David José Marques Vicente

Licenciado em Ciências da Engenharia Electrotécnica e de Computadores

Distributed Algorithms for Target Localization in Wireless Sensor Networks Using Hybrid Measurements

Dissertação para obtenção do Grau de Mestre em

Engenharia Electrotécnica e de Computadores

Orientador: Marko Beko, Investigador Auxiliar, UNINOVA
Co-orientador: Rui Miguel Henriques Dias Morgado Dinis, Professor Associado com Agregação, FCT-NOVA

Júri

Presidente: Rodolfo Alexandre Duarte Oliveira
Arguente: João Pedro Castilho Pereira Santos Gomes
Vogal: Marko Beko

FACULDADE DE CIÊNCIAS E TECNOLOGIA
UNIVERSIDADE NOVA DE LISBOA

Março, 2017

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Ao Pinto e ao Paulo, que mereciam mais tempo.

Acknowledgements

Gostaria de agradecer ao meu orientador, o Professor Marko Beko. Pela disponibilidade e paciência ao longo de todo o percurso que levou à escrita desta dissertação. Pelo à-vontade que demonstrou logo desde o início. E por manter um espírito positivo mesmo quando as situações não corriam da melhor forma.

Quero também agradecer ao meu colega Slavisa Tomic. Por toda a preciosa ajuda e tempo que disponibilizou, e pelas explicações esclarecedoras.

Um obrigado aos meus amigos, com quem tive a oportunidade de trabalhar várias vezes ao longo do curso: André Pontes, Afonso Ferreira, Diogo Basílio, Emanuel Sequeira, Guilherme Gaspar, Ivo Bernardino, João Matias, Ricardo Lima, Tiago Aleixo, Tiago Pina. Mais recentemente, Gonçalo Leandro e Pedro Mateus. Em especial, um obrigado ao Carlos Filipe, que me deu *carry* em várias práticas de electrónica.

Quero também agradecer aos restantes membros do auto-intitulado grupo do *SAL*, simplesmente por estarem presentes.

Aos meus colegas e amigos que tive o prazer de conhecer no decorrer deste percurso e com quem também tive a oportunidade de trabalhar, um obrigado, Flávio Jacinto, João Eusébio Henriques, Ana Filipa Sebastião.

Um obrigado à minha, felizmente, muito numerosa família que sempre me deu força e acreditou em mim. Todos os primos, primas, tios, tias, avó, avô, que de uma forma ou de outra contribuíram para o meu sucesso.

Um obrigado especial aos meus pais, António e Filomena, que todos os dias às 9:00h já estão a "lutar" para que eu e o meu irmão possamos estudar. Que sempre acreditaram e se esforçam incondicionalmente. Ao meu irmão, Henrique, que várias vezes cozinhou para mim quando mais uma hora de estudo fazia toda a diferença.

Quero também deixar um obrigado ao Carlos, à Isabel e ao João. Que me acolheram nestes últimos anos e me recebem todos os dias em sua casa.

E por fim, um obrigado muito especial à Ana. Que se esforça por nós desde o primeiro dia, e sem ela talvez ainda estaria a tentar ser aprovado a Análise Matemática I B.

Abstract

This dissertation addresses the target localization problem in wireless sensor networks (WSNs). WSNs is now a widely applicable technology which can have numerous practical applications and offer the possibility to improve people's lives. A required feature to many functions of a WSN, is the ability to indicate where the data reported by each sensor was measured. For this reason, locating each sensor node in a WSN is an essential issue that should be considered.

In this dissertation, a performance analysis of two recently proposed distributed localization algorithms for cooperative 3-D wireless sensor networks (WSNs) is presented. The tested algorithms rely on distance and angle measurements obtained from received signal strength (RSS) and angle-of-arrival (AoA) information, respectively. The measurements are then used to derive a convex estimator, based on second-order cone programming (SOCP) relaxation techniques, and a non-convex one that can be formulated as a generalized trust region sub-problem (GTRS). Both estimators have shown excellent performance assuming a static network scenario, giving accurate location estimates in addition to converging in few iterations.

The results obtained in this dissertation confirm the novel algorithms' performance and accuracy. Additionally, a change to the algorithms is proposed, allowing the study of a more realistic and challenging scenario where different probabilities of communication failure between neighbor nodes at the broadcast phase are considered. Computational simulations performed in the scope of this dissertation, show that the algorithms' performance holds for high probability of communication failure and that convergence is still achieved in a reasonable number of iterations.

Keywords: Target localization, wireless sensor network (WSN), received signal strength (RSS), angle of arrival (AoA), convex optimization, maximum likelihood (ML) estimation, second-order cone programming (SOCP) problem, generalized trust region sub-problem (GTRS), distributed computation, cooperative localization.

Resumo

Esta dissertação aborda o problema da localização de sensores alvo em redes de sensores sem fios. A tecnologia das redes de sensores sem fios é amplamente aplicável actualmente, tendo numerosas aplicações práticas e a capacidade de melhorar a vida das pessoas. Uma propriedade necessária a muitas funções neste tipo de redes, é a capacidade para indicar o local onde foram medidos os dados reportados por cada sensor. Desta forma, localizar cada sensor, numa rede de sensores sem fios, torna-se uma questão essencial a ser considerada.

Nesta dissertação, é apresentada uma análise de desempenho de dois algoritmos de localização recentemente propostos para redes de sensores sem fios cooperativas e em 3 dimensões. Os algoritmos testados fazem uso de medições de distâncias e ângulos, obtidos através da potência do sinal recebido e do ângulo de chegada do sinal, respectivamente. As medições são posteriormente usadas para deduzir um estimador convexo, baseado em técnicas para relaxar problemas através de programação cónica de segunda ordem, e um estimador não convexo, que pode ser formulado como um sub-problema generalizado de região de confiança. Ambos os estimadores demonstram um excelente desempenho, assumindo um cenário em que a rede é estática, fornecendo estimativas de localização com precisão para além de convergirem em poucas iterações.

Os resultados obtidos nesta dissertação confirmam o desempenho e precisão dos dois algoritmos recentemente propostos. Adicionalmente, foi proposta uma alteração aos algoritmos que permite estudar um cenário mais realista e desafiante, onde são consideradas diferentes probabilidades de falha de comunicação entre sensores vizinhos, durante a fase de difusão. As simulações computacionais levadas a cabo no âmbito desta dissertação, demonstram que o desempenho dos algoritmos se mantém para cenários com alta probabilidade de falhas de comunicação, e que a convergência continua a ser alcançável num número razoável de iterações.

Palavras-chave: Localização de sensores alvo, redes de sensores sem fios, potência do sinal recebido, ângulo de chegada, optimização convexa, estimação, método da máxima verosimilhança, programação cónica de segunda ordem, sub-problema generalizado de região de confiança, computação distribuída, localização colaborativa.

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Glossary

anchors — Sensor nodes whose locations are known from the very start. The information is either obtained by manual configuration during deployment, or by external means such as GPS. A reference node .

sensor — A device which is connected, via a RF communication medium, to the other devices in the network. Throughout this document the terms ’sensor’, ’sensor node’ and ’node’ all refer to the same thing and are used interchangeably .

targets — Sensor nodes in the network whose locations are unknown and have to be estimated .

Acronyms

AoA	angle of arrival.
BS	base station.
GPS	global positioning system.
GTRS	generalized trust region sub-problem.
IoT	Internet of Things.
LAN	local area network.
LoS	line-of-sight.
LS	least squares.
Mc	Monte Carlo.
MLE	maximum likelihood estimate.
NLoS	non-line-of-sight.
PDF	probability density function.
PLE	path loss exponent.
RF	radio frequency.
RSS	received signal strength.

RSSI	received signal strength indicator.
SDP	semidefinite programming.
SOCP	second-order cone programming.
SR-WLS	squared-range weighted least squares.
TDoA	time difference of arrival.
ToA	time of arrival.
WSNs	wireless sensor networks.

Notation

For reference purposes, some of the most common symbols used throughout the dissertation are listed below. Upper-case bold type, lower-case bold type and regular type are used for matrices, vectors and scalars, respectively.

$\mathbb{R}$	the set of real numbers
$\mathbb{R}^n$	n-dimensional real vectors
$\mathbb{R}^{m\times n}$	$m \times n$ real matrices
$\boldsymbol{A}^T$	the transpose of $\boldsymbol{A}$
$\boldsymbol{A}^{-1}$	the inverse of $\boldsymbol{A}$
$\boldsymbol{I}_n$	the $n \times n$ identity matrix
$\mathbf{0}_{m\times n}$	the $m \times n$ matrix of all zero entries
$\mathbf{1}_n$	the n-dimensional column vector with all entries equal to one
$\mathrm{diag}(\boldsymbol{x})$	the square diagonal matrix with the elements of vector $\boldsymbol{x}$ as its main diagonal, and zero elements outside the main diagonal
$\lVert\boldsymbol{x}\rVert$	the Euclidean norm of vector $\boldsymbol{x}$; $\lVert\boldsymbol{x}\rVert = \sqrt{\boldsymbol{x}^T\boldsymbol{x}}$, where $\boldsymbol{x} \in \mathbb{R}^n$ is a column vector
$p(\cdot)$	probability density function
$\mathcal{N}(\boldsymbol{\mu}, \Sigma)$	real-valued Gaussian distribution with mean vector $\boldsymbol{\mu}$ and covariance matrix Σ
$\sim$	distributed according to
$\approx$	approximately equal to
$\log_a(x)$	the base-a logarithm of x; when a is omitted it denotes the natural algorithm

CHAPTER 1

INTRODUCTION

1.1 Motivation

Wireless sensor networks (WSNs) have gone from just a vision to a widely applicable technology in merely three decades. Small, low-cost and low-power devices equipped with sensor(s), a microprocessor and a transceiver, monitor physical phenomena in an area of interest, communicating through wireless links. The observed physical or environmental conditions can be light, sound, temperature, vibration, pressure or even vital signs, to name a few [1, 15].

Such characteristics grant the possibility for WSNs to have numerous applications in areas like environmental monitoring, military and home domain. For example, farmers can use this type of technology to monitor the soil conditions, industrial bodies may use it to track greenhouse emissions, and it may provide battlefield advantage to the military thanks to its rapid deployment and self organization characteristics. Also pushing the research and development of WSNs is the currently trending concept called Internet of Things (IoT) [72], "a world-wide network of interconnected objects uniquely addressable, based on standard communication protocols" [30]. This vision of IoT is driving the usage of sensors in applications such as smart homes and intelligent transportation systems, acting as a bridge between the physical and digital world [3].

The number of sensor nodes in a WSN is often in the order of hundreds or thousands, all of which are densely deployed in an area of interest. With this type of scale, and sometimes due to the harshness or inaccessibility of the terrain, the positions of sensor nodes are most of the time not predetermined. Instead, random deployment with no fine control and little to no human intervention (*i.e.*, dropped from an air plane or mortar launched) is preferred over a manual one (*i.e.* deterministically) [31]. The downside of this approach is that it becomes impossible to predetermine and preprogram the geographical coordinates of each sensor node, as manual localization of every node would be prone to human error and unfeasible in a realistic amount of time. A problem arises however, as sensor nodes should be able to indicate where the gathered data was recorded, since a set of measurements without location information is only useful to compute simple statistics such as the average of said measurements.

Localization of sensor nodes in a WSN is required in many practical applications, namely: event detection (fires, floods) [62], monitoring (health care, industrial, agricultural, environmental) [16, 38, 56], exploration (underground, deep water, outer space) [23], and surveillance (intrusion detection) [28]. Furthermore, there are routing protocols that require the nodes' locations in order to make multi-hop communication decisions and energy-efficient routing [2, 9]. By knowing each node's location it is also possible to evaluate the node density and coverage of the interest area. Lastly, localization enables context-awareness, identification and correlation of gathered data, allowing statistics to be computed in terms of spatial sampling rather than the count of sensor readings [10].

All of above mentioned points make the localization of each node after the deployment phase a high priority issue. Therefore, localization techniques should be used to estimate each node's location so that the network is able to organize itself.

The most obvious solution to this matter would be equipping each node with a global positioning system (GPS) receiver, rendering the localization problem trivial (in favorable conditions). This method, however, is unsuitable for WSNs considering the constraints to which the devices are subject like low production cost, low complexity and low power consumption [49]. Furthermore, GPS requires line-of-sight (LoS) signal propagation between the satellites and the receiver resulting in possible obstruction by trees, tall buildings, etc. The system is also unreliable in specific environments like indoors and underground tunnels, as the signal cannot penetrate walls and soil very well [34]. Consequently, being able to perform localization using different approaches should be considered.

Another, more efficient, approach to localization in WSNs is to establish the position of sensor nodes whose location is unknown (targets), given some information about a sparse number of reference sensors (anchors), possibly equipped with GPS receivers, in addition to measured distances/angles between any pair of nodes. To obtain such measurements, one can exploit different characteristics of the radio frequency (RF) signals used by the nodes to communicate [49].

In summary, knowing the exact location of each sensor node is of utmost importance, making localization techniques a key factor in WSNs and motivating the development of efficient localization schemes.

1.2 Objectives and Contributions

The goal of this dissertation is to present an overview of the localization estimation in WSNs theme and to study two range-based, distributed and cooperative algorithms for target localization in WSNs proposed in [68]. The main contributions are:

- An implementation of two distributed localization algorithms, taking advantage of combined RSS/AoA measurements in a cooperative 3-D WSN, based on:
  - Second-order cone programming (SOCP) relaxation;
  - Generalized trust region sub-problem (GTRS) framework.
- An implementation of the algorithm based on SOCP, generalized for different and unknown transmit powers.
- A proposed change to the distributed localization algorithms, so that failure between neighbor nodes at the broadcast phase is considered.
- A computational experiment based on the Monte Carlo (Mc) method, providing a probabilistic interpretation of the algorithms' performances.
- A comparative analysis on the performance of the algorithms.
- A study of the algorithms' behavior in view of different probabilities of communication failure.

The work conducted in this dissertation resulted in the following submission:

D. Vicente, S. Tomic, M. Beko, and R. Dinis, "Performance Analysis of a Distributed Algorithm for Target Localization in Wireless Sensor Networks Using Hybrid Measurements in a Connection Failure Scenario", Submitted to *International Young Engineers Forum on Electrical and Computer Engineering*, May 2017.

1.3 Organization

The dissertation structure is briefly organized as follows:

Chapter 2 begins by describing the importance of localization in WSNs. Afterwards, the most relevant criteria used to classify localization schemes are discussed. The addressed criteria are: computational organization, dependency on anchor nodes, collaboration between target nodes, methods of estimating inter-node distance/angle or connectivity. Next, range combining techniques are explained. Throughout the chapter, many recent works on the subject are cited and contextualized.

Chapter 3 presents the general mathematical formulation of the localization problem, followed by the introduction of some of the techniques explained in the previous chapter. The studied algorithms are derived and then presented in the form of pseudo-code. Lastly, the contribution of this work is also presented.

Chapter 4 consists of the information relative to the carried computational experiments. Each experimental environment is explained and the respective results are displayed in the form of comparative plots.

Chapter 5 draws the conclusions and suggests future work directions.

Appendix A presents the submitted article, which is based on the contribution made in this work.

Chapter 2

State of the Art

2.1 Introduction

The idea of wireless positioning was initially conceived for cellular networks. In this dissertation we limit our discussion to sensor localization in WSNs. However, it is worth noting that, in practice, a base station (BS) or an access point in a local area network (LAN) can be considered as an anchor, while other devices such as cell phones, laptops, tags, etc., can be considered as targets. [59]

The goal of localization can be stated as: for each node with an unknown position, find and assign its geographic coordinates in the area of interest. For most WSNs applications, sensor data should be accompanied with an indication of the physical location where the data was recorded. Even if the accessible knowledge about positions of nodes is only approximate, there are many other tasks done in a WSN that benefit from it, namely: network services, location-based routing, data aggregation, etc [10].

While localization is a required feature to many functions in a WSN, it is not the real purpose of it. Therefore, localization should cost as little as possible in terms of network resources while still producing satisfactory results. In other words, the power cost, hardware cost, and deployment cost of nodes should be minimized [4].

It is possible to classify location discovery algorithms based on several criteria and used techniques. These algorithms should work with inexpensive off-the-shelf hardware, have minimal energy requirements, scale to large networks, and also achieve good accuracy independently of the surrounding environment, giving the solution in a short amount of time [46].

In this chapter several ways of classifying localization schemes are described. First, two means of conducting computational operations in the network are explained. Afterwards, the difference between using or not of anchor nodes in the network is explained.

Next, the contrast amidst collaborative and non-collaborative networks is detailed. Then, two ways of obtaining distance estimates between sensors are explained, followed by a discussion about the main ranging techniques used in range-based localization schemes. Finally, the most basic range-combining techniques are introduced, in addition to dedicating some attention to schemes based on hybrid measurements.

2.2 Classification of Localization Schemes

Localization schemes can be classified according to many criteria that reflect design choices and structural features. The distinctions are made generally when addressing known challenges in localization (computing organization, measurement techniques, existence of reference/anchor nodes, node mobility, position estimation accuracy, etc.). The rest of this section describes in more detail the most relevant criteria in the context of this dissertation.

2.2.1 Computational Organization

A typical issue in WSNs is the computational constraints to which this type of networks are subject. In order to maintain the network's overall cost low, sensor nodes are limited in power, computational capacities, and memory [1]. There are two approaches to address how computation is performed in the network [31], both are described next:

Centralized This approach assumes the existence of a sink node which acts as a central processor. Each sensor node sends the measured information (distance, orientation, connectivity) to the sink. This central node gathers all the measurements and solves the localization problem, forwarding the result back to the sensor nodes afterwards.

Distributed In this approach, computation is distributed between every sensor node. Targets usually use information from neighbor and anchor nodes to determine their position.

The centralized approach offers efficient processing and addresses the computational constraints present in low-cost sensor nodes. However, this way of tackling the problem suggests communication overhead, as each node has to make more transmissions in order to relay measured information, resulting in a higher battery consumption. Furthermore, this model introduces a single point of failure in the network, meaning that the entire localization process fails in case there is a sink node failure [50].

For large-scale and highly-dense networks, applying a distributed model becomes a preferable solution [17]. This approach requires higher processing capabilities from each sensor node but decreases communication overhead compared to the former model. Nonetheless, distributed algorithms are executed iteratively, which raises the energy consumption of the processing tasks and makes them prone to error propagation. The work in [49] states that, when the necessary number of iterations required for convergence is lower than the average number of hops to the central processor, the distributed approach is likely to be more energy-efficient. Examples of centralized algorithms are the MDS-MAP [60] and the semidefinite programming (SDP) based algorithm in [18]. Some exemplars of distribuited algorithms are the APS [43], the APIT [27], the bounding box [61] and the gradient [5].

2.2.2 Anchor-based and Anchor-free

It is possible to classify localization schemes regarding its use of anchor nodes. Anchors are a set of nodes that are aware of their coordinates *a priori*, either by using some external system like GPS or through manual configuration during deployment.

The goal of anchor-based schemes is to provide partial information of the entire network to a localization algorithm. This way, the algorithm can extrapolate the coordinates of non-anchors based on this information [58]. With anchor-based schemes the network's coordinate system can be aligned with a global coordinate system, making it possible to obtain absolute coordinates like latitude, longitude and altitude for each target. Still, GPS receivers are expensive and equipping anchors with them in high scale networks might not be viable. Plus, GPS requires LoS communication which typically does not work indoors as the signal cannot penetrate walls reliably and can be obstructed by environmental obstacles [34]. The alternative of pre-programming nodes with their locations is impractical, for the same scalability reasons, or even impossible, depending on the interest area's accessibility or the method of deployment. Another downside is that accuracy relies heavily on the number of anchor nodes and their geographic placement in the network. It has been found that localization accuracy improves if anchors are deployed in a convex hull around the network, but additional targets at centre of it is also beneficial [4]. Yet, the big advantage of using anchor nodes is greatly simplifying the task of obtaining coordinates for the unknown location nodes.

Anchor-free schemes, as the name implies, do not rely on *a priori* localization information from a set of nodes in order to make assumptions about the remaining nodes's positions [41, 53]. This type of localization scheme picks a set of reference nodes to establish a coordinate system, and estimates relative positions to it for the other nodes. One obvious advantage of not requiring anchors is the possibility of performing localization in environments where it is impossible to get GPS signal. However, if at least one of the reference nodes moves, positions have to be recomputed for all the nodes, even if no other nodes have moved [74].

Target Collaboration

In anchor-based localization schemes the number of anchors is often wanted to be as little possible for the reasons stated above. Considering that the wireless communication range of sensor nodes in the network is limited, this may lead to a situation where many target nodes can not communicate with anchors. Still, targets need to be within communication range of anchors in order to perform range measurements that are needed for localization estimation. However, in this conditions their ability to perform self-location is limited if not impossible. Collaboration between target nodes is the mechanism used to face this issue. A collaborative localization scheme allows target nodes to communicate with either anchors or targets, improving localization accuracy and coverage [76].

2.2.3 Range-based and Range-free

Localization schemes are in general classified as range-based or range-free with regard to the mechanisms used to estimate inter-node distances. Range-based schemes use absolute pairwise distance and/or orientation measurements for coordinates computation. Range-free schemes, on the other side, rely solely on connectivity information to locate the sensors. An overview of the two can be seen in Figure 2.1.

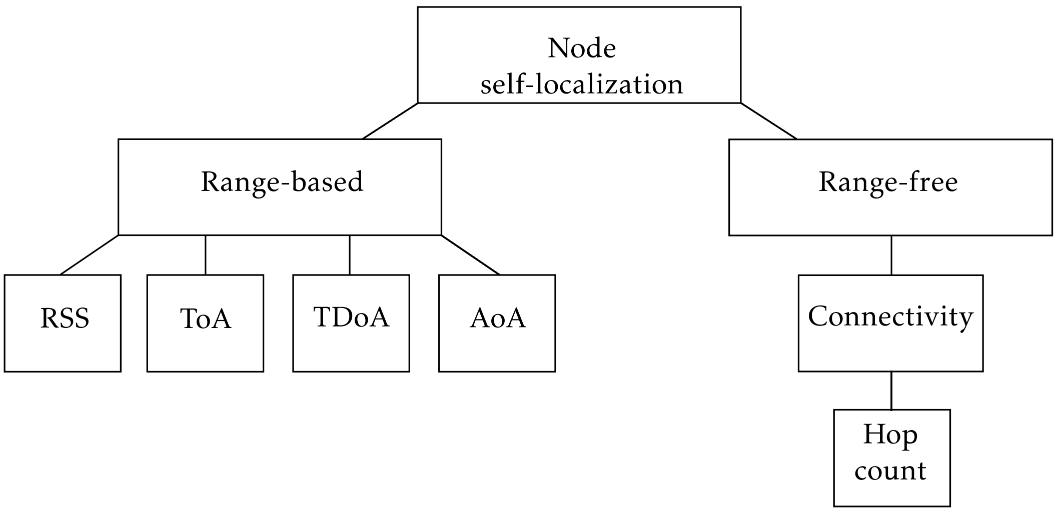


Figure 2.1: Overview of distance and connectivity methods

Range-based localization schemes use various ranging techniques to measure distances or angles. Each technique may require special hardware support, for example, angle of arrival techniques use radio or microphones arrays [19, 45, 78]. The need for extra hardware limits the applicability of these techniques and, consequently, of the corresponding localization schemes. Plus, it increases the overall cost of the network.

Range-free methods use merely radio connectivity between sensor nodes, not requiring special hardware to be installed in the nodes for that reason. Connectivity information is sampled over a long period of time making this methods robust against wireless channel fluctuations. This way accuracy is not affected by temporary variations of the channel. Some algorithms that use this method are [12, 27].

Overall, range-based localization techniques are more accurate but have higher computational costs and together with the additional hardware this results in increased energy requirements. On the other hand, range-free schemes are less accurate but do not require additional hardware and have smaller computational overhead [21].

Ranging techniques used in range-based schemes will be discussed in the next section.

2.3 Range Estimation Methods

The four ranging techniques described in this section, namely, received signal strength (RSS), time of arrival (ToA), time difference of arrival (TDoA), and angle of arrival (AoA), are the building blocks to range-based localization schemes. These techniques provide local information in terms of distance or orientation related to the neighbors of a node.

2.3.1 Received Signal Strength

Using RSS measurements is the most common ranging technique. It exploits the fact that a signal is attenuated with the travel distance from transmitter to receiver. The main advantage of this technique is that every sensor node has a radio, needed for communication, and is capable of obtaining the RSS of an incoming packet, not requiring additional hardware for it. All of this is done by most transceivers available for wireless networking applications, since they include circuitry to measure the received signal strength indicator (RSSI). Actually, many wireless standards and specifications demand the RSS information in order to ensure basic radio functions such as clear channel assessment, link quality estimation, handover, and resource management [77].

Conducted propagation measurements in a mobile radio channel show [55] that, the average received power P_r(dBm) at distance d from the transmitting antenna is approximated by

$$P_r = P_0 - 10\gamma \log_{10}\left(\frac{d}{d_0}\right) + X, \tag{2.1}$$

where

P_0 (dBm) is the power received at a close-in reference distance d_0 from the transmitting antenna,

γ is the path loss exponent (PLE), a value that depends on the environment (see Table 2.1),

X is the uncertainty factor whose value depends on the multipath fading and shadowing effects.

Considering that P_0 is normally fixed and known by the sensor nodes, it is possible to use RSS to estimate the distance with an error proportional to the uncertainty factor, and the RSSI measurement error. Figure 2.2 shows how RSS measurement errors and fading effects may affect distance estimation. Since, the further apart from each other the transmitter and receiver are, the worst is the quality of the estimate.

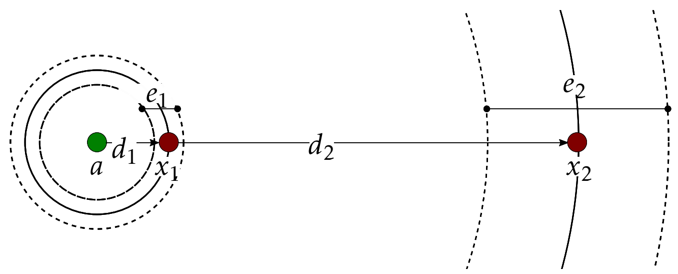


Figure 2.2: Effects of errors in RSS-based range measurements versus distance.

Table 2.1: Path Loss Exponent values for different environments [55]

Environment	Path Loss Exponent (PLE) γ
Free space	2
Urban area cellular radio	2.7 to 3.5
Shadowed urban cellular radio	3 to 5
In building line-of-sight	1.6 to 1.8
Obstructed in building	4 to 6
Obstructed in factories	2 to 3

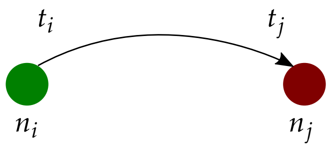
Using this technique for distance estimation offers the already mentioned benefit of not needing additional hardware. Consequently, this method saves power compared to others, since additional hardware means higher power consumption.

Localization using RSS based schemes has some disadvantages that cannot be overlooked. First, the accuracy of this methods is severely affected by shadowing and multipath effects, requiring multiple measurements to deal with them. Second, when there is an obstacle between two nodes, meaning that LoS communication is impossible, the signal suffers greater attenuation compared to its LoS counterpart. This means that in this situation the distance estimated using the RSS is not a good estimate of the actual distance. Moreover, unlike the first problem, this one cannot be corrected by realizing multiple measurements since the obstacle's effects will affect all of them [2]. Third, the parameters γ and X in model 2.1 depend on the surroundings, and obtaining estimates for them might prove to be difficult and sometimes even impossible. Finally, all sensor nodes should have their transceivers calibrated so that all of them share the same transmission power, or else, the RSSI values do not correspond to the actual RSS [71].

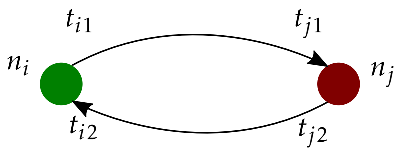
2.3.2 Time of Arrival

Time of arrival is a ranging method that exploits the proportional relationship between the amount of time a signal takes to propagate from one point to another and the distance between those two points (in this case, sensor nodes). It is possible to estimate distance, provided that the signal's propagation speed is known, using the following relation:

$$d = v \times t$$



(a) One-way ToA



(b) Two-way ToA

Figure 2.3: Illustration of different ToA methods

where d is the distance, v is the propagation speed, and t is the time taken by the signal to travel distance d. ToA measurements can be done in two distinct ways (see Figure 2.3):

One-way or Passive ToA

Passive ToA measures the trip time of a signal from the transmitter node i to the receiver node j. The distance is given by:

$$d_{ij} = v \times (t_j - t_i).$$

For this technique to work, the transmitter must embed the time stamp of transmission in the signal adding complexity and overhead to the signal. Plus, this method relies on synchronization and very accurate hardware to work. This method is the one used in GPS systems [29].

Two-way or Active ToA

In two-way ToA, the receiver node responds to the transmitter node and the round-trip time is used by the latter to estimate the distance between them. The result is sent back to the receiver.

The chronology of the process is the following: Sensor node i transmits a signal at its local time t_{i1}. The signal reaches sensor j at its local time t_{j1}. After some processing delay, sensor node j sends the signal back at its local time t_{j2}. The signal is received back at node i at its local time t_{i2}. The relations are:

$$\text{Total round-trip time} = t_{i2} - t_{i1}$$

$$\text{Processing delay suffered at node } j = t_{j2} - t_{j1}$$

$$\text{Round-trip time excluding delay} = (t_{i2} - t_{i1}) - (t_{j2} - t_{j1})$$

$$\text{time of flight} = \frac{(t_{i2} - t_{i1}) - (t_{j2} - t_{j1})}{2}$$

Therefore, the distance between the two nodes is:

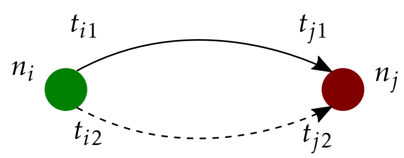


Figure 2.4: Illustration of TDoA.

$$d_{ij} = \frac{(t_{i2} - t_{i1}) - (t_{j2} - t_{j1})}{2} \times v$$

The disadvantage of this method is that it produces communication overhead, in addition to consuming energy in the process.

Using ToA to estimate distance between sensor nodes is not practical for traditional WSNs. The reason being, if RF signals are used to estimate distance, any small error in time measurement results in a large distance error. In order to obtain accuracy, high precision hardware is needed, which goes against the low-cost, low-complexity premise of WSNs to begin with. If on the other hand, slower propagation signals (*e.g.*, ultra sound) are used to estimate distance, less precise clocks are required. However, sensor nodes have to be equipped with additional hardware which is not desirable either. Nonetheless, algorithms using ToA have been developed [47].

2.3.3 Time Difference of Arrival

TDoA is a term often used to describe two different things in the context of localization. In this section however, range estimation methods, which are ways of measuring distance (or orientation), are being discussed. For this reason, we refer to TDoA as a method of estimation distance through the use of two different kinds of signal that have different propagation speeds. The *other* TDoA is discussed in Section 2.4.1 as a geometrical method of computing location.

TDoA uses two different kinds of signal to estimate distance. The transmitting node i transmits two different signals simultaneously or separated by some fixed time interval, $(t_{i2} - t_{i1})$. The goal is to use the fact that signals have different propagation speeds, this is illustrated in Figure 2.4. A regular line represents a RF signal, while in order to represent a signal with slower propagation speed a dashed line is used instead. For example, node i transmits a RF signal which is received by node j at time t_{j1}. Afterwards, node i waits for a period of time $t_{i2} - t_{i1}$, which is also known by node j, and transmits an ultrasound signal. Node j receives the signal at time t_{j2}. With this, the distance can be calculates as:

$$d_{ij} = (v_{rf} - v_{us})(t_{j2} - (t_{i2} - t_{i1}) - t_{j1})$$

where v_{rf} and v_{us} are the propagation speeds of the RF and ultrasound signal respectively. The most popular user of this technique is the Cricket system [52].

TDoA offers high accuracy if line-of-sight conditions are guaranteed. However, non-line-of-sight (NLoS) conditions have highly destructive effects to this measurement method. Additionally, ultrasonic waves especially are affected by atmospheric effects such as temperature, pressure and humidity. Finally, this ranging method also requires additional hardware [2].

2.3.4 Angle of Arrival

AoA methods are used to determine the orientation of propagation of a received signal. This methods typically use directional antennas or a special configuration of antenna arrays to estimate the AoA of a received signal [19, 45].

When directional antennas are used, they rotate about its axis in order to transmit in or receive from wanted directions. When an array of antennas is used, the antennas' position in the array is known and used to estimate orientation. The difference of ToA of the received signal at different antennas is then used to estimate the direction from which the signal arrived.

The accuracy of AoA techniques depends on the measurement accuracy, that is, on the precision and complexity of the equipment. For example, to achieve high accuracy with antenna arrays very sophisticated ones are required, increasing the cost of sensor nodes and consequently the overall cost of the network. Plus, the required spacing between antennas, so that spatial diversity is guaranteed, often lowers this method's applicability. One premise of WSNs is the tiny size of sensor nodes, thus, increasing the size of those nodes for localization purposes might not be an option. Furthermore, multi-path, scattering and NLoS conditions introduces more error to AoA techniques than it does to RSS or TDoA ones [2]. The authors in [44] proposed a APS algorithm that uses AoA measurements. In [42] a technique by which sensor nodes determine their positions in a WSN by obtaining angular bearings relative to a set of fixed beacon nodes is presented. Figure 2.5 illustrates how the measurement errors affect AoA estimation. It is possible to observe that an estimated angle may be considered a good measurement if the sensors are close to each other. But, for the case when the distance between sensors is high, the same estimated angle may induce to a position which is considerably far away from the real one.

2.4 Range Combining Techniques

Given range and orientation measurements obtained by using the methods explained in Section 2.3 it is possible to combine them and compute a sensor's location using some algorithm. The techniques can be simplistic (ignoring measurement noise), and have simple geometric interpretations. On the other hand, more realistic approaches consider

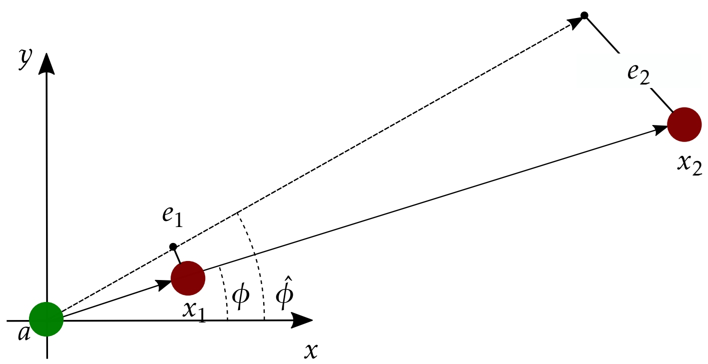


Figure 2.5: Effects of errors in AoA measurements versus distance.

the existence of noise, whose behavior can be known or not, and the possibility of existing overdetermined systems of equations (more measurements available than the minimum required). [59].

2.4.1 Geometric Approaches

Range and orientation measurements have geometrical interpretations which can be used to compute a node's position relative to other nodes. Using simple geometric methods such as intersecting enough circles, lines or hyperbolae (in 2-D), results in a position in space [74, 78].

Trilateration

Trilateration can be performed using RSS, ToA or TDoA measurements, and it is the most basic and intuitive method [2, 4, 46, 74]. To estimate the position of one node in a 2-D space, that node needs to know the position of three reference nodes and its distance to them (four, in 3-D). Each distance defines the radius of a circle with the respective neighbor at its center. If three of such circles (four, in 3-D) belonging to non-collinear(coplanar) neighbors intersect, the exact location of the node is defined. Figure 2.6 illustrated how trilateration is performed in a 2-D space.

Triangulation

Triangulation uses AoA measurements between the target node and two anchor nodes (three, in 3-D). Each AoA measurement represents a line in a 2-D space (plane, in 3-D), and the intersection of two lines (three planes) give a position in space [2, 4, 46, 74].

Another way of interpreting (in 2-D) can be seen in Figure 2.7. Two anchors and a target define a triangle. Using trigonometric relations, the location of the target can be computed using the anchors' locations and the respective pair-wise AoA measurements.

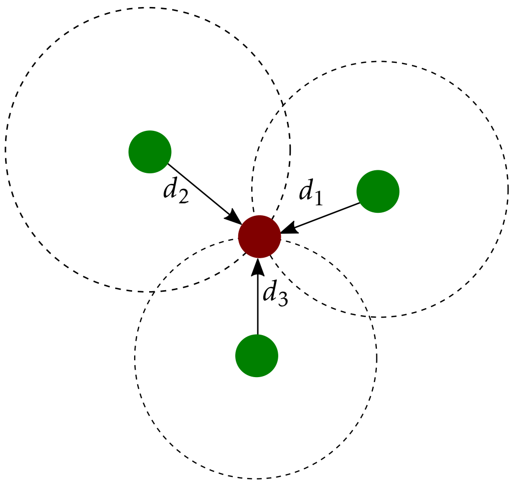
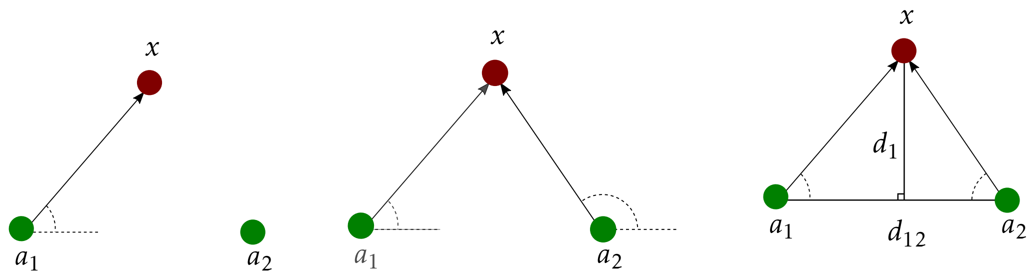


Figure 2.6: Illustration of trilateration.

The advantage of triangulation over trilateration is the fact that it requires less anchors to perform positioning. However, the disadvantages carried by the use of AoA measurement are still present (see Section 2.3.4).



(a) x's angle relative to a_1 (b) x's angle relative to a_2 (c) Using trigonometric relations to locate x

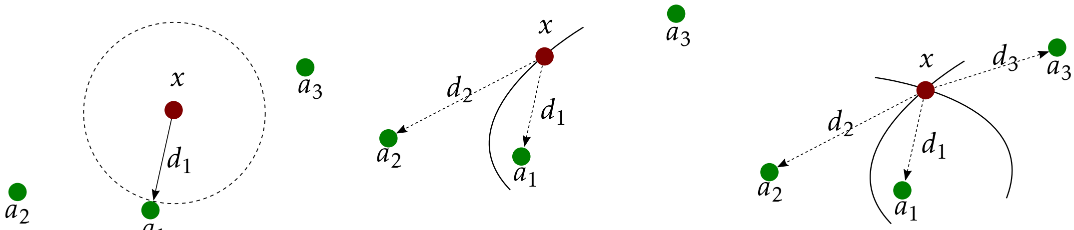
Figure 2.7: Illustration of the steps required to perform triangulation.

Hyperbolic Positioning

Often called multi node TDoA[1], hyperbolic positioning actually uses ToA as a mean of obtaining distance measurements. In a 2-D case, three synchronized anchors transmit signals at exactly the same time. The receiver (a target) measures the ToA of each incoming signal. Afterwards, the target computes the difference in ToA from each pair of received signals, *i.e.*, the pair-wise time difference of arrival. Each pair of time differences form a

[1]In reality, 'multilateration' is the most common term for this technique. However, 'multilateration' is also often used to refer to trilateration generalized for when more than three measurements are available, *i.e.*, an overdetermined system, which can be solved by least squares (see Section 2.4.2). Since denoting different things using a single term can lead to confusion (for me it certainty did), in this dissertation word 'multilateration' is not used by choice.

hyperbola in space, and the intersection of two hyperbolae (three in 3-D) is then used to locate the target node [2, 4, 46, 74], this procedure is exemplified in Figure 2.8.



(a) Distance to a_1 (b) ToA difference from a_1 and a_2 (c) Hyperbole intersection

Figure 2.8: Illustration of the steps required to perform hyperbolic positioning.

2.4.2 Optimization Based Approaches

Despite the intuitiveness of the geometric approaches, in reality range and orientation measurements are corrupted with errors and noise. Furthermore, there are situations where the number of obtained measurements is higher than the minimum required to perform positioning, *i.e.*, an over-determined system (see Figure 2.9). For example, when using trilateration, if a target has five measurements in a non-coplanar case, the intersection of the resulting five spheres does not result in a single point in space but in an infinite set of possible solutions. In situations like this, a way of determining the solution is minimizing the difference between the measurements and the known relationships between the data and the sensor nodes positions [63, 76].

Least Squares

Least squares (LS) is a unconstrained optimization problem with an objective function which is the sum of squares. It can be used to solve the localization problem considering that the observation error does not have a known distribution [33, 76].

If we consider a set of M targets in a three dimensional space, their positions can be defined by the matrix $\boldsymbol{X}$ ($\boldsymbol{X} \in \mathbb{R}^{3\times M}$). The objective is to estimate $\boldsymbol{X}$ using observations, $\boldsymbol{\theta}$, and the known positions of N anchors, $\boldsymbol{A}$ ($\boldsymbol{A} \in \mathbb{R}^{3\times N}$). It is assumed that the observations are range measurements between all sensor nodes which are within communication range of each other. We can consider that the measurements are function of the target's and anchor's positions corrupted by some observation error:

$$\boldsymbol{\theta} = f(\boldsymbol{X}, \boldsymbol{A}) + \boldsymbol{n}, \tag{2.2}$$

where $\boldsymbol{n}$ is the observation error and $f(\boldsymbol{X}, \boldsymbol{A})$ is a known relationship between the node positions and the measurements in the lack of any error. Using this information, we can find the solution which minimizes the squared error between measurements and the known relationship, *i.e.*:

$$\hat{\boldsymbol{X}} = \arg\min_{\boldsymbol{X}} (\boldsymbol{\theta} - f(\boldsymbol{X}, \boldsymbol{A}))^2. \tag{2.3}$$

Maximum Likelihood

LS assumes that the distribution of the measurement noise is not known. If on the other hand we have such information, it can be used for our benefit. That is, we can maximize the probability density function (PDF) of $\boldsymbol{\theta}$ given $\boldsymbol{X}$, called the likelihood function, obtaining the maximum likelihood estimate (MLE) [33, 76]. Or equivalently, minimize the negative[2] log-likelihood function[3]:

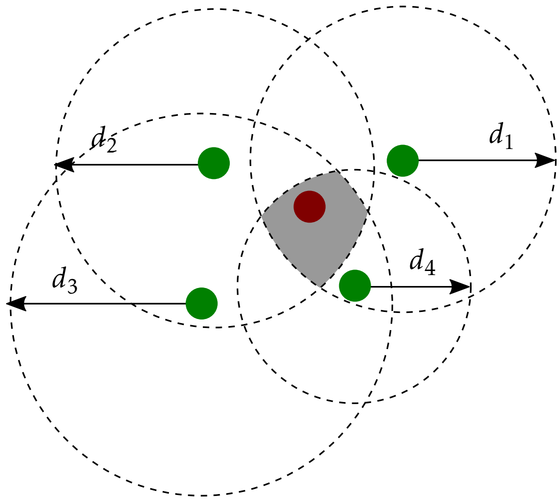
$$\hat{\boldsymbol{X}} = \arg\min_{\boldsymbol{X}} \{-\log(p(\boldsymbol{\theta}|\boldsymbol{X}, \boldsymbol{A}))\}. \tag{2.4}$$

Using optimization based approaches to find the solution to the localization problem present a drawback. The problem resides in the fact that $f(\boldsymbol{X}, \boldsymbol{A})$ is generally non-linear and non-convex. Considering this, a used solver, more often than not, might not be able to find the solution, *i.e.*, the targets' positions. The approach to this issue is to apply relaxations or use iterative techniques in order to find an approximation of the true solution.

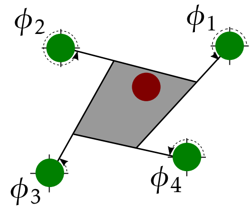
Recursive methods, such as Newton's method combined with gradient descent method, are often used to obtain the ML solution [33]. However, since the objective function may have numerous local optima, it is possible that local search methods may get trapped in them. This issue can be overcome by using approaches such as grid search methods and linear or convex relaxation techniques, which can also be used to provide good initial points for more accurate iterative algorithms [35, 36, 48]. Grid search methods solve the ML problem by forming a grid and passing each point of the grid through the ML objective function until the optimal point is found. This approach is suboptimal since it doesn't search for the solution in a efficient way; the result is a time-consuming method with computational complexity and memory requirements proportional to the grid size and number of unknown parameters. Less complex are the linear estimators such as the linear least squares. Methods of this type are very efficient regarding the processing time and computational complexity. Nonetheless they are based on heavy approximations so low accuracy is to be expected, especially in the presence of high noise levels [69]. Another way of tackling the issue is to employ convex relaxation techniques. The original non-linear and non-convex ML problem is transformed into a convex one. The advantage is that convergence to the globally optimal solution is guaranteed. Still, the obtained convex problem is a relaxed version of the original problem, therefore, its solution may not correspond to the original ML problem's solution [11].

[2] Negating the function comes from consistency, as conventionally in optimization theory (cost) functions are minimized.

[3] The logarithm is a monotonic function, so optimizing a function is the same as optimizing the logarithm of it. However, using the logarithm actually makes the function easier to handle and numerically more stable.



(a) Combining noisy distance measurements



(b) Combining noisy angle measurements

Figure 2.9: Use of an optimization method to combine noisy measurements in overdetermined systems.

2.4.3 Hybrid Localization

The most basic approaches of combining measurements rely on just one type of measurement. However, recently, hybrid systems that fuse different types of measurement have started to be implemented. The advantage of fusing different types of range measurements comes from the increase of available information for algorithms to work with. Hybrid systems take advantage of the strongest points of each ranging method, minimizing their individual drawbacks in the process [18, 39].

In [18], Doherty *et al.* proposed a way of estimating unknown node positions using connectivity-induced constraints and SDP, additionaly stating that fusing range and angle measurements would be beneficial. The recent works in [6, 25, 26] use hybrid systems that fuse RSS with two-way ToA measurements. A. Bahillo *et al.* proposed a hybrid RSS and two-way ToA scheme where the ToA ranging estimates are used as constraints for the RSS based technique [6]. To combine the measurements, the authors used a least squares approach. The work in [26] proposes a radio frequency identification system for indoor localization. The system uses RSS measurements to determine the bearing of target tags while two-way ToA is used to measure distance. The reason for this lies in the fact that ToA is less sensitive to multipath effects, and the system is designed to work indoors. In [25] the authors start by relying on RSS for rough positioning and afterwards use two-way ToA just on the most important links. This way, the problem of high communication load that usually comes with the use of two-way ToA is minimized.

Another combination of measurement types is proposed in [37]. The authors design a method which fuses TDoA and AoA measurements to estimate distance and angles

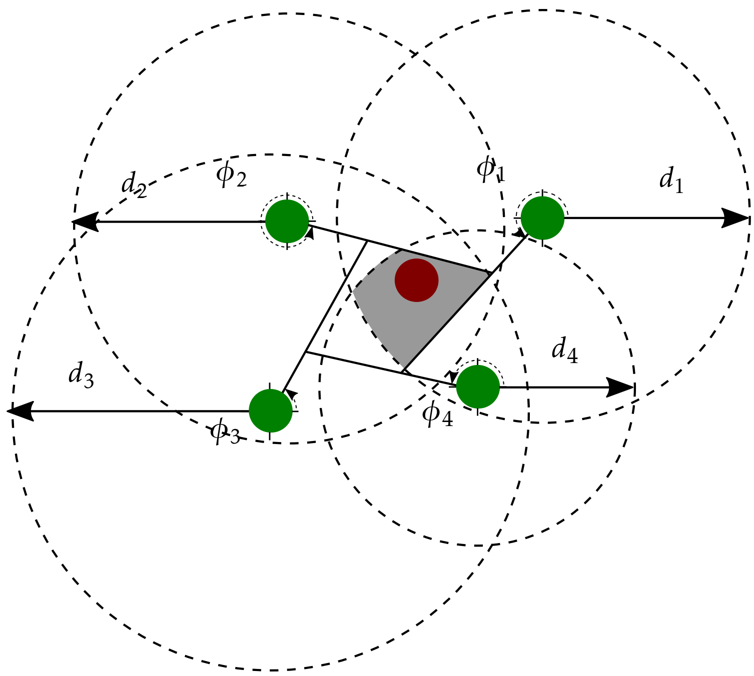


Figure 2.10: Use of optimization methods to combine noisy hybrid (distance and angle) measurements.

among sensors. The method's next step is to adjust the three-dimensional coordinate system based on local measurements and the fact that at least one of the reference sensors is aligned with the earth's center of gravity (as opposed to using digital compasses). The proposed algorithm is distributed and suitable for practical use on harsh environments.

Yet another combination of measurement types is the one which fuses RSS and AoA measurements. Figure 2.10 illustrates the advantages of using hybrid measurements compared to Figure 2.9. This particular combination is the one employed in the algorithm studied in Chapter 3 and onwards. Many localization algorithms that fuse RSS and AoA have been proposed. In [70] two hybrid RSS/AoA estimators were proposed. One by deriving a linear least squares system of equations, and the other by exploiting the statistical properties of the measurements and using the maximum likelihood criterion. L. Gazzah *et al.* proposed a NLOS estimator for localization in cellular networks using RSS and AoA [22]. The suggested algorithm uses scale factors that adjust the NLOS-corrupted measurements to approximate their LOS values. The authors in [14] developed an algorithm which uses WSN localization to locate a non-cooperative mobile target assuming NLOS conditions. In order to mitigate the effect of NLOS, the authors use an optimization approach to estimate the orientation of scatterers off which the target signals are reflected before reaching the sensors. In [57], a nonlinear LS estimator is used for localization in visible light communication systems based on light emitting diodes. The authors use AoA measurements as a initial point for an analytical learning rule based on the Newton-Raphson method. Afterwards the algorithm starts using RSS measurements. In [8], Biswas *et al.* added AoA measurements, to improve their previous approach which only used distance measurements when deriving a semi definite program based localization algorithm. Tomic *et al.* proposed cooperative and non-cooperative algorithms for centralized WSNs in [65], which used RSS and AoA hybrid measurements. In [66], the authors establish new relationships between the measurements and the unknown target location by using a spherical coordinate conversion and the available AoA observations, and derive a simple closed-form solution method.

CHAPTER 3

Hybrid Distributed Algorithms

3.1 Introduction

Two hybrid algorithms (in addition to a generalization) proposed in [68] are presented and studied, in order to ultimately propose a change which allows the creation and consideration of a more realistic scenario. The algorithms are distributed, anchor-based collaborative approaches, using hybrid RSS and AoA measurements. In the original work, the authors broke down a non-convex and computationally complex ML localization estimation problem into smaller local problems for each target in the network. Afterwards, using the least squares criterion, a local non-convex estimator was derived. This estimator approximates the local ML one for small noise levels. Next, Tomic *et al.* transformed the non-convex estimator into a convex SOCP estimator and into a squared-range weighted least squares (SR-WLS) one. The authors also generalized the SOCP estimator for the case where target transmit powers are different and unknown. The SOCP estimators were solved by interior-point algorithms, while the SR-WLS was solved using a bisection procedure. While the results show very high localization accuracy in just a few iterations, some strong assumptions were made such as that the network topology remains constant during the computational period. However, this assumption might not hold in practice, especially in dynamic environments where people and/or cars (or other objects) are passing by, blocking the LoS between nodes, or where weather conditions are constantly changing with time.

In this chapter, the target localization problem and RSS and AoA measurement models are explained in more detail. Then, the implemented distributed estimators are presented. Afterwards, a contribution is proposed by introducing a change to the algorithms in order to study their behavior in the presence of communication link failure between neighbor nodes at the broadcast phase.

3.2 Problem Statement

The first step to solve the localization problem is to formally state it. *The objective is to estimate the coordinates of M sensors (targets), given a priori the coordinates of N sensors (anchors) and pair-wise measurements between the nodes.* The WSN is composed of $M+N$ sensors nodes, with a communication range of R, randomly deployed over a cubic region of side B. This WSN can be represented as an Euclidean connected graph $\mathcal{G}(\mathcal{V},\mathcal{E})$ with the following properties:

- $\mathcal{V} = \{v_1, v_2, ..., v_{M+N}\}$ is the set of all sensor nodes, represented by the vertexes in the graph.
- $e = (i,j) \in \mathcal{E}$ if v_i is within communication range of v_j, that is, the distance between v_i and v_j is less or equal to R.

We can now label the set of targets and the set of anchors respectively as $\mathcal{T}$ ($|\mathcal{T}| = M$) and $\mathcal{A}$ ($|\mathcal{A}| = N$), where $|\bullet|$ represents the cardinality (the number of elements in a set) of a set. Their locations are denoted by $\boldsymbol{x}_1, \boldsymbol{x}_2, ..., \boldsymbol{x}_M$ and $\boldsymbol{a}_1, \boldsymbol{a}_2, ..., \boldsymbol{a}_N$ ($\boldsymbol{x}_i, \boldsymbol{a}_j \in \mathbb{R}^3, \forall i \in \mathcal{T}, \forall j \in \mathcal{A}$), respectively. The sets of all edges that represent target/anchor and target/target connections are defined as $\mathcal{E}_{\mathcal{A}} = \{(i,j) : \|\boldsymbol{x}_i - \boldsymbol{a}_j\| \leq r, \forall i \in \mathcal{T}, \forall j \in \mathcal{A}\}$ and $\mathcal{E}_{\mathcal{T}} = \{(i,k) : \|\boldsymbol{x}_i - \boldsymbol{x}_k\| \leq r, \forall i,k \in \mathcal{T}, i \neq k\}$, respectively. Lastly, we define a matrix $\boldsymbol{X} = [\boldsymbol{x}_1, \boldsymbol{x}_2, ..., \boldsymbol{x}_M]$ ($\boldsymbol{X} \in \mathbb{R}^{3\times M}$) as the matrix of unknown locations that must be estimated.

It is assumed that the range measurements are obtained from RSS information. When two sensors i and j are within communication range of each other, we can model the signal power received by i, P_{ij} (dBm), as:

$$P_{ij}^{\mathcal{A}} = P_{0i} - 10\gamma \log_{10} \frac{\|\boldsymbol{x}_i - \boldsymbol{a}_j\|}{d_0} + n_{ij}, \forall (i,j) \in \mathcal{E}_{\mathcal{A}}, \tag{3.1a}$$

$$P_{ik}^{\mathcal{T}} = P_{0i} - 10\gamma \log_{10} \frac{\|\boldsymbol{x}_i - \boldsymbol{x}_k\|}{d_0} + n_{ik}, \forall (i,k) \in \mathcal{E}_{\mathcal{T}}, \tag{3.1b}$$

following [55], where P_{0i} (dBm) represents the reference power at a distance d_0 ($\|\boldsymbol{x}_i - \boldsymbol{a}_j\| \geq d_0$, $\|\boldsymbol{x}_i - \boldsymbol{x}_k\| \geq d_0$) from the transmitting sensor, γ is the PLE between sensors i and j, and n_{ij} and n_{ik} are the log-normal shadowing terms modeled as $n_{ij} \sim \mathcal{N}(0, \sigma_{n_{ij}}^2)$, $n_{ik} \sim \mathcal{N}(0, \sigma_{n_{ik}}^2)$. For the sake of simplicity and without loss of generality, symmetric target/target RSS measurements are assumed[1], *i.e.*, $P_{ik}^{\mathcal{T}} = P_{ki}^{\mathcal{T}}, \forall (i,k) \in \mathcal{E}_{\mathcal{T}}, i \neq k$.

It is assumed that methods like the ones discussed in Section 2.3.4 are used to obtain the AoA measurements (in 3-D, both azimuth and elevation angles). Before the AoA measurements from different anchors can be used, digital compasses need to be implemented in each anchor, since the alignment information is needed [73, 75]. However, a digital compass introduces an error in the AoA measurements due to its static accuracy. For the sake of simplicity and without loss of generality, the angle measurement error and the alignment error are modeled as one random variable.

[1] It is readily seen that, if $P_{ik}^{\mathcal{T}} \neq P_{ki}^{\mathcal{T}}$, then it is enough to replace $P_{ik}^{\mathcal{T}} \leftarrow (P_{ik}^{\mathcal{T}} + P_{ki}^{\mathcal{T}})/2$ and $P_{ki}^{\mathcal{T}} \leftarrow (P_{ik}^{\mathcal{T}} + P_{ki}^{\mathcal{T}})/2$ when solving the localization problem.

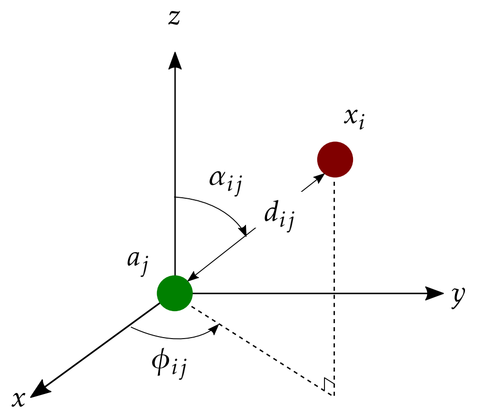


Figure 3.1: Illustration of a target's relative distance and angles to an anchor.

Figure 3.1 gives an representation of a target and an anchor locations in a 3-D space. We can define $\boldsymbol{x}_i = [x_{ix}, x_{iy}, x_{iz}]^T$ as the unknown coordinates of the i-th target, and $\boldsymbol{a}_j = [a_{jx}, a_{jy}, a_{jz}]^T$ as the known coordinates of the j-th anchor. Additionally, $d_{ij}^{\mathcal{A}}$, $\phi_{ij}^{\mathcal{A}}$ and $\alpha_{ij}^{\mathcal{A}}$ represent the distance, azimuth angle and elevation angle between the i-th target and the j-th anchor, respectively. As stated in [49] it is possible to obtain the ML estimate of the distance between two sensors from the RSS measurement model (3.1) in the following way:

$$\widehat{d_{ij}} = \begin{cases} d_0 10^{\frac{P_{0i} - P_{ij}^{\mathcal{A}}}{10\gamma}}, & \text{if } j \in \mathcal{A}, \\ d_0 10^{\frac{P_{0i} - P_{ij}^{\mathcal{T}}}{10\gamma}}, & \text{if } j \in \mathcal{T}. \end{cases} \tag{3.2}$$

We can model the azimuth and elevation angle measurements[2] by applying simple geometry, respectively as [75]:

$$\phi_{ij}^{\mathcal{A}} = \arctan\left(\frac{x_{iy} - a_{jy}}{x_{ix} - a_{jx}}\right) + m_{ij}, \text{ for } (i,j) \in \mathcal{E}_{\mathcal{A}}, \tag{3.3}$$

and

$$\alpha_{ij}^{\mathcal{A}} = \arccos\left(\frac{x_{iz} - a_{jz}}{||\boldsymbol{x}_i - \boldsymbol{a}_j||}\right) + v_{ij}, \text{ for } (i,j) \in \mathcal{E}_{\mathcal{A}}, \tag{3.4}$$

where m_{ij} and v_{ij} are the measurement errors of azimuth and elevation angles, respectively, modeled as $m_{ij} \sim \mathcal{N}(0, \sigma_{m_{ij}}^2)$ and $v_{ij} \sim \mathcal{N}(0, \sigma_{v_{ij}}^2)$.

We can now define the observation vector $\boldsymbol{\theta} = [\boldsymbol{P}^T, \boldsymbol{\phi}^T, \boldsymbol{\alpha}^T]^T$ ($\boldsymbol{\theta} \in \mathbb{R}^{3|\mathcal{E}_{\mathcal{A}}| + |\mathcal{E}_{\mathcal{T}}|}$), where

[2]The authors in [68] concluded that only the anchors need to perform the respective angle measurements as their simulations showed that there is no gain in also having the targets do it, and it would severely raise the overall network implementation costs.

$\boldsymbol{P} = [P_{ij}^{\mathcal{A}}, P_{ik}^{\mathcal{T}}]^T$, $\boldsymbol{\phi} = [\phi_{ij}^{\mathcal{A}}]^T$, $\boldsymbol{\alpha} = [\alpha_{ij}^{\mathcal{A}}]^T$, the conditional PDF is given as:

$$p(\boldsymbol{\theta}|\boldsymbol{X}) = \prod_{i=1}^{3|\mathcal{E}_{\mathcal{A}}|+|\mathcal{E}_{\mathcal{T}}|} \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left(-\frac{(\theta_i - f_i(\boldsymbol{X}))^2}{2\sigma_i^2}\right), \tag{3.5}$$

where

$$\boldsymbol{f}(\boldsymbol{X}) = \begin{bmatrix} \vdots \\ P_{0i} - 10\gamma \log_{10} \frac{\|\boldsymbol{x}_i - \boldsymbol{a}_j\|}{d_0} \\ \vdots \\ P_{0i} - 10\gamma \log_{10} \frac{\|\boldsymbol{x}_i - \boldsymbol{x}_k\|}{d_0} \\ \vdots \\ \arctan\left(\frac{x_{iy} - a_{jy}}{x_{ix} - a_{jx}}\right) \\ \vdots \\ \arccos\left(\frac{x_{iz} - a_{jz}}{\|\boldsymbol{x}_i - \boldsymbol{a}_j\|}\right) \\ \vdots \end{bmatrix}, \ \boldsymbol{\sigma} = \begin{bmatrix} \vdots \\ \sigma_{n_{ij}} \\ \vdots \\ \sigma_{n_{ik}} \\ \vdots \\ \sigma_{m_{ij}} \\ \vdots \\ \sigma_{v_{ij}} \\ \vdots \end{bmatrix}.$$

Maximizing the log of the likelihood function (3.5) with respect to $\boldsymbol{X}$ gives us the ML estimate, $\hat{\boldsymbol{X}}$, of the unknown locations [33], or equivalently:

$$\hat{\boldsymbol{X}} = \arg\min_{\boldsymbol{X}} \sum_{i=1}^{3|\mathcal{E}_{\mathcal{A}}|+|\mathcal{E}_{\mathcal{T}}|} \frac{1}{\sigma_i^2} \left[\theta_i - f_i(\boldsymbol{X})\right]^2. \tag{3.6}$$

The ML estimator in (3.6) has the property of being asymptotically efficient, *i.e.*, for large data records it approximates the minimum variance estimator [33]. Solving (3.6), however, is not possible, since (3.6) is non-convex and has no closed-form solution. Nevertheless, the LS problem in (3.6) can be solved in a distributed manner by applying certain approximations. The authors [68] proposed a convex relaxation technique leading to a distributed SOCP estimator that can be solved efficiently by interior-point algorithms [11], and a suboptimal estimator based on the GTRS framework leading to a distributed SR-WLS estimator, which can be solved *exactly* by a bisection procedure [7]. It was also showed that the proposed SOCP estimator can be generalized to solve the localization problem in (3.6) where, besides the target locations, their transmit powers are different and unknown.

3.2.1 Assumptions

The authors in [68] made some assumptions for the WSN (made for the sake of simplicity and without loss of generality), which are enumerated here for the sake of completeness:

(1) The network is connected and it does not change during the computation period[3];

[3] We address the scenario where this is not the case in Section 3.4

(2) Measurement errors for RSS and AoA models are independent, and $\sigma_{n_{ij}} = \sigma_n$, $\sigma_{m_{ij}} = \sigma_m$ and $\sigma_{v_{ij}} = \sigma_v$, $\forall (i,j) \in \mathcal{E}_{\mathcal{A}} \cup \mathcal{E}_{\mathcal{T}}$;

(3) The additional hardware for collecting the AoA measurements is installed at anchors exclusively;

(4) A coloring scheme of the network is available.

In assumption (1), it is assumed that the sensors are static and that there is no sensor/link failure during the computation period, and that there exists a path between any two sensors $i, j \in \mathcal{V}$. Assumption (2) is made for the sake of simplicity. Assumption (3) indicates that only anchors are suitably equipped to acquire the AoA measurements, due to network costs. Finally, assumption (4) implies that a coloring scheme is available in order to color (number) the sensors and establish a working hierarchy in the network. More precisely, the use of a second-order coloring scheme is assumed, meaning that no sensor has the same color (number) as any of its one-hop neighbors nor its two-hop neighbors [20]-[67], with this approach, energy is saved by avoiding message collision, and the execution time of the algorithm is reduced, since sensors with the same color can work in parallel[4].

3.3 Distributed Localization

Note that the problem in (3.6) only depends on the locations and pairwise measurements between the adjacent sensors. Thereby, assuming that estimations for the initial location of the targets are available, $\hat{\boldsymbol{X}}^{(0)}$, the problem in (3.6) can be divided, *i.e.*, the minimization can be performed independently by each target using only the information gathered from its neighbors. Hence, rather than solving (3.6) directly, which can be computationally exhausting (in large-scale WSNs), the problem is broken (3.6) into sub-problems, which can be solved locally (by each target) using an iterative approach. Consequently, target i updates its location estimate in each iteration, t, by solving the following local ML problem:

$$\hat{\boldsymbol{x}}_i^{(t+1)} = \arg\min_{\boldsymbol{x}_i} \sum_{j=1}^{3|\mathcal{E}_{\mathcal{A}_i}|+|\mathcal{E}_{\mathcal{T}_i}|} \frac{1}{\sigma_j^2}\left[\theta_j - f_j(\boldsymbol{x}_i)\right]^2, \ \forall i \in \mathcal{T}, \tag{3.7}$$

where $\mathcal{E}_{\mathcal{A}_i} = \{j : (i,j) \in \mathcal{E}_{\mathcal{A}}\}$ and $\mathcal{E}_{\mathcal{T}_i} = \{k : (i,k) \in \mathcal{E}_{\mathcal{T}}, i \neq k\}$ represent the set of all anchor and all target neighbors of the target i respectively, and the first $|\mathcal{E}_{\mathcal{A}_i}| + |\mathcal{E}_{\mathcal{T}_i}|$ elements of $f_j(\boldsymbol{x}_i)$ are given as:

[4]Note that the network coloring problem may be considered as an optimization problem where the goal is to minimize the number of different colors used. In graph theory, this is called a minimum vertex coloring and in particular the scheme used in this dissertation is a L(2,1)-coloring [13]. The minimum number of colors itself is called the chromatic number of a graph, and to find it is a NP-complete problem [32]. While in practice integer numbers are used instead of actual colors, the name traces historically to the problem of coloring countries in map. More recently, the problem reached popularity in the form of a game named Sudoku.

$$f_j(\boldsymbol{x}_i) = P_{0i} - 10\gamma \log_{10} \frac{\|\boldsymbol{x}_i - \hat{\boldsymbol{a}}_j\|}{d_0}, \text{ for } j = 1, ..., |\mathcal{E}_{\mathcal{A}_i}| + |\mathcal{E}_{\mathcal{T}_i}|,$$

with

$$\hat{\boldsymbol{a}}_j = \begin{cases} \boldsymbol{a}_j, & \text{if } j \in \mathcal{A}, \\ \hat{\boldsymbol{x}}_j^{(t)}, & \text{if } j \in \mathcal{T}. \end{cases}$$

3.3.1 Known P_{0i}'s

Method 1: Distributed SOCP Algorithm

Given $\hat{\boldsymbol{X}}^{(0)}$, and if the noise power is sufficiently small, from (3.1) we can write:

$$\lambda_{ij}\|\boldsymbol{x}_i - \hat{\boldsymbol{a}}_j\| \approx d_0, \ \forall i \in \mathcal{T}, \forall j \in \mathcal{E}_{\mathcal{A}_i} \cup \mathcal{E}_{\mathcal{T}_i}, \tag{3.8}$$

where

$$\lambda_{ij} = \begin{cases} 10^{\frac{P_{ij}^{\mathcal{A}} - P_{0i}}{10\gamma}}, & \text{if } j \in \mathcal{A}, \\ 10^{\frac{P_{ij}^{\mathcal{T}} - P_{0i}}{10\gamma}}, & \text{if } j \in \mathcal{T}. \end{cases}$$

Likewise, from (3.3) and (3.4) we respectively get:

$$\boldsymbol{c}_{ij}^T(\boldsymbol{x}_i - \boldsymbol{a}_j) \approx 0, \ \forall i \in \mathcal{T}, \forall j \in \mathcal{E}_{\mathcal{A}_i} \tag{3.9}$$

and

$$\boldsymbol{k}_{ij}^T(\boldsymbol{x}_i - \boldsymbol{a}_j) \approx \|\boldsymbol{x}_i - \boldsymbol{a}_j\|\cos(\alpha_{ij}^{\mathcal{A}}), \ \forall i \in \mathcal{T}, \forall j \in \mathcal{E}_{\mathcal{A}_i} \tag{3.10}$$

where $\boldsymbol{c}_{ij} = [-\sin(\phi_{ij}^{\mathcal{A}}), \cos(\phi_{ij}^{\mathcal{A}}), 0]^T$ and $\boldsymbol{k}_{ij} = [0, 0, 1]^T$. According to the LS criterion and (3.8), (3.9) and (3.10) each target updates its location by solving the following problem:

$$\begin{aligned} \hat{\boldsymbol{x}}_i^{(t+1)} = \arg\min_{\boldsymbol{x}_i} \sum_{j \in \mathcal{E}_{\mathcal{A}_i} \cup \mathcal{E}_{\mathcal{T}_i}} \left(\lambda_{ij}\|\boldsymbol{x}_i - \hat{\boldsymbol{a}}_j\| - d_0\right)^2 + \sum_{j \in \mathcal{E}_{\mathcal{A}_i}} \left(\boldsymbol{c}_{ij}^T(\boldsymbol{x}_i - \boldsymbol{a}_j)\right)^2 \\ + \sum_{j \in \mathcal{E}_{\mathcal{A}_i}} \left(\boldsymbol{k}_{ij}^T(\boldsymbol{x}_i - \boldsymbol{a}_j) - \|\boldsymbol{x}_i - \boldsymbol{a}_j\|\cos(\alpha_{ij}^{\mathcal{A}})\right)^2. \end{aligned} \tag{3.11}$$

However, the LS problem in (3.11) is still non-convex and as consequence has no closed-form solution. To convert (3.11) into a convex problem, we introduce auxiliary variables $r_{ij} = \|\boldsymbol{x}_i - \hat{\boldsymbol{a}}_j\|, \forall (i,j) \in \mathcal{E}_{\mathcal{A}} \cup \mathcal{E}_{\mathcal{T}}$, $\boldsymbol{z} = [z_{ij}]$, $\boldsymbol{g} = [g_{ij}]$, $\boldsymbol{p} = [p_{ij}]$, where $z_{ij} = \lambda_{ij}^{\mathcal{A}} r_{ij} - d_0, \forall (i,j) \in \mathcal{E}_{\mathcal{A}} \cup \mathcal{E}_{\mathcal{T}}$, $g_{ij} = \boldsymbol{c}_{ij}^T(\boldsymbol{x}_i - \boldsymbol{a}_j)$, and $p_{ij} = \boldsymbol{k}_{ij}^T(\boldsymbol{x}_i - \boldsymbol{a}_j) - r_{ij}\cos(\alpha_{ij}^{\mathcal{A}}), \forall (i,j) \in \mathcal{E}_{\mathcal{A}}$. We get:

$$\underset{\boldsymbol{x}_i, \boldsymbol{r}, \boldsymbol{z}, \boldsymbol{g}, \boldsymbol{p}}{\text{minimize}} \ \|\boldsymbol{z}\|^2 + \|\boldsymbol{g}\|^2 + \|\boldsymbol{p}\|^2$$

subject to

$$\begin{aligned} r_{ij} &= \|\boldsymbol{x}_i - \hat{\boldsymbol{a}}_j\|, \ \forall (i,j) \in \mathcal{E}_{\mathcal{A}} \cup \mathcal{E}_{\mathcal{T}}, \\ z_{ij} &= \lambda_{ij} r_{ij} - d_0, \ \forall (i,j) \in \mathcal{E}_{\mathcal{A}} \cup \mathcal{E}_{\mathcal{T}}, \\ g_{ij} &= \boldsymbol{c}_{ij}^T(\boldsymbol{x}_i - \boldsymbol{a}_j), \ \forall (i,j) \in \mathcal{E}_{\mathcal{A}}, \\ p_{ij} &= \boldsymbol{k}_{ij}^T(\boldsymbol{x}_i - \boldsymbol{a}_j) - r_{ij}\cos(\alpha_{ij}^{\mathcal{A}}), \ \forall (i,j) \in \mathcal{E}_{\mathcal{A}}. \end{aligned} \tag{3.12}$$

Additionally introducing epigraph variables e_1, e_2 and e_3, as well as applying a second-order cone constraint relaxation of the form $||\boldsymbol{z}||^2 \leq e_1$, we obtain:

$$\underset{\boldsymbol{x}_i,\boldsymbol{r},\boldsymbol{z},\boldsymbol{g},\boldsymbol{p},e_1,e_2,e_3}{\text{minimize}} \quad e_1 + e_2 + e_3$$

subject to

$$\begin{gathered}
||\boldsymbol{x}_i - \hat{\boldsymbol{a}}_j|| \leq r_{ij},\ \forall(i,j) \in \mathcal{E}_{\mathcal{A}} \cup \mathcal{E}_{\mathcal{T}}, \\
z_{ij} = \lambda_{ij} r_{ij} - d_0,\ \forall(i,j) \in \mathcal{E}_{\mathcal{A}} \cup \mathcal{E}_{\mathcal{T}}, \\
g_{ij} = \boldsymbol{c}_{ij}^T(\boldsymbol{x}_i - \boldsymbol{a}_j),\ \forall(i,j) \in \mathcal{E}_{\mathcal{A}}, \\
p_{ij} = \boldsymbol{k}_{ij}^T(\boldsymbol{x}_i - \boldsymbol{a}_j) - r_{ij}\cos(\alpha_{ij}^{\mathcal{A}}),\ \forall(i,j) \in \mathcal{E}_{\mathcal{A}}, \\
\left\| \begin{bmatrix} 2\boldsymbol{z} \\ e_1 - 1 \end{bmatrix} \right\| \leq e_1 + 1, \left\| \begin{bmatrix} 2\boldsymbol{g} \\ e_2 - 1 \end{bmatrix} \right\| \leq e_2 + 1, \left\| \begin{bmatrix} 2\boldsymbol{p} \\ e_3 - 1 \end{bmatrix} \right\| \leq e_3 + 1.
\end{gathered} \tag{3.13}$$

The problem in (3.13) is an SOCP problem, which can be efficiently solved by the `CVX` MATLAB package [24] for specifying and solving convex programs. In the further text, we will refer to (3.13) as “SOCP”.

Method 2: Distributed SR-WLS Algorithm

The relation in (3.8) can be rewritten as:

$$\lambda_{ij}^2 ||\boldsymbol{x}_i - \hat{\boldsymbol{a}}_j||^2 \approx d_0^2,\ \forall(i,j) \in \mathcal{E}_{\mathcal{A}} \cup \mathcal{E}_{\mathcal{T}}. \tag{3.14}$$

Weights $\boldsymbol{w} = [\sqrt{w_{ij}}]$ are also introduced as a way of giving more importance to nearby links, where

$$w_{ij} = 1 - \frac{\widehat{d_{ij}}}{\sum_{(i,j)\in\mathcal{E}_{\mathcal{A}}\cup\mathcal{E}_{\mathcal{T}}} \widehat{d_{ij}}}.$$

The relation in (3.10) is also changed, substituting $||\boldsymbol{x}_i - \hat{\boldsymbol{a}}_j||$ with $\widehat{d_{ij}}$ described in (3.2). This way, according to the WLS criterion and (3.14), (3.9) and (3.10) each target updates its location by solving the following problem:

$$\begin{aligned}
\hat{\boldsymbol{x}}_i^{(t+1)} = \arg\min_{\boldsymbol{x}_i} & \sum_{j\in\mathcal{E}_{\mathcal{A}_i}\cup\mathcal{E}_{\mathcal{T}_i}} w_{ij}\left(\lambda_{ij}^2 ||\boldsymbol{x}_i - \hat{\boldsymbol{a}}_j||^2 - d_0^2\right)^2 \\
& + \sum_{j\in\mathcal{E}_{\mathcal{A}_i}} w_{ij}\left(\boldsymbol{c}_{ij}^T(\boldsymbol{x}_i - \boldsymbol{a}_j)\right)^2 \\
& + \sum_{j\in\mathcal{E}_{\mathcal{A}_i}} w_{ij}\left(\boldsymbol{k}_{ij}^T(\boldsymbol{x}_i - \boldsymbol{a}_j) - \widehat{d_{ij}}\cos(\alpha_{ij}^{\mathcal{A}})\right)^2.
\end{aligned} \tag{3.15}$$

Similar to 3.11,the above WLS estimator is also non-convex and has no closed-form solution. Still, this estimator can be expressed (3.15) as a quadratic programming problem whose *global* solution can be computed efficiently [7]. Rewriting it in matrix form and applying the substitution $\boldsymbol{y}_i = [\boldsymbol{x}_i^T, ||\boldsymbol{x}_i||^2]^T, \forall i \in \mathcal{T}$, (3.15), results in:

$$\hat{\boldsymbol{y}}_i^{(t+1)} = \arg\min_{\boldsymbol{y}_i} ||\boldsymbol{W}(\boldsymbol{A}\boldsymbol{y}_i - \boldsymbol{b})||^2$$

subject to

$$\boldsymbol{y}_i^T \boldsymbol{D} \boldsymbol{y}_i + 2\boldsymbol{l}^T \boldsymbol{y}_i = 0, \tag{3.16}$$

where $\boldsymbol{W} = \text{diag}\left(\left[w_{ij \in \mathcal{E}_{\mathcal{A}_i} \cup \mathcal{E}_{\mathcal{T}_i}}, w_{ij \in \mathcal{E}_{\mathcal{A}_i}}, w_{ij \in \mathcal{E}_{\mathcal{A}_i}}\right]\right)$,

$$\boldsymbol{A} = \begin{bmatrix} \vdots & \vdots \\ -2\lambda_{ij}^2 \hat{\boldsymbol{a}}_j^T & \lambda_{ij}^2 \\ \vdots & \vdots \\ \boldsymbol{c}_{ij}^T & 0 \\ \vdots & \vdots \\ \boldsymbol{k}_{ij}^T & 0 \\ \vdots & \vdots \end{bmatrix}, \boldsymbol{b} = \begin{bmatrix} \vdots \\ d_0^2 - \lambda_{ij}^2 \|\hat{\boldsymbol{a}}_j\|^2 \\ \vdots \\ \boldsymbol{c}_{ij}^T \boldsymbol{a}_j \\ \vdots \\ \boldsymbol{k}_{ij}^T \boldsymbol{a}_j + \widehat{d}_{ij}^{\mathcal{A}} \cos(\alpha_{ij}^{\mathcal{A}}) \\ \vdots \end{bmatrix},$$

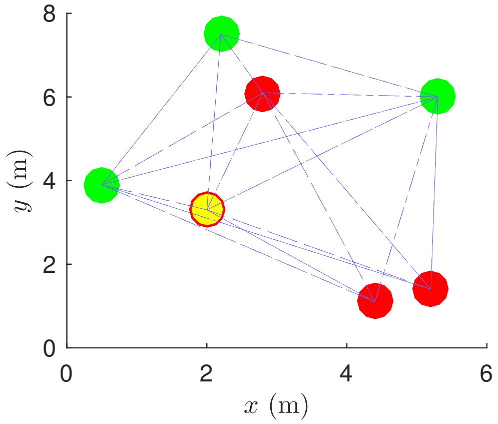
$$\boldsymbol{D} = \begin{bmatrix} \boldsymbol{I}_3 & \boldsymbol{0}_{3\times 1} \\ \boldsymbol{0}_{1\times 3} & 0 \end{bmatrix}, \boldsymbol{l} = \begin{bmatrix} \boldsymbol{0}_{3\times 1} \\ -1/2 \end{bmatrix},$$

i.e., $\boldsymbol{W} \in \mathbb{R}^{3|\mathcal{E}_{\mathcal{A}_i}|+|\mathcal{E}_{\mathcal{T}_i}| \times 3|\mathcal{E}_{\mathcal{A}_i}|+|\mathcal{E}_{\mathcal{T}_i}|}$, $\boldsymbol{A} \in \mathbb{R}^{3|\mathcal{E}_{\mathcal{A}_i}|+|\mathcal{E}_{\mathcal{T}_i}| \times 4}$, and $\boldsymbol{b} \in \mathbb{R}^{3|\mathcal{E}_{\mathcal{A}_i}|+|\mathcal{E}_{\mathcal{T}_i}| \times 1}$.

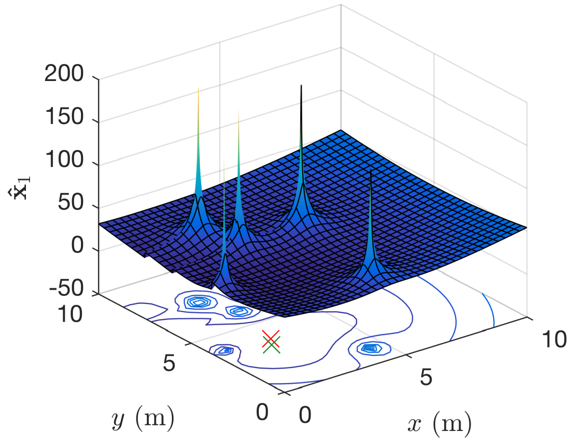
Both the objective function and the constraint in (3.16) are quadratic. A problem with this characteristics is known as GTRS [7, 40], and it can be solved *exactly* by a bisection procedure [7]. We denote (3.16) as "SR-WLS" in the remaining text.

The steps taken to derive both estimators can be summarized in the following way. First, the local non-convex ML estimator in (3.7) is approximated by a different non-convex estimator, (3.11) and (3.15) respectively. Both of the obtained estimators in (3.11) and (3.15) show objective functions with a much smoother surface in comparison to (3.7). This comes at a cost of introducing some bias with respect to the ML solution (see Fig 3.2). However, if the bias effect is small, employing a local search around the solution of (3.11) and (3.15) might be enough to reach the ML solution. The second step consist in converting (3.11) and (3.15) into a convex problem and GTRS framework, by employing the respective above procedures.

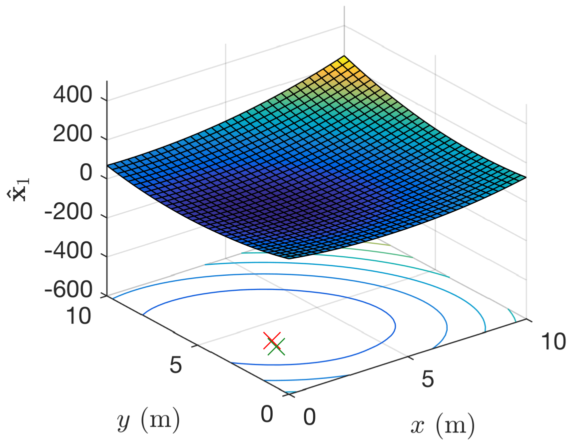
Fig. 3.2 (a) illustrates a sensor node performing self-localization in a 2-D WSN using RSS and AoA measurements. In (b) it is possible to observe a representation of the objective function in (3.7) where the true sensors' locations were used. In (c) and (d), the objective functions (3.11) and (3.15) are represented after only one iteration, and the estimated targets' locations were used. The target's real location was set to [2.0; 3.3] and was capable of communicating directly with three anchors and three targets. The noise standard deviation (STD) of RSS measurements was set to $\sigma_{n_{ij}} = 3$ dB and the noise STD of angle measurements was set to $\sigma_{m_{ij}} = 6$ deg, and the rest of the parameters follow the set-up described in Chapter 4. It is possible to observe, that in (b), the objective function is highly non-convex and its global minimum is located at [2.26; 3.69]. Due to non-convexity of the problem, recursive algorithms, such as gradient search method, might get trapped into a local minimum, causing large error in the location estimation process. On the



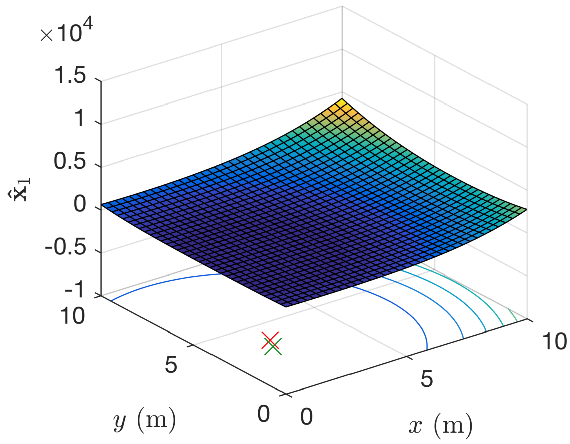
(a) Yellow sensor performing localization.



(b) Objective function in (3.7).



(c) Objective function in (3.11).



(d) Objective function in (3.15).

Figure 3.2: (a) Illustration of a sensor (in yellow) performing self-localization in a 2-D WSN (green dots are anchors, red dots are targets). RSS and AoA measurements are used. Figures (b) (c) and (d) show the respective objective functions in (3.7), (3.11) and (3.15) (minus the elevation angle terms) versus x (m) and y (m). The yellow sensor's real coordinates are projected, in the contours part of the plot, as a green cross in (b) (c) (d), each objective function's minimum is projected as a red cross.

other hand, it can be seen that the objective functions in (c) and (d) are much smoother. The estimated target's coordinates were obtained by solving the "SOCP" and "SR-WLS" algorithms, in (c) and (d) respectively, and their respective minimums are located in $[2.17; 3.77]$ and $[2.24; 3.93]$. This algorithms allow to obtain their respective objective functions' global minimums effortlessly far all targets via interior-point algorithms [11] and bisection procedure [7]. While the estimation accuracy depends on the tightness of the performed relaxation, we can conclude that the objective functions in (3.11) and (3.15) are excellent approximations of the original problem in (3.7) as it is shown in Chapter 4.

The authors in [68] proposed Algorithm 1, which summarizes the distributed SOCP and SR-WLS algorithms, where T_{max} is the maximum number of iterations and $\mathcal{C}$ the set of used colors in the coloring scheme. Algorithm 1 is distributed in the sense that there is no central processor in the network, its coordination is carried out according to the applied coloring scheme, information exchange occurs between two incident sensors exclusively, and data processing is performed locally by each target. Lines $5-7$ are executed simultaneously by all targets $i \in \mathcal{C}_c$, which may decrease the execution time of the algorithm. At Line 6, (3.13) is solved if the SOCP algorithm is employed, or (3.16) if SR-WLS algorithm is employed. Targets broadcast their location updates $\hat{\boldsymbol{x}}_i^{(t+1)}$ to their neighbors at Line 7, making this the sole information exchange phase of the algorithm. One can conclude that since $\hat{\boldsymbol{x}}_i^{(t+1)} \in \mathbb{R}^3$, the algorithm requires at most a broadcast of $3 \times T_{\max} \times M$ real values. Depending on which estimator is employed, in the remaining text, we label Algorithm 1 either as "SOCP" or as "SR-WLS".

Algorithm 1 The distributed SOCP/SR-WLS algorithm

Require: $\hat{\boldsymbol{X}}^{(0)}$, $T_{\max}$, $\mathcal{C}$, $\boldsymbol{a}_j$, $\forall j \in \mathcal{A}$
1: **Initialize:** $t \leftarrow 0$
2: **repeat**
3: **for** $c = 1, \ldots, \mathcal{C}$ **do**
4: **for all** $i \in \mathcal{C}_c$ (in parallel) **do**
5: Collect $\hat{\boldsymbol{a}}_j, \forall j \in \mathcal{E}_{\mathcal{A}_i} \cup \mathcal{E}_{\mathcal{T}_i}$
6: $\hat{\boldsymbol{x}}_i^{(t+1)} \leftarrow \begin{cases} \text{solve (3.13)}, & \text{if using SOCP algorithm,} \\ \text{solve (3.16)}, & \text{if using SR-WLS algorithm} \end{cases}$
7: Broadcast $\hat{\boldsymbol{x}}_i^{(t+1)}$ to $\hat{\boldsymbol{a}}_j, \forall j \in \mathcal{E}_{\mathcal{A}_i} \cup \mathcal{E}_{\mathcal{T}_i}$
8: **end for**
9: **end for**
10: $t \leftarrow t + 1$
11: **until** $t < T_{\max}$

3.3.2 Unknown P_{0i}'s

Antenna testing and calibration are often in practice not the priority in order to restrict the implementation costs. Furthermore, due to battery exhaustion with time, sensors' transmit powers, P_i's, might change over time. Therefore, P_i's are often not calibrated, *i.e.*, not known. Not knowing P_i implies that P_{0i} is not known in the RSS model (3.1); see [49] and the references therein.

The authors in [68] generalized the SOCP estimator for known P_{0i} for the case where P_{0i} is not known. More specifically, it is possible to rewrite (3.8) as follows:

$$\zeta_{ij}\|\boldsymbol{x}_i - \hat{\boldsymbol{a}}_j\| \approx \eta_i d_0, \forall i \in \mathcal{T}, \forall j \in \mathcal{E}_{\mathcal{A}} \cup \mathcal{E}_{\mathcal{T}}, \tag{3.17}$$

where $\eta_i = 10^{\frac{P_{0i}}{10\gamma}}$ and

$$\zeta_{ij} = \begin{cases} 10^{\frac{P_{ij}^{\mathcal{A}}}{10\gamma}}, & \text{if } j \in \mathcal{A}, \\ 10^{\frac{P_{ij}^{\mathcal{T}}}{10\gamma}}, & \text{if } j \in \mathcal{T}. \end{cases}$$

Employing the LS approach using (3.17), (3.9) and (3.10), each target updates its location by solving the following problem:

$$\begin{aligned} \left(\hat{\boldsymbol{x}}_i^{(t+1)}, \eta_i\right) = \underset{\boldsymbol{x}_i,\eta_i}{\arg\min} \sum_{j\in\mathcal{E}_{\mathcal{A}_i}\cup\mathcal{E}_{\mathcal{T}_i}} \left(\zeta_{ij}\|\boldsymbol{x}_i - \hat{\boldsymbol{a}}_j\| - \eta_i d_0\right)^2 \\ + \textstyle\sum_{j\in\mathcal{E}_{\mathcal{A}_i}} \left(\boldsymbol{c}_{ij}^T(\boldsymbol{x}_i - \boldsymbol{a}_j)\right)^2 + \sum_{j\in\mathcal{E}_{\mathcal{A}_i}} \left(\boldsymbol{k}_{ij}^T(\boldsymbol{x}_i - \boldsymbol{a}_j) - \|\boldsymbol{x}_i - \boldsymbol{a}_j\|\cos(\alpha_{ij}^{\mathcal{A}})\right)^2. \end{aligned} \tag{3.18}$$

In a similar way to what is done in Section 3.3.1, we can obtain the following SOCP estimator:

$$\underset{\boldsymbol{x}_i,\eta_i,\boldsymbol{r},\boldsymbol{z},\boldsymbol{g},\boldsymbol{p},e_1,e_2,e_3}{\text{minimize}} \quad e_1 + e_2 + e_3$$

subject to

$$\begin{gathered} \|\boldsymbol{x}_i - \hat{\boldsymbol{a}}_j\| \le r_{ij}, \ \forall (i,j) \in \mathcal{E}_{\mathcal{A}} \cup \mathcal{E}_{\mathcal{T}}, \\ z_{ij} = \zeta_{ij} r_{ij} - \eta_i d_0, \ \forall (i,j) \in \mathcal{E}_{\mathcal{A}} \cup \mathcal{E}_{\mathcal{T}}, \\ g_{ij} = \boldsymbol{c}_{ij}^T(\boldsymbol{x}_i - \boldsymbol{a}_j), \ \forall (i,j) \in \mathcal{E}_{\mathcal{A}}, \\ p_{ij} = \boldsymbol{k}_{ij}^T(\boldsymbol{x}_i - \boldsymbol{a}_j) - r_{ij}\cos(\alpha_{ij}^{\mathcal{A}}), \ \forall (i,j) \in \mathcal{E}_{\mathcal{A}}, \\ \left\|\begin{bmatrix} 2\boldsymbol{z} \\ e_1 - 1 \end{bmatrix}\right\| \le e_1 + 1, \left\|\begin{bmatrix} 2\boldsymbol{g} \\ e_2 - 1 \end{bmatrix}\right\| \le e_2 + 1, \left\|\begin{bmatrix} 2\boldsymbol{p} \\ e_3 - 1 \end{bmatrix}\right\| \le e_3 + 1. \end{gathered} \tag{3.19}$$

The problem in (19) is a classical SOCP, where the objective function and equality constraints are affine, and the inequality constraints are second-order cone constraints [11].

The SOCP algorithm for unknown P_i's is summarized in Algorithm 2. All targets $i \in \mathcal{C}_c$ may run Lines 5 − 10 concurrently, which might reduce the running time of the algorithm. At Line 6, (3.19) is solved a number of times equal to S, after which the ML estimate of P_{0i}, $\widehat{P_{0i}}$, starts to be calculated. From this moment targets switch to solving (3.13) as if P_{0i} is known. Line 7 is introduced to avoid the oscillation in the location estimates. At Line 10, the location updates, $\hat{\boldsymbol{x}}_i^{(t+1)} \forall i \in \mathcal{T}$, are broadcasted to neighbors of i. In the remaining text, we label Algorithm 2 as "uSOCP".

3.4 Link Failure Scenario

For the purpose of evaluating the effects of communication failure between nodes of the network, it is necessary to redesign the algorithms proposed in [68] to reflect such

Algorithm 2 The distributed uSOCP algorithm

Require: $\hat{\boldsymbol{x}}_i^{(0)}, \forall i \in \mathcal{T}, \boldsymbol{a}_j, \forall j \in \mathcal{A}, \mathcal{C}, S, P_0^{\text{Low}}, P_0^{\text{Up}}, T_{\max}$

Initialize: $t \leftarrow 0$
repeat
  for $c = 1, \ldots, \mathcal{C}$ **do**
    for all $i \in \mathcal{C}_c$ (in parallel) **do**
      Collect $\hat{\boldsymbol{a}}_j, \forall j \in \mathcal{E}_{\mathcal{A}_i} \cup \mathcal{E}_{\mathcal{T}_i}$
      $\hat{\boldsymbol{x}}_i^{(t+1)} \leftarrow \begin{cases} \text{solve (3.19)}, & \text{if } t < S, \\ \text{solve (3.13) using } \widehat{P_{0i}}, & \text{if } t \geq S \end{cases}$
      if $\frac{\|\hat{\boldsymbol{x}}_i^{(t+1)} - \hat{\boldsymbol{x}}_i^{(t)}\|}{\|\hat{\boldsymbol{x}}_i^{(t)}\|} > 1$ **then**
        $\hat{\boldsymbol{x}}_i^{(t+1)} \leftarrow \hat{\boldsymbol{x}}_i^{(t)}$
      end if
      Broadcast $\hat{\boldsymbol{x}}_i^{(t+1)}$ to $\hat{\boldsymbol{a}}_j, \forall j \in \mathcal{E}_{\mathcal{A}_i} \cup \mathcal{E}_{\mathcal{T}_i}$
    end for
  end for
  $t \leftarrow t + 1$
  if $t > S$ **then**
    for all $i \in \mathcal{T}$ (in parallel) **do**
      $\widehat{P_{0i}} \leftarrow \frac{\sum_{j \in \mathcal{E}_{\mathcal{A}_i} \cup \mathcal{E}_{\mathcal{T}_i}} P_{ij} + 10\gamma \log_{10} \frac{\|\hat{x}^{(t)} - \hat{\boldsymbol{a}}_j\|}{d_0}}{|\mathcal{E}_{\mathcal{A}_i}| + |\mathcal{E}_{\mathcal{T}_i}|}$
      if $\widehat{P_{0i}} < P_0^{\text{Low}}$ **then**
        $\widehat{P_{0i}} \leftarrow P_0^{\text{Low}}$
      else if $\widehat{P_{0i}} > P_0^{\text{Up}}$ **then**
        $\widehat{P_{0i}} \leftarrow P_0^{\text{Up}}$
      end if
    end for
  end if
until $t < T_{\max}$

scenario. Algorithms 3 and 4 are the updated versions of Algorithms 1 and 2 respectively, where P_f is the probability of link failure. A probability of information exchange failure between two incident sensors is added in the form of conditional statement, as can be seen in Lines 7 – 12 and 10 – 15 of Algorithms 3 and 4, respectively. A sample from a standard uniform distribution is taken each time a sensor tries to send its updated location estimate to one of its neighbors. That sample is then compared to a certain probability fixed at start of the computational period. This comparison decides if the information dissemination succeeds or not, to that particular neighbor.

3.5 Complexity Analysis

The overall performance of a localization algorithm can be inferred from the trade off analysis between the estimation accuracy and computational complexity. In this section, the computational complexity of the considered algorithms is presented.

Algorithm 3 The proposed change to the distributed SOCP/SR-WLS algorithm

Require: $\hat{\boldsymbol{X}}^{(0)}$, $T_{\max}$, $\mathcal{C}$, u, P_f, $\boldsymbol{a}_j$, $\forall j \in \mathcal{A}$

Initialize: $t \leftarrow 0$

repeat

  for $c = 1, \ldots, \mathcal{C}$ **do**

    for all $i \in \mathcal{C}_c$ (in parallel) **do**

      Collect $\hat{\boldsymbol{a}}_j, \forall j \in \mathcal{E}_{\mathcal{A}_i} \cup \mathcal{E}_{\mathcal{T}_i}$

      $\hat{\boldsymbol{x}}_i^{(t+1)} \leftarrow \begin{cases} \text{solve } (3.13), & \text{if using SOCP algorithm,} \\ \text{solve } (3.16), & \text{if using SR-WLS algorithm} \end{cases}$

      for all $j \in \mathcal{E}_{\mathcal{A}_i} \cup \mathcal{E}_{\mathcal{T}_i}$ **do**

        $u \leftarrow \mathcal{U}(0,1)$

        if $u > P_f$ **then**

          Broadcast $\hat{\boldsymbol{x}}_i^{(t+1)}$ to $\hat{\boldsymbol{a}}_j, \forall j \in \mathcal{E}_{\mathcal{A}_i} \cup \mathcal{E}_{\mathcal{T}_i}$

        end if

      end for

    end for

  end for

  $t \leftarrow t + 1$

until $t < T_{\max}$

According to [51], the worst case computational complexity of an SOCP is:

$$\mathcal{O}\left(\sqrt{L}\left(m^2 \sum_{i=1}^{L} n_i + \sum_{i=1}^{L} n_i^2 + m^3\right)\right), \tag{3.20}$$

where L is the number of the second-order cone constraints, m is the number of the equality constraints, and n_i is the dimension of the i-th second-order cone.

Assuming that $N_{\max}$ is the maximum number of steps in the bisection procedure, Table 3.1 provides a summary of the worst case computational complexities of the considered algorithms.

It is possible to conclude from Table 3.1 that the size of neighborhood fragments have a larger weight on the computational complexity of a distributed algorithm compared to the total number of sensors in a WSN. Having a fully connected network, *i.e.*, $|\mathcal{E}_{\mathcal{A}_i}| + |\mathcal{E}_{\mathcal{T}_i}| = M + N - 1, \forall i \in \mathcal{T}$, is theoretically possible. In practice however, the size of the neighborhood fragments are much smaller, due to energy restrictions (limited R). This makes distributed algorithms a preferable solution in large-scale and highly-dense networks, since adding more sensors in the network will not have a severe impact on the size of neighborhood fragments. As anticipated, Table 3.1 also shows that the distributed SOCP algorithms are computationally more demanding than the SR-WLS one. This result is not surprising, since the SOCP approach employs sophisticated mathematical tools, whereas the SR-WLS approach applies the bisection procedure to solve the localization problem. Nevertheless, higher complexity of the proposed SOCP algorithms is justified by their superior performance in terms of the estimation accuracy and convergence, as we will see in Chapter 4.

Algorithm 4 The proposed change to the distributed uSOCP algorithm

Require: $\hat{\boldsymbol{x}}_i^{(0)}$, $\forall i \in \mathcal{T}$, $\boldsymbol{a}_j$, $\forall j \in \mathcal{A}$, u, P_f, $\mathcal{C}$, S, P_0^{Low}, P_0^{Up}, $T_{\max}$

Initialize: $t \leftarrow 0$
repeat
  for $c = 1, \ldots, \mathcal{C}$ **do**
    for all $i \in \mathcal{C}_c$ (in parallel) **do**
      Collect $\hat{\boldsymbol{a}}_j, \forall j \in \mathcal{E}_{\mathcal{A}_i} \cup \mathcal{E}_{\mathcal{T}_i}$
      $\hat{\boldsymbol{x}}_i^{(t+1)} \leftarrow \begin{cases} \text{solve (3.19)}, & \text{if } t < S, \\ \text{solve (3.13) using } \widehat{P_{0i}}, & \text{if } t \geq S \end{cases}$
      if $\frac{\|\hat{\boldsymbol{x}}_i^{(t+1)} - \hat{\boldsymbol{x}}_i^{(t)}\|}{\|\hat{\boldsymbol{x}}_i^{(t)}\|} > 1$ **then**
        $\hat{\boldsymbol{x}}_i^{(t+1)} \leftarrow \hat{\boldsymbol{x}}_i^{(t)}$
      end if
      for all $j \in \mathcal{E}_{\mathcal{A}_i} \cup \mathcal{E}_{\mathcal{T}_i}$ **do**
        $u \leftarrow \mathcal{U}(0,1)$
        if $u > P_f$ **then**
          Broadcast $\hat{\boldsymbol{x}}_i^{(t+1)}$ to $\hat{\boldsymbol{a}}_j, \forall j \in \mathcal{E}_{\mathcal{A}_i} \cup \mathcal{E}_{\mathcal{T}_i}$
        end if
      end for
    end for
  end for
  $t \leftarrow t + 1$
  if $t > S$ **then**
    for all $i \in \mathcal{T}$ (in parallel) **do**
      $\widehat{P_{0i}} \leftarrow \frac{\sum_{j \in \mathcal{E}_{\mathcal{A}_i} \cup \mathcal{E}_{\mathcal{T}_i}} P_{ij} + 10\gamma \log_{10} \frac{\|\hat{\boldsymbol{x}}^{(t)} - \hat{\boldsymbol{a}}_j\|}{d_0}}{|\mathcal{E}_{\mathcal{A}_i}| + |\mathcal{E}_{\mathcal{T}_i}|}$
      if $\widehat{P_{0i}} < P_0^{\text{Low}}$ **then**
        $\widehat{P_{0i}} \leftarrow P_0^{\text{Low}}$
      else if $\widehat{P_{0i}} > P_0^{\text{Up}}$ **then**
        $\widehat{P_{0i}} \leftarrow P_0^{\text{Up}}$
      end if
    end for
  end if
until $t < T_{\max}$

Table 3.1: Computational complexity of the considered algorithms

Algorithm	Complexity
SOCP	$T_{\max} \times M \times \mathcal{O}\left(\left(\max_i \left\{3|\mathcal{E}_{\mathcal{A}_i}| + |\mathcal{E}_{\mathcal{T}_i}|\right\}\right)^{3.5}\right)$
SR-WLS	$T_{\max} \times M \times \mathcal{O}\left(N_{\max} \times, \max_i \left\{3|\mathcal{E}_{\mathcal{A}_i}| + |\mathcal{E}_{\mathcal{T}_i}|\right\}\right)$
uSOCP	$T_{\max} \times M \times \mathcal{O}\left(\left(\max_i \left\{3|\mathcal{E}_{\mathcal{A}_i}| + |\mathcal{E}_{\mathcal{T}_i}|\right\}\right)^{3.5}\right)$

Chapter 4

Computational experiments

4.1 Introduction

In this chapter the aspects of the performed numerical simulations are clarified first. Next, a set of results that confirms the performance of the recently proposed algorithms in [68] are presented. Finally the impact that the introduced changes have on each algorithm's convergence is illustrated. More precisely, the algorithms' behavior is investigated for the cases when 50%, 70% and 90% of link failure probability is present.

To study each of the algorithms presented in Chapter 3, various MATLAB® scripts were written. All of the algorithms based on SOCP were solved by using the MATLAB package `CVX` [24] where the solver is SeDuMi [64].

4.2 Framework

A set of computational experiments based on the Monte Carlo (M_c) method were executed, allowing to obtain a statistical interpretation of the algorithms' performance.

Initially, $M_c = 500$ random sensor locations were generated inside a cubic area of side $B = 20$. Only networks that formed a connected graph were considered. The reference distance was set to $d_0 = 1$ m and the communication range of each sensor to $R = 6.5$ m. For each Monte Carlo (M_c) run, a number of M sensors were defined as targets while the remaining N were set as anchors. Figure 4.1 illustrates one such generated networks. The reference power for each sensor was sampled from a uniform distribution on an interval $[P_0^{\text{Low}}, P_0^{\text{Up}}]$, i.e., $P_{0i} \sim \mathcal{U}[P_0^{\text{Low}}, P_0^{\text{Up}}]$ dBm. In order to account for a realistic measurement model mismatch and since in reality knowledge of the PLE is imperfect, the true PLE was drawn from $\gamma_{ij} \sim \mathcal{U}[2.7, 3.3], \forall (i,j) \in \mathcal{E}_{\mathcal{A}} \cup \mathcal{E}_{\mathcal{T}}, i \neq j$, while the assumed value of the PLE within the estimation process was fixed to $\gamma = 3$. Noise realizations between any pair of

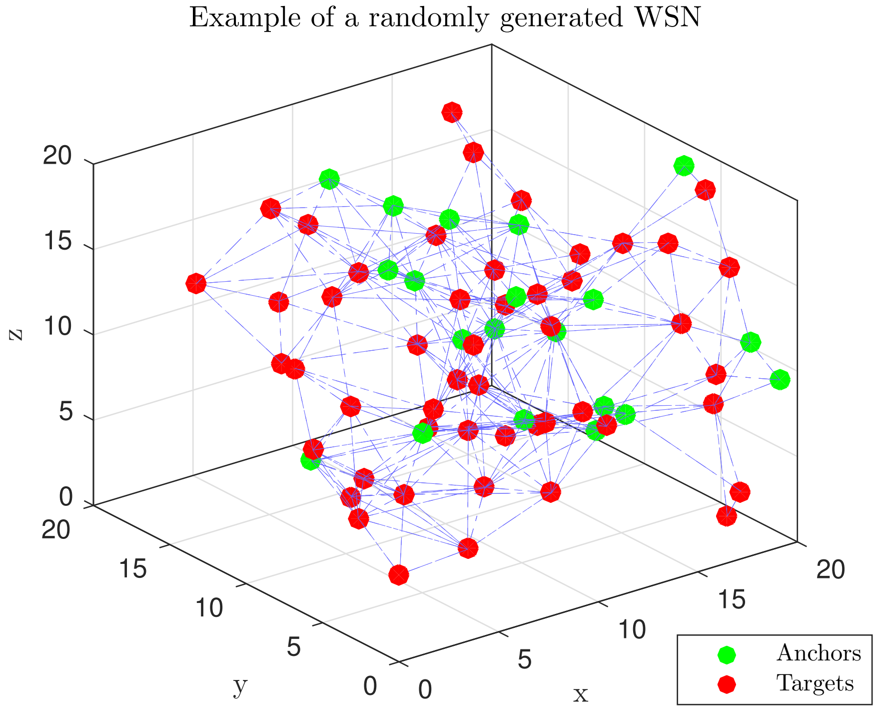


Figure 4.1: A randomly generated WSN with $N = 20$, $M = 50$, $R = 6.5$ and $d_0 = 1$.

neighbors, $\forall(i,j) \in \mathcal{E}_\mathcal{A} \cup \mathcal{E}_\mathcal{T}, i \neq j$, in each M_c run were generated. The algorithms also assume that the initial targets' locations, $\hat{\boldsymbol{X}}^{(0)}$, is in the cubic volume's center, i.e., $\hat{\boldsymbol{x}}_i^{(0)} = [\frac{B}{2}, \frac{B}{2}, \frac{B}{2}]^T \ \forall i \in \mathcal{T}$. All algorithms were tested for three cases of link failure probability P_f, specifically $50, 70$ and 90%.

In order to compare performances, the metric used was the normalized root mean square error (NRMSE), which is defined as

$$\text{NRMSE} = \sqrt{\frac{1}{MM_c}\sum_{i=1}^{M_c}\sum_{j=1}^{M}\|\boldsymbol{x}_{ij} - \widehat{\boldsymbol{x}}_{ij}\|^2},$$

where $\widehat{\boldsymbol{x}}_{ij}$ stands for the estimate of the true location of the j-th target, $\boldsymbol{x}_{ij}$, in the i-th Monte Carlo run.

4.3 Results

The performance of the considered algorithms, when $N = 20$ and $M = 50$, is illustrated in Figure 4.2 which compares NRMSE versus t. All considered algorithms show better results as t grows, as anticipated. Furthermore, the effects of not knowing that the P_{0i}'s in the "uSOCP" can be noticed as its curve gets saturated at $t = 3$. At this point the algorithm starts estimating P_{0i}'s, and continues as if P_{0i}'s are known. This fact explains the sudden curve drop after $t = 3$. It is possible to observe that the "uSOCP" algorithm shows excellent performance, almost achieving the lower bound provided by its counterpart for known P_i's. While the"SR-WLS" method does not perform as good as than the

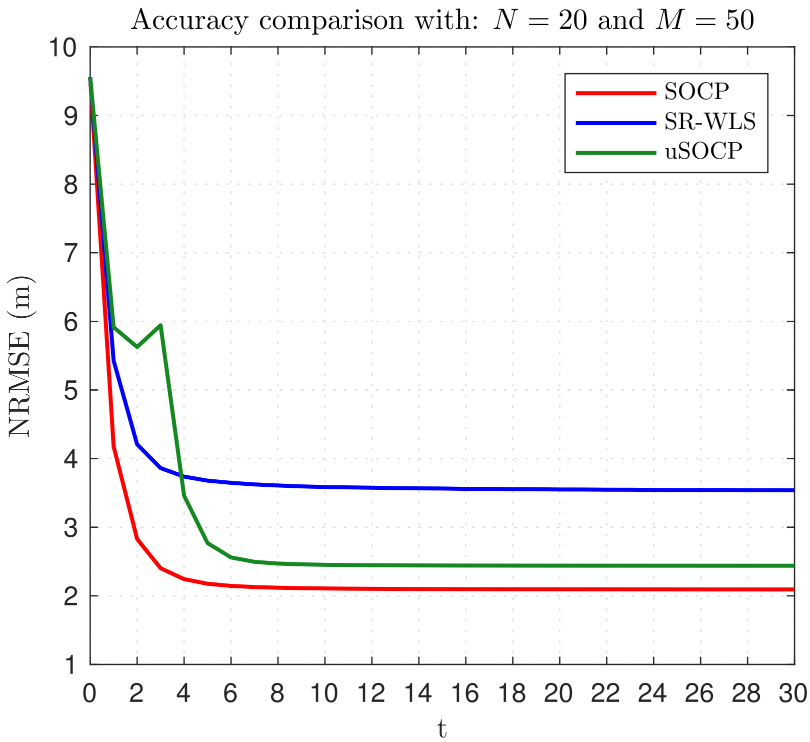


Figure 4.2: NRMSE versus t comparison, when $N = 20$, $M = 50$, $R = 6.5$ m, $\sigma_{n_{ij}} = 3$ dB, $\sigma_{m_{ij}} = 6$ deg, $\sigma_{v_{ij}} = 6$ deg, $\gamma_{ij} \in \mathcal{U}[2.7, 3.3]$, $\gamma = 3$, $B = 20$ m, $P_{0i} \in \mathcal{U}[-12, -8]$ dBm, $d_0 = 1$ m, $M_c = 500$.

"SOCP" and "uSOCP" methods it is important to note that the both the latter methods are computationally more demanding due to the use of sophisticated mathematical tools. We can conclude that a simple algorithm such as the one based on bisection procedure can produce good enough estimation accuracy that shouldn't be overlooked. It is also possible to perceive that in all algorithms the majority of performance changes take place in the first few iterations ($t \leq 10$), and that the performance gain is negligible afterwards. This result is very important because it shows that a low number of signal transmissions is required, which might enhance the utilization efficiency of the radio spectrum, a precious resource for wireless communications. Additionally it shows that the algorithms are energy efficient, as the communication phase is much more expensive (in terms of energy) than the data processing one [49].

For the scenario when $N = 30$ and $M = 50$, it is possible to see a plot of NRMSE versus t in Figure 4.3. A comparison between Figure 4.2 and 4.3 reveals that the performance of all algorithms improves significantly as more anchors are introduced into the network. This is intuitively expected, since with more reference nodes (higher N) more reliable information and more AoA measurements are available in the network.

Figure 4.4 shows the NRMSE versus t performance of the three algorithms when $N = 20$ and $M = 60$. From Figs. 4.2 and 4.4 we can note that a 20% increase in the number of targets does not impact negatively the performance of any algorithm.

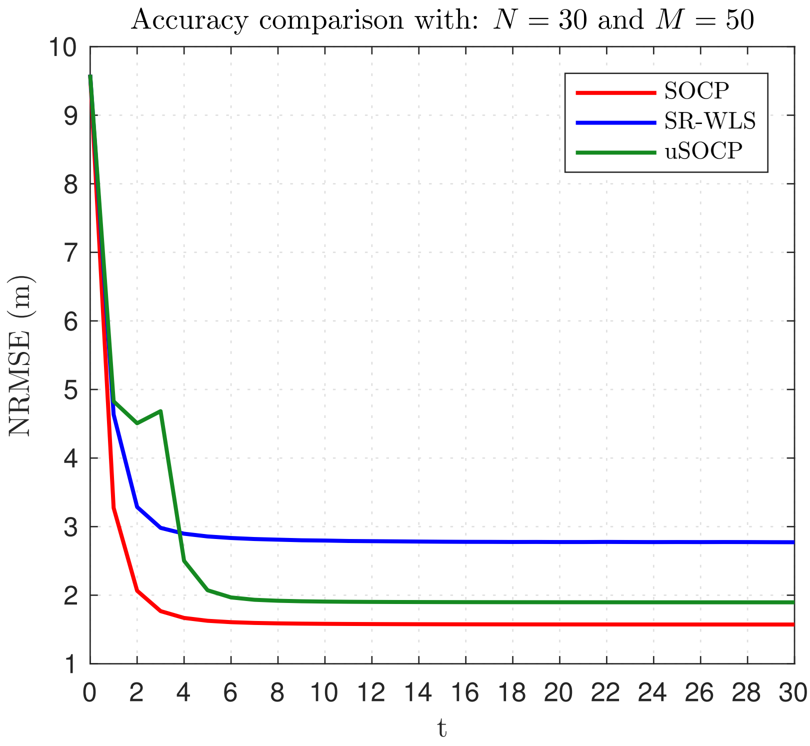


Figure 4.3: NRMSE versus t comparison, when $N = 30$, $M = 50$, $R = 6.5$ m, $\sigma_{n_{ij}} = 3$ dB, $\sigma_{m_{ij}} = 6$ deg, $\sigma_{v_{ij}} = 6$ deg, $\gamma_{ij} \in \mathcal{U}[2.7, 3.3]$, $\gamma = 3$, $B = 20$ m, $P_{0i} \in \mathcal{U}[-12, -8]$ dBm, $d_0 = 1$ m, $M_c = 500$.

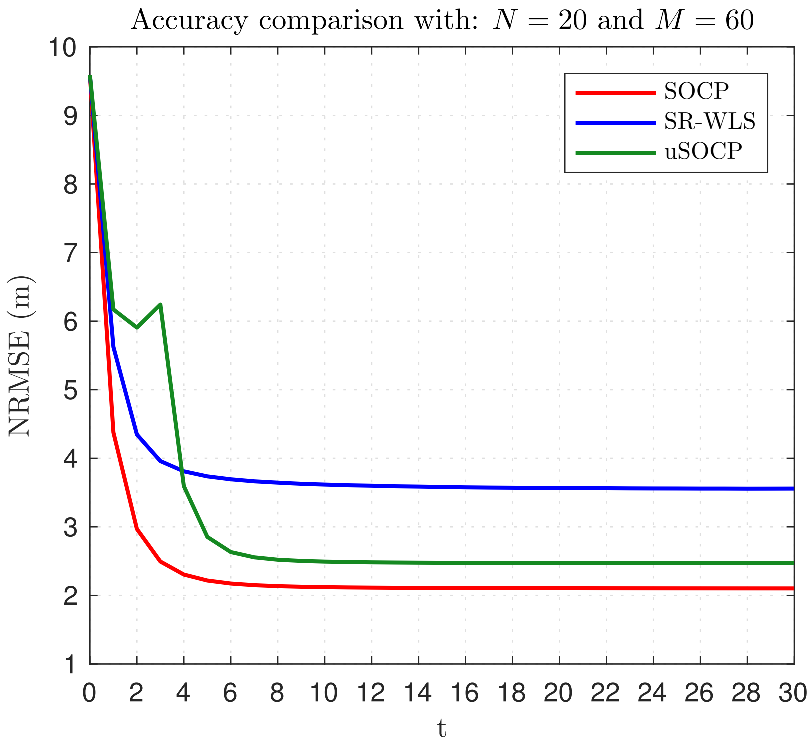


Figure 4.4: NRMSE versus t comparison, when $N = 20$, $M = 60$, $R = 6.5$ m, $\sigma_{n_{ij}} = 3$ dB, $\sigma_{m_{ij}} = 6$ deg, $\sigma_{v_{ij}} = 6$ deg, $\gamma_{ij} \in \mathcal{U}[2.7, 3.3]$, $\gamma = 3$, $B = 20$ m, $P_{0i} \in \mathcal{U}[-12, -8]$ dBm, $d_0 = 1$ m, $M_c = 500$.

Figures 4.5, 4.6 and 4.7 display the impact that the noise has on RSS and AoA measurements, and, consequently, on the algorithms' performances. This experiment compares the NRMSE versus $\sigma_{n_{ij}}$ (dB), $\sigma_{m_{ij}}$ (deg) and $\sigma_{v_{ij}}$ (deg), when $N = 20$, $M = 50$, $R = 6.5$ m, and $T_{\max} = 30$ for all algorithms. It is possible to observe as the any measurement error increases that the performances of "SOCP" and "uSOCP" deteriorate. We can observe that the condition of RSS measurements affects more the performance of the studied algorithms, while the error in the azimuth and elevation angle measurements have minimal impacts on the performance. This can be explained by the fact that since all sensors are at most separated by $R = 6.5$ m, a small error in AoA measurements can not affect the performances considerably (see Figure 2.5). RSS measurements, however, have a difficult to predict behavior that comes from the nature of RF signal propagation [49]. Still, the the performance loss is about 15% for the "SOCP" and "uSOCP", and 10% for the "SR-WLS", which is relatively low for the considered error span. Algorithm "SR-WLS" shows a identical behavior to the other algorithms when the RSS is varied. However, the effects of AoA errors are negligible in Figure 4.6, while in Figure 4.7 it appears that the effects of increasing the error decreases its impact. This is most likely a false result, and more data points would be required to reach a conclusion. Nonetheless, the weights used in "SR-WLS" favor shorter pair-wise distances, which can explain why the AoA errors have such a minimal impact, like explained before.

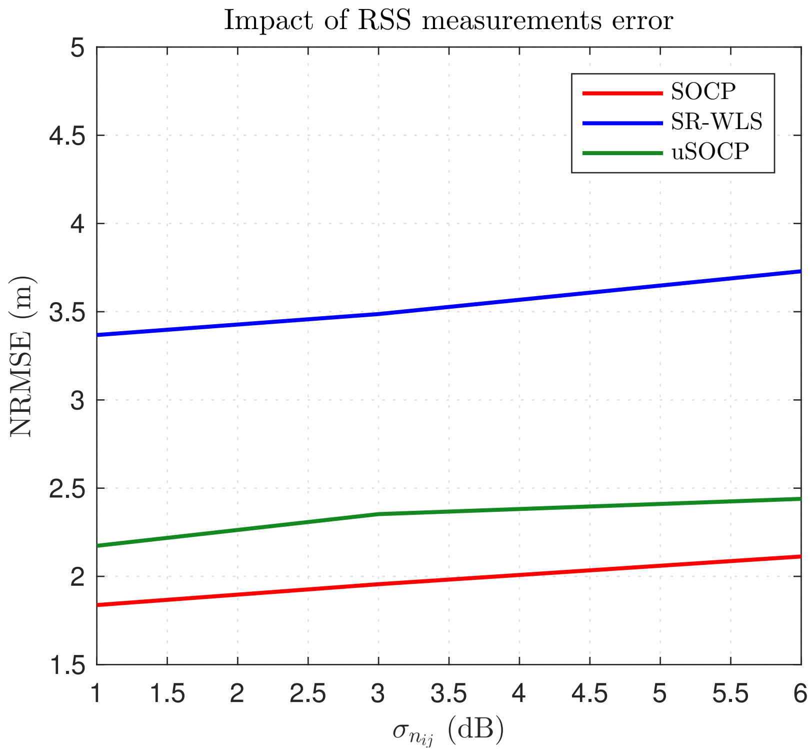


Figure 4.5: NRMSE versus $\sigma_{n_{ij}}$ (dB) comparison, when $N = 20$, $M = 50$, $R = 6.5$ m, $\sigma_{m_{ij}} = 1$ deg, $\sigma_{v_{ij}} = 1$ deg, $\gamma_{ij} \in \mathcal{U}[2.7, 3.3]$, $\gamma = 3$, $T_{\max} = 30$, $B = 20$ m, $P_{0i} \in \mathcal{U}[-12, -8]$ dBm, $d_0 = 1$ m, $M_c = 500$.

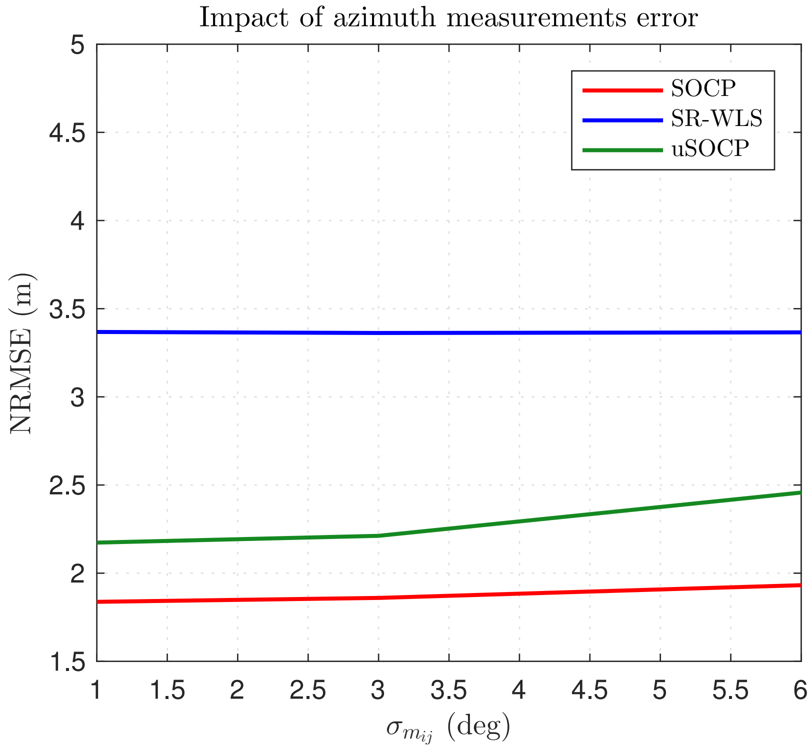


Figure 4.6: NRMSE versus $\sigma_{m_{ij}}$ (deg) comparison, when $N = 20$, $M = 50$, $R = 6.5$ m, $\sigma_{n_{ij}} = 1$ dB, $\sigma_{v_{ij}} = 1$ deg, $\gamma_{ij} \in \mathcal{U}[2.7, 3.3]$, $\gamma = 3$, $T_{\max} = 30$, $B = 20$ m, $P_{0i} \in \mathcal{U}[-12, -8]$ dBm, $d_0 = 1$ m, $M_c = 500$.

Figure 4.8, Figure 4.9 and Figure 4.10 illustrate the NRMSE (m) versus t comparison for different probabilities of link failure for the "SOCP", "SR-WLS" and "uSOCP" algorithms respectively. The figures exhibit that more iterations are required for the algorithm to converge when the link failure probability is increased, as expected. We see that although the propagation of the updated information is slowed down, all targets eventually collect the complete information form their neighbors It can be noticed that both algorithms behave similarly and probabilities of link failure up to 70% are easily mitigated by a small increase of iterations required for convergence. As expected, the performance of both algorithms worsens with the probability of 90% of link failure. However, this setting is more theoretical than practical, since it basically means that there is nearly no communication between sensors, or in other words, that the network does not work. Nevertheless, the algorithms still converge if enough iterations are made. This confirms the robustness of the proposed algorithms in [68].

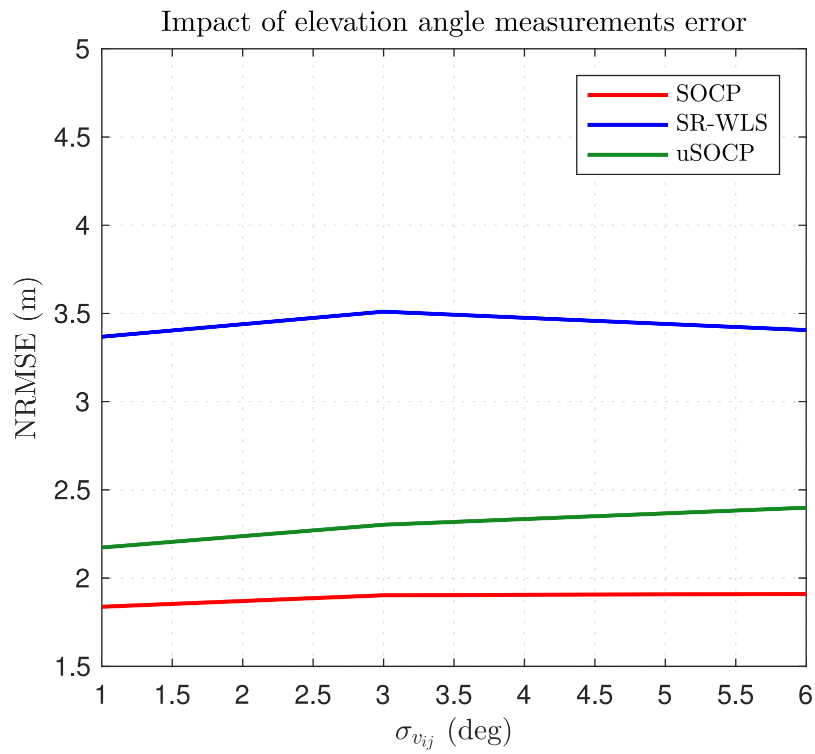


Figure 4.7: NRMSE versus $\sigma_{v_{ij}}$ (deg) comparison, when $N = 20$, $M = 50$, $R = 6.5$ m, $\sigma_{n_{ij}} = 1$ dB, $\sigma_{m_{ij}} = 1$ deg, $\gamma_{ij} \in \mathcal{U}[2.7, 3.3]$, $\gamma = 3$, $T_{\max} = 30$, $B = 20$ m, $P_{0i} \in \mathcal{U}[-12, -8]$ dBm, $d_0 = 1$ m, $M_c = 500$.

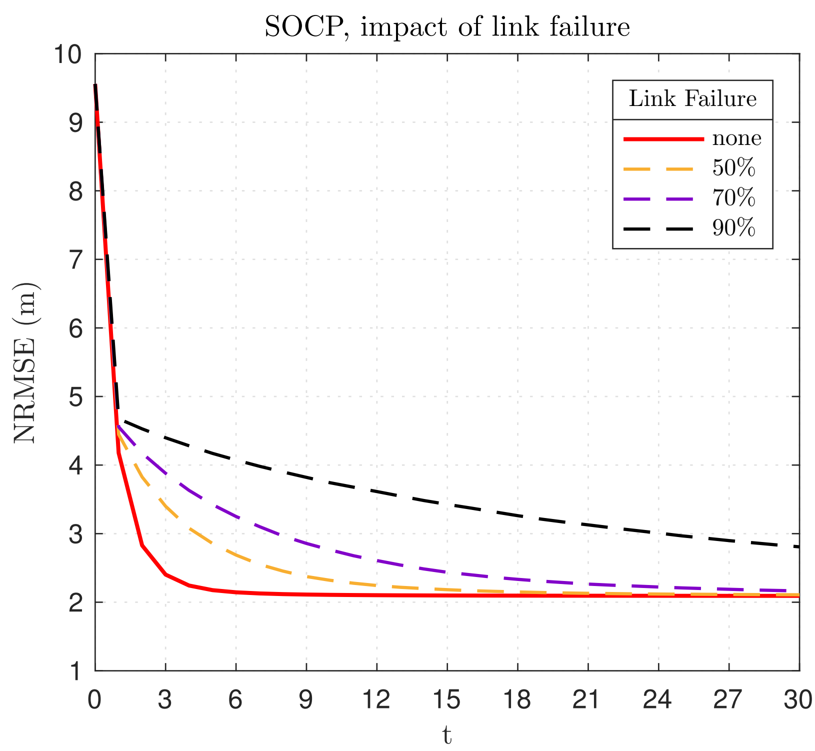


Figure 4.8: NRMSE versus t comparison for "SOCP", when $N = 20$, $M = 50$, $R = 6.5$ m, $\sigma_{n_{ij}} = 3$ dB, $\sigma_{m_{ij}} = 6$ deg, $\sigma_{v_{ij}} = 6$ deg, $\gamma_{ij} \in \mathcal{U}[2.7, 3.3]$, $\gamma = 3$, $B = 20$ m, $P_{0i} \in \mathcal{U}[-12, -8]$ dBm, $d_0 = 1$ m, $M_c = 500$, varying P_f.

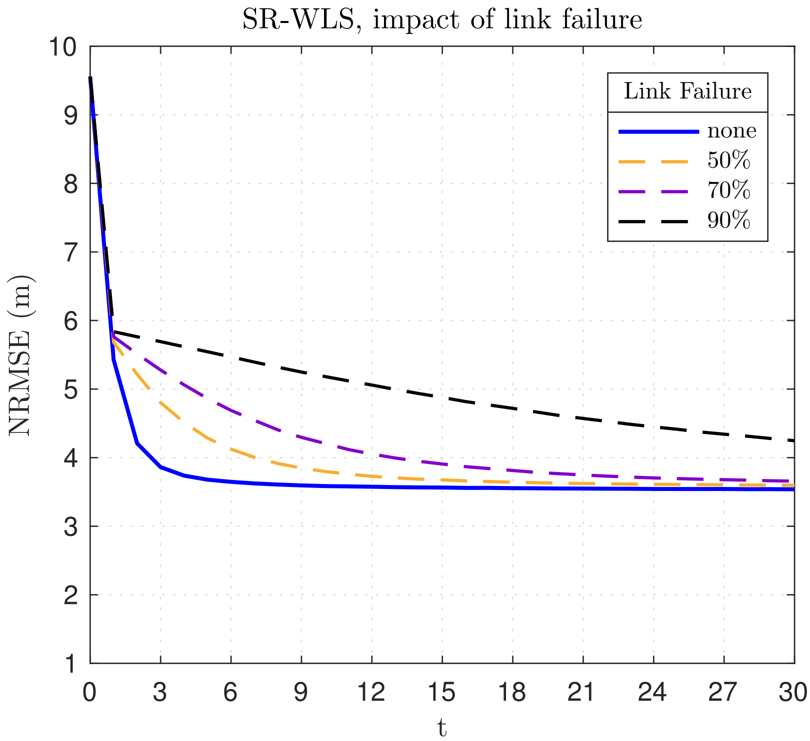


Figure 4.9: NRMSE versus t comparison for “SR-WLS”, when $N = 20$, $M = 50$, $R = 6.5$ m, $\sigma_{n_{ij}} = 3$ dB, $\sigma_{m_{ij}} = 6$ deg, $\sigma_{v_{ij}} = 6$ deg, $\gamma_{ij} \in \mathcal{U}[2.7, 3.3]$, $\gamma = 3$, $B = 20$ m, $P_{0i} \in \mathcal{U}[-12, -8]$ dBm, $d_0 = 1$ m, $M_c = 500$, varying P_f.

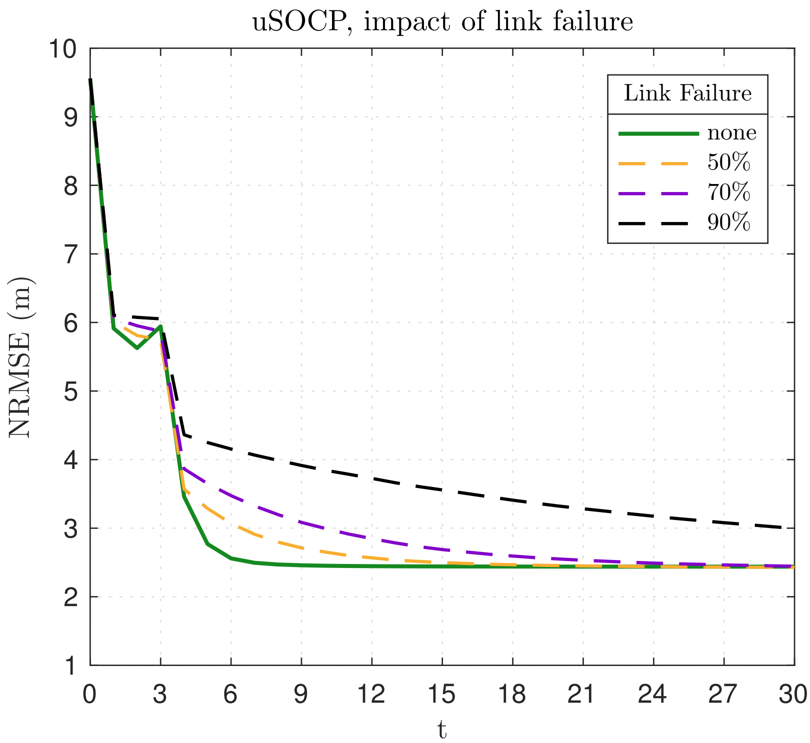


Figure 4.10: NRMSE versus t comparison for “uSOCP”, when $N = 20$, $M = 50$, $R = 6.5$ m, $\sigma_{n_{ij}} = 3$ dB, $\sigma_{m_{ij}} = 6$ deg, $\sigma_{v_{ij}} = 6$ deg, $\gamma_{ij} \in \mathcal{U}[2.7, 3.3]$, $\gamma = 3$, $B = 20$ m, $P_{0i} \in \mathcal{U}[-12, -8]$ dBm, $d_0 = 1$ m, $M_c = 500$, varying P_f.

Chapter 5

Conclusions and Future Work

5.1 Introduction

This chapter summarizes the attained results and the contribution made in the scope of this dissertation in Section 5.2. Future research on this topic and some foreseen directions are discussed in Section 5.3.

5.2 Conclusions

Target localization in WSNs is a problem that drives the development of highly sophisticated and complex algorithms with computational and energy efficiency as the main issues.

Two novel algorithms, namely "SOCP" and "SR-WLS", in addition to a generalization of the former, designated "uSOCP", were implemented and studied. The attained results confirm the ones stated in [68], as almost exact parity was achieved. Since the implemented algorithms ran on different networks, the existent differences may be possible explained by the method used to generate WSNs, which was undocumented and therefore is impossible to compare.

Link failure is a known problem in wireless communications, that should be addressed when designing algorithms for WSNs. An algorithm with inability to still converge in view of failed transmissions may result in the exhaustion of the network's battery resources as the amount of retransmissions go up. For this reason, the implemented algorithms were tested in a more practical scenario, by alleviating one of the strong assumptions made by the authors in [68]. All algorithms were tested against moderate to high probability of communication failure at the broadcast phase, enabling the observation of their ability to converge in face of a high strain scenario. It was indeed shown that the algorithms are very robust, and behave positively in a more practical scenario.

5.3 Future Work

Localization in WSNs offer many interesting possibilities for future research, some of those (in the context of this dissertation) are mentioned below.

While computational simulations allow to obtain results relatively fast and using few monetary resources, validating all potential algorithms and ideas through experimental setup would be of great interest. An example of target localization research backed by experimental results is the Cricked System, found in [54].

In this dissertation the problem of link failure is considered, a problem which may lead to isolated islands of sensors. In this situations, sensors might find itselves with no or very scarce information, and therefore unable to compute acceptable location estimation. Algorithms that are specially designed for this situation could improve energy efficiency in the network.

This work considered the target localization problem using combined RSS and AoA measurements. Employing other combinations to form different hybrid systems in order to solve the localization problem might be of interest for future research as well.

The algorithms studied and modified in this dissertation employed a second-order coloring scheme as a simple MAC protocol. This issue was not in the main scope of the dissertation, and probably not exploited to its fullest potential, as the design of better protocols might lead to error and time-execution reduction. Protocols with a higher degree of intelligence might produce better estimation accuracy and at the same time increase the convergence rate of an algorithm (*e.g.*, protocols such that targets with the highest number of anchor neighbors work first propagation a better estimation in the network).

Another future research direction might be the development of new and adaptation of the presented algorithms to more challenging scenarios such as indoor localization in severe NLoS environments. NLoS can negatively impact estimation accuracy in a significant way, especially in cases where the configuration of the environment is not known, *i.e.*, when it is not known *a priori* which links in LoS conditions and which are in NLoS ones. However, false detection may make it difficult to distinguish between LoS and NLoS links. So, instead of disregarding the NLoS links, it would be of interest to exploit the property of positive NLoS bias, which is known to be much larger than the measurement noise.

It might also be of interest to investigate the case where targets limit the number of nodes they cooperate with, especially in large-scale WSNs. If a target finds itself in a situation where it has a high number of neighbors, selecting only a certain number of its them might be a possibility. One example would be choosing only the nearest ones such that the computational burden is decreased and that its estimation accuracy remains unaffected. Disregarding possibly noisy links could also be an option. The main challenge of such approach would be designing an intelligent neighbor-selecting strategy, in way that noisy observations wouldn't mislead a target to disregard a potentially good link and maintain a bad one.

Finally, since this dissertation assumed omnidirectional antenna directivity, the set of all possible position solutions form belongs to the area formed by an intersection of multiple circle-shaped contours. This assumption works well in all considered scenarios, however it might be an oversimplification of the problem, since the antenna radiation pattern is non-isotropic in practice (e.g., antenna radiation pattern depends on antenna geometry configuration shape and dimension, dielectric material, combination (antenna array), and signal wavelength). Consequently, in reality, the area formed by the intersection of non-circular power contours, determined by the antenna pattern, form the set of all possible solutions. Thus, taking the antenna pattern into consideration when deriving a localization scheme should also be considered.

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Appendix

A

Article Submitted to *International Young Engineers Forum on Electrical and Computer Engineering, May 2017*

Performance Analysis of a Distributed Algorithm for Target Localization in Wireless Sensor Networks Using Hybrid Measurements in a Connection Failure Scenario

David Vicente*, Slavisa Tomic†, Marko Beko‡ and Rui Dinis§
*Departamento de Engenharia Electrotécnica, FCT-UNL, 2829-516 Caparica, Portugal
Email: dj.vicente@campus.fct.unl.pt
†ISR/IST, LARSyS, Universidade de Lisboa, 1049-001 Lisbon, Portugal
Email: stomic@isr.ist.utl.pt
‡Universidade Lusófona de Humanidades e Tecnologias, Campo Grande 376, 1749 - 024 Lisboa, Portugal
CTS/UNINOVA, Campus da FCT/UNL, Monte de Caparica, 2829-516 Caparica, Portugal
Email: beko.marko@ulusofona.pt, mbeko@uninova.pt
§Departamento de Engenharia Electrotécnica, FCT-UNL, 2829-516 Caparica, Portugal
Instituto de Telecomunicações, Av. Rovisco Pais 1, Torre Norte, piso 10, 1049 - 001 Lisboa, Portugal
Email: rdinis@fct.unl.pt

***Abstract*—This paper presents a performance analysis of two recently proposed distributed localization algorithms for cooperative 3-D wireless sensor networks (WSNs) in a more realistic scenario. The tested algorithms rely on distance and angle measurements obtained from received signal strength (RSS) and angle-of arrival (AoA) information, respectively. The measurements are then used to derive a convex estimator, based on second order cone programming (SOCP) relaxation techniques, and a non-convex one that can be formulated as a generalized trust region sub-problem (GTRS). Both estimators have shown excellent performance assuming a static network scenario, giving accurate location estimates in addition to converging in few iterations. Here, we test their performance considering different probabilities of communication failure between neighbour nodes at the broadcast phase. Our simulations show that their performance holds for high probability of communication failure and that convergence is still achieved in a reasonable number of iterations.**

I. Introduction

Following great advances in the last three decades, wireless sensor networks (WSNs) became a technology applicable in a vast number of different areas such as commercial, military and health care. Its use goes from monitoring and event detection to inventory management [1]. A currently trending concept called internet of things (IoT) is also pushing the research and development of WSNs and driving the usage of sensors in applications such as smart homes [2], [3].

In some applications, such as monitoring soil conditions in large agricultural fields, it is preferred to rely on random sensor deployment (i.e., dropped from an air plane) since the number of nodes in a WSN can be in the order of hundreds or thousands. In cases like this, the localization of each node after their deployment becomes a high priority issue. Knowing the exact location of each sensor node is of utmost importance if more than simple statistics is wanted. Localization enables context-awareness, identification and correlation of gathered data, allowing statistics to be computed in terms of spatial sampling rather than just the count of sensor readings [4].

One could equip all of the network nodes with a global positioning system (GPS) receiver, rendering the localization problem trivial. However, GPS offers poor performance in some environments like indoors, dense forests, and caves, since it requires unobstructed line-of-sight between satellites and the receiver [5]. Furthermore, devices in a WSN should have low production cost, low complexity, and low power consumption. Consequently, being able to perform localization using different approaches should be considered. One clever way is to exploit different characteristics of the radio frequency (RF) signals used by the nodes to communicate.

An approach to localization in WSNs is to establish the position of sensor nodes whose location is unknown (targets), given some information about a sparse number of reference sensors (anchors) and measured distances/angles between pairs of nodes. RF signals can be used to estimate distances and orientation. Typical ways of obtaining such information are techniques like time-of-arrival, time-difference-of-arrival, received-signal-strength (RSS), angle-of-arrival (AoA), or a combination of some, depending on the hardware at hand [6]–[11].

It is possible to classify a network's localization scheme regarding its computational organization as either centralized or distributed [12]. While the former requires a central processor to estimate the positions of all nodes, in the latter all nodes perform local processing and compute their own positions. A centralized approach offers a trade-off where the problem of having little computational power in each node is solved while introducing communicating overhead. This way of addressing the issue may cause a bottleneck at the central processor as the network grows larger. Therefore, it may not be suitable for large-scale WSNs. In an opposite way, distributed schemes rely on local processing and intercommunication between neighbours making it more suitable for large scale networks. The downsides however, are the increased vulnerability to error propagation, since these algorithms run iteratively, and rise of energy consumption in the computation period.

Position estimation in sensor networks has been a recurrent researched theme in the last years. In [13] a recursive algorithm was proposed to estimate the location of nodes with help of a few reference nodes. Using convex constraints, it was shown in [14] how to limit a sensor's location estimate to a certain radius and angle range from a another sensor. With device simplicity in mind, the authors in [15] compared proximity, RSS, and quantized RSS based techniques in localization. It was shown in the article that it would be possible to use an 8-level quantized RSS as a method to estimate distance. Adopting a semi-definite programming (SDP) relaxation technique, it was described in [16] that convex programming methods offer good results when solving the localization problem. Following that publication, further SDP relaxations were presented in [17], sacrificing accuracy for speed. In [18], Srirangarajan et al. derived a distributed approach using a second-order cone programming (SOCP) relaxation with cooperation between unknown location nodes. The authors in [19] proposed algorithms based on AoA measurements. A hybrid RSS/AoA approach was presented in [20] for indoor localization with the aid of a visible light infrastructure.

The authors in [21] proposed distributed cooperative localization algorithms. Cooperative in the sense that any two neighbour nodes can communicate regardless of being targets or anchors. This approach is beneficial in WSN where the number of anchor nodes is often desired to be as low as possible. By taking advantage of combined RSS/AoA measurements, the authors designed algorithms based on SOCP and generalized trust region sub-problems (GTRS) framework. While the results show very high localization accuracy in just a few iterations, some strong assumptions were made such as that the network topology remains constant during the computational period. However, this assumption might not hold in practice, especially in dynamic environments where people and/or cars (or other objects) are passing by, blocking the line of sight between nodes, or where weather conditions are constantly changing with time.

In this paper, we tackle a more realistic localization problem, by alleviating this assumption. More precisely, we introduce probability of link failure between neighbour nodes at the broadcast phase. This affects how new information spreads throughout the network, creating a more realistic scenario. We investigate how the two algorithms proposed in [21] behave for the case of 50%, 70% and 90% of link failure probability.

The adopted notation in this paper is described as follows: Upper-case bold type, lower-case bold type and regular type are used for matrices, vectors and scalars, respectively. $\mathbb{R}^n$ denotes the n-dimensional real Euclidean space. The operator $(\bullet)^T$ denotes matrix transpose and $\otimes$ the Kronecker product. The normal (Gaussian) distribution with mean μ and variance σ^2 is denoted by $\mathcal{N}(\mu, \sigma^2)$. $\text{diag}(\boldsymbol{x})$ denotes a square diagonal matrix in which the elements of vector $\boldsymbol{x}$ form the main diagonal of the matrix, and the elements outside the main diagonal are zero. The N-dimensional identity matrix is denoted by $\boldsymbol{I}_N$ and the $M \times N$ matrix of all zeros by $\mathbf{0}_{M\times N}$ (if no ambiguity can occur, subscripts are omitted). $\|\boldsymbol{x}\|$ denotes the vector norm defined by $\|\boldsymbol{x}\| = \sqrt{\boldsymbol{x}^T\boldsymbol{x}}$, where $\boldsymbol{x} \in \mathbb{R}^n$ is a column vector.

The remainder of this work is organized as follows. In Section II we present the problem formulation. For the sake of completeness, Section III presents the distributed estimators proposed in [21]. Section IV describes the changes made to the algorithms in order to introduce probability of link failure. In Section V we discuss the performance effects that our changes introduced. Finally, Section VI summarizes the main conclusions.

II. Problem formulation

Let us formally state the location estimation problem in WSNs. We consider a WSN composed $M + N$ sensors randomly distributed over a cubic region of interest, where M is the number of targets and N the number of anchors, all with a communication range of R(m). Formally, the network can be represented by a connected graph, $\mathcal{G}(\mathcal{V}, \mathcal{E})$, with $|\mathcal{V}| = M+N$ vertices and $|\mathcal{E}|$ edges, with $|\bullet|$ representing the number of elements in a set, i.e. its cardinality. The set of targets and the set of anchors are respectively labelled as $\mathcal{T}$ ($|\mathcal{T}| = M$) and $\mathcal{A}$ ($|\mathcal{A}| = N$), and their locations are denoted by $\boldsymbol{x}_1, \boldsymbol{x}_2, ..., \boldsymbol{x}_M$ and $\boldsymbol{a}_1, \boldsymbol{a}_2, ..., \boldsymbol{a}_N$ ($\boldsymbol{x}_i, \boldsymbol{a}_j \in \mathbb{R}^3$, $\forall i \in \mathcal{T}$ and $\forall j \in \mathcal{A}$), respectively. Each edge, $\langle i, j\rangle$, in the set $\mathcal{E}$ represents a pairwise node connection. This means that the distance between nodes represented by vertices v_i and v_j is less or equal to R, therefore communication between them is possible. As a result, all target/anchor and target/target connections (edges) are defined as the sets $\mathcal{E}_\mathcal{A} = \{(i, j) : \|\boldsymbol{x}_i - \boldsymbol{a}_j\| \le R, \forall i \in \mathcal{T}, \forall j \in \mathcal{A}\}$ and $\mathcal{E}_\mathcal{T} = \{(i, k) : \|\boldsymbol{x}_i - \boldsymbol{x}_k\| \le R, \forall i, k \in \mathcal{T}, i \ne k\}$, respectively.

For all the nodes whose location is unknown we define $\boldsymbol{X} = [\boldsymbol{x}_1, \boldsymbol{x}_2, ..., \boldsymbol{x}_M]$ ($\boldsymbol{X} \in \mathbb{R}^{3\times M}$) as the matrix of their coordinates. The algorithms considered in [21] determine these locations by using a hybrid system that fuses range and angle measurements.

In [21], it is assumed that the range measurements are obtained from RSS information. When two sensors i and j are within communication range of each other, we can model the signal power received by i, P_{ij} (dBm), as:

$$P_{ij}^{\mathcal{A}} = P_{0i} - 10\gamma \log_{10} \frac{\|\boldsymbol{x}_i - \boldsymbol{a}_j\|}{d_0} + n_{ij}, \forall (i, j) \in \mathcal{E}_\mathcal{A}, \quad \text{(1a)}$$

$$P_{ik}^{\mathcal{T}} = P_{0i} - 10\gamma \log_{10} \frac{\|\boldsymbol{x}_i - \boldsymbol{x}_k\|}{d_0} + n_{ik}, \forall (i, k) \in \mathcal{E}_\mathcal{T}, \quad \text{(1b)}$$

following [22], where P_{0i} (dBm) represents the reference power at a distance d_0 ($\|\boldsymbol{x}_i - \boldsymbol{a}_j\| \ge d_0$, $\|\boldsymbol{x}_i - \boldsymbol{x}_k\| \ge d_0$)

from the transmitting sensor, γ is the path loss exponent (PLE) between sensors i and j, and n_{ij} and n_{ik} are the log-normal shadowing terms modelled as $n_{ij} \sim \mathcal{N}(0, \sigma^2_{n_{ij}})$, $n_{ik} \sim \mathcal{N}(0, \sigma^2_{n_{ik}})$. For the sake of simplicity and without loss of generality, we assume that the target/target RSS measurements are symmetric.

The RSS measurement model (1) can be used to estimate the distance between two sensors as follows [12]:

$$\widehat{d}_{ij} = \begin{cases} d_0 10^{\frac{P_{0i} - P^{\mathcal{A}}_{ij}}{10\gamma}}, & \text{if } j \in \mathcal{A}, \\ d_0 10^{\frac{P_{0i} - P^{\mathcal{T}}_{ij}}{10\gamma}}, & \text{if } j \in \mathcal{T}. \end{cases} \tag{2}$$

The authors in [21] considered that the anchors exclusively had means of measuring AoA (both azimuth and elevation angles), possible by being equipped with antenna arrays or a directional antenna [6], [23], [24]. These angle measurements can be modelled as

$$\phi^{\mathcal{A}}_{ij} = \arctan\left(\frac{x_{i\mathrm{y}} - a_{j\mathrm{y}}}{x_{i\mathrm{x}} - a_{j\mathrm{x}}}\right) + m_{ij}, \text{ for } (i,j) \in \mathcal{E}_{\mathcal{A}}, \tag{3}$$

and

$$\alpha^{\mathcal{A}}_{ij} = \arccos\left(\frac{x_{i\mathrm{z}} - a_{j\mathrm{z}}}{\|\boldsymbol{x}_i - \boldsymbol{a}_j\|}\right) + v_{ij}, \text{ for } (i,j) \in \mathcal{E}_{\mathcal{A}}, \tag{4}$$

where $\boldsymbol{x}_i = [x_{i\mathrm{x}}, x_{i\mathrm{y}}, x_{iz}]^T$ and $\boldsymbol{a}_j = [a_{j\mathrm{x}}, a_{j\mathrm{y}}, a_{jz}]^T$ are respectively the unknown coordinates of the i-th target and the known coordinates of the j-th anchor. The distance, azimuth angle and elevation angle between the i-th target and the j-th anchor are represented as $d^{\mathcal{A}}_{ij}$, $\phi^{\mathcal{A}}_{ij}$ and $\alpha^{\mathcal{A}}_{ij}$ respectively, while m_{ij} and v_{ij} are the measurement errors of azimuth and elevation angles, respectively modelled as $m_{ij} \sim \mathcal{N}(0, \sigma^2_{m_{ij}})$ and $v_{ij} \sim \mathcal{N}(0, \sigma^2_{v_{ij}})$.

Given the observation vector $\boldsymbol{\theta} = [\boldsymbol{P}^T, \boldsymbol{\phi}^T, \boldsymbol{\alpha}^T]^T$ ($\boldsymbol{\theta} \in \mathbb{R}^{3|\mathcal{E}_{\mathcal{A}}|+|\mathcal{E}_{\mathcal{T}}|}$), where $\boldsymbol{P} = [P^{\mathcal{A}}_{ij}, P^{\mathcal{T}}_{ik}]^T$, $\boldsymbol{\phi} = [\phi^{\mathcal{A}}_{ij}]^T$, $\boldsymbol{\alpha} = [\alpha^{\mathcal{A}}_{ij}]^T$, and the Gaussian noise assumption, the conditional probability density function (PDF) is given as:

$$p(\boldsymbol{\theta}|\boldsymbol{X}) = \prod_{i=1}^{3|\mathcal{E}_{\mathcal{A}}|+|\mathcal{E}_{\mathcal{T}}|} \frac{1}{\sqrt{2\pi\sigma^2_i}} \exp\left(-\frac{(\theta_i - f_i(\boldsymbol{X}))^2}{2\sigma^2_i}\right), \tag{5}$$

where

$$\boldsymbol{f}(\boldsymbol{X}) = \begin{bmatrix} \vdots \\ P_{0i} - 10\gamma \log_{10} \frac{\|\boldsymbol{x}_i - \boldsymbol{a}_j\|}{d_0} \\ \vdots \\ P_{0i} - 10\gamma \log_{10} \frac{\|\boldsymbol{x}_i - \boldsymbol{x}_k\|}{d_0} \\ \vdots \\ \arctan\left(\frac{x_{iy} - a_{jy}}{x_{i\mathrm{x}} - a_{j\mathrm{x}}}\right) \\ \vdots \\ \arccos\left(\frac{x_{iz} - a_{jz}}{\|\boldsymbol{x}_i - \boldsymbol{a}_j\|}\right) \\ \vdots \end{bmatrix}, \ \boldsymbol{\sigma} = \begin{bmatrix} \vdots \\ \sigma_{n_{ij}} \\ \vdots \\ \sigma_{n_{ik}} \\ \vdots \\ \sigma_{m_{ij}} \\ \vdots \\ \sigma_{v_{ij}} \\ \vdots \end{bmatrix}.$$

In statistics, the maximum likelihood estimator (MLE) is commonly used, since it has the property of being asymptotically efficient (for large enough data records) [25]. The ML estimate is obtained by maximizing the log of the likelihood function (5) with respect to $\boldsymbol{X}$, i.e.:

$$\hat{\boldsymbol{X}} = \arg\min_{\boldsymbol{X}} \sum_{i=1}^{3|\mathcal{E}_{\mathcal{A}}|+|\mathcal{E}_{\mathcal{T}}|} \frac{1}{\sigma^2_i} \left[\theta_i - f_i(\boldsymbol{X})\right]^2. \tag{6}$$

However, since the MLE is non-convex and has no closed-form solution, Tomic, et al., [21] proposed to solve a tight approximation of (6) using a distributed approach. This can be done either by employing a convex relaxation technique, leading to a distributed SOCP estimator that can be solved efficiently by interior-point algorithms [26], or a suboptimal estimator based on the GTRS framework, leading to a distributed squared range weighted least squares (SR-WLS) estimator, which can be solved *exactly* by a bisection procedure [27].

III. Distributed localization

The problem in (6) is only dependent on the locations and pairwise measurements between the adjacent sensors. In light of this, Tomic, et al., [21] broke (6) into sub-problems and derived a local ML estimator, solved iteratively by each target independently.

$$\hat{\boldsymbol{x}}^{(t+1)}_i = \arg\min_{\boldsymbol{x}_i} \sum_{j=1}^{3|\mathcal{E}_{\mathcal{A}_i}|+|\mathcal{E}_{\mathcal{T}_i}|} \frac{1}{\sigma^2_j} \left[\theta_j - f_j(\boldsymbol{x}_i)\right]^2, \ \forall i \in \mathcal{T}, \tag{7}$$

where t in the iteration index, $\mathcal{E}_{\mathcal{A}_i} = \{j : (i,j) \in \mathcal{E}_{\mathcal{A}}\}$ and $\mathcal{E}_{\mathcal{T}_i} = \{k : (i,k) \in \mathcal{E}_{\mathcal{T}}, i \neq k\}$ represent the set of all anchor and all target neighbours of the target i respectively, and the first $|\mathcal{E}_{\mathcal{A}_i}| + |\mathcal{E}_{\mathcal{T}_i}|$ elements of $f_j(\boldsymbol{x}_i)$ are given as:

$$f_j(\boldsymbol{x}_i) = P_{0i} - 10\gamma \log_{10} \frac{\|\boldsymbol{x}_i - \hat{\boldsymbol{a}}_j\|}{d_0}, \text{ for } j = 1, ..., |\mathcal{E}_{\mathcal{A}_i}| + |\mathcal{E}_{\mathcal{T}_i}|,$$

with

$$\hat{\boldsymbol{a}}_j = \begin{cases} \boldsymbol{a}_j, & \text{if } j \in \mathcal{A}, \\ \hat{\boldsymbol{x}}^{(t)}_j, & \text{if } j \in \mathcal{T}. \end{cases}$$

The following step carried out by the authors involved some approximating to (7) by two different non-convex estimators (one for each algorithm). This approximations were motivated by the need to obtain a smoother estimator, with the pay-off being an introduction of some bias with respect to the ML solution. These smoother, but still non-convex, estimators we then converted into a convex problem and GTRS framework, respectively. In Section III-A and Section III-B, we present what resulted from those relaxations, for more details on the processes see [21].

A. Distributed SOCP Algorithm

The first relaxation is done by employing SOCP techniques, resulting in

$$\underset{\boldsymbol{x}_i,\boldsymbol{r},\boldsymbol{z},\boldsymbol{g},\boldsymbol{p},e_1,e_2,e_3}{\text{minimize}} \quad e_1 + e_2 + e_3$$

subject to

$$\begin{gathered}
\|\boldsymbol{x}_i - \hat{\boldsymbol{a}}_j\| \le r_{ij},\ \forall(i,j) \in \mathcal{E}_\mathcal{A} \cup \mathcal{E}_\mathcal{T},\\
z_{ij} = \lambda_{ij} r_{ij} - d_0,\ \forall(i,j) \in \mathcal{E}_\mathcal{A} \cup \mathcal{E}_\mathcal{T},\\
g_{ij} = \boldsymbol{c}_{ij}^T(\boldsymbol{x}_i - \boldsymbol{a}_j),\ \forall(i,j) \in \mathcal{E}_\mathcal{A},\\
p_{ij} = \boldsymbol{k}_{ij}^T(\boldsymbol{x}_i - \boldsymbol{a}_j) - r_{ij}\cos(\alpha_{ij}^\mathcal{A}),\ \forall(i,j) \in \mathcal{E}_\mathcal{A},\\
\left\|\begin{bmatrix} 2\boldsymbol{z} \\ e_1 - 1\end{bmatrix}\right\| \le e_1 + 1, \left\|\begin{bmatrix} 2\boldsymbol{g} \\ e_2 - 1\end{bmatrix}\right\| \le e_2 + 1, \left\|\begin{bmatrix} 2\boldsymbol{p} \\ e_3 - 1\end{bmatrix}\right\| \le e_3 + 1.
\end{gathered} \tag{8}$$

where

$$\lambda_{ij} = \begin{cases} 10^{\frac{P_{ij}^\mathcal{A} - P_{0i}}{10\gamma}}, & \text{if } j \in \mathcal{A},\\ 10^{\frac{P_{ij}^\mathcal{T} - P_{0i}}{10\gamma}}, & \text{if } j \in \mathcal{T},\end{cases}$$

$$\boldsymbol{c}_{ij} = [-\sin(\phi_{ij}^\mathcal{A}), \cos(\phi_{ij}^\mathcal{A}), 0]^T,$$

$$\boldsymbol{k}_{ij} = [0, 0, 1]^T,$$

r_{ij}, z_{ij}, g_{ij} and p_{ij} are auxiliary variables, while e_1, e_2 and e_3 are epigraph variable [26]. This type of problem can be efficiently solved by the CVX package [28]. From now on, we will refer to (8) as “SOCP”.

B. Distributed SR-WLS Algorithm

The problem can also be relaxed if rewritten as a quadratic program with a quadratic constraint, leading to

$$\hat{\boldsymbol{y}}_i^{(t+1)} = \underset{\boldsymbol{y}_i}{\arg\min}\ \|\boldsymbol{W}(\boldsymbol{A}\boldsymbol{y}_i - \boldsymbol{b})\|^2$$

subject to

$$\boldsymbol{y}_i^T \boldsymbol{D} \boldsymbol{y}_i + 2\boldsymbol{l}^T \boldsymbol{y}_i = 0, \tag{9}$$

where $\boldsymbol{W} = \boldsymbol{I}_3 \otimes \operatorname{diag}(\boldsymbol{w})$,

$$\boldsymbol{A} = \begin{bmatrix} \vdots & \vdots \\ -2\lambda_{ij}^2 \hat{\boldsymbol{a}}_j^T & \lambda_{ij}^2 \\ \vdots & \vdots \\ \boldsymbol{c}_{ij}^T & 0 \\ \vdots & \vdots \\ \boldsymbol{k}_{ij}^T & 0 \\ \vdots & \vdots \end{bmatrix}, \boldsymbol{b} = \begin{bmatrix} \vdots \\ d_0^2 - \lambda_{ij}^2 \|\hat{\boldsymbol{a}}_j\|^2 \\ \vdots \\ \boldsymbol{c}_{ij}^T \boldsymbol{a}_j \\ \vdots \\ \boldsymbol{k}_{ij}^T \boldsymbol{a}_j + \widehat{d}_{ij}^\mathcal{A} \cos(\alpha_{ij}^\mathcal{A}) \\ \vdots \end{bmatrix},$$

$$\boldsymbol{D} = \begin{bmatrix} \boldsymbol{I}_3 & \boldsymbol{0}_{3\times1} \\ \boldsymbol{0}_{1\times3} & 0 \end{bmatrix}, \boldsymbol{l} = \begin{bmatrix} \boldsymbol{0}_{3\times1} \\ -1/2 \end{bmatrix},$$

with $\boldsymbol{y}_i = [\boldsymbol{x}_i^T, \|\boldsymbol{x}_i\|^2]^T, \forall i \in \mathcal{T}$, and $\boldsymbol{w} = [\sqrt{w_{ij}}]$ where

Fig. 1. The added connection failure scenario to the distributed SOCP/SR-WLS algorithm proposed by [21]

Require: $\hat{\boldsymbol{X}}^{(0)}$, $T_{\max}$, $\mathcal{C}$, u, P_f, $\boldsymbol{a}_j$, $\forall j \in \mathcal{A}$

1: **Initialize:** $t \leftarrow 0$
2: **repeat**
3: **for** $c = 1, ..., \mathcal{C}$ **do**
4: **for all** $i \in \mathcal{C}_c$ (in parallel) **do**
5: Collect $\hat{\boldsymbol{a}}_j, \forall j \in \mathcal{E}_{\mathcal{A}_i} \cup \mathcal{E}_{\mathcal{T}_i}$
6: $\hat{\boldsymbol{x}}_i^{(t+1)} \leftarrow \begin{cases} \text{solve (8), if SOCP} \\ \text{solve (9), if SR-WLS} \end{cases}$
7: **for all** $j \in \mathcal{E}_{\mathcal{A}_i} \cup \mathcal{E}_{\mathcal{T}_i}$ **do**
8: $u \leftarrow \mathcal{U}(0, 1)$
9: **if** $u > P_f$ **then**
10: Broadcast $\hat{\boldsymbol{x}}_i^{(t+1)}$ to $\hat{\boldsymbol{a}}_j$
11: **end if**
12: **end for**
13: **end for**
14: **end for**
15: $t \leftarrow t + 1$
16: **until** $t < T_{\max}$

$$w_{ij} = 1 - \frac{\widehat{d}_{ij}}{\sum_{(i,j)\in\mathcal{E}_\mathcal{A}\cup\mathcal{E}_\mathcal{T}} \widehat{d}_{ij}}.$$

This type of problem, where the objective function and the constraint are both quadratic, is known as GTRS [29], and can be solved *exactly* by a simple bisection method [27]. In the remaining of this paper we will refer to (9) as “SR-WLS”.

IV. Redesigned Algorithm

In order to evaluate the effects of communication failure between nodes of the network, it is necessary to redesign the algorithm proposed in [21] to reflect such scenario. Figure 1 summarizes the redesigned algorithm, where T_max is the maximum number of iterations, C is the number of colours (see [21]), and P_f is the probability of link failure which can be given values from $[0, 1]$. A probability of information exchange failure between two incident sensors is added in the form of conditional statement, as can be seen in lines 7-12. A sample from a standard uniform distribution is taken each time a sensor tries to send its updated location estimate to one of its neighbours. That sample is then compared to a certain probability fixed at start of the computational period. This comparison decides if the information dissemination succeeds or not, to that particular neighbour.

V. Performance Results

In this section we explain the aspects of the numerical simulations performed and present a set of results that illustrate impact on performance of the introduced changes in terms of convergence. For benchmark purposes, we use the SOCP and SR-WLS algorithms proposed in [21] without link failures as a lower bound for time of convergence. All of the presented algorithms were solved by using the MATLAB package CVX [28].

For our computational experiments we used the Monte Carlo (M_c), method in order to have a probabilistic interpretation of the algorithms' performance. We generated $M_c = 500$ random sensor locations, as well as noise samples for each of the range and angle measurements. For each Monte Carlo (M_c) run, $M = 50$ targets and $N = 20$ anchors were randomly deployed inside a cubic area of side $B = 20$. Only networks that formed a connected graph were considered. We set the reference distance to $d_0 = 1$ m and the communication range of a sensor to $R = 6.5$ m. The reference power for each sensor was sampled from a uniform distribution on an interval $[P_0^{\text{Low}}, P_0^{\text{Up}}]$, i.e., $P_{0i} \sim \mathcal{U}[P_0^{\text{Low}}, P_0^{\text{Up}}]$ dBm. In order to account for a realistic measurement model mismatch and since in reality knowledge of the PLE is imperfect, the true PLE was drawn from $\gamma_{ij} \sim \mathcal{U}[2.7, 3.3], \forall (i,j) \in \mathcal{E}_{\mathcal{A}} \cup \mathcal{E}_{\mathcal{T}}, i \neq j$, while the assumed value of the PLE within the estimation process was fixed to $\gamma = 3$. The algorithms also assume that the initial targets' locations, $\hat{\boldsymbol{X}}^{(0)}$, is in the cubic volume's centre, i.e., $\hat{\boldsymbol{x}}_i^{(0)} = [\frac{B}{2}, \frac{B}{2}, \frac{B}{2}]^T \; \forall i \in \mathcal{T}$. Both algorithms were tested for three cases of link failure probability P_f, specifically $50, 70$ and 90%.

As a performance metric we use the normalized root mean square error (NRMSE), defined as

$$\text{NRMSE} = \sqrt{\frac{1}{MM_c} \sum_{i=1}^{M_c} \sum_{j=1}^{M} \|\boldsymbol{x}_{ij} - \widehat{\boldsymbol{x}}_{ij}\|^2},$$

where $\widehat{\boldsymbol{x}}_{ij}$ stands for the estimate of the true location of the j-th target, $\boldsymbol{x}_{ij}$, in the i-th Monte Carlo run.

Fig. 2 and Fig. 3 illustrate the NRMSE (m) versus t comparison for different probabilities of link failure for the SOCP and SR-WLS algorithms respectively. The figures exhibit that more iterations are required for the algorithm to converge when the link failure probability is increased, as expected. We see that although the propagation of the updated information is slowed down, all targets eventually collect the complete information form their neighbours. It can be noticed that both algorithms behave similarly and probabilities of link failure up to 70% are easily mitigated by a small increase of iterations required for convergence. As expected, the performance of both algorithms worsens with the probability of 90% of link failure. However, this setting is more theoretical than practical, since it basically means that there is nearly no communication between sensors, or in other words, that the network does not work. Nevertheless, the algorithms still converge if enough iterations are made. This confirms the robustness of the proposed algorithms in [21].

VI. Conclusion

Link failure is a known problem in wireless communications, that should be addressed when designing algorithms for WSNs. An algorithm with inability to still converge in view of failed transmissions may result in the exhaustion of the network's battery resources as the amount of retransmissions go up. In this work we tested the algorithms proposed in [21] in a more practical scenario, by alleviating one of the strong assumptions made by the authors. The two recently proposed distributed algorithms were tested against moderate to high probability of communication failure at the broadcast phase. We have shown that the algorithms are very robust, and behave positively in a more practical scenario.

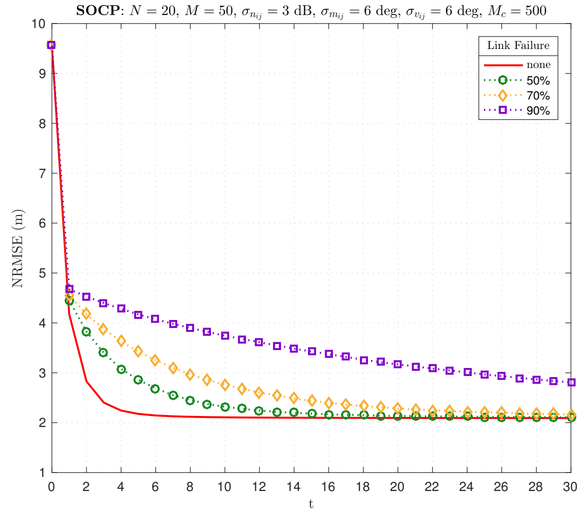

Fig. 2. SOCP, NRMSE versus t comparison for different values of P_f, when $N = 20$, $M = 50$, $R = 6.5$ m, $\sigma_{n_{ij}} = 3$ dB, $\sigma_{m_{ij}} = 6$ deg, $\sigma_{v_{ij}} = 6$ deg, $\gamma_{ij} \in \mathcal{U}[2.7, 3.3]$, $\gamma = 3$, $B = 20$ m, $P_{0i} \in \mathcal{U}[-12, -8]$ dBm, $d_0 = 1$ m, $M_c = 500$.

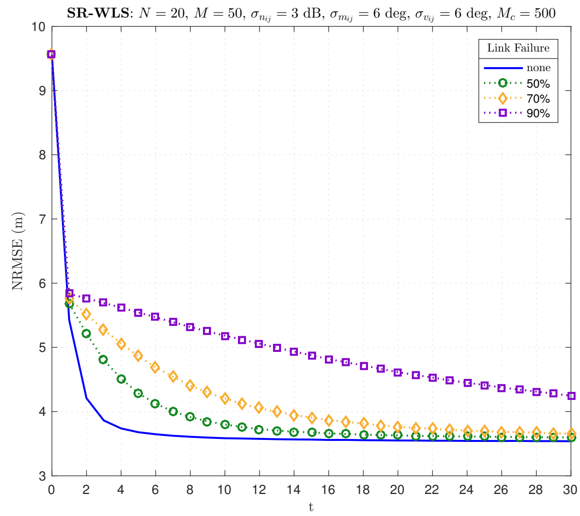

Fig. 3. SR-WLS, NRMSE versus t comparison for different values of P_f, when $N = 20$, $M = 50$, $R = 6.5$ m, $\sigma_{n_{ij}} = 3$ dB, $\sigma_{m_{ij}} = 6$ deg, $\sigma_{v_{ij}} = 6$ deg, $\gamma_{ij} \in \mathcal{U}[2.7, 3.3]$, $\gamma = 3$, $B = 20$ m, $P_{0i} \in \mathcal{U}[-12, -8]$ dBm, $d_0 = 1$ m, $M_c = 500$.

Acknowledgment

This work was partially supported by Fundação para a Ciência e a Tecnologia under Projects PEst-OE/EEI/UI0066/2014 (UNINOVA) and UID/EEA/50008/

2013 (Instituto de Telecomunicações), Program Investigador FCT under Grant IF/00325/2015 and Grant SFRH/BD/91126/2012.

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