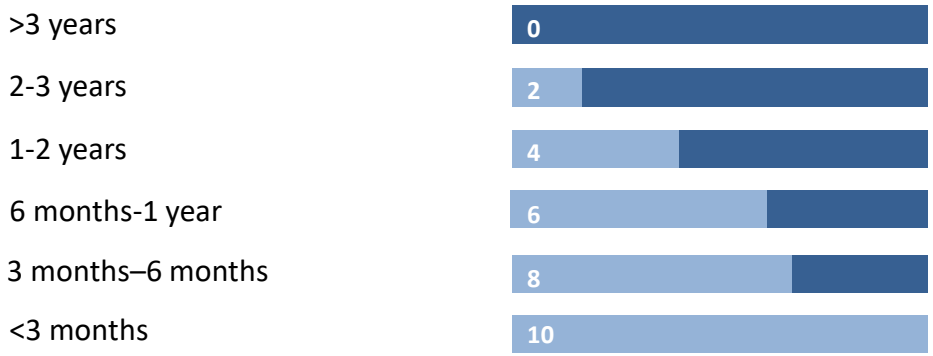


Points given as Caixagest

INCOME SOURCES - STABLENESS

Guiso, Jappelli and Terlizzese (1996), Guiso and Jappelli (1998), and Palia, Qi and Wu (2009) find that investors with more uncertain labour income, facing tighter borrowing constraints buy more insurance and tend to participate and invest less in equity markets.

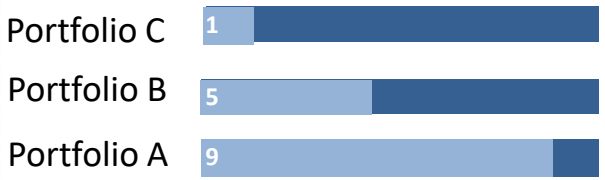
SAVINGS AMOUNT



As a consequence, background risk is particularly relevant for the young who have very little buffer savings and have still a long horizon over which earnings can be affected by persistent labour income shocks.

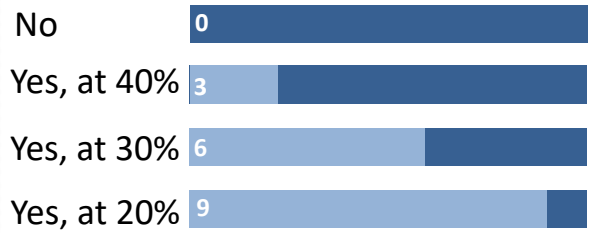
THE QUESTIONS CHOSEN FOR GAMMA CALIBRATION WERE THE FOLLOWING AND POINTS WERE GIVEN SUCH THAT HIGHER POINTS SUGGEST HIGHER RISK AVERSION

INVESTMENT PORTFOLIO



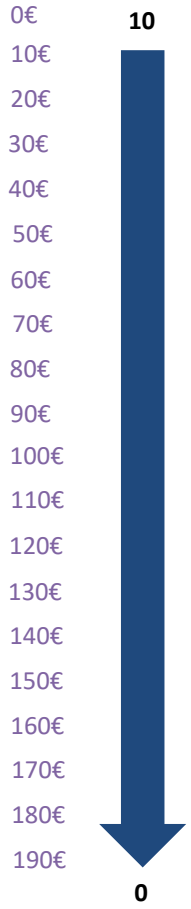
Points given as Caixagest

LIQUIDITY CONSTRAINTS



Asked in 75% of our benchmark of risk profile questionnaires.

LOTTERY



RESULT FROM THE QUESTIONNAIRE

RISK LOVING
1 MINIMUM

Relatively young (the first bracket of ages goes from 18 to 35) and low-educated investor, but whose finances are stable and has no liquidity constraints.

70 MAXIMUM
RISK AVERSE

Older and higher-educated investor, with liquidity constraints, unstable finances and more concerned with the stability of its investments than its returns.

IN THE STATIC MODEL (EXCEL) THE INTERVAL CHOSEN FOR GAMMA WAS FROM 10 TO 140

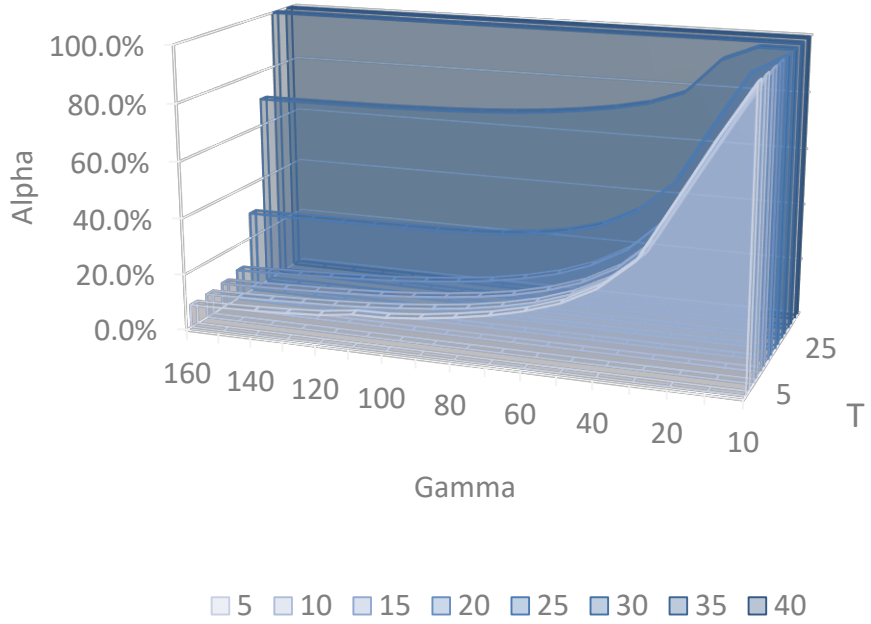


Figure XXXVIII - Alpha as a dependent variable of gamma and T
Source: authors' own figure

Fixed Inputs	
$r_f = 2\%$	$\mu_M = 8.81\%$
$\sigma_M = 8.86\%$	$\mu_W = 2\%$
$\sigma_W = 5.19\%$	$s = 20\%$
$Wage_0 = €48K$	$Wealth_0 = €118\,K$
$\Delta t = \frac{T}{50}$	

Gamma influence depends on the investment horizon

When the investment horizon is smaller, gamma has a bigger importance on determining the optimal alpha, whereas in larger horizons, the length of the investment seems to surpass the effect of the risk aversion coefficient.

Gamma calibration

For all time periods considered, a gamma below 10 yields an optimal alpha above 100%, which would not be a problem if we had considered the possibility of short selling, which we left out for the sake of simplicity. Moreover, as the risk aversion coefficient increases, the optimal value for alpha converges, in all the scenarios tested. Even though it may not be clear from the graph this conversion starts for a gamma around 70.

Thus, the gamma interval for this model is **10 – 70.**

The importance of time steps

After performing some tests, we found out that whenever T/t_steps was above 0.5 the alpha did not change much or was almost always equal to 1. This led us to the conclusion that the number of time steps needs to be high enough to "support" the time horizon, otherwise there is too much noise. Put differently, if we have for example T=20, we need to have sufficient intermediate time steps in between to allow for a more accurate calculation of alpha. Having this in mind, we define 50 time steps as a reasonable number, because T is usually lower than 50.

IN THE DYNAMIC MODEL (MATLAB) THE INTERVAL CHOSEN FOR GAMMA WAS ALSO FROM 10 TO 70

- To allow for comparison between models, the interval for gamma in this model was also set between 10-70 , which is reasonable having in mind the graphs depicted to the right.
- Even though in some situations, alpha is 100% or, if we had allowed, would probably be higher, this is normal, because the Sharpe ratio of the portfolio M is high, approximately 0.8.

Fixed Inputs	
$r_f = 2\%$	$\mu_M = 8.81\%$
$\sigma_M = 8.86\%$	$\mu_W = 2\%$
$\sigma_W = 5.19\%$	$Age\ end = 82$
$X_0 = 4$[1]	

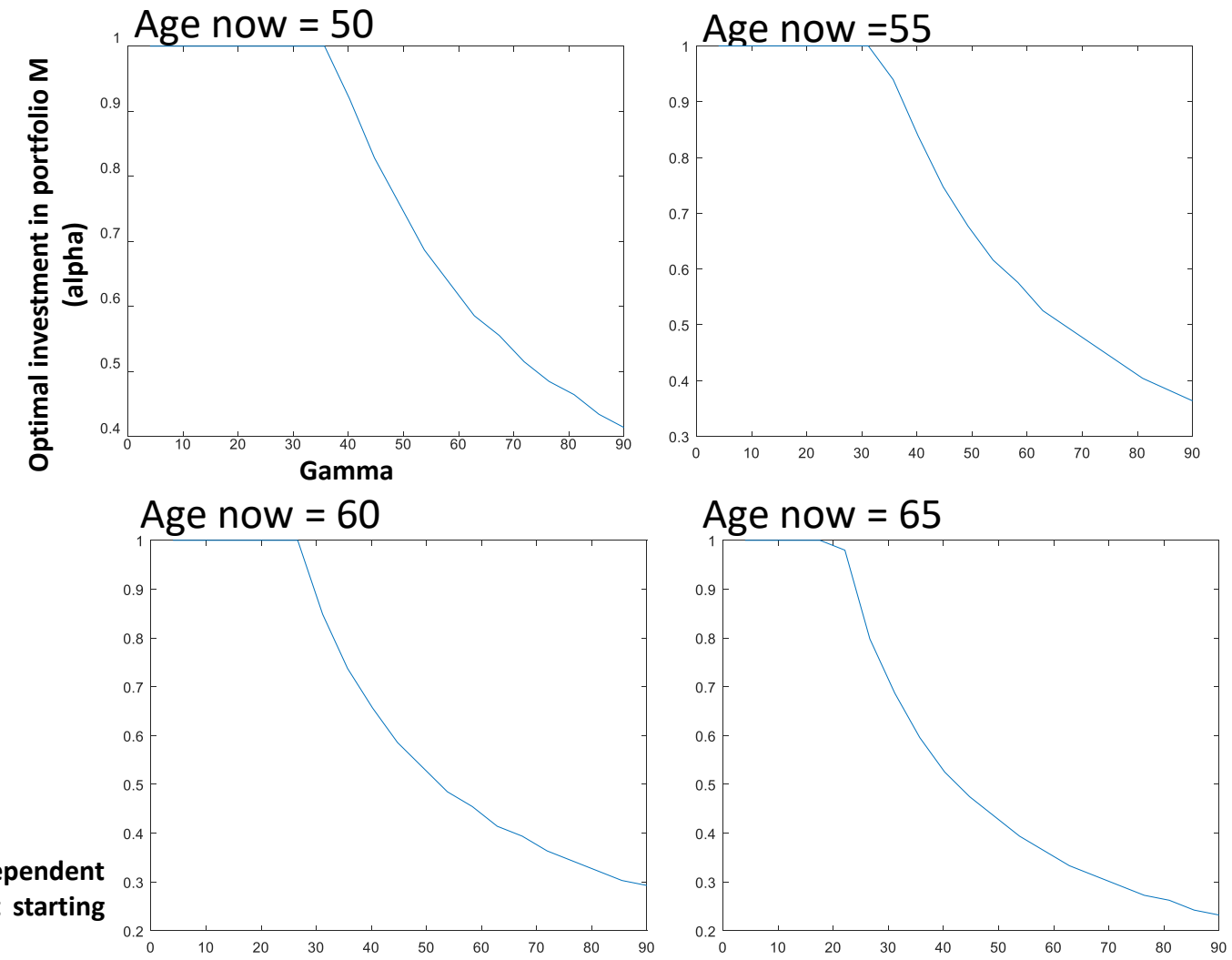


Figure XXXIX - Alpha as a dependent variable of gamma for different starting ages. Source: authors' own figure

[1]Total wealth normalized by wage.

THE WAY TO CONVERT THE POINTS OBTAINED IN THE QUESTIONNAIRE TO BE USED IN THE MODEL IS THE FOLLOWING

$$\gamma = \gamma_{min} + \frac{number\ of\ points}{maximum\ number\ of\ points} * (\gamma_{max} - \gamma_{min}) \quad \text{(XVIII)}$$

Variables	Definitions
γ_{min}	the lower bound of the appropriate gamma interval, which we assumed to be 10 in both models
$number\ of\ points$	refers to the number of points obtained in the questionnaire by the respondent, which depends on risk aversion
$maximum\ number\ of\ points$	the highest number of points that is possible to obtain in the questionnaire, which is 70
γ_{max}	the upper bound of the appropriate gamma interval which is 70

AFTER TESTING BOTH MODELS WITH ONE OF THE RESPONDENTS' ANSWERS WE GOT THE FOLLOWING RESULTS

Auxiliary calculation - Gamma

1

INPUTS	POINTS
Age = 49 years	2
Bachelors Economics/Finance	10
Savings for 1-2 years	4
100% of income is stable	0
Chose portfolio C	1
Would never liquidate (with 40%, 30% and 20% loss)	0
Total	**17**

2

$$\gamma = \gamma_{min} + \frac{total\ points\ quest.}{maximum\ number\ of\ points} * (\gamma_{max} - \gamma_{min})$$

γ = 10 + ($\frac{17}{70}$)*(70-10) = 25

$r_f = 2\%$
$\sigma_M = 8.86\%$
$\sigma_W = 2\%$
$Wage_0 = €48K$

$\mu_M = 8.81\%$
$\mu_W = 2\%$
$s = 20\%$
$Wealth_0 = €118\,K$
$T = 33$ (until 82 years)
$\gamma = 25$

Matlab

α = 100%

Excel

α = 84%

Analysing both models' answers, the optimal investment allocation to the risky asset for this individual is between 84% and 100%. In the interpretation we need to have in mind that this portfolio M has a great Sharpe ratio and thus individuals want to invest heavily in it.

Inputs Effect on Alpha

In this section we will test how the models react to changes in inputs (these changes effects' on alpha) and provide possible explanations for these results.

THERE IS AN OPPOSITE EFFECT OF WAGE AND WEALTH ON ALPHA, WHILE THE FORMER MOVES IN THE SAME DIRECTION THE LATTER MOVES IN THE OPPOSITE

Figure XL – Initial wage's effect on alpha
Source: authors' own figure

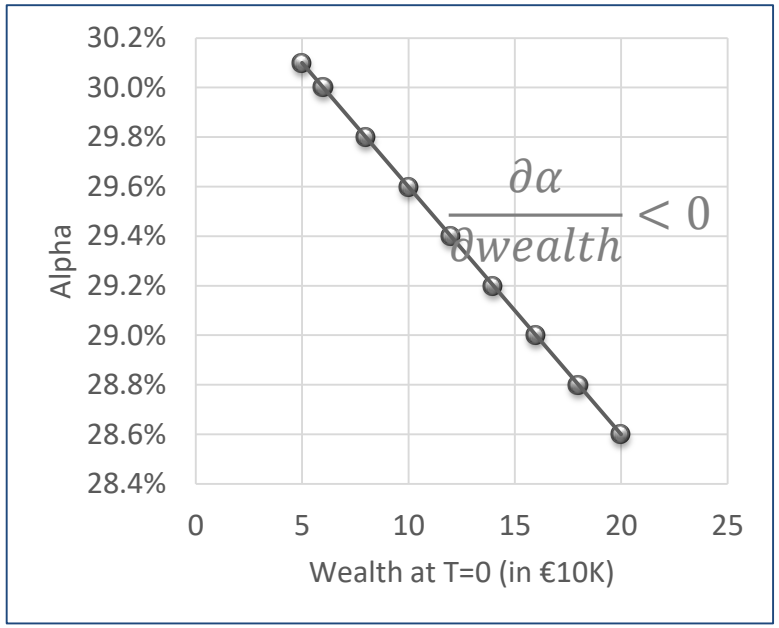


Figure XLI – Initial wealth's effect on alpha
Source: authors' own figure

Fixed Inputs	
$r_f = 2\%$	$\mu_M = 8.81\%$
$\sigma_M = 8.86\%$	$\mu_W = 2\%$
$\sigma_W = 2\%$	$s = 20\%$
$T = 10$	$\gamma = 40$
$\Delta t = \frac{T}{50}$	

Higher wage, higher alpha

Higher wage, means that the investor has a higher human capital, which is seen as a lower risk asset and thus he/she can invest more in the risky asset.

Retirement is seen as a risk free asset

Clearly, this future retirement income acts as a substitute for risk-free asset holdings and induces the investor to hold more stocks. The agent with little wealth will then tilt his financial portfolio more aggressively towards equities than the agent with a large amount of financial wealth, simply because the poorer investor already has a relatively larger risk-free asset position from her retirement income.

Consumption and Portfolio Choice over the Life Cycle

A DECREASE IN X IS THE SAME AS A DECREASE IN WEALTH OR AN INCREASE IN WAGE, SO THE EFFECT IS THE SAME AS IN EXCEL

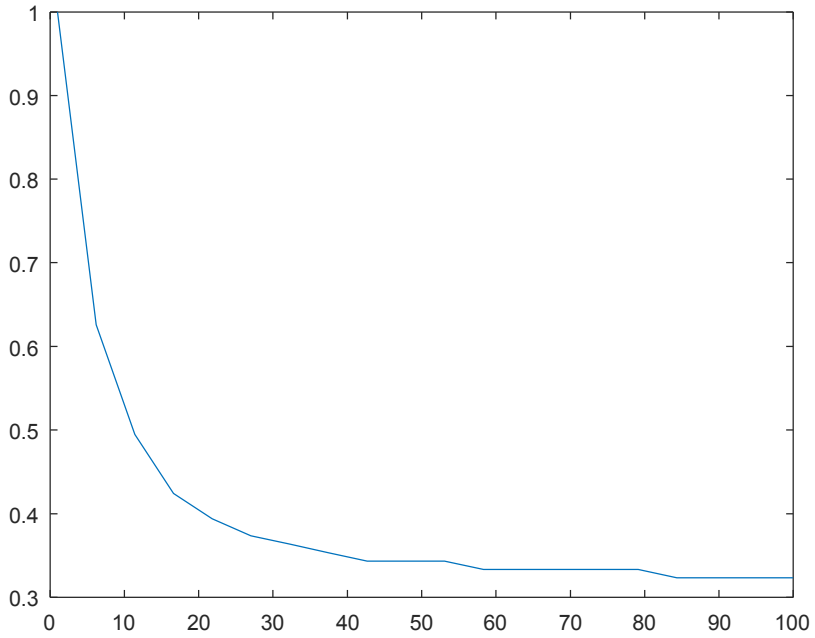


Figure XLII – X0 effect on alpha
Source: authors' own figure

Fixed Inputs	
$r_f = 2\%$	$\mu_M = 8.81\%$
$\sigma_M = 8.86\%$	$\mu_W = 2\%$
$\sigma_W = 2\%$	$\gamma = 40$
$T = 10$	

Higher wage, lower X, higher alpha

Since X is defined as total wealth divided by wage, if wage increases the investor has a higher value of human capital which is seen as a risk free asset and thus he invests more in the risky asset. (An increase in wage implies an increase in the denominator and thus a decrease in X)

Retirement is seen as a risk free asset

Same as before.

AS WAGE VOLATILITY INCREASES, THE EFFECT ON ALPHA IS NOT LINEAR

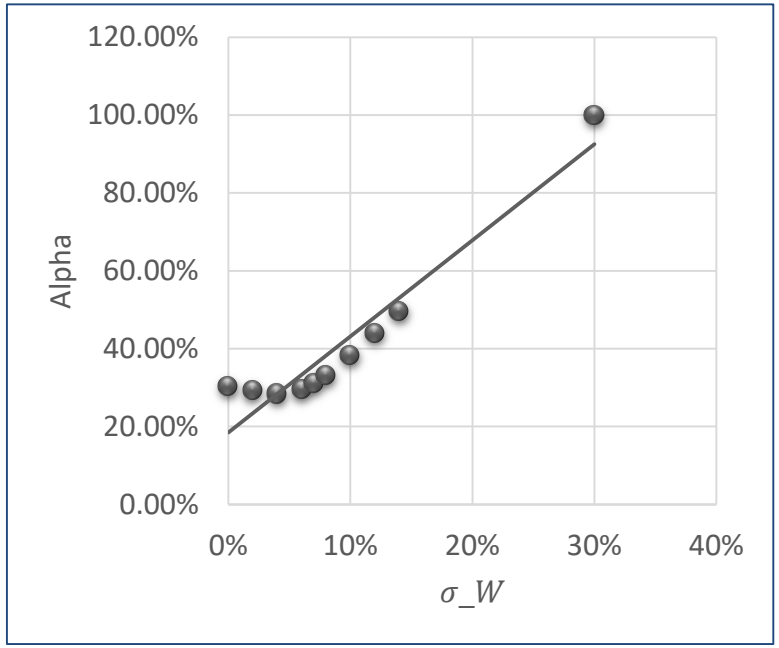


Figure XLIII – Wage volatility effect on alpha
Source: authors' own figure

Fixed Inputs	
$r_f = 2\%$	$\mu_M = 8.81\%$
$\sigma_M = 8.86\%$	$\mu_W = 2\%$
$T = 10$	$s = 20\%$
$\Delta t = \frac{T}{50}$	$Wage_0 = €118\,K$
	$W_0 = €48\,K$
	$\gamma = 40$

The wage volatility has different impacts during the life of the agent

1. An increase in the wage volatility is seen as an increase in his/her wage when the agent is younger. Thus, for higher investment periods *(age end-age now)*, an increase in the wage volatility is understood by the model as an increase in the human capital, which is seen as a lower risk asset, and consequently the agent can allocate more wealth to the risky asset.
2. An increase in the wage volatility is seen as an increase in the his/her risk when the agent is older. Thus, for smaller investment periods an increase in the wage volatility leads to a lower allocation of wealth to the risky asset, because in case of a loss the agent has a smaller period to to recover.

Therefore, the wage volatility has a greater impact as the investment period decreases.

AS WAGE VOLATILITY INCREASES, THE EFFECT ON ALPHA IS NOT LINEAR

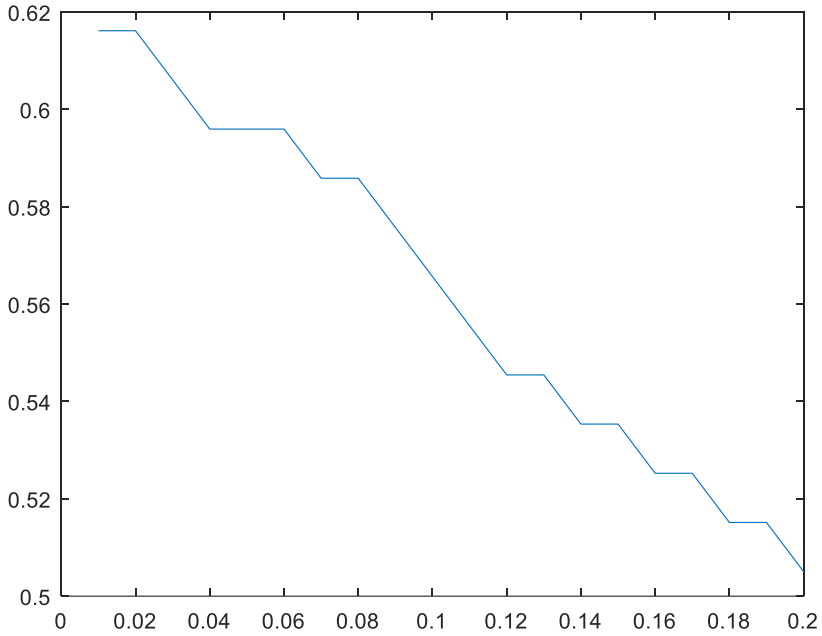


Figure XLIV – Wage volatility effect on alpha for T=5
Source: authors' own figure

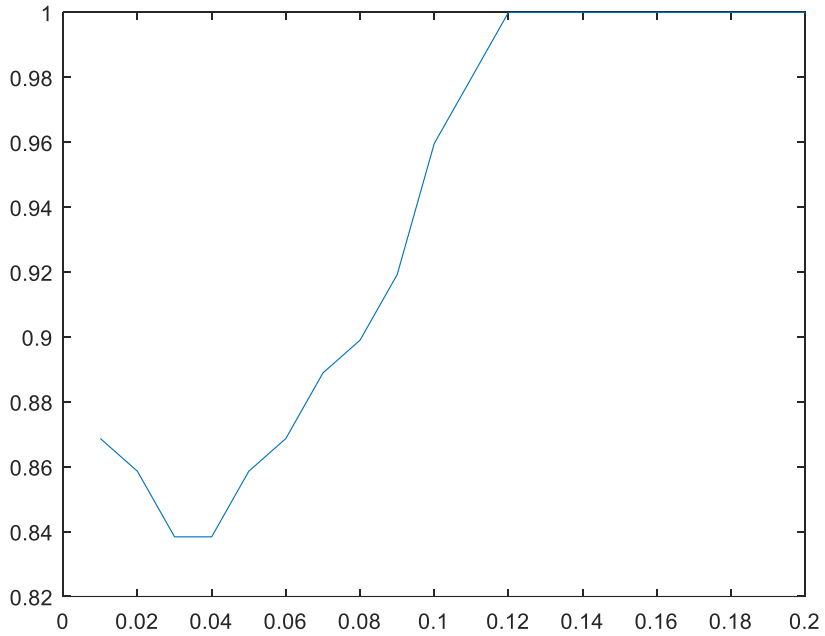


Figure XLV – Wage volatility effect on alpha for T=10
Source: authors' own figure

The same explanation as for the excel model. For lower time horizons the effect is the more intuitive one which is higher wage volatility, lower alpha. While for higher time horizons the effect is the opposite.

Fixed Inputs	
$r_f = 2\%$	$\mu_M = 8.81\%$
$\sigma_M = 8.86\%$	$\mu_W = 2\%$
$T = 5$	$X_0 = 3$
	$\gamma = 40$

Fixed Inputs	
$r_f = 2\%$	$\mu_M = 8.81\%$
$\sigma_M = 8.86\%$	$\mu_W = 2\%$
$T = 10$	$X_0 = 3$
	$\gamma = 40$

AS WAGE AVERAGE GROWTH INCREASES, ALPHA INCREASES

Figure XLV – Wage average growth effect on alpha in Excel
Source: authors' own figure

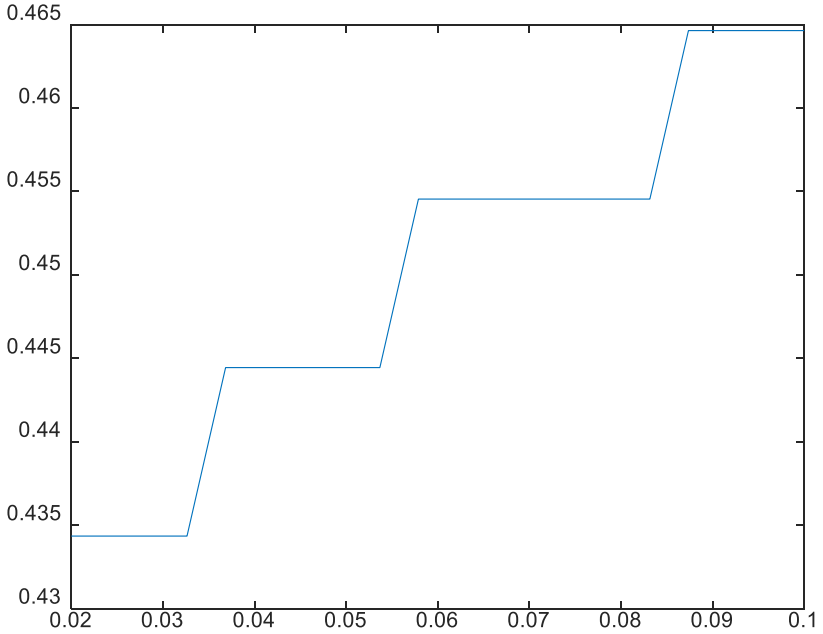


Figure XLVI – Wage average growth effect on alpha in Matlab
Source: authors' own figure

When wage average growth of the agent increases, this change is understood by the model as an increase in the human capital, which is seen as a lower risk asset, and consequently the agent can allocate more wealth to the risky asset.

Fixed Inputs	
$r_f = 2\%$	$\mu_M = 8.81\%$
$\sigma_M = 8.86\%$	$s = 20\%$
$\sigma_W = 2\%$	$Wage_0 = €48\,K$
$T = 10$	$Wealth_0 = €118\,K$
$\Delta t = \frac{T}{50}$	$\gamma = 40$

Fixed Inputs	
$r_f = 2\%$	$\mu_M = 8.81\%$
$\sigma_M = 8.86\%$	$\mu_W = 2\%$
$\sigma_w = 2\%$	$\gamma = 40$
$X_0 = 3$	
$T = 10$	

AS SAVINGS INCREASE, ALPHA INCREASES

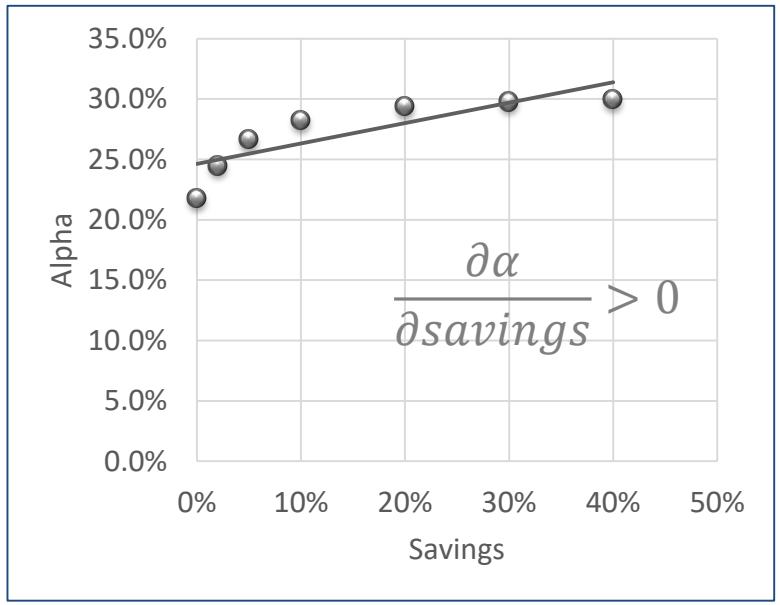


Figure XLVII – Savings rate effect on alpha
Source: authors' own figure

Fixed Inputs	
$r_f = 2\%$	$\mu_M = 8.81\%$
$\sigma_M = 8.86\%$	$\mu_W = 2\%$
$\sigma_W = 2\%$	$Wage_0 = €48\,K$
$T = 10$	$Wealth_0 = €118\,K$
	$\gamma = 40$

Savings increase, alpha increases

If the savings rate increases it means that the investor consumes less and has more to invest thus he can invest more.

Note: In the Matlab model, the savings rate is not an input, it is defined by the model based on the maximization of utility.

The impact in alpha does seem to be high

With savings changing from 0 to 50% the effect on alpha does not seem to be huge. This could be happening because the initial level of wealth is relatively high, thus the impact of each additional savings does not have a huge impact as it would probably have for people with little wealth.

Profiles

This section was included because we had some problems with the questionnaire application, as described above. Thus we will create hypothetical investors to test both models.

IN THIS FIRST PART WE DECIDED TO USE THE RETURN AND VOLATILITY OF THE PORTFOLIO M THAT WE OBTAINED IN THE INPUTS SECTION

Assumptions Matlab Model

- The profiles represent agents between 50 and 60 years old.
- The profiles represent agents with different levels of wealth (X_0=4; X_0 =6).
- The investors have wages with different volatilities (σ_w=2%; $\sigma_w = 5\%$).
- The investors have different levels of risk aversion ($\gamma = 10$ − extremely risk loving; $\gamma = 70$ − extremely risk averse).
- The last investment period was at the age of 82 (life expectancy in Portugal).
- The combination of the different inputs results in 16 (2^4) investor profiles.

Assumptions Excel Model

- Same assumptions.
- Savings were assumed to be 20%.

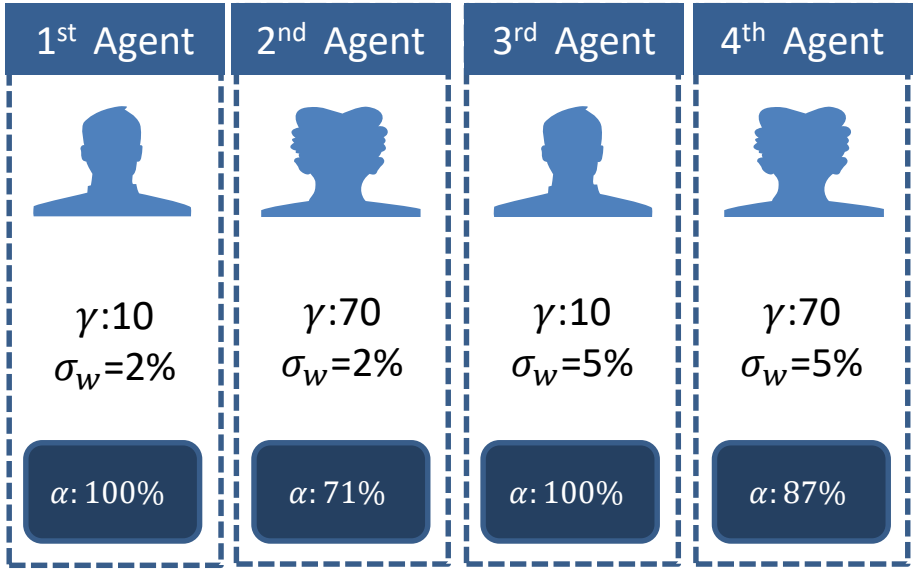
Investors' Goal

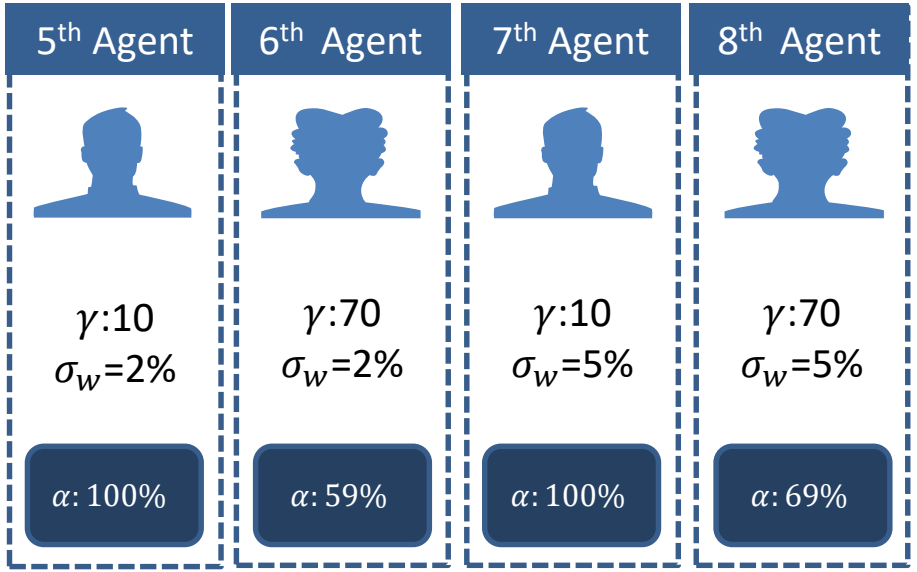
↓

Maximize lifetime utility

Inputs	
Age now	50,60
X_0	4,6
σ_w	2%,5%
rf	2%
μ_M	8.81%
σ_M	8.86%

THIS SUMMARIZES THE OPTIMAL INVESTMENT ALLOCATION SUGGESTED BY THE DYNAMIC MODEL FOR AGENTS WITH 50 YEARS AND WEALTH OF 4 AND 6



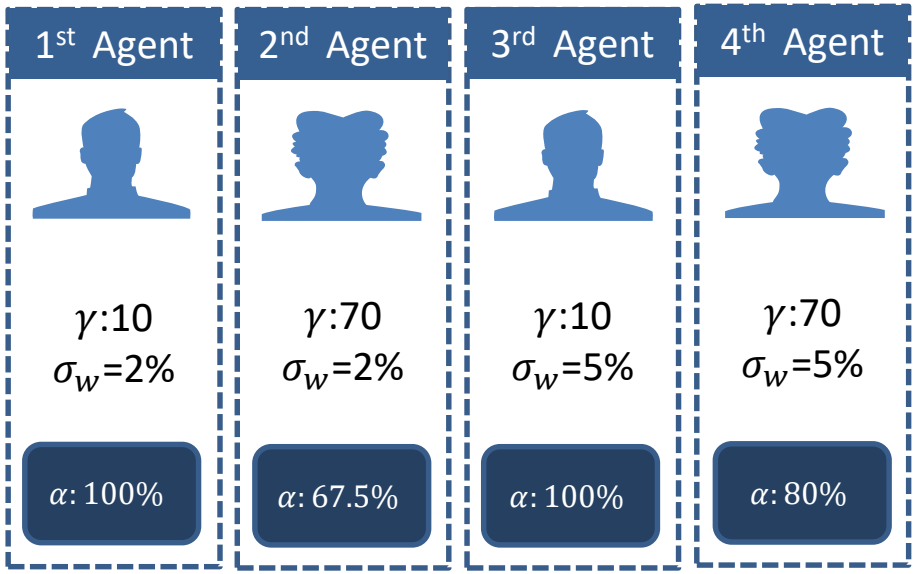


- The agents have a normalized **wealth of 4** and are 50 years old.
- The risk aversion increases from the 1st to the 2nd agent and consequently the exposure to the risky asset decreases same logic applies to the 3rd and 4th agents.
- When the wage of the agent becomes more volatile his/her allocation to the risky asset increases for the same gamma (2nd vs 4th agent).

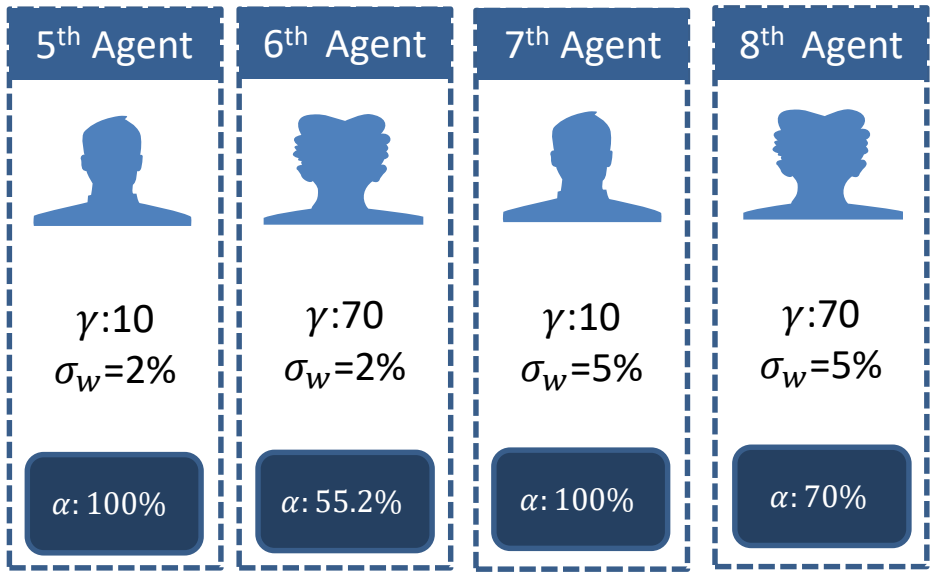
- The agents have a normalized **wealth of 6** and are 50 years old.
- The risk aversion increases from the 5th to the 6th agent and consequently the exposure to the risky asset decreases same logic applies for the 7th and 8th agent.
- When the normalized wealth of the agent increases, all else equal he/she invests less on the risky asset (2nd vs 6th agent for example).[1]

[1]This result is only possible to check for a gamma of 70, but in a gamma of 10 the same could be happening and not being perceptible because we are limiting the alpha to 100% not allowing short selling restrictions.

THIS SUMMARIZES THE OPTIMAL INVESTMENT ALLOCATION SUGGESTED BY THE STATIC MODEL FOR AGENTS WITH 50 YEARS AND WEALTH OF 4 AND 6[1]



- The agents have a normalized **wealth of 4** and are 50 years old.
- The risk aversion increases from the 1st to the 2nd agents and consequently the exposure to the risky asset decreases.
- When the wage of the agent (2nd and 4th) becomes more volatile his/her allocation to the risky asset increases for the same risk aversion coefficient, γ, (67.5% →80%, $\gamma = 70$). The volatility of the wage as no effect for agents 1 and 3, given they are extremely risk loving.



- The agents have a normalized **wealth of 6** and are 50 years old.
- γ increases from the 5th to the 6th agents and consequently the exposure to the risky asset decreases (100% → 52.83%).
- When the wealth of the agent increases, as we have discussed before, the optimal proportion of the portfolio invested in the risky asset decreases, as agents 2 and 6 illustrate.

[1]The static model created does not have X0 as input. Thus, the results shown apply to a case where total wealth and wage yield a ratio of X0. Different results can be shown for different values of wealth and wage chosen.

THIS SUMMARIZES THE OPTIMAL INVESTMENT ALLOCATION SUGGESTED BY THE DYNAMIC MODEL FOR AGENTS WITH 60 YEARS AND WEALTH OF 4 AND 6

9th Agent	10th Agent	11th Agent	12th Agent
			
γ:10 σ_w:2%	γ:70 σ_w:2%	γ:10 σ_w:5%	γ:70 σ_w:5%
α: 100%	α: 59%	α: 100%	α: 62%

- The agents have a normalized **wealth of 4** and are 60 years old.
- The risk aversion increases from the 9th to the 10th agent and consequently the exposure to the risky asset decreases.
- An investor with a higher number of expected years ahead to live, will, *ceteris paribus*, invest less safely, i.e. a higher alpha, α. This goes in accordance with literature, as longer investment periods have more opportunities to recover, in the event of a pullback in the equity market.

13th Agent	14th Agent	15th Agent	16th Agent
			
γ:10 σ_w:2%	γ:70 σ_w:2%	γ:10 σ_w:5%	γ:70 σ_w:5%
α: 100%	α: 48%	α: 100%	α: 49%

- The agents have a normalized **wealth of 6** and are 60 years old.
- The risk aversion increases from the 13th to the 14th agent and consequently the exposure to the risky asset decreases same logic applies to the 15th and 16th agents.
- When the wage of the agent (14th vs 16th agent) becomes more volatile his/her allocation to the risky asset increases for the same gamma.

THIS SUMMARIZES THE OPTIMAL INVESTMENT ALLOCATION SUGGESTED BY THE STATIC MODEL FOR AGENTS WITH 60 YEARS AND WEALTH OF 4 AND 6

1st Agent	2nd Agent	3rd Agent	4th Agent
			
γ:10 σ_w=2%	γ:70 σ_w=2%	γ:10 σ_w=5%	γ:70 σ_w=5%
α: 100%	α: 23.4%	α: 100%	α: 31.1%

5th Agent	6th Agent	7th Agent	8th Agent
			
γ:10 σ_w=2%	γ:70 σ_w=2%	γ:10 σ_w=5%	γ:70 σ_w=5%
α: 100%	α: 21.92%	α: 100%	α: 27.5%

- Investors have 22 years ahead of investment (60 years old) and normalized **wealth of 4**.
- The share invested in the risky asset decreases as the risk aversion coefficient increases, as seen before.
- For gamma of 10, even with a lower investment period which would suggest a lower alpha, alpha continues to be 100%, which suggests that (1) as some authors state 10 is too low for a risk aversion interval; (2) short selling restrictions could be considered .

- Agents are 60 years old and normalized **wealth of 6**.
- As discussed before, the effect of an increase in wage volatility originates an increase in alpha, α (see agents 6 and 8).
- Last but not the least, it is important to reinforce what we have seen before: in the static model considered, a $\gamma = 10$ yields, under reasonable inputs, a risky share percentage of 100.

IF WE ANALYSE BOTH MODELS, WE CAN CREATE A RANGE OF ALPHAS FOR EACH SPECIFIC PROFILE. SOME EXAMPLES ARE PRESENTED BELOW.

X_0 →

Age ↓

Age now = 50; X_0 = 4; σ_W = 2%

Gamma = 10	Gamma = 70
Range for alpha: **100%**	Range for alpha: **68% - 71%**

For an agent with this characteristics both models suggest 100% in the risky asset.

For an agent with this characteristics excel suggests 68% while Matlab suggests 71%.

Age now = 50; **X_0 = 6**; σ_W = 2%

Gamma = 10	Gamma = 70
Range for alpha: **100%**	Range for alpha: **55% - 59%**

Age now = 50; X_0 = 4; σ_W = 5%

Gamma = 10	Gamma = 70
Range for alpha: **100%**	Range for alpha: **87% - 100%**

For an agent with this characteristics both models suggest 100% in the risky asset.

For an agent with this characteristics Matlab suggests 87% while excel suggests 100%.

Age now = 50; X_0 = 6; σ_W = 5%

Gamma = 10	Gamma = 70
Range for alpha: **100%**	Range for alpha: **69% - 100%**

Age now = 60; X_0 = 4; σ_W = 2%

Gamma = 10	Gamma = 70
Range for alpha: **100%**	Range for alpha: **24% - 59%**

Age now = 60; X_0 = 6; σ_W = 2%

Gamma = 10	Gamma = 70
Range for alpha: **100%**	Range for alpha: **22% - 48%**

Age now = 60; X_0 = 4; σ_W = 5%

Gamma = 10	Gamma = 70
Range for alpha: **100%**	Range for alpha: **31% - 62%**

Age now = 60; X_0 = 6; σ_W = 5%

Gamma = 10	Gamma = 70
Range for alpha: **100%**	Range for alpha: **28% - 49%**

IF WE ANALYSE BOTH MODELS, WE CAN OBSERVE SOME DIFFERENCES AND SIMILARITIES

Matlab	Excel
• **Matlab model is more sensitive to changes in wealth**, the same change in X_0 usually causes a higher increase/decrease in the exposure to the risky asset in Matlab than in Excel. • When the investment horizon is higher (age now = 60), **Matlab suggests a lower difference in allocation between agents with gamma of 10 vs 70** (goes from 100 to around 60%) than Excel (goes from 100 to around 20%) .	• It is a more conservative model, because it does not allow for alpha adjustment during the investment period, **thus the optimal alphas given by this model are lower than the ones given by Matlab.**

When the agent is highly risk loving (γ = 10) models are not sensitive to a change in the investment period (age now of 50 vs 60), they always advise the investor to allocate 100% to the risky asset, which may be explained by the huge Sharpe ratio of portfolio M ((8.81%(r_p) - 2%(r_f)/8.86%(σ_p) = 0.8). Since it is highly unlikely that investors would accept to allocate 100% of their wealth to this portfolio, this calibration has then to be adjusted by Caixagest.

THE PROFILES CONFIRM THE EFFECT OF AGE, VOLATILITY OF WAGE, RISK AVERSION AND WEALTH ON CAPITAL ALLOCATION (ALPHA)

Age now

When the investor starts investing at a lower age he/she allocates a higher weight of his/her wealth to the risky asset.

Risk Aversion (γ)

When the risk aversion parameter increases the agent invests less in the risky asset, because he/she is more risk averse.

Normalized wealth for wages (X0)

When the wage of the investor decreases, the normalized wealth increases and the agent invests less in the risky asset.[1]

Wage's Volatility (σ_w)

The agent invests more in the risky asset when the his/her wage becomes more volatile, because we are considering a great interval between the start and end of the investment.[1]

- ✓ These relations between the inputs (T, γ, x0, σ_w) and the alpha (exposure to the risky asset) are in accordance to what was expected and reported before.
- ✓ Both models are very sensitive to the magnitude of the inputs. Thus in the next slides we decided to test a change in the return and volatility of the risky asset. They were changed to 6.5% and 15%, respectively. This Sharpe ratio of 0.3 is a more realistic approximation of what happens in the markets and was the one used by Cocco and Gomes (2005) – the paper in which we based our Matlab model. As expected all agents invest less in the risky asset.

[1]These relations which are less intuitive were explained in the "inputs effect on alpha" section.

IF WE CHANGE THE RETURN AND THE VOLATILITY OF THE RISKY ASSET THE SENSITIVITY OF THE MODEL CHANGES DRASTICALLY

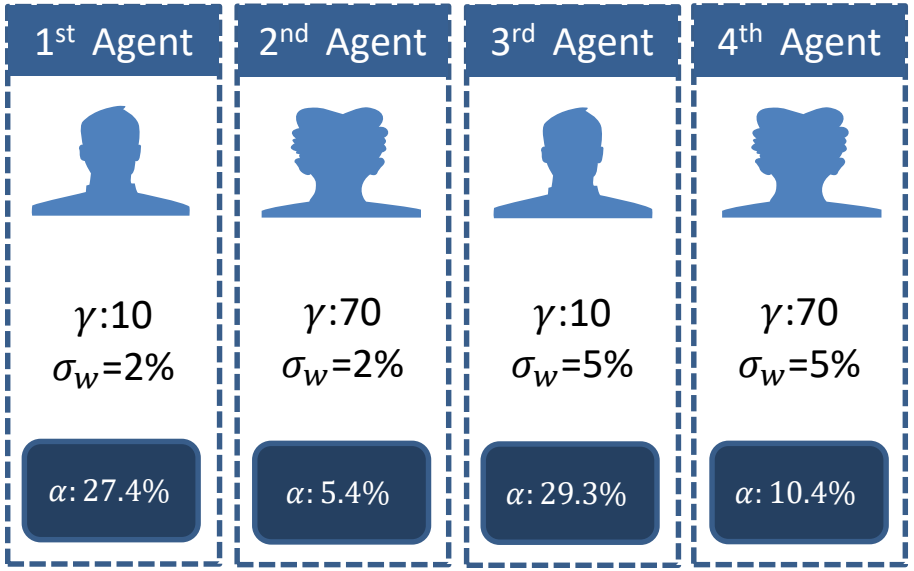
9th Agent	10th Agent	11th Agent	12th Agent
			
γ:10 σ_w:2%	γ:70 σ_w:2%	γ:10 σ_w:5%	γ:70 σ_w:5%
α: 71%	α: 13%	α: 72%	α: 15%

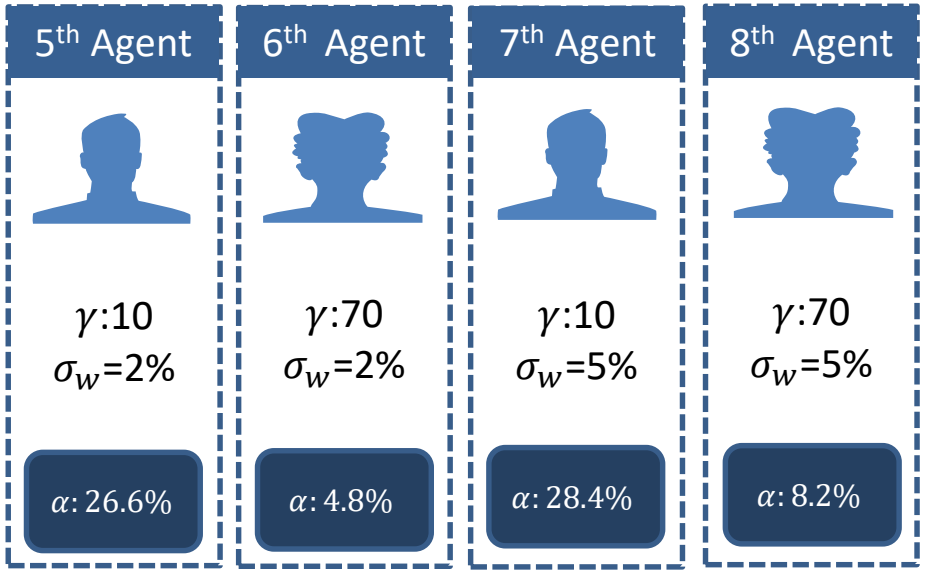
- The agents have a normalized **wealth of 4** and are 50 years old.
- The risk aversion increases from the 9th to the 10th agent and consequently the exposure to the risky asset decreases same logic applied to the 11th and 12th agents.
- When the wage of the agent (10th vs 12th agents) becomes more volatile his/her allocation to the risky asset increases for the same gamma.

13th Agent	14th Agent	15th Agent	16th Agent
			
γ:10 σ_w:2%	γ:70 σ_w:2%	γ:10 σ_w:5%	γ:70 σ_w:5%
α: 60%	α: 11%	α: 61%	α: 12%

- The agents have normalized **wealth of 6** and are 50 years old.
- Same conclusions regarding changes in alpha due to changes in risk aversion and in wage volatility.
- Now, for gamma = 10 (risk loving) the risky share is no longer 100%, which is related to the lower Sharpe ratio.

IF WE CHANGE THE RETURN AND THE VOLATILITY OF THE RISKY ASSET THE SENSITIVITY OF THE MODEL CHANGES DRASTICALLY

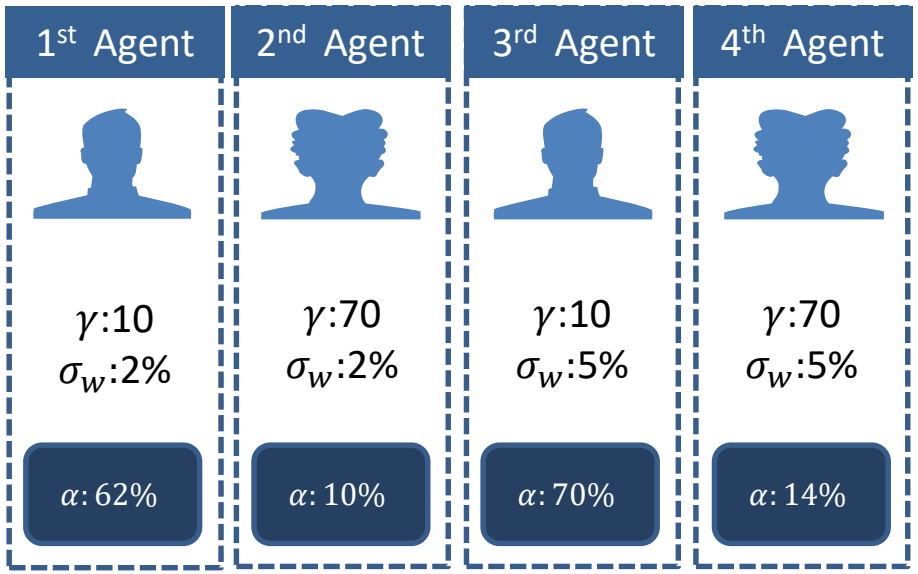




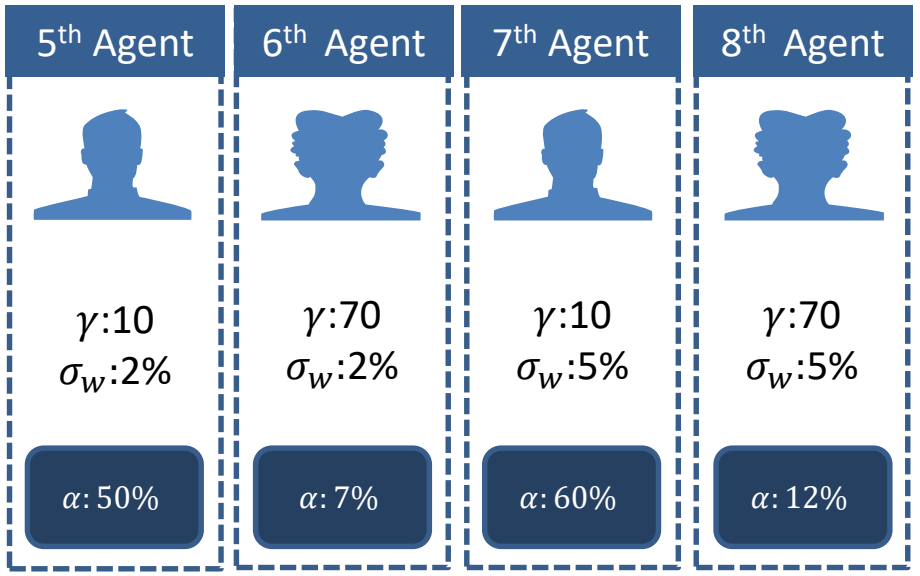
- The agents have a normalized **wealth of 4** and are 50 years old.
- The agents still behave in the same way, i.e. increase alpha when their wage becomes less stable or their risk aversion coefficient is smaller, and decrease it when their wage value decreases (normalized wealth increases).

- The agents have a normalized **wealth of 6** and are 50 years old.
- The biggest difference now is that the Sharpe Ratio of the risky portfolio is no longer close to 1. It is now a less stable, and thus more risky, investment so all agents will tilt their portfolio towards the riskless asset.

IF WE CHANGE THE RETURN AND THE VOLATILITY OF THE RISKY ASSET THE SENSITIVITY OF THE MODEL CHANGES DRASTICALLY

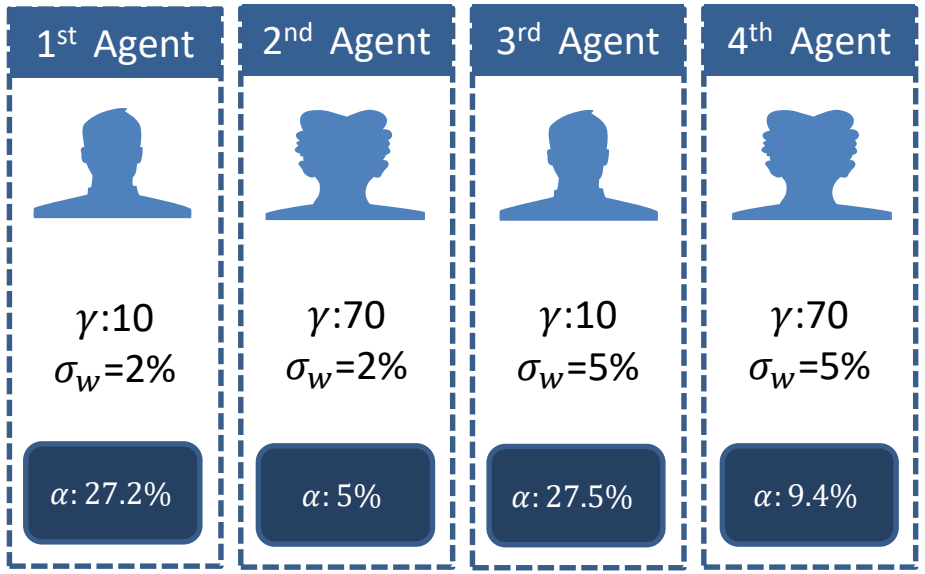


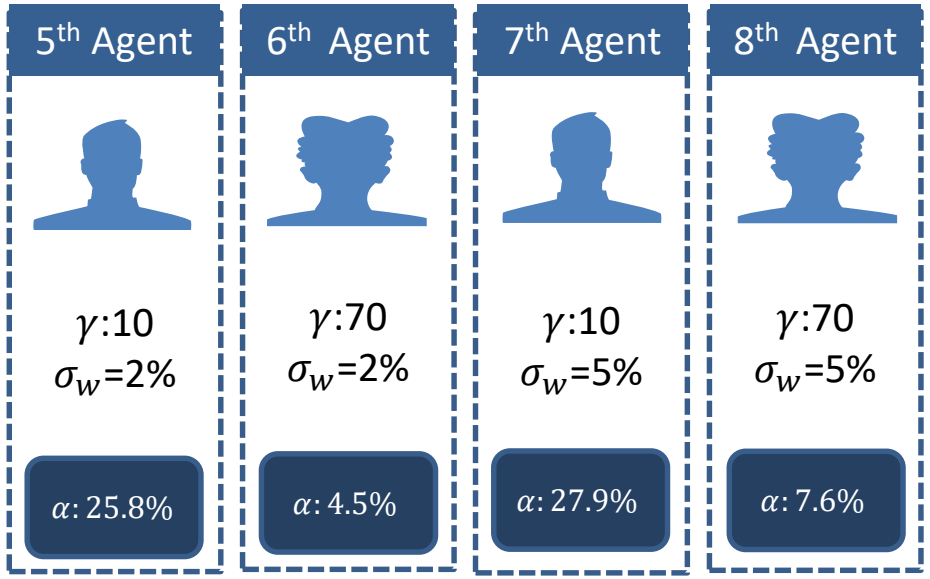
- The agents have a normalized **wealth of 4**, the agents are 60 years old.
- The jumps in alpha from a change in wage volatility are lower than the ones with the previous Sharpe ratio (10 → 14%), suggesting that if the portfolio is already risky a more volatile wage will not have a great impact. The riskiness of the portfolio is more relevant.



- The agents have a normalized **wealth of 6**, the agents are 60 years old.
- The risk aversion increases from the 5th to the 6th agent and consequently the exposure to the risky asset decreases same logic applies to the 7th and 8th agents.
- When the wage of the agent (6th vs 8th agent) becomes more volatile his/her allocation to the risky asset increases for the same gamma.

IF WE CHANGE THE RETURN AND THE VOLATILITY OF THE RISKY ASSET THE SENSITIVITY OF THE MODEL CHANGES DRASTICALLY





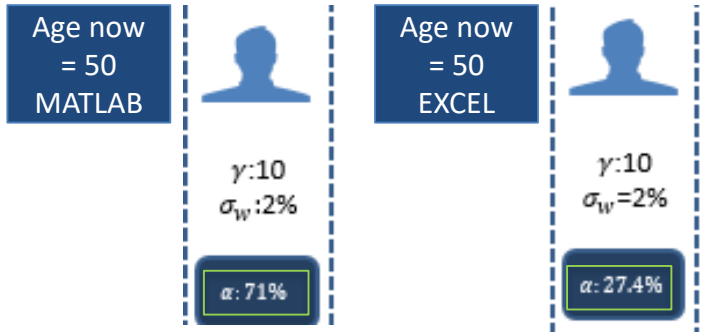
- The agents have a normalized **wealth of 4**, the agents are 60 years old.
- The agents still behave in the same way, i.e. increase alpha when their wage becomes less stable or their risk aversion coefficient is smaller, and decrease it when their wage value decreases. Moreover, increasing the number of expected years to live, still increases the optimal risky share.

- The agents have a normalized **wealth of 6**, the agents are 60 years old.
- There is a big change in investment allocation comparing with the previous slides for the Sharpe Ratio of 0.8. The alpha that was before reasonable for gamma = 70 (around 20-30%) is now suggested for the risk loving agent, showing the impact of the Sharpe Ratio change.

IF WE USE A SHARPE RATIO THAT IS USUALLY ASSOCIATED TO THE MARKET, MODELS' ANSWERS ARE A LITTLE BIT DIFFERENT

Main conclusions:

- For gamma = 10, both models now give a alpha different than 100%, at both ages and both σ_W. The alpha value now goes as low as 5%, compared to the previous situation (with the Sharpe ratio of 0.8) in which the minimum value was 22%.
- σ_w does not seem to have a huge impact on capital allocation, at least through the change considered from 2% to 5%. We opted not to use more extreme values of σ_w, because they are not common (check figures XXXV and XXXVI in the inputs section)
- The range of values for alpha is now larger than before.
  Examples:



Caixagest portfolio is highly diversified across investment products, sectors and countries (inputs section: figures XXIX - XXIV) and that is why it has the Sharpe ratio of 0.8. This suggests that the wealth manager (Caixagest as a whole) has the ability to create a portfolio with a better performance than the usual market portfolio. However we need to be aware that this is for this historical period and that the portfolio has some exchange risk that we are not evaluating fully and could have an impact in the risk and return of the portfolio.

The Procedure

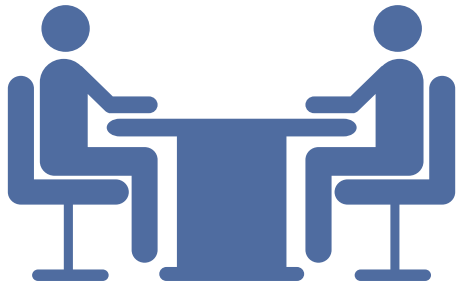
In this section we will explain briefly how the procedure will work from the individual questionnaire until the allocation of optimal portfolio. There are mainly three steps where the role of the wealth manager is crucial.

SUMMING UP…

THE PROCEDURE THAT WE SUGGESTED IS COMPOSED OF THE MODELS CREATED BY US BUT ALSO BY THE HELP OF THE WEALTH MANAGER

1st Step

The client completes the questionnaire and Caixagest collects the information regarding age, savings, wage, wealth. The wealth manager extracts the gamma from summing up the points for the questions regarding age, academic qualifications, income sources-stableness, savings amount, the question regarding which portfolio to invest, liquidity constraints and the lottery.



2nd Step

After collecting the information, the wealth manager converts the points obtained in the questionnaire into a gamma to be used in the models. The wealth manager introduces the inputs of the client into the models (Matlab and Excel) and the models give the optimal alpha according to the clients' characteristics.

3rd Step

The wealth manager analyses the alpha obtained by the model and adjusts it according to the sectors to which clients are exposed.

During the investment period, the allocation to the risky asset is updated whenever the investment period changes or the client asks to.

Suggestions

In this section we will suggest some topics which we were not able to explore fully, yet we believe deserve some attention and, possibly, further development. Being the first one the goals based investment model in which utility for the investor is derived from the achievement of goals, the second one, the possibility of considering that the investor makes an insurance to his human capital and thus has a higher risk capacity and the third one is the simple one objective model.

Goals Based Investing

In this section we will present a model based on the individual and his/her goals. The utility for the investor is derived from the achievement of goals. We will also present a different approach to the client in which the client can see the probability of achieving his/her goals in different scenarios

PERFORMANCE IS MEASURED BY THE CLIENT'S PROGRESS TOWARDS ACHIEVING EACH GOAL

- Goals-based investing is oriented around the **individual investor**. Investment strategies are specifically designed around each **client's specific goals**. **Performance is measured by the client's progress towards achieving each goal**.
- Goals-based investing also recognizes that investors have multiple and sometimes conflicting goals. Rather than pool all client assets into a single portfolio, we create a separate portfolio "bucket" for each goal. This approach takes into account the time horizon for each goal.

Risk → **In this theory, risk is considered as the failure of not achieving the desired goals.**

Benefits of Goal Based Investing

- Improve clients' relationship and their trust.
- Clients with this approach give more importance to risk in terms of their specific goals rather than just short term volatility and consequently the impact on their investments.
- Proposed risk measures differ from traditional measures, because they are based on events and do not require specification of a time interval to evaluate the performance of a portfolio.
- Different levels of risk tolerance for separate goals rather than an overall risk profile

Investors are focused on their specific goals and will not switch their financial advisors during recessions.

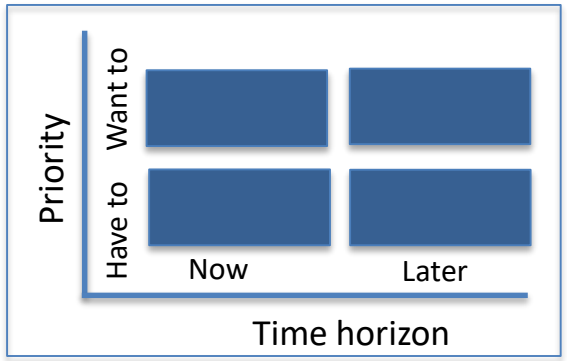


Figure XLVIII – Prioritizing financial goals- listing the clients' goals by priority and time horizon.
Source: Dan Nevins, October 2003, Goals-based Investing: Integrating Traditional and Behavioral Finance, SEI Investments also published in The Journal of Wealth Management, Volume 6, number 4

THE STRATEGY FOR THE GROWTH FOCUSED PORTFOLIOS IS MUCH MORE AGGRESSIVE THAN THE STRATEGY APPLIED TO THE STABILITY FOCUSED PORTFOLIOS

Portfolios used in goals based investing

The **stability-focused** portfolios are designed for investors who need to **protect against losses while having a comfortable level of growth** (wealth preservation). Strategies offered are Short Term, Defensive, Conservative and Moderate. The portfolios are managed to a drawdown target, with the objective being to avoid a loss that exceeds a target percentage under most market conditions. (For example: the drawdown target of the Defensive Portfolio is -10%. In a poor market scenario, the fund would be managed in order to avoid breaking a 10% loss from its highest valuation.)

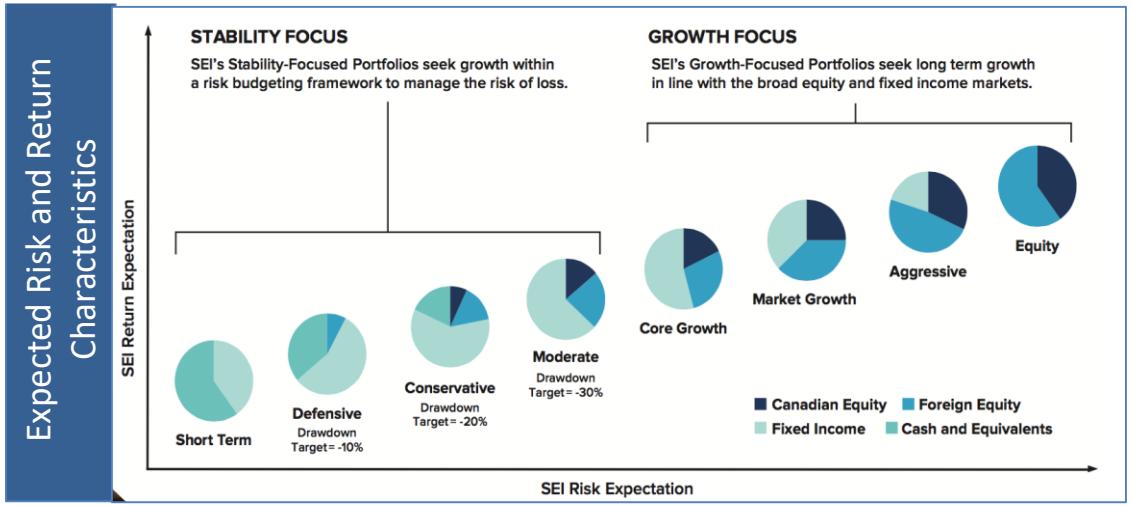
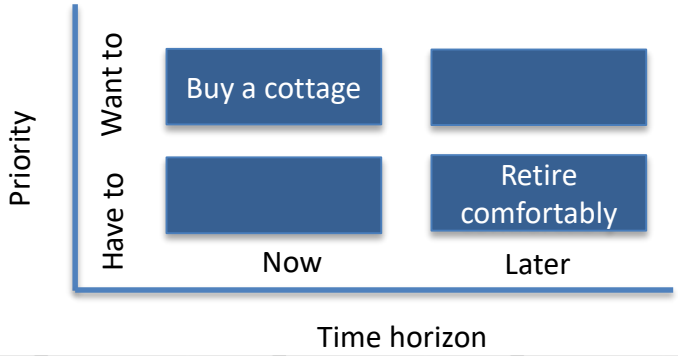


Figure XLIX – Expected risk and return characteristics. Source: Dan Nevins, October 2003, Goals-based Investing: Integrating Traditional and Behavioral Finance, SEI Investments also published in The Journal of Wealth Management, Volume 6, number 4

The **growth-focused** portfolios are designed for investors who seek to **accumulate wealth based on their risk tolerance** (wealth accumulation). These portfolios are actively managed to guarantee the highest possible return for a giver risk tolerance and they include equity, developed and emerging markets, and across large and small-cap stocks. The strategies offered are Core Growth, Market Growth, Aggressive and Equity.

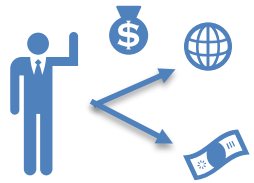
Goal	Buy a cottage	Retire comfortably
Strategy	Grow Assets	Capital growth and maximize total return
Components	Components in line with goals: shorter-term	Broad mix (Regional and Global Developed Equities, Emerging Market Equities, Large and Small Cap Equities)



IN ORDER TO ACHIEVE BOTH GOALS THE INVESTOR SHOULD INVEST AT LEAST 50% IN THE RISKY ASSET IN THE FIRST PERIOD AND 60% IN THE SECOND PERIOD

An investor(30y) pretends to invest 60,000€ in order to achieve 2 goals

1. In 10 years make a trip with 25,000€
2. Retire at the age of 65 with 2,000€ per month (PV(65years; rf=1,49%)=126,999€).



The investor will split their money to invest between two different time periods, firstly the investor will invest 20,000€ and after invests the remaining. The considered retirement period is 17 years because the average life expectancy in Portugal is 82 years.

In order to compute the probability of the agent achieving his goals, the following steps were taken:

1. The returns of the risky asset (portfolio M) and the risk free were computed using the σ and μ provided by Caixagest (8.81% and 8.86%).
2. The value of the portfolio consists in the return of the risk free asset plus the return of the risky asset. For each scenario it was considered 19 portfolios with alphas between 0 and 100%.

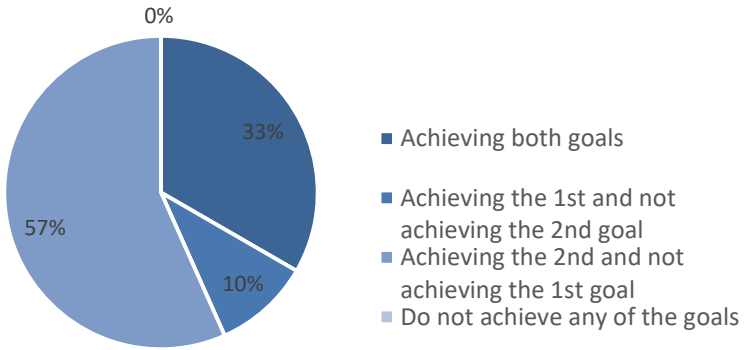


Figure L – Probabilities of achieving goals
Source: authors' own calculations

In the cases where the investor only achieves the first goal (a trip of 25,000€) he had to invest at least 85% of his wealth in the risky asset.

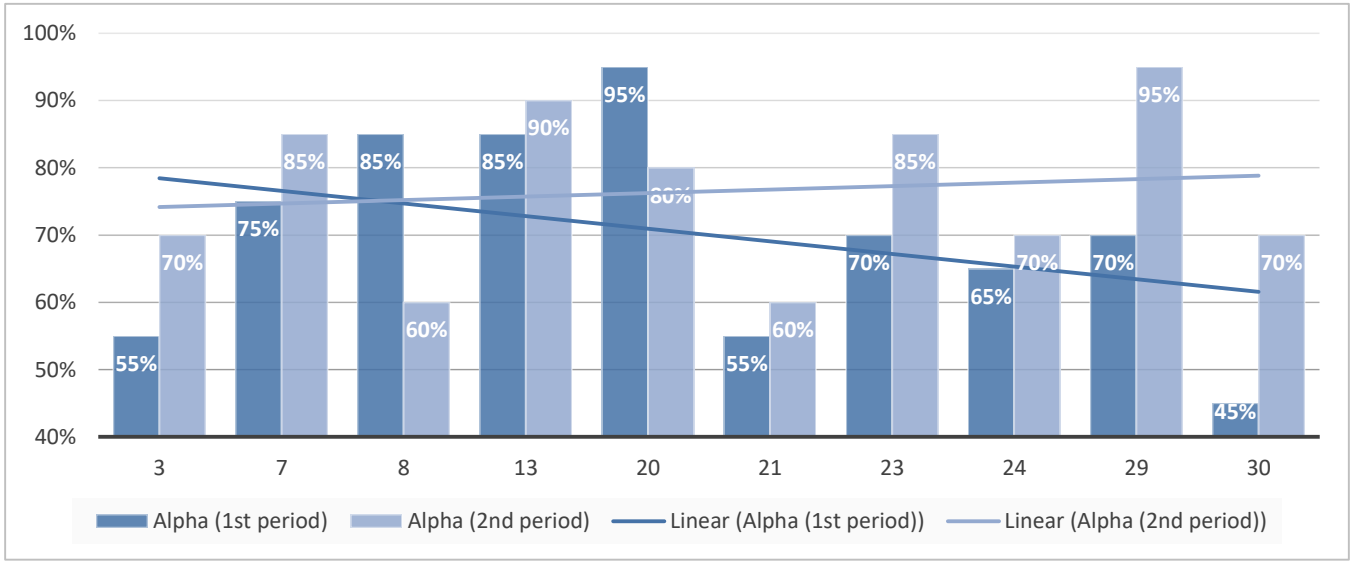


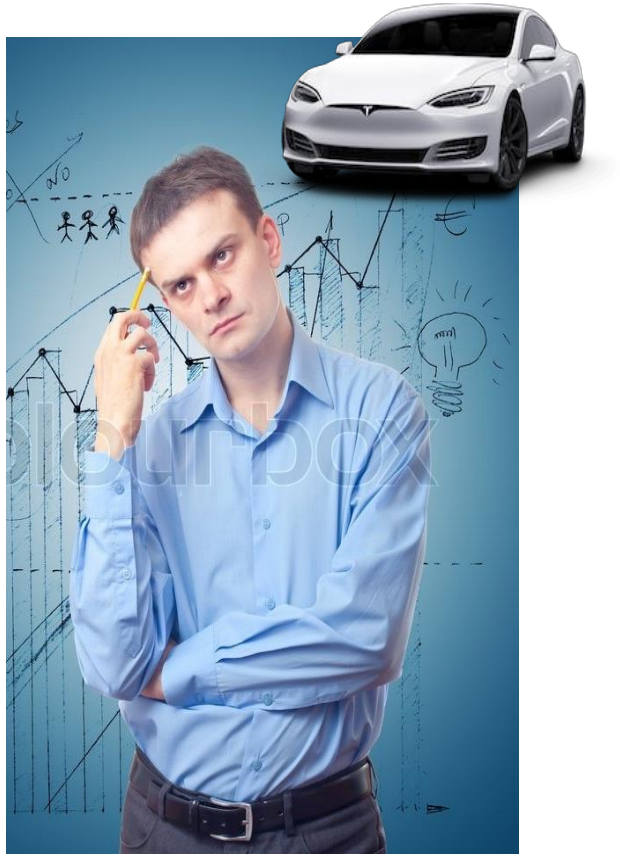
Figure LI –Cases in which both goals were achieved Source: authors' own calculations

A simple one-objective model

This section discusses a simple model for one objective only. The following section will provide further suggestions for this model and others.

THE OBJECTIVE IS THE CLIENT MEETING A CERTAIN CAPITAL REQUIREMENT WITHIN HIS (HER) SPECIFIED TIME HORIZON

Goals-based investing aims to meet the investor's financial capital needs. It differs from the previous models because its objective is not to maximize intertemporal wealth nor wealth at maturity: it aims to meet a certain requirement, *K*. It follows a completely different logic.



An example

- Eusébio is now 30 years old and is a middle officer in a Portuguese bank.
- He wants to buy a €50k Tesla at 35.
- He is committed to investing 15% of his income every year.
- He makes €35k a year and is willing to invest €25k immediately.
- Can he make it happen with Caixagest's M?
- How much we advise him to invest depends on his attitude towards risk (γ).

Most likely yes

- But there is an estimated (low) probability Eusébio's portfolio won't be enough.
- *And what if he wants €5k more for extras?*
- Not that likely anymore…

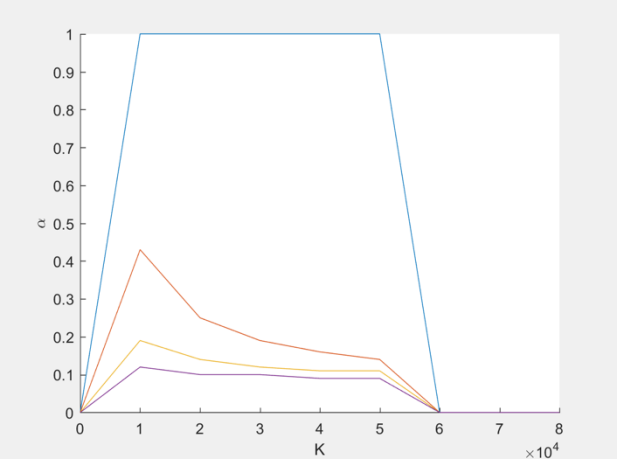


Figure LII - Optimal allocations to M (blue γ=10, violet γ=5).
Source: authors' own calculation

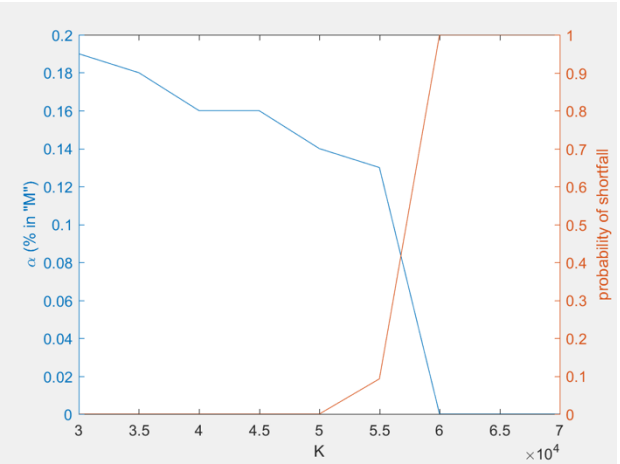


Figure LIII - Optimal allocations and probability of shortfall (γ=5)
Source: authors' own calculations

A NEW UTILITY FUNCTION THAT TAKES THE OBJECTIVE INTO ACCOUNT IS CALLED FOR

The CRRA function is good when the objective is maximizing wealth but it is lacking when the objective is matching a given level of wealth. In this model, we use the same static framework as before with an updated utility function that features disappointment aversion.

Köszegi and Rabin (2009)

- Prospect theory proposes a utility function that depends
  - positively on meeting and/or beating expectations and
  - negatively on not meeting expectations
- Barberis (2013) refers to this publication as a seminal work on prospect theory applied to finance but recognizes that this field is far from being developed.
- Indeed, Köszegi and Rabin only provide necessary conditions to the utility function but do not reveal a final specification of the model.

In this model, great marginal utility is attached to states where objective *K* is close to being reached. Risk-averse individuals require being closer to their objective in order to derive utility; risk-taking investors attach more value to expanding their wealth beyond *K*, whereas risk-averse would rather not take unnecessary risks.

Parameters λ· were included in order to calibrate the curvature of the utility function and the impact of σ_X.

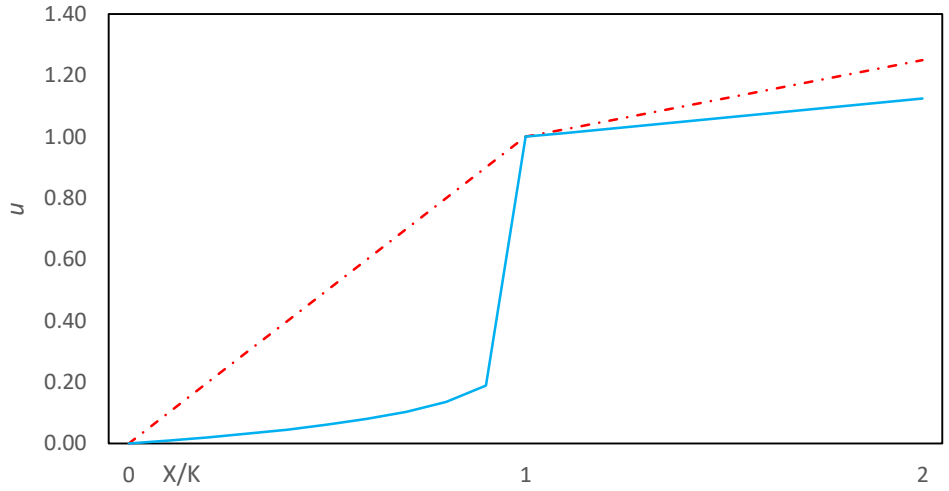


Figure LIV - New utility function (blue γ=20, violet γ=0). X and K are as defined before. Source: authors' own calculations

This function meet's Köszegi and Rabin's (2009) conditions. $\sigma_X = 0$ for simplicity

$$u = \begin{cases} -\left(-\left(\frac{X}{K} - 1\right)\right)^{\lambda^-} + 1 - \gamma\lambda_\gamma\sigma_X & X < K \\ 1 + \lambda^+\left(\frac{X}{K} - 1\right) - \gamma\lambda_\gamma\sigma_X & X \geq K \end{cases}$$

$$\lambda^- = 1/(1-\gamma) \qquad \lambda^+ = \frac{1}{4}\left(1 - \frac{\gamma}{\gamma_{\max}}\right) \qquad \lambda_\gamma = 1 \qquad \text{(XIX)}$$

DIFFERENT MODELS, DIFFERENT VALIDATION—AND DIFFERENT TRAPS AS WELL

- Results display significant variability within the selected intervals of γ. Most of the higher-alpha regions as time increases are due to the effect of λ^+.
- Each time horizon has its own "potential" K – hence the "clouds" in the second chart.
- Alpha collapses when the riskless rate goes beyond the input market rate of 6%.
- But **these results are not credible**: look at the shorter-maturity portion of the first chart – in such short maturities, the model suggests investing 100% in "M" but the probability of shortfall is actually converging to 1.

```
T=10;
g=10;
w0=35000; w00=w0; w0=1;
x0=25000; x00=x0; x0=x0/w00;
sav=.15;
rf=.01;
muM=.06;
sM=.09;
muW=.021;
sW=.023;
nsimul=4000;
K=140000; K00=K; K=K/w00;
epsW=xlsread('epsW.xlsx');
epsM=xlsread('epsM.xlsx');
```

Fixed variables
(unless otherwise indicated)

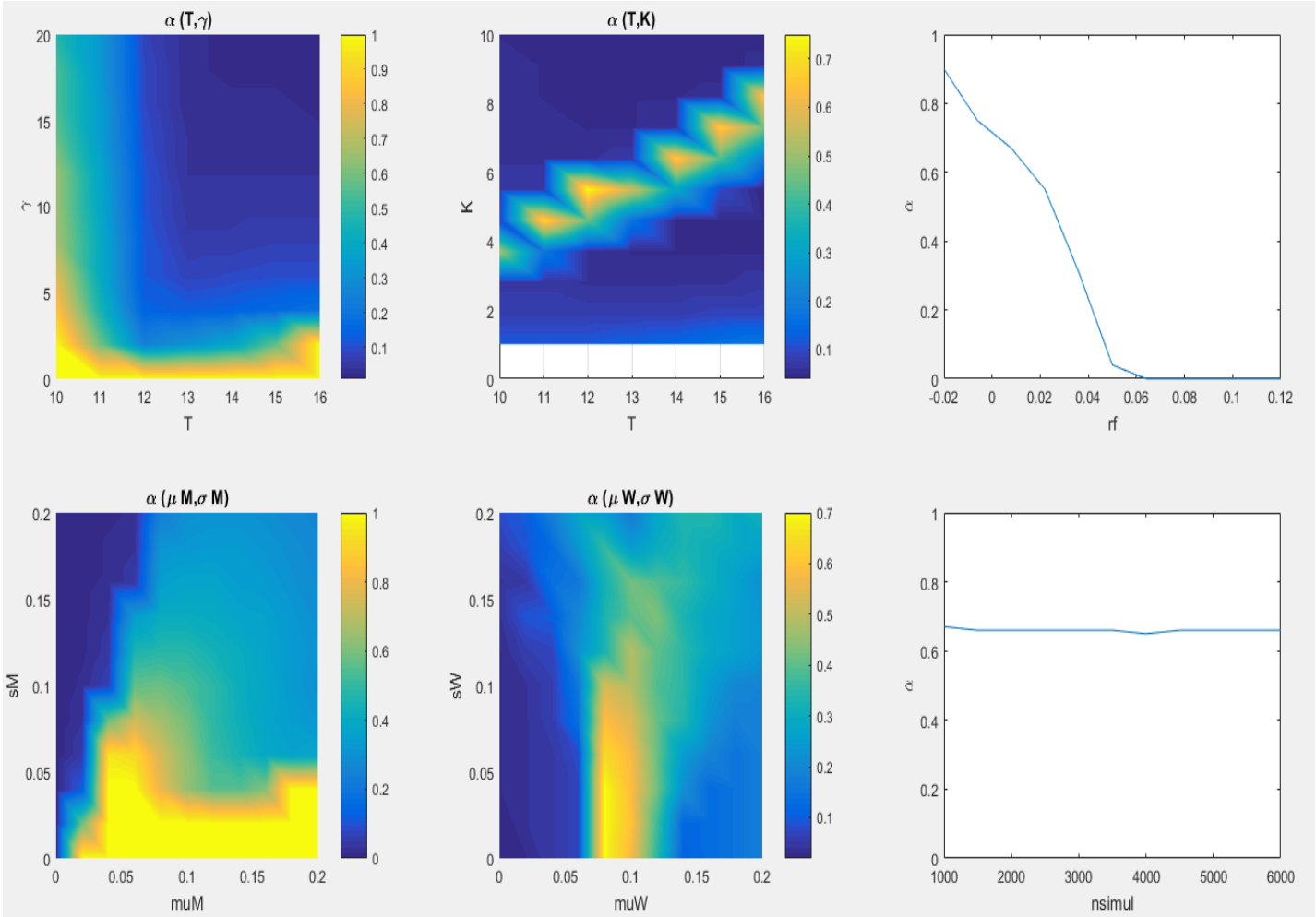


Figure LV – Results of the tests. Source: authors' own figure

RESULTS GAIN ROBUSTNESS BY INCLUDING A RESTRICTION ON MAXIMUM PROBABILITY OF LOSS (10%)

$$u = \begin{cases} 0 & P(X < K) \geq 10\% \\ \text{previous utility} & P(X < K) < 10\% \end{cases} \quad \text{(XX)}$$

- Results display significant variability within the selected intervals of γ.
- Most of the higher-alpha regions as time increases are due to the effect of λ^+.
- Each time horizon has its own "potential" *K* – hence the spike in the first chart and the "clouds" in the second (cf. previous slide);
- for *K*=4, lower maturities are not advised to invest at all, since investing that high an objective would be wishful thinking (cf. example).
- Alpha collapses when the riskless rate goes beyond the input market rate of 6%; the change for lower of r_f values is due to holding all other variables constant while adding the restriction above.
- On average, more market return and more wage return lead to riskier investments, as discussed before.

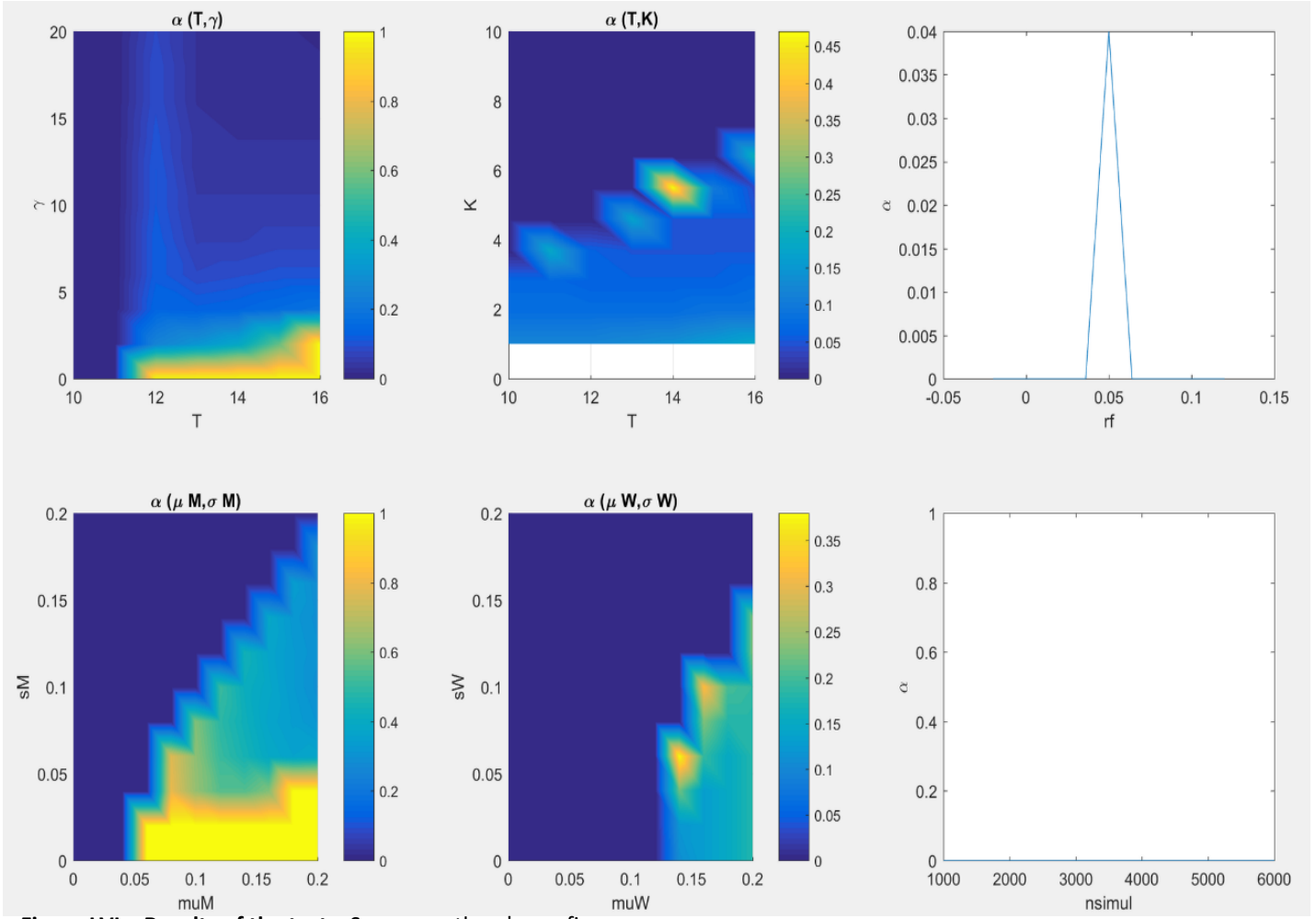


Figure LVI – Results of the tests. Source: authors' own figure

Incorporating a Life Insurance

In this section we will present a model that incorporates a life insurance in order to hedge the biggest threat to human capital – the mortality risk. The main goal of the model described is to maximize the investor's overall utility, which depends on his/her utility in two distinct states, namely the dead and alive states.

MOTIVATION – HOW TO HEDGE THE INVESTOR´S TOTAL WEALTH

- An investor's total wealth can be divided in two parts: One being readily tradable **financial assets** and the other **human capital.**
- Human capital contains a **unique mortality risk** in the form of the loss of future income and wages, in the event of the wage earner's death. However, this risk can be hedged with life insurance. Consequently, human capital affects both optimal asset allocation and demand for life insurance.
- From the **view of an individual investor's portfolio**, these two decisions (i.e. life insurance and asset allocation decisions) must be jointly determined as they serve as risk substitutes.
- **Life insurance** is a perfect hedge for human capital in the event of death since the terms of life insurance and human capital have a negative 100% correlation in the "alive" (consumption) state versus "dead" (bequest) state.
- If insurance pays off at the end of the year, human capital does not, and vice versa. Thus, the combination of the two provides **great diversification** to an investor´s total portfolio.

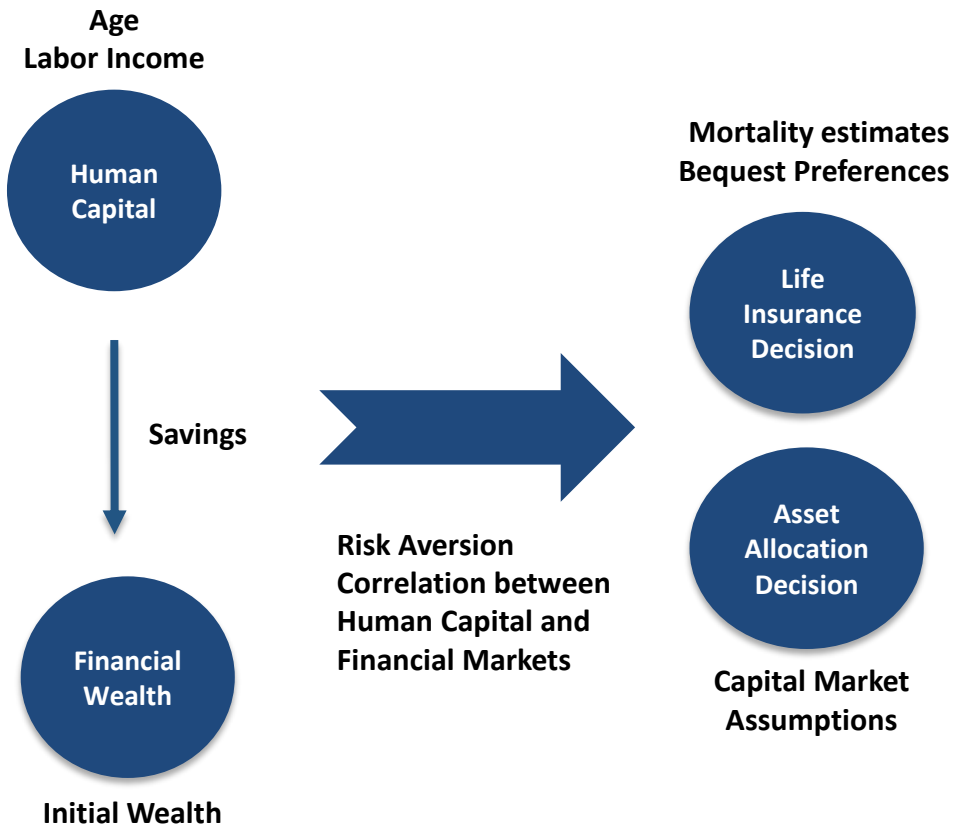
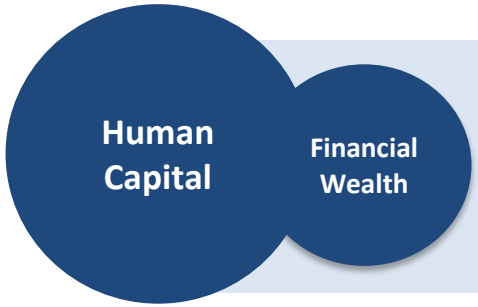
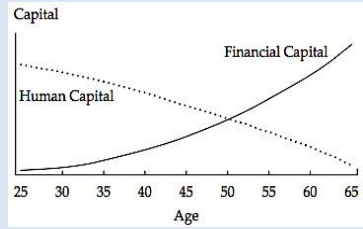


Figure LVI – Schematics of the paper's model. Source: Peng Chen, CFA, Roger G. Ibbotson, Moshe A. Milevsky, and Kevin X. Zhu, 2006, Human Capital, Asset Allocation, and Life Insurance, Working paper

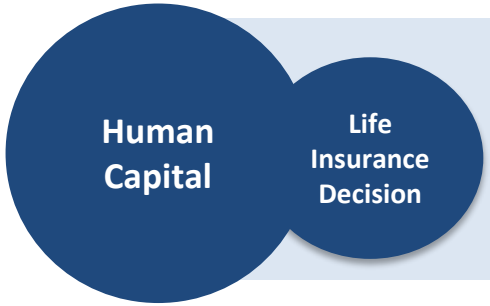
RELATIONSHIPS BETWEEN KEY CONCEPTS OF THE MODEL



Typically, **young** investors tend to hold **more human capital than financial capital** as they have more years to work, than those older workers do, and have had fewer years to save financial wealth. Conversely, older investors tend to have more financial capital than human capital.



Optimal asset allocation depends on the risk–return features of assets and on the **flexibility of the individual's labour income.** In the model, the investor adjusts the financial portfolio to compensate for no tradable risk exposures in human capital (Merton 1971; Bodie, Merton, and Samuelson 1992; Heaton and Lucas 1997; Jagannathan and Kocherlakota 1996; Campbell and Viceira 2002).



Smith and Buser (1983) found that the **optimal amount of insurance** depends on two components: **the expected value of human capital and on the risk–return characteristics of the insurance contract**. Ostaszewski (2003) went further by saying that life insurance is the business of human capital securitization.

MODEL BY PENG CHEN, CFA, ROGER G. IBBOTSON, MOSHE A. MILEVSKY, AND KEVIN X. ZHU (2009)

- Solid understanding of the actuarial factors that affect the pricing of a life insurance contract.
- A one year renewable term policy premium is paid by the investor at the beginning of the year and protects his/her human capital for duration of that year.
- At the beginning of each period, it is assumed an investor decides on asset allocation and insurance purchase and is paid his/her labor income
- The goal is then to maximize the overall utility, which comprises the utility from the two states previously indicated

OPTIMIZATION PROBLEM

Compute the optimal **amount of life insurance** (the face value of life insurance – death benefit) and the **allocation to risky assets**, so as to maximize the end-year utility of total wealth, weighted by both the alive and dead states, subject to certain budget constraints, as detailed below:

$$\boldsymbol{Max}_{(\boldsymbol{\theta_x},\boldsymbol{\alpha_x})} = \boldsymbol{E}[(\mathbf{1}-\boldsymbol{D}) * (\mathbf{1}-\mathbf{q}_{\boldsymbol{x}}) * \boldsymbol{U_{alive}}(\boldsymbol{W_{x+1}}+\boldsymbol{H_{x+1}}) + \boldsymbol{D} * \mathbf{q}_{\boldsymbol{x}} * \boldsymbol{U_{dead}}\,(\boldsymbol{W_{X+1}}+\boldsymbol{\theta_X})\,] \quad \text{(XXI)}$$

where,

$\boldsymbol{\theta_x} = amount\ of\ life\ insurance$
$\boldsymbol{\alpha_x} = allocation\ to\ the\ risky\ asset$
$\boldsymbol{D} = relative\ strenght\ of\ utility\ of\ bequest$
$\mathbf{q}_{\boldsymbol{x}} = subjective\ probability\ of\ death\ at\ the\ end\ of\ year\ x+1\ conditional\ on\ being\ alive\ at\ age\ x$
$(\mathbf{1}\text{-}\mathbf{q}_{\boldsymbol{x}}) = subjective\ probability\ of\ survival$
$\boldsymbol{W_{x+1}} = wealth\ level\ at\ age\ x+1$
$\boldsymbol{H_{x+1}} = human\ capital\ at\ age\ x+1$
$\boldsymbol{U_{alive}}\ and\ \boldsymbol{U_{dead}} = utility\ functions\ associated\ with\ the\ alive\ and\ death\ states$

MODEL BY PENG CHEN, CFA, ROGER G. IBBOTSON, MOSHE A. MILEVSKY, AND KEVIN X. ZHU (2009)

$$Max_{(\theta_x,\alpha_x)} = E\left[(1-D)*(1-\mathfrak{q}_x)*U_{alive}(W_{x+1}+H_{x+1}) + D*\mathfrak{q}_x*U_{dead}(W_{X+1}+\theta_X)\right]$$

Link the asset allocation decision with the life insurance purchase decision into one framework by **incorporating human capital**

The investor's human capital can be viewed:

- As a **"stock"** if its correlation with a given financial market is high and the volatility of the person's labor income is high;
- Or as a **"bond"** if both the correlation and the volatility are low.

In between this, Human capital is a **diversified portfolio** of stocks and bonds plus idiosyncratic risk.

Model the **labor income volatility** and its **correlation with financial market** volatility

As the correlation term between shocks to income and risky assets **increases**, the optimal allocation to risky assets and optimal quantity of life insurance will **decline.**

WHY?

The higher the correlation, the **higher the discount rate** used to compute human capital based on future income, implying a **lower valuation** of human capital and thus, **less insurance demand.**

Take into consideration the impact **of bequest motive** on asset allocation and life insurance

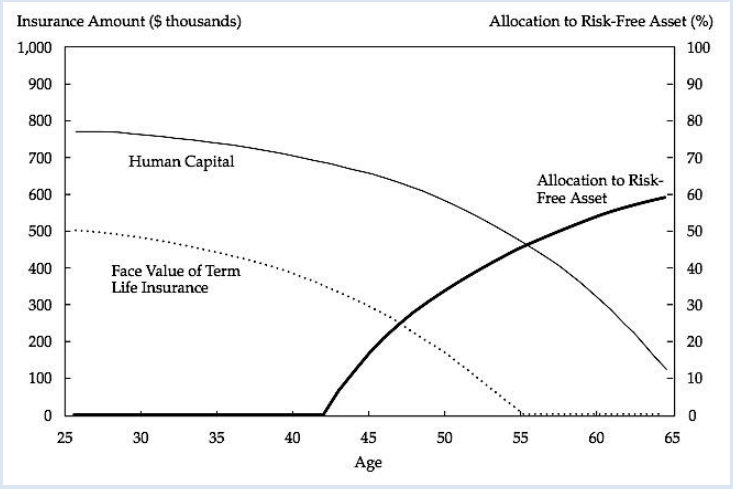
Bequest preference is arguably the most important factor other than human capital when evaluating life insurance demand.

Investors who weight bequest more (higher *D*) are likely to purchase **more** life insurance.

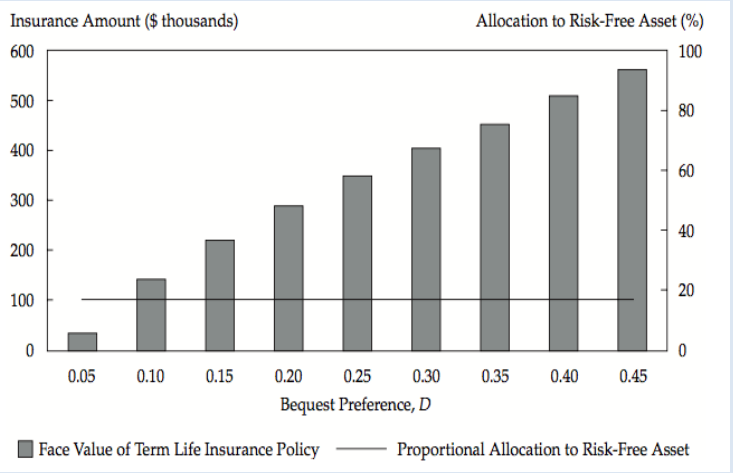
Model the **investor´s subjective survival probability**

A unique aspect of this model is considering the subjective survival probability ($1 - q$), investors with low subjective survival probability will tend to buy more life insurance.

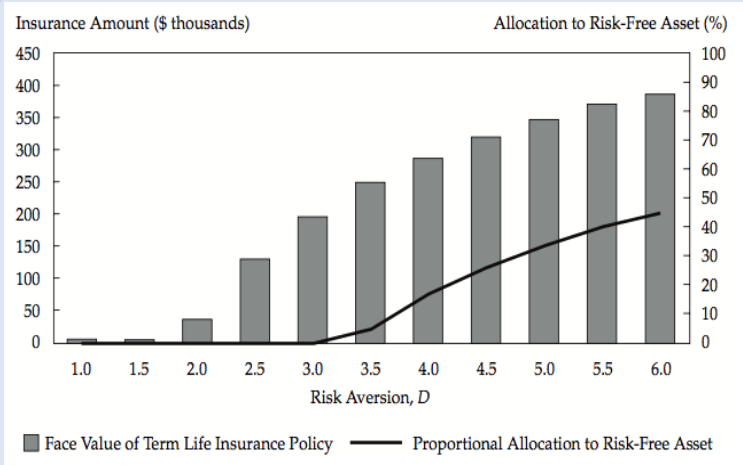
MODEL BY PENG CHEN, CFA, ROGER G. IBBOTSON, MOSHE A. MILEVSKY, AND KEVIN X. ZHU (2009) - FINDINGS



- Human capital decreases as the investor ages, reducing the demand for life insurance.
- Financial wealth and savings increase overtime. Consequently it will affect the allocation of riskless assets, which will increase during the given period.



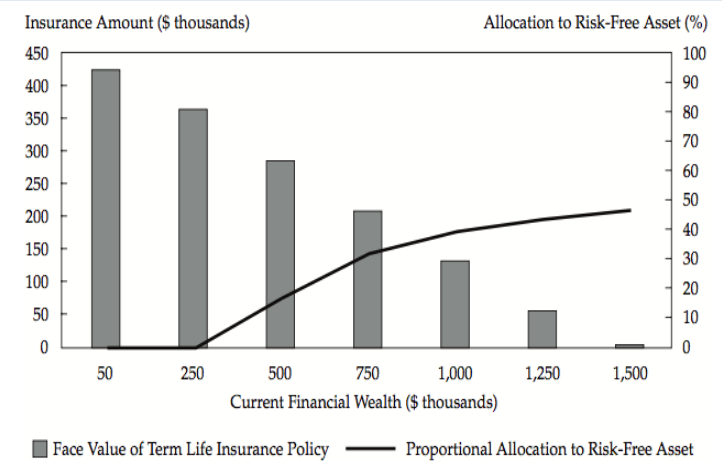
- As bequest motive desire increases, the insurance demand will also increase.
- The proportional allocation to riskless assets will be almost constant during the years, making those both factors irrelevant when doing the asset allocation.



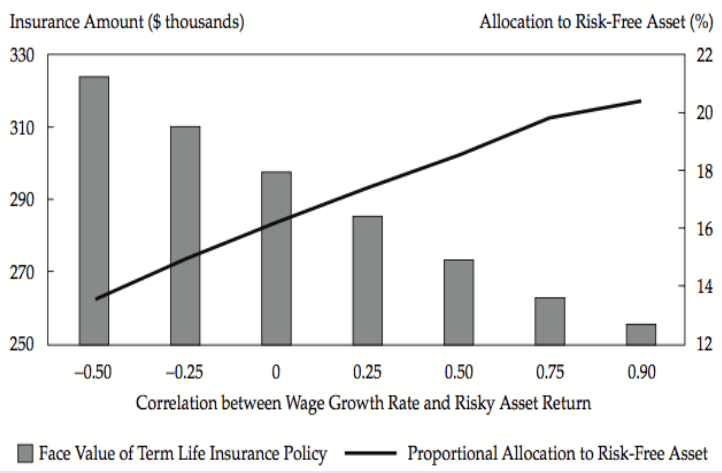
- The optimal amount of insurance increases with risk aversion.
- In this way, risk averse investors should invest more in riskless assets and buy more insurance contrary to those that are considered to be risk loving.

Figure LVII – Graphs conclusion from the paper. Source:Peng Chen, CFA, Roger G. Ibbotson, Moshe A. Milevsky, and Kevin X. Zhu, 2006, Human Capital, Asset Allocation, and Life Insurance, Working paper

MODEL BY PENG CHEN, CFA, ROGER G. IBBOTSON, MOSHE A. MILEVSKY, AND KEVIN X. ZHU (2009) - FINDINGS



- The optimal allocation when considering riskless assets increases with initial wealth, which is not consistent with CRRA utility functions, however wealth in this case accounts for financial wealth and human capital. Consequently, an increase in financial wealth will increase total wealth, while reducing the percentage of human capital on the total wealth. Due to the fact that human capital is less risky than risky assets.
- In what concerns to the optimal insurance demand, it will decrease with an increase of financial wealth.
- Summing up, an investor with more financial wealth than human capital is considered to be conservative, as he will have more allocation of wealth in riskless assets and less demand for life insurance.



- As the correlation between wage growth rate and risky asset return increases, less diversified will be the portfolio and hence, higher risk will rely on. In order to reduce this risk, an investor can invest more in risk-free assets.
- An increase in the correlation between those two assets will decreased the value of human capital and thus will decrease the optimal amount of life insurance.
- To conclude, as stock market and wage income are more and more correlated, the investor will be more conservative when allocates his assets and therefore will demand for less life insurance.

Figure LVIII – Graphs conclusion from the paper. Source: Peng Chen, CFA, Roger G. Ibbotson, Moshe A. Milevsky, and Kevin X. Zhu, 2006, Human Capital, Asset Allocation, and Life Insurance, Working paper

MODEL BY PENG CHEN, CFA, ROGER G. IBBOTSON, MOSHE A. MILEVSKY, AND KEVIN X. ZHU (2009) – MAIN TAKEAWAYS

- The decision of asset allocation and life insurance must be made jointly.
- Human Capital volatility and its correlation with other assets can affect both decisions overtime.
- The demand for insurance can be affected by the survival subjective probability and bequest preferences. However, in what concerns to optimal asset allocation they might have little impact.
- Risk averse investors should buy more life insurance and invest more in riskless assets when compared with risk loving investors.

Individual Parts

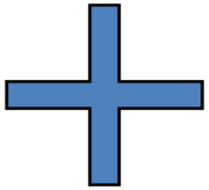
In this part of the work project each member of the group will delve into one subject of the all project. The topics are: Incentive questionnaires, Hedging strategies, Equity premium and goals based investing, Marketing or financial concepts videos. Each member will also share his/her view of the work project.

INDIVIDUAL PART - HUGO

WORKING PROJECT OVERVIEW

- Group work;
- Assistance from Caixagest employees who taught us valuable things and challenged the way we approached the challenges;
- Being involved in a big organization and getting an insight into what a career in Wealth Management is;
- A more hand-to-work, somehow experience to Financial Consulting;
- Understanding how difficult our task was and how much work is still left to be done in the sector, as a whole;
- The steep learning curve this project has made me go through – from the social to the analytical part of Wealth Management, as well as a deeper understanding of how the business works in Portugal and what are its biggest barriers.

- Sometimes, the whole group felt unhelpful as the task on hand was extremely more difficult than expected, which was also a pro as it challenged us and made us more aware of deadlines and workload;
- Hard to find consistent literature, as for all aspects considered (i.e. creation of questionnaire, gamma calibration and utility function) different papers would present really different approaches;
- Unable to test risk aversion through more efficient manners, as discussed before;
- Much more can be developed in terms of both risk aversion calibration and optimal utility functions/investment models.



- Future work projects with Caixagest could be focused on exploring the challenges we faced in our work project, such as:

I. How to extract optimal risk aversion signals;
II. Hedging strategies considering the risky portfolio;
III. How to shape the risky portfolio according to each clients believes and/or interests: sectors, currency, etc;
IV. Different portfolios for different investment goals;
V. Different utility functions that prioritize the achievement of investment goals;

RISK AVERSION QUESTIONNAIRE – A FURTHER, MORE EFFICIENT APPROACH

As discussed before, several studies have reinforced the idea that very few methods have proven to be efficient when modelling risk aversion. Both qualitative and quantitative questions have been studied for several years, both having qualities and drawbacks associated to them.

WHY?

Not only does risk aversion depends on the monetary amount involved (Hans P. Binswanger, 1980), it may also depend on which side of the market the individual is in – whether the choice involves buying or selling (Steven J. Kachelmeier and Mohamed Shehata, 1992); Regardless of the approach chosen, it is believed that the degree of risk aversion that would explain certain behaviours in low paying payoffs would yield absurd degrees of risk aversion, under high paying payoffs: Hartog et al (1997) prove that other aspects influence risk aversion such as gender, whether the individual is a public or private sector employee, an entrepreneur, among other factors;

Harrison and Rutsröm (2008) study 5 different procedures (Multiple Price Listing, Random Lottery Pairs, Ordered Lottery Selection, Becker-DeGroot-Marschak and Trade-off). Because both the questions and incentives differ among procedures, not all lead to the same degree of risk aversion, considering the same individual;

Holt and Laury, 2002, use lottery choices, under both low and high payoffs, where the interviewed individual has indeed to pay. The authors find that even under low payoffs, most individuals are risk averse and this risk aversion increases strongly when payoffs are scaled by factors of 20, 50 and 90. They find that individuals overestimate their risk aversion;

Most of these studies also fail to differentiate loss aversion from risk aversion, as “the aggravation that one experiences in losing a sum of money appears to be greater than the pleasure associated with gaining the same amount" (Kahneman and Tversky, 1979).

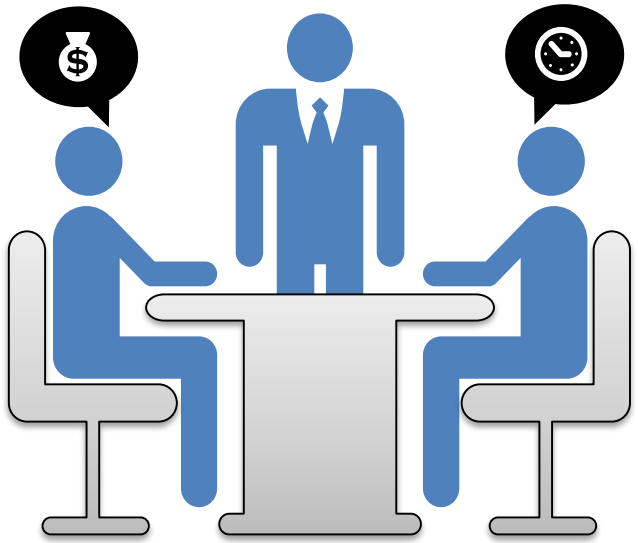
RISK AVERSION QUESTIONNAIRE – A FURTHER, MORE EFFICIENT APPROACH

Holt and Laury (2002) also find that "*subjects facing hypothetical choices cannot imagine how they would actually behave under high-incentive conditions*", hence reinforcing the unreality of extracting valid signals of risk aversion through qualitative answers, as well from quantitative answers where the subject is not involved and not actually dealing with risk.

Thus, and even though there is still research to be done ahead, optimally Caixagest could try to involve its new clients in an experience where these would be faced with actual lotteries.

An initial thought could be to offer a small monetary reward, to new clients, and have them managing it so one could understand their preferences, believes, etc.

Thus, and given the findings discussed, a possible idea, which would have to be carefully studied, could be to admit a certain risky share to begin with, regardless of the client. Upon this, in the initial periods, clients and portfolio managers would be in constant conversations so as to define the best strategies and with this, extract the client's degree of risk aversion.



It is quite surprising and disappointing to me that almost 40 years after the establishment of the concept of risk aversion by Pratt and Arrow, our profession has not yet been able to attain a consensus about the measurement of risk aversion. Without such a consensus, there is no hope to quantify optimal portfolios, efficient public risk prevention policies, optimal insurance deductibles, and so on. It is vital that we put more effort on research (…) this line of research is not in fashion these days, and it is a shame. ------- Gollier, 2001

WORKING PROJECT OVERVIEW

Learnings

- The **role of Wealth Management** in a bank, and its strong relation with financial markets;
- The fear and mistrust in **financial markets in Portugal**. However, it is needed a change in the Portuguese's mindset, since there are **two main incentives to invest**: the low rates of return in deposits and an insufficient support from the social security;
- How **investing in portfolios** can be seen as an alternative way to maintain the desired quality of living standards, while enhancing financial markets activity;
- How **technology is changing and improving** banking services, e.g. robot advisors replacing account managers;
- **The importance of human capital** and how it can be hedged.

Challenges

- The difficulty of the procedure **to assess an investor profile**, its characteristics and desires, through a questionnaire
- The **choice and measurement of inputs** for each model, driven from the individual investor questionnaire
- The level of risk aversion of an individual investor, and its translation into a **gamma**
- The necessity of making the **models valid and applicable to real investors**
- The incorporation of the **bequest motive** in the suggested model, given its relevance in an investor´s life
- **Lack of time to implement** all the desire models

"IS TRADITIONAL ASSET ALLOCATION ENOUGH?"

In traditional asset allocation, an investors seeks low correlation between asset classes, which enhances diversity while reducing volatility. And that is why Caixagest provides to his clients a hugely diversified portfolio, the **portfolio M**.

However, an investor can….

1. Have too **much exposure** to a specified sector
2. Have a desire to **limit his maximum amount** of loss

1. Hedging exposures

- ✓ Imagine an investor that has an exposure to the gas and oil market, since it is his source of labour income
- ✓ To hedge its background risk, he can **sell futures of gas**, offsetting his exposure to the energy sector that is present in portfolio M
- ✓ The purpose is not to gain from favourable price movements but prevent losses from potentially unfavourable changes, maintaining the predetermined result
- ✓ The optimal number of futures = $\frac{hedge\ ratio * size\ of\ position\ to\ be\ hedged}{size\ of\ one\ futures\ contract}$, where hedge ratio $= \rho_{S,F}\frac{\sigma_S}{\sigma_F}$ (XXII)

Advantages :

- Pricing is easier to understand
- Fees
- Liquidity

Disadvantages :

- Highly leveraged
- Short-term investments
- Requires constant monitoring

"Futures are financial contracts that obligate the buyer to purchase an asset or the seller to sell an asset, such as a physical commodity or a financial instrument, at a predetermined future date and price. Futures contracts detail the quality and quantity of the underlying asset; they are standardized to facilitate trading on a futures exchange"

2. Limit his maximum amount of loss

- ✓ The idea now is to protect an investor during negative market movements, offering a consistent downside protection.
- ✓ By trading **put and call options** around an underlying asset, the risk is reduced and a range of investment outcomes is provided due to the limits proposed by the exercise prices of each option.
- ✓ Moreover, the cost of purchase the put options on the underlying asset can be offset by selling call options, being less expensive and more cost-effective.

Advantages :

- Minimize the impact of market disruptions or downturns
- Provide higher risk-adjusted returns, by going long on equities and using options to protect capital during volatile markets

Disadvantages :

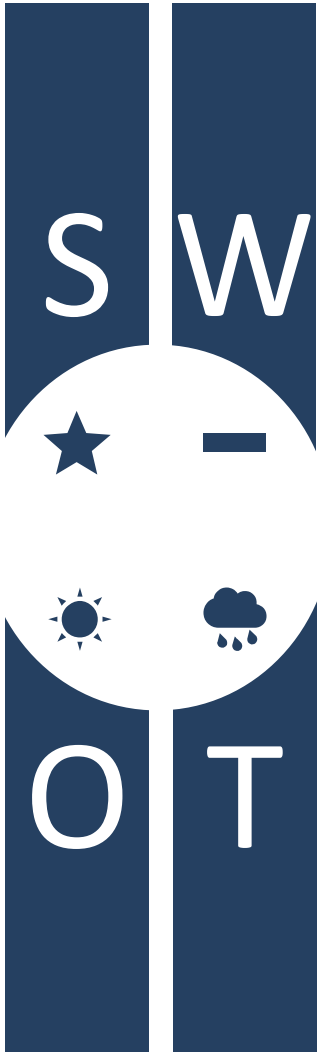
- Since it provides a significant downside protection, investors can sacrifice some returns in growing markets

"A Put is an option contract giving the owner the right, but not the obligation, to sell a specified amount of underlying asset at a set price within a specified time, while a Call is an option that gives the owner the right but not the obligation to buy a specified amount of underlying asset at a set price within a specified time"

TO CONCLUDE

Caixagest can incorporate this hedging strategies when presenting the optimal portfolio to an investor, according with his goals, in different and many ways. Consequently, the bank can provide an even better range of customized investment solutions.

WORKING PROJECT OVERVIEW



Strengths

- Caixagest' collaborators were **available to help us** in most situations.
- We had the opportunity to **use the theoretical knowledge** obtained in university.
- Having the **midterm presentation** allowed us to improve some topics with the feedback given by collaborators and align objectives.
- The opportunity to **work in the bank's building** allowed us to resolve problems quickly.

Weaknesses

- In the beginning it was **quite difficult to understand the complete process of investment allocation**, at first it looked like it lacked some structure. In the end it turned out to be a good challenge for us, since we had to make it more structured.
- The **lack of time and knowledge in programming** to make models even more accurate and insert restrictions that could improve them.

Opportunities

- Improve the process of **gamma calibration**.
- Explore **goals based investing and bequest motive**.
- **Low interest rates** in deposits could allow to market alternative investments easily.
- **Low commission charged by Caixagest** compared to other banks that operate in Portugal.

Threats

- **High level of risk aversion** among individuals.
- Portuguese **lack of financial knowledge**.
- The recent events of the financial crisis had a significant impact in **people's confidence on banks**, thus making it harder to change minds and people's vision of wealth management.
- In Portugal **corruption** is highly associated with banks.

INDIVIDUAL PART – CATARINA CAVACO

VIDEOS ARE USED IN THE FINANCIAL INDUSTRY WITH DIFFERENT PURPOSES, LET'S EXPLORE SOME OF THEM

LifeSkills
Created with BARCLAYS

Public service campaign focused on teaching young people basic skills to succeed in their professional life.

HOW?

Half minute videos show casting the personal and authentic stories of young people (e.g. teaching skills such as body language).

Bank of America

Teaching financial literacy that has an impact in lifecycle objectives such as buying a house.

HOW?

Animated YouTube videos in a playlist called #BetterMoneyHabits with 57 videos.

HSBC

Inspiring stories and advice videos to help people in their businesses.

HOW?

YouTube videos usually no longer than 3 minutes based on different topics, for example there is a series called "Focus on Entrepreneurs" with 10 videos.

YouTube Caixa Geral de Depositos

Most banks have YouTube channels. CGD also has a channel, but the videos uploaded are used to show the events and meetings the bank had and not to convey a message as the examples shown.

Next slide…

Will present an idea of a video that I developed. The idea was to incorporate the video into the questionnaire, to be shown if the respondent said it had no financial knowledge (as explained in the questionnaire section).[1]

[1]The video is Portuguese because it makes more sense having in mind the target audience.

Separate Media file
This slide refers to the video which was uploaded as a media file, since it was not possible to upload it in pdf. The quality of the video may be somewhat compromised because the file had to be compressed.

EXPERIENCE-INDIVIDUAL PART-MARIA

WORKING IN CAIXAGEST WAS A GREAT EXPERIENCE THAT ALLOWED ME TO DEVELOP IMPORTANT SKILLS SUCH AS TEAMWORK AND DECISION MAKING

1 First, we defined the most relevant characteristics of investors based on existing questionnaires and relevant literature. Then, we adjusted Caixagest's questionnaire and justified our changes.

2 Created two models which match the client profile to a portfolio allocation: a dynamic model (Matlab) and a static model (Excel).

Biggest challenges

- Identify the correct way to **capture a client's level of risk aversion**.
- Construct a model that is valid and returns a specific allocation according to the client's profile.
- Calibrate the model parameters, namely the gamma.
- Understand and justify the behavior of the inputs and their impact on the alpha, i.e. the allocation to the risky asset.
- The lack of programing skills and time did not allow us to incorporate the goals based investing approach or a bequest motive in the models.

Lessons learned

- Investors should be more exposed to the risky asset than they currently are. **Portuguese investors remain very conservative** when it comes to investing their savings, although they stand out in financial knowledge.
- Moreover, **robot advisors and artificial intelligence** are a concern to the industry, since these automatically collect information from clients and construct a portfolio based on their goals and specific characteristics.

CONCLUSION

The possibility to work on a consulting project as a thesis allowed me to **apply the knowledge** that I have been acquiring at Nova and to **face problems** that I will be facing in my daily work life such as lack of information or regulation restrictions. it was very important to receive feedback throughout the process to improve and introduce new ideas in the presentation. As a result, I feel that I am a **more resilient person** than I was before and that I improved my ability to produce back from adversity.

The easiest part

We found several questionnaires from other banks (Goldman Sachs, Morgan Stanley, Bank of America, RBS), which **we used as benchmarks** to draw the structure of our questionnaires (number of questions, risk profiles, multiple choices vs open questions, etc).

What would I change

I would **reverse the order of the tasks.** It would be much easier if we started by constructing the model and tested its sensibility to the inputs afterwards. Then, we would design the questionnaire to capture the most appropriate inputs.

MEHRA AND PRESCOTT WERE THE FIRST AUTHORS TO DECLARE THE HIGHER RETURN OF STOCKS OVER GOVERNMENT BONDS AS EQUITY PREMIUM PUZZLE

EQUITY PREMIUM PUZZLE

It is defined as equity returns minus bond returns and reflects the relative risk of stocks compared to "risk-free" government bonds. **Stocks' higher real returns over government bonds is seen as a puzzle**: given the higher performance of stocks over government bonds, why anyone would hold bonds?

Objections

Survivorship bias

Jurion and Goetzmann argued that it was important to understand that U.S. equity markets never closed (permanently/temporarily). Thus, estimating risk premiums from the most successful country as Mehra and Prescott did (real returns of 4.3% annually versus a median of .8% for other countries) and **ignoring the evidence from stock markets that did not survive** the full sample period, will create an upward bias in estimates of expected returns.

Irrational behavior

The puzzle only occurs because investors show an irrational behaviour. The authors introduced the concept "Narrow framing", which is the idea that investors evaluate every risk separately: they do not take into account the low correlation of a stock portfolio with other components of wealth. This is, **investors do not consider the benefits of diversification** and therefore require a higher risk premium than rational models would predict.

Longer time period

Siegel extended the data back to 1802, until 1997. During this period the equity premium was 5.3% per year, **1.3 pp less than the equity premium reported by Mehra and Prescott (from 1889 to 1985)**.

Siegel finds that real returns in the short-term fixed income market have fallen dramatically over time: from 5.4 percent (1802-1871), to 3.3 percent (1872-1925), and 0.7 percent since 1926.

V S

- The period studied by Mehra and Prescott includes the stock market crash and following Great Depression (1929-1933), when stocks lost ~80% of their value.
- Gelien, Hirose and Tso computed replicated returns for Germany and Japan through WWII and concluded that the premium was higher than for the U.S.

SHOWING TO THE CLIENTS 3 SCENARIOS CONVEYS THEM MORE CONFIDENCE BUT ALSO THE POTENTIAL OF INVESTING IN THE RISKY ASSET

1 Based on the fact that Portuguese investors remain very conservative when it comes to invest their savings, Caixagest can use the following approach proposed to show to its clients the probability of achieving their goals and the benefits that arise from diversification.

2 An investment simulator that returns the evolution of possible clients portfolios across time, divided into investment phases according to the number of goals a client has. The inputs used in the simulator are the investment period, client wealth, return of the risky asset and risk free and the respective standard deviations. The output is 30 simulations, each one including the evolution of 19 portfolios with different alphas.

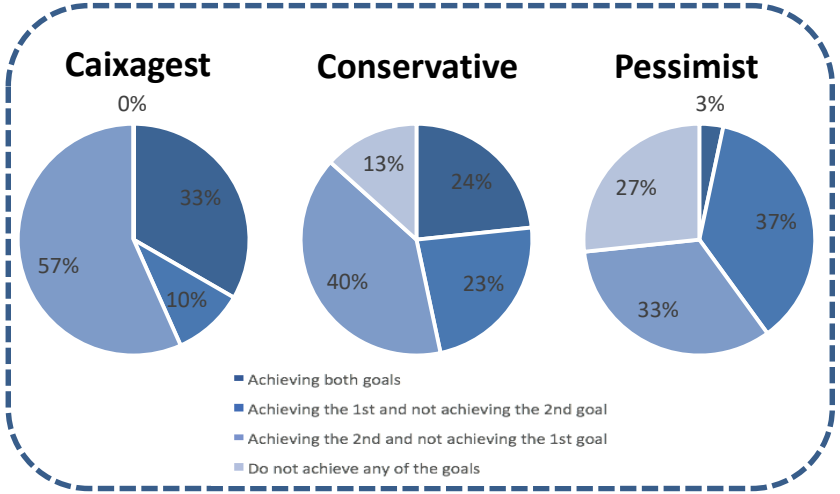
3 **I would suggest** proposing to clients three scenarios (Caixagest, Conservative and Pessimist), including exposures required to achieve the goals of each scenario. These were constructed based on different portfolios, with different sharpe ratios (0.8, 0.5, 0.3 respectively).

Advantages

- Clients have an idea of the investment they would have to make to achieve their objectives.
- Clients see a possible evolution of their investment over the years with gains and losses and be aware of the risks involved in their investments.
- Very simple, flexible and intuitive model that the wealth managers can adjust according to the objectives of their client.

Disadvantages

- Difficult to identify the optimal alpha.
- Does not represent the exact evolution of the portfolio.



Probabilities of achieving the goals

Risks

Caixagest Scenario: If a client chooses to leave the money in the bank for 35 years at the risk-free instead of not investing the €60,000 in Caixagest's portfolio, at retirement he/she would have €100,685 FV(€60,000;1.49%). In this scenario the investor never looses money.[1]

Conservative Scenario: The investor does not achieve its goals in 4 of the 30 simulations. The investor only loses money in 1 simulation. In the other 3 simulations the investor would need to invest at least 65% of its wealth in the risky asset.[2]

Pessimist Scenario: The probability of not achieving any of the goals is 23% (8/30). In these 8 cases, investors lose money 87.5% of the cases and in the cases that they do not lose they need to invest 55% in the risky asset.[3]

([1,2,3]assuming the same investment problem we presented in the suggestions section).

INDIVIDUAL PART – BRUNO

WORKING PROJECT OVERVIEW

Project's achievements

- We provided Caixagest with 3 different portfolio-building models serving different purposes: intertemporal consumption for utility maximization, wealth maximization and meeting a given wealth objective.
- We describe several concrete suggestions which shouldn't be too difficult for a follow-up team to model and implement.
- Working on the project allowed us to develop a sense of practicality that is likely to be useful for the follow-up team. We are available for the follow-up team to consult with us.

Project's shortfalls

- Follow-up work is required if implementation is to happen: models should include other features (view section about suggestions) and may require additional fine-tuning.

Opportunities ahead

- Since we have left significant material, follow-up professionals don't have to spend too much time deciding on frameworks (e.g. relevant variables, simulation vs. binomial, dynamic vs. static). They can focus directly on fine tuning and details (e.g. calibrating utility functions for risk-aversion, accounting for age-varying risk aversion, including multiple objectives, including retirement as an objective).
- Afterwards, implementation can be passed over to IT and the marketing department for direct implementation.
- We believe implementation will be a great competitive advantage for Caixa because it will position it in the forefront of robot-advisory in Portugal.

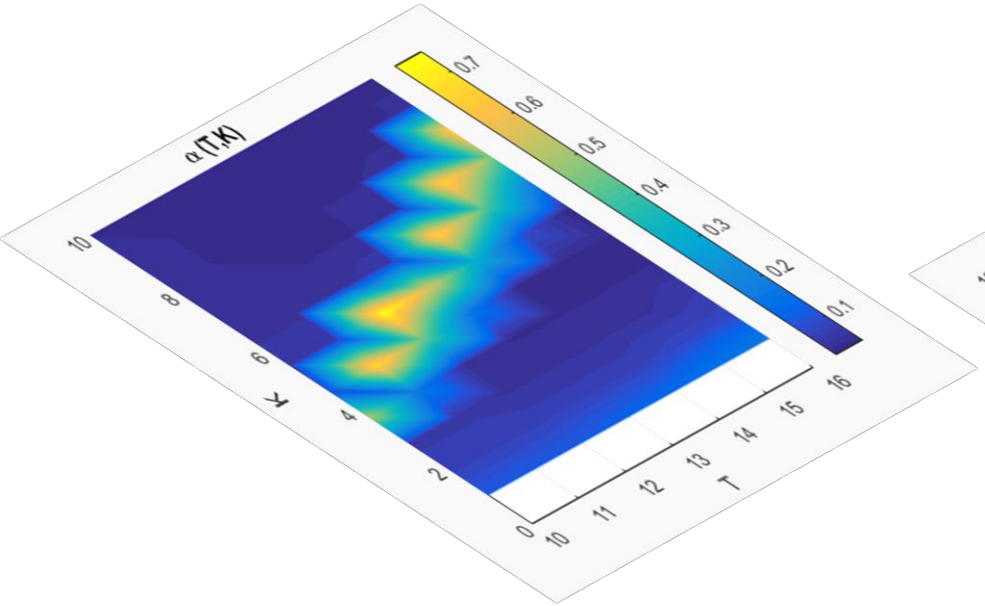
Personal achievements and learnings

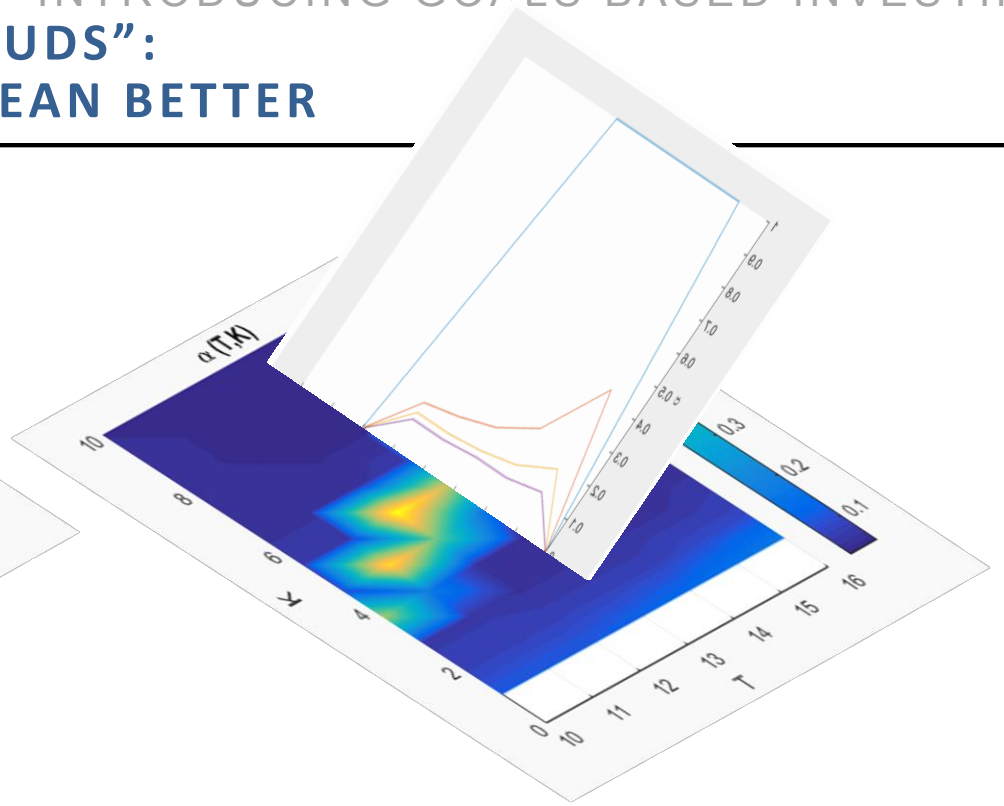
- Learned about the banking industry and some of its trends, both in Portugal and abroad.
- Improved my computational skills and critical sense.
- Experienced the professional work environment in a bank for the first time in my life.
- Learned about how different people work by observing the group members, the advisor and Caixagest employees.
- Learned that having a sufficiently diverse work group is crucial for keeping the team productive and motivated.
- Learned about the importance of project pre-planning in order to efficiently conciliate meetings, efficiently divide tasks and exchange ideas and integrate different opinions.

Other personal opinions

- I was privileged to participate in this project because it was basically quant work and it is extremely unlikely for someone without a PhD to ever be a quant in a bank; the project was much more interesting and versatile than I expected from reading the syllabus.
- The technical aspects of the project were easy (possibly because we used relatively simple frameworks) but work intensive.
- Working with the intertemporal model was the most challenging part of the project; the help of our academic advisor was crucial.
- I wish we could have made significant advances in the next stage and achieved more directly implementable, practical results.

A CLOSER LOOK INTO "CLOUDS": MORE DOESN'T ALWAYS MEAN BETTER





Back to the example…

K (€)	30,000		40,000		**50,000**		60,000		70,000	
Alpha	19%	0%	16%	0%	14%	0%	0%	0%	0%	0%
x0	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000	25,000
E[x1]	25,590	25,375	25,556	25,375	25,533	25,375	25,375	25,375	25,375	25,375
E[x2]	31,686	31,199	31,609	31,199	31,558	31,199	31,199	31,199	31,199	31,199
E[x3]	38,029	37,222	37,901	37,222	37,815	37,222	37,222	37,222	37,222	37,222
E[x4]	44,663	43,453	44,470	43,453	44,342	43,453	43,453	43,453	43,453	43,453
E[x5]	57,368	55,722	57,105	55,722	56,930	55,722	55,722	55,722	55,722	55,722
P(X<K)	0	0	0	0	0	0	1	1	1	1

- "Clouds" happen because there is a potential level of *K* for every maturity.
- For very low levels of *K*, alpha=0 because there is no need to take risk to achieve the objective.
- If *K* is "easily" attainable with alpha=0, alpha>0 because there are opportunities to expand wealth beyond *K*.
- As *K* approaches from its potential, alpha decreases because using more M will not only be exploiting possible surpluses – but it will also account for potential losses from taking more risk.
- Aiming for values of *K* too high is wishful thinking and should be reflected as a no-go signal rather than desperately high alphas – alpha returns to 0.

CONSIDERING NEW MODELS: PRELIMINARY THOUGHTS ON AN INTEGRATED APPROACH WITH EXOTIC PREFERENCES AND PROSPECT THEORY

We have discussed models that maximize wealth subject to risk preferences and a simple model that aims to meet a specific wealth objective subject to risk preferences. More models are needed in order to effectively respond to other needs of the client.

What is still needed

- Expanding the application of disappointment aversion (already existing in the simple one-objective model but not robust – intertemporal effects are not taken into account)
- Including multiple objectives & accurately account for retirement options

Theoretical updates by Backus et al. (2005)

- Authors compile methods for processing utility functions
- They discuss exotic, time additive preferences, i.e., variations of

$$V_0(U_t) = E_0 \sum_{t=1}^{T} [\delta \beta^t U_t(C_t, \gamma)]$$

where E_0 is the expectations operator at t=0.

- The authors discuss different features of such preferences and variations of additive preferences and suggest numerical optimization, which is similar to what we have previously shown.
- The originally described applications by these authors have the scope of macroeconomic theory. (The authors go as far as concluding that their work is essentially a branch of Neocalassical economics.) Building on these updates is useful but still lacking in terms of practical application by an asset manager such as Caixagest. We propose the following:

Improvements to do from here

- A new model could work on the basis of forward-looking expectations from Monte Carlo simulations by including additive preferences
- Departing from concepts compiled by Backus et al. (2005), we can experiment with different ways of accounting for precautionary savings and disappointment aversion.
- We can also recalibrate the utility function to account for asset management-specific restrictions, such as maximum loss and aversion to ambiguity (varying probabilities of states) – cf. Gilboa et al. (1989) for a static framework.
- We can also finally incorporate relevant updates by Köszegi and Rabin (2009) regarding changes in expectations. (We have previously experimented with this approach in the first intertemporal model, but dropped the application in favor of the gains from using the log-approximation.)
- In other words, we are able to reconcile our previous quantitative methodology with exotic preferences, prospect theory and other practical requirements by Caixagest. This tailored, integrated approach is yet to be fully developed by academia.

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