

How Progressive are Portugal's Taxes?*

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Abstract

In this work project, we use micro data from the Household Budget Survey wave of 2010 to measure the extent of progressivity of income and VAT taxes, together with social security contributions. We use the withholding tax rates to convert the net-at-source-income reported by the individuals in the survey into gross income, and then, through a careful implementation of the details of the income tax system, compute the net income. The identification of tax units and use of gross income allows for improvements on previous literature. We use the Reynolds-Smolensky and Kiefer indices to compute the progressivity of the several components of the tax system, i.e., *(i)* social security contributions; *(ii)* income taxes, including tax credits and deductions; *(iii)* withholding taxes, and, *(iv)* value added tax on final consumption goods and services. Behavioural responses to taxation are also inspected. Income tax is found to be progressive, with withheld taxation being more progressive than final taxes. Tax deductions are found to be more regressive than tax credits, and VAT is found to be regressive.

Keywords: Tax Progressivity, Public Finance, Portugal, Income taxes, VAT taxes

JEL Classification: H22, H23, H24, H31

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1 Introduction

While the level of the tax burden is issue of constant polemic, there is not one tax burden but a distribution of tax burdens across income and demographic characteristics. Stable or decreasing tax burdens for the whole may conceal increasing effective tax rates for a part of the population. The graduation of taxes across income and demographic characteristics is an essential feature of public finance policy design, with consequences across a range of objectives from equity to stabilization (Auerbach and Feenberg, 2000). Regardless then of whether policymakers wish to enhance or dampen progressivity, it is relevant to know to what extent a society's taxes are progressive.

Progressivity is generally defined as effective average tax rates increasing with income. Progressivity contributes to income redistribution and is usually sought by inequality-averse polities and policymakers.

This work project seeks to estimate the progressivity of the tax system in Portugal as it was in 2009. This section introduces the article. Subsequently, a framework of concepts and notation is defined. Section 3 reviews literature and examines the range of progressivity indices. Section 4 presents the portuguese tax system, facilitating a discussion on key issues in tax policy design. Section 5 introduces the dataset and presents our net-gross conversion model and tax unit identification strategy. Section 6 presents evidence of behavioural responses to taxation, namely bunching at kinks and notches. Finally, Section 7 presents results of our tax simulation model and computes progressivity indices for the tax system as a whole as well as individual taxes and a number of counterfactual scenarios. Section 8 concludes and presents limitations and areas for further research.

2 Framework

Individuals earn and spend income, y , which is distributed in the population according to a density function, $f(y)$ with the corresponding cumulative distribution function (CDF), $F(y)$.

Income distributions may also be expressed in terms of income shares, $\phi(y)$, with the respective cumulative version, $\Phi(y)$, as expressed in (1). The progression of $\Phi(y)$ on $F(y)$ is called the Lorenz curve, $L(F) = \Phi$. (Cowell: 20)

$$\Phi(y) = \frac{\int_0^y \phi(y) dy}{\int_0^\infty \phi(y) dy} = \frac{\int_0^y y \cdot f(y) dy}{\int_0^\infty y \cdot f(y) dy} \quad (1)$$

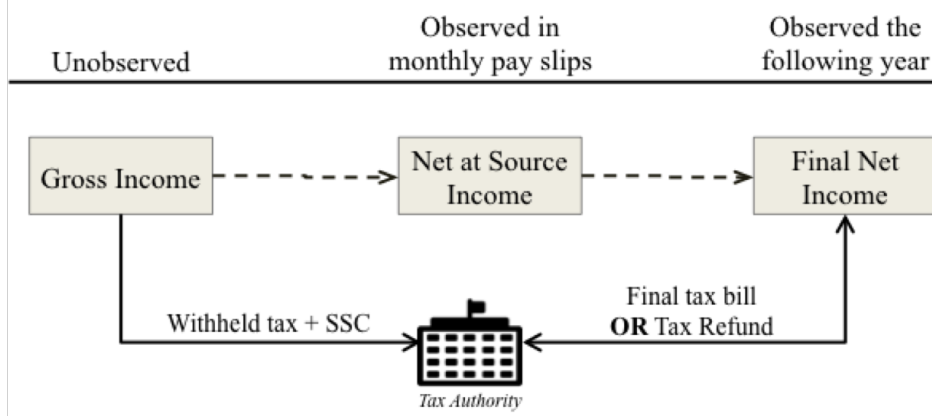
Gross income, y^g is before-tax income, while net-of-tax income is denoted, y^n . Households expenditure c is also taxed. Personal income tax, $t(y)$, as well as social security contributions, $s(y)$ are a function of income, while value-added tax is a function of expenditure, $t(c)$. There are also a range of excise taxes for different kind of goods, including petrol tax, alcohol tax, car tax, circulation tax and stamp duty.

In Portugal as in most OECD countries, personal income taxation takes place in two moments - first upon payment, i.e., with withholding taxes, and then by submitting a tax return in April/May of the following year, after which they may have to pay additional tax or receive a refund. This has become a common feature of tax systems around the world, with all OECD countries except Switzerland and France adopting the practice (OECD

2015).¹

Withheld tax creates an intermediate income level, termed ‘net-at-source income’ or y^s . The flow from gross income to net income through net-at-source income is described by Figure 1.

Figure 1: Timeline of disposable income and tax payments



The withheld tax function, $\tau(y; P, D)$, and the income tax liability function, $t(y; P, D)$, reflect that $t(y)$ and $\tau(y)$ vary not only by income, but also by demographic, D , and professional characteristics, P . The relation of net and net-at-source income to gross income are so defined:

$$y^n = y^g - t(y; P, D) - s(y), \quad \text{and} \quad y^s = y^g - \tau(y; P, D) - s(y) \quad (2)$$

For simplicity, we will refer to them hereafter as $t(y)$ and $\tau(y)$.

The sum of individual incomes, y_i , in a household, h , is the household income, y_h . Taxes are filed by tax unit, u . Tax units are subsets of households, and therefore tax unit income, y_u , is the sum of individual income in the tax unit.

We consider five categories, k , of income, y_k : employment, y_1 , self-employment, y_2 , pension, y_3 , capital, y_4 , and property income, y_5 . These correspond to the taxonomy in Portuguese tax law, with the exception of increases in wealth which is unavailable in data.

The tax liability function, $t(y)$, allows one to compute the marginal tax rate, $t'(y)$, as its derivative and the average tax rate, $\frac{t(y)}{y^g}$, as the ratio of tax liability on gross income. Progressivity occurs where average rates are increasing with income. An equivalent definition of progressivity is where marginal rates exceed average rates (Musgrave and Thin, 1948: 498).

When analysing VAT progressivity, it is relevant to pursue this analysis for consumption as well as income. Thus, the average rate of tax on consumption is the ratio between tax and consumption, $\frac{t(c)}{c}$.

Beyond setting tax rates and the income range over which they apply, policymakers also define tax expenditures. These can be either tax deductions, δ , which are subtracted from an individual’s gross income to form her adjusted gross income, y^a , or tax credits, χ , which are subtracted from an individual’s tax liability $t(y)$. Furthermore, couples who file their tax returns jointly benefit from a ‘couple quotient’, γ . In effect, this divides adjusted income by the number of income-earners in the tax unit, computes tax liability

¹France was planning to introduce a withholding tax system as of 1 January 2018. *Vide* EY, op. cit.

and multiplies it again by γ . Thus, setting γ as the size of the tax unit, we can obtain expressions for adjusted income and tax liability:

$$y^a = y^g - \delta, \quad \text{and} \quad t(y) = \gamma \times t\left(\frac{y^a}{\gamma}\right) - \chi \quad (3)$$

3 Literature Review: Measuring Progressivity

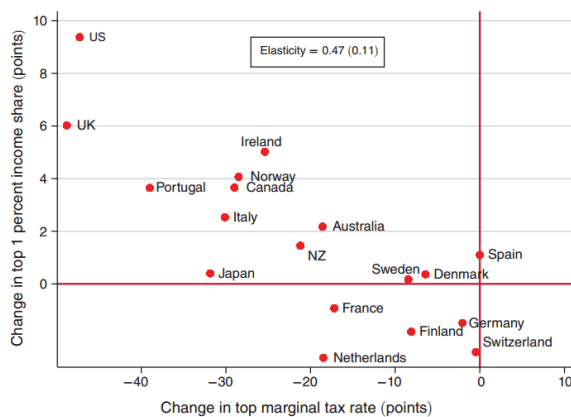
Tax progressivity is sought for both instrumental and intrinsic purposes - namely, reducing inequality and promoting tax fairness.

It is clear that taxes and transfers play an important role in dampening inequality. Joumard et al. (2012) estimate that in the late 2000s, taxes and transfers reduced about inequality by about 25%, with about a quarter of this directly attributable to taxes.

However, the past 30 years has witnessed large-scale fiscal transformation with many authors suggesting a decrease in progressivity, particularly in personal income taxes (PIT). Peter et al. (2009: 2) find that globally “the GDP-weighted average top statutory marginal PIT rate fell from 62% in 1981 to 42.9% in 1991 and by 2005 reached a twenty five year low of 36.4%.”

As Piketty et al. (2014: 253) demonstrate, Portugal was one of the countries who reduced the top marginal PIT rate the most as well as one of the countries where the top 1% income share increased the most. Piketty et al.’s chart demonstrating a strong correlation between reductions in the top marginal PIT rate and increases in the top 1%’s income share is reproduced in Figure 2 below .

Figure 2: Changes in Top Income Shares and Top Marginal Tax Rates



Source: Piketty et al., 2014: 254

Meanwhile, lower income marginal rates have not been reduced at a similar rate, resulting in a convergence of both marginal and average tax rates. One expression of this is the reduction of the number of tax brackets. 10 times fewer countries had more than 20 PIT brackets in 2001-2005 than in 1981-1985. This reduction in brackets has been a driver of income inequality. (Fitoussi and Saraceno 2010: 4-5)

The impact that less redistributive taxes have played in increasing inequality is well grounded in the literature. (Ballarino et al.: 44) There are a variety of ethical, economic

and other reasons why policymakers are averse to inequality and therefore seek tax progressivity. Inequality is negatively correlated with all forms of political, civic, social and cultural participation, as well as with levels of political legitimacy. (Van de Werfhorst et al., 2012) Top income shares have been found to be correlated with lower human well-being (Burkhauser et al., 2016), while the seminal *The Spirit Level* has documented exhaustively the correlations between income inequality and worse social outcomes (Wilkinson and Pickett, 2009) From an economic point of view, Robert Gordon calls rising inequality “the most important [headwind] quantitatively in holding down the growth of our future income” (Gordon 2012: 17).

The ‘ability-to-pay’ principle is the most common principle behind tax fairness and is verified by tax burden increasing with ability to pay (i.e. income). While observing the progression of the average tax rate allows one to conclude if a tax is progressive, it is not in itself a measure of progressivity. It may not be straightforward to order tax systems by progressivity or describe the extent of their progressivity. To do this, we need ‘a scalar numerical representation’ of progressivity. (Cowell: 7)

Indices are such a representation. Quantifying progressivity has the advantage of permitting comparisons across time, tax systems or tax system components.

This section will present the range of distributional progressivity indices proposed in the literature, as well as discuss literature estimating these indices. ²

3.1 Gini-based indices

The most common measure of inequality is the Gini index, $G(y)$. Gini compares an income distribution expressed through a Lorenz curve, $L(y)$ with the Lorenz curve that would obtain in perfect equality, E . As all individuals earn the same in perfect equality, income share is equal to population share, such that $\Phi(y) = F(y)$. Figure 3 shows the application of this for HES data.

G is the area (A) between L and E divided by the area beneath E (illustrated in Figure 3 by areas A and B). As both $\Phi(y)$ and $F(y)$ range from 0 to 1, the area beneath E is 0.5. Rearranging, one can obtain Gini only in terms of the distribution’s L curve:

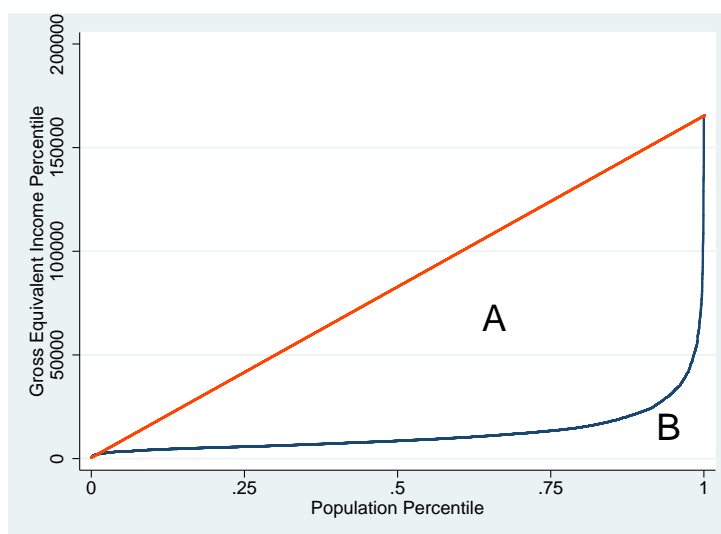
$$G(y) = \frac{\int_0^1 (E(y) - L(y))dy}{\int_0^1 E(y)dy} = \frac{A}{A + B} = 2A = 1 - 2B = 1 - 2 \int_0^1 L(y)dy \quad (4)$$

Additionally, some authors measure concentrations of tax burden in a similar way to Gini indices. Tax burden share, $\theta(y)$, is calculated by dividing tax liabilities, $t(y)$, by total tax liabilities ($\int_0^\infty t(y)$). The tax share CDF, $\Theta(y)$, can be plotted against $F(y)$ to form a Lorenz curve for tax shares, $T(y)$. It follows that the Gini index for taxes, $G(t(y))$, is defined as:

$$G(t(y)) = 1 - 2 \int_0^1 T(y)dy \quad (5)$$

²There are also structural progressivity indices. They differ from distributional indices by analysing statutory rather than effective rates. Peters et al. (2009) and Doerrenberg and Peichl (2014) are examples of papers using these kinds of index. However, we have opted for effective tax rates as these are a better indication of progressivity as effectively experienced by households, and as the dataset allows us to simulate tax expenditures which are not adequately encompassed by structural progressivity indices.

Figure 3: Lorenz Curve and Gini Index



Source: Authors' calculations. Annual household gross income calculated from HBS 2010

Table 1 lists the seven main Gini-based progressivity indices used in the literature. These are broadly distinguished between redistribution indices, focused on comparing net and gross Gini, and relative concentration indices, focused on comparing the concentration of income with the concentration of tax burden.

Redistribution indices “do not always offer consistent results” (Kiefer: 507). This is illustrated by a simulation of how different indices would be affected by changes in the tax system, income distribution or both. In doing so, he demonstrates the disparity of tax progressivity representations by different indices. In one example, “EP indicates that the tax system is responsible for a 7.4 percent increase in income inequality, whereas PO indicates a 13.5 percent decrease in inequality.” (ibid: 503)

This disparity must be met by the researcher with care and deliberation in the choice of the progressivity index with which to work. Each index is related to a different definition of progressivity, and it is not enough to pick one or more arbitrarily. (Kiefer, 1984)

Kiefer argues that “all three [redistribution indices] have a certain intuitive appeal”, but that Reynolds-Smolensky is “the most appropriate because it alone judges taxes and tax changes which make equal improvements in social welfare (...) to be equally progressive regardless of the distribution of pretax income.” (Kiefer: 509) This work project shall thus use the Reynolds-Smolensky index, following Braz and Correia da Cunha (2009) and Joumard et al. (2012).

As for relative concentration indices, Kiefer (1984) notes that they have the “convenient property that the measured progressivity of a group of taxes is equal to the weighted average of the progressivity indexes for the individual taxes, where the weights are the effective tax rates of the respective taxes.” (Kiefer: 499) However, this seems to ignore that individual taxes may not be equally progressive. For instance, VAT may be regressive and should thus have a negative index score, however by simply applying its weight, one would conclude that VAT is progressive.

The Suits index differs from the Kakwani and Khetan-Poddar indices in the sense that it integrates over income rather population. The difference between this approach and P

Table 1: Gini-based tax progressivity indices

Index	Formula	Source	Progressivity condition
Redistribution Indices			
Effective Progression (EP)	$\frac{1-G(y^n)}{1-G(y^g)}$	Musgrave and Thin, 1948	$EP > 1$
Pechman-Okner (PO)	$\frac{G(y^n)-G(y^g)}{G(y^g)}$	Pechman and Okner, 1974	$PO < 0$
Reynolds-Smolensky (RS)	$G(y^n) - G(y^g)$	Reynolds and Smolensky, 1977	$RS < 0$
Relative Concentration Indices			
Kakwani (P)	$G(t(y)) - G(y^g)$	Kakwani, 1977	$P > 0$
Khetan-Poddar (KP)	$\frac{1-G(y^g)}{1-G(t(y))}$	Khetan and Poddar, 1976	$KP > 1$
Suits (S)	$1 - 2 \int_0^\infty F(y)[t(y)]L'(y)$	Suits, 1977 ³	
Stroup (SH)	$\frac{G(t(y))-G(y)}{1-G(y)}$	Stroup, 2005	$SH > 0$

Source: Authors' construction, based on Kiefer (1984) and cited papers.

or KP is that it attributes weights to the difference between the concentration of taxes and gross income, with this weight increasing proportionately with income across the population. Thusly, the Suits index oversamples the wealthy because of their disproportionately large income shares. (Stroup and Hubbard, 2011: 20-22)

Relative concentration indices are insensitive to the average tax rate but sensitive to changes in income distribution even when the tax structure is unchanged. Stroup considers this 'useful' as the progressive rate structure will cause redistribution to be amplified by the increased inequality (Stroup 2005: 207). However, relative concentration indices "do not measure the effects of the tax system *on* income distribution." (Kiefer: 506) Their meaning is unclear beyond a "restatement of their definitions: they provide indications (...) of the cumulative distribution of taxes relative to the cumulative distribution of income." (ibid: 507)

In conclusion, it is preferable to use an index which directly measures the redistributive effect of taxes on income. Thus, the Reynolds-Smolensky index will be used. To ease presentation, and given Ginis are often presented as percentages, the index will be presented as percentage points (equivalent to multiplying the result by 100).

3.2 Equally-Distributed Equivalent Level of Income

While the most popular inequality index, the Gini index is not without its critics. Many have criticized the Gini index for its relative sensitivity to transfers at different income levels. The Gini index attaches more weight to changes in income distribution near the middle rather than at the tails of the distribution. (Cowell, 2011: 26; Kiefer, 1984: 500, Atkinson 1970: 256) As the late Atkinson puts it, "it is not clear that such a weighting

would necessarily accord with social values”.

Another criticism made of Gini is that it is not decomposable, in the sense that it is possible for Gini indices to register an increase in inequality in subgroups and a decrease in inequality overall. (Cowell: 64) These criticisms have motivated a debate on other measures of inequality. In this sub-section I will focus on Atkinson’s index.

Atkinson (1970) proposes a social welfare function (SWF) that is nondecreasing in incomes, symmetric, additive, strictly concave and with constant elasticity. In this social welfare function, if a person’s income increases by 1%, her welfare weight drops by $\varepsilon\%$ of its former value. ε is the SWF’s ‘inequality aversion’, indicating how strongly the society values equality.

Atkinson introduces the concept of “equally distributed equivalent” level of income, y_{ede} . This concept is the level of income that, distributed equally among individuals, would yield the same utility-level as the existing income distribution. For inequality-averse societies (with $\varepsilon > 0$), y_{ede} will be smaller than mean income, \bar{y} . Atkinson’s index, $I(\varepsilon, y)$, for a given ε , is defined by the difference from 1 of this quotient, representing how much less income would be necessary to maintain utility levels if income were equally distributed. This takes the following form, where y_x is the x’tth income level:

$$I(\varepsilon, y) = 1 - \frac{y_{ede}}{\bar{y}} = 1 - \left[\sum_x \left(\frac{y_x}{\bar{y}} \right)^{1-\varepsilon} f(y_x) \right]^{\frac{1}{1-\varepsilon}} \quad (6)$$

Societies with more inequality *ceteris paribus* have lower utilities and therefore would require an even smaller share of current income equally distributed to maintain utility.

Kiefer (1984) presents two progressivity indices based on Atkinson’s index. The first, $BD(\varepsilon), y$, developed by Blackorby and Donaldson (1983) takes the structure of EP in taking the quotient of net and gross income inequality. The second, $K(\varepsilon, y)$, which Kiefer proposes, follows the form of the Reynolds-Smolensky in taking the difference between net and gross income inequality. The forms of these are displayed in Equation 7 below.

$$BD(\varepsilon) = \frac{1 - I(\varepsilon, y^n)}{1 - I(\varepsilon, y^g)} - 1, \quad \text{and} \quad K(\varepsilon) = I(\varepsilon, y^g) - I(\varepsilon, y^n) = \left(\frac{y_{ede}^n}{\bar{y}^n} \right) - \left(\frac{y_{ede}^g}{\bar{y}^g} \right) \quad (7)$$

For the same reasons as *RS* was preferred to other Gini-based indices, so K will be used as the preferred y_{ede} based index. As I ranges from 0 to 1, it can be presented as a percentage, with K being presented as percentage points (equivalent to multiplying the result by 100). This is done in order to ease presentation.

4 The Portuguese tax system

This section will present the main features of the Portuguese tax system and use it to introduce key issues in tax policy design.

4.1 Personal Income Tax

The PIT schedule, described in Table 2, applies marginal tax rates over different intervals of the income scale, called tax brackets. The marginal tax rate, $t'(y)$, applies only to

income within such a bracket, with an average rate of tax being calculated for the lower bound of the tax bracket. The progressivity of the tax system on adjusted gross income is demonstrated by marginal taxes being greater than average taxes, $t'(y) > \frac{t(y)}{y}$.

Table 2: Personal income tax schedule in mainland Portugal, 2009

y^a	$t'(y)$	$\frac{t(y)}{y}$
$y^a \leq 4,755$	10.5%	–
$4,755 < y^a \leq 7,192$	13.0%	10.5%
$7,192 < y^a \leq 17,836$	23.5%	11.3%
$17,836 < y^a \leq 41,021$	34.0%	18.6%
$41,021 < y^a \leq 59,450$	36.5%	27.3%
$59,450 < y^a \leq 64,110$	40%	30.2%
$y^a > 64,110$	42%	30.9%

Source: Pricewaterhouse Coopers, Guia Fiscal 2009. All incomes in €.

Without tax brackets, marginal rates would apply over an individual’s entire income. Thus, if an individual were to increase her income from €4,755 to €4,756, their tax liability would increase from €499.28 to €618.28 leaving their net income reduced from €4,255.72 to €4137.72. This discontinuity in the net income function is called a ‘notch’ and represents a marginal rate above 100%. (Slemrod 2013: 259) The condition for a ‘notch’ is as follows:

$$\frac{dy^n}{dy^g} < 0 \Leftrightarrow \frac{d(y^g - t(y))}{dy^g} < 0 \Leftrightarrow 1 - t'(y) < 0 \Leftrightarrow t'y > 1 \quad (8)$$

Withholding tax for employment and pension income does not work with tax brackets, creating a notch at each new tax rate. Self-employment and property income face notches of their own on the €10,000 threshold for withheld tax exemption, after which withheld tax is applied over all income of that kind.

Notches introduce “an incentive for moving from a region above the cutoff to a point just below the cutoff, thereby creating a hole in the earnings distribution on the high-tax side and excess bunching in the earnings distribution on the low-tax side of the notch point.” (Kleven and Waseem, 2013: 670) While there is empirical evidence of bunching before the cutoff, strong frictions keep individuals from adjusting their labour supply optimally. (Kleven and Waseem, 2013: 722)

Withholding tax is a relevant feature in a tax system, constituting for some a pre-payment of their tax liability and for others an interest-free loan to the government. The effects of withholding tax have been widely studied, inducing reduced income tax consciousness (Van Wagstaff, 1965), reduced compliance (Shepanski and Shearer, 1995) and increasing consumption (Shapiro and Slemrod, 1995). Per the permanent income hypothesis, there should be no effect on consumption from a change in “the timing of cash flow with essentially no impact on lifetime resources” (ibid: 274). However, Shapiro and Slemrod find that consumers spend this transitory income.

Withheld tax rates differ according to kind of income, professional and demographic characteristics. For instance, withheld tax on employment income, $\tau(y_1)$, differed on 4 variables as well as income level - whether the individual was disabled, married, the single earner in the marriage, and how many dependent individuals were under his charge.

Table 3: Tax expenditures in Portugal, 2009

Tax Expenditure	Formula	Can estimate?
Tax Deductions, δ_k		
Social Security Contributions, δ_{11}	$\max\{s(y_{1i}), 3888\}$	Yes
Pay in lieu of notice to employer, δ_{12}	N/A	No
Union dues, $\delta_{13} = w$	$\max\{1.5w, 0.01y_i^g\}$	No
Insurance for exacting professions, δ_{14}	$\min\{\delta_{14}, 2096.10\}$	No
Simplified Accounting Deduction, δ_{21}	$\delta_{21} = 0.3y_{2i}^g$	Yes
Pension Allowance, δ_{31}	€6000 if $y_{3i} \leq \text{€}30,000$. Tapers off at €76.153, 85	Yes
House Upkeep Costs, δ_{41}	$\min\{\delta_{41}, y_{4i}\}$	Yes
Property Tax, δ_{42}	N/A	No
Tax Credits, χ		
Taxpayer Credit, χ_1	$\chi_1 = 247.5 \times \gamma$	Yes
Single Parent Family, χ_2	$\chi_2 = \text{€}360$ if single parent family	Yes
Dependent, χ_3	$\chi_3 = \text{€}180 \times \text{dependents}$	Yes
Dependent aged below 3, χ_4	$\chi_4 = \text{€}180 \times \text{dependentes aged below 3}$	Yes
Healthcare, χ_5	$\chi_5 = 0.3 \times c_{\text{healthcare}}$	Yes
Education, χ_6	$\chi_6 = \min\{0.3 \times c_{\text{education}}; 720 * n_h\}$	Yes
Mortgage, χ_7	$\chi_7 = \min\{0.3 \times c_{\text{mortgage}}; \text{depends on tax bracket}\}$	Yes
Old-Age Homes, χ_8	$\chi_8 = \min\{0.3 \times c_{\text{old-age homes}}; 382.50\}$	No

Source: Authors' construction, based on the PIT tax code applicable in 2009 and PwC's *Guia Fiscal*. Some formulae have been simplified for the sake of presentation.

Capital income is subject to different forms and rates of tax depending on their origin. While interest on capital injections, royalties, technical assistance and equipment rental is subject to retention taxes of 15% like property taxes; dividends, interest on deposits, bonds, repo transactions, and life insurance payouts are subject to a 'liberatory tax' of 20%. Earnings from participating in private equity funds or florest real estate investment funds pay a liberatory tax of 10%.

Liberatory taxes allows a tax unit to pay all the tax on a given income upon payment, exempting it from reporting capital income in its tax return. Capital income can be reported, in which case it works like a withholding tax. If an individual's marginal tax rate is below the liberatory tax rate, it is optimal for a tax unit to report this capital income as it will get a tax refund.

Tax progressivity analysis should look at how the tax system affects disposable income. Thus, estimates of progressivity should not merely reflect unobserved gross and net income but also the intermediate net-at-source position that holds between one's paycheck and tax return the following year. This is an innovation not found elsewhere in the literature.

Tax progressivity analysis should also concern itself with tax expenditures, which "tend to benefit the well-off" (Joumard et al., 2012: 6) Table 3 describes the main tax expenditures in Portugal in 2009, and whether they can be calculated using HBS data. In Portugal, the sum of tax credits cannot exceed the tax bill meaning that it can never be the case that $t(y^a) < \chi$.⁴

⁴The reader may recall that there are four kinds of taxable income considered in this article: 1 - employment; 2 - self-employment, 3 - pension income, 4 - property income, 5 - capital income.

4.2 Social Security Contributions

Social Security contributions (SSC) are not a tax, but it is customary to include them in tax progressivity analysis. (Piketty and Saez, 2006: 8).

In 2009, SSC for generally-employed individuals, $s(y_1^g)$, was 11%, with employed pensioners paying 7.8% if old-age pensioners and 8.4% if disability pensioners.

Self-employment incomes below 6 times the social benefits index, i.e. €2,515.32 are exempt from SSC. Additionally, if an individual earns over 12 times the social benefits index (or *indexante de apoios sociais* (IAS)) in employment income, or any amount in old age or disability pension, her self-employment income is exempt from SSC.⁵

In 2009, self-employed individuals could choose between two SSC rates (25.4% and 32% offering different levels of social protection) applied over a fixed tax base, defined as multiples of the IAS. An individual paid SSC on the tax base closest to her adjusted income, y_2^g , though she could request a change to neighbouring tax bases. This creates notches as within a tax base group, the marginal effective tax rate is 0% and increasing the tax base causes a discontinuity in the tax function. These fixed tax bases are described in Appendix 2.

Several other regimes existed for a range of different labour market situations. However, none of these can be applied on our dataset. Pension income, property income and capital income do not pay SSC.

4.3 Value-Added Tax

VAT is usually assumed to be regressive. As a consumption tax, the wealthy's lower marginal propensity to consume results in a lower lower effective VAT rate with respect to income.

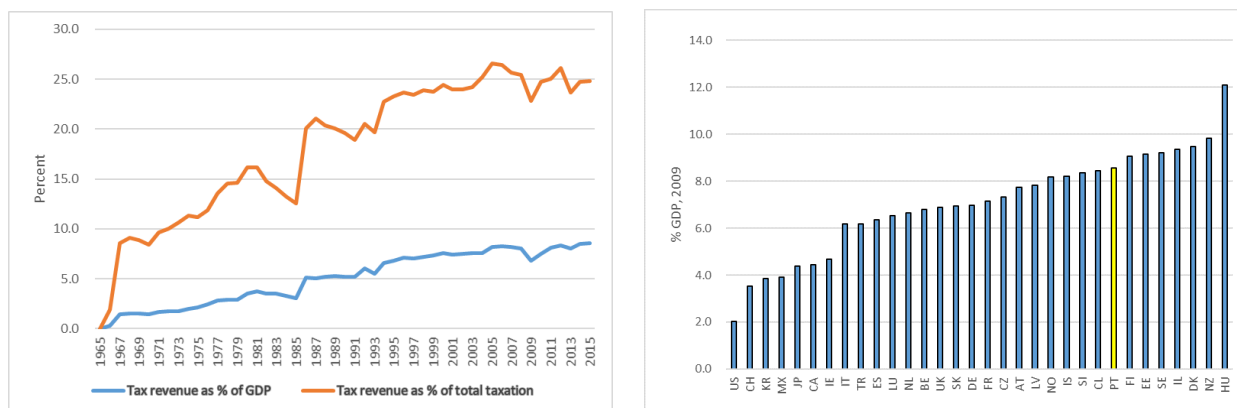
Policymakers have used reduced rates for 'essential goods' to attenuate VAT's regressivity. In 2009, the VAT schedule included reduced rates at 5% and 12%, and a standard rate of 20%.

However, this is not enough to counteract the tendency of the marginal propensity to consume to fall with income. O'Donoghue et al. (2004) compare VAT progressivity across many countries. The authors find that Portuguese VAT is the most regressive of 12 EU countries. The progressivity of VAT in Portugal was also studied by Braz and Correia da Cunha (2009), who show how VAT revenues have evolved over time and how they compare with other countries. Their graphs are reproduced and updated with the latest data for OECD countries in Figure 4 below.

Using the 2005 Household Budget Survey (HBS), Braz and Correia da Cunha (2009) compute effective VAT rates with respect to income and consumption and a Reynolds-Smolensky index for VAT. They find that while VAT is progressive on expenditure, declining average propensity to consume cause VAT to be regressive on income. Two previous studies on VAT progressivity in Portugal are also identified, one (Rodrigues et al. 2002) matching their findings and another (Albuquerque and Neves 1994) finding VAT to be progressive in income after excluding the first income quartile.

⁵The social benefits index is a threshold that is used by Social Security as a base with which to compute several low-income support transfers, as well as several tax system details, such as this one.

Figure 4: VAT revenues in Portugal: evolution and comparison with other countries



Source: OECD Statistics

5 Data and Methodology

Calculating tax progressivity indices requires data on the distribution of gross and net income. Martins (2016) calculated the progressivity of income tax using the few data points obtainable from official statistics. However, it is preferable to estimate indexes on microdata from a sample “large enough to permit estimation with small standard errors.” (Kiefer 1984: 501)

This work project employs the 2010 wave of the Household Budget Survey (HBS), known in Portuguese as the *Inquérito às Despesas das Famílias*. This survey includes information from 9,489 households and 24,383 individuals ranging from demographic and professional characteristics, income and expenditures.

The Household Budget Survey is a quinquennial survey, with 2010 being the latest available wave. Information is collected about all individuals in a household, with households being selected from randomly selected statistical areas belonging to a 2001 ‘Mother-Sample’. Information is collected through in-person interviews and, in the case of consumption data, by filing in survey papers. Information is recorded on consumption of 199 goods and services. Weights are estimated for each household using a corrected Horvitz-Thompson estimator.

The HBS reports all forms of monetary income as net-at-source (y^s). Evidence from SILC reveals that this is the type of income most individuals report when given the choice (Farinha Rodrigues, 2007). Both Braz and Correia da Cunha (2009) and Rodrigues et al. (2002) use these net at source incomes to perform their analysis. It is however “more suitable to use gross income for the calculation of the average rate” (Braz and Correia da Cunha: 78). Stroup too mentions that his index can be improved by using gross income instead of adjusted gross income. This work project will estimate gross income by creating a model to convert net-at-source income into gross income. In doing so, this work project goes a step further than these authors, therefore allowing for a better estimation of tax progressivity in all accounts.

When estimating household-level distributional indices such as $G(y)$ or $A(\varepsilon, y)$, it is necessary to equalise income to avoid the size of the household impacting its ranking on the income distribution. In Portugal, official Gini estimates use the OECD modified

Table 4: Household Budget Survey: descriptive statistics

	Sample		Population	
	n	%	n	%
Individuals	24,383	100%	10,561,614	100%
Female	12,738	52.24%	5,513,309	52.20%
Married	12,475	51.16%	5,218,318	49.41%
Lives with a spouse	13,224	54.32%	5,579,974	52.83%
Works	9,740	39.95%	4,488,548	42.5%
Households	9,489	100%	4,044,100	100%
Mainland Portugal	7,658	80.7%	3,869,537	95.68%
1 adult, 0 dependents	1,910	20.1%	862,257	21.3%
2 adults, 0 dependents	3,071	32.4%	1,172,390	29.0%
2 adults, 1 dependents	1,107	11.7%	537,056	13.3%
2 adults, 2 dependents	963	10.1%	400,547	9.9%
1 tax unit	7,073	74.54%	3,039,437	75.16%
2 tax units	1,904	20.07%	777,615	19.23%
> 2 tax units	489	5.15%	215,946	5.34%
Only employment income	3,206	33.8%	1,520,481	37.6%
Only pension income	3,011	31.7%	1,126,642	27.9%
Has self-employment income	1,307	13.8%	587,252	14.5%
Has capital or property income	1,139	12.0%	588,684	14.6%
\bar{y}_{1h}^s	€9,768.14	–	€11,377.66	–
\bar{y}_{2h}^s	€1,428.45	–	€1,593.27	–
\bar{y}_{3h}^s	€2,842.23	–	€2,710.56	–
\bar{y}_{4h}^s	€252.85	–	€322.06	–
\bar{y}_{5h}^s	€64.01	–	€86.75	–
\bar{c}_h^s	€18,492.87	–	€20,391.21	–

Source: HBS 2010

equivalence scale, which has also been applied in this work project. Furthermore, tax-exempt transfers must also be added to gross income before estimating distributional indices.

Table 4 describes key descriptive statistics about the HBS.

5.1 Net-Gross Conversion

Net incomes (y^n) are commonly converted into gross income (y^g) (and vice-versa) using microsimulation tools such as EUROMOD. However, Farinha Rodrigues (2007) demonstrates these are not accurate conversions of net-at-source income to gross income, providing an alternative methodology for this conversion involving a reconstruction of gross income from tax code information.⁶

Net-at-source income can be converted into gross income by calculating the applicable rates of SSC and withheld taxation for each individual (denoted here and hereafter as β)⁷

⁶As the reader may recall, there are five categories of income: employment (y_1), self-employment (y_2), pension (y_3), capital (y_4) and property income (y_5).

⁷All β notation shall have two sub-scripts, denoting kind of income (k) as defined above, and kind of

and dividing net-at-source income by the difference of these rates from 1. This produces the following system of equations:

$$y_1^s = y_1^g - s_1(y_1^g, y_3^g) - \tau_1(y_1^g) = y_1^s - \beta_{11}y_1^g - \beta_{12}y_1^gy_1^s \Leftrightarrow y_1^g = \frac{y_1^s}{(1 - \beta_{11} - \beta_{12})} \quad (9)$$

$$y_2^s = y_2^g - s_2(y_2^g) - \tau_2(y_2^g) = y_2^g - \beta_{21}y_2^g - \beta_{22}y_2^g \Leftrightarrow y_2^g = \frac{y_2^s}{(1 - \beta_{21} - \beta_{22})} \quad (10)$$

$$y_3^s = y_3^g - \tau_3(y_3^g) = y_3^g - \beta_{32}y_3^g \Leftrightarrow y_3^g = \frac{y_3^s}{(1 - \beta_{32})} \quad (11)$$

$$y_4^s = y_4^g - \tau_4(y_4^g) = y_4^g - \beta_{42}y_4^g \Leftrightarrow y_4^g = \frac{y_4^s}{(1 - \beta_{42})} \quad (12)$$

$$y_5^s = y_5^g - \tau(y_5^g) = y_5^g - \beta_{52}y_5^g \Leftrightarrow y_5^g = \frac{y_5^s}{(1 - \beta_{52})} \quad (13)$$

Employment Income

Employment income is earned and pays retention tax at an individual-level. Consequently, net-gross conversion must be operated on an individual level. As the income thresholds for progressing in the withheld tax rate are listed in gross terms, these too have to be converted to net income terms.

The 4 demographic characteristics and income levels yield 2,052 types of individuals, regarding employment and pension income alone. For instance, unmarried, employed individuals with no dependents face a withheld income tax schedule with 33 rates.

Per Portuguese law, dependents are either children, stepchildren or godchildren, both as minors and non-earnings young adults (up to age 26 earning below the minimum wage), as well as any individuals who are permanently incapacitated for work. We assume that all children in a household are dependents of a taxpayers in the household. There are no cases of permanently incapacitated people in the sample. Thus, the number of dependents can be adequately identified.

While SSC for employed individuals, $s(y_1^g)$, is generally 11%, employed pensioners face a reduced rate. Of the 6,511 pensioners in the sample, representing 2.4 million pensioners in the population, 281 worked with a weight of 4.6% on the pensioner population. This special case requires not only a change in the formula of translating net into gross income, but also calculating new net income thresholds for the withheld tax rate.

Self-employment income

Converting self-employment income is rather more complex. Self-employed individuals can opt for the organized accounting regimen or, if their income is below €150,000, the simplified regimen. Organized accounting works like normal business accounting, with net earnings being subject to taxation. The simplified regimen applies a tax deduction of 30% to services and 70% to goods. Available data does not allow us to distinguish those providing services from those selling goods, nor can the organized accounting regimen be applied. Thus, conversion will follow the simplified regimen for providing services.

taxation where 1 denotes the rate for social security contributions and 2 denotes retention tax.

Table 5: Conversion rules for self-employment income

Combines incomes?	$0 < y_2^g \leq 2,515.32$	$2,515.32 < y_2^g \leq 10,000$	$y_2^g > 10,000$
Yes (271 observations)	No SSC No VAT No withheld tax (88 observations)	No SSC No VAT No withheld tax (138 observations)	No SSC VAT Withheld tax (56 observations)
No (1226 observations)	$0 < y_2^s \leq 2,515.32$ No SSC No VAT No withheld tax (104 observations)	$2,515.32 < y_2^s \leq 10,000$ SSC No VAT No withheld tax (836 observations)	$y_2^s > 9000$ SSC VAT Withheld tax (634 observations)
	$0 < y_2^s \leq 2,515.32$	$1,876.43 < y_2^s \leq 8,083.33$	$y_2^s > 7,083.33$

Source: Authors' calculations. Annual individual income calculated from HBS 2010. All incomes stated in €'s

There are 271 individuals who accumulate enough of other kinds of income to have their self-employment income exempt from SSC. The minimum income threshold for self-employment SSC adds an additional 104 individuals who are exempt from social security contributions.

While individuals could choose the tax base and tax rate at which they would contribute to Social Security, information is not available on what people actually paid. One has then to assume that individuals paid the lower rate of contributions (25.4%) applicable in 2009, and did so against the the tax base associated with their income level. In the case of notches, I will assume that individuals chose to pay according to the lower tax base.

VAT and withheld tax is charged if self-employed incomes exceeds €10,000. Only 351 sampled individuals exceeded this threshold, weighing 176,526 individuals. Depending on one's occupation, VAT and withheld tax rates differ. For instance, while in 2009 the VAT rate and withheld tax rate for 'individual businessmen' was 20% and 10% respectively, 'liberal professionals' (lawyers, doctors, etc.) were exempt from VAT but faced a 20% withheld tax rate. These occupational differences cannot however be reliably identified through either the aggregate economic activity classifications (CAE) or the aggregate professional classifications (CNP) available in the dataset. Other special cases abound (members of governing bodies, royalties, technical assistance, etc.), however none of these cases can be differentiated using available data either. Consequently, all observations must be treated equally using a 20% VAT rate and 10% retention rate on incomes exceeding €10,000.

The six cases of net-gross conversion for self-employment income are summarized in Table 5.

Beyond the notches created by the fixed base SSC system, there are notches after the SSC exemption and after the VAT and withheld tax exemption. Notches imply an overlap between the net income range for each of the conversion cases presented above, making it possible for individuals within these net income ranges to fall within two distinct con-

version cases, causing ambiguity over how to convert these cases. These are summarized in Table 6 below.

Table 6: Notch-caused conversion ambiguities

Combines incomes $9000 < y_2^s \leq 10,000$	11 observations	0.03% of population	0.54% of self-employed
Does not combine incomes $7,083.33 < y_2^s \leq 8,083.33$	157 observations	0.61% of population	9.67% of self-employed
Does not combine incomes $1,876.43 < y_2^s \leq 2,515.32$	36 observations	0.12% of population	1.89% of self-employed
Total	204 observations	0.76% of population	12.10% of self-employed

Source: Authors' calculations. Annual individual income obtained from HBS 2010

Calibration to total income or mean income from administrative data could, in theory, resolve ambiguities. However, observed net-at-source self-employment income ($\sum_i y_{2i}^s$) is greater than gross self-employment income ($\sum_i y_{2i}^s$) in official statistics for both total (€6.49 billion in HBS v. €6.08 billion in *Autoridade Tributária*) and mean income (€9054 v. €6281). Conversion to gross income should increase total income, not reduce it. Consequently, no calibration is possible.

As individuals who report self-employment income have to fill in their self-employment income slip monthly, it is plausible that in this case, gross rather than net at source figures were given. This is so assumed.

Pension Income

A variety of pension income is available in the dataset, including old-age (both contributory and social), disability and survivor's pensions, as well as unemployment benefits, sickpay and education and training benefits such as scholarships. In order to distinguish between tax-paying and tax-exempt transfers, hereafter 'pensions' shall refer to tax-paying pensions while 'transfers' shall refer to tax-exempt benefits.

Property Income

According to article 8 of the Personal Income Tax Code, property income is defined as the rents received or made available to property-owners. According to Decree-Law 42/91, the retention tax on property income in 2009 was 15%.

As discussed above, the retention tax threshold creates a notch and notches create ambiguities in how to convert data. Furthermore, disparity between administrative and survey data do not allow for calibration in this case either. While tax authorities report €2.9 billion in property income, only €1.30 billion is reported in the survey. Oversampling at the top might approximate survey information to reality. As a means of approximating property income to administrative figures, all net-at-source income above €8,500 will be considered as equivalent to a gross income of €10,000.

Capital Income

Regardless of whether individuals include capital income in their tax return or settle for the liberatory tax, there is an immediate retention of capital income. However, without further information, one cannot settle on the applicable rate, $\tau'(y_4)$.

Capital income is only available in aggregate terms, making it impossible to convert each form of capital income in such a way as to calculate the specific rate for the specific kind of capital income, or even settle for a modal kind of capital income and its respective rate.

Calibration is once again implausible due to the large disparity between administrative and survey-reported data. While the survey reports €351 million, tax returns in 2009 reported only €78 million in capital income. The reason for such a dramatic over-reporting is because individuals need not report their capital income in their annual tax returns, as explained above. Furthermore, no administrative information was found of which was the modal category of capital income.

The PIT schedule for mainland Portugal has no rate below 10% in 2009, meaning a liberatory tax of 10% is always preferable to including capital income in one's tax return. When the liberatory tax rate is 20%, it would be rational for taxpayers with adjusted incomes below €7,192 to report their capital income. Assuming $\tau'(y_4) = 20\%$, with the possibility of incorporating capital income entirely in PIT returns, offers the best scope for analysing the effects of this feature of the tax system.

5.2 Tax Unit

Income in HBS is available in either household or individual terms. However, individuals can file tax returns in tax units, with some households containing several tax units.

It is necessary to correctly identify each tax unit in order to accurately calculate tax liability. This problem is also identified by Rodrigues et al. in a similarly-minded analysis of SILC data (Rodrigues et al., 2012: 172) Treating households as tax units would overestimate adjusted gross income, accessing higher levels of marginal tax and thus overestimating tax liability as well.

It is not sufficient to just match married couples. Since 2000, spouses in civil partnerships may file taxes jointly in Portugal. Data only shows, however, if individuals live with a spouse. While we cannot know whether they are in civil partnerships and eligible for filing jointly, all couples so identified in the HBS are assumed to file jointly.

While individual-level incomes can be added for all individuals in a tax unit, property and capital income are only available at a household-level. As no information allows us to know which individual brought this income to the household, property and capital income were allocated to individuals in the household proportionally to their respective employment, self-employment and pension income on total household income, ρ_i :

$$\rho_i = \frac{y_{1i}^g + y_{2i}^g + y_{3i}^g}{\sum_h y_{1i}^g + y_{2i}^g + y_{3i}^g} \quad (14)$$

We then impute individual capital and property income, y_{4i}^g , and y_{5i}^g , respectively, as $y_{ki}^g = \rho_i y_{k,h}^g$, $k = 4, 5$. Finally, we compute the tax unit's capital and property income as the sum of the imputed values for individuals in that tax unit.

A similar problem could occur with tax credits (χ), which are calculated on a number of characteristics which are only available at a household level such as number of dependents or consumption of specific goods. While the income share method could be used to impute these values for each individual and then aggregating to tax units, this is not necessary unless tax-unit level distributional analyses are desired. As $t(y)$ will be aggregated per household in any case, we need only household-level χ . Care must be taken, however, to adequately calculate limits as tax credit limits apply per tax unit and not per household.

6 Behavioural Responses to Taxation

Literature and theory both suggest that agents choose their labour supply bearing in mind the net income function, such that the marginal disutility of work equals marginal utility of net income. (Saez 2009) The behavioural response to the tax function can be verified by evidence of bunching (Saez, 2009: 1).

Bunching is evidence that taxes are distorting an individual's optimal labour supply. This is not only a static effect, as labour market agents will consider notches in negotiating pay rises. Like all distortions, this implies an efficiency loss which will be shared by workers, employers and, insofar as this efficiency loss may affect the price of goods and services, consumers.

6.1 Bunching at Notches

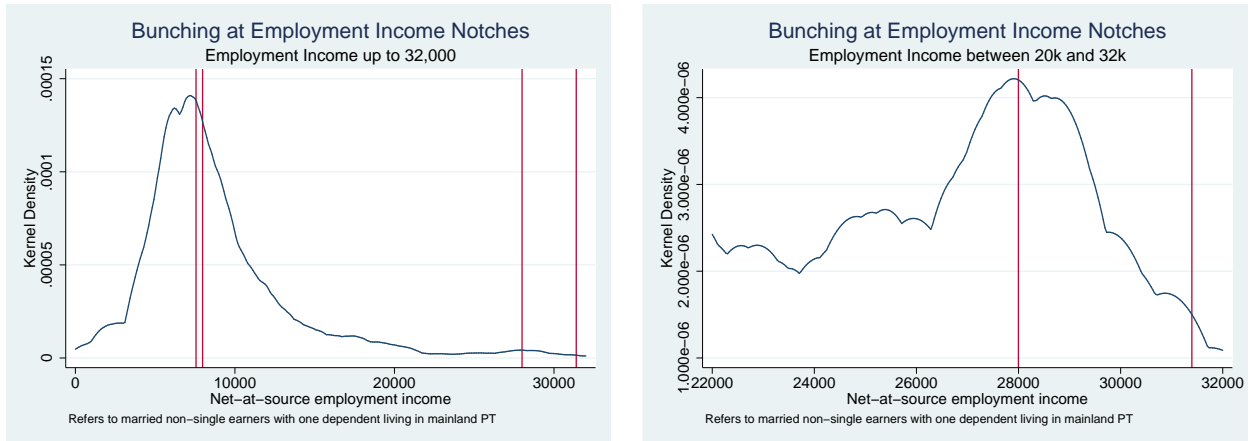
While most literature focuses on responses to kinks in the net income function, our review above of the Portuguese tax system in 2009 identified a variety of notches. These include:

- **Employment and pension income:** retention tax rates are applied as average rather than marginal taxes.
- **Self-employment income:**
 1. Exemption of VAT and retention tax on gross incomes below €10,000 ($y_2^g > 10,000$);
 2. Exemption on Social Security taxes for incomes below 50% of the Social Benefits Index ($y_2^g > 2,515.32$);
 3. Social Security fixed tax base system creating discontinuous tax 'plateaus' with 0% effective tax rate and discontinuities ($s'(y) > 1$) when crossing plateaus.
- **Property income:** Application of retention tax on income above €10,000.

Evidence of bunching is demonstrated graphically by inspecting kernel density histograms for humps. Bunching is also verified by estimating intensive elasticities of income with respect to the marginal tax rate.

However, measurement error in survey data and imputed data as resulting from the net-gross conversion model produces inconsistent estimates of bunching. (Hausman 1982) Furthermore, bunching is imprecise as agents do not fully control their incomes. In this case, we should expect bunching to happen not exactly before notches, but around them. (Saez 2009) For this reason, only graphical evidence of bunching around notches will be produced.

Figure 5: Bunching at notches in employment income



Source: Authors' calculations. Annual individual income obtained from HBS 2010

Bunching behaviour is only clearly identified using gross or adjusted income. However, all of the identified notches take place at withholding at source. This caused ambiguities in converting observed net-at-source incomes to unobserved gross income, and does not reveal the distribution of gross income around notches. Thus, all graphical evidence refers to net-at-source income, with notch areas marked by pairs of red lines.

Employment income

As different rates and therefore different notches are present depending on the various demographic characteristics, it is difficult to present notches for all of the over 2000 typologies of individuals. However, pensioners cannot adjust their labour supply to affect their pension income. Thus, we have opted to do so for the most frequent case in the employment income distribution: an individual, who is married, is not the single earner and lives in mainland Portugal with one dependent child.

Inspection of each of the 31 notch areas in employment income leaves only two as striking more substantial disruption in the density function. These are presented in Figure 5. Bunching is particularly significant around the first withheld tax notch where withheld tax is minimized, in line with Saez (2009).

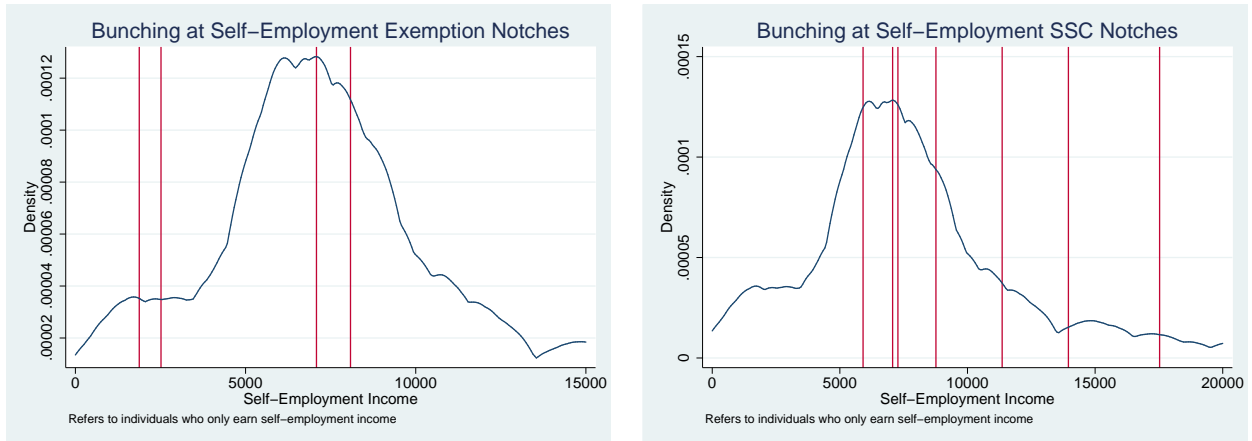
Self-employment income

Figure 6 shows the distribution of net-at-source self-employment income with notch areas marked by pairs of red lines. Some bunching does appear to exist before both exemption notches, while evidence is far less clear with the Social Security fixed tax base system, partly due to the width of notch areas.

Property income

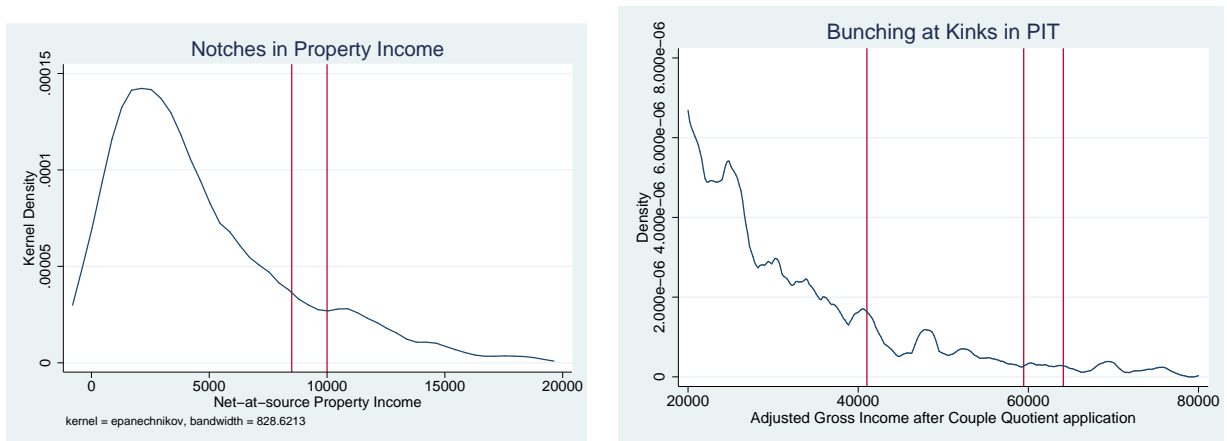
Property income above €10,000 pays retention tax of 15% which implies a net-at-source income of €8,500. Therefore, those receiving gross property incomes between €8500 and

Figure 6: Bunching at notches in self-employment income



Source: Authors' calculations. Annual individual income obtained from HBS 2010

Figure 7: Bunching at notches in property income and kinks in PIT



Source: Authors' calculations. Annual household income and tax unit adjusted income obtained from HBS 2010

10,000 will be in the same net-at-source density bin ($8500 < y_4^n \leq 10000$) as those receiving between €10,000 and €11,765.

The net-at-source distribution is shown in Figure 7. No bunching is evident, partly because the distribution appears to reach its maximum at much smaller property income.

6.2 Bunching at Kinks

It is also possible to view if there is bunching at kinks in the net income function, in line with Saez (2009). Kinks in the net income function represent changes in the marginal tax rate. Marginal tax rates apply over adjusted gross income, with respective application of couple quotient. Therefore, in order to visualize bunching at kinks, it is necessary to inspect kernel density graphs for adjusted gross income. Red lines were added at income levels where marginal tax rate progresses.

There is little evidence of bunching at kinks. In effect, the density reaches a peak at

€2825. While this may be construed as evidence of bunching, it is more likely to be a reflection of labour market structure. A small hump is visible in the adjusted gross income distribution just before the €41,021 threshold, as shown in Figure 7. This result, while weak, is robust to different income specifications (i.e. without tax deductions, without couple quotient)

7 Results

The successful conversion of net-at-source incomes to gross incomes constitutes *per se* an improvement from the literature. By using gross income rather than net-at-source income (as Braz and Correia did) or adjusted income (as used by Stroup), the effect of the retention system and of tax deductions can be incorporated into progressivity measures. This model also represents an improvement on Rodrigues et al. (2012) in terms of the identifying tax units. Comparisons between our estimates and tax authority statistics are included in Table 7 below.

Table 7: Comparison of Estimation Results with Tax Authority Statistics

Statistic	Estimated Results	Tax Authority Statistics	Comparison
u with $y^g > 0$	4,778,780	4,654,114	+2.68%
u with $y^a > 0$	3,926,367	4,159,754	-5.61%
u with $t(y) > 0$	1,445,797	2,007,821	-28.0%
% of u with $y^g > 0 \wedge t(y) > 0$	30.3%	43.1%	-12.9pp
$\sum y^g$	8.97×10^{10}	8.51×10^{10}	+5.41%
$\sum y_1^g$	6.00×10^{10}	5.68×10^{10}	+5.60%
$\sum y_2^g$	6.49×10^9	6.08×10^9	+6.69%
$\sum y_3^g$	2.14×10^{10}	1.85×10^{10}	+15.5%
$\sum y_4^g$	1.42×10^9	2.92×10^9	-51.3%
$\sum y_5^g$	4.34×10^8	7.8×10^7	+456%
$\sum \chi$	4.20×10^9	3.74×10^9	+12, 3%
$\sum \chi(\text{dependents})$	5.41×10^8	3.40×10^8	+59.1%
$\sum \chi(\text{healthcare})$	1.44×10^9	6.59×10^8	+119%
$\sum \chi(\text{mortgage})$	3.79×10^8	5.62×10^8	-32.6%
$\sum \chi(\text{education})$	3.04×10^9	2.99×10^8	+917%

Source: Annual population income. Authors' calculations using HBS 2010 and income tax statistics from *Autoridade Tributária*

Our tax simulation model appears to more accurately simulate the tax system than that presented in Rodrigues et al. (2012). Rodrigues et al. estimate 75.6% of households paid PIT in 2009 while administrative data for 2009 state that only 43.1% of fiscal units pay income tax. Our model compares favourably with this, as shown in Table 7, estimating 34.4% households and 27.3% of fiscal units paying income tax. While one would expect this to be due to traditional underreporting of income in surveys (Moore and Welniak 2000), Table 7 seems to suggest that overestimation of tax credits could also play a large role. Another reason for our model providing a smaller percentage of tax-paying tax units is the lack of oversampling at the top.

The reader will recall both Gini indices and Atkinson indices are calculated from equivalised income.

Personal income tax and social security taxes are pre-paid through retention tax, resulting in net-at-source income (y^s), and subsequently paid in full, resulting in net income (y^n). These are, therefore, two distinct distributions allowing for two distinct progressivity measures.

$$RS_{y^s} = G(y^s) - G(y^g), \quad \text{and} \quad K_{y^s}(\varepsilon) = I(\varepsilon, y^g) - I(\varepsilon, y^s) \quad (15)$$

The Gini index for net equivalised income ($G(y^n)$) was estimated at 0.369, which is slightly above INE's 'official' 0.337 obtained using SILC data.

It is estimated that, in 2009, RS_{y^n} was -4.80pp, while RS_{y^s} was -5.20pp, indicating a progressive tax system in both tax collection moments. This is also similar to the progressivity estimates obtained by Rodrigues et al. (2012) who estimate a RS index of -4.2. It is not entirely clear whether the authors' 'net income' is referring to final net income (y^n) or net-at-source income (y^s).

By isolating the effects of a tax on the income distribution, it is possible to estimate indices for individual taxes or various tax scenarios. In each case, the Gini/Atkinson index of gross equivalent income is subtracted from the Gini/Atkinson index of income net of such taxes. It is from this baseline distribution that tax liabilities must be subtracted in order to measure the specific redistributive effects of a tax.

7.1 Personal Income Tax

This section will demonstrate the measures of progressivity of the personal income tax in Portugal in 2009, as well as many features of this tax. The estimated progressivity indices are displayed in Table 8. The reader will recall that the progressivity conditions for the Reynolds-Smolensky and Kiefer indices are $RS < 0$ and $K > 0$.

Just as above net-at-source and net were distinguished, so can the PIT-related part of each of these have their progressivity estimated. Gross income subtracted only of withheld income tax and final income tax will be denoted by y^τ and $y^{t(y)}$.

$$y^s = y^g - \tau(y) - s(y), \quad \text{and} \quad y^n = y^g - t(y) - s(y) \quad (16)$$

$$y^\tau = y^s + s(y) = y^g - \tau(y), \quad \text{and} \quad y^{t(y)} = y^n + s(y) = y^g - t(y) \quad (17)$$

$$RS_{\tau(y)} = G(y^\tau) - G(y^g), \quad \text{and} \quad RS_{t(y)} = G(y^{t(y)}) - G(y^g) \quad (18)$$

A first inspection at the progression of the effective tax rate curves, in Figure 8 below, shows that these taxes are progressive, but does not permit comparison between two very similar curves.

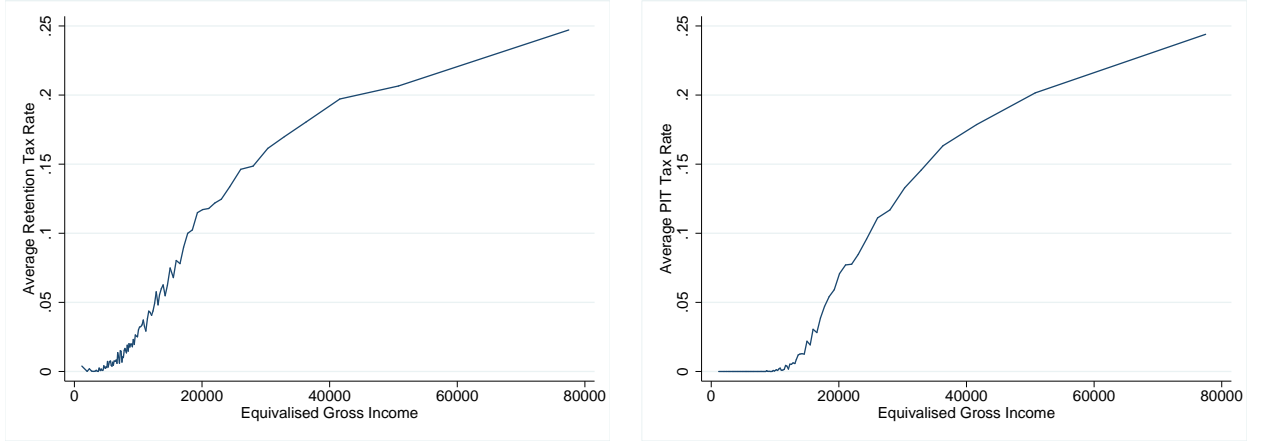
Tax liability and thus net income are affected by not only marginal taxes ($t'(y)$) but also tax deductions (δ_i) and tax credits (χ). These carry a distribution of tax savings which can be measured in order to investigate the progressivity of the tax expenditure system. Thus, a range of progressivity indices can be calculated.

Income net of personal income taxes is defined thusly:

$$y^{t(y)} = y^g - t(y) = y^g - [t(y^a) - \chi] = y^g + \chi - t(y^g - \delta) \quad (19)$$

Thus, if tax deductions and/or tax credits were not to exist, net income would be:

Figure 8: Progression of Effective Retention Tax Rates and PIT Tax Rates



Source: Authors' calculations, using HBS 2010. Annual household income, equivalised using the OECD modified scale.

$$y_{\delta=0}^{t(y)} = y^g + \chi - t(y^g), \quad \text{and} \quad y_{\chi=0}^{t(y)} = y^n - \chi, \quad \text{and} \quad y_{\delta+\chi=0}^{t(y)} = y^g - t(y^g) \quad (20)$$

The progressivity of tax deductions or tax expenditures should be understood as their contribution to PIT's progressivity index.⁸ This contribution can be estimated as difference of progressivity indices for the tax system with these features and for the tax system without these features. This simplifies to the gain/loss in inequality from abolishing these features, assuming no behavioural response to such tax reform. For instance, for tax deductions, the Reynolds-Smolensky index would have the following form:

$$RS_{\delta} = RS_{t(y);\delta} - RS_{t(y);\delta=0} = G(y^{t(y)}) - G(y^g) - [G(y_{\delta=0}^{t(y)}) - G(y^g)] = G(y^{t(y)}) - G(y_{\delta=0}^{t(y)}) \quad (21)$$

Table 8 presents the estimated progressivity indices for each of these features of personal income taxation.

7.2 Social Security Contributions

Social Security contributions (SSC), unlike PIT, is collected in one moment for each kind of income. Therefore, this allows us to estimate only one at-large value for SSC progressivity, which is found to be progressive in all progressivity index specifications.

Social Security in employment is proportional, with the exception being a lower rate for pensioners. The self-employed, however, face a notch-riddled fixed tax base structure, as explained above. One should expect this fixed tax base structure to be worse than a normal notch as not only does the implicit tax rate exceed 1 at the threshold ($s'(y) > 1$) but the implicit tax rate within the relevant income intervals is 0.

In order to obtain an estimate of the progressivity of this feature, one has to compare the existing system with a counterfactual system ($s_2(y) = \sigma_2(y)$). For simplicity, let

⁸Pfähler (1990) does this for the Kakwani index. His methodology cannot be emulated due to the significant differences between the Kakwani index and the Reynolds-Smolensky index.

this system be termed $\sigma(y)$ for all kinds of income. The best counterfactual is that self-employment SSC were to follow the same structure as SSC for employment, that is a constant marginal tax rate across the entire income range. Thus, the RS index for this case is defined by the following expression:

$$RS_{s_2(y)} = RS_{s(y)} - RS_{\sigma(y)} = G(y^g - s(y)) - G(y^g - \sigma(y)) \quad (22)$$

The estimated progressivity of SSC and of the fixed tax base system is presented in Table 8. The progressivity index for the latter is not significantly different from zero, indicating no effect on progressivity.

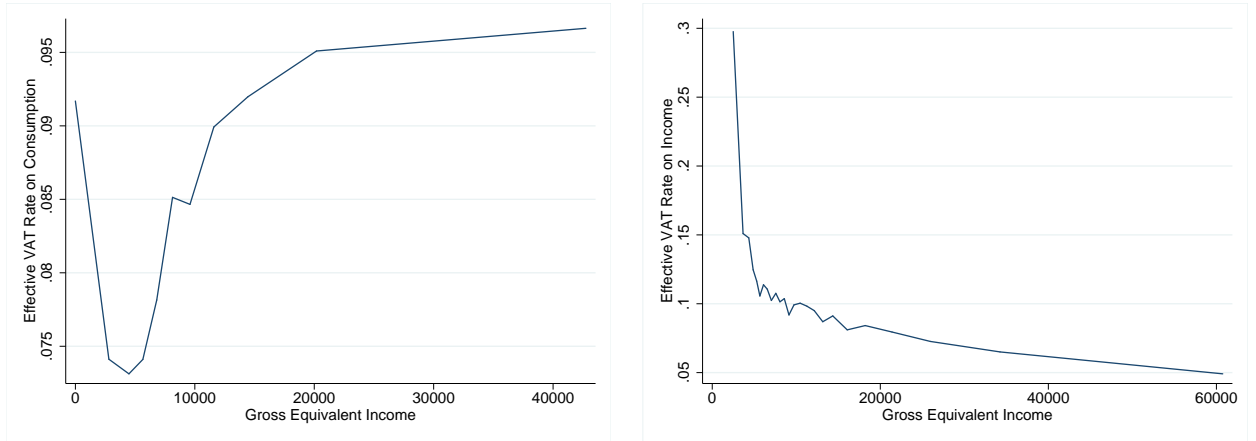
7.3 Value-Added Tax

In order to estimate the progressivity of VAT, it is necessary to attribute VAT rates to goods and services. While the extent of aggregation does not allow us to perfectly discriminate VAT rates on each good, it allows for significant detail.

Calculating progressivity for VAT assumes that its incidence falls entirely on consumers. While legal incidence of VAT is with businesses, theory suggests that economic incidence lies proportionally on the rigid side of the market. However, for the sake of this progressivity exercise, we will assume that VAT's incidence falls entirely on the consumer.

The progressivity of value-added tax is estimated in the literature over income and over consumption. The literature considers the progression of the average rate of VAT over income and over expenditure. This is illustrated in Figure 9 below.

Figure 9: Progression of Effective VAT tax rates on consumption and income



Source: Authors' calculations, using HBS 2010. Annual household income, equivalised using the OECD modified scale.

Visual inspection suggests that VAT is mostly progressive with regards to consumption, and generally regressive with regards to income. Inspection of the average propensity to consume, illustrated in Appendix 4, one witnesses that lower-income households consume a far greater proportion of their income (often above 100%) than their high-income counterparts.

The progressivity indices for VAT can be calculated by the same mechanism as for other simulations above. The regressivity of VAT is equal to the inequality loss that would result from VAT being abolished. The results for this is included in Table 8.

Table 8: Progressivity Indices Results

	<i>RS</i>	<i>K</i> (0.5)	<i>K</i> (1)	<i>K</i> (2)
Overall retention tax, y^s	-5.20 (0.15)	3.28 (0.33)	5.25 (0.39)	6.79 (0.00)
Overall tax, y^n	-4.80 (0.17)	3.16 (0.36)	4.88 (0.41)	5.93 (0.00)
Withheld income tax, $\tau(y)$	-4.23 (0.11)	2.75 (0.26)	4.27 (0.30)	5.11 (0.00)
Income tax, $t(y)$	-3.86 (0.13)	2.63 (0.29)	3.89 (0.33)	4.28 (0.00)
Tax deductions, δ	1.31 (0.02)	-0.71 (0.03)	-1.26 (0.06)	-1.83 (0.00)
Tax credits, χ	0.40 (0.03)	-0.16 (0.04)	-0.45 (0.05)	-1.02 (0.00)
Tax expenditures, $\delta + \chi$	0.13 (0.04)	0.00 (0.06)	-0.21 (0.08)	-0.87 (0.00)
SSC, $s(y)$	-0.57 (0.06)	0.32 (0.09)	0.65 (0.12)	1.20 (0.00)
Fixed Tax Base SSC, $\sigma(y)$	0.02 (0.02)	-0.02 (0.03)	-0.01 (0.04)	0.05 (0.00)
VAT, $t(c)$	0.85 (0.05)	0.56 (0.07)	1.06 (0.10)	2.47 (0.00)

Source: Authors' calculations from HBS data from annual equivalised income, using Distributional Analysis Stata Package (DASP). Standard errors are provided in parentheses.

8 Conclusion

This work project sought to find out how progressive Portugal's taxes were. Using data from the 2010 Household Budget Survey, it converted net-at-source income to gross income, identified tax units and calculated their respective tax liability. The Reynolds-Smolensky index and Kiefer indices were computed to measure progressivity not only of the tax system as a whole, but of individual taxes and even particular features of these.

This work project improves on a number of other progressivity estimates. Building a tailor-made net-gross conversion is an improvement on microsimulation tools such as EUROMOD or the use of adjusted gross or net-at-source income. While not without its flaws, the tax simulation model performs better against the benchmark of administrative data than other models for Portugal 2009. Finally, progressivity is analysed not only with regards to final tax payments but also with regards to retention taxes.

Portugal's tax system was found to be generally progressive. We found less inequality in net-at-source income than in net income. Tax deductions and tax credits were found to

be regressive, with tax deductions being more regressive than tax credits. Social security contributions were found to be progressive. Lastly, VAT was found to be regressive on income, while generally progressive on consumption. These findings are robust to different progressivity indices.

Behavioural response to taxes was also assessed through graphical inspection of y^s densities. Three cases of bunching were identified: (i) at the first $\tau'(y_1)$ threshold, (ii) at the €28,000 $\tau'(y_1)$ threshold, and (iii) at the $\tau'(y_2)$ exemption threshold.

Limitations and Further Research

One way to improve this research would be to implement it across years or tax jurisdictions (i.e. countries). This would provide a means of comparison and greatly enrich the interpretation of my findings. Another area for further research is implementing decomposition of the progressivity of different taxes into vertical redistribution (V), horizontal inequality or unequal treatment of equals (H) and reranking effects (R), as proposed by Aronson et al. (1994) and implemented by Braz and Correia da Cunha (2009) for VAT.

A third limitation of this work project is the quality of the dataset. While a more detailed survey might arguably provide further information, a better route would be administrative data. A number of countries provide administrative data on taxation which provide more accurate and detailed information on the income, tax liabilities and consumption of tax units. Furthermore, the Portuguese government does already provide administrative data on labour market (*Quadros de Pessoal*) and firm accounts (*Sistema Nacional de Contas Integradas* and *Informação Empresarial Simplificada*) to protocolled researchers.

Administrative data is particularly essential to extend this research to wealth and property taxes. Absence of these taxes is a fourth and major limitation to this article. Piketty and Saez note how taking these taxes into account changed the interpretation of the evolution of tax progressivity in France, United Kingdom and United States. (Piketty and Saez, 2006: 21) Other survey data (i.e. Household Finance and Consumption Survey [ISFF]) do not provide necessary information for the estimation of these taxes' progressivity, namely property valuation for tax purposes rather than market valuation.

References

- [1] Atkinson, Anthony B. "On the measurement of inequality." *Journal of economic theory* 2.3 (1970): 244-263.
- [2] Auerbach, Alan J., and Daniel Feenberg. "The significance of federal taxes as automatic stabilizers." No. w7662. National bureau of economic research, 2000.
- [3] Ballarino, G., et al. "Drivers of growing inequality, AIAS, GINI Intermediate Work Package 3 Report." Online: www.gini-research.org (2012).
- [4] Braz, Cláudia, and J. Correia da Cunha. "The redistributive effects of VAT in Portugal." *Economic Bulletin* (2009): 71-86.
- [5] Burkhauser, Richard V., Jan-Emmanuel De Neve, and Nattavudh Powdthavee. "Top Incomes and Human Well-being around the World." Discussion Paper No. 9677, IZA (2016).

- [6] Cabrita, Jorge, Perista, Heloísa, Rego, Raquel & Reinhard Naumann. “Portugal: Self-employed workers.” EurWORK (2009). Web. Accessed at <https://www.eurofound.europa.eu/observatories/eurwork/comparative-information/national-contributions/portugal/portugal-self-employed-workers> on 2nd May 2017.
- [7] Cowell, Frank. “Measuring inequality”. Oxford University Press, 2011.
- [8] Doerrenberg, Philipp, and Andreas Peichl. “The impact of redistributive policies on inequality in OECD countries.” *Applied Economics* 46.17 (2014): 2066-2086.
- [9] European Commission. “VAT rates applied in the Member States of the European Union”. European Commission. 2017. Web.
- [10] EY. “French Government Confirms Implementation of Withholding Tax from 2018.” EY. EY, n.d. Web. 22 Apr. 2017.
- [11] Fitoussi J.P., Saraceno F., (2010). “Inequality and macroeconomic performance”. Document de travail de l’OFCE No.13.
- [12] Gordon, Robert J. “Is US economic growth over? Faltering innovation confronts the six headwinds”. No. w18315. National Bureau of Economic Research, 2012.
- [13] Hausman, Jerry A. “Stochastic problems in the simulation of labor supply.” *Behavioral simulation methods in tax policy analysis*. University of Chicago Press, 1983. 47-82.
- [14] Joumard, I., M. Pisu and D. Bloch (2012), “Less Income Inequality and More Growth ? Are They Compatible? Part 3. Income Redistribution via Taxes and Transfers Across OECD Countries?”, OECD Economics Department Working Papers, No. 926, OECD Publishing.
- [15] Kiefer, Donald W. “Distributional tax progressivity indexes.” *National Tax Journal* (1984): 497-513.
- [16] Kleven, Henrik J., and Mazhar Waseem. “Using notches to uncover optimization frictions and structural elasticities: Theory and evidence from Pakistan.” *The Quarterly Journal of Economics* (2013): qjt004.
- [17] Martins, Ana. *Progressivity in the Portuguese personal income tax system*. Diss. 2016.
- [18] Moore, Jeffrey C., and Edward J. Welniak. “Income measurement error in surveys: A review.” *Journal of Official Statistics* 16.4 (2000): 331.
- [19] Musgrave, Richard Abel, and Tun Thin. “Income tax progression, 1929-48.” *Journal of political Economy* 56.6 (1948): 498-514.
- [20] OECD (2015), “Tax Administration 2015: Comparative Information on OECD and Other Advanced and Emerging Economies”, OECD Publishing, Paris.
- [21] O’donoghue, Cathal, Massimo Baldini, and Daniela Mantovani. “Modelling the redistributive impact of indirect taxes in Europe: an application of EUROMOD”. No. EM7/01. EUROMOD Working Paper, 2004.
- [22] Pfähler, Wilhelm. ”Redistributive effect of income taxation: decomposing tax base and tax rates effects.” *Bulletin of Economic Research* 42.2 (1990): 121-129.
- [23] Piketty, Thomas, and Emmanuel Saez. “The evolution of top incomes: a historical and international perspective.” No. w11955. National Bureau of Economic Research, 2006.
- [24] Piketty, Thomas, Emmanuel Saez, and Stefanie Stantcheva. “Optimal taxation of top labor incomes: A tale of three elasticities.” *American economic journal: economic policy* 6.1 (2014): 230-271.

- [25] Peter, Klara Sabirianova, Steve Buttrick, and Denvil Duncan. "Global reform of personal income taxation, 1981-2005: Evidence from 189 countries." (2009).
- [26] Pricewaterhousecoopers. "Guia Fiscal 2009". Pricewaterhousecoopers (2009).
- [27] Rodrigues, C., Parente, P. and Bago d'Álva, T. (2002), "Efeito Distributivo do aumento da taxa normal do IVA", Direcção Geral de Estudos and Previsão do Ministério das Finanças, Documento de Trabalho nº 30.
- [28] Rodrigues, C. F. "Income in EU-SILC-Net/gross conversion techniques for building and using EU-SILC databases." Comparative EU Statistics on Income and Living Conditions: Issues and Challenges (2007): 157-172.
- [29] Rodrigues, C.F. et al. (2012), "Desigualdades Económicas em Portugal", FFMS, Lisboa. (Economic Inequality in Portugal).
- [30] Saez, Emmanuel. "Do taxpayers bunch at kink points?." American Economic Journal: Economic Policy 2.3 (2010): 180-212.
- [31] Santos, Ana Sofia. "Os Insólitos Do IVA." Jornal Expresso. Imprensa, 6 Nov. 2009. Web. 22 Apr. 2017.
- [32] Schepanski, Albert, and Teri Shearer. "A prospect theory account of the income tax withholding phenomenon." Organizational Behavior and Human Decision Processes 63.2 (1995): 174-186.
- [33] Shapiro, Matthew D., and Joel Slemrod. "Consumer Response to the Timing of Income: Evidence from a Change in Tax Withholding." American Economic Review 85.1 (1995): 274-83.
- [34] Slemrod, Joel. "Buenas notches: lines and notches in tax system design." eJournal of Tax Research 11.3 (2013): 259.
- [35] Stroup, Michael D. "An index for measuring tax progressivity." Economics Letters 86.2 (2005): 205-213.
- [36] Stroup, Michael D., and K. E. Hubbard. An Improved Index and Estimation Method for Assessing Tax Progressivity. No. 13-14. Mercatus Center Working Paper, 2013.
- [37] Van de Werfhorst, Herman, et al. "Political and cultural impacts of growing inequalities." GINI Intermediate work package 5 (2012).
- [38] Van Wagstaff, Joseph. "Income tax consciousness under withholding." Southern Economic Journal (1965): 73-80.
- [39] Wilkinson, Richard G., and Kate Pickett. "The spirit level: Why more equal societies almost always do better." Vol. 6. London: Allen Lane, 2009.

9 Appendix

Stroup Index

Stroup (2005) proposes a tax progressivity index with the following formula:

$$SH = 1 - \frac{\int_0^1 T(y)dy}{\int_0^1 L(y)dy}$$

From the Gini equation, we know that:

$$G(y) = 1 - 2 \int_0^1 L(y)dy \quad G(t(y)) = 1 - 2 \int_0^1 T(y)dy$$

Rearranging for the expressions in Stroup's formula:

$$\int_0^1 L(y)dy = 0.5 \times (1 - G(y)) \quad \int_0^1 T(y)dy = 0.5 \times (1 - G(t(y)))$$

Substituting these into Stroup's index:

$$SH = 1 - \frac{0.5 \times (1 - G(t(y)))}{0.5 \times (1 - G(y))} = 1 - \frac{1 - G(t(y))}{1 - G(y)}$$

$$SH = \frac{1 - G(y) - 1 + G(t(y))}{1 - G(y)}$$

Simplifying, we find:

$$SH = \frac{G(t(y)) - G(y)}{1 - G(y)} = \frac{P}{1 - G(y)}$$

Self-employment SSC tax base

Table 9: Self-employment SSC tax base, monthly values

	$\times IAS$	Tax Base	y_2^a	y_2^g
0	0.5	€209.61	€146.73-€419.21	€209.61-€598.87
1	1	€419.22	€419.22-€524.02	€598.88-€748.59
2	1.5	€628.83	€524.03-€733.63	€748.60-€1,048.04
3	2	€834.44	€733.64-€943.24	€1,048.05-€1,347.48
4	2.5	€1,048.05	€943.25-€1152.85	€1,347.49-€1,646.92
5	3	€1,257.66	€1152.86-€1467.26	€1,646.93-€2,096.09
6	4	€1,676.88	€1467.27-€1886.48	€2,096.10-€2,694.97
7	5	€2,096.10	€1886.49-€2305.7	€2,694.98-€3,293.86
8	6	€2,515.32	€2305.71-€2934.53	€3,293.87-€4,192.19
9	8	€3,353.76	€2934.54-€3772.97	€4,192.20-€5,389.96
10	10	€4,192.20	€3772.98-€4611.41	€5,389.97-€6,587.73
11	12	€5,030.64	€4611.42-∞	€6,587.74-∞

Source: Authors' calculations based on phone interviews with Social Security authority.

Evolution of VAT schedule in mainland Portugal

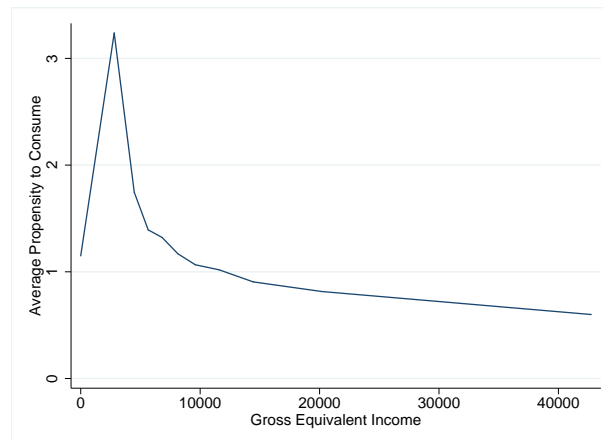
Average Propensity to Consume

Table 10: Evolution of VAT schedule in mainland Portugal

Date	Reduced Rate 1	Reduced Rate 2	Standard Rate	Increased Rate
01/01/1986	0	8	16	30
01/02/1988	0	8	17	30
24/03/1992	-	5	16	30
01/01/1995	-	5	17	-
01/07/1996	5	12	17	-
05/06/2002	5	12	19	-
01/07/2005	5	12	21	-
01/07/2008	5	12	20	-
01/07/2010	6	13	21	-
01/01/2011	6	13	23	-

Source: European Commission, 2017

Figure 10: Distribution of Average Propensity to Consume



Source: Authors' calculations, using HBS 2010. Annual household income, equivalised using the OECD modified scale.