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# Strategic Asset Allocation in Brazil

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## Abstract

We study the impact of asset returns' predictability on optimal portfolio allocation, considering investors concerned with a steady flow of long-term consumption. Relying on a monthly database for 2006-2016, the analysis focuses on the Brazilian context covering returns on (1) a real short-term asset, (2) a long-term asset, (3) a Brazilian stock index, and five other state variables. Predictability of long-term assets returns has a significant impact on their overall optimal demand. In addition, by using the S&P500 index we show that foreign stocks are more predictable than the Brazilian index for moderately conservative investors.

*JEL Classification:* G11

*Keywords:* Intertemporal hedging demand; Portfolio Choice; Predictability; Strategic asset allocation

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## 1 Introduction

Harry Markowitz (1952) pioneered the analysis of quantitative finance with the unprecedented article *Portfolio Selection*. In his work, the author shows the importance of the process of asset allocation through portfolio diversification. The mean-variance analysis of Markowitz, despite having proved to be a robust tool in the process of asset allocation and be widely used, has shown some inconsistencies with long-term asset allocation. This theory is based on the assumption that investors are only concerned with the wealth distribution a period ahead. Thus, the model is static and does not capture the investor's concern to maintain a flow of long-term consumption provided by its wealth.

Markowitz's model should be extended in order to take into consideration the long-term allocation process, namely the investors' need to reduce the risk of having their consumption affected over time. Paul Samuelson (1969) and Robert Merton (1969, 1971) started developing that line of research in the context of multi-period models. Because they do not present closed form solution, very complex numerical methods are necessary, which, for a long time, made the use of these models unviable. The computational revolution in the 1990s, alongside with the recent developments in numerical methods, made the implementation of multi-period models more appealing in the recent past.

Solutions of multi-period asset allocation models may differ significantly from those presented by static models, as summarized in Campbell and Viceira (2002). Particularly, when investment opportunities are not constant over time, long-term investors are concerned both with shocks to these opportunities and with shocks to their own wealth. In order to hedge this exposure, long-term investors should seek an intertemporal demand for protection in financial assets. This demand for protection is related to the agents' degree of risk aversion. Investors with relative risk aversion equal to one allocate their wealth purely myopically, not caring about shocks to investment opportunities. As the degree of risk aversion increases,

agents seek for assets that increase in value in case of deterioration in investment opportunities. These assets behave as insurance against the risks of falling returns on wealth.

Investment opportunities are not constant over time. Campbell (1999), Fama and French (1989), among others, provide empirical evidence supporting that. The real rate of short-term interest has changed over time, but, at the same time, the excess returns on stocks and bonds also show some predictability in their dynamics. The most important aspect in these findings is the mean reversion tendency of stocks in the long-run. This feature reflects the different perception of risk that short and long-term investors have.

In order to meet the long-term analysis using the mean-variance model, one could calculate long-run variances and apply them in the static model. However, such a solution would only serve investors who are only concerned with the average and volatility of their wealth in a single period of time. And yet, this investor would be limited to making a single allocation for that period, without the opportunity for rebalancing it over time.

Hereupon, this article replicates the dynamic model proposed by Campbell, Viceira, and Chan (2003). In particular, we study how the predictability and persistence of some selected Brazilian assets' returns affect the long-term wealth distribution of an investor who values a constant stream of consumption, and considers, in his decision, the variability of investment opportunities over time.

## 2 The Model

### 2.1 Securities

The model assumes that  $n$  securities are available to invest, and investors can distribute their wealth, after consumption, among them.  $R_{p,t+1}$  is the real portfolio return,  $\alpha_{i,t}$  is the portfolio weight on asset  $i$ , and  $R_{1,t+1}$  is the benchmark real return. We use a real short-term asset return as a benchmark and, then, calculate excess returns by using it. Though, the benchmark real return is not a risk-free asset.

$$R_{p,t+1} = \sum_{i=2}^n \alpha_{i,t} (R_{i,t+1} - R_{1,t+1}) + R_{1,t+1}. \quad (1)$$

## 2.2 State variables dynamics

Following Campbell *et al.* (2003), we assume a first-order vector autoregressive process VAR(1) to capture the dynamics of the relevant state variables<sup>1</sup>. Hereby, the vector of log excess returns,  $\mathbf{x}_{t+1}$ , is defined in the following way:

$$\mathbf{x}_{t+1} \equiv \begin{bmatrix} r_{2,t+1} - r_{1,t+1} \\ r_{3,t+1} - r_{1,t+1} \\ \vdots \\ r_{n,t+1} - r_{1,t+1} \end{bmatrix}, \quad (2)$$

where  $r_{i,t+1} \equiv \log(R_{i,t+1})$ . In this article,  $r_{1,t+1}$  is the real short-term return,  $r_{2,t+1}$  is the real return on nominal bonds, or long-term assets, and  $r_{3,t+1}$  is the real stock index return. The system is going to include three other state variables,  $s_{t+1}$ . Placing  $r_{1,t+1}$ ,  $\mathbf{x}_{t+1}$ , and  $s_{t+1}$  together, we get a  $m \times 1$  vector, the state vector:

$$\mathbf{z}_{t+1} \equiv \begin{bmatrix} r_{1,t+1} \\ \mathbf{x}_{t+1} \\ s_{t+1} \end{bmatrix}. \quad (3)$$

The first-order vector autoregressive system is equated in the following matrix form:

$$\mathbf{z}_{t+1} = \mathbf{\Phi}_0 + \mathbf{\Phi}_1 \mathbf{z}_t + \mathbf{v}_{t+1}, \quad (4)$$

$$\mathbf{v}_{t+1} \sim N(0, \mathbf{\Sigma}_v),$$

$$\mathbf{\Sigma}_v \equiv \text{Var}_t(\mathbf{v}_{t+1}) = \begin{bmatrix} \sigma_1^2 & \sigma'_{1x} & \sigma'_{1s} \\ \sigma_{1x} & \mathbf{\Sigma}_{xx} & \mathbf{\Sigma}'_{xs} \\ \sigma_{1s} & \mathbf{\Sigma}_{xs} & \mathbf{\Sigma}_{ss} \end{bmatrix}, \quad (5)$$

where,  $\mathbf{\Phi}_0$  is the  $m \times 1$  matrix of intercepts,  $\mathbf{\Phi}_1$  is the  $m \times m$  slope coefficients matrix, and  $\mathbf{v}_{t+1}$  represents the shocks to the state variables. These shocks are independent and identically distributed (*i.i.d.*). Consequently, they can be cross-sectionally correlated, although they are homoscedastic and independently distributed over time. By using a VAR system, we can

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<sup>1</sup> Authors such as Kandel and Stambaugh (1987), Campbell (1991, 1996), Hodrick (1992), and Barberis (2000) have been using this kind of dynamic specification.

analyse the dependence of expected asset returns on their previous values, even as on other variables that show to be good predictors.

Assuming these shocks are homoscedastic, it does not allow the state variables to predict changes in risk. Portfolio choice is only affected by their predictive power in expected returns. However, in the late 1980's and early 1990's some authors<sup>2</sup> have studied the capability of the state variables in predicting risk, and found out that these variables had little power in predicting risk. The state variables had bigger effects in predicting expected returns, dominating the former effect.

### 2.3 Preferences

Here, the investor is going to be assumed of having recursive Epstein-Zin (1989, 1991) preferences. The convenience this model of utility holds is held by its characteristic that allows to separate the definition of relative risk aversion from elasticity of intertemporal substitution. They are not necessarily the inverse of each other anymore, like in the older power utility model,  $\gamma \neq \psi^{-1}$ . Then, the utility function is expressed below:

$$U(C_t, E_t(U_{t+1})) = \left[ (1 - \delta)C_t^{\frac{1-\gamma}{\theta}} + \delta \left( E_t(U_{t+1}^{1-\gamma}) \right)^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}}, \quad (6)$$

where  $C_t$  represents consumption at time  $t$ ,  $\gamma$  holds for relative risk aversion,  $\psi$  is the elasticity of intertemporal substitution,  $\delta$  is the time discount factor,  $\theta$  is equal to  $\frac{(1-\gamma)}{(1-\psi^{-1})}$ , and  $E_t(\cdot)$  is the conditional expectation operator.

At time  $t$ , agents use all relevant information in order to make an optimal consumption and portfolio allocation over time. The investor is constrained to an intertemporal budget constraint:

$$W_{t+1} = (W_t - C_t)R_{p,t+1}, \quad (7)$$

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<sup>2</sup> Campbell (1987), Harvey (1989, 1991), and Glostene, Jagannathan, and Runkle (1993).

where  $W_t$  is wealth at time  $t$ . The Euler equation for consumption, subject to the intertemporal budget constraint, derived by Epstein and Zin (1989 and 1991) is expressed by:

$$E_t \left[ \left\{ \delta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \right\}^{\theta} R_{p,t+1}^{-(1-\theta)} R_{i,t+1} \right] = 1. \quad (8)$$

This equation is set for an asset  $i$ , and also for the portfolio. Consumption and portfolio decisions of investors must satisfy this Euler equation (8). When investment opportunities do not vary over time, the equilibrium decision should be a constant consumption-wealth ratio, and investors optimize their portfolios myopically. The allocation decision is made as if the investment horizon is only one period ahead. However, when investment opportunities vary over time, there are no exact analytical solutions to this equation, except for certain values of  $\gamma$  and  $\psi$ . Giovannini and Weil (1989) showed that, with  $\gamma = 1$ , the investor should follow a myopic rule, and that he chooses a constant ratio of consumption-wealth equal to  $(1 - \delta)$  when  $\psi = 1$ . Though, when  $\gamma = 1$ , a choice for consumption relative to wealth is not constant, unless  $\psi = 1$ , and likewise, with  $\psi = 1$ , the optimal allocation is not myopic, unless  $\gamma = 1$ . This is the case of the log utility investor,  $\gamma = \psi = 1$ .

### 3 Methodology of the solution

#### 3.1 An approximation

In this framework, we are going to use log returns. Campbell and Viceira (1999, 2001) got to the following expression of log portfolio returns<sup>3</sup>, accurate for continuous time:

$$r_{p,t+1} = r_{1,t+1} + \alpha'_t \mathbf{x}_{t+1} + \frac{1}{2} \alpha'_t (\sigma_x^2 - \Sigma_{xx} \alpha_t). \quad (9)$$

The vector  $\sigma_x^2 \equiv \text{diag}(\Sigma_{xx})$  contains the variances of excess returns.

The budget constraint in (7) is not linear. According to Campbell (1993, 1996), log-linearization around the unconditional mean of the log consumption-wealth ratio obtains

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<sup>3</sup> See Appendix A of Campbell *et al.* (2003) to look at the derivation of log portfolio returns.

$$\Delta w_{t+1} \approx r_{p,t+1} + \left(1 - \frac{1}{\rho}\right)(c_t - w_t) + k, \quad (10)$$

where  $\Delta w_{t+1}$  is the difference between  $w_{t+1}$  and  $w_t$ ,  $\rho \equiv 1 - \exp(E[c_t - w_t])$  and  $k \equiv \log(\rho) + (1 - \rho)\log\left(\frac{(1-\rho)}{\rho}\right)$ . As consumption is optimally chosen by the investor,  $\rho$  depends on the optimal level of  $c_t$  relative to  $w_t$ , and, thus, is endogenous. This form of budget constraint is exact if the elasticity of intertemporal substitution is equal to one and, in this case,  $c_t - w_t$  is a constant equal to  $(1 - \delta)$ .

By applying a second-order Taylor expansion to the Euler equation (8) around the conditional means of  $\Delta c_{t+1}$ ,  $r_{p,t+1}$  and  $r_{i,t+1}$ , one obtains a loglinearized Euler equation that becomes exact if consumption and return on assets have lognormal joint distribution ( $\psi = 1$ ). Hence, we get to a transformed log linearized Euler equation as follows:

$$E_t(r_{i,t+1} - r_{1,t+1}) + \frac{1}{2}Var_t(r_{i,t+1} - r_{1,t+1}) = \frac{\theta}{\psi}(\sigma_{i,c-w,t} - \sigma_{1,c-w,t}) + \gamma(\sigma_{i,p,t} - \sigma_{1,p,t}) - (\sigma_{i,1,t} - \sigma_{1,1,t}), \quad (11)$$

where,

$$\begin{aligned} \sigma_{i,c-w,t} &= Cov(r_{i,t+1}, c_{t+1} - w_{t+1}), \sigma_{1,c-w,t} = Cov(r_{1,t+1}, c_{t+1} - w_{t+1}), \sigma_{i,p,t} = Cov(r_{i,t+1}, r_{p,t+1}), \\ \sigma_{1,p,t} &= Cov(r_{1,t+1}, r_{p,t+1}), \sigma_{i,1,t} = Cov(r_{i,t+1}, r_{1,t+1}), \text{ and } \sigma_{1,1,t} = Var(r_{1,t+1}). \end{aligned}$$

Left-hand part of the equation (11) is the risk premium on asset  $i$  over asset 1, adjusted for Jensen's Inequality by adding half of the excess return variance. This equation relates the risk premium of asset  $i$  to its excess of covariance with consumption growth, to its excess of covariance with the return on the portfolio, and to the covariance of its excess return with the return on asset 1 (the last term is eliminated from the equation when asset 1 presents no risk). Since consumption growth and portfolio return are endogenous, this equation is a first-order condition that describes the optimal solution and not a solution on its own.

### 3.2 Solving the approximate model

In order to solve the model, Campbell and Viceira (2002) assume that the optimal rule for consumption and portfolio are presented as follows:



$$\boldsymbol{\alpha}_t = \mathbf{A}_0 + \mathbf{A}_1 \mathbf{z}_t, \quad (12)$$

$$c_t - w_t = b_0 + \mathbf{B}'_1 \mathbf{z}_t + \mathbf{z}'_t \mathbf{B}_2 \mathbf{z}_t. \quad (13)$$

The optimal rule for the portfolio is thus linear in the VAR state vector, however the optimal rule for consumption is quadratic.  $\mathbf{A}_0$   $(n-1) \times 1$ ,  $\mathbf{A}_1$   $(n-1) \times m$ ,  $b_0$   $1 \times 1$ ,  $\mathbf{B}_1$   $m \times 1$ , and  $\mathbf{B}_2$   $m \times m$  are matrices of constant coefficients to be determined. This is the generalization of the solution obtained by Campbell and Viceira (1999) for the multivariate case. It is noteworthy that only  $m + \frac{m^2-m}{2}$  elements of  $\mathbf{B}_2$  are to be determined.

In order to validate equations (12) and (13) and obtain the solution of the parameters, the conditional moments appearing in (11) are written as functions of the VAR parameters and the unknown parameters of (12) and (13). Next, we solve this resulting equation for the parameters that satisfy (11). Consequently, the conditional expectation of equality (11) is:

$$E_t(x_{t+1}) + \frac{1}{2} Var_t(x_{t+1}) = \mathbf{H}_x \boldsymbol{\Phi}_0 + \mathbf{H}_x \boldsymbol{\Phi}_1 \mathbf{z}_t + \frac{1}{2} \boldsymbol{\sigma}_x^2. \quad (14)$$

Campbell *et al.* (2003) show that the three conditional moments on the right-hand side of equation (11) can be written as linear functions of the state variables:

$$\boldsymbol{\sigma}_{c-w,t} - \sigma_{1,c-w,t} \mathbf{l} \equiv [\sigma_{i,c-w,t} - \sigma_{1,c-w,t}]_{i=2,\dots,n} = \boldsymbol{\Lambda}_0 + \boldsymbol{\Lambda}_1 \mathbf{z}_t, \quad (15)$$

$$\boldsymbol{\sigma}_{p,t} - \sigma_{1,p,t} \mathbf{l} \equiv [\sigma_{i,p,t} - \sigma_{1,p,t}]_{i=2,\dots,n} = \boldsymbol{\Sigma}_{xx} \boldsymbol{\alpha}_t + \boldsymbol{\sigma}_{1x}, \quad (16)$$

$$\boldsymbol{\sigma}_{1,t} - \sigma_{1,1,t} \mathbf{l} \equiv [\sigma_{i,1,t} - \sigma_{1,1,t}]_{i=2,\dots,n} = \boldsymbol{\sigma}_{1x}, \quad (17)$$

where  $\mathbf{l}$  is a vector of ones.

### 3.3 Optimal Portfolio Choice

After the log linearized Euler equation (11) being solved for the optimal portfolio rule:

$$\boldsymbol{\alpha}_t = \frac{1}{\gamma} \boldsymbol{\Sigma}_{xx}^{-1} \left[ E_t(x_{t+1}) + \frac{1}{2} Var_t(x_{t+1}) + (1-\gamma) \boldsymbol{\sigma}_{1x} \right] + \frac{1}{\gamma} \boldsymbol{\Sigma}_{xx}^{-1} \left[ -\frac{\theta}{\psi} (\boldsymbol{\sigma}_{c-w,t} - \sigma_{1,c-w,t} \mathbf{l}) \right]. \quad (18)$$

$E_t(x_{t+1}) + \frac{1}{2} Var_t(x_{t+1})$  and  $(\boldsymbol{\sigma}_{c-w,t} - \sigma_{1,c-w,t} \mathbf{l})$  are linear functions of  $\mathbf{z}_t$  in (14) and (15), respectively. The optimal portfolio choice is expressed as the sum of two components.

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<sup>4</sup>  $\mathbf{H}_x$  is a selection matrix that captures the excess vector of returns of the state vector  $\mathbf{z}_t$ .

The first term on the right of (18) is the myopic component. When the benchmark asset is considered not risky ( $\sigma_{1x} = 0$ ), the myopic allocation is ruled essentially by the Sharpe ratio (SR) of the risky assets: the vector of expected excess returns is adjusted by the inverse of the variance-covariance matrix of returns on risky assets and the inverse of the relative risk aversion coefficient. If asset 1 is risky, investors with  $\gamma \neq 1$  adjust their allocation by a term  $(1 - \gamma)\sigma_{1x}$ . This component does not depend on  $\psi$ .

The second term on the right of (18) represents the demand for protection. In this model, investment opportunities vary over time because the expected returns of the assets depend on the state variables. An investor who is more risk averse than a log utility investor, will seek protection against adverse changes in investment opportunities (Merton 1969, 1971). He will optimize his portfolio by having strategic positions in some assets. A logarithmic investor optimizes his allocation purely myopically, having no hedging demand. If  $\gamma = 1$ , then  $\theta = 0$ , therefore, the protection component disappears. Yet, when investment opportunities are constant, the demand for protection is zero for every level of risk aversion, corresponding to a VAR model with just a constant term.

By replacing (14) and (15) into (18):

$$\alpha_t \equiv \mathbf{A}_0 + \mathbf{A}_1 \mathbf{z}_t, \quad (19)$$

where,

$$\mathbf{A}_0 = \left(\frac{1}{\gamma}\right) \Sigma_{xx}^{-1} \left( \mathbf{H}_x \Phi_0 + \frac{1}{2} \sigma_x^2 + (1 - \gamma) \sigma_{1x} \right) + \left(1 - \frac{1}{\gamma}\right) \Sigma_{xx}^{-1} \left( \frac{-\Lambda_0}{1 - \psi} \right), \quad (20)$$

$$\mathbf{A}_1 = \left(\frac{1}{\gamma}\right) \Sigma_{xx}^{-1} \mathbf{H}_x \Phi_1 + \left(1 - \frac{1}{\gamma}\right) \Sigma_{xx}^{-1} \left( \frac{-\Lambda_1}{1 - \psi} \right). \quad (21)$$

Equation (19) confirms the initial assumption made by Campbell *et al.* (2003) for the initial guessing of the optimal portfolio rule. Coefficients matrices  $\mathbf{A}_0$  and  $\mathbf{A}_1$  show as functions of the parameters that describe preferences and the dynamics of the state variables, as well as of the consumption-wealth ratio parameters,  $\mathbf{B}_1$  and  $\mathbf{B}_2$ , through  $\Lambda_0$  and  $\Lambda_1$ . It is important to

note that the terms  $\left(1 - \frac{1}{\gamma}\right)$  in equations (20) and (21) picture the effect of intertemporal protection on optimal portfolio choice. Therefore, the demand for intertemporal protection affects the average allocation of the portfolio through  $\mathbf{A}_0$  and  $\mathbf{A}_1$ , and the sensitivity of the optimal allocation to variations in the state variables through  $\mathbf{A}_1$ .

Campbell *et al.* (2003) show that given  $\rho$ , the coefficient matrices  $\left(\frac{-\mathbf{A}_0}{1-\psi}\right)$  and  $\left(\frac{-\mathbf{A}_1}{1-\psi}\right)$  are independent of the intertemporal elasticity of substitution  $\psi$ . Hence, the optimal portfolio rule is independent for any  $\psi$  given  $\rho$ .  $\psi$  only affects portfolio choice as it defines  $\rho$ .

#### 4 Empirical Application

Previous section presented the academical background of strategic asset allocation. Here, such a set-up is used to assess how investors, who differ in their preferences for consumption and risk aversion, distribute their portfolio among the available assets provided by the model: a real short-term asset (STA), a long-term asset (LTA), and a stock. The VAR system describes the dynamics of investment opportunities using a real short-term asset return, an excess-return of a stock-market index, an excess-return of a long-term asset, and other state variables that help to forecast these excess-returns. Optimal allocation of the portfolio is computed for different levels risk-aversion  $\gamma$ . Assuming that  $\psi = 1$ ,  $c_t - w_t = 1 - \delta^5$ , and  $\delta = 0,95^6$  in annual terms, implies that the parameter  $\rho = \delta^7$ .

##### 4.1 Data Description

For the purpose of this study, log returns of portfolio assets, and other state variables are necessary. Our sample covers January 2006 to December 2016, using monthly frequency. In order to construct monthly log returns of the real short-term asset (STA), we obtained the

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<sup>5</sup> When  $\psi = 1$ , Campbell *et al.* (2003) show that consumption-wealth ratio equals  $1 - \delta$ .

<sup>6</sup> This value, 0,95, is obtained by  $\delta = e^{-\bar{s}r}$ , where  $\bar{s}r$  is the mean average of the CDI rate of return for the considered period. We get the same value for  $\delta$  when we consider the IDkA Pre 3M return instead.

<sup>7</sup> The numerical procedure used to compute the optimal allocation of assets is described in detail in Appendix B of Campbell, *et al.* (2003).

accumulated monthly nominal return over the CDI rate, which is a daily average of the overnight interbank loans in Brazil, adjusted by inflation –  $r_{cdi}$ . For the monthly log excess-return of stock-market index (Stock), we gathered monthly nominal returns over the Ibovespa index  $r_{ibov}$ , monthly nominal returns over the IbrX index  $r_{ibrx}$ , and monthly nominal returns over the S&P 500 index plus the exchange-rate returns between the U.S. Dollar and the Brazilian Real  $r_{spix}$  (SPXBRL). Monthly log excess returns of the long-term asset (LTA) were constructed using monthly nominal returns of a set of indexes that measure the behaviour of synthetic portfolios of Brazilian federal securities with constant term<sup>8</sup>: IDkA Pre 3M,  $r_{3m}$  (three months), IDkA Pre 1A,  $r_{1y}$  (one year), IDkA Pre 2A,  $r_{2y}$  (two years), IDkA Pre 3A,  $r_{3y}$  (three years), IDkA Pre 5A,  $r_{5y}$  (five years), IDkA IPCA 2A,  $r_{ipca2y}$  (two years), IDkA IPCA 3A,  $r_{ipca3y}$  (three years), IDkA IPCA 5A,  $r_{ipca5y}$  (five years). The 3 last indexes can be analysed as inflation-indexed bonds. Note that IDkA Pre 3M,  $r_{3m}$ , is also used as an alternative for the monthly log returns of the real short-term asset. Subsequently, the 12-month accumulated Ibovespa dividend-yield,  $s_{d-p}$ , the yield-spread between the IDkA Pre 5A and the IDkA Pre 3M,  $s_{spread}$ , the nominal return over the CDI,  $s_{ycdi}$ , the nominal return over the IDkA Pre 3M,  $s_{y3m}$ , the Emerging Markets Bond Index<sup>9</sup> (EMBI +),  $s_{embi}$ , and, finally, the 12-month accumulated return over the Brazilian Institute of Geography and Statistics' (IBGE) price index (IPCA-IBGE),  $s_{ipca}$ , were the state variables chosen to predict the returns<sup>10</sup>. All the variables adjusted by inflation used IPCA-IBGE.

In the next sections, we use combinations of these log returns and state variables.

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<sup>8</sup> Since January 2006, these indexes are constructed by the Brazilian Association of Financial and Capital Markets (ANBIMA).

<sup>9</sup> The EMBI + is an index, created by JPMorgan, based on debt securities issued by emerging countries. It shows the financial returns obtained each day by a selected portfolio of securities from these countries. The unit of measure is the base point. Ten basis points are equivalent to one-tenth of 1%. Points show the difference between the rate of return of emerging-market securities and that offered by securities issued by the US Treasury. This difference is the spread, or the sovereign spread, which is also known by risk of Brazil.

<sup>10</sup>  $s_{d-p}$ ,  $s_{spread}$ , and  $s_y$  are variables that have been identified as good return predictors by authors such as, respectively, Fama and Schwert (1977), Campbell (1987), and Glosten *et al.* (1993). Moreover,  $s_{embi}$ , and  $s_{ipca}$  have been identified as good return predictors by previous Brazilian empirical research.

## 4.2 Sample Analysis

Table 1.1 and Table 1.2 show the first and second moments for the historical series of the variables introduced before. In Table 1.1, stock returns and LTA returns are in the excess form over the CDI return. These same returns, in Table 1.2, are in the excess form over the IDkA Pre 3M return. Sample statistics are in percentage and in monthly frequency. Log mean excess-returns are adjusted by half of the variance to accommodate for Jensen's inequality. In both tables, LTA have the highest log mean excess-return and the lowest volatility, therefore, they also hold the highest Sharpe ratios, contrary to Campbell, *et al.* (2003) where stocks had the highest Sharpe ratio.

**Table 1.1 – Sample Statistics (Excess returns over CDI return)**

	CDI	Ibovespa	Ibrx	SPXBRL	IDkA Pre 1A	IDkA Pre 2A	IDkA Pre 3A	IDkA Pre 5A
<b>Mean</b>	0,416%	-0,231%	-0,060%	-0,099%	0,093%	0,168%	0,213%	0,301%
<b>SD</b>	0,342%	6,612%	6,317%	4,543%	0,406%	1,092%	1,849%	3,410%
<b>Sharpe Ratio</b>	-	-0,035	-0,009	-0,022	0,230	0,154	0,115	0,088
	IDkA IPCA 2A	IDkA IPCA 3A	IDkA IPCA 5A	EMBI +	IPCA	SPREAD	DIV	YIELD_CDI
<b>Mean</b>	0,217%	0,242%	0,259%	2,498%	0,480%	0,772%	3,590%	0,895%
<b>SD</b>	0,830%	1,217%	1,886%	0,819%	0,270%	3,398%	0,690%	0,186%
<b>Sharpe Ratio</b>	0,261	0,199	0,137	-	-	-	-	-

**Table 1.2 – Sample Statistics (Excess returns over IDkA Pre 3M return)**

	IDkA Pre 3M	Ibovespa	Ibrx	SPXBRL	IDkA Pre 1A	IDkA Pre 2A	IDkA Pre 3A	IDkA Pre 5A
<b>Mean</b>	0,429%	-0,244%	-0,073%	-0,112%	0,081%	0,155%	0,200%	0,288%
<b>SD</b>	0,354%	6,611%	6,314%	4,545%	0,372%	1,067%	1,827%	3,391%
<b>Sharpe Ratio</b>	-	-0,037	-0,011	-0,025	0,216	0,145	0,109	0,085
	IDkA IPCA 2A	IDkA IPCA 3A	IDkA IPCA 3A	EMBI +	IPCA	SPREAD	DIV	YIELD_3M
<b>Mean</b>	0,204%	0,229%	0,246%	2,498%	0,480%	0,772%	3,590%	0,908%
<b>SD</b>	0,802%	1,190%	1,864%	0,819%	0,270%	3,398%	0,690%	0,203%
<b>Sharpe Ratio</b>	0,254	0,192	0,132	-	-	-	-	-

It is worth noting that both of the previous tables have a negative log mean excess-return for stocks (Ibovespa, IbrX, and S&P 500). They have also the highest volatility.

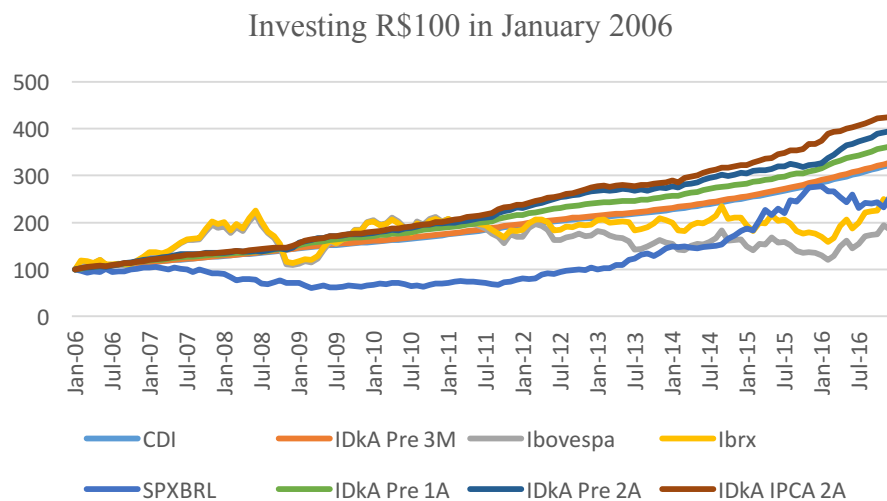
Table 2 contains mean nominal returns for CDI, IDkA Pre 3M, Ibovespa, SPXBRL, IDkA Pre 1A, IDkA Pre 2A and IDkA IPCA 2A, which are the assets considered in the

subsequent exercises. These returns are not in the log form, therefore, they were not corrected by half the variance. They are also not presented in the excess-return form.

**Table 2 – Nominal Returns**

	CDI	IDkA Pre 3M	Ibov	Ibrx	SPXBRL	IDkA Pre 1A	IDkA Pre 2A	IDkA IPCA 2A
<b>Mean</b>	0,899%	0,912%	0,663%	0,836%	0,798%	0,994%	1,069%	1,118%
<b>SD</b>	0,188%	0,205%	6,555%	6,253%	4,549%	0,474%	1,142%	0,871%
<b>Sharpe Ratio</b>	-	-	0,101	0,134	0,175	2,097	0,936	1,284

Once again, it can be seen that Ibovespa shows to be the worst investment, ex-post. This is clear when we analyse Figure 1. This graph shows the result of an investment of R\$100,00 in the beginning of January 2006 in each of the following eight assets: CDI, IDkA Pre 3M, Ibovespa, Ibrx, SPXBRL, IDkA Pre 1A, IDkA Pre 2A, and IDkA IPCA 2A.



*Figure 1*

In December 2016 the CDI strategy results in R\$325,95, the IDkA Pre 3M strategy results in R\$331.49, the Ibovespa strategy results in R\$180,02, the Ibrx strategy results in R\$231,53, and, even taking into account the exchange rate risk, the S&P500 strategy yields R\$249,58, exceeding both Brazilian stock-market indexes. All three assets that follow outperformed STA and stocks: IDkA Pre 1A, IDkA Pre 2A, and IDkA IPCA 2A. In December 2016, the IDkA Pre 1A strategy results in R\$368,34, the IDkA Pre 2A strategy results in R\$403,61, the IDkA IPCA 2A strategy results in R\$432,02.

### 4.3 VAR Estimation

There were made 391 combinations containing a real short-term asset, a long-term asset and a stock (Brazilian stock or foreign stock), and three other state variables by using the data presented in Section 4.1. Afterwards, for the purpose of this analysis, we selected four types of combinations: one that includes a long-term asset and a Brazilian stock-market index; a Brazilian stock-market index and a foreign equity index; a long-term inflation-indexed asset and a Brazilian stock-market index; and, finally, a combination that contains a long-term asset, a Brazilian stock-market index, and the same state variables used by Campbell *et al.* (2003), which are the 12-month accumulated dividend yield, the spread between a long-term asset and a short-term asset, and the benchmark nominal return. The selection process consisted in selecting the combinations with more statistical significant coefficients and highest excess-returns' R-squared in their VAR estimations. They are going to be named combination 1, combination 2, combination 3 and combination 4, respectively.

Tables 3 and 4 display the estimated VAR parameters and the covariance structure of unexpected returns, respectively. Tables 3 exhibit the estimated coefficients of the VAR setting, as well as their t-statistics, in parentheses, and the R-squared statistics. Tables 4 show, in the main diagonal, the standard deviations of the unexpected returns multiplied by 100, and the upper-diagonal elements are the cross-correlation coefficients of unexpected returns.

#### 4.3.1 VAR Estimation – Combination 1

In this combination, it was used the CDI real return as a real short-term variable, the IDkA Pre 1A excess return over CDI as a LTA, the Ibovespa excess-return over CDI as a stock-market index, and three other state variables: the 12-month accumulated dividend yield, the EMBI + spread, and the CDI nominal return. This is the first-type combination with more statistical significant variables and highest LTA and stock R-squared.

In Table 3.1, the second column presents the equation of the real short-term asset, where the only variable that explains its future returns is its one-year lagged returns<sup>11</sup>.

Third column of Table 3.1 shows the equation of the LTA,  $r_{1y,t+1}$ . Lagged LTA returns are not statistically significant in predicting its future returns. The EMBI + spread has a coefficient with t-statistics greater than 2, being the only variable with predictive power in this equation.

**Table 3.1 – VAR Parameters**

	$r_{cdi,t+1}$ (real)	$r_{1y,t+1}$ (excess)	$r_{ibov,t+1}$ (excess)	$s_{d-p,t+1}$	$s_{embi,t+1}$	$s_{ycdi,t+1}$
	(t-stat)	(t-stat)	(t-stat)	(t-stat)	(t-stat)	(t-stat)
$r_{cdi,t}$	0,629 (7,949)	0,246 (1,806)	3,164 (1,412)	-0,050 (-0,504)	-0,089 (-0,830)	-0,022 (-0,667)
$r_{1y,t}$	0,016 (0,309)	0,095 (1,061)	0,008 (0,005)	-0,001 (-0,021)	0,008 (0,110)	-0,013 (-0,619)
$r_{ibov,t}$	-0,005 (-1,688)	-0,007 (-1,266)	0,198 (2,179)	-0,009 (-2,203)	-0,012 (-2,741)	0,000 (-0,269)
$s_{d-p,t}$	-0,059 (-1,709)	0,048 (0,816)	0,691 (0,709)	0,889 (20,445)	-0,029 (-0,619)	-0,028 (-1,992)
$s_{embi,t}$	0,001 (0,033)	0,107 (2,027)	1,824 (2,093)	-0,026 (-0,669)	0,889 (21,268)	0,024 (1,867)
$s_{ycdi,t}$	0,225 (1,390)	-0,105 (-0,377)	-6,733 (-1,469)	-0,085 (-0,413)	0,339 (1,543)	0,793 (11,814)
<b>Constant</b>	0,002 (0,924)	-0,004 (-1,268)	-0,028 (-0,579)	0,005 (2,565)	0,001 (0,485)	0,002 (3,350)
<b>R-squared</b>	0,579	0,124	0,085	0,837	0,867	0,744

**Table 4.1 – Covariance Structure of Unexpected Returns**

	$r_{cdi}$	$r_{1y}$	$r_{ibov}$	$r_{d-p}$	$r_{embi}$	$r_{ycdi}$
$r_{cdi}$	0,227	0,023	0,012	-0,100	-0,088	0,498
$r_{1y}$	-	0,391	0,159	0,035	-0,272	-0,066
$r_{ibov}$	-	-	6,433	-0,740	-0,693	-0,223
$r_{d-p}$	-	-	-	0,287	0,536	0,123
$r_{embi}$	-	-	-	-	0,309	0,122
$r_{ycdi}$	-	-	-	-	-	0,094

Fourth column of Table 3.1 presents the log equation of the Ibovespa excess-return over the CDI. The remark addressed by Campbell *et al.* (2003) is also valid here: stock's excess-returns are difficult to predict – this equation has the lowest R-squared, 0,085. Only lagged stock returns and lagged risk of Brazil spread have predictive power.

<sup>11</sup> As a rule of thumb, a coefficient is significant if its t-statistic's absolute value is greater than 2.



Among these two excess-returns' equations the one that has the highest R-squared is the one of the LTA. Although, the number of variables that help to predict these asset returns is higher in the Ibovespa equation, with two statistically significant variables. The first two equations have only one statistically significant variable each.

Last three columns show the results of the estimated parameters for the state variables equations. Each of these three equations is well described by an AR (1) process. Their own lagged coefficients are positive and close to one, thus, presenting a dynamic persistence. The third state variable, CDI nominal return, has the lowest coefficient, 0,793.

Table 4.1 shows the covariance structure of the VAR system innovations. Unexpected returns of Ibovespa are negatively correlated with shocks to its 12-month accumulated dividend yield. This is in line with what was found in previous empirical research promoted by authors such as Campbell (1991), Campbell and Viceira (1999), and Stambaugh (1999). Shocks to Ibovespa returns are also negatively correlated with shocks to the EMBI + spread state variable, and with shocks to the CDI nominal returns. Finally, LTA unexpected returns are negatively correlated with shocks to the EMBI + spread.

#### 4.3.2 VAR Estimation – Combination 2

Combination 2 differs from combination 1 in the real short-term asset and in the long-term asset. They are substituted, respectively, by the log return of the IDkA Pre 3M and by the log excess-return of the S&P 500 over IDkA Pre 3M return.

The second column of Table 3.2 presents the log equation of the foreign stock asset. Its one-period lag and Brazilian stock's one-period lag are the only coefficients statistically significant in predicting real short-term asset's returns.

Log equation of the foreign stock has only the 12-month accumulated dividend yield coefficient statistically significant in predicting its returns. Its R-squared is the highest among the excess-returns' equations. Only the one-period lagged Ibovespa's coefficient of

Ibovespa's equation is statistically significant to predict its future returns. Among these excess-returns, the one with highest R-squared is the real foreign stock.

**Table 4.2 – VAR Parameters**

	$r_{3m,t+1}$	$r_{spx,t+1}$	$r_{ibov,t+1}$	$s_{d-p,t+1}$	$s_{embi,t+1}$	$s_{y3m,t+1}$
	(real)	(excess)	(excess)			
	(t-stat)	(t-stat)	(t-stat)	(t-stat)	(t-stat)	(t-stat)
$r_{3m,t}$	0,649	1,584	2,956	-0,050	-0,092	0,007
	(7,905)	(1,058)	(1,335)	-(0,505)	-(0,864)	(0,182)
$r_{spx,t}$	-0,006	-0,132	-0,031	0,001	-0,005	-0,001
	-(1,219)	-(1,507)	-(0,238)	(0,228)	-(0,785)	-(0,627)
$r_{ibov,t}$	-0,007	0,052	0,203	-0,009	-0,012	-0,003
	-(2,051)	(0,870)	(2,310)	-(2,248)	-(2,902)	-(1,585)
$s_{d-p,t}$	-0,060	2,173	0,891	0,889	-0,027	-0,040
	-(1,679)	(3,339)	(0,925)	(20,726)	-(0,589)	-(2,304)
$s_{embi,t}$	0,016	-0,718	1,629	-0,026	0,890	0,043
	(0,501)	-(1,198)	(1,839)	-(0,651)	(20,984)	(2,670)
$s_{ycdi,t}$	0,088	-4,607	-5,166	-0,065	0,283	0,657
	(0,537)	-(1,549)	-(1,175)	-(0,332)	(1,346)	(8,224)
<b>Constant</b>	0,002	-0,027	-0,043	0,005	0,002	0,003
	(1,464)	-(0,910)	-(0,979)	(2,704)	(0,723)	(4,271)
<b>R-squared</b>	0,564	0,130	0,080	0,837	0,867	0,665

**Table 4.2 – Covariance Structure of Unexpected Returns**

	$r_{3m}$	$r_{spx}$	$r_{ibov}$	$r_{d-p}$	$r_{embi}$	$r_{y3m}$
$r_{3m}$	0,239	-0,058	-0,018	-0,064	-0,063	0,539
$r_{spx}$	-	4,360	-0,103	0,114	0,130	0,102
$r_{ibov}$	-	-	6,450	-0,736	-0,698	-0,230
$r_{d-p}$	-	-	-	0,287	0,535	0,144
$r_{embi}$	-	-	-	-	0,309	0,127
$r_{y3m}$	-	-	-	-	-	0,117

Last three columns comprise the results of the estimated parameters for the state variables' equations. Similarly, from combination 1, the three equations representing the state variables are well described by an AR (1) process. Their lagged coefficients are positive and close to one, presenting a dynamic persistence. Here,  $s_{y3m}$  has the lowest coefficient, 0,657.

Table 4.2 presents the covariance structure of the innovations of the VAR system. Unexpected returns of Ibovespa are negatively correlated with shocks to the 12-month accumulated Ibovespa dividend yield, as well as with shocks to the nominal return of the

IDkA Pre 3M, and with shocks to the risk of Brazil spread. Conversely, innovations to the foreign stock are positively correlated with shocks to all three state variables.

Combination 3 does not have any lagged state variable with a statistical significant coefficient in its log excess-returns' equations. The same is true for combination 4, making it hard to use these combinations, once it limits our ability to explain portfolio allocations.

#### 4.4 Strategic Asset Allocations

In section 3.3 we show that the optimal allocation rule is linear in the vector of state variables. Therefore, optimal allocation among the chosen assets varies over time. One way to study this rule is to examine its mean and volatility. In order to analyse the magnitude of the effects of the optimal allocation, we calculated the myopic allocations (tactical effect) for each of these assets, as well as their demands for protection (strategic effect). Based on the estimated VAR parameters and the cross-correlation of residuals, the optimal portfolios were calculated for each of the four combinations of  $\psi = 1$ ,  $\delta = 0,95$  (at an annual frequency) and  $\gamma = 1$ ,  $\gamma = 2$ ,  $\gamma = 5$ , and  $\gamma = 20$ .

Both third and eighth columns in Table 5 show the portfolio allocation across the STA, LTA or foreign stock, and Brazilian stock for a log utility investor holding combination 1 and combination 2, respectively. In this case, the investor is going to decide merely myopically. When  $\gamma = 1$ , it can be seen from equation (18) that this allocation depends only on the inverse of the variance-covariance matrix of unexpected excess-returns and on the mean excess-returns. Therefore, the allocation decision is essentially based on the asset with the best risk-return relationship, that is, the asset with the highest Sharpe ratio (SR, Table 1).

For combination 1, the myopic allocation when  $\gamma = 1$  holds a long position in LTA of 6349,21%, a selling position in stock of 131,06% and a short position in STA of 6118,15% – Table 5. The largest excess-return position is in the asset with the highest SR in absolute value, 0,230, while the stock holds a negative 0,035 SR. In Table 6, we observe the absolute

value of the LTA to stock ratio of 48,45, probably due to the substantial positive estimated correlation of 15,9% between unexpected returns on stocks and bonds, shifting the exposure towards the asset with highest SR in absolute value.

Regarding combination 2, the myopic allocation when  $\gamma = 1$  has a short position in the foreign stock of 68,88%, a short position of 78,20% in the Brazilian stock and a long position in the STA of 247,07%. The largest excess-return's position is in the asset with the highest absolute value in the SR, which is the Ibovespa, 0,037, compared to the -0,025 foreign stock's SR. Here, the foreign stock to Brazilian stock ratio is 0,88.

**Table 5 – Mean Asset Demands for Combination 1 and Combination 2 (%)**

	Asset	Combination 1				Asset	Combination 2			
		$\gamma=1$	$\gamma=2$	$\gamma=5$	$\gamma=20$		$\gamma=1$	$\gamma=2$	$\gamma=5$	$\gamma=20$
Myopic Demand		-6118,15	-3008,43	-1142,59	-209,68		247,07	173,32	129,08	106,95
Hedging Demand	STA	0,00	-1589,02	-1309,38	-429,42	STA	0,00	77,94	98,20	41,76
Total Demand		-6118,15	-4597,45	-2451,98	-639,10		247,07	251,26	227,27	148,72
Myopic Demand		6349,21	3173,97	1268,83	316,26		-68,88	-34,27	-13,51	-3,13
Hedging Demand	LTA	0,00	1457,98	1205,58	399,56	Foreign Stock	0,00	-42,16	-51,18	-19,63
Total Demand		6349,21	4631,95	2474,40	715,82		-68,88	-76,43	-64,69	-22,76
Myopic Demand		-131,06	-65,54	-26,24	-6,58		-78,20	-39,05	-15,57	-3,82
Hedging Demand	Stock	0,00	131,04	103,81	29,86	Stock	0,00	-35,78	-47,02	-22,13
Total Demand		-131,06	65,50	77,57	23,28		-78,20	-74,83	-62,58	-25,95

**Table 6 – LTA and Foreign Asset to Brazilian Stock Ratios in Absolute Value**

LTA/Stock	Combination 1				Combination 2			
	$\gamma=1$	$\gamma=2$	$\gamma=5$	$\gamma=20$	$\gamma=1$	$\gamma=2$	$\gamma=5$	$\gamma=20$
Myopic LTA/Stock	48,45	48,43	48,36	48,05	0,88	0,88	0,87	0,82
Hedge LTA/Stock	-	11,13	11,61	13,38	-	1,18	1,09	0,89
Total LTA/Stock	48,45	70,72	31,90	30,75	0,88	1,02	1,03	0,88

Investors with risk aversion greater than one seek assets that increase in value when investment opportunities deteriorate. Such assets behave as insurance against a fall in the return of wealth, which would lead to a reduction in the pattern of consumption that wealth could provide. Assets with such characteristics are those that tend to have a lower risk over long horizons of time. Thus, investors with  $\gamma > 1$  seek, therefore, an intertemporal demand for protection. This intertemporal hedging demand tends to be higher in assets that show a

greater degree of predictability. It also tends to increase with the degree of persistence of state variables that prove to be good predictors of these asset returns.

In combination 1, since LTA have a large positive Sharpe ratio, investors hold, normally, long positions in these assets. Hence, an increase in expected LTA returns is understood as an improvement in the investment opportunity set. According to our VAR estimation, future LTA returns increase when the EMBI + spread increases (Table 3.1). Once LTA are considerably negatively correlated with the EMBI + spread by 27,2%, poor LTA returns are correlated with an improvement in future investment opportunities. Consequently, conservative investors with  $\gamma > 1$  can use LTA to hedge against the variation of their future returns, resulting in a positive intertemporal hedging demand.

Stock returns also have a positive intertemporal hedging demand. First, the EMBI + spread forecasts excess stock returns and LTA returns positively. Thus, an increase in the stock return represents an improvement in the investment opportunity set. Since stock returns are highly negatively correlated with changes in the risk of Brazil variable by -69,3%, this generates a positive intertemporal hedging demand by conservative investors with  $\gamma > 1$ . Still, there is a positive correlation between unexpected LTA returns and stock returns of 15,9%, so investors can offset their positive intertemporal hedging demand for LTA by taking short positions in stocks. Although, the model yields a positive intertemporal hedging demand for all levels of risk-aversion, meaning that the former effect more than compensates for the later. Here, investors with  $\gamma > 1$  have a positive total demand for stock, since intertemporal hedging demand overweighs myopic demand. Strategic effects surpass tactical effects, making it clear the importance of asset returns predictability in optimization.

Intertemporal hedging demand is smaller for stocks than for LTA, probably due to the fact that LTA returns are more predictable than stock returns – LTA R-squared is the highest amongst the two excess-return equations.

In what concerns combination 2, following the same reasoning above, since the Brazilian stock has a negative SR, investors hold, normally, short positions in these type of assets. Hence, an increase in expected Brazilian stock returns is understood as a deterioration in investment opportunities. Future Brazilian stock returns increase when the 12-month accumulated dividend yield increases (Table 3.2). Once the Brazilian stock is considerably positively correlated with the 12-month accumulated dividend yield by 11,4%, poor Brazilian stock returns are correlated with an improvement in future investment opportunities. Consequently, conservative investors with  $\gamma > 1$  can use the Brazilian stock to hedge against their future returns volatility, resulting in a negative intertemporal hedging demand.

Foreign stock returns also have a negative intertemporal hedging demand. There is a negative correlation between unexpected Brazilian stock returns and foreign stock returns of 10,3%, so investors can offset their negative intertemporal hedging demand for Brazilian stock by taking short positions in foreign stocks. The Brazilian Hedging stock to foreign Hedging stock ratio is bigger than 1 for moderately conservative investors, probably due to the former's R-squared, which is higher than the foreign stock's R-squared.

The same analysis of could have been made for combinations 3 and 4. There, LTA returns are, once again, more predictable than stock returns, and intertemporal hedging demand is bigger for LTA, than for stocks.

Figure 2.1 and Figure 2.2 plot the history of total and hedging asset allocations for  $\gamma = 5$  and  $\psi = 1$  across the 3 assets of combination 1 and combination 2, respectively. Hedging demands are less volatile than total demands for both combinations. Myopic allocations are responsible for changing the sign of demands. Campbell *et al.* (2003) show that hedging demands only change sign in harsh events. Trying to follow such strategies is almost undoable due to transactions costs which investors are subject to.

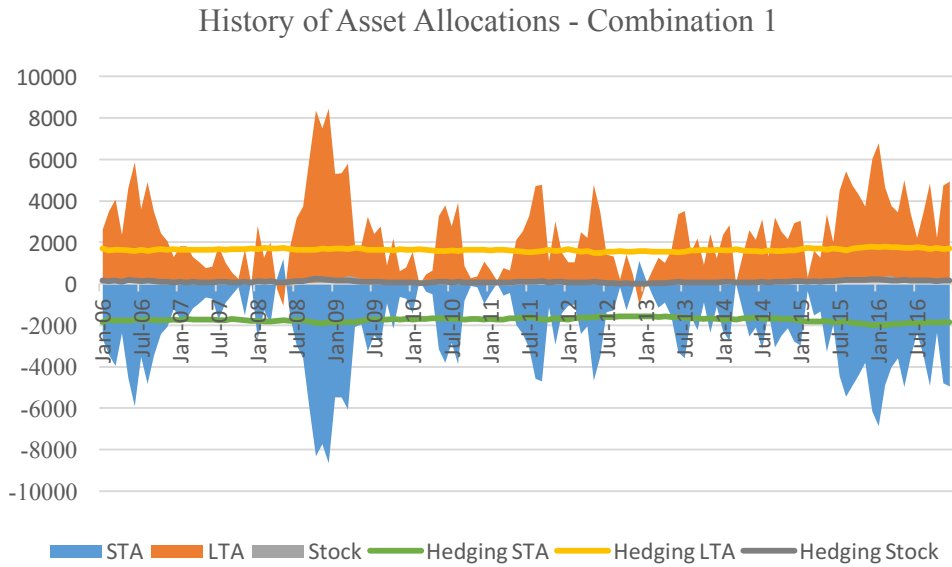


Figure 2.1

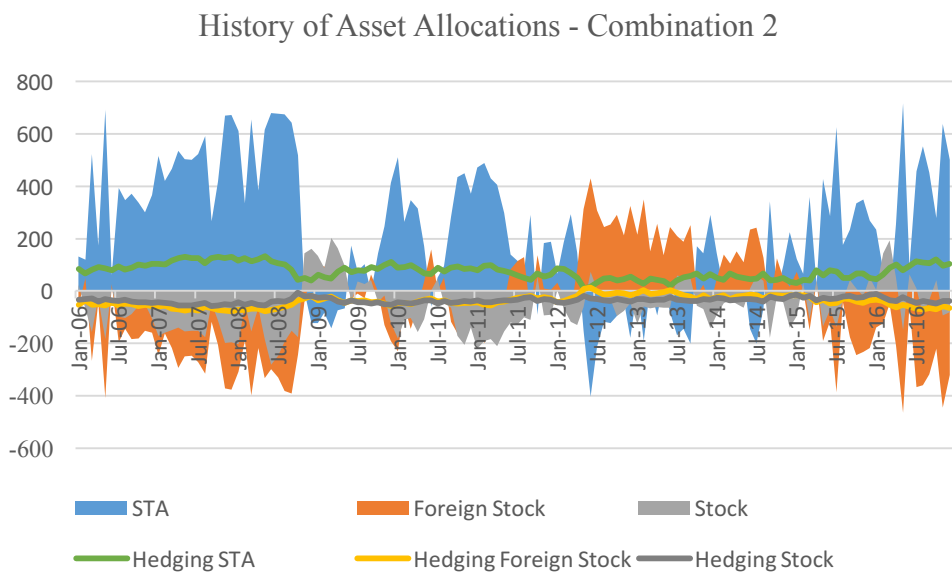


Figure 2.2

## 5 Conclusion

We study the predictability of asset returns and its impact on long-term investors' financial decisions. Intertemporal hedging demands are strategically relevant in optimal portfolio allocation, contrary to myopic optimal allocations where they play a tactical role.

The state variables used to predict asset returns affect this demand for protection through the correlation of their shocks to unexpected returns on stocks and long-term assets,

and their persistence. Risk of Brazil, as measured by the EMBI + spread, and the 12-month accumulated dividend yield has the highest correlations amongst all the state variables. These state variables are the most important in defining the optimal portfolio demands.

We consider two types of asset combinations. First we use only Brazilian assets and perform the predictability analysis. We innovate in combination 2 by replacing one of the Brazilian components, the Long Term Assets (LTA), by a foreign index (the S&P500).

In the first combination, the long-term asset was the responsible for intertemporal hedging demand. This demand arises from the high correlation between shocks to the variable that help to predict returns, EMBI + spread, and shocks to long-term assets. We find that the total optimal demand (strategic plus myopic) for long-term assets is positive. In this combination, intertemporal hedging demand is positive, playing an important strategic role for conservative investors, as opposed to the negative myopic allocation. Corroborating past empirical research, we find that LTA returns are more predictable than stocks in Brazil, so that intertemporal hedging demand is greater for LTA than for stocks.

Our analysis innovates by comparing investment in Brazilian assets with the possibility of investing in a foreign index. In our set-up the foreign index is more predictable for moderately conservative investors, than the Brazilian stocks. As a consequence, moderately conservative investors use S&P 500 for intertemporal hedging.

This research is not free of limitations. Long-term investors have wealth, but labour income is not modelled. Short-selling is not constrained. The results constitute an approximation for investors with elasticity of intertemporal substitution different than one. The VAR system is estimated without corrections for small-sample biases and Bayesian priors were not used. Although, Banbura, Giannone, and Reichlin (2010) have shown that a Bayesian vector auto regression model is more appropriate for modelling large data sets. Finally, the authors consider that investors know all parameters of the model.



As the ANBIMA historical series will become longer, it will be worth to reassess the demands for the assets with more robust estimates of the VAR parameters.

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