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Design Philosophy Study on CFRP RC Structures

Dissertação para obtenção do Grau de Mestre em
Engenharia Civil - Perfil de Estruturas

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Acknowledgments

I would like to thank all who in one way or another contributed in the completion of the present dissertation. First, I give thanks to God for being present in my life, for protecting and giving me strength to make it through every day.

I would like to express my deepest gratitude to my advisors, Dr.-Ing. Maria Patricia Garibaldi, Technische Universität Dresden- Faculty of Civil Engineering, Dr. -Ing. Frank Jess, Technische Universität Dresden- Faculty of Civil Engineering and Dr.-Ing. Carlos Chastre, Universidade Nova de Lisboa- Faculdade de Ciências e Tecnologia for their continuous guidance and support.

Lastly, I must thank my family for their love and unfailing support, I thank my father and my grandmother for their constant words of encouragement, my friends Priscilla and Miguel, my sisters Barbara and Maria Clara for their patience and willingness to help me at any stage of this dissertation and specially my mother who continuously supports and encourages me and my decisions. This accomplishment would not have been possible without them. Thank you.

Lorena Silveira

Abstract

The present dissertation was developed with the main purpose of comparing the different design philosophies existing for prestressed reinforced concrete members, as well as analyzing a solution that is both safe and economically viable.

Since one of the major problems in civil engineering infrastructures is due to corrosion, a study was conducted to examine the feasibility of replacing the material used as reinforcement and prestress. The steel, which for many decades has been used shall be replaced by FRP, particularly CFRP. However, since carbon presents a brittle behavior, a new design methodology suggested by fib and by the Canadian code is presented so that the failure is not due to the reinforcement but due to the concrete, which, although it is not a ductile material, presents more ductility than materials with carbon fibers.

In this way, a code has been developed so that, through the balance of the cross section, it calculates the amount of reinforcement required, following the American (ACI) and European code (Eurocode) design methodology. Another code for prestressed reinforced concrete members was elaborated to analyze the prestress system used and the material used as reinforcement and tendons.

The European code has proved to be economically viable since, for the same applied moment, it requires a smaller amount of reinforcement. As for the bonded and unbonded system the results show that the systems with unbonded tendons are only feasible when the concrete members are subjected to high moments. It is also observed that depending on the initial force applied on the prestress, the results may be favorable regarding to the amount of reinforcement required, however the amount of reinforcement can be significantly increased when the initial force of the prestress is highly increased.

Keywords: Reinforced concrete, Ultimate limit state, ACI, Eurocode, CFRP, Bonded and unbonded tendons, Prestress.

Resumo

A presente dissertação foi elaborada com o principal objectivo de comparar as diferentes filosofias de dimensionamentos existentes para peças de betão armado pré-esforçado, bem como analisar uma solução que seja ao mesmo tempo, segura e economicamente viável.

Uma vez que um dos principais problemas nas infraestruturas de engenharia civil, ocorre devido a corrosão. Foi elaborado um estudo para analisar a viabilidade de substituição do material usado como armadura ordinária e cabos de pré-esforço. O aço, que por muitas décadas vem sendo usado, passa a ser substituído por FRP, nomeadamente CFRP. No entanto, uma vez que o carbono apresenta um comportamento frágil, uma nova metodologia de dimensionamento sugerida pela FIB é apresentada, para que a rotura não se dê pela armadura ordinária, mas sim pelo betão, que mesmo não sendo um material dúctil, apresenta mais ductilidade que os elementos feitos com fibras de carbono.

Deste modo, foi elaborado um código que através do equilíbrio da secção transversal, calcula a quantidade de reforço necessária através da metodologia de dimensionamento do código americano (ACI) e do código Europeu (Eurocódigo). Um segundo código foi elaborado para peças de betão armado pré-esforçadas para analisar o sistema mais adequado para o pré-esforço, bem como o material mais vantajoso.

O código europeu mostrou-se economicamente mais viável, uma vez que para o mesmo momento fletor requer uma menor quantidade de armadura. Quanto ao sistema aderente e não aderente os resultados mostram que os sistemas com pré-esforço não aderente só são viáveis para um momento atuante muito elevado. Observa-se também que, dependendo da força inicial que aplicamos no pré-esforço, os resultados podem ser bem vantajosos relativamente a quantidade de armadura necessária, mas, no entanto, pode aumentar significativamente a quantidade de reforço necessário quando aumentamos em demasia a força inicial aplicada nos cabos.

Palavras-chave: Betão Armado, Estado Limite Último, ACI, Eurocódigo, CFRP, Sistema de Pré-Esforço Não Aderente, Sistema de Pré-Esforço Aderente

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Abbreviations and Symbols

Abbreviations

EC2	Eurocode 2
ACI	American Concrete Institute
RC	Reinforced Concrete
FRP	Fiber Reinforced Polymers
CFRP	Carbon Fiber Reinforced Polymers
VBA	Visual Basic for Applications

Symbols

A_{Ed}	Design value of an accidental action and $A_{Ed} = \gamma_1 A_{Ek}$
A_{Ek}	Characteristic value of seismic action
A_p	Area of a prestressing tendon or tendons
A_s	Area of tension steel reinforcement
b	Width
d	Effective depth of a cross-section
E	Effect of actions
E_d	Design value of effect of actions
E_f	Modulus of elasticity of FRP
E_p	Modulus of elasticity of tendons
E_s	Modulus of elasticity of reinforcement
f_{cd}	Design concrete strength
f_{ck}	Characteristic compressive cylinder strength of concrete at 28 days
F_p	Prestress force
F_s	Steel reinforcement force

F_c	Compression force according to EC2
C	Compression force according to ACI
f_{fd}	Design value of tensile strength for FRP
f_{fk}	Characteristic value of tensile strength of FRP reinforcement
f_{pk}	Characteristic tensile strength of prestressing
f_y	Yield strength of reinforcement
f_{yd}	Design yield strength of reinforcement
f_{yk}	Characteristic yield strength of reinforcement
f'_c	Concrete strength
G_{jk}	Characteristic value of permanent action j
h	Overall depth of a cross-section
P^∞	Prestress force after all losses
Q_{1k}	Characteristic value of the leading variable action
M_{Ed}	Design moment
M_{Rd}	Resistant moment
x	Depth of the neutral axis
ΔP_{c+s+r}	Long term losses due to creep, shrinkage and relaxation at location x, at time t
ε_{cu}	Ultimate compressive strain in the concrete
ε_{p0}	Initial strain of tendon
ε_c	Compressive strain in the concrete
ε_{cu2}	Ultimate compressive strain in the concrete according to Table 3.1 EC2
ε_{c2}	Strain at reaching the maximum strength according to Table 3.1 EC2
ε_s	Steel strain
ε_p	Prestress strain
ε_{uk}	Characteristic strain of reinforcement or prestressing steel at maximum load
γ_c	Partial factor for concrete
γ_f	Partial factor for FRP
γ_s	Partial factor for reinforcement

γ_p	Partial factor for tendons
σ_p	Stress applied to the tendon

1 Introduction

1.1 Objectives and Scope

Textile reinforced concrete, carbon concrete composites or CFRP RC are different names for the same type of modern development. The use of high performance fibers as a replacement for reinforcement steel. Among the advantages are the high strength, high durability, excellent fatigue resistance, a principal non-corrosive characteristic and more. Barriers for practical application include high price, required adaptation of production technology and also lack of experience in practical design or lack of design rules itself. One of the most promising materials as a replacement for steel are carbon fibers. The economic aspect of design solution is strongly connected to the required amount of reinforcement and the security factor. The use of high safety factor leads to expensive infrastructures. A conservative approach might be as safe as uneconomic. The blind transfer of RC design rules to Carbon fiber reinforcement might be as simple as unsafe, because the switch from ductile steel reinforcement to brittle carbon fibers will have fundamental impact on the load bearing behavior and especially on the observable failure types. For design with CFRP various guidelines and recommendations exists already, but they differ fundamentally in the numbers chosen for safety factors and safety concepts. The aim of this study is to show the impact on design results caused by using different design concepts and to create a conceptual design for both the structures described above and the elements to be used.

1.2 Thesis outline

The presented thesis is divided into five chapters.

Chapter 1: Theme Description, previously developed work, motivations and objectives to be achieved with the realization of this work.

Chapter 2: The design theory for reinforced concrete structures according to the American and European code. Comparison between the different approaches for the flexural design in the ultimate limit state.

Chapter 3: History of prestress and its different systems. The influence of the choice of types of tendons, whether bound or unbound. Physical and mechanical characteristics of the material used for the ultimate limit state. Introduction to a new design theory suggested by fib.

Chapter 4: Presenting the results obtained with the help of VBA program. Comparison in the amount of reinforcement needed when using the American or European approach. Comparison between bonded and unbonded tendons and also carbon tendons and rebars. Effect of the initial prestress force for a prestressed rectangular section of reinforced concrete.

Chapter 5 - Conclusions taken from the realization of this thesis and suggestion for future works.

2 Design philosophy

The aim of this study is to understand and compare the different approaches based on the ACI 318 and ACI 314 [1-3] and the Eurocode 2 [4]. The principles adopted for both codes are the same, the concrete member must be in equilibrium, which means that the internal forces must balance the external load.

However, they present differences in the design procedure which leads to significant differences in the results for reinforced concrete structures in bending.

For prestressed reinforced concrete structures another comparison is made, and different types of reinforcement and prestress are used.

Once one of the most important aspects of this study is to evaluate the feasibility of CFRP structures, and since the Eurocode does not provide a design procedure for this type of material and so, for the conventional steel prestressed structure the EC2 was followed and for CFRP ones the design procedure given by FIB [5] was followed.

In this chapter are described the procedures given by the previously-mentioned codes and different assumptions are considered to evaluate where these differences come from.

2.1 ACI 314 & ACI 318 flexure design philosophy

The approach suggested by the ACI [2] is that there is a reduction in the resistant moment and not a reduction of the capacities of the material. Differently than suggested by Eurocode 2 [4]

To calculate the resistant moment, i.e. the design strength of a given cross section, the considered assumptions are:

- Bernoulli hypothesis, plan sections remain planes after loading. The strain in concrete and reinforcement is assumed to be directly proportional to the distance from the neutral axis. Linear strain distribution;
- The static equilibrium and the strain compatibility must be satisfied;
- Stress in reinforcement below the yield point shall be taken as the strain times the modulus of elasticity, E_s linear behavior. For strains, greater than that corresponding to ϵ_y , stress is considered as f_y ;
- For the flexural calculation, the tensile strength of concrete is neglected;
- Concrete strength = f'_c ;
- Yield strength of reinforcement = f_y ;
- Concrete ultimate compressive strain $\epsilon_{cu} = 0.003$;

2.1.1 Design concrete compressive strength

The compressive strength is given by the strength of the concrete, which means that the ultimate load is the one that causes the failure. Figure 2.1 shows the stress-strain curve of the concrete, therefore a simplification of the real curve is given in the ACI code [6] and the stress-stress curve for concrete is the one shown in Figure 2.2.

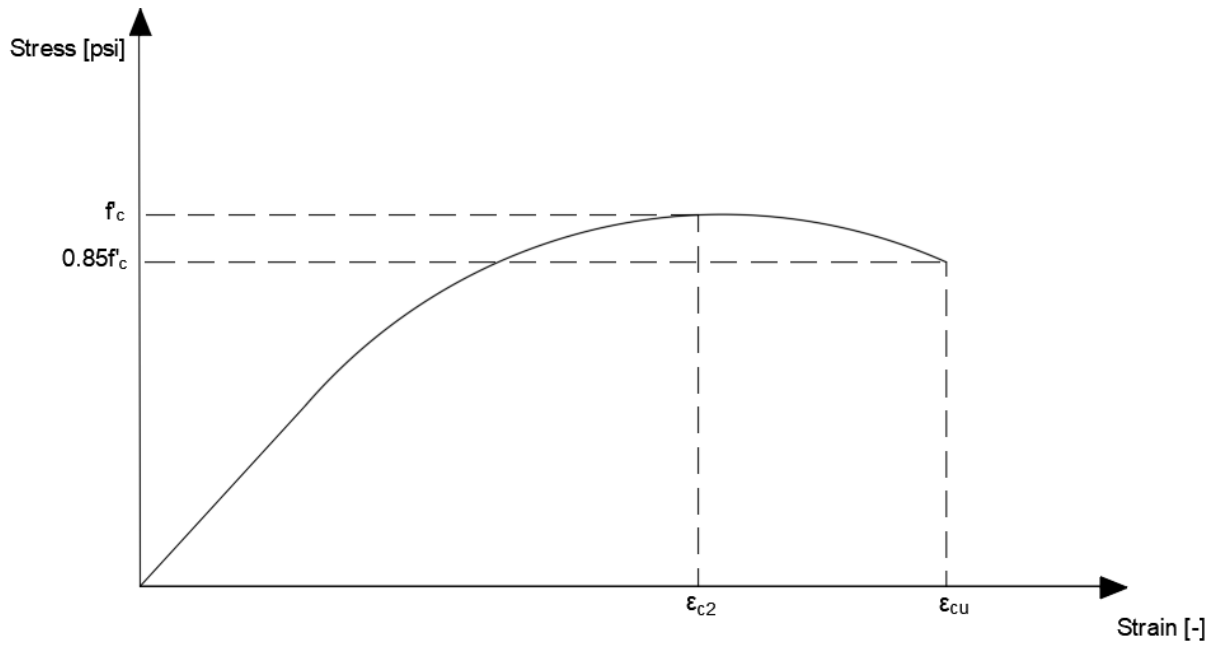


Figure 2.1 - Stress-Strain curve for concrete adapted from ACI [6]

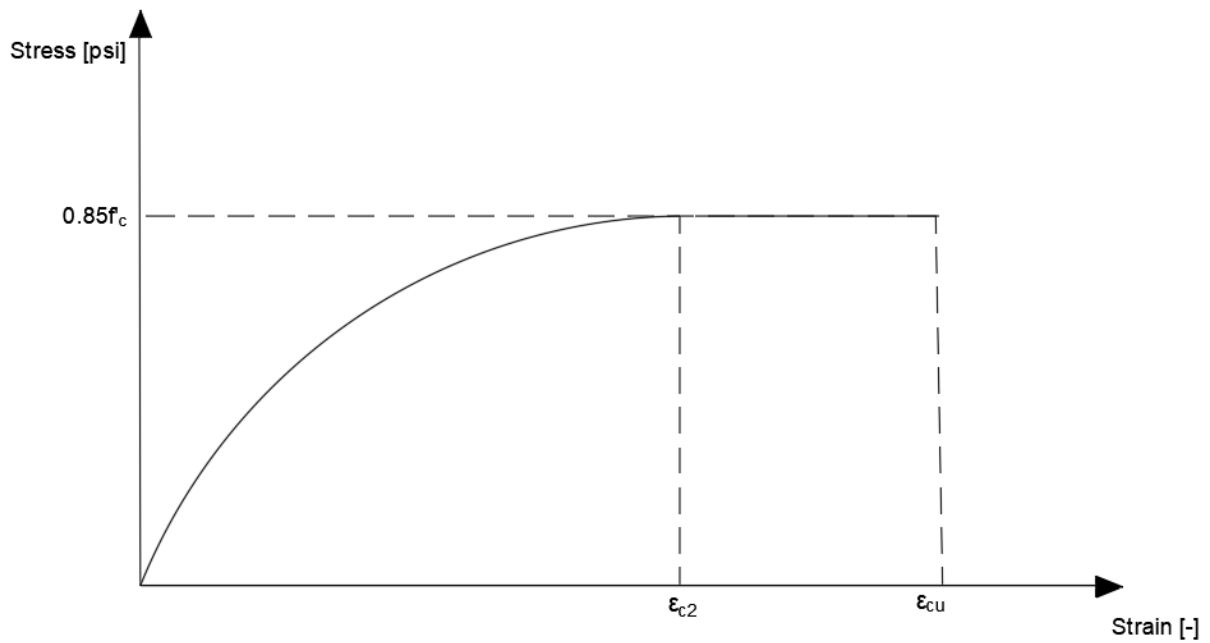


Figure 2.2 – Parabolic-rectangular stress-strain curve for concrete adapted from ACI [6]

To calculate the stress of the concrete curve (Figure 2.2) the equation shown in the expression (12) must be used:

$$f_c = \begin{cases} 0.85f'_c \left[2 \left(\frac{\epsilon_c}{\epsilon_{c2}} \right) - \left(\frac{\epsilon_c}{\epsilon_{c2}} \right)^2 \right] & \text{for } 0 \leq \epsilon_c \leq \epsilon_o \\ 0.85f'_c & \text{for } \epsilon_o \leq \epsilon_c \leq \epsilon_{cu} \end{cases} \quad (1)$$

2.1.2 Design flexural tensile strength of steel

For the steel used as reinforcement the following behavior of the material was considered (Figure 2.3). It presents an initial linear branch that satisfies the Law of Hook and after reaching the point of yield, the stress remains constant on a horizontal plateau.

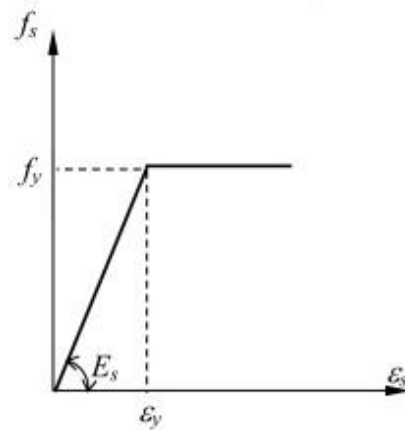


Figure 2.3 - Stress-strain curve for steel adapted from ACI [7]

2.1.3 Design procedure

For the section analysis the scheme presented in the following figure is given by ACI Committee Report [7] and it depends on the type of the state that has been chosen, Ultimate, Figure 2.4 (c) or Service,

Figure 2.4 (d). This code also presents a simplification, the rectangular stress block due to its easy calculation process. Figure 2.4(e).

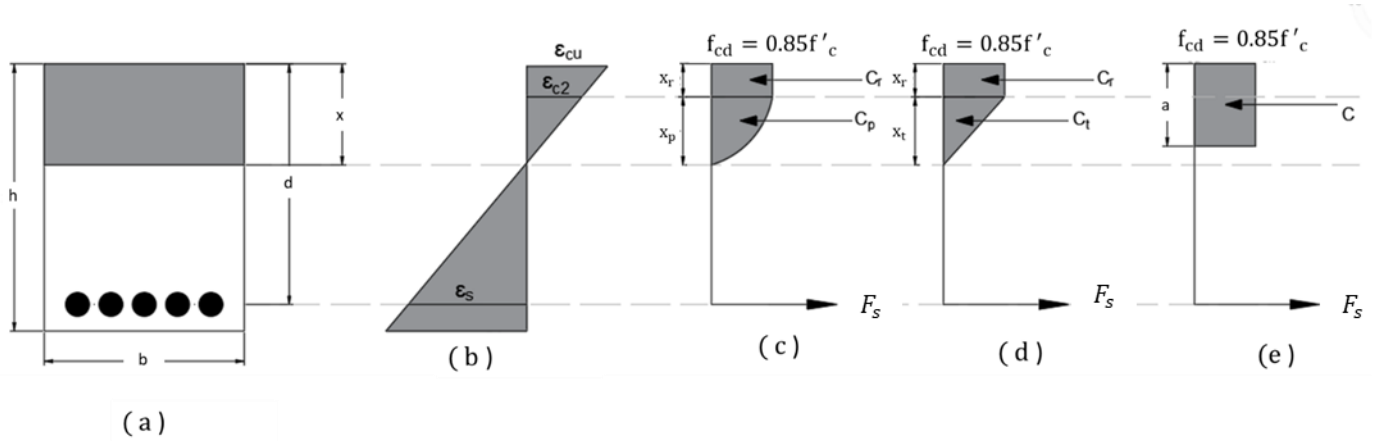


Figure 2.4 - Internal stress and strain distribution for a rectangular section under flexure at ULS;(a) Cross Section; (b) Strain Diagram; (c) Parabola-rectangular stress block; (d) bilinear stress block; (e) Rectangular stress block. Adapted from ACI [7].

When the compression reinforcement is considered the following scheme is adopted, (Figure 2.5) and therefore, the compression force given by the reinforcement helps the compression force given by the concrete. The tensile force must balance both forces, the concrete and the steel ones.

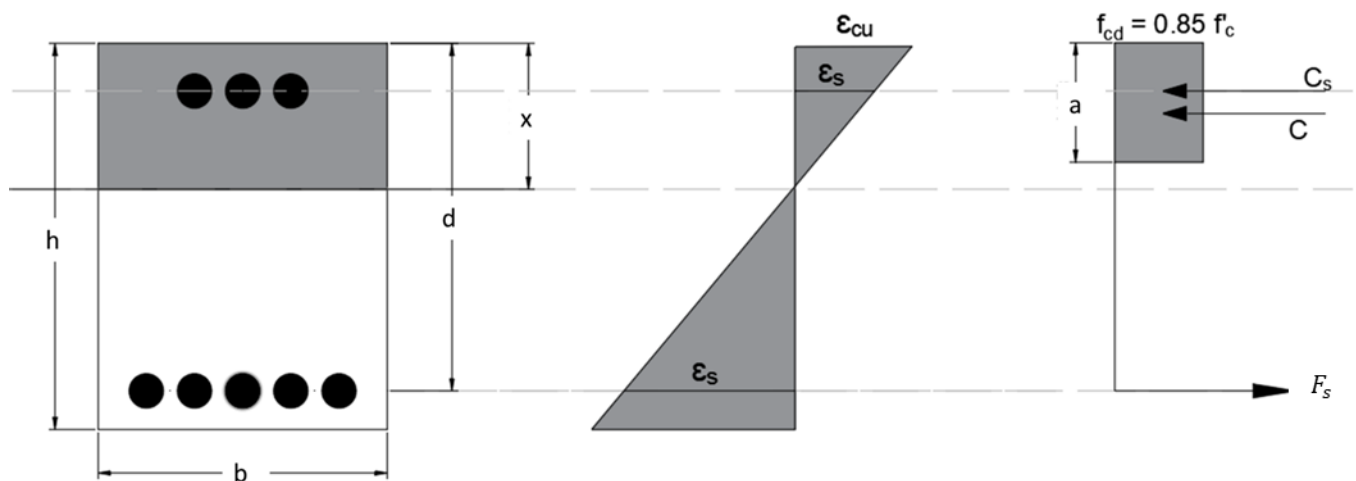


Figure 2.5 - Rectangular stress block considering compression reinforcement [8]

However, for this case, it was considered a reinforced concrete member under flexure and compression reinforcement were not included in the equilibrium.

Below is presented the calculation scheme considering the rectangular stress-block.

For a singly reinforced rectangular beam, an initial strain is always assumed for the concrete and steel. Knowing the initial strain, it is possible to calculate the depth of the neutral axis that satisfies the balanced strain condition through the following expression:

$$x = \frac{\varepsilon_c}{\varepsilon_c + \varepsilon_s} \cdot d \quad (2)$$

where:

x is the depth of the neutral axis;

ε_c is the concrete strain;

ε_s is the steel strain;

d is distance from extreme compression fiber to centroid of tension reinforcement;

By calculating through the numerical integration the stress block diagram, it is possible to obtain the resulting compression force, C :

$$C = 0.85f'_c\alpha b \quad (3)$$

where :

f'_c is specified compressive strength of concrete;

b is the width of rectangular cross section;

$\alpha = x \cdot \beta_1$;

β_1 - reduction coefficient obtained from the following parameter table (Table 2.1) provided by ACI [9]

Table 2.1 – Concrete stress block parameters given by ACI [9]

f'_c [MPa]	≤ 8	35	42	49	≥ 56
α	0.72	0.68	0.64	0.60	0.56
β	0.425	0.400	0.375	0.350	0.325
β_1	0.85	0.8	0.75	0.7	0.65
$\gamma = \alpha / \beta_1$	0.85	0.85	0.85	0.86	0.86

To satisfy the equilibrium condition the internal forces must balance, hence it is possible to write the following expression:

$$F_s = C \quad (4)$$

Where:

C is the resulting compression force given by equation (2)

T is the tensile force of reinforcement given by:

$$F_s = A_s f_y \quad (5)$$

Where:

A_s is the area of tension steel reinforcement;

f_y is the specified yield stress of non-prestressed steel reinforcement;

Thus, it is possible to write the following expression:

$$A_s f_y = 0.85 f'_c \beta_1 x b \quad (6)$$

Thus, introducing Eq. (2) and Eq. (4) into Eq. (3) the following expression can be derived and therefore it is possible to obtain the necessary amount of reinforcement, A_s :

$$A_s = \frac{0.85 f'_c \beta_1 x b}{f_y} \quad (7)$$

The nominal moment is the capacity of the structure to resist the applied loads and it can be calculate by the internal forces of the cross section, as can be seen in the following expression:

$$M_n = A_s f_y \left(d - \frac{\beta_1 x}{2} \right) \quad (8)$$

Once the nominal moment has been calculated, the reduction factor is applied and the following condition must be satisfied:

$$M_u \leq \phi M_n \quad (9)$$

Where:

ϕ is the reduction factor;

M_n is the nominal moment capacity;

M_u is the factored moment at section;

The procedure that has been demonstrated above serves for the simplification suggested by the ACI[7] to use rectangular stress block instead of parabolic one. Since the shape of the stress block changes when it comes to the bilinear situation, it is necessary to calculate the application point for the new resulting compression force. Instead of an integration of a constant function the integral of a polynomial function of the first degree must be calculated and thus the compressive force is given by:

$$C_t = \frac{f'_c \cdot b \cdot x_t}{2} \quad (10)$$

Where x_t represents the compressed zone until it reaches f'_c .

Regarding the application point of the force, as it is known, the resulting force on the triangular part, before the concrete strain reaches the ultimate strain, is applied to a third of the extreme compression fiber ($c_t/3$). On the other hand, the constant part of the stress, after the strain reaches its ultimate point, is calculated in the same way the resulting force is applied in the middle of the stress block.

After the calculation of the resulting compression force and the point of application for the linear stress block, it is possible to calculate the nominal moment by the following equation (10) and thus proceed to check the ultimate limit state through equation (8).

$$M_n = C_t \cdot z_t \cdot C_r \cdot z_r \quad (11)$$

Where:

C_t is the resulting compression force before the ultimate strain is reached;

z_t is the distance between the force resulting from the triangular stress block and the tensile force;

C_r is the resulting compression force after the ultimate strain is reached;

z_r is the distance between the force resulting from the rectangular stress block and the tensile force;

For the present study, the bilinear scheme has been used and it does not include compression reinforcement.

2.1.4 Safety factors

The ACI code [7] reduces the nominal strength and therefore a reduction factor must be used depending on the failure type. The strength reduction factor can be seen in Figure 2.6.

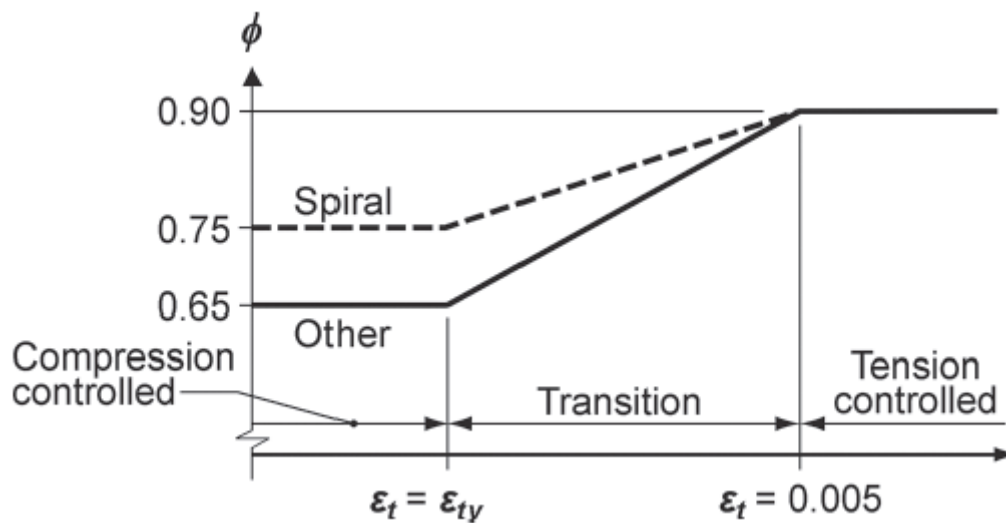


Figure 2.6 - Strength reduction factor ACI 318 [2]

Where:

- Compression-controlled: $\Phi = 0.65$
- Tension-controlled: $\Phi = 0.90$
- Transition zone: $\Phi = 0.65 + (\epsilon_{sy} - 0.002) (250/3)$

2.1.5 Load combinations

According to ACI [2] the load combinations considered are presented in Table 2.2 and the required strength should be bigger than the factored loads.

Table 2.2 – Load combinations

Loading combination	Primary load
$U = 1.4D$	D
$U = 1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$	L
$U = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.5W)$	L _r or S or R
$U = 1.2D + 1.0W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R)$	W
$U = 1.2D + 1.0E + 1.0L + 0.2S$	E
$U = 0.9D + 1.0W$	W
$U = 0.9D + 1.0E$	E

**Required strength U shall include internal load effects due to reactions induced by prestressing with a load factor of 1.0.

Where :

U = the design (ultimate) load

D = dead load

F = fluid load

T = self straining force

L = live load

Lr = roof live load

H = lateral earth pressure load, ground water pressure.

S = snow load

R = rain load

W = wind load

E = earthquake load

2.2 Eurocode 2

Contrary to what is provided by the ACI code, presented in the previous section, the Eurocode 2 is not based on reducing the bearing capacity of the reinforced concrete member but on the contrary, the resistant moment is already calculated with the reduced capacities of the materials that are affected by a safety factor.

To calculate the resistant moment, design strength, of a given cross section the assumptions considered are:

- Bernoulli hypothesis, flat sections remain flat after loading. The strain in concrete and reinforcement is assumed to be directly proportional to the distance from the neutral axis. Linear strain distribution;
- The static equilibrium and the strain compatibility must be satisfied;
- Stress in reinforcement below the yield point shall be taken as the strain times the modulus of elasticity, linear behavior. For strains, greater than that stress, is considered as f_{yd} ;
- For the flexural calculation, the tensile strength of concrete is neglected;
- Design concrete strength = f_{cd} ;
- Design yield strength of steel reinforcement = f_{yd} ;
- Concrete ultimate compressive strain $\varepsilon_{cu} = 0.0035$;

2.2.1 Design compressive strength of concrete

- Stress-strain relationship for the design of the cross section

According to EN-1992-1-1 (3.1.7) there are three options for the stress-strain distribution and stress block: (1) Parabola-rectangle, Figure 2.7; (2) Bi-linear and (3) Rectangular. The stress-strain distribution that has been used in this thesis is the parabola-rectangle as shown.

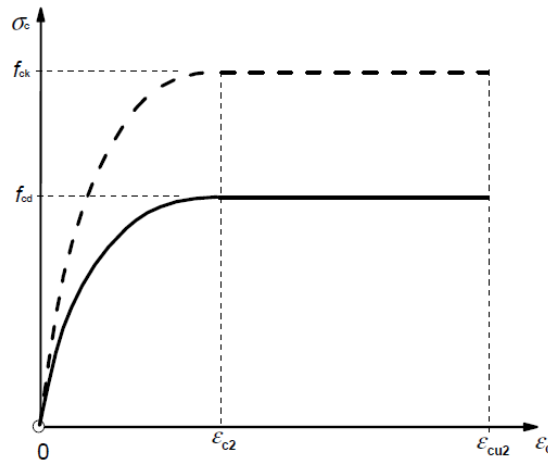


Figure 2.7 - Parabola-rectangle stress-strain diagram for concrete under compression [4]

In which the design value of concrete compressive strength is given by the following expression:

$$f_{cd} = \frac{\alpha_{cc} f_{ck}}{\gamma_c} \quad (12)$$

Where:

γ_c is the partial safety factor for concrete;

α_{cc} is the coefficient taking account of long term effects on the compressive strength and of unfavorable effects resulting from the way the load is applied.

The value of α_{cc} can be found in the National Annex and therefore it is considered as 0.85 in the German Annex and 1.0 in the Portuguese Annex.

The curve shown in Figure 2.7 can be obtained by the following expressions:

$$\sigma_c = f_{cd} \left[1 - \left(1 - \frac{\varepsilon_c}{\varepsilon_{c2}} \right)^n \right] \text{ for } 0 \leq \varepsilon_c \leq \varepsilon_{c2} \quad (13)$$

$$\sigma_c = f_{cd} \text{ for } \varepsilon_{c2} \leq \varepsilon_c \leq \varepsilon_{cu2} \quad (14)$$

Regarding the stress block the parabola-rectangle is used (Figure 2.8) in this study to represent the most realistic behavior of the concrete member.

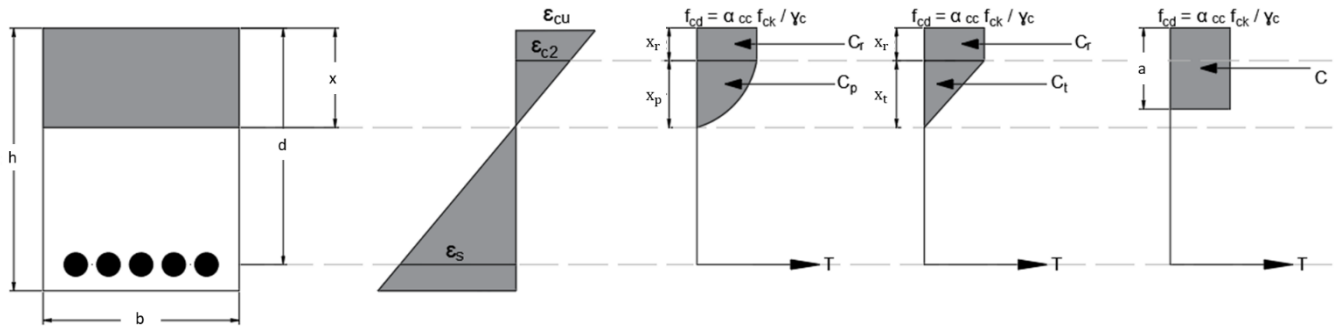


Figure 2.8 - Parabola-rectangle, bilinear and rectangular stress block adapted from Eurocode 2 [4]

2.2.2 Design flexural tensile strength of steel

Regarding the steel reinforcement the following stress-strain distribution is given by EN-1992-1-1(3.2.7) and the assumption B with the horizontal branch has been used for the flexural design, Figure 2.9.

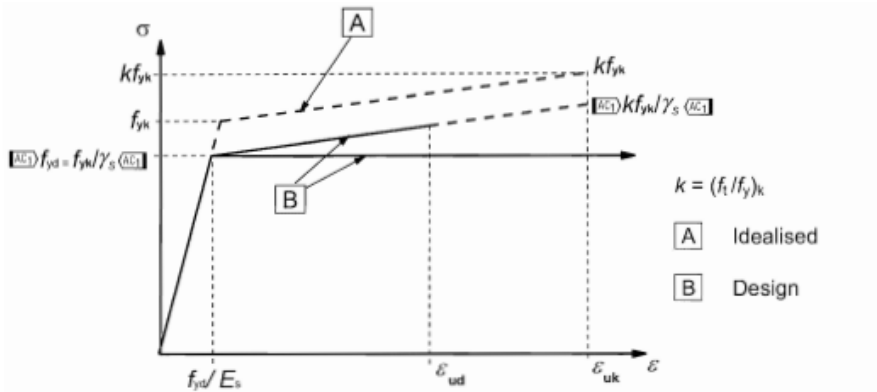


Figure 2.9 - Idealized and design stress-strain diagrams for reinforcing steel[4]

The value of the design yield strength of reinforcement is given by the following expression:

$$f_{yd} = \frac{f_{yk}}{\gamma_s} \tag{15}$$

2.2.3 Safety factors

The safety factors concept given by Eurocode considers that the material capacity must be reduced and then the nominal moment is compared with the applied factored moment.

To calculate the used values previously mentioned, f_{cd} and f_{yd} , respectively, the Design value of concrete compressive strength and the Design yield strength of reinforcement, the following safety factors recommended by EC2 should be:

- Concrete : $\gamma_c = 1.5$
- Steel : $\gamma_s = 1.15$

2.2.4 Design procedure

According to EC2, the verification of a singly reinforced beam involves analyzing the strain in the concrete member when it is subjected to a load. Thus, 4 scenarios can be considered to describe a state of stress-strain of a reinforced concrete member when subjected to bending moment. In Figure 2.10, and Table 2.3 the behavior of the cross section can be observed when the strains reach its limit. When the strain of the concrete or the steel reaches the ultimate strain, it is considered that the concrete member reached its limit and therefore fails.

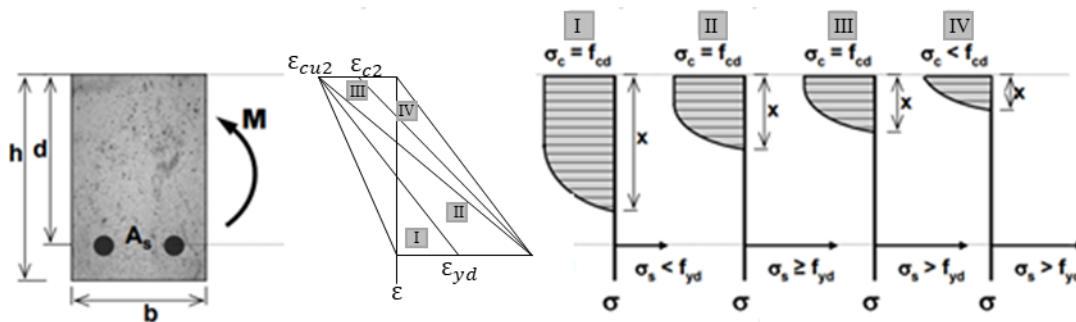


Figure 2.10 - Stress distribution when the reinforced concrete member is subjected to bending moment [10]

Table 2.3 - Stress-strain distribution for concrete crushing and steel failure

Case scenario	Concrete	Steel	Failure
I	$\epsilon_c = \epsilon_{cu2}; \sigma_c = f_{cd}$	$0 < \epsilon_s < \epsilon_{yd}; \sigma_s < f_{yd}$	Concrete
II	$\epsilon_c = \epsilon_{cu2}; \sigma_c = f_{cd}$	$\epsilon_{yd} < \epsilon_s < \epsilon_{ud}; \sigma_s \geq f_{yd}$	Concrete
III	$\epsilon_{c2} \leq \epsilon_c \leq \epsilon_{cu2}; \sigma_c = f_{cd}$	$\epsilon_s = \epsilon_{ud}; \sigma_s \geq f_{yd}$	Steel
IV	$\epsilon_c \leq \epsilon_{cu2}; \sigma_c < f_{cd}$	$\epsilon_s = \epsilon_{ud}; \sigma_s \geq f_{yd}$	Steel

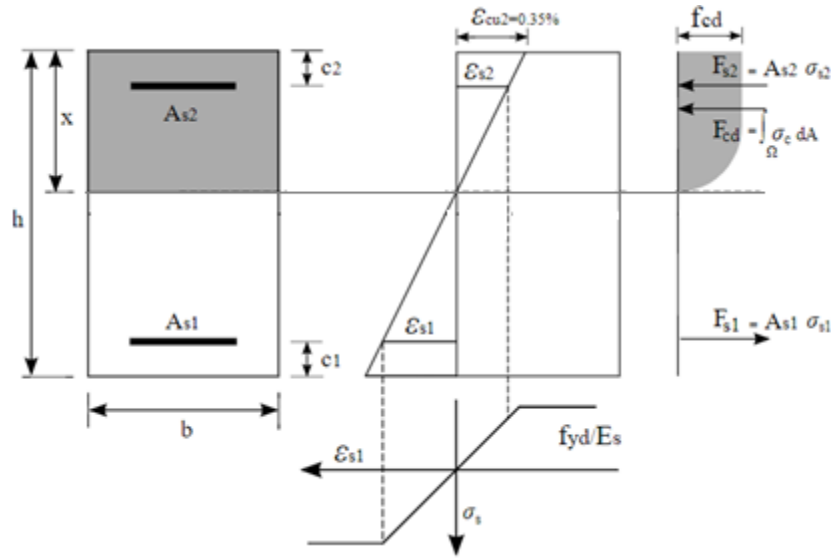


Figure 2.11 – Stress block considering compression reinforcement [11]

The scheme (Figure 2.11) is shown to demonstrate the case scenario for a concrete beam with compression reinforcement and how to calculate its resistance capacity. The main difference from the previous scheme, (Figure 2.10), is that, a compression force due to the compression reinforcement force is included in the equilibrium.

By knowing the initial strain, it is possible to calculate the position of the neutral axis using the same expression used in section 2.1.3 equation (1).

Considering the stress block that has been mentioned, Figure.2.8, for a given cross section, the compression force that is given by the integration of the stress-block is calculated. The stress considered has the expression (12) or (13), depending on the strain value. And the resulting compression force is given by:

$$F_c = \int_A \sigma_c dA \quad (16)$$

The compression force is given by the sum of the two parts: the compression force after the strain reaches ε_{cu2} , represented by the rectangular portion of the stress-block and a second part, due to the parabolic portion, when the strain has not reach ε_{cu2} yet.

Integrating the rectangular part of the stress block we get:

$$C_r = f_{cd} \cdot \frac{\varepsilon_{cu2} - \varepsilon_{c2}}{\varepsilon_{cu2}} \cdot b \cdot x \quad (17)$$

As for the force resulted of the parabolic stress-block, the force is obtained through the following equation:

$$C_p = f_{cd} \cdot \frac{n}{n+1} \cdot \frac{\varepsilon_{c2}}{\varepsilon_{cu2}} \cdot b \cdot x \quad (18)$$

Thus, the resulting compression force is given by:

$$F_c = C_r + C_p \quad (19)$$

The position of the compression force to the neutral axis can be calculated by solving the following integral:

$$x - c = \frac{\int_A \sigma_c \cdot y \, dA}{\int_A \sigma_c \, dA} \quad (20)$$

In order to satisfy the balance of longitudinal forces we have:

$$\sum H = 0 \rightarrow F_c = F_s \quad (21)$$

Where:

F_c is the resulting compression force;

F_s is the tensile force;

$$F_s = \sigma_s A_s \quad (22)$$

and from Figure 2.9 the stress in the steel reinforcement is given by:

$$\sigma_s = \begin{cases} f_{yd} & \text{for } \varepsilon_s \geq \varepsilon_{yd} \\ \varepsilon_s E_s & \text{for } \varepsilon_s < \varepsilon_{yd} \end{cases} \quad (23)$$

After calculating the compression force and the force due to the reinforcement, it is possible to calculate the resistant moment of the cross section:

$$M_{Rd} = F_c \cdot z = F_s \cdot z \quad (24)$$

And the nominal moment:

$$\mu = \frac{M_{Rd}}{bd^2 f_{cd}} \quad (25)$$

After the above calculations have been completed, it is necessary to check the condition of the ultimate limit state.

$$M_{Ed} \leq M_{Rd} \quad (26)$$

. Where:

M_{Ed} - is the design moment

M_{Rd} - is the resistance moment

To calculate the design moment the applied load may be esteemed through one of the following four combinations according to Eurocode, depending in which environment situation the structure is.

Persistent/transient design situations:

$$E_d = E \sum_{j=1}^m \gamma_{Gj} G_{jk} + P + \gamma_{Qi} Q_{ik} + \sum_{i=2}^n \gamma_{Qi} \psi_{0i} Q_{ik} \quad (27)$$

$$E_d = E \sum_{j=1}^m G_{jk} + P + A_d + (\psi_{11} \text{ ou } \psi_{21}) Q_{ik} + \sum_{i=2}^n \psi_{2i} Q_{ik} \quad (28)$$

Seismic design situations:

$$E_d = E \sum_{j=1}^m G_{jk} + P + A_{Ed} + \sum_{i=1}^n \psi_{2i} Q_{ik} \quad (29)$$

Where:

E Effect of actions;

E_d is the design value of effect of actions;

G_{jk} is the characteristic value of permanent action j;

Q_{1k} is the characteristic value of the leading variable action;

A_{Ek} Characteristic value of seismic action;

A_{Ed} is the design value of an accidental action and $A_{Ed} = \gamma_1 A_{Ek}$;

P Relevant representative value of a prestressing action;

γ_{Gj} Partial factor for permanent action j;

γ_{Qi} Partial factor for variable action i;

γ_1 Importance factor (see EN 1998);

ψ_0 Factor for combination value of a variable action;

ψ_1 Factor for frequent value of a variable action;

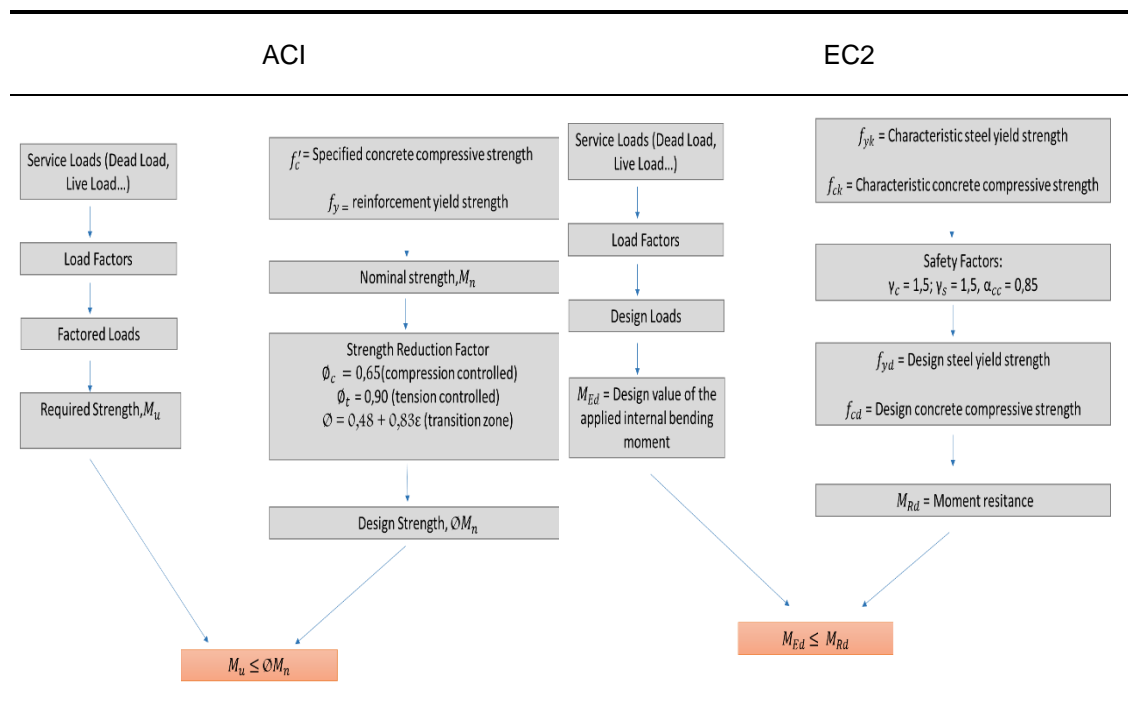
ψ_2 Factor for quasi-permanent value of a variable action;

2.3 Comparison- ACI codes and Eurocode 2

The procedure followed in this section is for reinforced concrete members without prestress. The cross section is known and so is the bending moment. Axial forces were not considered.

To summarize the different flexural design approaches given by the European code and the American code, the following table is presented, Table 2.4:

Table 2.4 - ACI and EC2 design comparison



3 Prestressed reinforced concrete members

In this chapter the different materials used for reinforced concrete members with prestress and its history will be described briefly. Also, different types of prestress systems and the flexural design method in the ultimate limit state will be presented.

In order to optimize prestressed reinforced concrete members, three types of structures were studied. conventional steel reinforced concrete structures, CFRP structures and finally mixed structures (CFRP and steel).

3.1 Prestress history

The benefits of prestressing capacities have been used from decades. The earliest date goes back to the beginning of the 18th century, a timber bridge made of timber by Stephen Harriman Long, Figure.3.1. [12].



Figure 3.1 - Prestressed wooden bridge as designed by Stephen H. Long [12]

Prestressed concrete members were well developed a few years later by Peter H. Jackson who obtained the first patent for prestressed concrete structure: a sidewalk made of prestressed cast iron and wrought iron[12].

His patent consisted of large plates of cast iron working as the upper chord of a small truss, tie rods of wrought iron working as the lower chord, and metallic stanchions working as vertical posts Figure 3.2 Peter H. Jackson created new techniques for bridges, trusses and prestress.

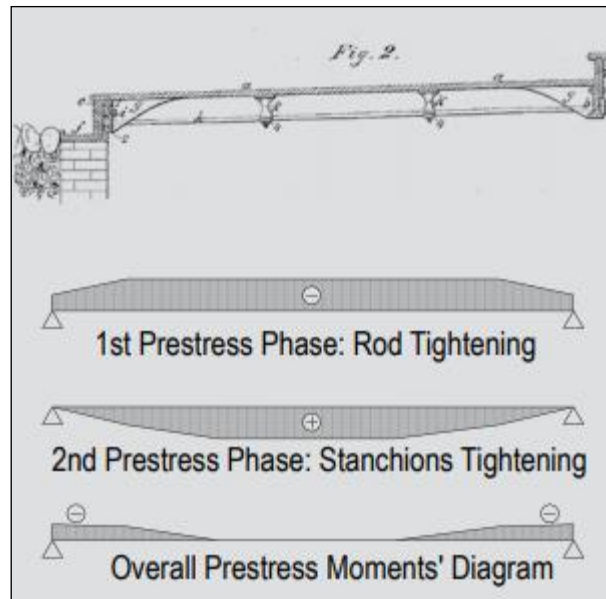


Figure 3.2 - Prestress cast iron sidewalk as designed by Peter H. Jackson-Diagrams[12]

In the 19TH century, Eugène Freyssinet, a structural engineer who propose to replace three old suspension bridges over Allier River (France) using high quality concrete with the cost of a single one. Le Veudre Bridge (1910), Boutiron Bridge, and Châtel-deNeuvre Bridge. However, deformation problems were detected shortly after its construction and in 1940 the bridge was destroyed during the war. In 1928 Freyssinet patent his ideas and an addendum was made. In 1930 Freyssinet demonstrates his knowledge and a few conclusions of his study are shown as an example:[12]

- the recommendation to use high-quality concrete;
- very high strength steel (wires);
- a variety of methods to tension the wires;
- the possibility of precasting several long elements on only one beam of wires followed by cutting them to the desired length;
- high strength concrete to reduce to a minimum the loss of initial prestress;
- high initial stress of tensioned steel;

An expansion of prestressed concrete occurred in Europe due to the World War I.

In 1937, the German engineer Ewald Hoyer, secured four patents about precast prestressed concrete beams.

In 1939 Freyssinet secured a patent for what is known nowadays as post-tensioning system. The system includes the anchorage device, the duct, the jack and the tendon. Figure 3.3.

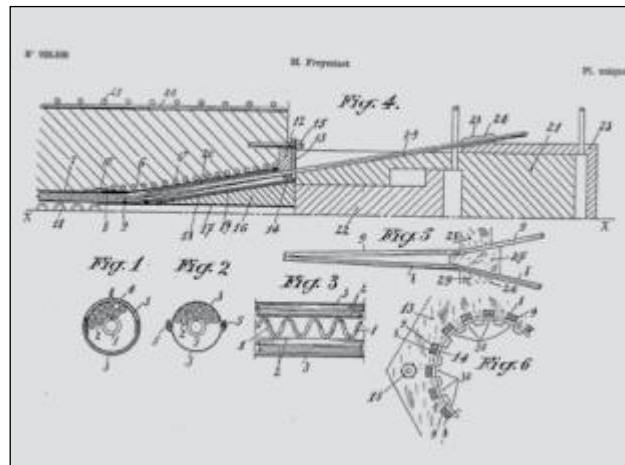


Figure 3.3 - Prestress cast iron sidewalk as designed by Peter H. Jackson- [12]

Another contribution was made by the Belgian Gustav Magnel to solve the problem of the steel relaxation and its influence on prestressed concrete members. Corrosions are one of the main problems of steel, and in order to solve the problem of steel tendons and rebars, important research on the use of new material was and still is nowadays conducted at many European universities.

3.2 Prestress systems

Concrete members present a good behavior when subjected to compression forces, however they have a very poor behavior when subjected to tension, so the tensile strength may be neglected and therefore a solution must be provided to compensate the lack of tensile strength. Steel and FRP are materials used to compensate the lack of tension forces in the concrete beam, either as conventional reinforcement or as prestressed cables, since these materials have a high tensile strength.

Prestressed cables are a good alternative to solve the lack of tensile strength. By tensioning high-strength tendons, compression stress is induced before loading, so after loaded (by equilibrium) the tensile forces will balance the compressive forces, which means that the tensile stresses will reduce or even be canceled as shown in Figure 3.4. Consequently, less reinforcement will be required.

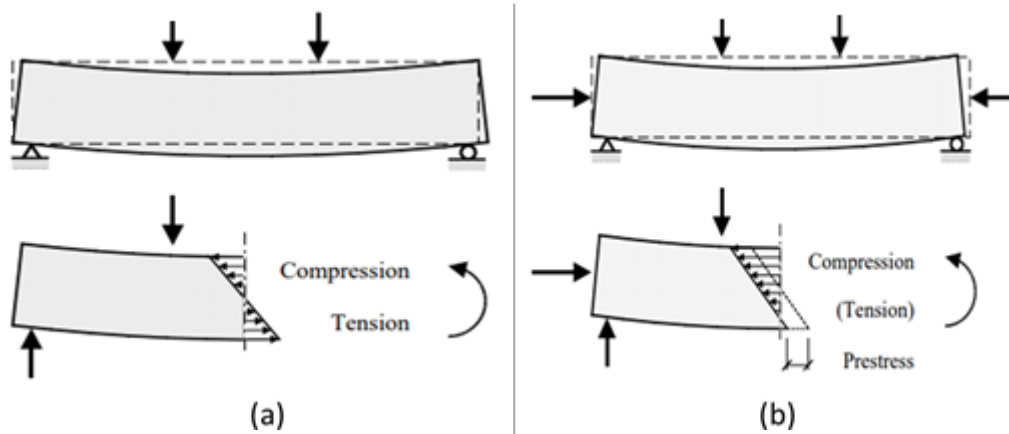


Figure 3.4 – (a) Concrete member subjected to bending without prestress; (b) concrete member subjected to bending with prestress- tensile strength reduction[13]

Advantages of prestressed concrete members:

- Increased spans;
- Higher slenderness of structural elements;
- Lighter structures;
- Improved in-service and long-term behavior;
- Smaller deflections;
- Rational use of high-strength concrete and steel;

There are two ways to tension the prestress, pre-tensioning and post-tensioning. In this dissertation only the post tensioning system was evaluated comparing bonded and unbonded tendons. Figure 3.5, summarize some of the prestress existing types.

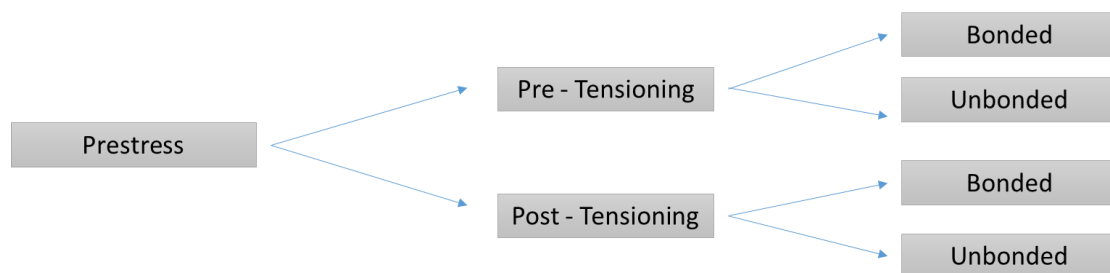


Figure 3.5 - Prestress systems

In the post-tensioning system, the prestress is tensioned after the hardening of the concrete member. When the concrete has acquired the necessary strength, the transfer of stress is carried out at the ends by anchors.

The following Figure 3.6 presents both system previously mentioned.

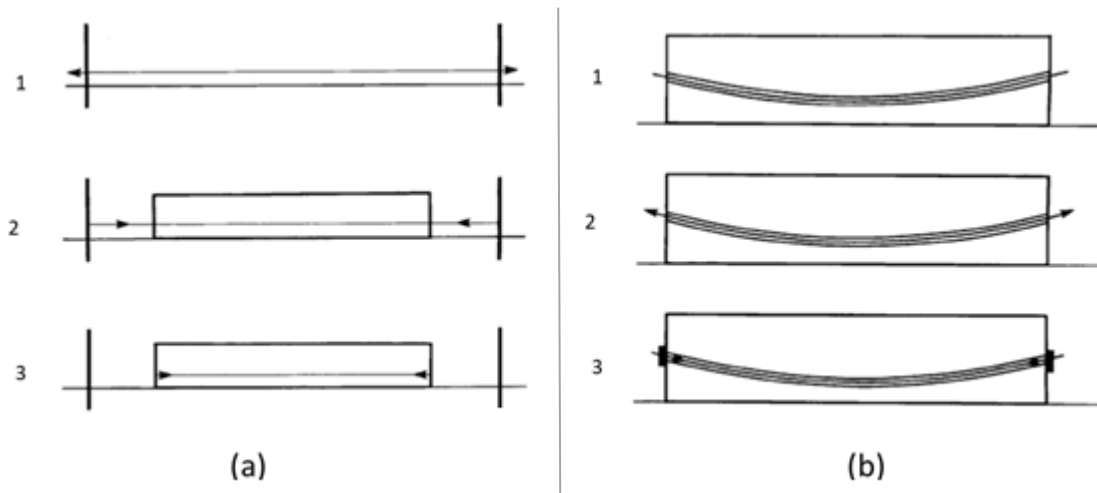


Figure 3.6 - Prestress systems: (a) pre-tensioning; (b) post-tensioning [14]

3.3 Bonded tendons

As shown before, the tendon used on prestressing may be bonded or unbonded. Bonded tendons have a main feature that consists in tendons forming a bond in the concrete structure by grouting over its length. The grout, most of the time is a cementitious matrix to be applied after stressing the strands. After the grout hardens the longitudinal movement is not possible, so the function of the grout is:

- Provides a continuous bond between the strand and the duct.
- Increase protection against corrosion, since it creates a physical barrier.
- Provides an environment non-conducted for corrosion through its alkalinity.

By compatibility, the force in the strand is directly related with the deformation of the surrounding concrete.

The losses by friction must be considered and it is higher in short tendons due to the high friction between the strand and its housing.

The process to build a prestressed beam is shown in Figure 3.7:

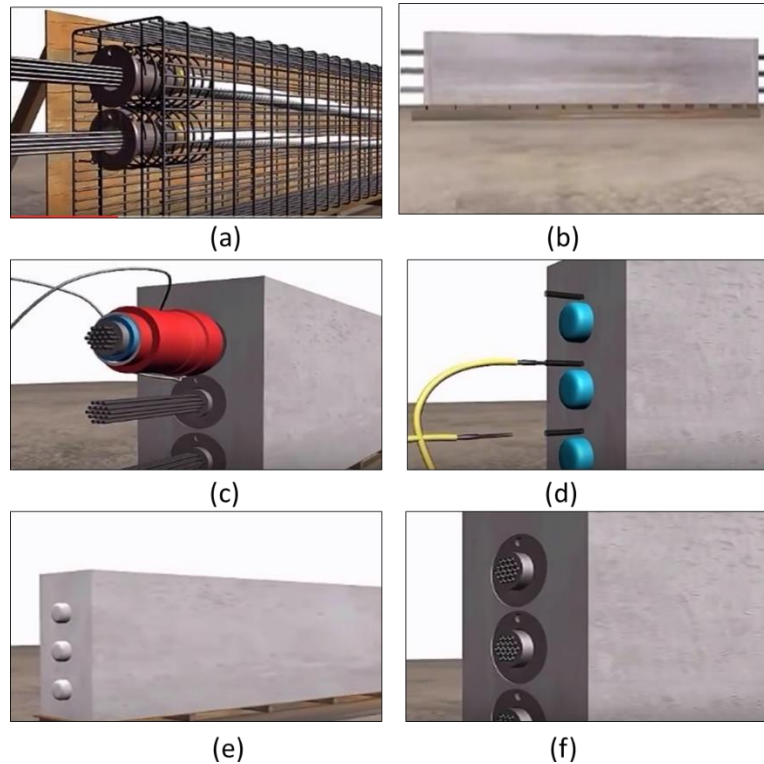


Figure 3.7 - Process for the manufacture of a bonded prestressed RC beam with post tension system (a) Placing prestress cables, (b) Concreting and hardening of concrete, (c) Anchoring process, (d) Grouting process, (e) Hardening of the concrete beam, (f) Anchoring detail [15]

Bonded Advantages:

- Develops higher ultimate flexural strength;
- Does not depend upon the anchorage after grouting;
- Localizes the effects of damage;
- Simple technique for demolishing or providing future opening in slab

For the design of bonded tendons, in this dissertation, the flexural design has been made according to Eurocode with the following assumptions:

- Perfect bond between tendon and concrete;
- Concrete strain capacity is assumed as 0.0035;
- Strain compatibility between prestress strain and concrete strain;
- Linear strain behavior;
- Tensile concrete strength may not be considered;
- A plane section remains plane after deformations;

- Perfect bond between reinforcement bars and concrete;
- The deformations due to the shear forces may be neglected;

3.4 Unbonded tendons

The prestress is designated as unbonded if the tendons are not bonded to the concrete member and therefore a longitudinal displacement is allowed relatively to the concrete. The compressive force goes only and directly from the anchors to the concrete

For unbonded tendons, as shown in Figure 3.8, there is a coating material that prevents the corrossions of strands and reduces the friction effect and a plastic sheathing to encase and it acts as a bond breaker, protecting against damage by mechanical handling and barrier against intrusion of moisture and chemicals.

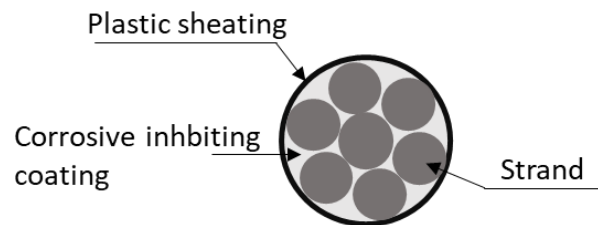


Figure 3.8 - Cross-section of wire unbonded tendon

When the concrete member is loaded a flexural deformation occurs and it leads to an increase in the length of the tendon. By compatibility of deformation the elongation and the strain are directly related. In that way, an increment of the stress must be considered, and it can be calculated considering the deformation and safety factors.

Unbonded Advantages:

- Provides greater available lever arm;
- Reduces friction losses;
- Simplifies pre-fabrications of tendons;
- Grouting not required;
- Can be constructed faster;

The analysis regarding unbonded tendons relies on the following assumptions:

- No friction between concrete and tendons;

- Tensile concrete strength may be ignored;
- A plane section remains plane after deformations;
- Perfect bond between reinforcement bars and concrete;
- The deformations due to the shear forces may be neglected;
- The stress in an unbonded tendon is constant over its full length;
- Concrete creep and tendon relaxation are not considered in the present study;
- Stress-Strain relationship is linear for both concrete and steel;

3.5 FRP

One of the major problems faced by civil engineers for construction with reinforced concrete members is the corrosion to which it is subjected, and it is directly related to their durability.

Thus, new types of materials have been studied to replace the conventional reinforcement and prestress. In this dissertation will be studied the FRP material and, more specifically the CFRP.

FRP (Fiber-reinforced polymer) are composite materials composed of aramid fibers (AFRP), basalt (BFRP), carbon (CFRP) or glass (GFRP). They can be used in aerospace, automotive, marine and construction industries, since it presents a high tensile strength.[16]

- **AFRP**

Aramid fibers are synthetic materials and it is known by their high strength and good heat-resistance. These fibers present a high modulus of elasticity but a low density. On the other hand, it has a higher cost than the other fibers such as glass and basalt, and therefore are less used as reinforcement in construction. In addition, the fibers of aramid also absorb moisture and must be impregnated with a polymer matrix

- **BFRP**

Basalt fibers are fibers with a high temperature and abrasion resistance, it has almost the same price of the glass fibers, however, the lack of guidelines for basalt by the ACI, Canadian Code and FIB makes them less competitive in the market than the other fibers.

- **CFRP**

Carbon fibers present high tensile strength, higher than all other fibers. They also feature lightweight and low thermal expansion. However, because it is a material with an elevated quality also presents a cost to match.

- **GFRP**

Glass fibers have high electrical insulation properties, good heat resistance and have the lowest cost and therefore are widely used for reinforcing structures. It has a much lower modulus of elasticity than steel.

The figure below (Figure 3.9) shows the comparison between the tensile strength of different materials used in the field of civil engineering. It should be noted that carbon has a modulus of elasticity similar to that of steel.

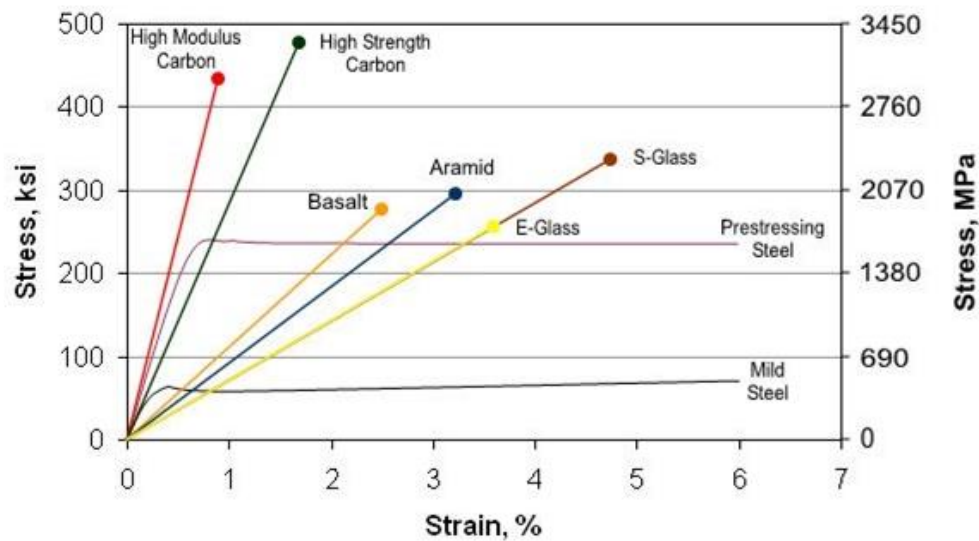


Figure 3.9 – Tensile strength comparison for different materials [17]

Within all of the aforementioned fibers, the most interesting for the use of prestressing are the CFRPs because they have the highest strength and modulus of elasticity. One of the main problems with CFRP tendons is the fact that their anchoring is not a simple process for the post-tensioning or pre-tensioning systems. Each supplier produces tendons with different characteristics and for that reason to find a perfect solution is almost impossible. Another problem is the lack of lateral resistance that FRP tendons present, making it difficult to find an optimal anchoring system, because any lateral force imposed can damage the tendon, since it is a fragile material with a bad lateral resistance. Although some studies mention mechanical anchorage as a possible solution because it is easy to install [17], while some others, [18] suggest that anchorage systems of bonded type are more adequate to avoid a concentration of stress, distributing the load evenly from the anchorage to the FRP tendon.

Mechanical anchors work by friction between the tendon and the interior of the anchorage. For the case of the mechanical anchorage type, a compression force is applied perpendicularly to the tendon axis. [18]. While bonded anchoring systems consist of wrapping the FRP tendon in an adherent material with the aim of distributing stress along the anchor's length, as it can be seen in Figure 3.10 since the high concentration at the end of the anchor can be very destructive [19].

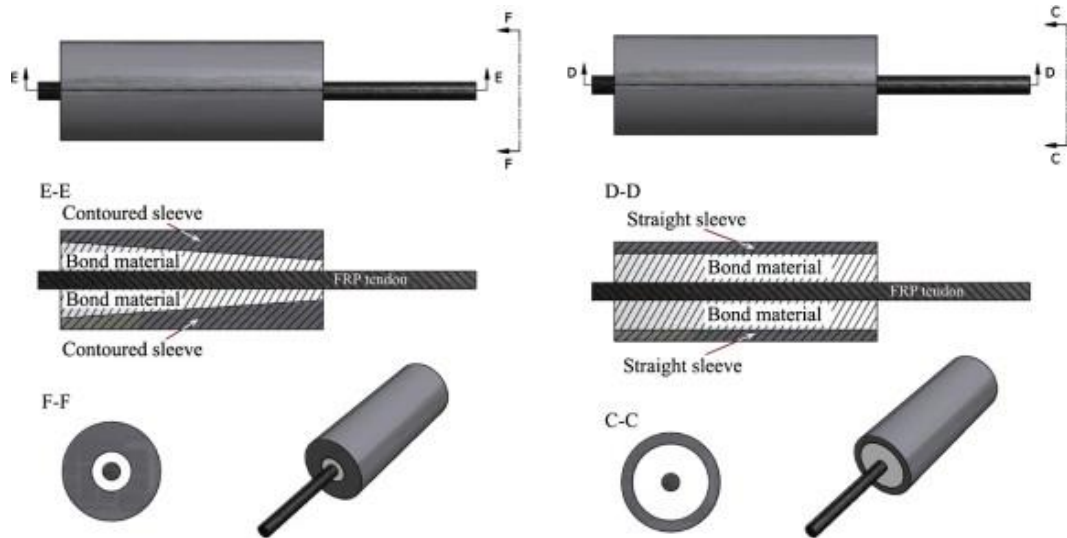


Figure 3.10 – Bond type anchorage system [18]

3.5.1 FRP Structures

The use of FRP in civil engineering structures has been growing considerably in recent decades, especially in pedestrian bridges.

The first pre-stressed bridge built, Lunenshe Gasse Bridges, was built in 1980 Dusseldorf, Germany [20]. It is composed by twelve GFRP tendons. Figure 3.11.



Figure 3.11 - Lunenshe Gasse Bridges [20]

The first highway bridge was also built in Dusseldorf, Germany, in 1986, The Ulenbergstrasse bridges (Figure 3.12(a)) consisting of two spans and was also made in GFRP, Figure 3.12(b) [20].

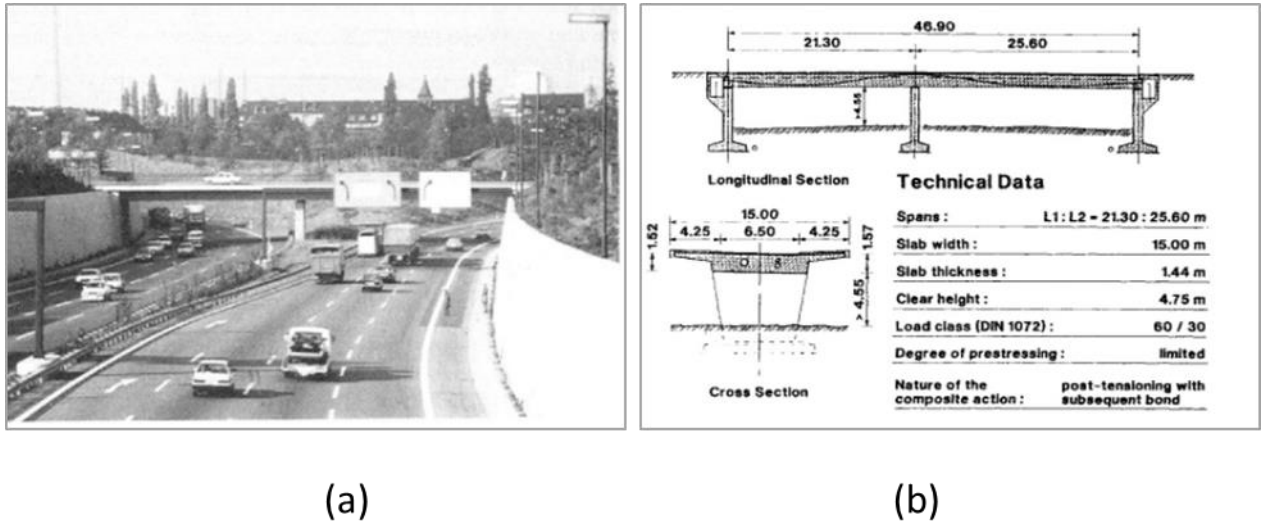


Figure 3.12 – (a) The Ulenbergstrasse bridges and (b) Technical data [20]

Relatively to bridges with CFRP, the first CFRP bridge with prestressed tendons was made in Japan in 1988 [21]. The first highway bridge with CFRP prestress and fiberoptic sensors was made in Alberta, Canada in 1993, the Beddington Bridges Trail, Figure 3.13.








Figure 3.13 – The Beddington Bridges Trail, Canada [21]

This bridge is composed of two spans 22.83m and 19.23m and each consisting of 13 bulb-tee sections precast and prestressed. Two types of CFRP cables were used, one produced by Tokyo Rope and the other by Mitsubishi Kasei [22].

The following table shows some examples of CFRP bridges (Table 3.1) with data provide from ACI Code[21].

Table 3.1 - CFRP Bridges examples

Bridge	Country	Year	Picture
<p>Shinmiya Bridge</p> <p>First application of Carbon cable as prestressed material</p>	Japan	1988	 A black and white photograph of the Shinmiya Bridge, a concrete bridge with a single span, crossing a river. The bridge has a simple design with a railing on top.
<p>Rapid City Bridge</p> <p>Precast and post-tensioned bridge.</p> <p>Length: 9m Span: 5.2m</p>	United States	1992	 A black and white photograph of the Rapid City Bridge, a precast and post-tensioned bridge. The bridge is shown in a construction or testing phase, with a large concrete structure supported by a temporary structure.
<p>Tsukude Golf Country Club Bridge</p> <p>Pedestrian bridge with a single span of 99.0m</p>	Japan	1993	 A black and white photograph of the Tsukude Golf Country Club Bridge, a long pedestrian bridge with a single span. The bridge is shown in a landscape setting with trees and a body of water.
<p>Beddington Trail Bridges</p> <p>Highway bridge with precast and prestressed concrete members.</p>	Canada	1993	 A black and white photograph of the Beddington Trail Bridges, a highway bridge with precast and prestressed concrete members. The bridge is shown from a low angle, looking down the road towards the bridge structure.
<p>The West Mill Bridge</p> <p>High way bridge with span of 10.0m</p>	United Kingdom	2002	 A black and white photograph of The West Mill Bridge, a highway bridge with a span of 10.0m. The bridge is shown from a side angle, highlighting its concrete structure and railing.

3.6 Ultimate limit state

3.6.1 Compression force

The compression force, since it only depends on the strain, is calculated the same as shown in section 2.2.4, equation (16) and (17).

3.6.2 Prestress force

The force of the prestress varies over time and it suffers instantaneous losses and long-term losses, however, it is considered a permanent action since in a short time it tends to its limit value.

According to EC2 the procedure for calculating the prestress force is given as follows:

- Maximum force P_{max} imposed at the active end.

$$P_{max} = A_p \cdot \sigma_{p,max} \quad (30)$$

Where:

A_p is the area of a prestressing tendon or tendons;

$\sigma_{p,max}$ is the Maximum prestress stress;

$$\sigma_{p,max} = \min \{ k_1 \cdot f_{pk} ; k_2 \cdot f_{p0.1k} \} \quad (31)$$

$$k_1 = 0.8$$

$$k_2 = 0.9$$

- The value of the initial prestress force P_{m0} :

$$P_{m0} = A_p \cdot \sigma_{p,m0}$$

where:

$\sigma_{p,m0}$ is the stress in the tendon immediately after tensioning or transfer;

$$\sigma_{p,m0} = \min \{ k_1 \cdot f_{pk} ; k_2 \cdot f_{p0.1k} \} \quad (32)$$

$$k_1 = 0.75$$

$$k_2 = 0.85$$

As previously mentioned, the bonded and unbonded tendons present different approaches and thus the prestress forces are presented for each type of tendons.

- **In the case of bonded tendons, we have:**

$$F_p = A_p \cdot \sigma_p \quad (33)$$

With:

The prestress stress given by:

$$\sigma_p = \begin{cases} \frac{0.9f_{pk}}{\gamma_p} & \text{for } \varepsilon_p \geq \varepsilon_{pd} \\ \varepsilon_p E_p & \text{for } \varepsilon_p < \varepsilon_{pd} \end{cases} \quad (34)$$

For the case of the carbon we do not have yield plateau and thus the tension can only be calculated linearly until reaching the ultimate strain.

- **In the case of unbonded tendons we have:**

$$\sigma_p \cong \frac{P_\infty}{A_p} + \Delta \sigma_{p,ULS} \quad (35)$$

Therefore, equation (33) into equation (30) it is possible to obtain the force of prestress that is given by:

$$F_p = P_\infty + \Delta \sigma_{p,ULS} \cdot A_p \quad (36)$$

Where:

F_p is the prestress force;

A_p is the area of a prestressing tendon or tendons;

σ_p is the stress applied to the tendon;

f_p is the tensile strength of prestressing;

f_{pk} is the characteristic tensile strength of prestressing;

γ_p is the partial factor for actions associated with prestressing;

P_∞ is the prestress force after all losses;

$\Delta \sigma_{p,ULS}$ is the increase of the stress from the effective prestress to the stress in the ultimate limit state and it can be considered as 100 MPa according to EC2, section 5.10.8

The strain distribution scheme is shown in the following Figure 3.14:

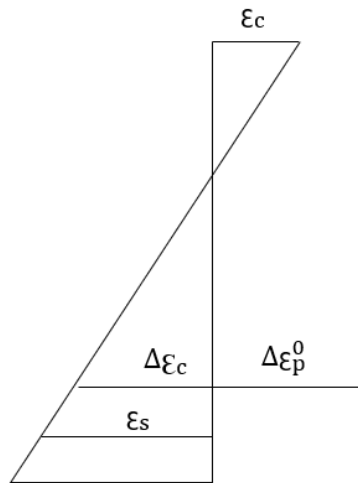


Figure 3.14 - Strain distribution for prestressed concrete beam

The prestress strain is the sum between the initial strain and the strain of tendon available in flexure. Given by:

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p0} \quad (37)$$

The initial strain is given by the following equation:

$$\varepsilon_{p0} = \frac{P_\infty}{A_p \cdot E_p} \quad (38)$$

And the strain in the prestress can be obtained by geometry :

$$\frac{\Delta\varepsilon_p}{d-x} = \frac{\varepsilon_c}{x} \quad (39)$$

The method used is the one provided by the EC, considering a parabolic-rectangular stress block and some values assumed as given values, such as: the pre-stress used, the type of concrete, the type of steel and a cross-section with a given geometry. Thus, the only unknown, is the need or not of extra reinforcement to satisfy the balance.

The calculation of the cross-section resistance capacity is a result of the equilibrium of the strains in the failure through the iterative method. Considering that the strain of concrete reaches the ultimate strain.

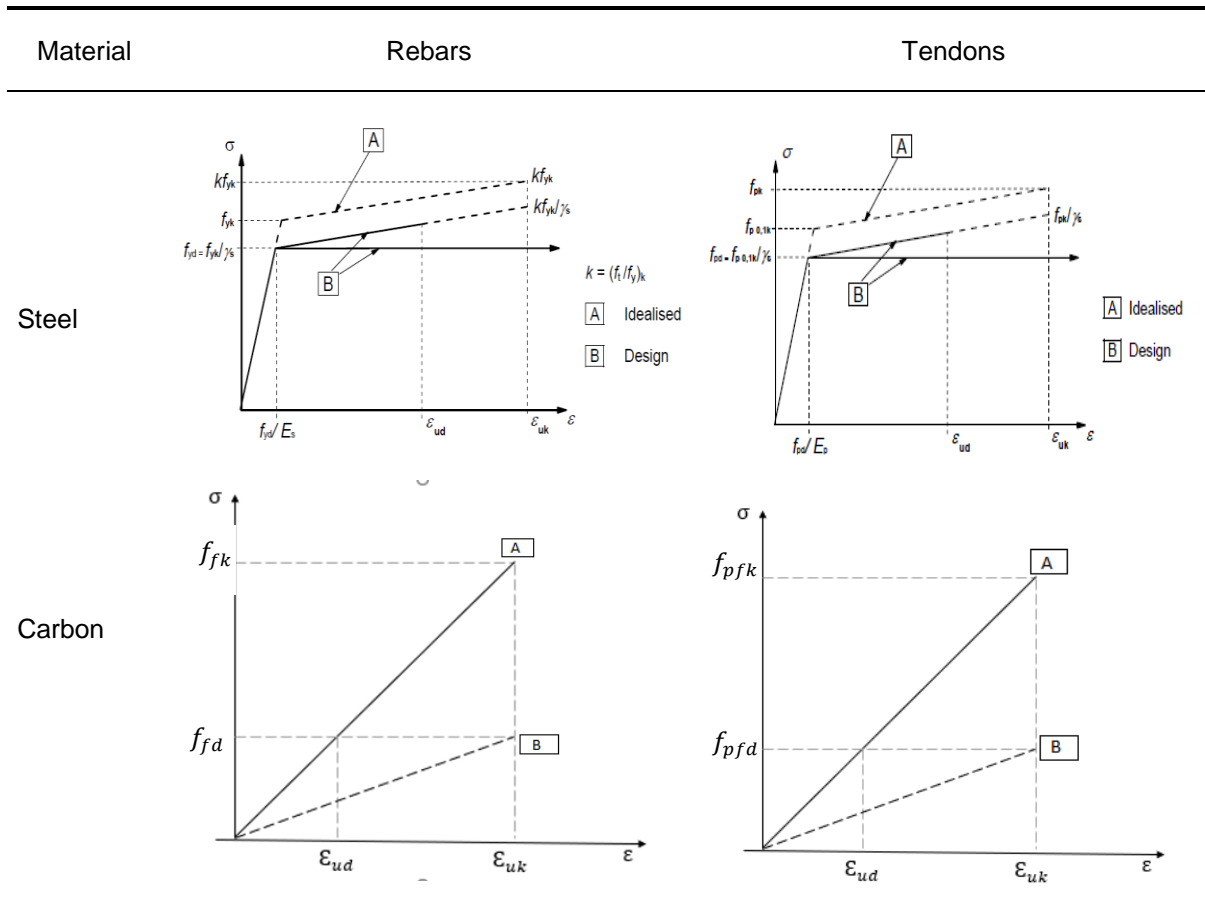
This method is based on initiating the process with ultimate strain for concrete and reinforcement. Depending on the need for a greater or lesser compression zone to satisfy the balance of forces, an

adjustment in the strain to reach the equilibrium is made. Once the strain is found, stress and the corresponding forces may be calculated.

3.6.3 Materials

Regarding the concrete properties used in this chapter they are the same as the ones presented in section 3.2 and for the reinforcement and prestress the following stress-strain distribution is considered. And as it can be seen, Table 3.2, steel and CFRP present different behavior that shall be considered in the design procedure. For the steel, it is possible and preferable to design after the yield point while for CFRP material this point does not exist, therefore the concrete member fails abruptly if the failure is due to CFRP reinforcement. The Table 3.2 shows these differences.

Table 3.2 - Steel and carbon. Comparison of stress-strain curves



The values of the safety factors used for each material of the prestressed reinforced concrete structure are presented in Table 3.3. For steel and concrete the values considered are given by EC2 [4] and for CFRP the safety factors considered are given by FIB [5]

Table 3.3 - Material safety factors given by FIB[5] and EC2 [4]

Concrete	Steel (tendons/rebars)	CFRP (Tendons)	CFRP (Rebars)
1.5	1.15	1.3	1.8

3.6.4 ULS – Flexural Verification

The ultimate limit state is a state in which the carrying capacity of the structure is exceeded, and therefore the safety of the structure and especially of human lives are compromised. It should be noted that the structures can be dimensioned for the ultimate limit state or for the service state, and the distinction between the two is made through the risk. Ultimate Limit State can lead to human and material losses, while the service state only leads to the discomfort of the user of the structure.

The verification of the ultimate limit state sums up the calculation of the amount of reinforcement required when the reinforced concrete member is subjected to a given moment.

For the ultimate limit state to be verified, the following condition must be satisfied:

$$E_d \leq R_d$$

Where:

E_d is the value of the applied load and R_d the resistant capacity;

As for the calculation of R_d , resistant capacity of the structure, a cross section analysis is made and as shown in Figure 3.15, these forces must be considered.

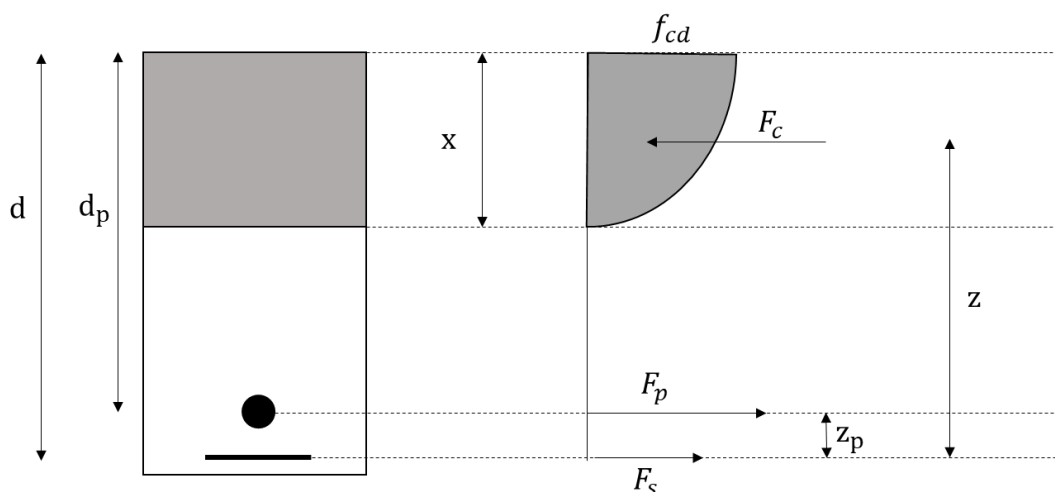


Figure 3.15 - Internal forces for calculation of the cross-section resistance capacity

The verification of the ultimate limit state was done through the balance of the forces. Assuming an initial strain, ultimate strain and through an iterative process it was possible to find the neutral axis and the forces that balance the applied loads as well.

$$\sum H = 0 \quad (40)$$

$$\sum M = 0 \quad (41)$$

Where:

H are the horizontal forces;

M is the moment in the cross section;

Doing the equilibrium of the moments at the point where the reinforcement is applied, it is possible to write the following:

$$M_{Rd} = F_c \cdot Z + F_p \cdot zp \quad (42)$$

Where:

M_{Rd} is the resistant moment;

F_c is the resulting compression force;

Z is the level arm, the difference between the compression force and the tensile force in the reinforcement rebars;

F_p is the force in prestressed cables;

Zp is the level arm, the difference between the tensile force in rebars and the prestress force;

The equilibrium given by the equation (41) the balance of the longitudinal forces is obtained.

$$F_c - F_p - F_s = 0 \quad (43)$$

With the expression (45), the tensile force in the steel reinforcement can be found and so the amount of reinforcement.

4 Parametric study for optimization of RC structures in bending with CFRP

4.1 General considerations

CFRP are materials with high performance fibers that have been used in civil construction in order to replace the conventional steel used as reinforcement and pre-stress. The lack of uniformity in the guidelines and the high cost are some of the characteristics that prevent this material from expanding exponentially in its use for the structures. The feasibility of the solution is directly related to the amount of material used, which may vary depending on the type of reinforcement, the design procedure and the value of the safety factor.

Several recommendations [5], [7], [23], [24] for CFRP structures exist today, but they do not agree with the value of the safety factor or with the design procedure itself.

Thus, a parametric study has been conducted and it is presented in this paper to show the impact on design results caused by using different design concepts, for the possible failure modes, for the type of material used as reinforcement and tendons and for the type of prestress system (bonded or unbonded). Prestressed and non-prestressed sections are analyzed in the following chapter.

The main goal of this dissertation is to analyze and optimize the design procedure for CFRP reinforced concrete structures.

4.2 The code developed

In order to help on solving the problems that have been mentioned, a study that used Visual Basic Application (VBA) was developed and in this way several other programs were created.

Thus, the results obtained by the program were based on the balance of forces and therefore for a cross-section, with a given geometry and known mechanical properties, it was possible to calculate the amount of reinforcement required that satisfies the equilibrium condition.

Several codes for different case scenarios were elaborated. A code in which the rules suggested by the Eurocode 2 [4] were followed (Appendix A). A second code for the design of reinforced concrete structures, but according to the ACI Standard [2], [3] design procedure and it is presented in the (Appendix B). The two codes previously mentioned are for non-prestressed reinforced concrete structures and it has the main goal to compare the design procedure given by these two codes.

The third one is for prestressed reinforced concrete structures” and it presents three different approaches (subparts). The first one, in which the applied moment or section width varies (Appendix C); a second one in which the value of the prestress force is variable and finally a design variation in which the strength of the material and its strain are reduced.

The codes are subdivided into 4 subparts: Input, Constitutive Laws, Calculation process and Output. And to demonstrate what was developed in a brief way it is presented a flowchart in Figure 4.1 to summarize the codes presented in the appendices.

Input- In this section all variables are entered manually. Program users have the freedom to choose the type of concrete and steel they want to work with, as well as the cross-section geometry and the limit strains of each material in a database table. Each code has its own table and properties to vary. Regarding the CFRP material the user must decide the specified values, which means, the safety factor is applied after, during the iteration process.

Constitutive Laws- In this section is defined the type of stress-strain diagram used to calculate the stress at the point of interest.

Calculation process- The part of the calculation itself, where the loop is drawn and by an iterative process seeks the balanced solution. The process in program 1 and 2 generates values for μ from 0 to 0.48 with the increment of 0.01 and μ is directly proportional to the applied moment. Regarding the program for prestressed members, the value of the input becomes the applied moment and in a second case the width of the cross section. And finally, the last one, varying the value of the initial force of the prestress.

With the limit strain defined as input it is possible to calculate the neutral axis and consequently the stress. After determining tensions, it is possible to calculate the resulting compression force and the tensile force applied to the reinforcement. Once these forces have been calculated, we only have to calculate the moment at one of these points and we can obtain the resistant moment of the cross section. This moment is compared later with the applied moment and if the difference between them is less than 10^{-5} we consider that the moments are equal and as such, we find the solution and the loop ends. If the applied moment is smaller than the resistant one we have to follow the strain curve of the concrete and we divide the strain value by half in each iteration, reducing the compressed zone until the equilibrium is found. If the applied moment is greater than the resistant one, we elaborate the same procedure but following the steel strain way.

Output – The output of the program consists of many different variables and we can chose among those to have the desired plot. In this case it is possible to analyze the amount of reinforcement and the variation of the strain in relation to the variation of the applied moment, section width or initial force of the prestress.

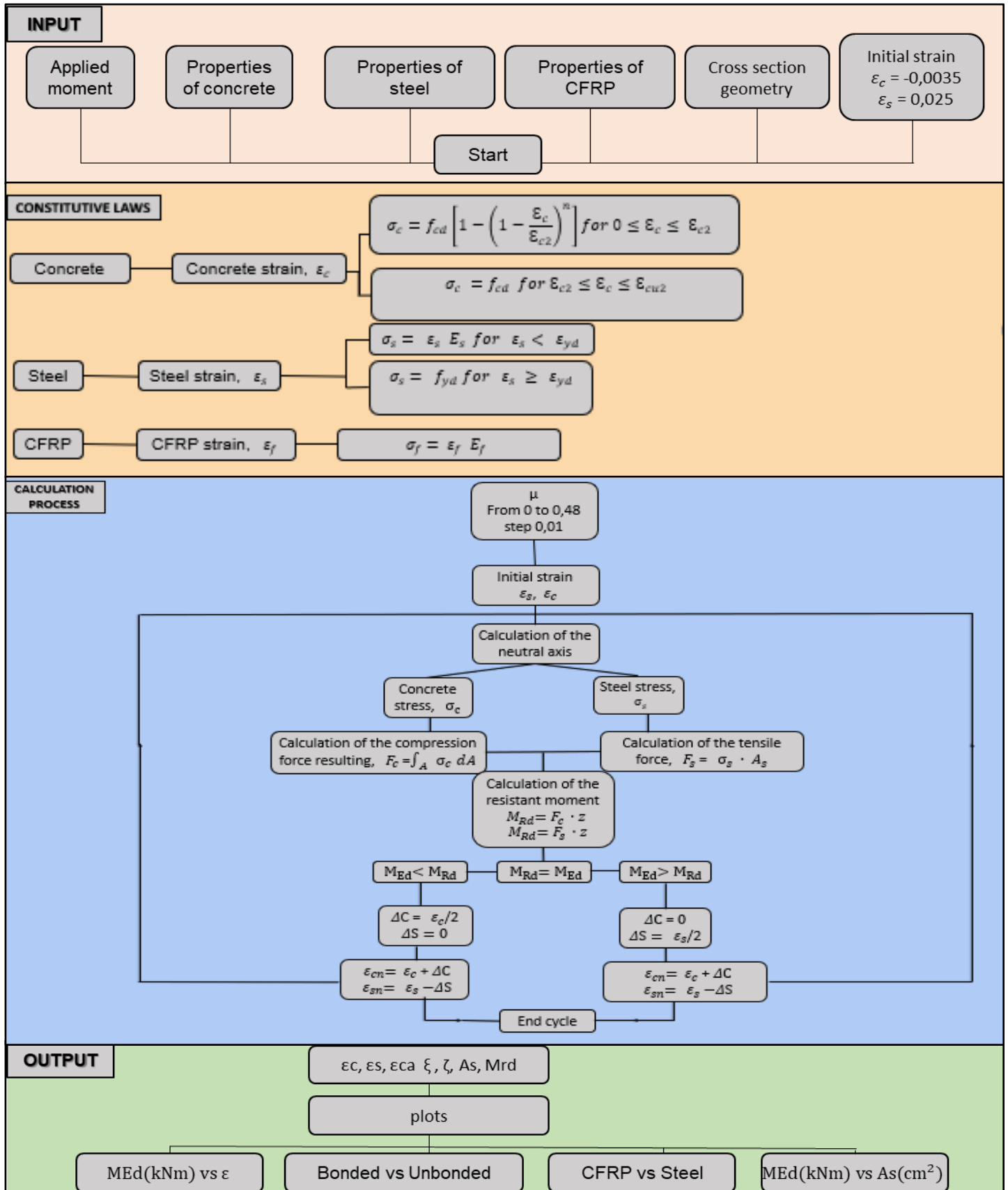


Figure 4.1 - Flowchart of the cycle performed by VBA code

4.2.1 Properties of materials

The properties of the CFRP tendons used in the code are shown in Table 4.1

Table 4.1 – Properties of CFRP tendons

CFRP Tendons	
f_{pk} (MPa)	2690
$f_{p0,1k}$ (MPa)	2421
f_{pd} (MPa)	2022.56
E_f (GPa)	155
ε_{uk} (‰)	17,35
γ_f	1.33
A_p (cm ²)	11.0544
P_{m0} (kN)	1275,46
ΔP_{c+s+r} (%)	10

On the other hand, the type of tendon selected is steel, the data considered are as follows in Table 4.2.

Table 4.2 – Properties of steel tendons

Steel Tendons	
f_{pk} (MPa)	1770
$f_{p0,1k}$ (MPa)	1593
f_{pd} (MPa)	1539,13
E_s (GPa)	195
ε_{uk} (‰)	25
γ_s	1.15
A_p (cm ²)	11.0544
P_{m0} (kN)	1275,46

ΔP_{c+s+r} (%)	10
------------------------	----

For the type of ordinary reinforcement, it is possible for the user to choose CFRP or steel and the characteristics of the rebar of each material are in Tables 4.3 and Table 4.4 respectively.

Table 4.3 - Properties of CFRP rebars

Carbon_Rebars	
f_{fk} (MPa)	2550.0
f_{fd} (MPa)	1416,67
E_f (GPa)	158,0
ε_{fk} (‰)	16,14
ε_{fd} (‰)	8,97
γ_f	1.8

Table 4.4 - Properties of CFRP rebars

Steel_rebars	
f_{yk} (MPa)	500
f_{yd} (MPa)	434,78
E_s (GPa)	200
ε_{uk} (‰)	25
ε_{yd} (‰)	2.174
γ_s	1.15

Where:

A_p is the area of a prestressing tendon or tendons;

E_f is the elastic modulus of FRP;

E_s is the elastic modulus of steel;

f_{fd} is the design value of tensile strength for FRP and ε_{fd} is its respective strain;

f_{fk} is the characteristic value of tensile strength of FRP reinforcement and ε_{fk} is its respective strain;

f_{yd} is the design yield strength of reinforcement and ε_{yd} is its respective strain;

f_{yk} is the characteristic yield strength of reinforcement;

ε_{uk} is the characteristic strain of reinforcement or prestressing steel at maximum load;

ΔP_{c+s+r} (%) Are the long-term losses due to creep, shrinkage and relaxation at location x, at time t;

γ_f partial safety factor for FRP;

γ_s partial safety factor for Steel;

Since, as previously mentioned, there is no European standard for the design of FRP structures, the values of these safety factors (Table 4.1-Table 4.4) were taken from previous studies [25] and guideline [5].

Case 1

In the case 1 the parameters of the cross-section as well as the properties of the concrete to be used are always constant and considered as input for the program. Table 4.5 and Table 4.6.

Table 4.5 – Geometry of the cross section for case 1

Cross_Section	
h(m)	1.1
dp(m)	1.075
d(m)	1.153
b(m)	0.96
e(m)	0.49

Table 4.6 – Concrete properties for case 1

Concrete	
f_{ck} (MPa)	30
f_{cd} (MPa)	17
ε_{c2}(‰)	2
ε_{cu2}(‰)	3.5
n	2

Variations are made relatively to type and material of the reinforcement and prestress as shown below, Table 4.7

Table 4.7 – Table used to choose the type of prestress system, tendon material, rebar material and concrete strength class for case 1

Properties	
Tendon_Type	Bonded → Bonded or Unbonded
Tendon_Material	Carbon → CFRP or Steel
Rebar_Material	Carbon → CFRP or Steel
Concrete_Type	C30/37

The code also allows the variation of the concrete strength but in this case, it was not considered for this dissertation.

Case 2

This case considers that the width of the cross section varies for a given and constant applied bending moment, $M_{Ed} = 3500\text{kNm}$

Table 4.8 - Geometry of the cross section for case 2

Cross_Section	
h(m)	1.1
dp(m)	1.075
d(m)	1.153
$M_{Ed}[\text{kNm}]$	3500

Table 4.9 - Concrete properties for case

Concrete	
f_{ck} (MPa)	30
f_{cd} (MPa)	17
ϵ_{c2} (‰)	2
ϵ_{cu2} (‰)	3.5
n	2

After this, as shown before, variations are made relatively to type and material of the reinforcement and prestress as shown below:

Table 4.10 - Table used to choose the type of prestress system, tendon material, rebar material and concrete strength class for case 1

Properties		
Tendon_Type	Bonded	→ Bonded or Unbonded
Tendon_Material	Carbon	→ CFRP or Steel
Rebar_Material	Carbon	→ CFRP or Steel
Concrete_Type	C30/37	

4.3 ACI 314/318 vs Eurocode 2 analysis

As presented in Chapter 2, ACI 318 [1] and the ACI 314 [3] codes have a different approach than and EC2 [4] when it comes to the dimensioning of reinforced concrete structures in the ultimate limit state.

Previous work made by KLEBER BARROS [26] has shown that these different design approaches suggested by EC2 [4] and ACI [1], lead to a difference of results when analyzing the amount of reinforcement needed. Thus, several comparisons were elaborated in order to understand the factor that influences these processes the most, creating several assumptions and different cases as seen in the following tables. The reinforced concrete member in question had a rectangular cross-section with known geometry and mechanical properties.

For this study it was compared the design of a simply supported reinforced concrete beam, subjected to an applied load (Figure 4.2) and rectangular cross section (Figure 4.3) with the following characteristics.

It is important to point out that the following results are for a given cross section and specific materials and therefore it should not be generalized to other cases without a more detailed study.

Data provided

- Cross section geometry
 - High: 0.80m
 - Width: 0.6m
 - Length: 9,0m
- Material
 - Concrete type: C30/37

Steel reinforcement: B500- A500NR

$E = 210\text{GPa}$

Varying

- Applied moment varying from $\mu = 0$ to $\mu = 0.48$

Objective

- Amount of reinforcement required

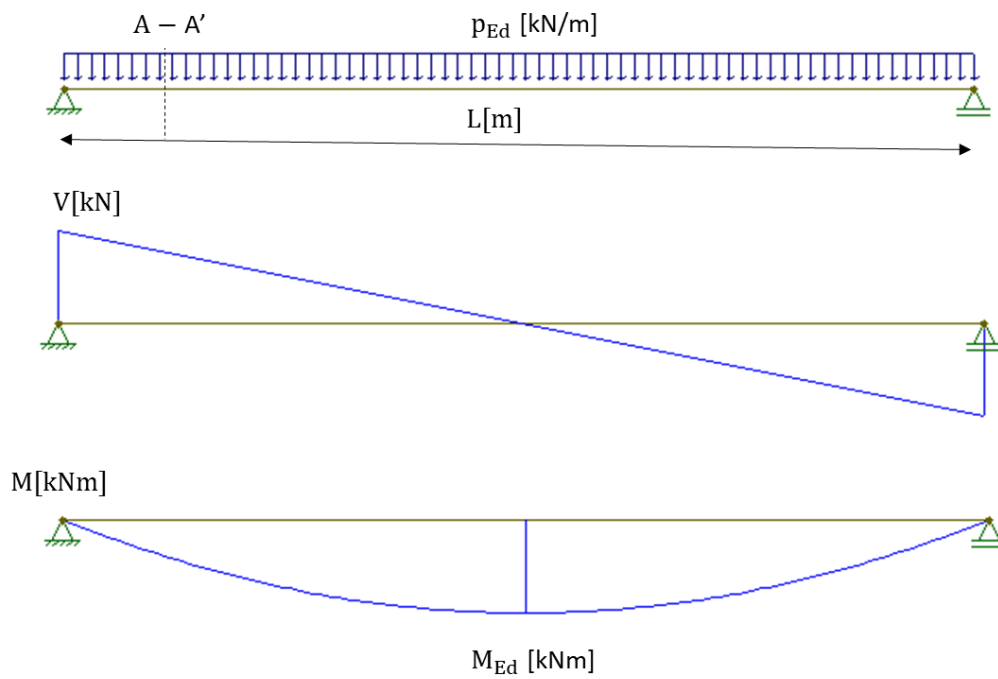


Figure 4.2 - Applied load, shear and bending moment diagrams

A-A'

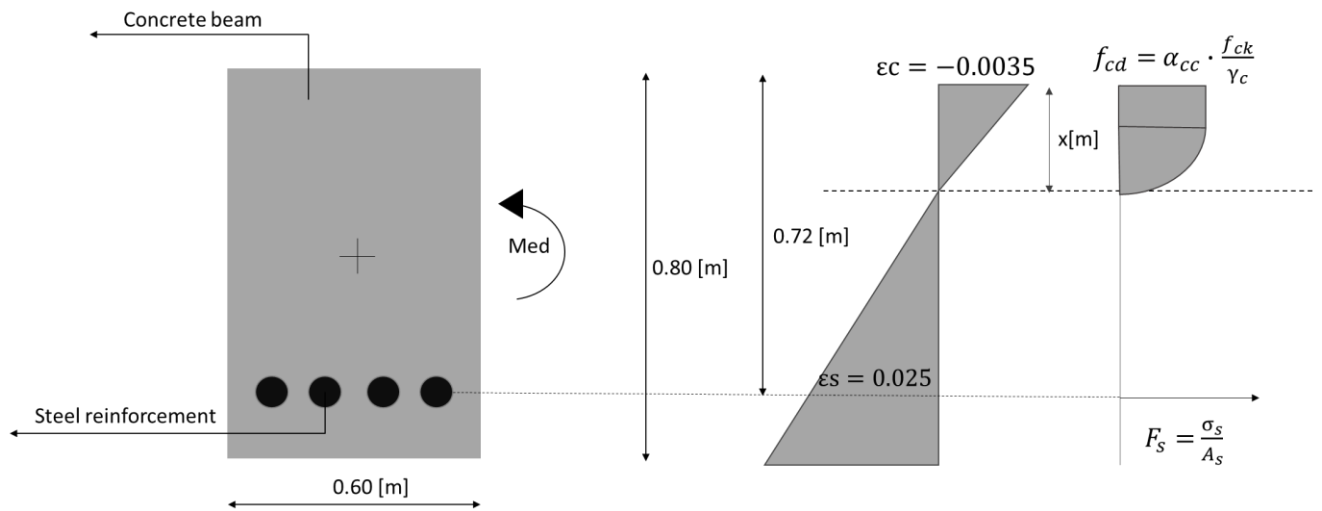


Figure 4.3 - Cross section and stress- strain distribution adapted from EC2 considering failure due to concrete crushing [4]

In the case of ACI, the following scheme was considered, and it can be seen in Figure 4.4.

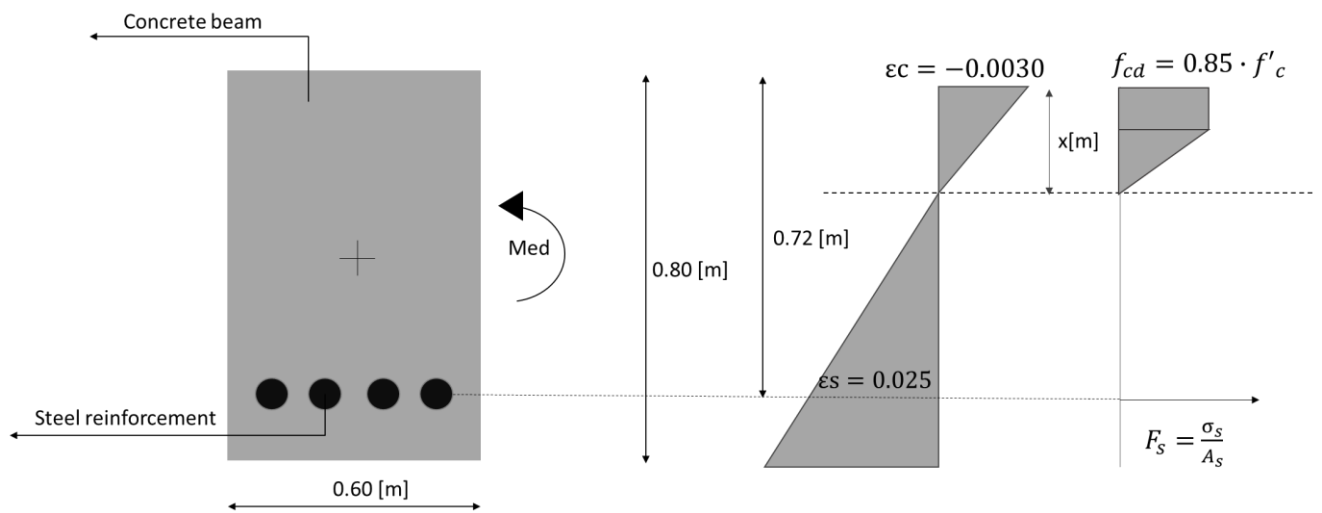


Figure 4.4 - Cross section and stress- strain distribution adapted from ACI considering failure due to concrete crushing [7]

The first code was elaborated following the parameters suggested by the EC2 [4] and a second one following the ACI Standard [1], however in order to analyze different parameters the ultimate strain, the safety factor and the design procedure itself changes.

Four cases are presented below to compare EC2 [4] and ACI 318[1].

4.3.1 Case A - ACI 318 and Eurocode 2 design procedure

In case A the comparison is made regarding the Eurocode 2 [4] and ACI [1],[2],[3] design procedure. For this study the hypotheses considered are in the following Table 4.1. The main goal of this study was to prove the difference found in the previous work [26]. And for this, all parameters were maintained as suggested by the codes. With this study it is also possible to compare the Eurocode in two European countries (Portugal and Germany) where α_{cc} can take different values according to the national annex, 1.0 in the Portuguese case and 0.85 for German one.

Table 4.11 - Assumptions, case A

ACI 318	Eurocode 2
Stress block bilinear	Stress block parabolic-rectangular
Perfect bond between the concrete and the reinforcement	Perfect bond between the concrete and the reinforcement
$\epsilon_{cu} = 0.003$	$\epsilon_{cu} = 0.0035$
$\Phi_t = 1/\gamma_s = 0.90 \rightarrow \gamma_s \cong 1.11$	$\gamma_s = 1.15$
$\Phi_c = 1/\gamma_c = 0.65 \rightarrow \gamma_c \cong 1.54$	$\gamma_c = 1.5$

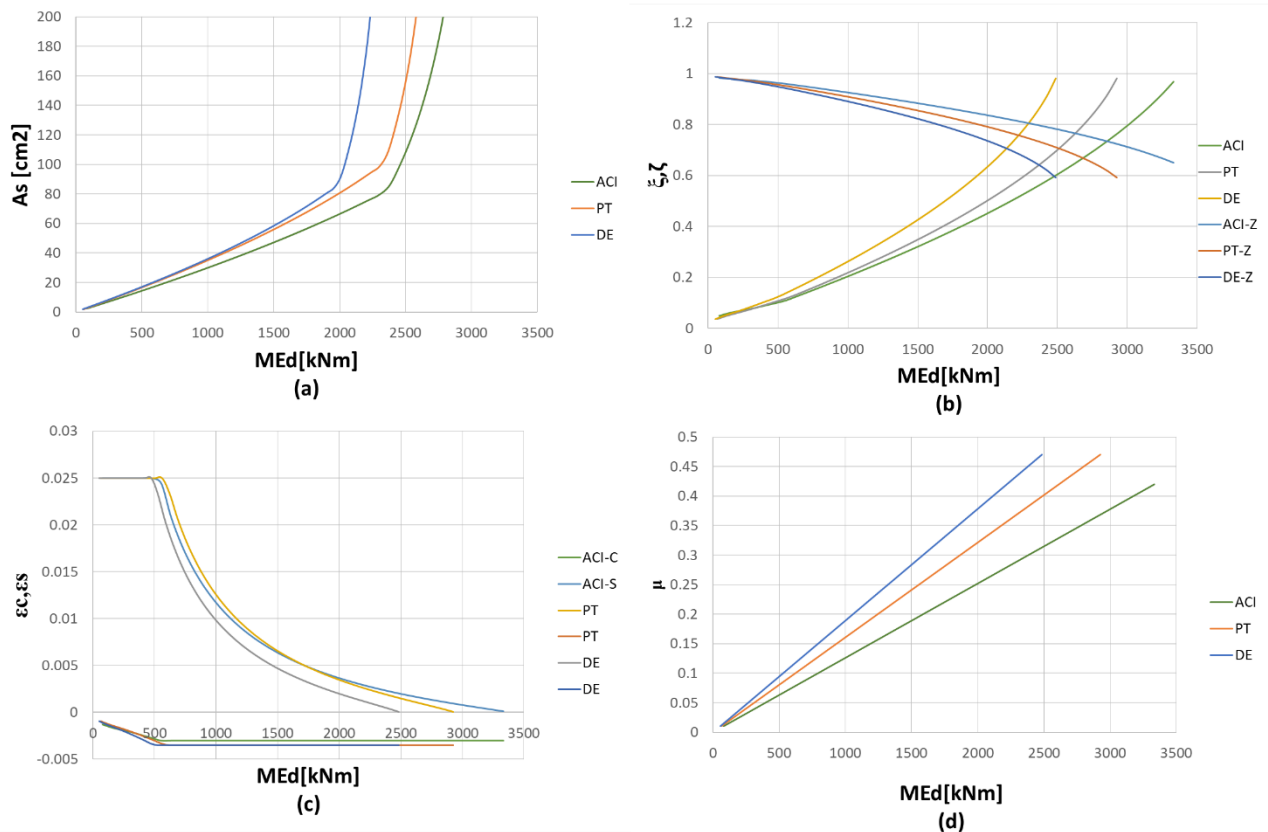


Figure 4.5 - Results for ACI [1] and Eurocode [4] design with the Portuguese National Annex (PT) and with the German one (DE); (a) Amount of reinforcement required with the variation of the applied moment; (b) Depth of the neutral axis and level arm varying the applied moment; (c) Strain distribution with the variation of the applied moment; (d) Nominal moment varying the nominal moment

Throughout the observation of Figure 4.5 some conclusions can be drawn. In Figure 4.5 (a) it is possible to observe the amount of reinforcement required in function of the different applied moment. The three curves that are presented refers to the European Code with the Portuguese and German Annex and the last one with the ACI Code.

It is observed that for a reduced moment there is small variation in the amount of reinforcement between the three curves, but from a certain point this difference increases considerably, being this point where the concrete member has reduced ductility and as such the designer should not do the project in that zone. For the same applied moment, it should be noted that the most conservative Code is the Eurocode with the German national annex since it always requires a greater amount of reinforcement than the other curves. When it comes to Figure 4.5 (b), the curve corresponding to the neutral axis of the Portuguese annex is lower than the German one, since the Portuguese one presents a higher compressive strength and to maintain the same applied moment it has to present a larger level arm for the moment calculation, such conclusions can be drawn and are consistent with what was theoretically expected. In Figure 4.5 (c) the strains are presented and in the case of the concrete strain they are very similar, except for the ACI curve which presents an ultimate strain of 0.003 instead of 0.0035. Regarding

the steel strain, for the Portuguese case, which presents a lower depth of the neutral axis, it requires a greater strain than the German annex and also greater than the ACI code. The graph of Figure 4.5 (d) shows a value of μ varying the applied moment and it is observed that for the German annex the reduced ductility point of the structure is reached before the Portuguese and the ACI ones.

It is noteworthy that in this study at no time was considered the compression reinforcement since, in general, it is usual to make checks for simply reinforced concrete beams. One of the problems faced by structural engineers is the use of single or double reinforced concrete beams, ie whether or not to use compression reinforcement. The issue is raised because at high moments the stress of the tensioned reinforcement is below the yield and thus is not economically feasible, making it more cost effective to adopt a solution with compression reinforcement. [27];[28].

As can be seen in Figure 2.10 it is possible to identify 4 different zones depending on the strain of steel and concrete.

According to [26,27] different situations can occur regarding to the ductility of reinforced concrete members without compression reinforcement which are directly related to the four phases mentioned in Table 2.3.

Situation I

The bending moment is high ($\mu > 0.3$), the concrete has crushed and it reached the ultimate strain. on the other hand, steel has not reached the yield yet. In this case the structure presents a brittle behavior.

Situation II

The bending moment is medium ($0.1 < \mu < 0.25$), the concrete has crushed and it reached the ultimate strain. on the other hand, steel is between yielding and failure. In this case the structure presents the adequate ductility.

Situation III

The bending moment is low ($0.05 < \mu < 0.1$), the concrete strain is between the strain at reaching the maximum strength and the ultimate strain on the other the steel has reached the ultimate strain.

Situation IV

The bending moment is low ($\mu < 0.1$), the concrete strain has not reached the maximum strain yet, however steel is at the ultimate strain.

For both situations, III and IV the beam is not under much stress, the reinforcement is little requested.

According to LNEC Tables, [26], section 3.2, for simply reinforced rectangular beams the reinforced concrete beam is only economically viable if:

$$\alpha \leq \alpha_{lim} = 0.617$$

$$\mu \leq \mu_{lim} = 0.371$$

$$\omega \leq \omega_{lim} = 0.499$$

μ is the nominal moment, given by:

$$\mu = \frac{M_{Rd}}{bd^2f_{cd}} \quad (45)$$

ω is the percentage of tensioned mechanical reinforcement:

$$\omega = \frac{A_s f_{yd}}{bd f_{cd}} \quad (46)$$

α is the value of the relative depth of the neutral axis given by:

$$\alpha = \frac{x}{d} \quad (47)$$

The limit values correspond to the steel reaching the yield ($\varepsilon_s = \varepsilon_{yd}$).

After considering these situations it is possible to observe in Figure 4.5 that where there are great differences in the results it is in zone I because with the ACI the end of the ductility is reached before. After this point there is no great interest of comparison since the design preference is in zone II and not I.

Portugal and Germany follow the same code, Eurocode, however there are small differences in their National Annexes, one of these differences is how the value of the strength is calculated, and it leads to small differences in the results as seen in Figure 4.5. The Portuguese way is safer, requires more reinforcement than the German way, but still requires less reinforcement than the ACI.

4.3.2 Case B - Eurocode 2 and ACI 318 with Eurocode parameters

In order to understand the differences in results between EC2 and ACI for the amount of reinforcement required, it follows the case B, the results are presented in Figure 4.6 and its objective was to analyze the influence of the design, which means, the procedure to apply the safety factor, comparing the process given by EC2 [4] and ACI [1]. As shown in the Table 4.2, all the features remain the same, the same stress block, the same ultimate strain and the same safety factor, the only parameter that was changed was the design procedure. All the European parameters using the ACI procedure. The difference regarding the required amount of reinforcement, as it can be seen in Figure 4.6 is high.

Thus, for case B, the applied moment continues to be what varies, however, the strain limit for the ACI case becomes the Eurocode, as well as the stress block and safety factors as shown below:

Table 4.12 - Assumptions, case B

ACI	Eurocode
Stress block parabolic-rectangular	Stress block parabolic-rectangular
Perfect bond between the concrete and the reinforcement	Perfect bond between the concrete and the reinforcement
$\epsilon_{cu} = 0.0035$	$\epsilon_{cu} = 0.0035$
$\Phi_t = 1/\gamma_s = 1/1.15 \cong 0.8696$	$\gamma_s = 1.15$
$\Phi_c = 1/\gamma_c = 1/1.5 \cong 0.6667$	$\gamma_c = 1.5$

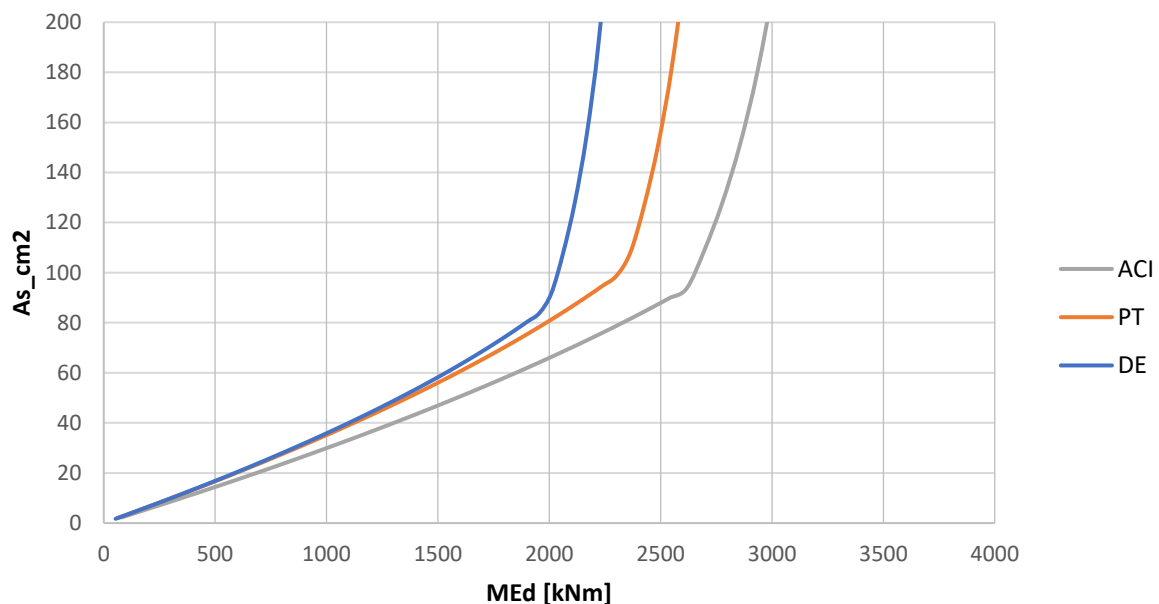


Figure 4.6 - Difference between ACI and Eurocode- PT (Portuguese National Annex) and DE (German National Annex), changing the procedure of the design. Amount of reinforcement required

Analyzing Figure 4.6 it is possible to observe that, even though all parameters are the same and the only difference is the design procedure itself, the great difference observed in Figure 4.5 (a) is maintained and thus the design procedure and not the variants themselves may be the cause of the differences presented in the three procedures considered.

4.3.3 Case C- ACI procedure and the effect of the ultimate strain

To evaluate the effect of the ultimate strain of the concrete, the analysis of case C was elaborated. The present case differs from the previous ones since the comparison is made only with the design provided

by ACI [1] and it is only to analyze the differences obtained when the ultimate strain of the concrete changes. The following plot, Figure 4.7 was considered in order to understand if the difference shown in Figure 4.5 is due to the difference in the concrete ultimate strain.

This case was elaborated following the ACI [1] and only the ultimate strain (Table 4.3) varied and analyzing the following chart, it can be observed that before the point of interest, that is, before reaching the yield point, the difference between the amount of reinforcement required for the two situations are identical and leads us to conclude that despite a small difference in strain distribution Figure 4.7 (b) this difference is due to the variation of the concrete ultimate strain and that even if there is such strain variation, it does not lead to the great difference initially mentioned in the previous work [26]

Table 4.13 - Assumptions, case C

ACI	ACI
Stress block bilinear	Stress block bilinear
Perfect bond between the concrete and the reinforcement	Perfect bond between the concrete and the reinforcement
$\epsilon_{cu} = 0.003$	$\epsilon_{cu} = 0.0035$
$\Phi_t = 1/\gamma_s = 0.90$	$\Phi_t = 1/\gamma_s = 0.90$
$\Phi_c = 1/\gamma_c = 0.65$	$\Phi_c = 1/\gamma_c = 0.65$

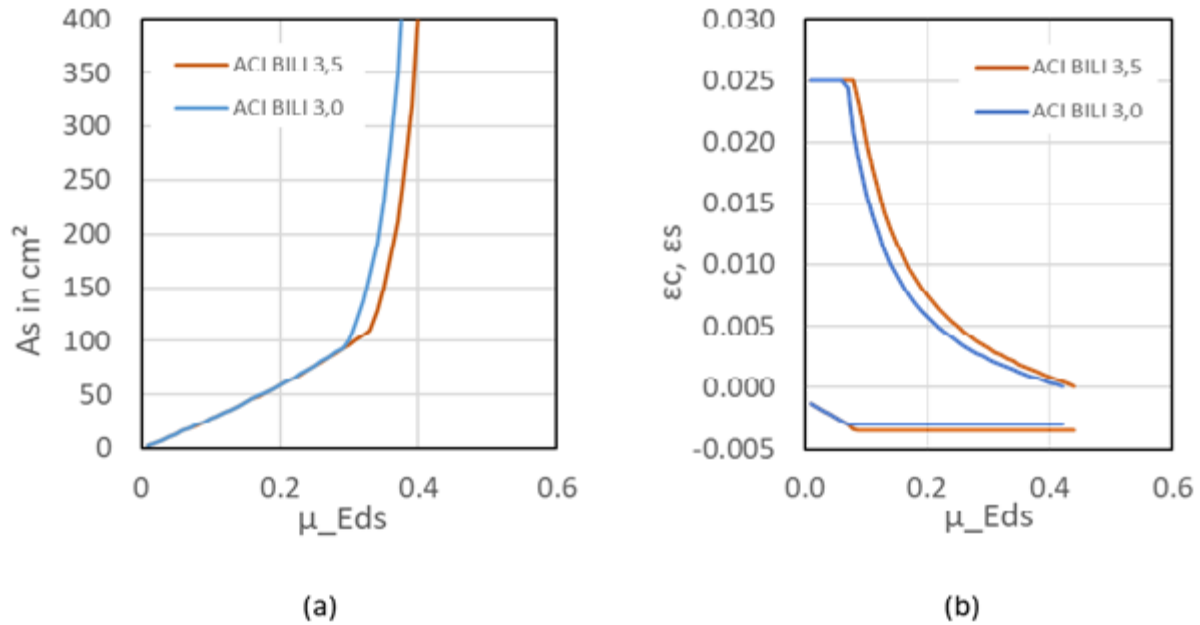


Figure 4.7 - Results for variation of the ultimate limit strain. ACI procedure considering the bilinear (bili) stress block. (a) Amount of reinforcement required; (b) Strain distribution

4.3.4 Case D- ACI comparison for effect of the safety factor

In case D, the main goal was to see if the large difference shown in the first graph was due to a small difference in the safety factor value. Thus, the following assumptions have been taken, Table.4.4, following the ACI design process and the results are shown below in Figure 4.8.

Table 4.14 - Assumptions, case D

ACI	ACI
Stress block bilinear	Stress block bilinear
Perfect bond between the concrete and the reinforcement	Perfect bond between the concrete and the reinforcement
$\epsilon_{cu} = 0.003$	$\epsilon_{cu} = 0.003$
$\Phi_t = 1/\gamma_s = 1/1.15 \cong 0.8696$	$\Phi_t = 1/\gamma_s = 0.90 \cong 1/1.11$
$\Phi_c = 1/\gamma_c = 1/1.5 \cong 0.6667$	$\Phi_c = 1/\gamma_c = 0.65 \cong 1/1.54$

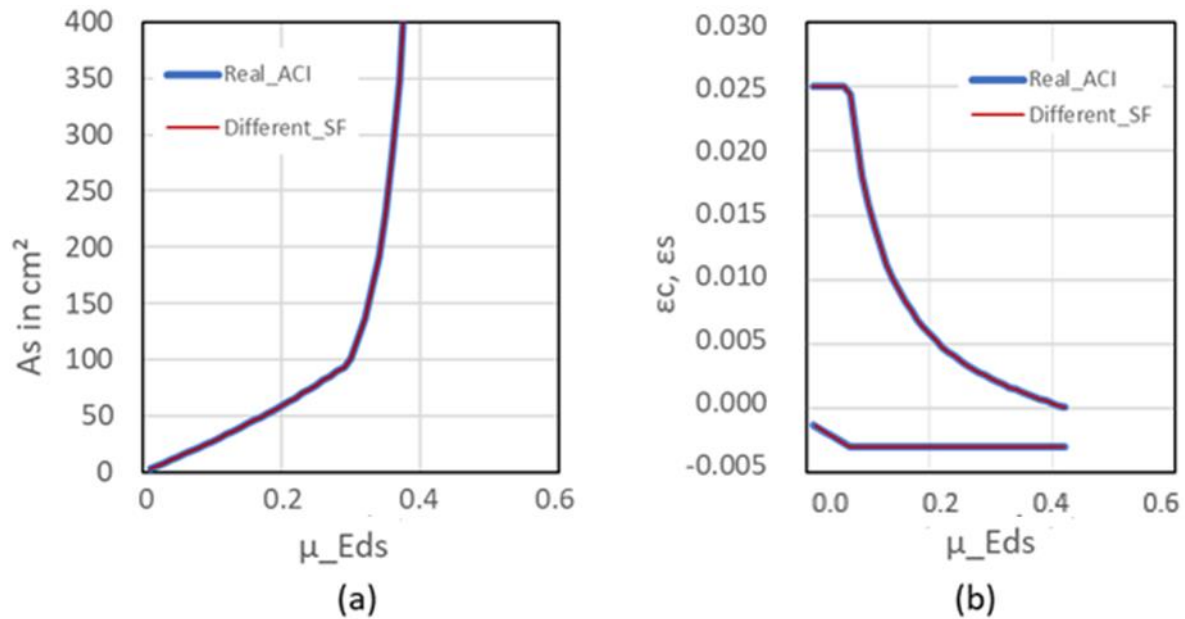


Figure 4.8 - Variation of the safety factor. ACI design. (a) Amount of reinforcement required; (b) Strain distribution

As can be seen from the graph in the Figure 4.8, this small difference shown in Table 4.14 of the variation of the safety factor has no influence on the results. All the curves presented are superimposed, the required amount of reinforcement curve and the curve of the strains distributions. Thus, we can conclude that this parameter, safety factor, is not the cause of the difference presented in the previous work.

4.4 Analysis of prestressed reinforced concrete members

The analysis of structures with prestress was also done for a simply supported beam, as shown in section 3, Figure 3.15.

Two different cases were analyzed for prestressed reinforced concrete structures:

- Case 1- Varying the applied bending moment (M_{Ed}) from 0 to 5000 kNm
- Case 2 - Varying the width (b) from 0.45m to 1.50m

In each case we have 3 types of structures Figure 4.9-4.11 to analyze:

- Conventional steel prestressed reinforced concrete structures.

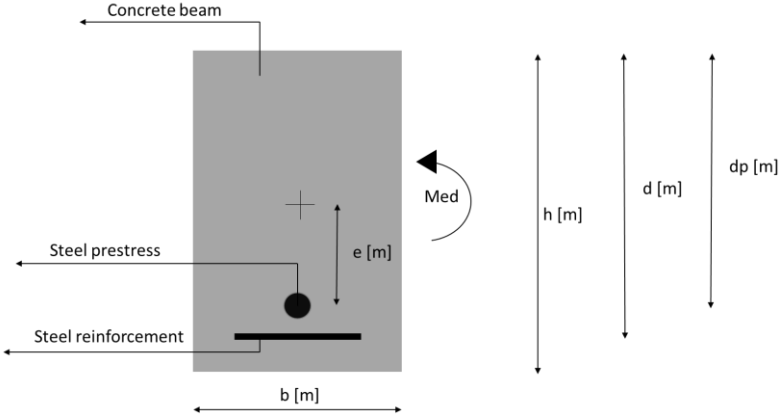


Figure 4.9- Prestressed RC beam- Steel rebar and steel tendons

- Mixed structures: steel + carbon

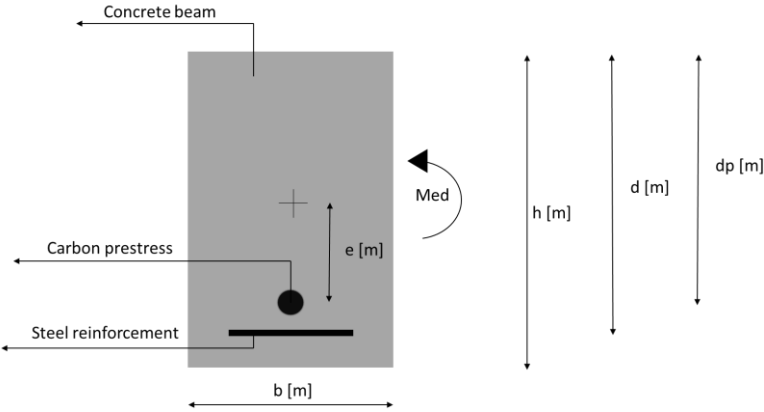


Figure 4.10 – Prestressed RC beam - Steel rebar and CFRP tendons

- CFRP prestressed reinforced concrete structures

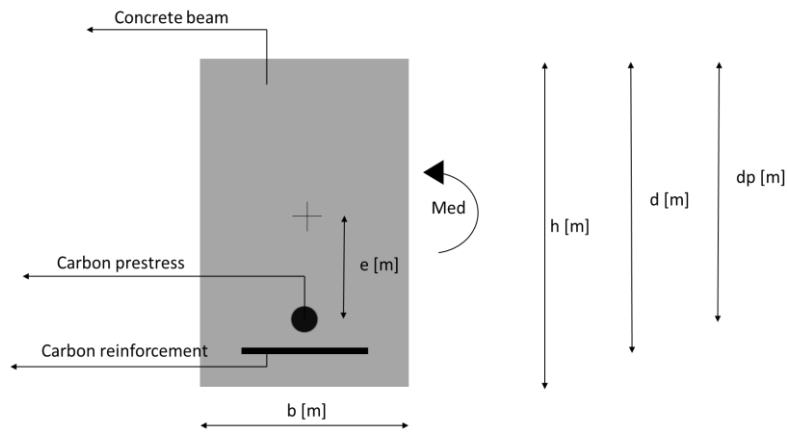


Figure 4.11- Prestressed RC beam - CFRP rebar and carbon tendons

For the case study considered, the prestress has the following configuration, the parabolic one, equivalent loads, axial, transverse and bending moment diagrams, as shown in Figure 4.12.

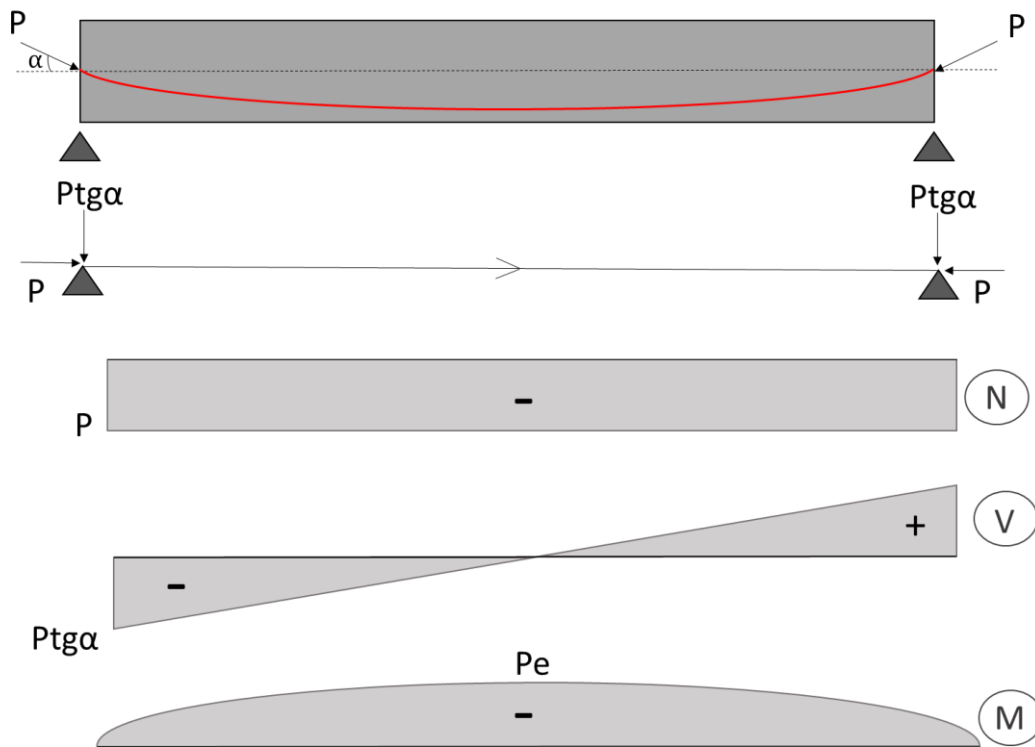


Figure 4.12 – Prestress configuration, loads and diagrams

4.5 Bonded vs Unbonded tendons - Case 1

As mentioned in section 3, bonded and unbonded prestress have different characteristics and ways of design. In order to understand the difference in these results, comparisons were made between bonded and unbonded tendons for the different combinations presented in the following Table 4.15.

Table 4.15- Rebars-tendons combinations

Combination	Rebar material	Tendon material
SS	Steel	Steel
SC	Steel	Carbon
CC	Carbon	Carbon

The following figures are showing the main differences regarding the amount of reinforcement for every type of structure:

The following chart Figure 4.13 refers to the conventional prestressed reinforced concrete structure. Being pre-stress and reinforcement both steels. In this graph we have the amount of reinforcement for the case of the bonded and unbonded tendons as well as the respective strains, for the steel and for the concrete when the applied moment varies.

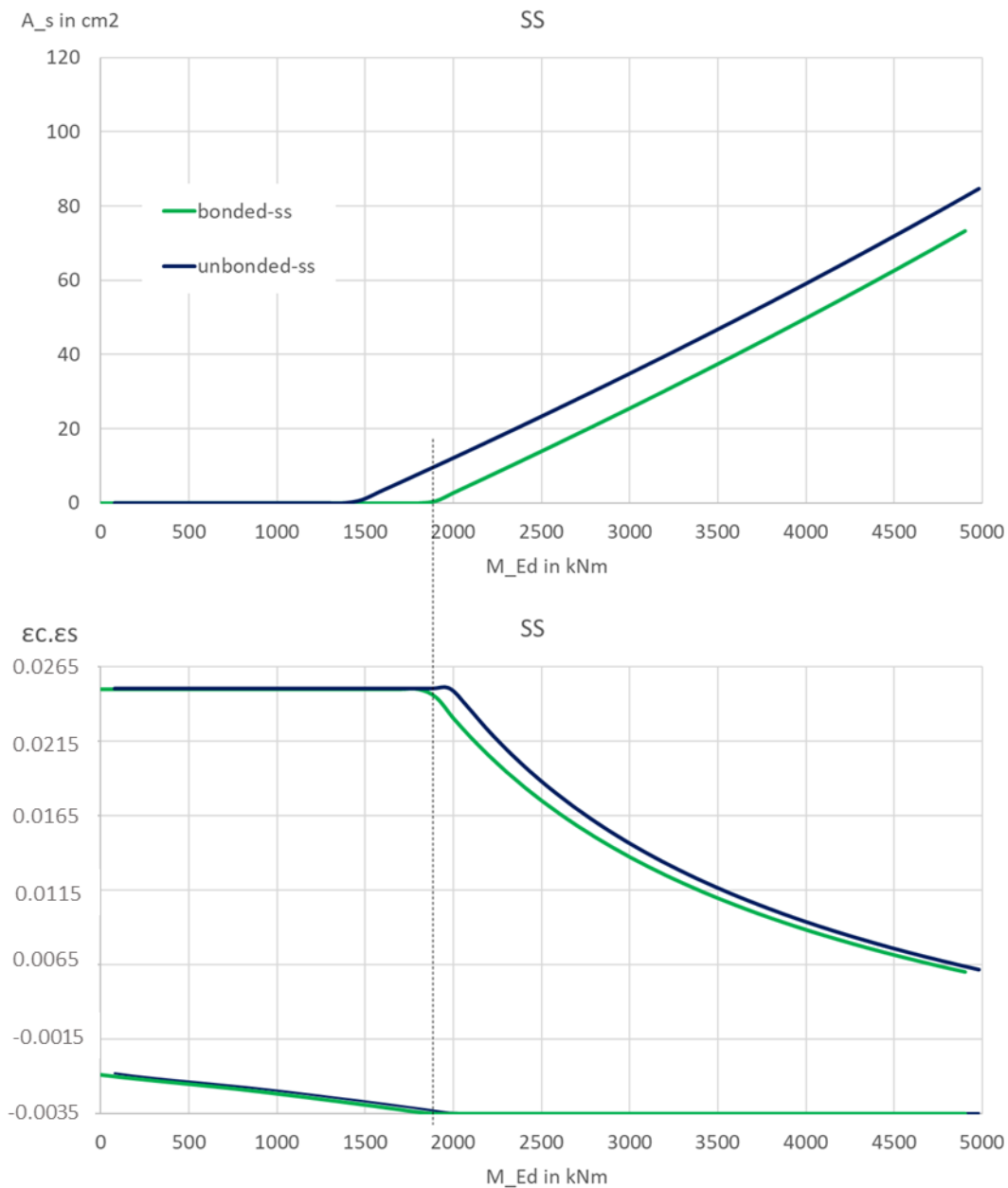


Figure 4.13 - Comparison for steel rebars and tendons varying the applied bending moment

With this graph it is possible to observe that up to the balanced point it is indifferent the use of bonded or unbonded tendons, however once it reaches the balanced point, the bonded tendons require less reinforcement than the unbonded ones. A large strain reduction of the reinforcement is also observed when the material used as prestress is steel, having no strain limitation. However, when it is changed to CFRP, it reduces drastically to approximately 0.007 for bonded tendons because the restriction is made by the CFRP tendon, in the bonded case because of the compatibility of strains.

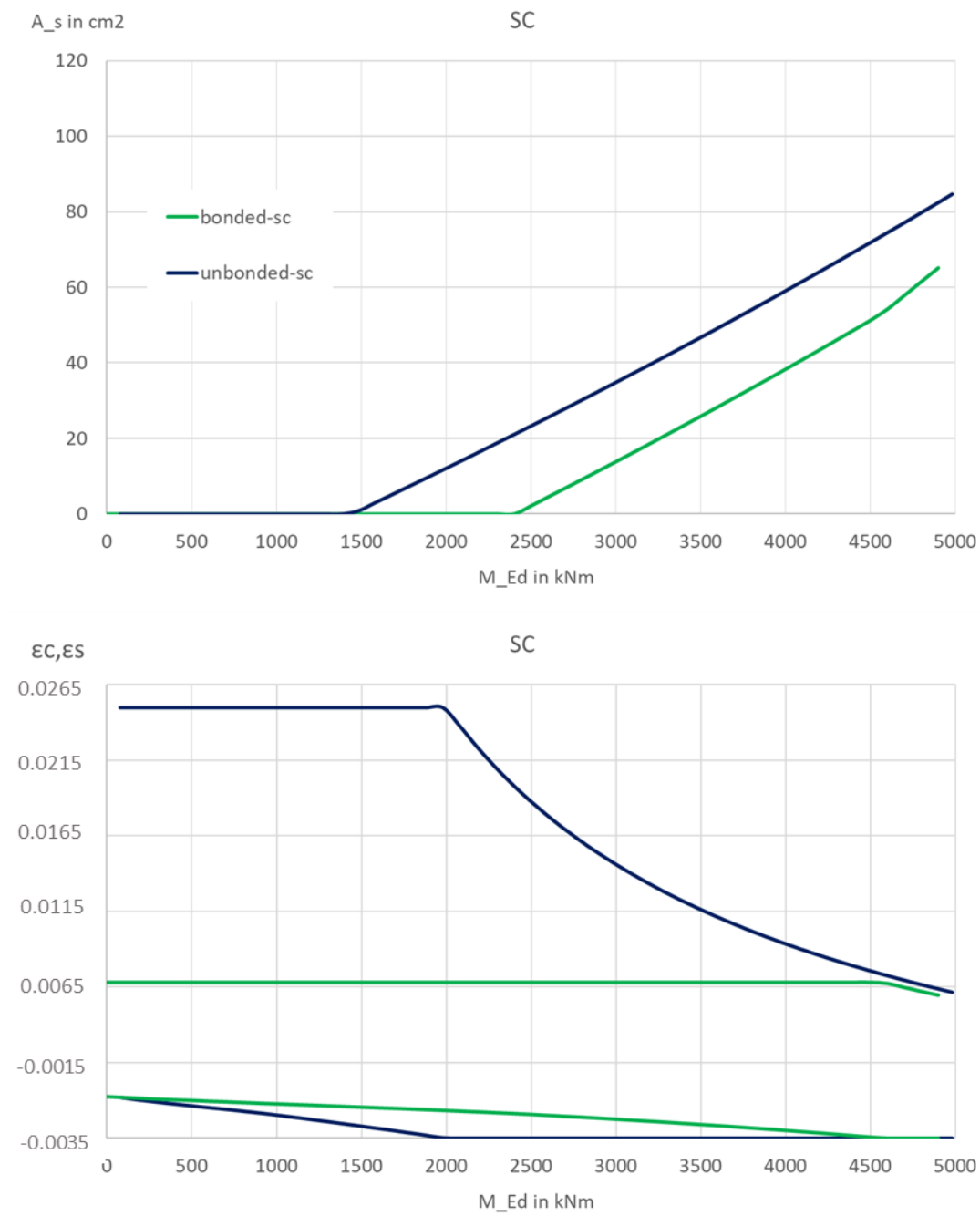


Figure 4.14 - Comparison for steel rebars and CFRP tendons varying the applied bending moment

The graph showed in Figure 4.14 refers to the same variations as the graph of Figure 4.13 however the prestress used in this case are CFRP tendons. Thus, the strains of the bonded tendons are reduced to a value a little higher than the limit strain of the carbon, because once bonded to the concrete, it has to verify the compatibility of the strain. For the unbonded case, it can be observed that the strain of the reinforcement does not lead to any change, and it is equal to 0.025. The trend mentioned above remains the same. Up to some point, the type of tendon is indifferent however after that limit, the bonded tendons are more advantageous in the amount of reinforcement required.

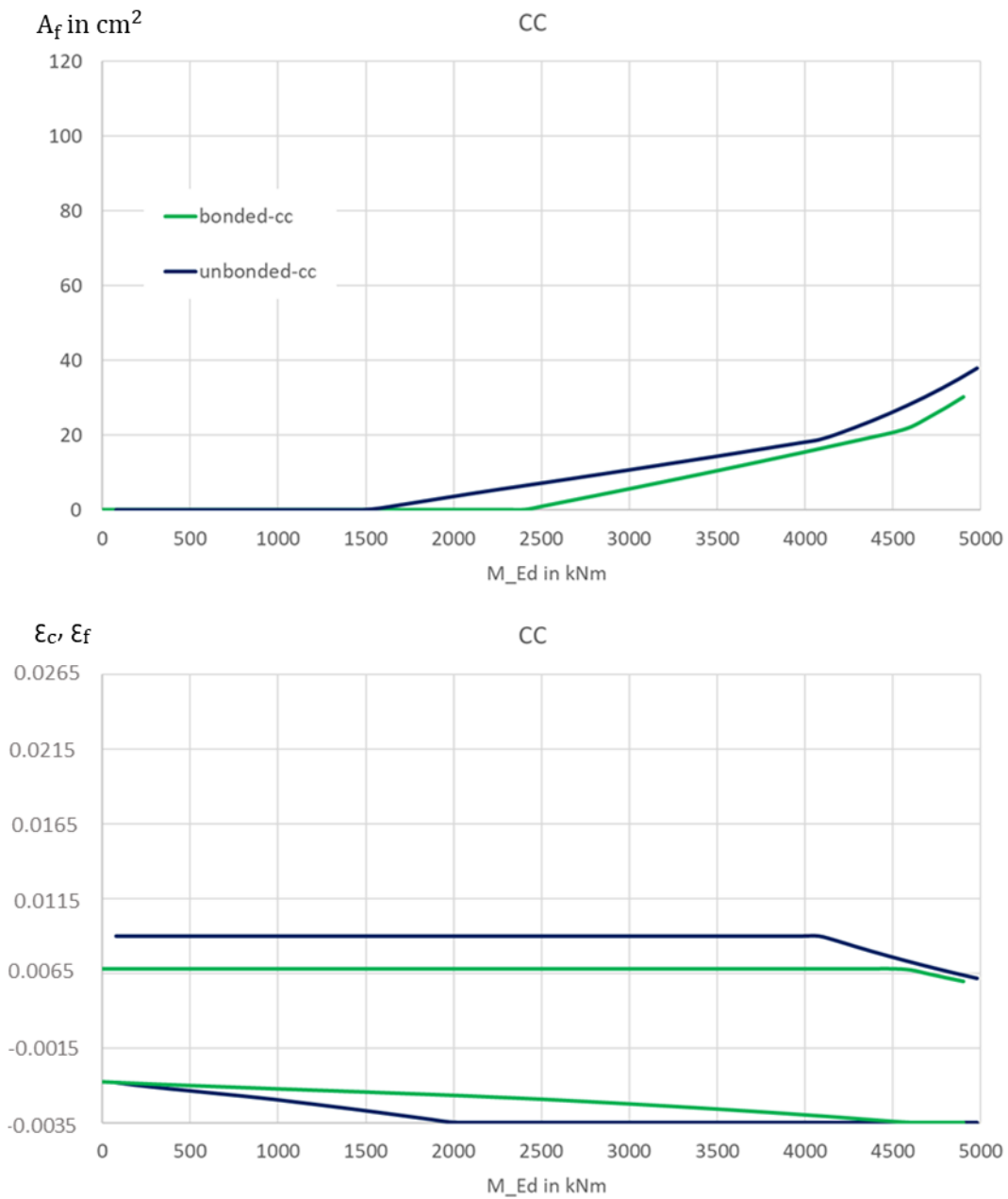


Figure 4.15 - Comparison for CFRP rebars and tendons varying the applied bending moment

For the last comparison, Figure 4.15, we have both reinforcement and prestress in CFRP. The strain in both cases are reduced to the carbon limit strain however the results confirm the expected, leading to the conclusion that the bonded tendons are more feasible related to the amount of reinforcement required.

4.6 Bonded vs Unbonded tendons - Case 2

The following graphs Figure 4.16-4.18 are related to case 2. The study was developed to see the influence of beam width on the two types of tendons: Bonded and Unbonded for different materials.

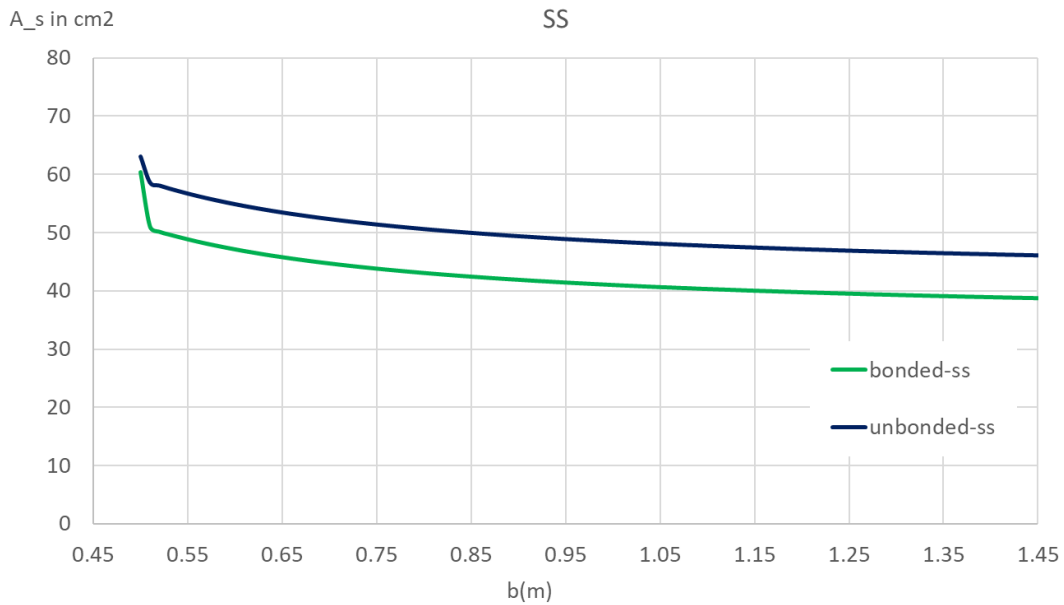


Figure 4.16 - Comparison for steel rebars and tendons varying the width

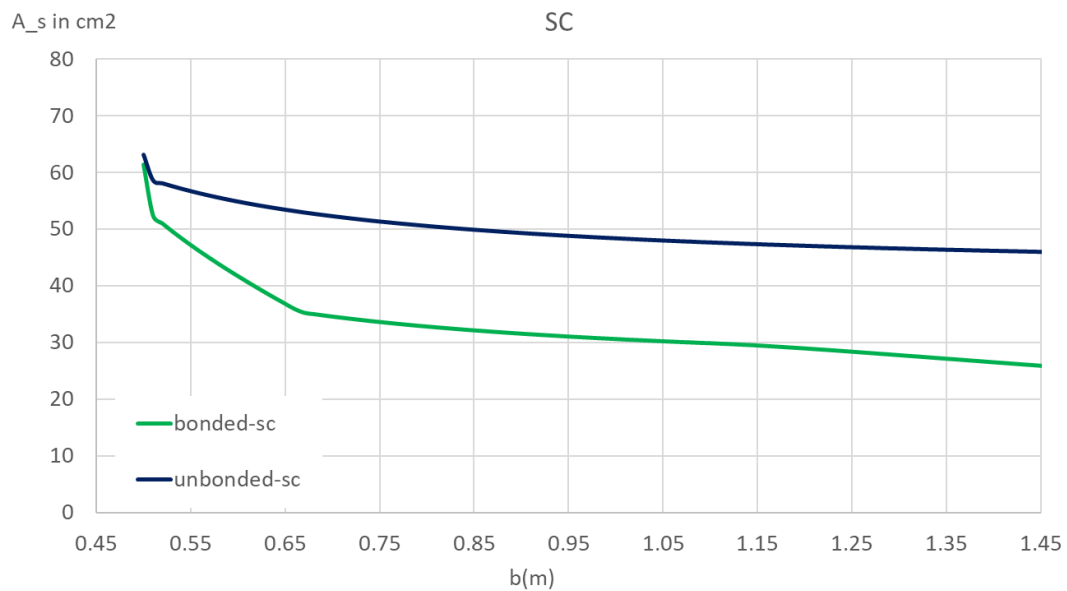


Figure 4.17 - Comparison for steel rebars and CFRP tendons varying the width

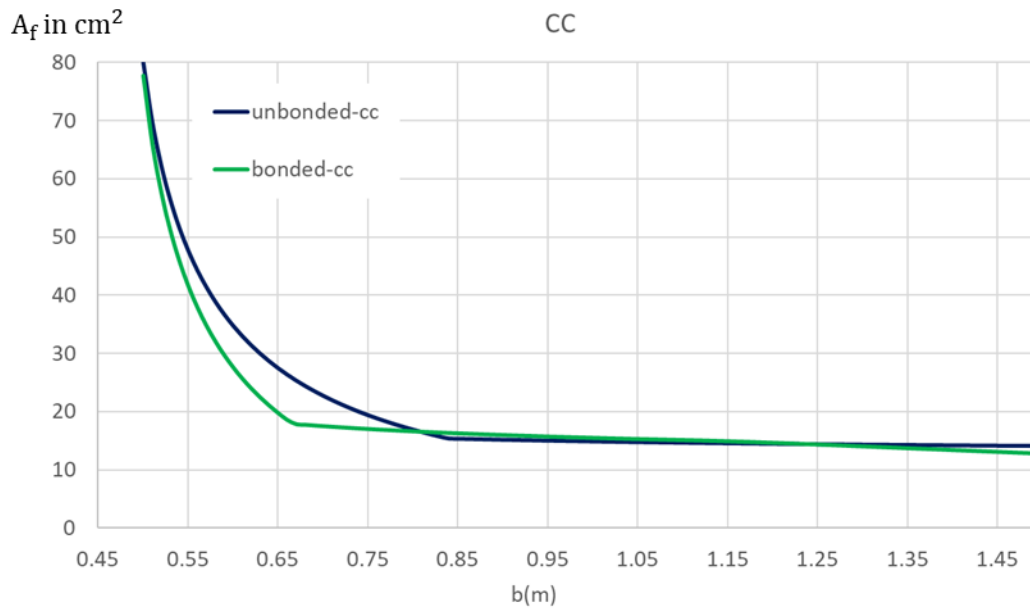


Figure 4.18 - Comparison for CFRP rebars and tendons varying the width

When analyzing Figures 4.18, 4.19 and 4.20, the same trend is observed for all three cases: bonded tendons require less amount of reinforcement than unbonded tendons in all situations. However, from a certain width of the beam to the CFRP structures it is indifferent the type of tendons that is used when analyzing the amount of reinforcement required.

4.7 Bonded tendons

The results presented in this section are comparisons made only for bonded tendons and its main objective is to show differences for different types of reinforcement and prestress, varying the applied moment. And the scheme given previously in Figure 4.9-4.12 represents the structures that have been studied.

In all cases presented, Figure 4.19 and Figure 4.20 cases with carbon are only more advantageous when the moment is above a given value. However, for high moment the use of CFRP can lead to a reduction of almost 40% when compared to the beams using steel as reinforcement and prestress.

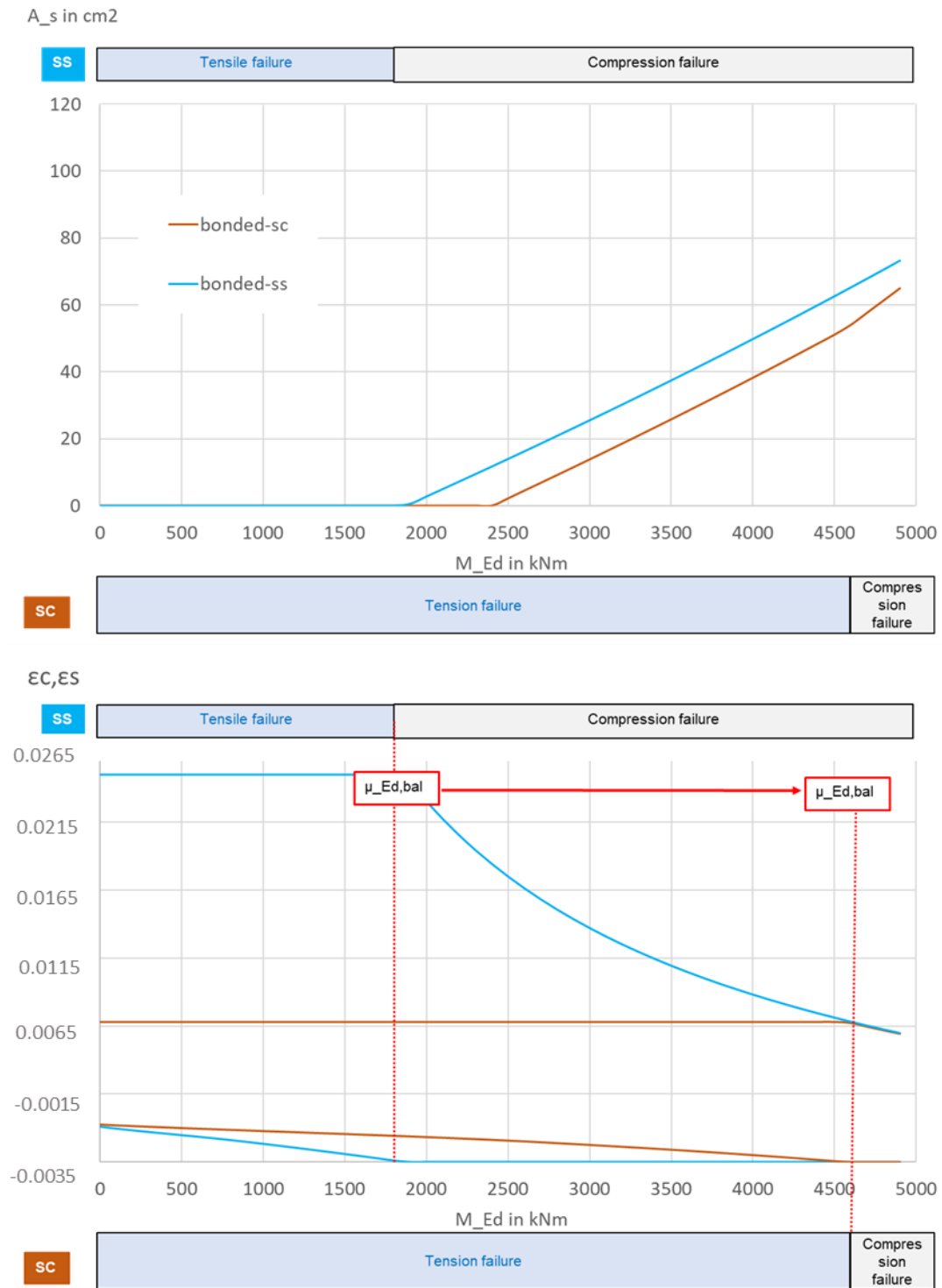


Figure 4.19 - Comparison of bonded tendons. Effectiveness of steel tendons and rebars compared to CFRP tendons

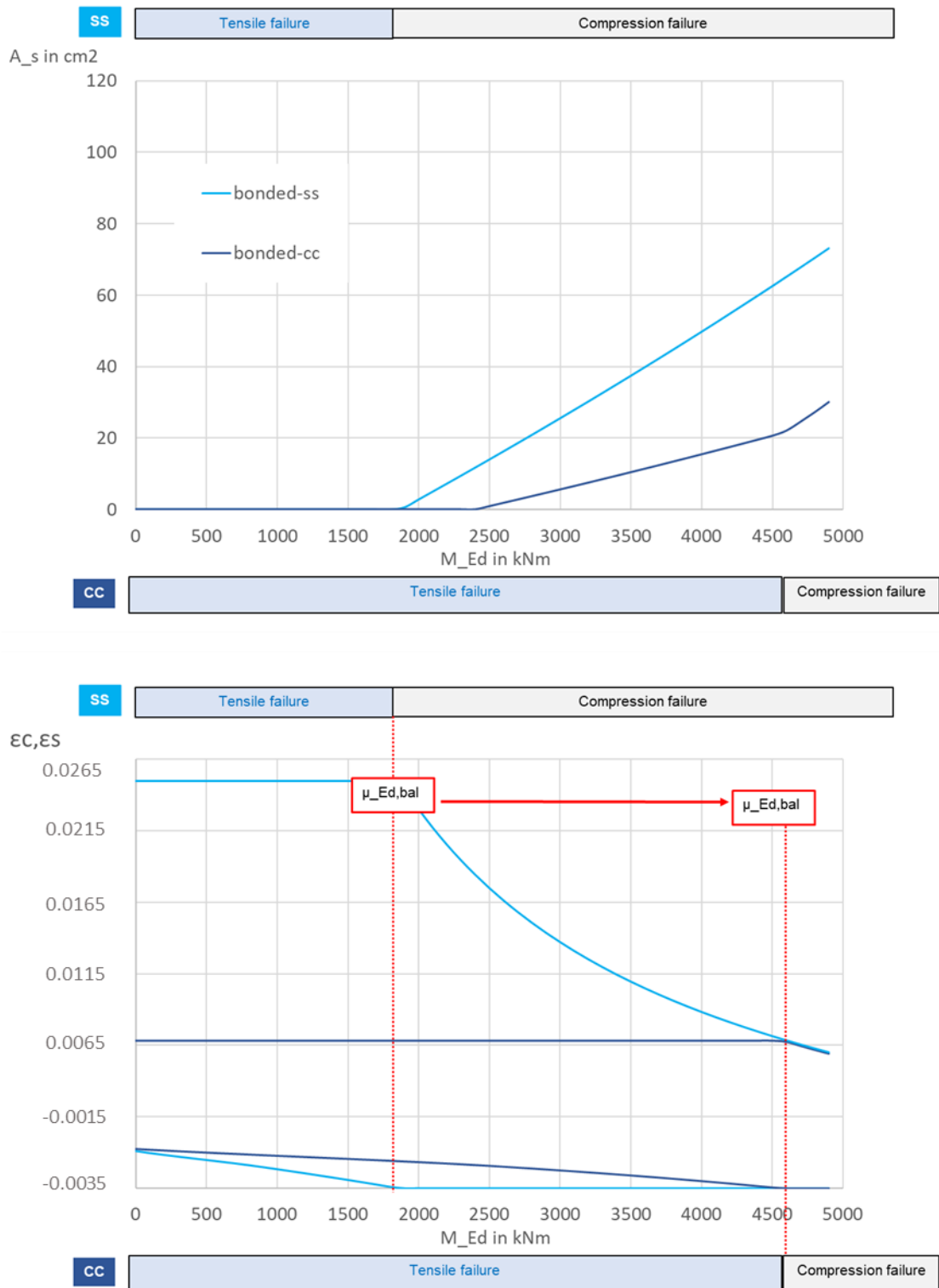


Figure 4.20 - Comparison of bonded tendons. Effectiveness of steel tendons and rebars compared to CFRP tendons and rebars

4.8 Unbonded post tensioning

Similar to the previous section, a prestress study with unbonded tendons was developed. The main objective was to analyze the behavior of a prestressed reinforced concrete beam using CFRP and compare with beams using conventional steel.

Analyzing the figure below Figure 4.21 it is possible to observe that for the combination SC and SS, the behavior is exactly the same, as there is no compatibility restriction relatively to the strain of the tendon, because the tendon is not connected to the concrete. That is the reason that the strain shown in Figure 4.21 for the reinforcement is the same regardless of whether the tendon used is made of carbon or steel.

Observing the second figure, Figure 4.22, a different behavior is identified. The amount of reinforcement required by the CC combination is much lower than is required for the steel. Significant strain reduction is observed, not due to the capability restriction but because of the CFRP itself which presents a much lower strain than steel.

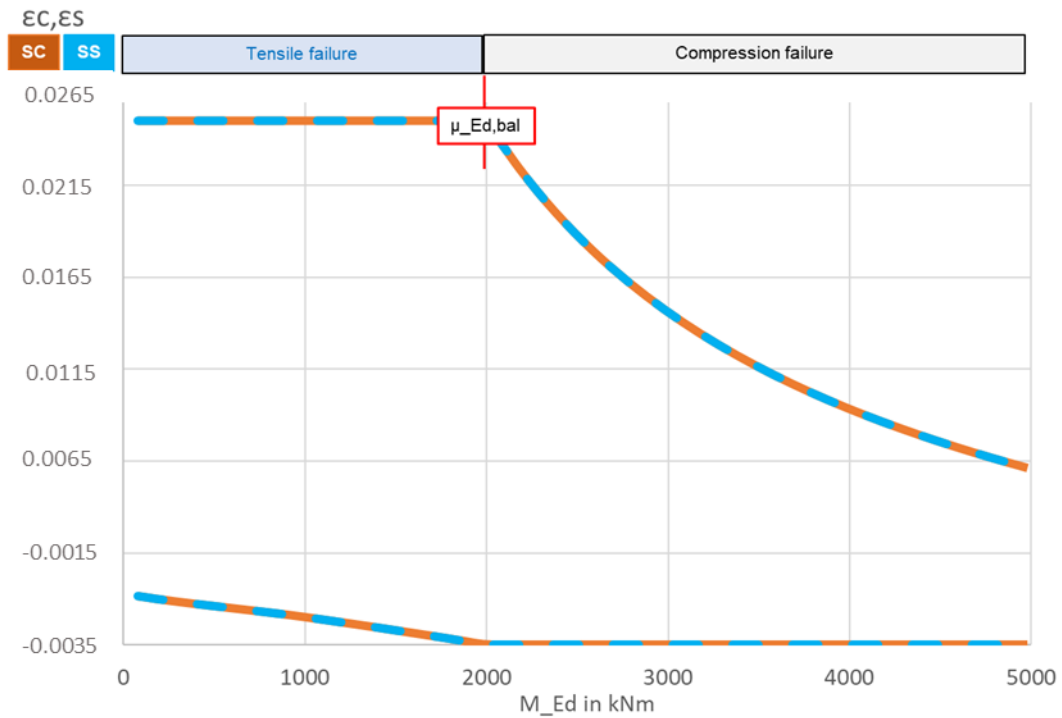
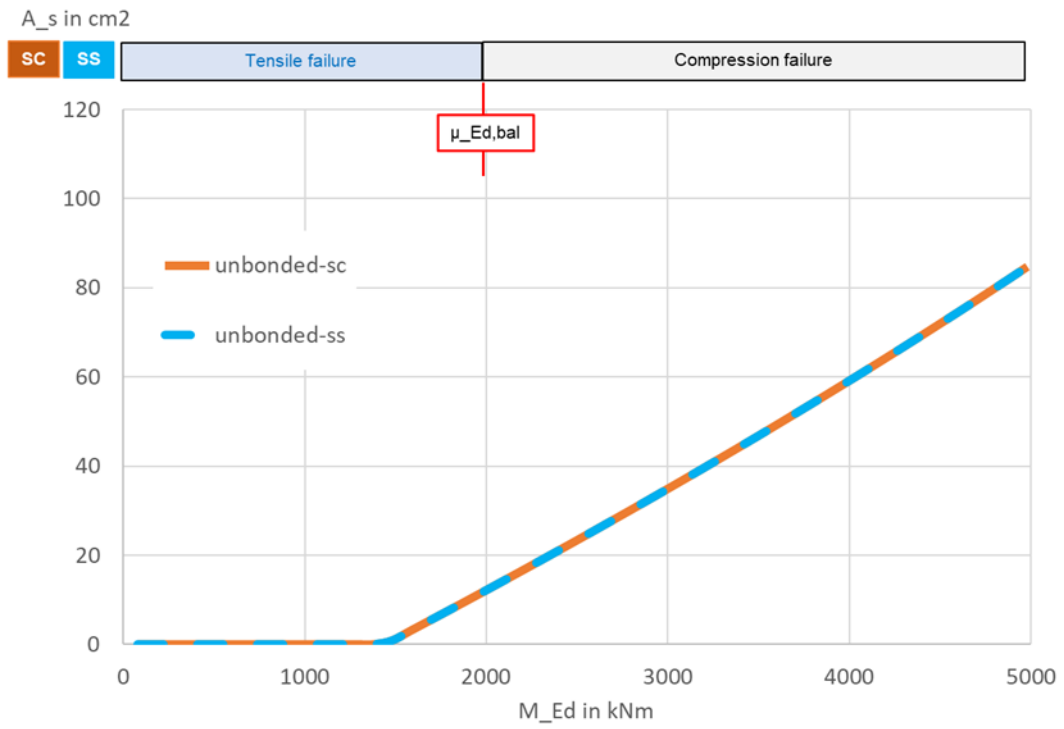
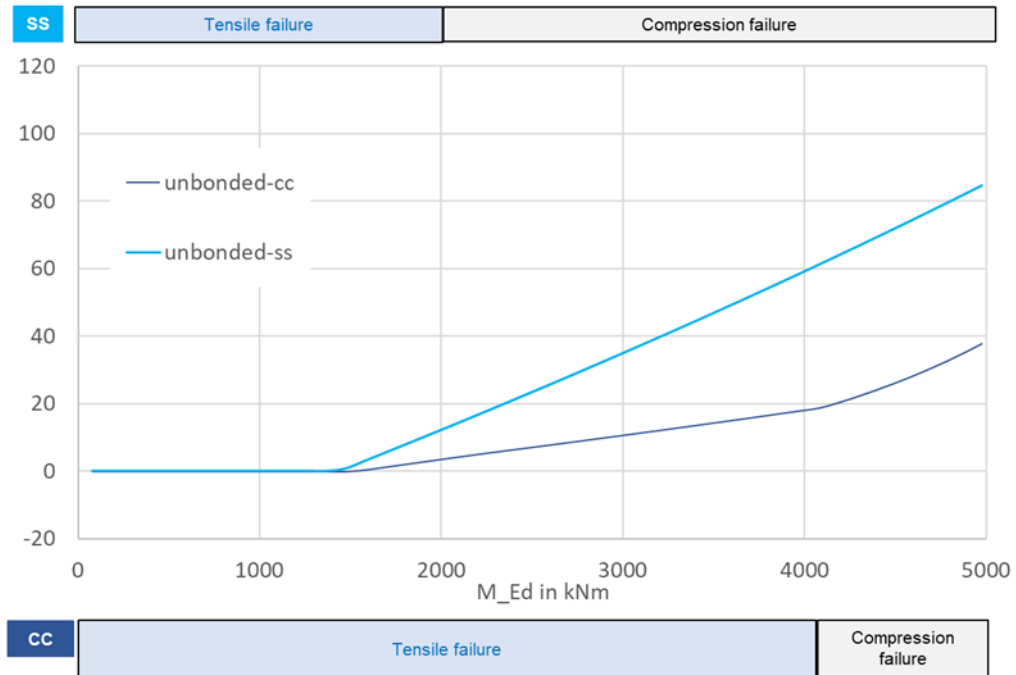


Figure 4.21 - Comparison of unbonded tendons. Effectiveness of steel tendons and rebars compared to carbon tendons

A_s in cm²



EC,ES

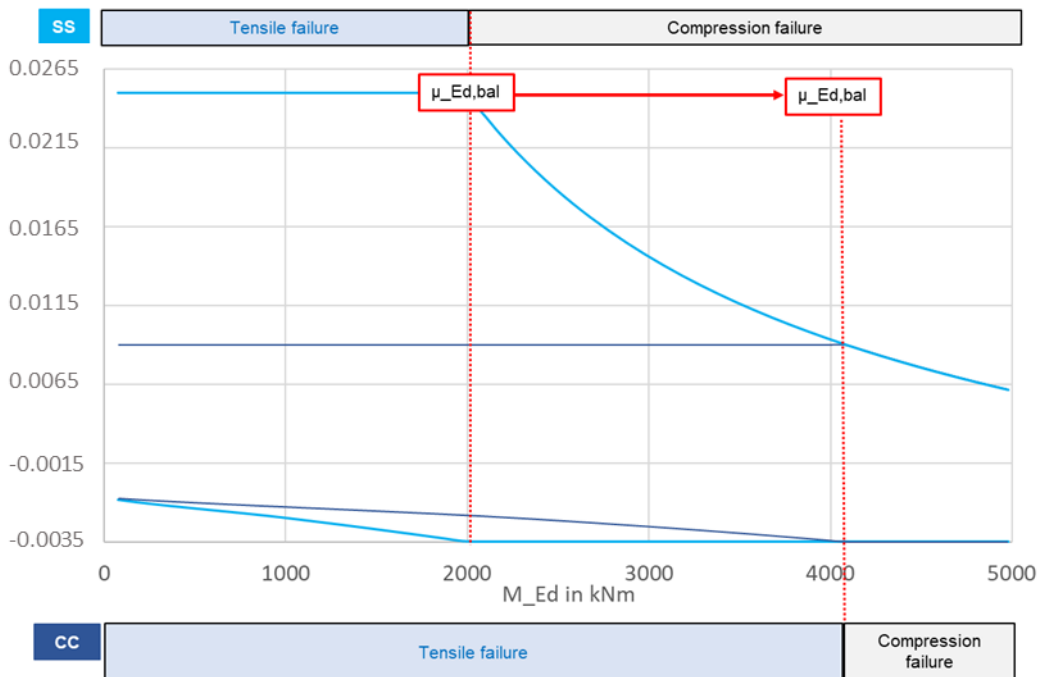


Figure 4.22 - Comparison unbonded tendons. Effectiveness of steel tendons and rebars compared to carbon tendons and rebars

4.9 Influence of the initial prestress in the amount of reinforcement

Pre-stress is considered an advantageous aspect for civil engineering infrastructures and also from an economic point of view.

In order to analyze the effect of pre-stress on the amount of reinforcement required, the following study was developed for pre-stress cables with bonded tendons.

Through analysis of the graphs below, Figure 4.23 it is possible to draw some conclusions relating strain with the amount of reinforcement required. For the mixed and carbon structures we can observe that there is a decrease in the amount of reinforcement as we increase the force of the prestress. However, after some limit there is an exponential increase in the amount of required reinforcement and when we simultaneously analyze what happens with the strain in the reinforcement, we can observe that it is not used, which is economically unviable the structure since the reinforcement is much less expensive than the prestress. For the case of the conventional prestressed reinforced concrete structure with steel as in the two previously mentioned structures there is a reduction in the amount of reinforcement required with the increase of the force of the prestress, despite of that, after a certain point, the amount of reinforcement and strain remain constant and this fact happens because the applied moment does not require more reinforcement since this verifies the ULS for the moment that, in this case, was applied. Thus, it was concluded that the increase of the force of the prestress in carbon structures is advantageous up to a certain point, since from this point on, the amount of reinforcement increases exponentially. For the case scenario with steel reinforcement and prestress it goes up to the minimum limit point and it maintains constant in a horizontal plateau.

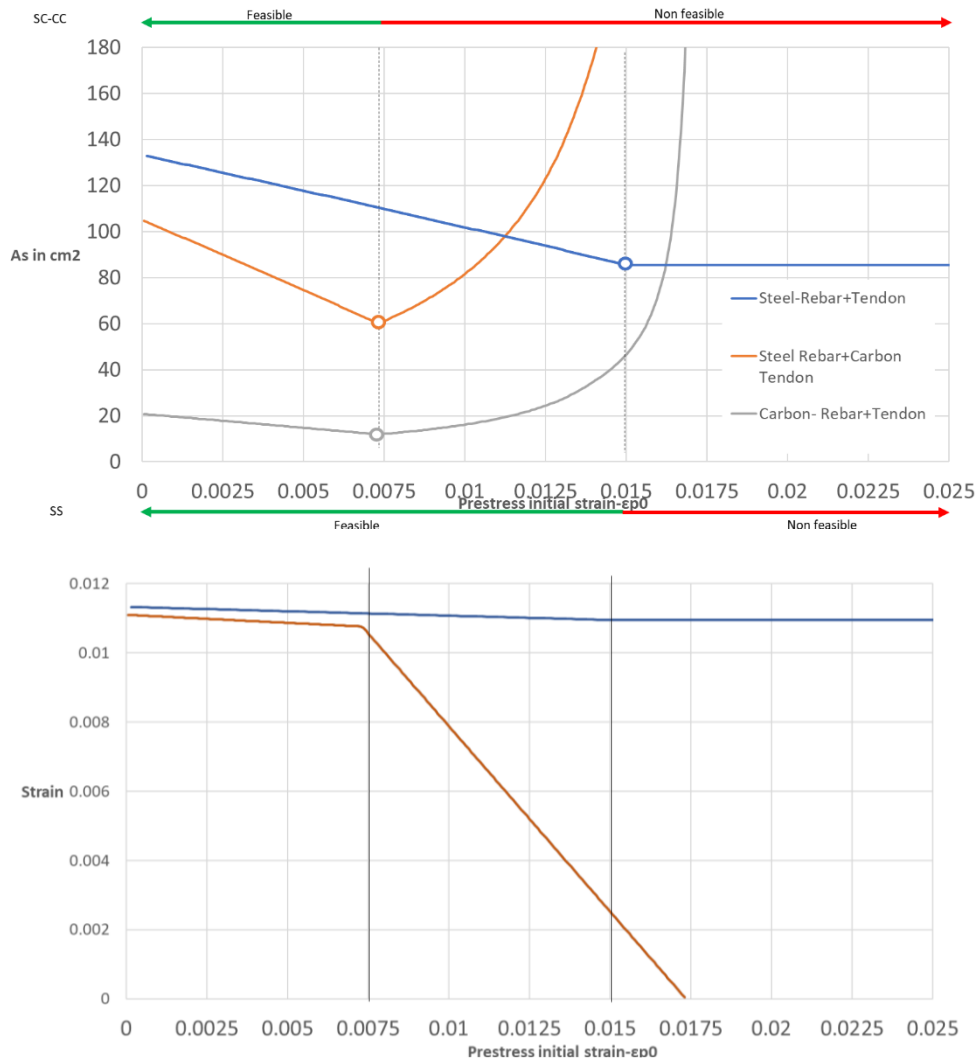


Figure 4.23 - Effect of the initial force of the prestress

4.10 New design approach

The case that follows is only for the CC combination, because it presents both the carbon reinforcement and the carbon tendons, and therefore has a sudden rupture if it fails through the carbon, considering that this is a material with a brittle behavior.

Studies have been made and according to the document given by fib [5], by the Canadian code [27] and also by BALAFAS and BURGOYNE [29], the failure is preferable by the concrete and not by the carbon. The failure is not as abrupt and catastrophic as the failure of the CFRP material and therefore, failure by the concrete should be preferred because it still has a deformation capacity. When compared to steel it is not significant but when compared to carbon, it is quite advantageous because it provides some obvious signs before the failure of the structure.

The compression failure occurs when the concrete crushes and tensile strain in the FRP are smaller than the ultimate strain. Compression failure in these situations occurs more smoothly, when compared to an over-reinforced concrete beam with internal steel reinforcement because, as observed in the stress strain graph of CFRP tendons, FRP materials usually present larger strains than yield strain of the typical steel prestressing tendons, thus the beam will present great deformations before the failure.

According to FIB [5], if the failure occurs due to the concrete crushing, then it does not matter the value of the safety factor that we use for the rebar. However, if the failure is due to rupture of the carbon rebar, then we must use the value of the safety factor associated with these rebars.

The failure due to concrete crushing, implies that we do not need a safety factor so high for the strain of the reinforcement, since it will never reach the ultimate limit strain and so, from a certain point, the safety coefficient can be reduced to the unit.

Thus, the results presented in Figure 4.24 are for the cases A and B, shown in Table 3.2 in section 3.6.3 and with the respective modification for the safety factors. In A there is a reduction of the strain limit due to the safety factor and in case B this does not occur.

After a careful analyze of the graph, Figure 4.24, we can observe that in an initial phase the results are very similar to the amount of reinforcement needed for both cases. They are practically the same. However, after the balanced point, the case where the safety factor is higher it requires more reinforcement as it was expected. Since the results do not show great differences later studies can be made to understand what happens in the transition zone and the exact point where the new method is advantageous. Nevertheless, for the study in question, it is concluded that the variation of stress-strain distribution type and choice of safety value do not justify the work that the calculation gives when analyzing the amount of reinforcement. Therefore, since it is the most practical way and because it does not present major differences in the amount of reinforcement, the conventional way of calculation is more advantageous.

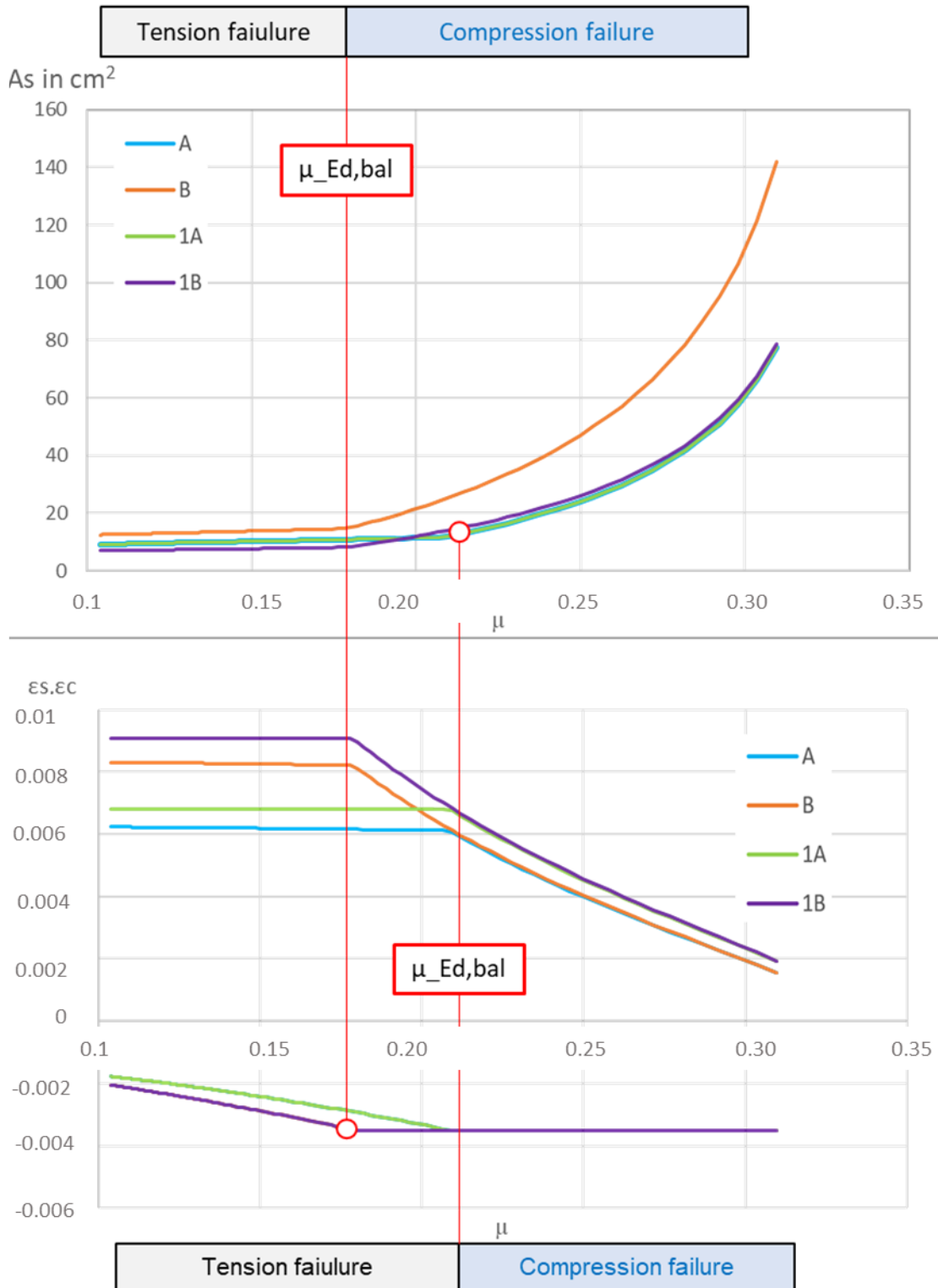


Figure 4.24 - Analysis of the new design approach

5 Conclusions

This chapter presents the results of this work and presents proposals for further work in this field of study.

The demand for safe and economically advantageous infrastructures has been and still is one of the problems faced by civil engineers, so new design approaches must be constantly updated as new application materials emerge in order to obtain a balance between safety and cost.

In this project the Excel program has been used to compare the design of reinforced concrete beams with and without prestress cables and to analyze their flexural behavior.

Several criteria can influence the cost of the infrastructure and one of them is the amount of material used, whether concrete, reinforcement or cables of prestress. Focusing on these aspects the present study was developed and with it, some conclusions can be drawn and used for new design methods.

A comparative study was elaborated in order to understand why the American and the European codes require different amount of reinforcement for the same applied moment, and with the results obtained with this study it is evidenced that the greatest difference is in the procedure itself and not in the physical and mechanical properties presented by the material.

As to the type of material used as reinforcement and prestress to be used it was concluded that carbon for both situations significantly reduces the amount of material required.

Another aspect to be considered when dealing with prestressed reinforced concrete beams are the types of prestressed to be used and the unbonded tendons have proved to be less viable than the bonded ones, regardless of the type of material used as reinforcement and as prestress. Once concluded that the bonded tendons, from this point of view, are more advantageous, the influence that the initial force of the prestress has on the reinforcement were analyzed. The rebars are considered economically viable when they fully utilized, i.e. the maximum possible strain. By varying the initial force of the prestress, a reduction in the amount of reinforcement is observed until a certain point in which the amount of reinforcement begins to grow exponentially. In the case of prestress with CFRP and in the case of prestress using steel, the amount of reinforcement remains the same after a certain point.

To sum up, the design must be done carefully since different parameters can lead to different results. The CFRP are materials that are worth investing in their research, because as it was observed with this study, it presents great advantages when it comes to corrosion, degradation of structure and amount of material required.

It is important to point out that the results obtained and the conclusions that have been taken in this dissertation are for a given cross section and specific materials and therefore it should not be generalized to other cases without a more detailed study.

For later works an optimum cost approach would be interesting, as well as other types of effects on prestressed reinforced concrete, deflection, shear and torsion. Also, the anchoring system for FRP tendons, since it is one of the biggest problems for FRP prestressed cables.

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Appendices

Appendix A

```

Sub ec()

    Dim j As Double
    Dim w As Worksheet

    Set w = Sheets("EUROCODE")
    w.Select
    w.Range("a17").Select
    l = 0

    For miu = 0.01 To 0.48 Step 0.01
        'MIU > 0.47- NO SOLUTION

        'ActiveCell.Value = miu
        'ActiveCell.Offset(1, 0).Select

        'Definir Variaveis

        d = Folhal.Cells(12, 5).Value
        b = Folhal.Cells(13, 5).Value
        fcd = Folhal.Cells(10, 14).Value
        n = Folhal.Cells(13, 13).Value
        ec2 = Folhal.Cells(11, 14).Value
        eci = Folhal.Cells(11, 8).Value
        esi = Folhal.Cells(12, 8).Value
        fyd = Folhal.Cells(10, 18).Value
        fyk = Folhal.Cells(9, 18).Value
        E_s = Folhal.Cells(12, 18).Value
        Gs = Folhal.Cells(4, 18).Value

        'Calcular Mrd

        'tensao limite do aço

        sigma_sd = fyk / Gs

        'tensao acco
        sigma_s = esi * E_s

        If sigma_s < sigma_sd Then
            sigma_sf = sigma_s
        Else
            sigma_sf = sigma_sd
        End If

        x = Abs(eci) * d / (Abs(eci) + esi)

        If Abs(eci) <= Abs(ec2) Then
            sigma = (1 - ((1 - (Abs(eci) / Abs(ec2))) ^ n)) * fcd
        Else
            sigma = fcd
        End If

        If Abs(eci) <= Abs(ec2) Then
            a = (1 - ((Abs(ec2 * 1000) / Abs(eci * 1000)) * ((1 - (1 - (Abs(eci * 1000) / Abs(ec2 * 1000))) ^ (n + 1)) / (n + 1))))
        Else
            a = 1 - (Abs(ec2 * 1000) / (Abs(eci * 1000) * (n + 1)))
        End If

        If Abs(eci) <= Abs(ec2) Then
            ka = (1 - (Abs(eci * 1000) / 8)) / (3 - (Abs(eci * 1000) / 2))
        Else
            ka = 1 - (((3 * Abs(eci * 1000)) / (3 * Abs(eci * 1000) - 2)) * ((3 * ((eci * 1000) ^ 2) - 2) / (6 * ((1000 * eci) ^ 2))))
        End If
    Next miu
End Sub

```

```

Do While (Abs(delta_M1) > 10 ^ -5)

    'Calcular Mrd

    'tensao limite do ago
    sigma_sd = fyk / Gs

    'tensao aco
    sigma_sl = esnl * E_s

    If sigma_sl < sigma_sd Then
        sigma_sf1 = sigma_sl
    Else
        sigma_sf1 = sigma_sd
    End If

    x1 = Abs(ecn1) * d / (Abs(ecn1) + esnl)

    If Abs(ecn1) <= Abs(ec2) Then
        signal = (1 - ((1 - (Abs(ecn1) / Abs(ec2))) ^ n)) * fcd
    Else
        signal = fcd
    End If

    If Abs(ecn1) <= Abs(ec2) Then
        al = (1 - ((Abs(ec2 * 1000) / Abs(ecn1 * 1000)) * ((1 - (1 - (Abs(ecn1 * 1000) / Abs(ec2 * 1000))) ^ (n + 1)) / (n + 1))))
    Else
        al = 1 - (Abs(ec2 * 1000) / (Abs(ecn1 * 1000) * (n + 1)))
    End If

    If Abs(ecn1) <= Abs(ec2) Then
        kal = (1 - (Abs(ecn1 * 1000) / 8)) / (3 - (Abs(ecn1 * 1000) / 2))
    Else
        kal = 1 - (((3 * Abs(ecn1 * 1000)) / (3 * Abs(ecn1 * 1000) - 2)) * ((3 * ((ecn1 * 1000) ^ 2) - 2) / (6 * ((ecn1 * 1000) ^ 2))))
    End If

    Fc1 = al * x1 * b * signal
    Z1 = d - kal * x1
    Mrd1 = Fc1 * Z1

    ks11 = x1 / d
    zeta1 = Z1 / d
    Fsl = Fc1
    A_sl = Fsl / sigma_sf1

    delta_M1 = Med - Mrd1

    If caminho = "C" Then
        DeltaC2 = Delta_C / (2 ^ i)
    Else
        DeltaC2 = 0
    End If

    If caminho = "S" Then
        DeltaS2 = Delta_S / (2 ^ i)
    Else
        DeltaS2 = 0
    End If

    If delta_M1 < 0 Then
        ecn1 = ecn1 - DeltaC2
        esnl = esnl + DeltaS2
    Else
        ecn1 = ecn1 + DeltaC2
        esnl = esnl - DeltaS2
    End If
    -----
    If Fc1 > Fsl Then
        a_slf = 0
    Else
        a_slf = 10000 * A_sl
    End If

    i = i + 1
Loop

Cells(17 + 1, 1).Value = miu
Cells(17 + 1, 2).Value = ecn1
Cells(17 + 1, 3).Value = esnl
Cells(17 + 1, 4).Value = ks11
Cells(17 + 1, 5).Value = zeta1
Cells(17 + 1, 6).Value = a_slf
Cells(17 + 1, 7).Value = delta_M1
Cells(17 + 1, 8).Value = Med
Cells(17 + 1, 9).Value = Mrd1

l = l + 1

Next

End Sub

```

Appendix B

```
Sub ACI()
```

```
Dim j As Double
Dim m As Worksheet
```

```
Set m = Sheets("REAL_ACI")
m.Select
m.Range("a17").Select
l = 0
```

```
For miu = 0.01 To 0.48 Step 0.01
'MIU > 0.47- NO
```

```
|
'Definir Variaveis
```

```
d = Folha9.Cells(12, 5).Value
b = Folha9.Cells(13, 5).Value
fck = Folha9.Cells(9, 14).Value
n = Folha9.Cells(13, 13).Value
ec3 = Folha9.Cells(11, 14).Value
ecu3 = Folha9.Cells(12, 14).Value
eci = Folha9.Cells(11, 8).Value
esi = Folha9.Cells(12, 8).Value
fyk = Folha9.Cells(9, 18).Value
Es = Folha9.Cells(12, 18).Value
Gama_s = Folha9.Cells(4, 18).Value
Gama_c = Folha9.Cells(3, 18).Value
ecu = Folha9.Cells(12, 13).Value
et = 5
fyk = Folha9.Cells(9, 18).Value
E_s = Folha9.Cells(12, 18).Value
ey = (fyk / Es)
fi_conc = 0.65
fi_steel = 0.9
```

```
'Calcular Mrd
```

```
sigma_sd = fyk
```

```
'tensao aco
sigma_s = esi * E_s
```

```
If sigma_s < sigma_sd Then
sigma_sf = sigma_s
Else
sigma_sf = sigma_sd
End If
```

```
x = Abs(eci) * d / (Abs(eci) + esi)
```

```
If Abs(eci) <= Abs(ec3) Then
sigma = (eci / ec3) * fck
Else
sigma = fck
End If
```

```
If Abs(eci) <= Abs(ec3) Then
a = 0.5 * (eci / ec3)
Else
a = (Abs(eci) - 0.5 * Abs(ec3)) / (Abs(eci))
```

```
End If
```

```
If Abs(eci) <= Abs(ec3) Then
ka = 1 / 3
Else
ka = (((eci) ^ 2) - (eci * ec3) + (((ec3) ^ (2)) / 3)) / ((2 * Abs(eci)) * (Abs(eci) -
```



```
Fc = a * x * b * sigma
Z = d - ka * x
Mrd = Fc * Z

'factor fi

'compression controlled
If esi <= ey Then
Mrdf = fi_conc * Mrd
Else
Mrdf = Mrd
End If

'transition
If Abs(eci) > Abs(ec3) And esi < et Then
fi_t = 0.48 + 83 * esi
Mrdf = fi_t * Mrd
Else
Mrdf = Mrd
End If

'tension controlled
If esi >= et Then
Mrdf = fi_steel * Mrd
Else
Mrdf = Mrd
End If

ksi = x / d
zeta = Z / d
Fs = Fc
a_s = Fs / sigma_sf

'calcular Med

Med = miu * (b * (d ^ 2) * fck)

'Calcular primeira linha

Delta_M = Med - Mrdf

If miu >= 0.5 Then
a_s = "no solution"
End If

If Delta_M < 0 Then
caminho = "C"
Else
caminho = "S"
End If

If caminho = "C" Then
Delta_C = 0.5 * eci
Else
Delta_C = 0
End If

If caminho = "S" Then
Delta_S = 0.5 * esi
Else
Delta_S = 0
End If
```

```

If Delta_M < 0 Then
ecn = eci - Delta_C
Else
ecn = eci + Delta_C
End If

If Delta_M > 0 Then
esn = esi - Delta_S
Else
esn = esi + Delta_S
End If

'Calcular segunda linha

i = 1

delta_M1 = Delta_M
ecn1 = ecn
esn1 = esn

Do While Abs(delta_M1) > 10 ^ -5 And miu < 0.5

'Calcular Mrd

sigma_sd = fyk

'tensao aco
sigma_sl = esn1 * E_s

If sigma_sl < sigma_sd Then
sigma_sfl = sigma_sl
Else
sigma_sfl = sigma_sd
End If

x1 = Abs(ecn1) * d / (Abs(ecn1) + esn1)

If Abs(ecn1) <= Abs(ec3) Then
signal = (ecn1 / ec3) * fck
Else
signal = fck
End If

If Abs(ecn1) <= Abs(ec3) Then
a1 = 0.5 * (ecn1 / ec3)
Else
a1 = (Abs(ecn1) - 0.5 * Abs(ec3)) / (Abs(ecn1))

End If

If Abs(ecn1) <= Abs(ec3) Then
kal = 1 / 3
Else
kal = (((ecn1) ^ (2)) - (ecn1 * ec3) + (((ec3) ^ (2)) / 3)) / ((2 * Abs(ecn1)) * (Abs(ecn1) - 0.5 * Abs(ec3)))
End If

```

```

Fcl = al * xl * b * sigmal
Zl = d - kal * xl
Mrdl = Fcl * Zl

'factor fi

'compression controlled
If esn <= ey Then
Mrdf1 = fi_conc * Mrd
Else
Mrdf1 = Mrdl
End If

'transition
If Abs(ecn) > Abs(ec3) And esn < et Then
fi_t = 0.48 + 83 * esn
Mrdf1 = fi_t * Mrdl
Else
Mrdf1 = Mrdl
End If

'tension controlled
If esn >= et Then
Mrdf1 = fi_steel * Mrdl
Else
Mrdf1 = Mrdl
End If

ksil = xl / d
zetal = Zl / d
miul = Mrdl / (b * (d ^ 2) * fck)
Fsl = Fcl
A_sl = Fsl / sigma_sfl

delta_M1 = Med - Mrdf1

If caminho = "C" Then
DeltaC2 = Delta_C / (2 ^ i)
Else
DeltaC2 = 0
End If

If caminho = "S" Then
DeltaS2 = Delta_S / (2 ^ i)
Else
DeltaS2 = 0
End If

If delta_M1 < 0 Then
ecn1 = ecn1 - DeltaC2
esn1 = esn1 + DeltaS2
Else
ecn1 = ecn1 + DeltaC2
esn1 = esn1 - DeltaS2
End If

If Fcl > Fsl Then
a_slf = 0
Else
a_slf = A_sl * 10000
End If

```

```
        i = i + 1

    Loop

    Cells(17 + 1, 1).Value = miu
    Cells(17 + 1, 2).Value = ecnl
    Cells(17 + 1, 3).Value = esnl
    Cells(17 + 1, 4).Value = ksil
    Cells(17 + 1, 5).Value = zetal
    Cells(17 + 1, 6).Value = a_slf
    Cells(17 + 1, 7).Value = delta_M1
    Cells(17 + 1, 8).Value = Med
    Cells(17 + 1, 9).Value = Mrdl

    l = l + 1

Next

End Sub
```

Appendix C

```

Sub BON_UNB_B_SOL_A_MED()

'NEW SHEET
sheet_name_to_create = "New_Sheet"

Sheets.Add after:=Sheets(Sheets.Count)
Sheets(ActiveSheet.Name).Name = sheet_name_to_create

Range("B3").Select
ActiveCell.FormulaR1C1 = "b(m)"
Range("C3").Select
ActiveCell.FormulaR1C1 = "Med"
Range("D3").Select
ActiveCell.FormulaR1C1 = "Mrd"
Range("e3").Select
ActiveCell.FormulaR1C1 = "ec"
Range("f3").Select
ActiveCell.FormulaR1C1 = "es"
Range("g3").Select
ActiveCell.FormulaR1C1 = "ksi"
Range("h3").Select
ActiveCell.FormulaR1C1 = "zeta"
Range("i3").Select
ActiveCell.FormulaR1C1 = "miu"
Range("j3").Select
ActiveCell.FormulaR1C1 = "As(cm2)"
Range("k3").Select
ActiveCell.FormulaR1C1 = "Delta_M"
Range("L3").Select
ActiveCell.FormulaR1C1 = "Delta_ep"
Range("M3").Select
ActiveCell.FormulaR1C1 = "ep0"
Range("N3").Select
ActiveCell.FormulaR1C1 = "epf"

Dim j As Double
Dim w As Worksheet

Set w = Sheets("DATA")
w.Select
w.Range("a17").Select
l = 0

'input

d = Folhal.Cells(5, 7).Value
dp = Folhal.Cells(4, 7).Value
fyk = Folhal.Cells(3, 19).Value
Gs = Folhal.Cells(5, 19).Value
eci = Folhal.Cells(6, 11).Value
E_s = Folhal.Cells(4, 19).Value
fp0lk = Folhal.Cells(4, 15).Value
E_p = Folhal.Cells(5, 15).Value
sigma_p0 = Folhal.Cells(19, 15).Value
ec2 = Folhal.Cells(5, 11).Value
fod = Folhal.Cells(4, 11).Value
Ap = Folhal.Cells(9, 15).Value
n = Folhal.Cells(7, 10).Value
Zp = d - dp
Ned = Folhal.Cells(11, 3).Value
Pf = Folhal.Cells(14, 15).Value
e = Folhal.Cells(7, 7).Value
Mpe = Folhal.Cells(12, 3).Value
type_bond = Folhal.Cells(3, 3).Value
epd = Folhal.Cells(7, 15).Value
sigma_pd = Folhal.Cells(18, 15).Value
eud = Folhal.Cells(6, 19).Value
tendon_type = Folhal.Cells(4, 3).Value
rebar_type = Folhal.Cells(5, 3).Value
b = 0.96
euk = Folhal.Cells(7, 19).Value

```

```

If type_bond = "Bonded" Then

If tendon_type = "Steel" And rebar_type = "Steel" Then
  'initial strain- ep0
  ep0 = Abs(sigma_p0) / E_p

  ' delta_ep

  Delta_max = euk - Abs(ep0)

  ' inicial steel strain

  esmax = (Delta_max * d) / dp

  'neutral axis

  esi = esmax

  End If

If tendon_type = "Carbon" And rebar_type = "Carbon" Then
  'initial strain- ep0
  ep0 = Abs(sigma_p0) / E_p

  ' delta_ep
      Delta_max = epd - Abs(ep0)
      esmax = (Delta_max * d) / dp
  ' inicial steel strain
  If esmax < eud Then
    esi = esmax
  Else
    esi = eud
  End If

  End If

If tendon_type = "Carbon" And rebar_type = "Steel" Then
  'initial strain- ep0
  ep0 = Abs(sigma_p0) / E_p

  ' delta_ep

  Delta_max = epd - Abs(ep0)

  ' inicial steel strain

  esmax = (Delta_max * d) / dp

  'neutral axis

  esi = esmax

  End If

  'rebars

  If rebar_type = "Steel" Then
    sigma_s = esi * E_s
    sigma_sd = fyk / Gs
    If sigma_s < sigma_sd Then
      sigma_sf = sigma_s
    Else
      sigma_sf = sigma_sd
    End If
  End If

```

```
For Med = 0.1 To 5000 Step 100
```

```
x = (Abs(eci) * d) / (Abs(eci) + esi)
```

```
delta_ep = ((dp - x) / (d - x)) * (esi)
```

```
' prestress
```

```
sigma_p = Abs(sigma_p0) + delta_ep * E_p
```

```
'Mrd
```

```
x = Abs(eci) * d / (Abs(eci) + esi)
```

```
If tendon_type = "Steel" Then
```

```
  If sigma_p > sigma_pd Then
```

```
    sigma_pf = sigma_pd
```

```
  Else
```

```
    sigma_pf = sigma_p
```

```
  End If
```

```
End If
```

```
If tendon_type = "Carbon" Then
```

```
  If sigma_p < sigma_pd Then
```

```
    sigma_pf = sigma_p
```

```
  End If
```

```
End If
```

```
Fp = Ap * sigma_pf
```

```
ep = delta_ep + ep0
```

```
If Abs(eci) <= Abs(ec2) Then
```

```
sigma = (1 - ((1 - (Abs(eci) / Abs(ec2))) ^ n)) * fcd
```

```
Else
```

```
sigma = fcd
```

```
End If
```

```
If Abs(eci) <= Abs(ec2) Then
```

```
A = (1 - ((Abs(ec2 * 1000) / Abs(eci * 1000)) * ((1 - (1 - (Abs(eci * 1000) / Abs(ec2 * 1000))) ^ (n + 1)) / (n + 1))))
```

```
Else
```

```
A = 1 - (Abs(ec2 * 1000) / (Abs(eci * 1000) * (n + 1)))
```

```
End If
```

```
If ec2 = -0.002 And eci = -0.0035 And n = 2 Then
```

```
  If Abs(eci) <= Abs(ec2) Then
```

```
    ka = (8 + Abs(eci * 1000)) / (24 + 4 * Abs(eci * 1000))
```

```
  Else
```

```
    ka = (3*Abs(eci*1000)*Abs(eci*1000) + 4*Abs(eci*1000) + 2) / (6*Abs(eci * 1000) * Abs(eci * 1000) + 4 * Abs(eci * 1000))
```

```
  End If
```

```
Else
```

```
  If Abs(eci) <= Abs(ec2) Then
```

```
    ka = 1 - (1 / a1) * [0.5 + ((ec2*ec2)/(eci*eci)) * (((1-ecnl/ec2)^(n+1))-1)/(n+1) - (((1-eci/ec2)^(n+2))-1)/(n+2)]
```

```
  Else
```

```
    ka = 1 - (1 / a1) * (0.5 - ((ec2 * ec2) / ((ecnl * ecnl) * (n + 1))) + ((ec2 * ec2) / ((eci * eci) * (n + 2))))
```

```
  End If
```

```

End If

Fc = A * x * b * sigma
Z = d - ka * x
Mrd = Fc * Z - Fp * Zp
ksi = x / d
zeta = Z / d
Fs = Fc - Fp

    miu = Med / (b * d * d * fcd)

    If miu < 0.47 Then
        omega = 1 - (1 - 2 * miu) ^ 0.5
        A_s = ((omega * b * d * sigma) / sigma_sf) - Fp / sigma_sf * 10000
    Else
        A_s = "no solution"
    End If

    If A_sl < 0 Then
        A_sf = 0
    Else
        A_sf = A_s
    End If

'first line

Delta_m = Med - Mrd

If Delta_m < 0 Then
    caminho = "C"
Else
    caminho = "S"
End If

If caminho = "C" Then
    Delta_C = 0.5 * eci
Else
    Delta_C = 0
End If

If caminho = "S" Then
    Delta_s = 0.5 * esi
Else
    Delta_s = 0
End If

If Delta_m < 0 Then
    ecn = eci - Delta_C
Else
    ecn = eci + Delta_C
End If

If Delta_m > 0 Then
    esn = esi - Delta_s
Else
    esn = esi + Delta_s
End If

'second line

I = 1

Delta_Ml = Delta_m
ecn1 = ecn
esn1 = esn

```



```

Do While Abs(Delta_Ml) > 10 ^ -5

    miu = Med / (fcd * b * d * d)
    xl = (Abs(ecnl) * d) / (Abs(ecnl) + esnl)

    delta_epl = ((dp - xl) / (d - xl)) * (esnl)

    ' stress of rebars

If rebar_type = "Steel" Then
    sigma_sl = esnl * E_s
    sigma_sd = fyk / Gs
    If sigma_sl < sigma_sd Then
        sigma_sfl = sigma_sl
    Else
        sigma_sfl = sigma_sd
    End If
End If

If rebar_type = "Carbon" Then
    sigma_sl = esnl * E_s
    sigma_sd = fyk / Gs
    If sigma_sl < sigma_sd Then
        sigma_sfl = sigma_sl
    End If
End If

'stress of prestress
sigma_pl = Abs(sigma_p0) + delta_epl * E_p

If tendon_type = "Steel" Then

    If sigma_pl > sigma_pd Then
        sigma_pfl = sigma_pd
    Else
        sigma_pfl = sigma_pl
    End If
End If

If tendon_type = "Carbon" Then

    If sigma_pl < sigma_pd Then
        sigma_pfl = sigma_pl
    End If

End If

Fpl = Ap * sigma_pfl
epl = delta_epl + ep0

```

```

' Mrd

If Abs(ecn1) <= Abs(ec2) Then
  sigmal = (1 - ((1 - (Abs(ecn1) / Abs(ec2))) ^ n)) * fcd
Else
  sigmal = fcd
End If

If Abs(eci) <= Abs(ec2) Then
  al = (1 - ((Abs(ec2 * 1000) / Abs(ecn1 * 1000)) * ((1 - (1 - (Abs(ecn1 * 1000) / Abs(ec2 * 1000))) ^ (n + 1)) / (n + 1))))
Else
  al = 1 - (Abs(ec2 * 1000) / (Abs(ecn1 * 1000) * (n + 1)))
End If

If ec2 = -0.002 And eci = -0.0035 And n = 2 Then

  If Abs(eci) <= Abs(ec2) Then
    kal = (8 + Abs(ecn1 * 1000)) / (24 + 4 * Abs(ecn1 * 1000))

  Else
    kal = (3*Abs(ecn1*1000)*Abs(ecn1*1000) + 4*Abs(ecn1*1000) + 2) / (6*Abs(ecn1*1000)*Abs(ecn1*1000) + 4 * Abs(ecn1 * 1000))
  End If

Else
  If Abs(eci) <= Abs(ec2) Then
    kal = 1 - (1 / al) * [0.5+((ec2*ec2)/(ecn1*ecn1))*[(((1-ecn1/ec2)^(n+1))-1)/(n+1))-(((1-ecn1/ec2)^(n+2))-1)/(n+2)]]

  Else
    kal = 1 - (1 / al) * (0.5 - ((ec2 * ec2) / ((ecn1 * ecn1) * (n + 1))) + ((ec2 * ec2) / ((ecn1 * ecn1) * (n + 2))))
  End If
End If

Fcl = al * x1 * b * sigmal
Zl = d - kal * x1

Mrdl = Fcl * Zl - Fpl * Zp
ksil = x1 / d
zetal = Zl / d
Fsl = Fcl - Fpl

miu = Med / (b * d * d * fcd)
If miu < 0.47 Then
  omega = 1 - (1 - 2 * miu) ^ 0.5
  A_s1 = ((omega * b * d * sigmal) / sigma_sfl) - Fpl / sigma_sfl * 10000

Else
  A_s1 = "no solution"
End If

If A_s1 < 0 Then
  A_sfl = 0
Else
  A_sfl = A_s1
End If

If ecn1 = eci Then
  miu_bal = miu
End If

```

```

Delta_M1 = Med - Mrd1

If caminho = "C" Then
DeltaC2 = Delta_C / (2 ^ I)
Else
DeltaC2 = 0
End If

If caminho = "S" Then
DeltaS2 = Delta_s / (2 ^ I)
Else
DeltaS2 = 0
End If

If Delta_M1 < 0 Then
ecn1 = ecn1 - DeltaC2
esn1 = esn1 + DeltaS2
Else
ecn1 = ecn1 + DeltaC2
esn1 = esn1 - DeltaS2
End If

I = I + 1
Loop

'plots

Sheets("New_Sheet").Select
Range("a1: h1").Select
ActiveCell.Offset(1, 0).Range("i4").Select
ActiveCell.Value = miu

Sheets("New_Sheet").Select
Range("a1: h1").Select
ActiveCell.Offset(1, 0).Range("e4").Select
ActiveCell.Value = ecn1

Sheets("New_Sheet").Select
Range("a1: h1").Select
ActiveCell.Offset(1, 0).Range("f4").Select
ActiveCell.Value = esn1

Sheets("New_Sheet").Select
Range("a1: h1").Select
ActiveCell.Offset(1, 0).Range("g4").Select
ActiveCell.Value = ks11

Sheets("New_Sheet").Select
Range("a1: h1").Select
ActiveCell.Offset(1, 0).Range("h4").Select
ActiveCell.Value = zetal

Sheets("New_Sheet").Select
Range("a1: h1").Select
ActiveCell.Offset(1, 0).Range("j4").Select
ActiveCell.Value = A_sfl

```

```

Sheets("New_Sheet").Select
Range("a1: h1").Select
ActiveCell.Offset(1, 0).Range("k4").Select
ActiveCell.Value = Delta_M1

Sheets("New_Sheet").Select
Range("a1: h1").Select
ActiveCell.Offset(1, 0).Range("d4").Select
ActiveCell.Value = Mrdl

Sheets("New_Sheet").Select
Range("a1: h1").Select
ActiveCell.Offset(1, 0).Range("c4").Select
ActiveCell.Value = Med

Sheets("New_Sheet").Select
Range("a1: h1").Select
ActiveCell.Offset(1, 0).Range("l4").Select
ActiveCell.Value = delta_ep1

Sheets("New_Sheet").Select
Range("a1: h1").Select
ActiveCell.Offset(1, 0).Range("m4").Select
ActiveCell.Value = ep0

Sheets("New_Sheet").Select
Range("a1: h1").Select
ActiveCell.Offset(1, 0).Range("n4").Select
ActiveCell.Value = ep1

Sheets("New_Sheet").Select
Range("a1: h1").Select
ActiveCell.Offset(1, 0).Range("b4").Select
ActiveCell.Value = b

Sheets("New_Sheet").Select
Range("a1: h1").Select
ActiveCell.Offset(1, 0).Range("a4").Select
ActiveCell.Value = miu_bal

l = l + 1

```

Next

Else

```

'UNBONDED
If tendon_type = "Steel" Then
    esi = Folhal.Cells(7, 19).Value
    ep0 = Abs(sigma_p0) / E_p
End If

```

```

If tendon_type = "Carbon" Then
    ' ep0
    ep0 = Abs(sigma_p0) / E_p

```

End If

```

'rebars

If rebar_type = "Steel" Then
esi = Folhal.Cells(7, 19).Value
sigma_s = esi * E_s
sigma_sd = fyk / Gs
If sigma_s < sigma_sd Then
sigma_sf = sigma_s
Else
sigma_sf = sigma_sd
End If
End If

If rebar_type = "Carbon" Then
esi = Folhal.Cells(6, 19).Value
sigma_s = esi * E_s
sigma_sd = fyk / Gs
If sigma_s < sigma_sd Then
sigma_sf = sigma_s
End If
End If

For b = 0.5 To 1.5 Step 0.01

'stress of prestress
sigma_p = Abs(sigma_p0) + delta_ep * E_p

If tendon_type = "Steel" Then

If sigma_p > sigma_pd Then
sigma_pf = sigma_pd
Else
sigma_pf = sigma_p
End If
End If

If tendon_type = "Carbon" Then

If sigma_p <= sigma_pd Then
sigma_pf = sigma_p
End If

End If

' Mrd

x = Abs(eci) * d / (Abs(eci) + esi)

If Abs(eci) <= Abs(ec2) Then
sigma = (1 - ((1 - (Abs(eci) / Abs(ec2))) ^ n)) * fcd
Else
sigma = fcd
End If

If Abs(eci) <= Abs(ec2) Then
A = (1 - ((Abs(ec2 * 1000) / Abs(eci * 1000)) * ((1 - (1 - (Abs(eci * 1000) / Abs(ec2 * 1000))) ^ (n + 1)) / (n + 1))))
Else
A = 1 - (Abs(ec2 * 1000) / (Abs(eci * 1000) * (n + 1)))
End If

```

```

If ec2 = -0.002 And eci = -0.0035 And n = 2 Then

    If Abs(eci) <= Abs(ec2) Then
        ka = (8 + Abs(eci * 1000)) / (24 + 4 * Abs(eci * 1000))
    Else
        ka = (3 * Abs(eci*1000)*Abs(eci*1000)+4*Abs(eci*1000)+2)/(6*Abs(eci * 1000) * Abs(eci * 1000) + 4 * Abs(eci * 1000))
    End If

Else
    If Abs(eci) <= Abs(ec2) Then
        ka = 1 - (1 / a1) * [0.5+((ec2*ec2)/(eci*eci))*[(((1-ecn1/ec2)^(n+1))-1)/(n+1))-(((1-eci/ec2)^(n+2))-1)/(n+2)]]
    Else
        ka = 1 - (1 / a1) * (0.5 - ((ec2 * ec2) / ((ecn1 * ecn1) * (n + 1))) + ((ec2 * ec2) / ((eci * eci) * (n + 2))))
    End If
End If

Fc = A * x * b * sigma
Z = d - ka * x
Mrd = Fc * Z
ksi = x / d
zeta = Z / d
Fs = Fc

Medf = Med + Mpe - Pf * e

'lst line

Delta_m = Medf - Mrd

If Delta_m < 0 Then
    caminho = "C"
Else
    caminho = "S"
End If

If caminho = "C" Then
    Delta_C = 0.5 * eci
Else
    Delta_C = 0
End If

If caminho = "S" Then
    Delta_s = 0.5 * esi
Else
    Delta_s = 0
End If

If Delta_m < 0 Then
    ecn = eci - Delta_C
Else
    ecn = eci + Delta_C
End If

If Delta_m > 0 Then
    esn = esi - Delta_s
Else
    esn = esi + Delta_s
End If

```

```

'2nd line

I = 1

Delta_M1 = Delta_m
ecnl = ecn
esnl = esn

Do While (Abs(Delta_M1) > 10 ^ -5 And I <= 1022) And miu < 0.47

miu = Medf / (fcd * b * d * d)
x1 = (Abs(ecnl) * d) / (Abs(ecnl) + esnl)

delta_epl = ((dp - x1) / (d - x1)) * (esnl)

' stress of rebars

If rebar_type = "Steel" Then
    sigma_sl = esnl * E_s
    sigma_sd = fyk / Gs
    If sigma_sl < sigma_sd Then
        sigma_sfl = sigma_sl
    Else
        sigma_sfl = sigma_sd
    End If
End If

If rebar_type = "Carbon" Then
    sigma_sl = esnl * E_s
    sigma_sd = fyk / Gs
    If sigma_sl <= sigma_sd Then
        sigma_sfl = sigma_sl
    End If
End If

'stress of prestress
sigma_pl = Abs(sigma_p0) + delta_epl * E_p

If tendon_type = "Steel" Then

    If sigma_pl > sigma_pd Then
        sigma_pfl = sigma_pd
    Else
        sigma_pfl = sigma_pl
    End If
End If

If tendon_type = "Carbon" Then

    If sigma_pl <= sigma_pd Then
        sigma_pfl = sigma_pl
    End If

End If

x1 = Abs(ecnl) * d / (Abs(ecnl) + esnl)

If Abs(ecnl) <= Abs(ec2) Then
    signal = (1 - ((1 - (Abs(ecnl) / Abs(ec2))) ^ n)) * fcd
Else
    signal = fcd
End If

```

```

If Abs(eci) <= Abs(ec2) Then
al = (1 - ((Abs(ec2 * 1000) / Abs(ecn1 * 1000)) * ((1 - (1 - (Abs(ecn1 * 1000) / Abs(ec2 * 1000))) ^ (n + 1)) / (n + 1))))
Else
al = 1 - (Abs(ec2 * 1000) / (Abs(ecn1 * 1000) * (n + 1)))
End If

If ec2 = -0.002 And eci = -0.0035 And n = 2 Then

    If Abs(eci) <= Abs(ec2) Then
        kal = (8 + Abs(ecn1 * 1000)) / (24 + 4 * Abs(ecn1 * 1000))

        Else
            kal = (3 * Abs(ecn1 * 1000)*Abs(ecn1*1000)+4*Abs(ecn1*1000)+2)/(6*Abs(ecn1*1000)*Abs(ecn1*1000) + 4 * Abs(ecn1 * 1000))
        End If

    Else
        If Abs(eci) <= Abs(ec2) Then
            kal = 1 - (1 / al) * [0.5+((ec2*ec2)/(ecn1*ecn1))*((((1-ecn1/ec2)^(n+1))-1)/(n+1))-(((1-ecn1/ec2)^(n+2))-1)/(n+2)]]

            Else
                kal = 1 - (1 / al) * (0.5 - ((ec2 * ec2) / ((ecn1 * ecn1) * (n + 1))) + ((ec2 * ec2) / ((ecn1 * ecn1) * (n + 2))))
            End If
        End If

        Fc1 = al * x1 * b * sigmal
        Z1 = d - kal * x1

        Mrd1 = Fc1 * Z1
        ks1l = x1 / d
        zeta1 = Z1 / d

        Medf = Med + Mpe - Pf * e
        miu = Medf / (b * fcd * d * d)

        If miu < 0.47 Then
            omega = 1 - (Sqr(1 - (2 * miu)))
            A_s1 = ((omega * b * d * sigmal + Pf) / sigma_sf1) * 10000
        Else
            A_s1 = "no solution"
        End If

        If A_s1 < 0 Then
            A_sf = 0
        Else
            A_sf = A_s1
        End If

        Delta_M1 = Medf - Mrd1

        If caminho = "C" Then
            DeltaC2 = Delta_C / (2 ^ I)
        Else
            DeltaC2 = 0
        End If

        If caminho = "S" Then
            DeltaS2 = Delta_s / (2 ^ I)
        Else
            DeltaS2 = 0
        End If

        If Delta_M1 < 0 Then
            ecn1 = ecn1 - DeltaC2
            esn1 = esn1 + DeltaS2
        Else
            ecn1 = ecn1 + DeltaC2
            esn1 = esn1 - DeltaS2
        End If

        If ecn1 = eci Then
            miu_bal = miu
        End If

        I = I + 1

Loop

```



```
'plots

Sheets("New_Sheet").Select
Range("a1: h1").Select
ActiveCell.Offset(1, 0).Range("i4").Select
ActiveCell.Value = miu

Sheets("New_Sheet").Select
Range("a1: h1").Select
ActiveCell.Offset(1, 0).Range("e4").Select
ActiveCell.Value = ecn1

Sheets("New_Sheet").Select
Range("a1: h1").Select
ActiveCell.Offset(1, 0).Range("f4").Select
ActiveCell.Value = esn1

Sheets("New_Sheet").Select
Range("a1: h1").Select
ActiveCell.Offset(1, 0).Range("g4").Select
ActiveCell.Value = ksil

Sheets("New_Sheet").Select
Range("a1: h1").Select
ActiveCell.Offset(1, 0).Range("h4").Select
ActiveCell.Value = zetal

Sheets("New_Sheet").Select
Range("a1: h1").Select
ActiveCell.Offset(1, 0).Range("j4").Select
ActiveCell.Value = A_sf

Sheets("New_Sheet").Select
Range("a1: h1").Select
ActiveCell.Offset(1, 0).Range("k4").Select
ActiveCell.Value = Delta_M1

Sheets("New_Sheet").Select
Range("a1: h1").Select
ActiveCell.Offset(1, 0).Range("d4").Select
ActiveCell.Value = Mrdl

Sheets("New_Sheet").Select
Range("a1: h1").Select
ActiveCell.Offset(1, 0).Range("c4").Select
ActiveCell.Value = Medf

Sheets("New_Sheet").Select
Range("a1: h1").Select
ActiveCell.Offset(1, 0).Range("l4").Select
ActiveCell.Value = delta_epl

Sheets("New_Sheet").Select
Range("a1: h1").Select
ActiveCell.Offset(1, 0).Range("m4").Select
ActiveCell.Value = ep0

Sheets("New_Sheet").Select
Range("a1: h1").Select
ActiveCell.Offset(1, 0).Range("n4").Select
ActiveCell.Value = epl

Sheets("New_Sheet").Select
Range("a1: h1").Select
ActiveCell.Offset(1, 0).Range("b4").Select
ActiveCell.Value = b

Sheets("New_Sheet").Select
Range("a1: h1").Select
ActiveCell.Offset(1, 0).Range("a4").Select
ActiveCell.Value = miu_bal
```

1 = 1 + 1

```

Next
End If

'new sheet and plots

Range("b3:N103").Select
ActiveSheet.Shapes.AddChart2(240, xlXYScatterLines).Select
ActiveChart.SetSourceData Source:=Range("New_Sheet!$B$3:$N$103")

Range("b3:N103").Select
Selection.Borders(xlDiagonalDown).LineStyle = xlNone
Selection.Borders(xlDiagonalUp).LineStyle = xlNone
With Selection.Borders(xlEdgeLeft)
.LineStyle = xlContinuous
.ColorIndex = 0
.TintAndShade = 0
.Weight = xlThin
End With
With Selection.Borders(xlEdgeTop)
.LineStyle = xlContinuous
.ColorIndex = 0
.TintAndShade = 0
.Weight = xlThin
End With
With Selection.Borders(xlEdgeBottom)
.LineStyle = xlContinuous
.ColorIndex = 0
.TintAndShade = 0
.Weight = xlThin
End With
With Selection.Borders(xlEdgeRight)
.LineStyle = xlContinuous
.ColorIndex = 0
.TintAndShade = 0
.Weight = xlThin
End With
With Selection.Borders(xlInsideVertical)
.LineStyle = xlContinuous
.ColorIndex = 0
.TintAndShade = 0
.Weight = xlThin
End With
With Selection.Borders(xlInsideHorizontal)
.LineStyle = xlContinuous
.ColorIndex = 0
.TintAndShade = 0
.Weight = xlThin
End With
Range("b3:N3").Select
With Selection.Interior
.Pattern = xlSolid
.PatternColorIndex = xlAutomatic
.ThemeColor = xlThemeColorAccent6
.TintAndShade = 0.399975585192419
.PatternTintAndShade = 0
End With
Range("b3:N103").Select
With Selection
.HorizontalAlignment = xlCenter
.VerticalAlignment = xlBottom
.WrapText = False
.Orientation = 0
.AddIndent = False
.IndentLevel = 0
.ShrinkToFit = False
.ReadingOrder = xlContext
.MergeCells = False
End With
Range("E1").Select
End Sub

```