A Work Project, presented as part of the requirements for the Award of a Master Degree in Finance from the NOVA – School of Business and Economics.

# OPTIMAL CONCENTRATION FOR VALUE AND MOMENTUM PORTFOLIOS

LEONOR MARIA DE SANTA MARTA GRANGER BAPTISTA

STUDENT 2291

A Project carried out on the Master in Finance Program, under the supervision of:

Professor Fernando Anjos

06/01/2017

Optimal Concentration for Value and Momentum Portfolios


**Abstract**

This paper aims to verify the persistence of the profitability of the Momentum strategy, first implemented by Richard Driehaus in the 1980's. Furthermore, the paper will test the impact of changing several parameters of the strategy on its profitability. A combination of the Momentum strategy with a Value-oriented one will also be analyzed, with a view to assess the outperformance of this aggregate portfolio. The results are in line with Jedadeesh and Titman (2001), there is still evidence for its profitability in recent years, except in times of severe volatility. Additionally, there is an improvement in combining the two strategies.



**Keywords:** Momentum, Value, Investment, Strategies

## I. Introduction

The main purpose of this paper is the continuation of the study of Jegadeesh and Titman (1993, 2001) on the Momentum strategy, whose results challenge the widely accepted market efficiency theory, established in 1969 by Eugene Fama (Efficient Market Hypothesis). In this light, the Momentum strategy shouldn't be profitable because it states that it's impossible to continuously beat the market given that all the relevant information is already reflected in stock prices. In theory, investors should make informed decisions, and having access to the information on this strategy and its profitability, enough individuals should be investing in it causing its abnormal returns to disappear.

With that as a starting point, I extended the analysis period up until 2015 and additionally designed and tested several variations to the zero-cost Momentum portfolio. Namely, tests were done for portfolios with different number of companies and different holding periods, as well as different ranking periods and lags. The objective being to analyze the possible impact that these parameters may have and optimize the strategy. Lastly, an analysis was done for a combination of the Momentum strategy with a Value-oriented strategy, maintaining a zero-cost portfolio, with the purpose of testing whether this new portfolio would result in a larger Sharpe Ratio than the stand-alones, expected due to their individual profitability and the anticipated uncorrelation between the two, which would increase diversity and so decrease volatility.

This paper will start with a literature review of the historical progress and studies of both Momentum and Value strategies, afterwards it will continue with the development of the hypothesis to be tested and the methodology used. The results and discussions will be separated in three parts: Momentum's profitability over time; detailed analysis of Momentum and combination of both strategies. The two final sections will be for the limitations to the paper and the conclusion. A more detailed reference list can be found at the end.

**II. Literature Review**

Momentum is an investment strategy that consists in buying the best performing companies, and shorting the worst performers. The idea behind it is in line with the saying of "the trend is your friend", which means that you expect that the companies will continue to follow the path that they've been having in the short-term. To follow this strategy, an investor needs to choose which group of stocks he will consider. For example, if his choice fell on the American stock market, he then would need to choose the number of companies in wish to invest, the number of months relevant for the ranking of the companies and the holding period. Also, he must determine if he would like to have a lag. A lag is when he waits two weeks or a month for example before investing, and this is often done due to short-term return reversals, which is when a company inverts its current path in the very short-term (under one month) as shown in de Groot *et al* (2011).

The Momentum strategy can be traced back to the 1950's with Richard Donchian's innovative trend following ideas. His strategy was used for commodity trading and it involved using moving averages and investing based on the higher and lower values. However, more commonly, Richard Driehaus is considered to be the father of the strategy, since in the 1980's he implemented it to run his funds. His idea was against Wall Street's practice at the time, of "buy low, sell high", he instead followed the concept of "buy high, sell higher". Since then, there have been many papers that attempt to explain and recreate this strategy, in Jegadeesh and Titman in 1993 and 2001, in Asness et al (2013), Daniel Moskowitz (2015) and many others. The reason why this strategy persists is still under discussion; the two better accepted theories being that its returns are simply a compensation of risk or that it's due to behavioral tendencies, see Jegadeesh and Titman (2001) for more discussion on the several explanations. However, all of them agree that it has statistically significant abnormal returns in periods between 1945 and 2015.

A more recent variation of this strategy is the Alpha Momentum, which was first documented in Grundy and Martin (1998). This new strategy came as a way of improving the Standard Momentum, by increasing its returns and decreasing its volatility, as was demonstrated in Hühn and Scholz (2013). In this strategy, the variable used to rank the companies is no longer past returns, but instead it's the alpha that represents the abnormal return and can be calculated with models such as the CAPM (Capital Asset Pricing Model), the Fama-French three-factor, or the Fama-French-Carhart four-factor model, represented in Equations I, II and III, respectively.

The CAPM model was developed by Sharpe (1964) and Lintner (1965), and its purpose was to establish a connection between a company's returns and the market. The tested hypothesis was that the returns would equal the risk-free rate plus a coefficient of the market's excess return (MKTRF stands for Market minus Risk Free). This coefficient could be positive if the company had a positive correlation with the market's excess return, or negative otherwise. Currently, it is more common to assume that the returns of a company are equal to an abnormal return plus the risk-free rate, plus the coefficient of the market's excess return. This abnormal return, commonly referenced as alpha, will be the indicator used for the Alpha Momentum in this paper. The CAPM, despite being questioned in papers such as Black (1972), which state that the market's excess return is not the only relevant factor, is still widely used for its simplicity.

**Equation I – CAPM Alpha (Jensen's Alpha)**

$$\alpha_p = r_p - \left[r_f + \beta_p MKTRF + \varepsilon\right]$$

The Fama-French three-factor Model came as an extension to the CAPM, and was developed by Eugene Fama and Kenneth French (1992), with the purpose to model stock returns, but with more descriptive variables to improve the model's explanatory power. Besides using the market's excess return as factor, it adds a size and a value element, written in Equation II as SMB (Small-Minus-Big) and HML (High-Minus-Low), respectively. The size factor represents the difference in returns between firms with small and large Market Capitalizations, since smaller firms have historically outperformed bigger firms. The value factor represents the difference in returns between value firms and growth firms, since, following the same reasoning, historically firms with high book-to-market ratios, value firms, have outperformed firms with low book-to-market ratios, growth firms.

**Equation II – Fama-French three-factor Alpha**

$$\alpha_p = r_p - [r_f + \beta_{mktrf} MKTRF + \beta_{smb} SMB + \beta_{hml} HML + \varepsilon]$$

The Fama-French-Carhart 4-factor model developed by Carhart (1997) adds another element, a Momentum factor written in Equation III as UMD (Up-Minus-Down), and represents the difference in returns between companies with previously high returns and companies with previously low returns, commonly referred to as winners minus losers.

**Equation III – Fama-French-Carhart four-factor Alpha**

$$\alpha_p = r_p - [r_f + \beta_{mktrf} MKTRF + \beta_{smb} SMB + \beta_{hml} HML + \beta_{umd} UMD + \varepsilon]$$

Besides analyzing the optimal concentration of Momentum portfolios, I will also study combination portfolios that include Value stocks. A portfolio of Value stocks is built by buying the companies that are undervalued with the expectation that the market will correct this undervaluation and the price will rise, and short the companies that are overvalued following the same logic. Benjamin Graham and David Dodd first established it in 1928 while teaching in Columbia Business School, resulting in the publication of their book *Security Analysis* in 1934. Following the approach of several papers that studied this strategy, such as Asness *et al* (2013), I use the ratio of price-to-book to understand whether there is some overvaluation or undervaluation of a company in a point in time. I use this variable because, as concluded in this paper, it can predict future returns based on its present value, however, it has also been done with other accounting ratios such as Price-to-Earnings, for example in Truong (2009).

As a comparison and evaluation measure of the different portfolios, I will use the ratio developed by Sharpe (1966), known as Sharpe Ratio, since it takes into account both the excess return when comparing to the risk-free rate and the volatility.

**Equation IV – Sharpe Ratio**

$$Sharpe\ Ratio = \frac{r_p - r_f}{\sigma_p}$$

## III. Hypothesis Development:

What I propose to do can be divided in three main goals. Firstly I want to expand the study of the Momentum strategy to the end of 2015, to study if the abnormal returns found in Jagadeesh and Titman (1993, 2001) until 1998 are still observable. Secondly, I want to calibrate the model, by varying four parameters of the strategy and finding their optimal values. The chosen parameters are: ranking period, lag before investing, holding period and number of companies to invest in. For this, I will analyze both a Standard Momentum and the Alpha Momentum, being that for the latter I will use the CAPM Model's alpha as the ranking variable.

The range of values for those parameters are based on other papers on the Momentum strategy, for example, Jegadeesh and Titman (2001) use a six-months ranking period. Also very important is the choice of stock exchanges, which this paper follows studies such as Jegadeesh and Titman (2001) and Fisher *et al* (2016), which use all stocks in the NYSE, AMEX and NASDAQ stock exchanges.

Finally, I want to analyze whether an investor would benefit from combining a Standard Momentum strategy with a Value strategy, using the price-to-book accounting ratio as ranking variable.

For all the strategies previously mentioned, all portfolios will have an equally weighted long and short component, with an equal number of firms in each. As consequence, all portfolios will be zero-cost, a long-short dollar-neutral strategy. During this paper, all variables (returns, standard deviations, Sharpe Ratios) have been annualized unless specified otherwise.

I expect to find that the strategy is still profitable, although this might happen at a lower level due to the widening awareness of its existence. When combining it with a Value strategy, I expect that they will be uncorrelated and if this happens then there might be benefits of combining the two at least in terms of a lower volatility, and if large enough this should compensate the expected drop in returns due to the expectation that the Value strategy will yield lower returns, result found in Fisher *et al* (2016), and lead to a higher Sharpe Ratio.

Additionally, for the Standard Momentum strategy, I aim to choose the best variation when considering the parameters described previously. When combining it with the Value strategy, I also aim to choose the best concentration in terms of how many companies to invest in, while always maintaining an equal weight between both strategies. The reason why we expect there to be an optimal concentration level and not a monotonic increase/decrease of Sharpe Ratio is because it is expected that both the Momentum's returns and risk will decrease with an increase in diversification, and we aim to find the best values for this trade-off. Another reason for there to be an optimal concentration is due to transaction costs, which increase with the increase of diversification, and so it can offset a rise in returns if it's not large enough. This paper doesn't formally address transaction costs, however a consideration is made to them in the Limitations section.

Throughout this paper, references to the number of companies always pertain to one leg of the strategy. For example, a point in a graph representing a Sharpe Ratio for a strategy with 100 companies means that the strategy has 100 companies long and 100 companies short, and if this happens for the combination of strategies, it means 100 companies long and 100 companies short on both portfolios.

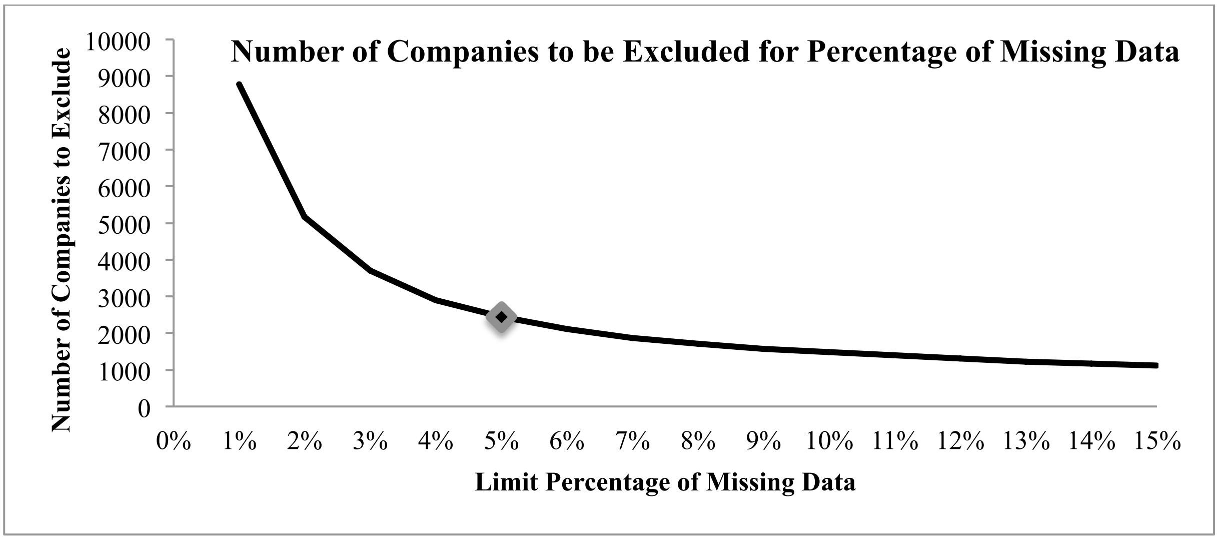
## IV. Methodology:

In general terms, there were three steps that I followed in order to get the final results: firstly I downloaded the data from Wharton University of Pennsylvania's database WRDS ("The Global Standard for Business Research"); secondly, using the programming software Matlab (Matrix Laboratory), I processed and cleaned the data; lastly, using the same software, I created an algorithm to implement the strategy.

More specifically, the data downloaded was for all stocks in the NYSE, NASDAQ and AMEX, which consisted of around 30,000 stocks, from 1965 until the end of 2015. To control for survivorship bias, companies that leave the stock exchange are still considered. The data needed for each stock is the entry and exit from its respective stock exchange, its monthly price and monthly number of shares outstanding to calculate its monthly Market Capitalization, and monthly price to book ratios. The daily data needed was its holding period return, which accounts for events such as stock splits and dividends, and resulted in a total of 80 million data points. To identify the stocks I used the database's unique identifier, called the PERMNO, since it doesn't change with time for any given company, contrary to the Ticker or Company Name, which may and frequently do change. Other data needed was the risk-free rate, the market return, small-minus-big factor, high-minus-low factor and up minus down factor, all available in the same database.

The data processing was a relatively complex part and very work-intensive, as often is the case when the data to handle is large. Firstly I organized and cleaned the data: for example, concerning the daily returns, if the company had no return information, it was removed from the analysis. Following the same logic, if the company had too many missing values, or "holes", it was also removed. For this purpose I used 5% as reference, so only companies with less than 5% missing returns were considered. For the model to work, the missing returns had to be replaced, which I did, in each occurrence, with the minimum between zero and the average of the closest previous and following return available, to be conservative. I chose to use this value as reference because it seems to give a good trade-off between the benefit of accuracy and loss in sample size, as can be seen in Figure I, a lower percentage would result in more accuracy but would lead to an exponential decrease in the number of companies considered.

**Figure I - Number of Companies to be excluded for Percentage of Missing Data**




After processing the data, I calculated the weekly returns from the daily if all returns for that week are available; if not available, that week is not considered in the model for that company. I used the same process to calculate the risk free, market, SMB, HML and UMD.

Finally, having all data necessary, I calculated the results for three different strategies: the Standard Momentum, the Alpha Momentum, and finally a Value strategy. The idea behind all three is similar, although in the first two I use only firms with a Market Cap higher than 100million USD and in the third I change this limit to 1million USD instead. Optimally, they should both have a lower limit of 200million to exclude penny stocks, which have historically displayed a higher volatility and lower liquidity, studied in Liu *et at* (2011).

However, this couldn't be done due to the lack of sufficient data that fits this criterion. Afterwards, I ranked the companies based on a chosen indicator (described below) from the selected past months (the ranking period) and chose the top and bottom companies, then after a lag, I calculated the return as if I had taken long/short positions on the chosen companies, and then held that portfolio for the holding period. I tested this strategy for all combination of variables in Table I:

**Table I – Combination of different parameters to be tested in Momentum strategy**

| Variable | Tested Values |
|---|---|
| **Ranking Period** | 6, 9, 11, 13 and 16 months |
| **Lag** | 2 weeks and 1 month |
| **Holding Period** | 1, 2 and 3 months |
| **Number of Companies** | Between 5 and 400, with intervals of 5 |

The chosen indicator is a function of the type of strategy: the Standard Momentum strategy uses as indicator the cumulative return of the ranking period; the Alpha Momentum uses the Jensen's alpha obtained by regressing the weekly excess returns with the market excess return for the ranking period; and finally the third uses the last price-to-book ratio available of the ranking period.

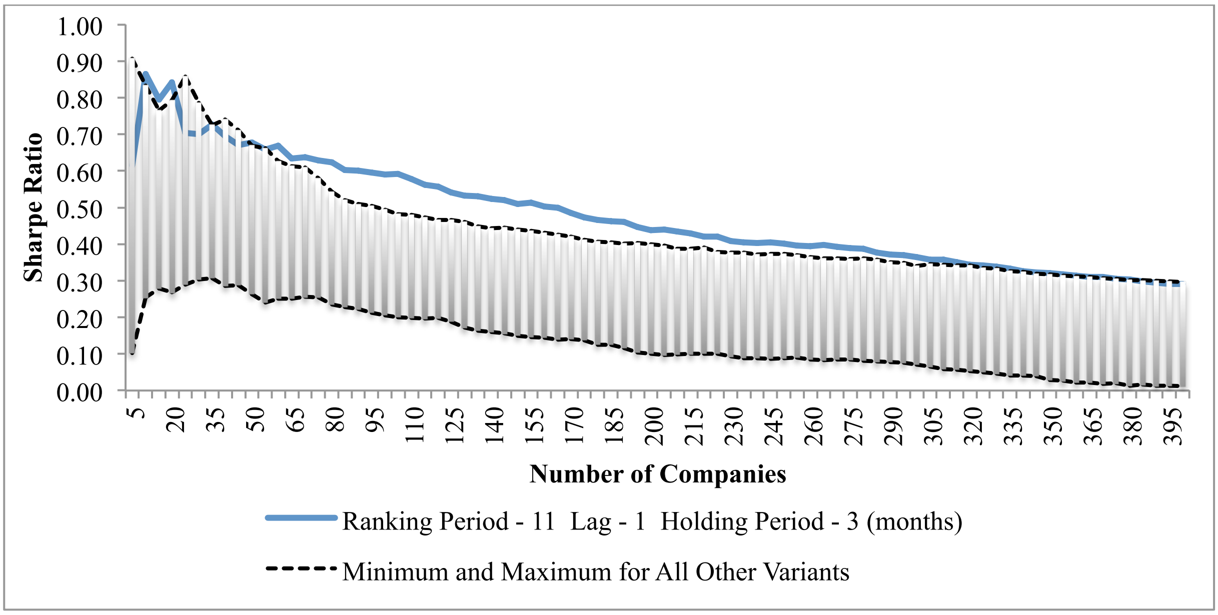
As said before, the purpose of changing so many variables was to optimize the calibration of the model. For each of them I calculate an average return, the standard deviation, and the Sharpe Ratio. The combination shown below had the highest overall Sharpe Ratio:

| |
|---|
| **Base Strategy:** Ranking Period – 11; Lag – 1; Holding Period – 3 |

As can be seen in Figure II, this strategy has an overall outperformance except for highly concentrated portfolios, in which case it competes with a variant that only differs in the ranking period, which is 13 months instead of 11. The variant with a 2-week lag instead of 1 month had very similar results, and so it is not included in Figure II but can be found in Figure IV.B in the Annexes.

For the rest of this paper, this optimal strategy will be considered the Base Strategy due to its overall outperformance.

**Figure II - Sharpe Ratio of different variations of Momentum strategy per number of companies to invest in**




## V. Results

i. Momentum's Profitability Over Time

In this section I wanted to study the abnormal returns of the Momentum strategy in different periods, and for that purpose I used Jensen's alpha from the CAPM model. Firstly, the test for the entire period of 1965 to 2015 resulted in a statistically significant abnormal return. Additionally, both tests for the periods of Jegadeesh and Titman's papers, 1965 to 1989 and 1990 to 1998 (paper of 1993 and 2001, respectively), also proved significantly greater than zero, which is what was expected, since in both papers they arrived at the same conclusion. However, the strategy for the period of 1999 to 2015 does not yield a statistically significant abnormal return. As this result was not expected, I split this period into sub periods to study it in more detail, and found that within this period occurred two financial crisis, in

2002 and 2008, which seem to be what is causing this return not to be significant, due to the high volatility observed in the American markets at the time. When analyzing the post-crisis period of 2012-2015, it is once again possible to observe the statistically significant abnormal returns of this strategy. Results are shown in Table II.

**Table II - CAPM model regression for different time periods**

**Linear Regression**

| | | 1965 -2015 | 1965 -1989 | 1990 -1998 | 1999 -2015 | 1999 -2007 | 1999 -2011 | 2012 -2016 |
|---|---|---|---|---|---|---|---|---|
| **Regression Statistics** | *R* | 0.138 | 0.067 | 0.248 | 0.207 | 0.155 | 0.268 | 0.115 |
| | *R-square* | 0.019 | 0.005 | 0.061 | 0.043 | 0.024 | 0.072 | 0.013 |
| | *Adjusted R-square* | 0.014 | -0.005 | 0.034 | 0.029 | -0.017 | 0.050 | -0.032 |
| | *S* | 0.142 | 0.081 | 0.115 | 0.207 | 0.260 | 0.241 | 0.097 |
| | *N* | 209 | 102 | 37 | 70 | 26 | 46 | 24 |
| ***Regression*** | *F* | 3.996 | 0.458 | 2.286 | 3.043 | 0.593 | 3.391 | 0.294 |
| | *p-level* | 0.047 | 0.500 | 0.140 | 0.086 | 0.449 | 0.072 | 0.593 |
| **Intercept** | *Coefficient* | 0.043 | 0.032 | 0.091 | 0.038 | 0.032 | 0.005 | 0.090 |
| | *Standard Error* | 0.010 | 0.008 | 0.022 | 0.025 | 0.051 | 0.035 | 0.023 |
| | *t Stat* | 4.271 | 3.983 | 4.189 | 1.503 | 0.620 | 0.132 | 3.929 |
| | *p-level* | 0.000 | 0.000 | 0.000 | 0.137 | 0.541 | 0.896 | 0.001 |
| | *H0 (5%)* | *rejected* | *rejected* | *rejected* | *accepted* | *accepted* | *accepted* | *rejected* |
| **MKTRF** | *Coefficient* | -0.258 | -0.068 | -0.503 | -0.553 | -0.451 | -0.752 | -0.203 |
| | *Standard Error* | 0.129 | 0.100 | 0.333 | 0.317 | 0.586 | 0.408 | 0.375 |
| | *t Stat* | -1.999 | -0.676 | -1.512 | -1.744 | -0.770 | -1.842 | -0.542 |
| | *p-level* | 0.047 | 0.500 | 0.140 | 0.086 | 0.449 | 0.072 | 0.593 |
| | *H0 (5%)* | *rejected* | *accepted* | *accepted* | *accepted* | *accepted* | *accepted* | *accepted* |

ii. Detailed Analysis of Momentum

a. Momentum and Market Volatility

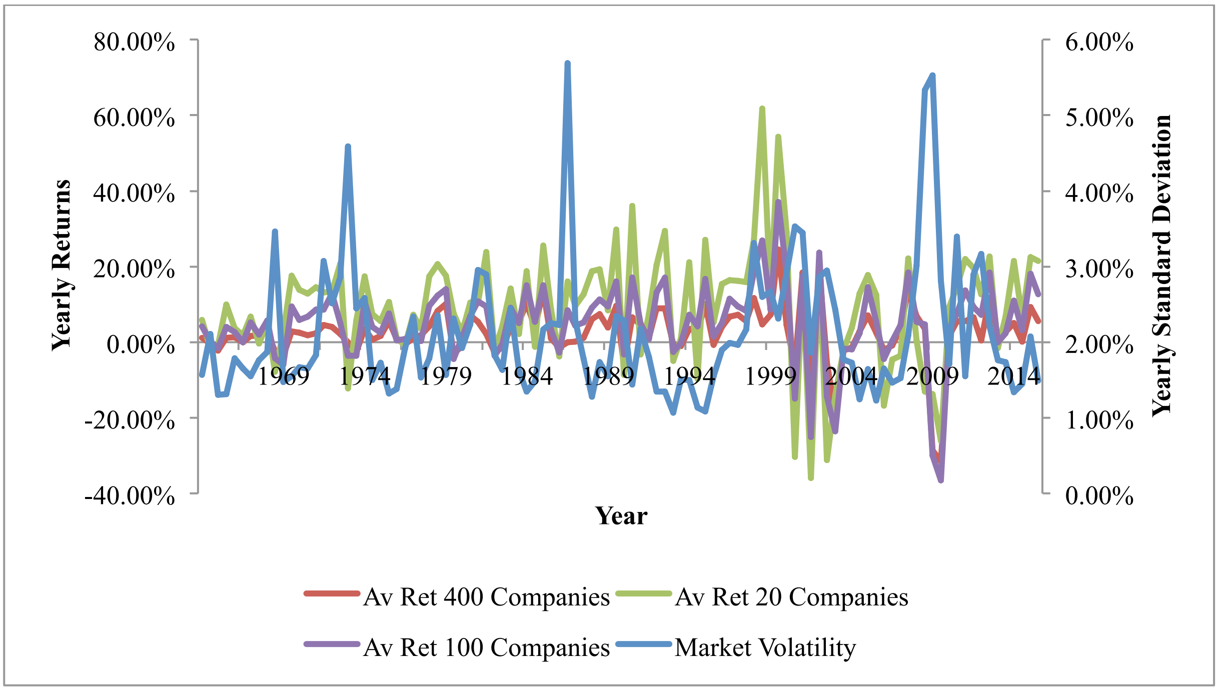
It was necessary to study in more depth the relationship between the returns of the strategy and the market volatility, to evaluate if, in fact, the reason why the Momentum strategy failed to yield significant abnormal returns in the period surrounding the two financial crisis was the increase in volatility. The results appear to show that there is a negative correlation between the strategy's returns and the market's volatility. When testing for this hypothesis, as seen in Table III, the returns with the strategy using 400 companies proved significant using the critical value of -1.67, corresponding to a confidence level of 95% of a one-tail test when testing for negative significance, and the returns of the strategies with 20 and 100 companies proved significant using the critical value of -1.30, corresponding to a confidence level of 90% of a one-tail test when testing for negative significance.

**Table III - Correlation of strategy returns with market volatility for different portfolio concentration levels**

| | Correlation | T-statistics |
|---|---|---|
| 400 companies | -0.17745 | -1.82102 |
| 20 companies | -0.13319 | -1.35719 |
| 100 companies | -0.14382 | -1.46774 |

Another conclusion that could be taken from both Table III and Figure III is that the 3 concentrations' returns seem to have a more accentuated negative correlation in times of high volatility, which I defined as the highest volatility decile in the sample (Table VII in the Annexes), but also that when this happens, the strategy with only 20 companies, which intuitively should have a larger crash in its returns due to its naturally high volatility, doesn't seem to fall much lower from the other two. In fact, this more concentrated portfolio seems to differ more from the others in times of high returns and less in times of low returns. This means that in times of high volatility, changing the strategy to have more companies in order to decrease its risk wouldn't have the effect desired. The negative relation between volatility and Momentum returns is also analyzed in Wang and Xu (2015) and Daniel and Moskowitz (2015). The volatility in Figure III was calculated using the overall market's returns for each six-month period, and Figure III.A in the Annexes shows the same behavior using the Volatility Index (VIX) for the years 2000-2017.

**Figure III - Strategy Returns for different portfolio concentration levels and market volatility**

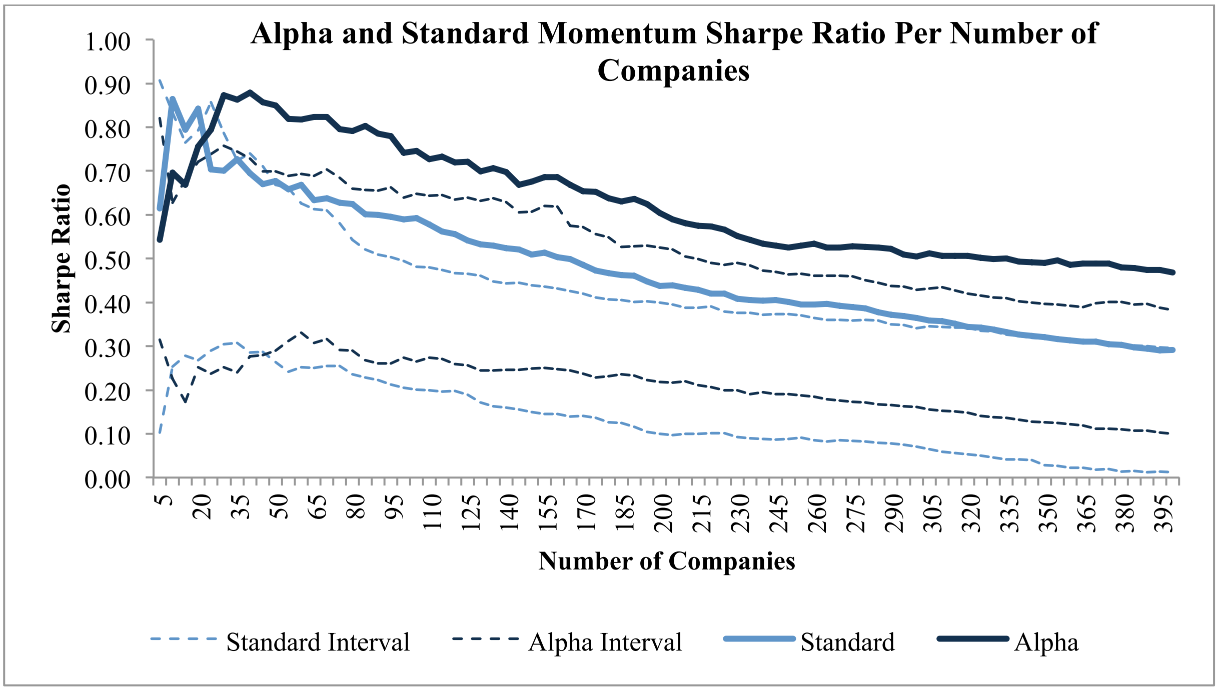



b. Alpha and Standard Momentum

Also interesting was the comparison between the Standard Momentum and the more recent Alpha Momentum strategy. The Alpha Momentum outperforms the Standard one for almost any number of companies, with the exception of very concentrated portfolios, and also for any scenario, as it's possible to see in the figure that the interval and the best variation are higher. Figure IV shows the best combination of factors of both the Alpha and the Standard Momentum, for each number of companies. The best variation happens to be equal for both, which is Ranking Period – 11, Lag – 1, Holding Period – 3 (months), and is represented in the Alpha and Standard lines. The dashed lines represent the interval (maximum and minimum) for all other variations of each strategy.

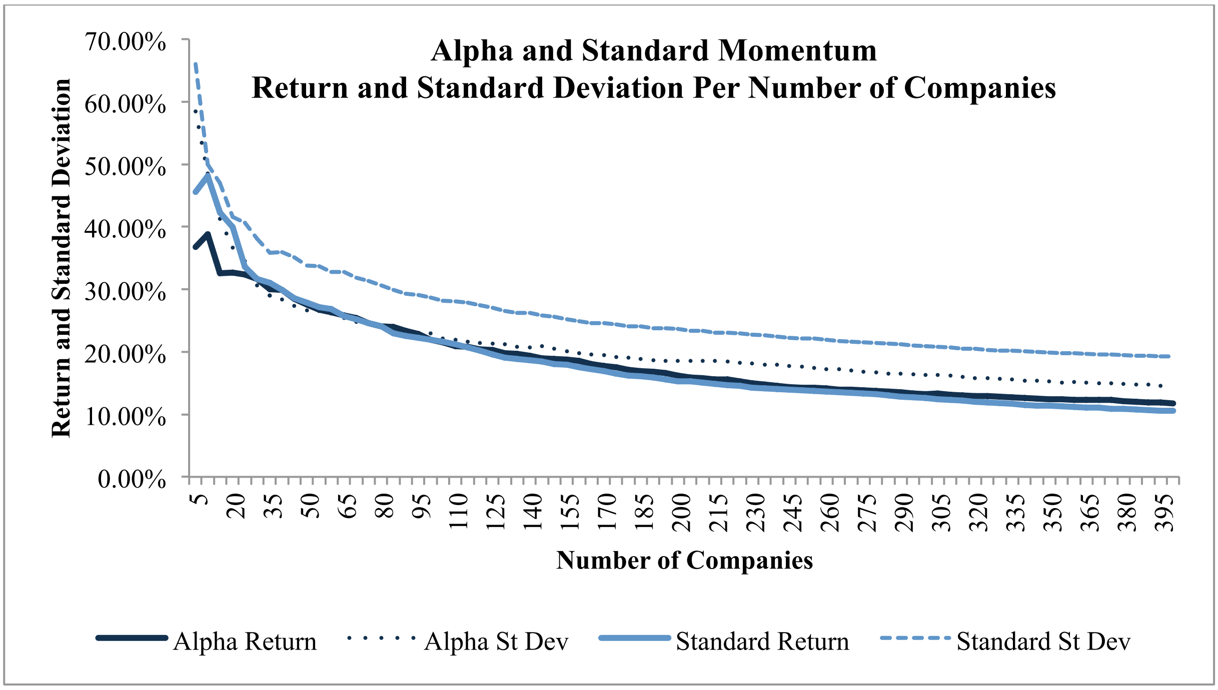
In these graphs I didn't include the variation of Months – 11 months, Lag- 2 weeks, Holding Period – 3 months since its results are very close to the ones shown in the continuous lines in Figure IV. Figure IV.B in the Annexes shows these variations.

**Figure IV - Standard and Alpha Momentum Sharpe Ratio for different variations per number of companies**




The Alpha Momentum has a better performance due to its much lower volatility, as can be seen in Figure V. Here again it's observable the exception of very concentrated portfolios.

**Figure V – Alpha and Standard Momentum's returns and standard deviations per Number of Companies. Parameters: Ranking Period – 11; Lag – 1; Holding Period – 3 (months)**




c. Momentum and Fama-French models

Finally, I conducted an analysis to see how the Standard Momentum strategy compares to the Fama-French three-factor model and Fama-French-Carhart four-factor model. The reason behind this analysis was to test whether the risk composition was different for a very concentrated portfolio (20 stocks long, 20 stocks short) differed from the one of a more diversified portfolio (400 stocks long, 400 stocks short).

I started with the three-factor model, and the results are shown on the left part of Tables IV and V. In these results the intercept and the HML factor are significant in both variants, while the MKTRF is significant in the 400 companies variant but not the 20 variant, and the SMB is significant in the 20 companies variant but not the 400 variant. This implies that these two portfolios do have different risk compositions: the performance of the more concentrated portfolio is not correlated to market returns, whilst that of the more diversified portfolio is not correlated to company size portfolios. Also, in the three-factor model, all betas are negative, while both intercepts are positive. For the SMB factor, this means that large-cap stocks outperform small-cap stocks, and that our portfolio is mostly composed of large-cap stocks. For HML it means that stocks with low book-to-market ratios outperform stocks with high ratios, which means our portfolio is mostly composed of growth stocks and not value stocks. Finally, for MKTRF it means that our portfolio is negatively correlated with the market's returns.

Although the three-factor models have high F-values, which means that the model has a better fit than an intercept-only model would have and there is some explanatory power, I also ran for a Fama-French-Carhart four-factor model, which gave different results, shown in the right of Tables IV and V. For both variances, the new factor, UMD is highly significant and the inclusion of it increases the R-square and the F-values. This is exactly what we would expect since the new variable is a Momentum factor itself. The differences came in the other variables: in the 400 companies variant, the SMB factor became significant, while both the

Intercept and the MKTRF factors lost their significance, suggesting that these factors were capturing some of the influence of the UMD factor. For the 20 companies variant, all factors are now significant.

**Table IV – Fama-French three-factor model (left) and Fama-French-Carhart four-factor model (right) for portfolio with 400 companies**

**Linear Regression** — Momentum strategy for 400 companies, Fama-French three-factor model

| **Regression Statistics** | |
|---|---|
| *R* | 0.303 |
| *R-square* | 0.092 |
| *Adjusted R-square* | 0.078 |
| *S* | 0.092 |
| *N* | 209 |

| **ANOVA** | *d.f.* | *SS* | *MS* | *F* | *p-level* |
|---|---|---|---|---|---|
| *Regression* | 3 | 0.18 | 0.06 | 6.90 | 0.000 |
| *Residual* | 205 | 1.73 | 0.01 | | |
| *Total* | 208 | 1.90 | | | |

| | *Coefficient* | *Standard Error* | *t Stat* | *p-level* | *H0 (5%)* |
|---|---|---|---|---|---|
| **Intercept** | 0.023 | 0.007 | 3.457 | 0.001 | *rejected* |
| **MKTRF** | -0.264 | 0.096 | -2.751 | 0.006 | *rejected* |
| **SMB** | -0.194 | 0.144 | -1.355 | 0.177 | *accepted* |
| **HML** | -0.441 | 0.121 | -3.643 | 0.000 | *rejected* |

**Linear Regression** — Momentum strategy for 400 companies, Fama-French-Carhart four-factor model

| **Regression Statistics** | |
|---|---|
| *R* | 0.828 |
| *R-square* | 0.686 |
| *Adjusted R-square* | 0.680 |
| *S* | 0.054 |
| *N* | 209 |

| **ANOVA** | *d.f.* | *SS* | *MS* | *F* | *p-level* |
|---|---|---|---|---|---|
| *Regression* | 4 | 1.31 | 0.33 | 111.27 | 0.00 |
| *Residual* | 204 | 0.60 | 0.00 | | |
| *Total* | 208 | 1.90 | | | |

| | *Coefficient* | *Standard Error* | *t Stat* | *p-level* | *H0 (5%)* |
|---|---|---|---|---|---|
| **Intercept** | -0.005 | 0.004 | -1.31 | 0.193 | *accepted* |
| **MKTRF** | -0.046 | 0.058 | -0.79 | 0.431 | *accepted* |
| **SMB** | -0.194 | 0.085 | -2.30 | 0.023 | *rejected* |
| **HML** | -0.169 | 0.073 | -2.32 | 0.021 | *rejected* |
| **UMD** | 1.138 | 0.058 | 19.64 | 0.000 | *rejected* |

**Table V - Fama-French three-factor model (left) and Fama-French-Carhart four-factor model (right) for portfolio with 20 companies**

**Linear Regression** — Momentum strategy for 20 companies, Fama-French three-factor model

| **Regression Statistics** | |
|---|---|
| *R* | 0.367 |
| *R-square* | 0.135 |
| *Adjusted R-square* | 0.122 |
| *S* | 0.194 |
| *N* | 209 |

| **ANOVA** | *d.f.* | *SS* | *MS* | *F* | *p-level* |
|---|---|---|---|---|---|
| *Regression* | 3 | 1.20 | 0.40 | 10.62 | 0.000 |

**Linear Regression** — Momentum strategy for 20 companies, Fama-French-Carhart four-factor model

| **Regression Statistics** | |
|---|---|
| *R* | 0.677 |
| *R-square* | 0.459 |
| *Adjusted R-square* | 0.448 |
| *S* | 0.154 |
| *N* | 209 |

| **ANOVA** | *d.f.* | *SS* | *MS* | *F* | *p-level* |
|---|---|---|---|---|---|
| *Regression* | 4 | 4.10 | 1.03 | 43.26 | 0.000 |

| | | | | | |
|---|---|---|---|---|---|
| *Residual* | 205 | 7.73 | 0.04 | | |
| *Total* | 208 | 8.93 | | | |

| | *Coefficient* | *Standard Error* | *t Stat* | *p-level* | *H0 (5%)* |
|---|---|---|---|---|---|
| **Intercept** | 0.094 | 0.014 | 6.62 | 0.00 | *rejected* |
| **MKTRF** | -0.018 | 0.203 | -0.08 | 0.93 | *accepted* |
| **SMB** | -1.148 | 0.304 | -3.78 | 0.00 | *rejected* |
| **HML** | -1.136 | 0.256 | -4.435 | 0.00 | *rejected* |

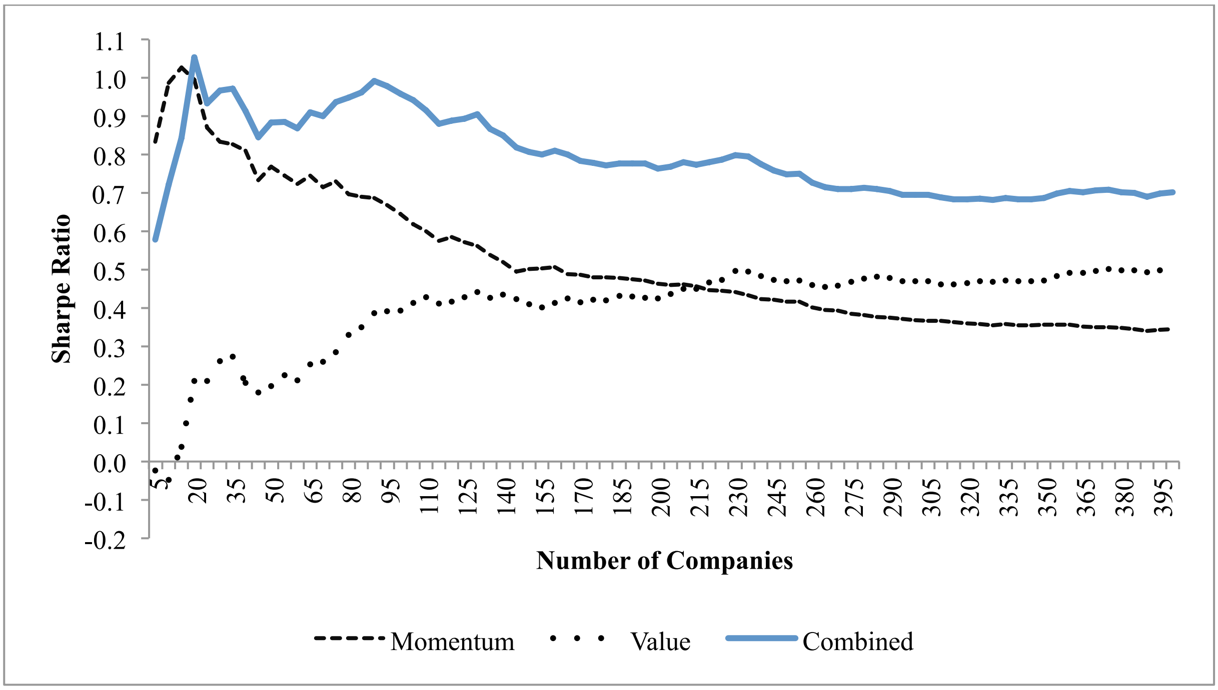
| | | | | | |
|---|---|---|---|---|---|
| *Residual* | 204 | 4.83 | 0.02 | | |
| *Total* | 208 | 8.93 | | | |

| | *Coefficient* | *Standard Error* | *t Stat* | *p-level* | *H0 (5%)* |
|---|---|---|---|---|---|
| **Intercept** | 0.048 | 0.012 | 4.01 | 0.00 | *rejected* |
| **MKTRF** | 0.333 | 0.164 | 2.02 | 0.04 | *rejected* |
| **SMB** | -1.148 | 0.241 | -4.77 | 0.00 | *rejected* |
| **HML** | -0.700 | 0.207 | -3.38 | 0.00 | *rejected* |
| **UMD** | 1.822 | 0.165 | 11.05 | 0.00 | *rejected* |

iii. Combination of both strategies

The last analysis that I've done in the search for a better performing variation of the Momentum strategy was to combine the Base Strategy with a Value Strategy with the same parameters and number of companies. Figure VI shows the stand-alone Sharpe Ratio of each strategy, and the equivalent for the combined portfolio. For almost any number of companies, the Sharpe Ratio is much higher when combining the two strategies than when investing in just one, and also that the Momentum's Sharpe Ratio only exceeds that of the combination due to a sizable underperformance for the Value strategies for highly concentrated portfolios.

**Figure VI - Sharpe Ratio for stand-alone strategies and combined strategy**



When analyzing the case of 100 companies in particular, it's clear that the reason behind this increase in Sharpe Ratio is the significant negative correlation between the two strategies, which can be seen in Table VI and Figure VII. This negative correlation was also found in Asness *et al* (2013).

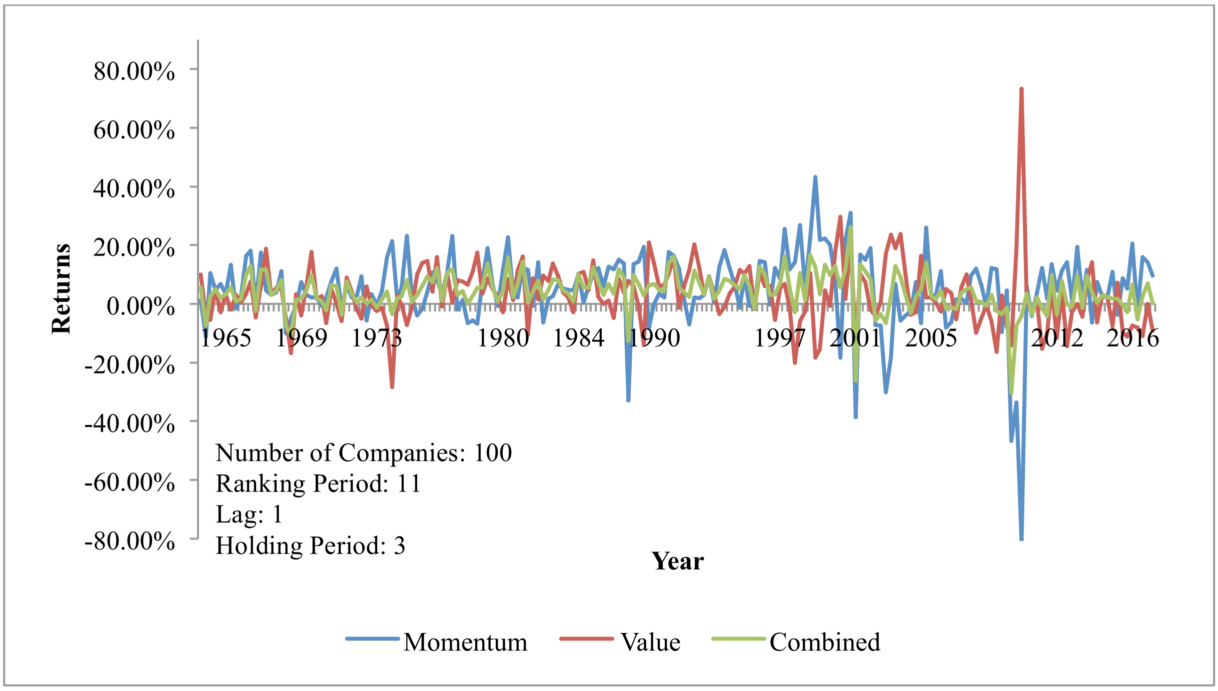
**Table VI - Return, volatility and Sharpe Ratio for each stand-alone strategy and for the combined strategy. Correlation between Momentum and Value strategies and its significance.**

| | Average Return | Volatility | SR |
|---|---|---|---|
| Momentum | 21.53% | 26.10% | 0.646 |
| Value | 12.93% | 20.94% | 0.394 |
| Combined | 17.17% | 13.04% | 0.959 |

| Correlation | T-statistics |
|---|---|
| -0.402 | -6.028 |

For the variations analyzed, the optimal concentration for a Momentum-only portfolio seems to be around 30 companies long plus 30 companies short, which led to an overall Sharpe Ratio of around 0.8. This number of companies is substantially lower than that of the optimal combined portfolio of Value and Momentum, which is around 90 companies for each leg. This happens because the Value strategy has an optimal concentration of 230 companies at a Sharpe Ratio of around 0.47, after which it stagnates. The estimated optimal Momentum concentration differs from most studies done on the Momentum strategy, since the average concentration observed in other studies is a decile of the three stock exchanges used here, for example in Jegadeesh and Titman (2001), which nowadays results in around 500 companies, slightly larger than the most diversified portfolio analyzed in this paper.

**Figure VII - Returns for stand-alone strategies and for the combined strategy**

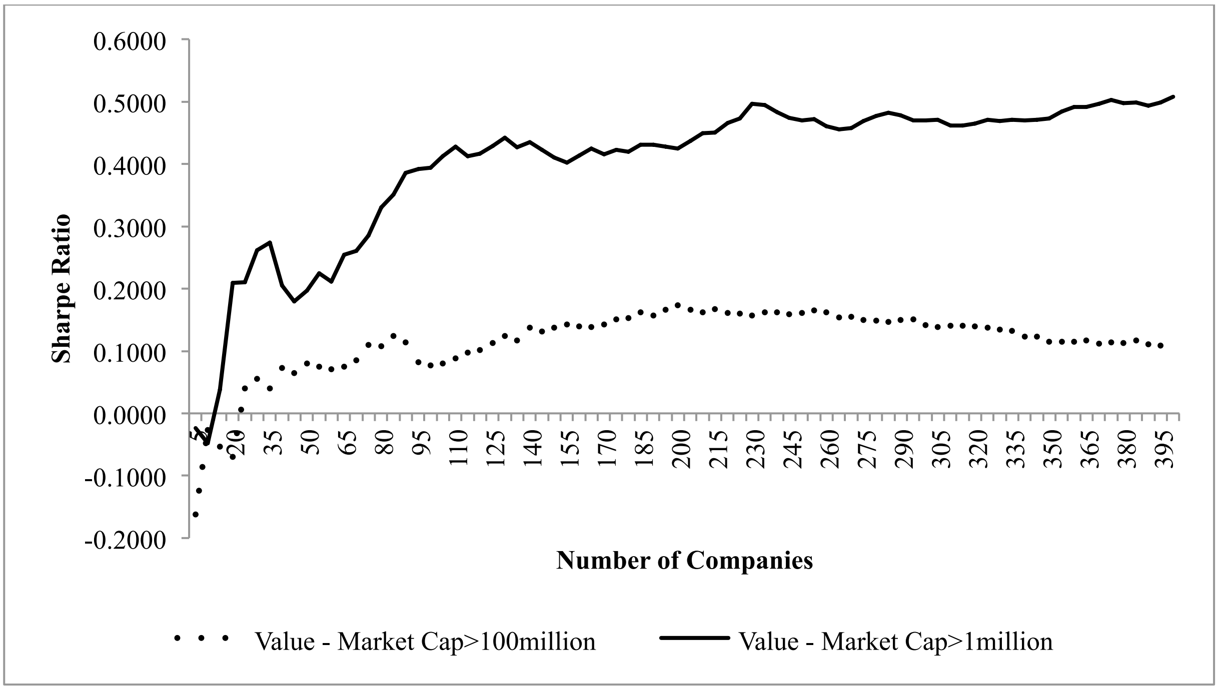



**Note: Returns above are quarterly**

## VI. Limitations

Although most of the results obtained were in line with what was expected, there were some limitations, which were made less severe due to the high number of companies in the sample. An example of this is the amount of missing data from the database: although the database is one of the largest, and has an immeasurable amount of information, there were still many cases of firms that had missing returns, or cases where the return was available, but the Market Cap wasn't. The biggest challenge in terms of data was for the price-to-book variable, which out of the 30,000 companies in the stock exchanges considered only 20,000 had values, and only starting in 1970. This limitation had a negative impact in another situation: when creating the Value strategy to combine it with the Momentum, I tried to impose the Market Cap lower limit of 100 million as done before, however since in this case the number of data available was much smaller, this resulted in a too controlled strategy, and for example if I wanted to long and short 400 companies, I could have only 500 that fit the market cap criteria, and so I had to lower the lower the Market Cap limit to 1 million only, so that the ranking of price-to-books would have enough companies with the data necessary, for all points in time. This is shown in Figure VIII, in the dotted line there are not enough companies in the sample for the strategy to be done effectively, and so the Sharpe Ratio is very low for any number of companies, and in the solid line there are enough companies in the sample, and the strategy has a high Sharpe Ratio. This shows that withdrawing the condition in this case has more pros than cons: it could have happened that the increase in volatility that originates from including too many penny-stocks wouldn't have compensated the increase in return, but as a matter of fact, in this case the difference is almost completely driven by an increase in return. Figures VIII.B and VIII.C in the Annexes file display their average returns and standard deviations, and the Sharpe Ratio of the combined strategy if both had been limited to a Market Cap superior to 100 million USD, respectively.

**Figure VIII - Sharpe Ratio for Value strategy when limited to a Market Cap higher than 1 million and 100 million**




An important consideration to have when studying any investment strategy are the transaction costs. This paper didn't take into consideration the eventual impact of transaction costs in the strategy return. However, considering the results of Frazzini *et al* (2015) and the fact that the abnormal returns found in this paper are so significant, I do not consider that a relevant limitation.

*"We conclude that the main capital market anomalies – size, value, and momentum – are robust, implementable, and sizeable in the face of transactions costs."*
(Frazzini *et al*, 2015: Page 1)

**VII. Conclusion**

Overall, the Standard Momentum strategy during the period between 1965 and 2015 has a 1.4% monthly statistically significant abnormal return as given by the Jensen's Alpha from the CAPM model, a result similar to the one found in Jegadeesh and Titman (2001) of 1.24% significant abnormal monthly return between 1965 and 1998. The sub periods of 1965 to 1989 and 1990 to 1998 also have significant abnormal returns for the variation of the strategy studied in this paper, however the final period of 1999 to 2015 does not give a significant overall abnormal return. Yet, by separating this, the period between 2012 and 2015 does yield a significant abnormal return of 2.9% monthly, while the period between 1999 and 2011 doesn't because of the high volatility in the markets due to the financial crisis of 2002 and 2008. The Alpha Momentum strategy has a higher Sharpe Ratio than the Standard for the overall period, with the exception of highly concentrated portfolios

When combining the Standard Momentum with a Value strategy, there is a negative correlation between the two, and although the Value part of the portfolio reduces returns, the decrease in volatility compensates and there is a large increase in Sharpe Ratio when comparing with any of the two strategies alone.

**VIII. Bibliography**

**Asness, Clifford S., Andrea Frazzini, and Lasse H. Pedersen.** 2014. "Quality Minus Junk." *CFA Institute*, Vol. 44, No. 1

**Asness, Clifford S., Tobias J. Moskowitz and Lasse H. Pedersen**. 2013. "Value and Momentum Everywhere." *The Journal of Finance*, Vol. 68, No. 3, Pages 929–985

**Barroso, Pedro and Pedro Santa-Clara.** 2015. "Managing Momentum risks: Momentum has its Moments " *Journal of Financial Economics*, Vol. 116, Issue 1, Pages 111-120

**Black, Fischer**. 1972. "Capital Market Equilibrium With Restricted Borrowing." *The Journal of Business*, Vol. 45, No. 3, Pages 444-455

**Blitz, David, Joop Huij and Martin Martens.** 2011. "Residual Momentum." *Journal of Empirical Finance*, Vol. 18, No. 3, Pages 506–521

**Carhart, Mark M.** 1997. "On Persistence in Mutual Fund Performance." *The Journal of Finance*, Vol. 52, No. 1, Pages 57-82

**Daniel, Kent and Tobias J. Moskowitz.** 2015. "Momentum Crashes." *Journal of Financial Economics*, Vol. 122, No. 2, Pages 221–247

**De Groot, Wilma, Joop Huij and Weili Zhou.** 2011. "Another Look at Trading Costs and Short-Term Reversal Profits." *Journal of Banking & Finance*, Vol. 36, No. 2, Pages 371–382

**Fama, Eugene F.** 1969. "Efficient Capital Markets: A Review of Theory and Empirical Work." *The Journal of Finance*, Volume 25, Issue 2, Pages 383-417

**Fama, Eugene F. and Kenneth R. French.** 1992. "The Cross-Section of Expected Stock Returns." *The Journal of Finance*, Vol. 47, No. 2, Pages 427−465

**Fama, Eugene F. and Kenneth R. French.** 2015. "A Five-Factor Asset-Pricing Model." *Journal of Financial Economics*, Vol. 116, No. 1, Pages 1–22

**Fisher, Gregg, Ronnie Shah and Sheridan Titman.** 2016. "Combining Value and Momentum." *Journal Of Investment Management*, Vol. 14, No. 2, Pages 33–48

**Frazzini, Andrea, Ronen Israel and Tobias J. Moskowitz.** 2012. "Trading Costs of Asset Pricing Anomalies." Fama–Miller Working Paper, Chicago Booth Research Paper No. 14-05

**Grundy, Bruce D. and J. Spencer Martin.** 1998. "Understanding the Nature of the Risks and the Source of the Rewards to Momentum Investing." *The Review of Financial Studies*, Volume 14, issue 1, pages 29-78

**Hühn, Hannah Lea and Hendrik Scholz.** 2013. "Alpha Momentum and Price Momentum." Available at SSRN: https://ssrn.com/abstract=2287848

**Jegadeesh, Narasimhan and Sheridan Titman**. 1993. "Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency." *The Journal of Finance*, Vol. 48, No. 1, pp. 65-91

**Jegadeesh, Narasimhan and Sheridan Titman**. 2001. "Profitability of Momentum Strategies: An Evaluation of Alternative Explanations." *The Journal of Finance*, Vol. 56, No. 2, pp. 699-720

**Lintner, John**. 1965. "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets." *The Review of Economics and Statistics*, Vol. 47, No. 1, Pages 13-37

**Liu, Qianqiu, S. Ghon Rhee and Liang Zhang.** 2011. "On the Trading Profitability of Penny Stocks." *24th Australasian Finance and Banking Conference 2011 Paper.* Available at SSRN: https://ssrn.com/abstract=1917300

**Sharpe, William F.** 1964. "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk." *The Journal of Finance,* Vol. 19, No. 3, Pages 425-442

**Sharpe, William F.** 1966. "Mutual Fund Performance." *The Journal of Business*, Vol.39, No. 1, Part 2: Supplement on Security Prices. Pages 119-138

**Wang, Kevin Q. and Jianguo Xu.** 2015. "Market Volatility and Momentum." *Journal of Empirical Finance*, Vol. 30, Pages 79–91

**Truong, Cameron.** 2009. "Value investing using price earnings ratio in New Zealand." *University of Auckland Business Review*, Vol. 11, No. 1