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# The determinants of teacher effectiveness in Portuguese schools 

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#### Abstract

This work project analyses the impact of teachers on student exam scores and the determinants of teacher effectiveness in Portuguese schools. The main findings are that teachers are an important component of student achievement and that unobservable characteristics explain most differences in teacher quality. Having a master's degree has no impact on teacher quality. Unlike their female counterparts male teachers exhibit losses in their teaching effectiveness over the years. We also find positive and significant peer effects between teachers.


JEL Classification: I210

Keywords: teacher value added; education; observables; national exams.

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## I. Introduction

It is well established among researchers, the educational community, and policy makers, that teachers are a fundamental element in the student's learning process. But how can we measure this impact? How relevant is it for a student to have a better or worse teacher? And what exactly makes a good teacher? Can we use observable characteristics to identify good teachers? The answer to such questions involves a two-step approach: first is it necessary to obtain a measure of teacher quality, then analyse what determines it. In the first step, researchers often resort to Teacher Value Added estimation techniques. Unfortunately such research is highly dependent on extensive datasets that are seldom available outside the US and UK. It is relevant to conduct such an analysis in different educational contexts. Therefore we employ data from the Portuguese public school system to investigate the impact that teacher quality has on Portuguese Language and Mathematics national exam scores, our measures of student achievement, and access the impact that different teacher observable characteristics have on teacher value added. We do this by estimating the teacher value added for two-year periods corresponding to the $5^{\text {th }}$ and $6^{\text {th }}$ grades.

We find that teachers are indeed a relevant determinant of student achievement with up to 13\% percent of the variation in student test scores in the Portuguese Language exam being determined by Portuguese Language teachers and up to $16 \%$ of the variation in Mathematics test scores being explained by Mathematics teachers. Teacher value added also shows a relevant degree of variability, with a 1 S.D. improvement in our measure of teacher quality leading to an increase of 3.94 points out of 100 in $6^{\text {th }}$ grade exam scores for Portuguese Language and 6.9 points for Mathematics over the two year period a teacher follows a student. This correspond to impacts equivalent to 0.246 SD in the exam score distribution for Portuguese Language and
0.31 SD for Mathematics. ${ }^{1}$ Noticing that most studies are performed with yearly exams (instead of exams every two years like we do) our results are comparable with other estimates in the literature. Aaronson et al. (2007) find that a 1SD increase in teacher quality has an impact of 0.15 SD in Mathematics yearly test scores using data from grades 3 to 11. Rockoff (2004) finds an impact of 0.1 SD in both English and Mathematics yearly test scores from grades 2 to 6 . More recently, Chetty, Friedman, and Rockoff (2014) find an impact of 0.14 SD for Mathematics and 0.1 SD for English using grades 3 to 8 .

We then use teacher observable characteristics to test if they are able to predict our measure of teacher quality. Using traditional specifications to access the determinants of teacher effectiveness we find that at most $1.2 \%$ of teacher quality can be explained by variables such as experience, gender and training. This has relevant policy implications. In particular, in the Portuguese case, teachers are allocated to public schools according to a national ranking based on teacher observable characteristics, with more experience and higher graduation grades giving priority in the allocation process. There is no compelling evidence on the literature (Gordon, Kane and Staiger (2006), Hanushek, Kain, O'Brien and Rivkin (2005)) of the relationship of such variables with educational outcomes nor that allocating teachers according to them improves student achievement, a result confirmed here.

However, we show that, unlike what is pointed by most literature, experience is indeed relevant after the first years of teaching but not in a positive manner. Once we make the distinction between male and female teachers, we find that male teachers experience a negative and statistically significant decrease in quality after 17 years of teaching. This suggests that experience is most likely capturing human capital depreciation associated with aging, that is not significant for female teachers.

[^0]In order to fully employ the measure of teacher quality obtained, we then extend this analysis to investigate to which degree teacher peer effects are relevant for an individual teacher effectiveness. We also investigate to which extent having a differentiated ability to teach different kinds of students affects teacher performance as well as if there are gains/losses in teaching in a school where a teacher differentiated teaching ability matches the one of his/her colleagues.

In Section II we revise the literature on the determinants of teacher effectiveness and measurement of teacher value added, in Section III a description of the Portuguese Educational system is presented, in Section IV we describe the data used, in Section V we expose our empirical strategy both at the level of the estimation of teacher value added and its determinants and in Section VI we analyse the results obtained. Finally, in Section VII we conclude.

## II. Literature Review

The idea that education is conjointly produced by community, families, schools, and students in an input-output relationship can be traced back to the "Coleman Report" (Coleman, (1968)), which emphasized the importance of student peer effects and family's socioeconomic background as being the major determinants of educational output. These relations would be latter on explored by economists under the concept of educational production function (Hanushek (1979)) and research expanded the focus to several other educational inputs such as schools and teachers.

Although there is a wide consensus around the educational community and literature that teachers are a decisive educational input (Hanushek, (2011), Hanushek, Kain, O'Brien, and Rivkin, (2005), Aaronson, Barrow and Sander (2007), Rockoff (2004)) ${ }^{2}$, there is not consensus

[^1]among scholars on what exactly are the characteristics of an effective teacher. In a comprehensive review Hanushek (2003) identifies a split in the literature with $41 \%$ of the studies finding experience to be significantly related with teacher performance. Authors such as Rockoff (2004), find that having more than 10 years of experience brings positive gains in teacher effectiveness ${ }^{3}$ while Staiger and Rockoff (2010), Hanushek, Kain, O'Brien and Rivkin (2005), find that experience is only relevant in the first years of teaching. A common result found by authors such as Hanushek and Rivkin (2006, 2004), as well as Gordon, Kane and Staiger (2006), is that the level of certification is not relevant, a result that we also find for Portuguese schools. Overall this points to the notion that teachers do matter, but the characteristics of an effective teacher are mainly unobservable.

The disentanglement of the effects of the inputs of the education production function is not an easy task, and the impact of teachers is no exception. Such estimation is complicated mainly due to a problem of non-randomness and limited data- as it is so common in Economics of Education. Indeed if teachers were simply randomly assigned to each student - with enough observations per teacher - the determination of most effective teachers could be obtained simply by looking at average student's test results at the end of every schoolyear (Harris, (2009)). However this is not the case. Family's housing and schooling decisions are based on household's preferences and endowment (Tiebout (1956)). There is considerable evidence that families react and enrol their children in schools that they perceive as being better, for instance using school rankings based on average scores in national exams (Nunes, Reis and Seabra (2015), Portugal). Also, there is evidence that there is student sorting within schools, at least in the USA, with least qualified teachers being assigned to underperforming students, and in teacher allocation between schools with more qualified and experienced teachers preferring to

[^2]move to schools with higher achieving student populations (Clotfelter, Ladd, and Vigdor (2006), Greenberg and McCall, (1974), Hanushek, Kain, and Rivkin (2004)).

However even if educational inputs are determined by schools and families choices this nonrandomness could be easily overcome with enough data on all the inputs affecting the educational production function over time, including student's individual ability (Todd and Wolpin, (2003)), which unfortunately are hard to obtain.

In order to deal with this difficulties researchers usually employ Teacher-Value Added (TVA) models and a growing body of literature has been expanding under their use. Although different specifications can be used, TVA models have the common features of explaining current student achievement as a function of lagged student achievement measures and contemporaneous school and family inputs (including socioeconomic variables). Assuming that education is a cumulative process the lagged student achievement measures should capture previous educational inputs. The choice of the specification used depends mainly on the kind of data the researcher has at its disposal (Todd and Wolpin, (2003)).

There isn't however consensus over the applicability of such models with authors such as Gordon, Kane, and Staiger (2006) and Hanushek (2009) advocating not only for their feasibility but that teacher selection using TVA models can significantly increase student achievement. In contrast, Corcoran (2010) and Baker et al. (2010) advocate that teacher value added is an inadequate approximation to teacher quality.

In order to access whether TVA models provide unbiased measures of teacher impact on student test scores Kane and Staiger (2008) conducted an experimental evaluation using data generated in Los Angeles Unified School District where teacher effects were estimated in a preexperimental period (with non-random assignment between students and teachers) and with random assignment. The authors were able to conclude that the usual teacher-value added models are unbiased and relatively accurate predictors of the causal impact of a teacher on
student performance in the short-term finding also significant decay in the impact of a teacher over the years. ${ }^{4}$

Chetty, Friedman, and Rockoff (2014) use tax records, teacher turnover events across schools and classes, and purposely omitted parent characteristics to access the unbiasedness of TVA estimates. They find not only that TVA estimates are unbiased predictors of a teacher's impact on student achievement but also that students have positive long-term benefits for being assigned to high Value-Added teachers, such as higher college attendance rates and salaries. These results favour the use of TVA models.

In an influential study, Rothstein (2008) develops a falsification method based on the idea that current TVA measures cannot "affect" past student scores. Using administrative data on public schools in North Carolina the author shows that this "effect" occurs from $5^{\text {th }}$ grade teachers to learning in $4^{\text {th }}$ grade - which obviously has no causal relationship - and concludes that commonly used TVA models do not obtain causal effects of teacher impacts on student test scores namely because classroom assignments are not independent conditional on typical controls such as student lagged scores and socioeconomic characteristics. However, in a recent study Goldhaber and Chaplin (2015) argue that Rothstein's falsification test can be helpful to identify the existence of student tracking but that this tracking can be the result of lagged achievement, which is a variable that is widely used in TVA models; therefore failing this test may not imply bias. In a theoretical exposition and simulations the authors show that the test will often falsify TVA models that are unbiased and will not falsify TVA models that are biased. The estimation of teacher value added outside the USA and UK is fairly limited ${ }^{5}$. Regarding the Portuguese case, Sousa (2016) analyses the determinants of teacher effectiveness for Portuguese secondary schools using $9^{\text {th }}$ and $12^{\text {th }}$ grade exam scores. The author's findings

[^3]suggest that teachers are responsible for $49 \%$ of the variation in students' performance after controlling for student's lagged test scores, gender, social support, age and internet access. The author also finds that gender, experience, and distance from home are significative determinants of teacher value added.

## III. Portuguese Educational system

The majority of Portuguese students attends public schools ( $87.5 \%$ of the student population in $2014 / 2015)^{6}$. Education is compulsory until the age of 18 and is divided into two stages. The first stage, Ensino Básico, is divided into three cycles. The first one goes from the $1^{\text {st }}$ to the $4^{\text {th }}$ grade, the second comprises $5^{\text {th }}$ and $6^{\text {th }}$ grade and the third cycle goes from $7^{\text {th }}$ to $9^{\text {th }}$ grade. In the second stage, Ensino Secundário, students complete their secondary education and can have access to tertiary education. In the period under study, students performed national exams for Mathematics and Portuguese Language in the end of $4^{\text {th }}, 6^{\text {th }}, 9^{\text {th }}$ and $12^{\text {th }}$ grade. ${ }^{7}$ We use the $6^{\text {th }}$ grade exams as dependent variable and $4^{\text {th }}$ grade exams as control in our estimations.

In primary education students are allocated to a single teacher that teaches most of the materials. From $5^{\text {th }}$ grade onwards students have several courses, namely Mathematics and Portuguese Language, each taught by a different teacher. Students are allocated to schools according to parent's preferences, with students whose home or parent's working location is closer to a given school being given priority to that school.

Regarding teachers, they are allocated to schools based on their preferences, with more experienced teachers and with higher grades upon graduation having priority relative to the other ones. ${ }^{8}$ Within a school, student-teacher pairing is up to the school principal, under the

[^4]administrative principle that it is preferable that teachers and students are placed with the same teacher over the years (e.g. the same math teacher in the $5^{\text {th }}$ and $6^{\text {th }}$ grade), a practice named "pedagogical continuity".

## IV. Data and Descriptive Statistics

In order to produce teacher value-added estimations this study uses an administrative dataset managed by DGEEC ${ }^{9}$. This dataset contains data at the student and teacher levels such as student socioeconomic characteristics as well as records about the school and courses attended. Regarding teacher characteristics it contains information such as teacher's education, training and years of experience.

Student test scores were obtained by merging this dataset with another dataset that belongs to JNE - Júri Nacional de Exames - the entity responsible for managing the Portuguese national exams. These standardized exams are produced by the Ministry of Education and Science and are administered annually in different disciplines depending on the grade students are attending.

## Sample restrictions

To its full extent the dataset contains data from the school year 2006/2007 up to 2014/2015. However, since Portuguese $6^{\text {th }}$ grade students are graded on a scale that goes from 1 to 5 up to the school year 11/12, during this period students' $6^{\text {th }}$ grade exam cannot be approximated by a continuous function. From school year $11 / 12$ until $14 / 15,6^{\text {th }}$ grade exam scores are reported on a scale ranging from 0 to 100 . Therefore we restrict our teacher value-added estimations to these school years. The data comprises four cohorts: the first one, which we will denote cohort 1012, did the $4^{\text {th }}$ grade exam in the school year 09/10 and the $6^{\text {th }}$ grade exam in the school year 11/12

[^5]and the last one, which we will denote cohort 1315 , did the $4^{\text {th }}$ grade exam in the school year $12 / 13$ and the $6^{\text {th }}$ grade exam in the school year 14/15.

Also in order to isolate the effect of a teacher we start by restricting our sample to the cases where a given student $i$ has the same teacher in both the $5^{\text {th }}$ and $6^{\text {th }}$ grade. This is a strong restriction as it excludes students that fail in the $5^{\text {th }}$ grade. Also, it is reasonable to consider that students and teachers that are matched two different years may have specific characteristics, for instance, for being in schools with high turnover rates where teachers don't want to be more than one year. Last but not least, the econometric methods used imply the usage of students for which a $6^{\text {th }}$ and $4^{\text {th }}$ grade exam can be matched.

As pointed out by Kane and Staiger (2002), the measurement error associated with teacher fixed-effects estimations can be problematic: the noise associated with a low number of observations per teacher will lead to the wrong conclusion that best and worst performing teachers will be the ones with a lower number of observations. Therefore based on Aaronson et al. (2007) we restrict our estimations to teachers with a minimum number of 15 observations.

## Sample and Population characteristics

In Table 1 we compare our restricted sample with all students for which a $6^{\text {th }}$ grade exam is recorded from the school year 11/12 onwards, our population of interest. The results presented refer to Portuguese Language exams. The same results apply to the Mathematics exams. Summarizing, our restricted sample includes students that performed a $6^{\text {th }}$ grade exam from the school year $11 / 12$ up to $14 / 15$, had the same teacher on the $5^{\text {th }}$ and $6^{\text {th }}$ grade and this teacher had a minimum of 15 observations between these school years.

Since our group of analysis involves a very large sample, selected by imposing restrictions to the populations, instead of applying the typical t-tests we follow Imbens (2015) and analyse the normalized difference between the mean characteristics of the population and the mean
characteristics of the group of students selected ${ }^{10}$. A difference higher than 0.25 is considered large (Imbens and Woolridge (2007)). This is defined as:

$$
\Delta_{X, k}=\frac{\overline{X_{p, k}}-\overline{X_{s, k}}}{\sqrt{\left(S_{p, k}^{2}+S_{s, k}^{2}\right) / 2}}
$$

where $\overline{X_{p, k}}$ and $\overline{X_{s, k}}$ refer to the average values of variable k in the population and restricted sample respectively, and $S_{p, k}^{2}$ and $S_{s, k}^{2}$ the corresponding variances.

As we can see from the last column of Table 1, there are no significant differences between our restricted sample mean characteristics and the population characteristics with the exception of Age and our measure of the stability of the teaching staff ${ }^{11}$. In the sample we have younger students and schools with a more stable teaching staff.

The restricted sample has 119657 observations, $40.4 \%$ of the population, distributed over 804 schools. Each observation in the restricted sample corresponds to a $4^{\text {th }}$ grade, $6^{\text {th }}$ grade exam score pair that a given student $i$ obtained. Regarding Mathematics we end up with 96494 students $-40.5 \%$ of the population - and 819 schools from the original 882 .

[^6]Table 1 - descriptive statistics

| Variable <br> Student level | Description | Mean |  | Normalized difference |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Population | Restricted Sample |  |
| $A_{i t}^{6}$ | 6th grade exam score | 57.32 | 59.29 | -0.11 |
| national | both parents are Portuguese nationals | 0.94 | 0.96 | -0.06 |
| unemp | one parent is unemployed | 0.16 | 0.14 | 0.08 |
| female | if the student is female | 0.49 | 0.5 | -0.03 |
| age | age in the 6th grade | 10.83 | 10.63 | 0.33 |
| computer | if the family owns a computer | 0.7 | 0.69 | 0.04 |
| mother_higher | the mother has higher education than high-school | 0.17 | 0.18 | -0.04 |
| father_higher | the father has higher education than high-school | 0.11 | 0.11 | -0.03 |
| SS_a | receives social support of type $\mathrm{a}^{12}$ | 0.21 | 0.22 | -0.03 |
| $S S \_b$ | receives social support of type $\mathrm{b}^{12}$ | 0.24 | 0.2 | 0.08 |
| stability_sch | stability of the teaching staff in a school | 0.93 | 0.98 | -0.28 |
| Number of students |  | 296088 | 119657 |  |
| Number of schools |  | 867 | 804 |  |

Note: All variables with the exception of Exam6, age, age_sch and stability_sch are dummy variables.

The restrictions imposed lead to a reduction in the number of teachers being analysed. The number of teachers reduces from 5687 to 3038 for Portuguese and from 5374 to 2497 for Mathematics. In the Population, $25 \%$ of the teachers have less than 22 observations for Portuguese and 24 observations for Mathematics

## V. Empirical Approach

## Teacher Value Added

The Teacher Value Added is obtained by estimating a model of the form:

$$
\begin{equation*}
A_{i t}^{6}=\theta A_{i, t-2}^{4}+\boldsymbol{\alpha} \boldsymbol{X}_{\boldsymbol{i t}}+\boldsymbol{\delta} \boldsymbol{S}_{\boldsymbol{k}(i, t)}+\mu_{j(i, t)}+\sum_{\boldsymbol{m}=\mathbf{1}}^{\boldsymbol{M}} \rho_{m(i, t)} C_{m(i, t)}+\epsilon_{i t} \tag{1}
\end{equation*}
$$

Where $A_{i t}^{6}$ is the $6^{\text {th }}$ grade exam score for student $i$ in year $t, A_{i, t-2}^{4}$ is the exam score in $4^{\text {th }}$ grade, $\boldsymbol{X}_{\boldsymbol{i t}}$ is a vector of student- characteristics in year t described in table 1 . Student $i$ is allocated to

[^7]school $k$ in years $t$ and $t-1$ and $\boldsymbol{S}_{\boldsymbol{k}}$ is a vector of school inputs. The fixed effect of teacher $j$ assigned to student $i$ in years $t$ and $t-1$ is measured explicitly by $\mu_{j}$, the estimated teacher-fixed effect, that is our measure of teacher quality. In other to add robustness to our estimations, cohort level dummies $\boldsymbol{C}_{\boldsymbol{m}}$ are included in the model. These capture possible differences in the exams difficulty over time. School level characteristics $\boldsymbol{S}_{\boldsymbol{k}}$ include the average number of students in the school over the years nstudents_sch, the proportion of students in the school that receive school subsidies, $S S \_a \_s c h$ and $S S \_b \_s c h$; the percentage of students whose mother has tertiary education, mother_higher_sch - the average students age, age_sch; and a measurement of the stability of the teaching staff, stability_sch, presented above.

Implicitly the model assumes that student-level characteristics only impact student achievement on the year the exam is performed with the objective of not increasing the parameters of an already demanding model. The characteristics included however are highly stable over the course of these two years.

Each model is estimated separately for Mathematics and Portuguese Language. The OLS estimation of the model is equivalent to applying the typical within-estimator in the sense that it also produces the same estimators for the coefficients (Cameron and Trivedi (2009)), with the advantage of not considering teacher-fixed effects as nuisance parameters and explicitly including them in the model instead of subtracting the within group average to eliminate them. Cluster-robust standard errors are employed at the school-level in order to take into consideration likely correlations between observations within each school. ${ }^{13}$

## Determinants of Teacher Value Added

[^8]In order to assess the determinants of teacher quality we use the teacher value added measure obtained using equation (1) for teacher j teaching the subject $b$-Mathematics or Portuguese Language - $\widehat{\phi_{J b}}=T V A_{j}$ and we regress them on a vector of teacher observable characteristics. We include both Mathematics and Portuguese Language teachers in the same regression adding also a dummy variable to differentiate between them ${ }^{14}$ :

$$
\begin{equation*}
T V A_{j}=\beta_{0}+\beta_{1} \mathrm{P}_{\mathrm{j}}+\beta_{2} T_{j}+\beta_{3} F_{j}+\beta_{4} E_{j}+\beta_{5} B_{j}+u_{j} \tag{2}
\end{equation*}
$$

where $P_{j} ; T_{j} ; F_{j}$ and $B_{j}$ are dummy variables. $P_{j}$ is equal to one if teacher $j$ changed school in the time period being considered ( denominated changed in regression outputs); $\mathrm{T}_{\mathrm{j}}$ is equal to one if teacher j has an educational level higher than a bachelor degree (higher); $\mathrm{F}_{\mathrm{j}}$ is equal to one if teacher j is female (female) and $\mathrm{B}_{\mathrm{j}}$ is equal to one if he/she teaches mathematics (mathematics). $E_{j}$ is a continuous variable that denotes the number of years of experience (exper). The model was further extended to allow an interaction between experience and gender.

## Peer effects and teacher differential

Using the teacher value added specification presented on the previous section we extend our analysis to investigate whether teacher peer effects can affect a given teacher effectiveness. We also look at the specific impact of a specific characteristic not previously studied in the literature, which we label teacher differential. In order to do this we estimate the model:

$$
\begin{equation*}
T V A_{j}=\beta_{0}+\beta_{1} \mathrm{D}_{\mathrm{j}}+\beta_{3} \overline{T V A_{j k}}+\beta_{4} \overline{D_{j k}}+\beta_{5}\left(\mathrm{D}_{\mathrm{j}} * \overline{D_{j k}}\right)+\boldsymbol{\rho} \boldsymbol{M}_{\boldsymbol{k}}+\boldsymbol{\theta} \boldsymbol{G}_{j}+u_{j} \tag{3}
\end{equation*}
$$

Where $D_{j}$ is the teacher differential, a proxy to the extent a teacher has differentiated teaching capabilities for different kinds of students. $\overline{T V A_{J k}}$ (mean_tva) is the average value added of teacher $j$ colleagues (working in school $k$ ) and $\overline{D_{j k}}$ (mean_differential) represents the school level teacher differential constructed as the average differential of teacher $i$ 's colleagues. An

[^9]interaction term $\left(\mathrm{D}_{\mathrm{j}} * \overline{D_{J k}}\right)$ is included. Since it is considered that teacher value added may influence heterogeneously different types of students we include $\boldsymbol{M}_{\boldsymbol{k}}$, a 1 by 4 vector of the proportion of students with each score from 2 to 5 in the $4^{\text {th }}$ grade Portuguese exams ${ }^{15}$ and $\boldsymbol{G}_{\boldsymbol{j}}$, a vector of teacher observable characteristics presented in equation (2).

The teacher differential $D_{j}$ is obtained by estimating the teacher value added model (1) using subsamples in order to include only low-achieving students (defined as having $4^{\text {th }}$ grade exam scores 1 and 2) or only high-achieving students (defined as having $4^{\text {th }}$ grade exam scores 4 and 5) and taking the difference between the later estimation and the former.

Such an analysis takes into consideration three possible sources of peer effects: from the average effectiveness of the colleagues' $\overline{T V A_{J k}}$ - from their teaching profile $\overline{D_{J k}}$ and from the degree of complementarity between one's teaching profile and one's colleagues $\mathrm{D}_{\mathrm{j}} * \overline{D_{J k}}$. Since our objective is to capture the extent to which the teaching staff effectiveness in a given school may affect an individual teacher's performance, it is important that these indicators $\overline{T V A_{J k}}$ and $\overline{D_{J k}}$ are not contaminated by the teachers own heterogeneity indicator. That is why for every teacher these indicators are calculated excluding each teacher's own values. Also teacher j individual differential $D_{j}$ captures the effect due to the teacher own heterogeneity. School level clustered standard errors are used to consider possible correlation between teacher unobservable characteristics within a school.

Variable $D_{j}$ allows us to understand to which extent teachers that are unequally efficient with different types of students are better teachers on average or not and to which extent working with colleagues with different teaching profiles affects one's teaching ability. The answer to such question is not immediate. One could consider the case of a teacher that puts an extra effort in teaching high achievers since they may provide more immediate and exciting outcomes. If

[^10]this is the case this teacher may have a higher value-added for these students. On the other way around, if we assume a concave educational production function with some degree of substitutability between educational inputs, then with low achievers there is more room for an extra effort from the teacher to have a significant positive impact in student achievement and therefore increase the teacher value added. Obviously a causal interpretation from $D_{j}$ would require the assumption that there are no external factors that affect this proxy for a teacher effectiveness profile and average teacher quality. Even if that is not the case such analysis would still have predictive and interpretative relevance.

The major drawback of such approach is that the inclusion of a teacher colleague's average effectiveness $\overline{T V A_{k}}$ raises relevant questions regarding possible endogeneity problems. It is not unlikely that better teachers may simply be attracted to specific schools and therefore the estimator will be positively biased. However, teacher allocation process in Portugal is relatively well defined and dependent on a teacher's preferences, graduation grade and experience, the latter one of these is controlled in our model. Also, if teacher preferences are determined by the quality of the students studying in the school, that is controlled for in our model, by the percentage of students having each possible $4^{\text {th }}$ grade exam score.

## VI. Results

## Impact of teacher Value Added

In Table 2 we present the estimation results for equation (1). All models presented use a minimum of 15 observations per teacher. The first two models use Portuguese Language exams with and without teacher Fixed-Effects and the last two use Mathematics exams with and without teacher Fixed-Effects. ${ }^{16}$

[^11]Table 2 - Estimation Results

|  | PT (1) | PT (2) | MAT (3) | MAT (4) |
| :---: | :---: | :---: | :---: | :---: |
| $A_{i, t-2}^{4}$ | 10.79*** | 10.86*** | 13.392*** | 13.22*** |
| 1113.cohort | -7.754*** | -7.978*** | -3.362 | -3.34 |
| 1214.cohort | -0.229 | -0.428** | 4.357 | 3.98 |
| 1315.cohort | 9.134*** | 9.271*** | 9.824*** | 10.24*** |
| stability_sch | 0.130 | -0.149 | -7.162** | -2.3 *** |
| SS_a_sch | 5.596 | 6.656*** | 5.360 | 20.6*** |
| SS_b_sch | -8.804** | -4.474*** | -7.162 | -1.53** |
| mother_higher_sch | 6.227 | $5.155^{* * *}$ | 19.55** | 18.3*** |
| age_sch | -2.266 | -2.479*** | 0.575 | $-2.42 * * *$ |
| nstudents_sch | -0.00371 | 0.000188 | -0.00723 | 0.001** |
| $S S_{-} a$ | -1.859*** | -1.954*** | $-3.547 * * *$ | $-3.87 * * *$ |
| SS_b | -3.200*** | -3.369*** | -6.525*** | -6.99*** |
| female | 2.674*** | $2.681^{* * *}$ | -0.316*** | $-0.25 * *$ |
| unemp | -0.406*** | $-0.541 * * *$ | $-1.130 * * *$ | $-1.53 * * *$ |
| national | 0.302 | $0.702^{* * *}$ | 0.612** | 0.83*** |
| computer | 1.090*** | $0.608^{* * *}$ | $1.713 * * *$ | $1.17 * * *$ |
| father_higher | 2.356*** | $2.500^{* * *}$ | 4.196*** | 4.54*** |
| mother_higher | $3.263 * * *$ | $3.475^{* * *}$ | 5.810 *** | 6.12*** |
| age | -2.465*** | -2.582*** | -4.77*** | -4.41*** |
| _cons | 70.28*** | 72.09 *** | 51.13 | 74.81*** |
| Teacher FE | YES | NO | YES | NO |
| $N$ | 119657 | 119657 | 91719 | 91719 |
| $R^{2}$ | 0.481 | 0.430 | 0.5145 | 0.4301 |
| adj. $R^{2}$ | 0.467 | 0.430 | 0.501 | 0.43 |
| AIC | 926644.6 | 937710.7 | 761950.6 | 776646.9 |
| BIC | 916738.5 | 937904.6 | 762139.1 | 776835.4 |
| F | 2483.5 | 2628.6 | 3390.59 | 3542.36 |

Source: authors' calculations
The usage of school and teacher fixed effects is dependent on teachers moving between schools.
This would imply that most of the teachers in our sample would be lost. Besides that, several authors present scepticism about the idea of using school-fixed effects to conduct an analysis with policy implications (Harris 2009) since the teacher effect being captured is not the teacher value added relative to the mean teacher in a country or school district but relative to the mean teacher in the school. Producing accountability measures based on these types of models could generate incentives for teachers to compete between each other within schools. ${ }^{17}$

The F-test for the joint significance of teacher effects reveals they are jointly significant at a $1 \%$ significance level. We find that up to $13 \%$ percent of the variation in student test scores in Portuguese Language exam is determined by Portuguese Language teachers and up to $16 \%$ of

[^12]the variation in Mathematics test scores is explained by Mathematics teachers. ${ }^{18}$ By looking at Table 3 we can see that teachers can have a determinant impact on a student's performance specially in Mathematics: If the $10 \%$ worse teachers were replaced by teachers of the same quality of the best $10 \%$, that would have an impact of at least 10.04 points in the $6^{\text {th }}$ grade national exam for Portuguese and 17.33 points for Mathematics on average.

By looking at the dummy cohorts we see a significant variation in the difficulty of both the Mathematics and Portuguese Language exams. ${ }^{19}$

From a policy point of view there has been a trend in Portugal to increase the stability of the teaching staff ${ }^{20}$. It is therefore relevant to notice that the stability of the teaching staff (stability) is not individually statistically significant under any specification for Portuguese Language. This variable was constructed before any sample restrictions were imposed and it is the probability that a given Portuguese or Mathematics teacher that teaches a school in year t was found in that same school in year t -1. This indicates therefore that, at least for students that are matched with the same teacher in the $5^{\text {th }}$ and $6^{\text {th }}$ grade, being in a school with a stable teaching

Table 3 - Distribution of teacher value added

| Percentile <br> Rank | Portuguese | Mathematics | Descriptive <br> Statistics | Portuguese | Mathematics |
| :--- | :---: | :---: | :--- | :---: | :---: |
| 0.05 | -6.64 | -11.74 | Min | -14.78 | -26.44 |
| 0.10 | -5.16 | -8.94 | Max | 15.88 | 24.06 |
| 0.25 | -2.71 | -4.94 | Mean | -0.13 | -0.35 |
| 0.50 | -0.15 | -0.32 | Std. Dev. | 3.94 | 6.90 |
| 0.75 | 2.42 | 4.26 | Variance | 15.51 | 47.55 |
| 0.90 | 4.88 | 8.39 | Skewness | 0.03 | 0.01 |
| 0.95 | 6.52 | 10.62 | Kurtosis | 3.19 | 3.12 |

[^13]staff has no significant effect on the student's performance or a negative impact as it is the case with mathematics. It is possible, however, that the stability of the student own teacher - that is having the same teacher in the $5^{\text {th }}$ and $6^{\text {th }}$ grade - is relevant for the learning process, which cannot be captured by our model.

At the individual level we can see that students under lower socioeconomic conditions have on average worse performance. Individual socioeconomic variables such as $S S \_a, S S \_b$, unemp, father_higher, mother_higher and computer, account for up to $23 \%$ of the variation in 6th grade exam scores for Portuguese exams and $28.5 \% .^{21}$ We can also see that female students have on average more 2.674 points (on a scale from 0 to 100 ) on the national exam than male students for Portuguese, a result also find by Slater, Davies, and Burgess (2012) but underperform in mathematics on average by -0.316 . We can also see that an individual' mother tertiary education is more relevant for a student's performance on average than a father's tertiary education with the Wald test for their difference yielding a p -value close to 0 .

## Determinants of Teacher value added

In Table 4 model (2) is represented by column output (2). Column (1) reproduces the same model while adding an interaction term between experience and gender. We can see that connecting the student/teacher records with the teacher's characteristics does not allow the matching of all teachers. Table 5 presents descriptive statistics for the variables used. A striking result is that at most $1.2 \%$ of the variability in teacher value added can be explained by observable characteristics, a value in line with what is found in the literature for the variables included in the model. Leigh (2010), finds that variables such as having a master's degree, gender, and experience explain less than $1 \%$ of the variability in teacher quality.

[^14]Table 4 - estimation results for model (2) - Portuguese and Mathematics

|  | $\mathbf{( 1 )}$ | $\mathbf{( 2 )}$ |
| :--- | :---: | :---: |
| changed | -0.170 | -0.159 |
| higher | -0.414 | -0.390 |
| female | -0.364 | $1.539^{* * *}$ |
| exper | $-0.0596^{*}$ | 0.000370 |
| (female X exper) | $0.0708^{*}$ |  |
| mathematics | 0.0832 | 0.0667 |
| _cons | 0.137 | $1.473^{* * *}$ |
| $N$ | 3963 | 3963 |
| $R^{2}$ | 0.012 | 0.011 |
| adj. $R^{2}$ | 0.010 | 0.010 |
| AIC | 24771.5 | 24773.9 |
| BIC | 24815.5 | 24811.6 |
| F | 6.828 | 7.557 |

Using a more comprehensive dataset Aaronson et al. (2007) finds that a model that includes tenure and advanced degrees indicators explain at most $1 \%$ of the variation. When teacher ethnicity, experience, certifications and university ranking are included the $R^{2}$ increases to $7.7 \%$

Source: author's calculations using MISI and JNE p-values:

* $\mathrm{p}<0.1,{ }^{* *} \mathrm{p}<0.05,{ }^{* * *} \mathrm{p}<0.01$

Table 5 - Descriptive Statistics

| Variable | Mean | Std. Dev. | Min | Max |
| :--- | :---: | :---: | :---: | :---: |
| changed | 0.03 | 0.17 | 0.00 | 1.00 |
| higher | 0.08 | 0.28 | 0.00 | 1.00 |
| female | 0.84 | 0.36 | 0.00 | 1.00 |
| exper | 26.48 | 7.15 | 0.87 | 39.97 |
| mathematics | 0.49 | 0.50 | 0.00 | 1.00 |



Figure 1 - Decomposition of Experience effects by gender (male teachers decreasing)
decompose experience by gender a different result emerges. In Figure 1 we show the average marginal effects of experience decomposed by gender.

We can see that female teachers are indeed not affected by experience in a statistically significant manner. Male teachers on the other hand are negatively affected by experience in a
statistically significant manner after 21 years of experience, an effect that is accentuated over the years. Our sample includes 634 male teachers and the first quartile of experience is 21.8.

It is worth nothing that in this sense experience is most likely capturing the ageing process of the teacher and not the number of years working in the profession.

It seems that there is a depreciation in teaching abilities for male teachers over time which is not verified (at least in a significant manner) for female teachers. Since studies analysing teacher quality determinants analyse the effects of experience in an aggregate fashion, the negative signal provided by male teachers is not enough to show experience as being statistically relevant.

From a policy point of view this implies that reforms that delay the retirement of teachers, especially male teachers, may generate an opportunity cost in terms of human capital accumulation for students.

## Peer effects and teacher differential

In Table 6 we present the estimation output for the model in eq. (3) an extension of eq. (2). The models simply differ on the covariates being included. Using column (1) as our base of analysis we can see that mean teacher peer effect is individually statistically significant at a $1 \%$ significance level. An increase in one point in a given teacher colleague's value added increases this teacher value added by 0.772 points on average, ceteris paribus. In other words a 1SD improvement on the mean peer effectiveness improves a teacher value by $0.34 \mathrm{SD} .{ }^{22}$ Using other specifications, (columns (2) and (3)) results do not change.

[^15]Table 6 - Determinants of teacher value added

|  | $\mathbf{( 1 )}$ | $\mathbf{( 2 )}$ | $\mathbf{( 3 )}$ |
| :--- | :---: | :---: | :---: |
| teacher_differential | -0.00820 |  |  |
| mean_tva | $0.772^{* * *}$ | $0.776^{* * *}$ | $0.785^{* * *}$ |
| mean_differential | $-0.0405^{* *}$ | $-0.0407^{* *}$ |  |
| (teacher_diff X mean_diff) | $0.00490^{*}$ |  |  |
| changed | -0.156 | -0.164 | -0.164 |
| higher | -0.234 | -0.229 | -0.240 |
| female | -0.327 | -0.340 | -0.353 |
| exper | $-0.0617^{*}$ | $-0.0629^{*}$ | $-0.0635^{*}$ |
| (female Xexper) | $0.0705^{* *}$ | $0.0707^{* *}$ | $0.0706^{* *}$ |
| mathematics | 0.0840 | 0.113 | 0.110 |
| School_grades2_pt | $45.00^{* *}$ | $45.52^{* *}$ | $41.98^{* *}$ |
| School_grades3_pt | $43.78^{* *}$ | $44.20^{* *}$ | $41.04^{* *}$ |
| School_grades4_pt | $43.85^{* *}$ | $44.29^{* *}$ | $40.77^{* *}$ |
| School_grades5_pt | $46.74^{* *}$ | $47.28^{* *}$ | $43.84^{* *}$ |
| _cons | $-43.80^{* *}$ | $-44.19^{* *}$ | $-40.82^{* *}$ |
| $N$ | 3943 | 3943 | 3943 |
| $R^{2}$ | 0.134 | 0.133 | 0.132 |
| adj. $R^{2}$ | 0.131 | 0.130 | 0.130 |
| AIC | 24137.6 | 24138.2 | 24139.1 |
| BIC | 24231.8 | 24219.8 | 24214.5 |
| F | 59.07 | 68.67 | 73.60 |

Source: author's calculations using MISI and JNE p-values: * p<0.1, ** $\mathrm{p}<0.05,{ }^{* * *} \mathrm{p}<0.01$; In order to give robustness to our analysis schools where less than 5 teachers have an estimated teacher value added or with less than 10 teachers in the population records are excluded, the number of schools being analysed is 504 .

However we can see that there is a statistically significant negative relationship between the colleagues differential and a given teacher effectiveness: working with colleagues where the average difference between the value added for high achievers versus low achievers increases by one point decreases the individual teacher value added by 0.0405 points. So it is not only the colleague's average effectiveness that matters, the difference in effectiveness is also relevant.

Interestingly there are statistically significant gains from this asymmetry at a $10 \%$ significance level, although of a weaker magnitude, as can be seen by the interaction term, which suggests that there are gains of complementarity between a given teacher differential and her/his colleagues: even if a colleague's differential is undesirable this can be slightly mitigated for the individual teacher if her/his differential matches the one of the colleagues.

## VII. Conclusion

Under a teacher value added approach this study uses data on 119657 Portuguese students of Portuguese Language and 91719 students of Mathematics to produce the most comprehensive teacher value added estimation made in Portugal in terms of the number of schools, teachers and students considered. Also it is the first one for $5^{\text {th }}$ and $6^{\text {th }}$ grade teachers. We found that teachers are a relevant component of student achievement: Replacing the $10 \%$ worse teachers by teachers of the same quality of the best $10 \%$, would have an impact of at least 10.04 points in the $6^{\text {th }}$ grade national exam for Portuguese and 17.33 points for Mathematics on average (in a scale from 0 to 100).

The determinants of teacher effectiveness were analysed. We show that observable characteristics play a residual role in the determination of teacher value added. Also it was clarified the nature of the relevance of experience and effectiveness differences among male and female teachers. We show that the difference in effectiveness found between male and female teachers can be accounted by the depreciation of male teacher's effectiveness. This contradicts the general finding that experience is not relevant after the first years in the profession.

It was also shown the existence of relevant peer effects at the teacher level within a school as well as the existence of gains in having an appropriate matching between a teacher teaching profile and a school teaching profile.

Although measurement error was taken into account, further lines of research could be followed in order to tackle this common problem in a more sophisticated manner that would not require the restrictions in the number of observations per teacher. In order to produce such estimates researchers such as have been developing empirical Bayesian statistical methods that weight teacher estimates based on estimated measurement errors associated with them. Kane and Staiger (2008) and Chetty, Friedman, and Rockoff (2014) constitute some examples of such approach.

As we have shown the group of students in our estimation sample is mostly representative of the general populations. But since we impose that teachers follow the same student for at least two years, the teachers being captured have lower turnover rates. Therefore although our results can be extrapolated to the population of teachers that are associated with a given school (Professores do quadro de agrupamento) the extrapolation of results to high turnover teachers (such as Professores contratados) should be done with caution.

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[^0]:    ${ }^{1}$ The standard deviations in the 6th grade exam score distributions are 16.125 for Portuguese and 22.1057 for Mathematics

[^1]:    ${ }^{2}$ Hanushek, Kain, O'Brien, and Rivkin (2005) find that a one SD increase in teacher quality raises student test scores by 0.22 SD in math; Rockoff (2004) an impact of 0.1 SD in both reading and math; Aaronson, Barrow and Sander (2007) an impact of 0.13 grade equivalents in math scores. Hanushek, (2011), finds significative economic impacts from studying with higher quality teachers.

[^2]:    ${ }^{3}$ Specifically the author finds that reading and test scores differ on 0.17 SD on average between beginning teachers and teachers with more than 10 years of experience.

[^3]:    ${ }^{4}$ Unfortunately this study does not contemplate how the teacher's allocation process between schools can affect the bias of TVA estimates, since the randomization was made within each school.
    ${ }^{5}$ Leigh (2010) in Australia, constitutes a noteworthy examples.

[^4]:    ${ }^{6}$ DGEEC - Direcção Geral de Estatisticas da Educação e Ciência - state entity that provides quantitative support to the Ministry of Education
    ${ }^{7}$ During secondary education students also perform other national exams besides Mathematics and Portuguese Language.
    ${ }^{8}$ Teachers may be associated with a school district (Professores de Quadro de Agrupamento) or not (Professores Contratados). The former apply every four years to some of the 10 national administrative divisions of the Portuguese educational system, Quadros de Zona Pedagógica (QZP) and to schools within the QZP. The latter

[^5]:    apply every year directly to schools without having a permanent link to the school; if they are not placed they go to a pool of teachers that are used to answer to emergency staff needs along the school year.
    ${ }^{9}$ DGEEC - Direcção Geral de Estatisticas da Educação e Ciência - produces statistical and quantitative analysis regarding science and education.

[^6]:    ${ }^{10}$ As pointed by Imbens (2015), the t-statistic is equivalent to the normalized difference between means multiplied by the square root of the number of observations in the sample. Therefore large samples would always have a tendency to have very large $t$-statistics. In practice we are analysing the degree of overlap between the covariate distributions of the two groups.
    ${ }^{11}$ Calculated as the probability that a Portuguese Language or Mathematics teacher in school-year t in school k was also in that school in the year t-1.

[^7]:    ${ }^{12}$ Social support given to low income families where Type A subsidies are given to the poorest families. It fully supports school meals and provides a voucher of 13EUR for school materials. Type B subsidies give half these benefits.

[^8]:    ${ }^{13}$ As noted by Todd and Wolpin (2003) and Harris and Sass (2006) teacher value added models require the imposition of assumptions regarding the nature of the learning process that are often disregarded by researchers. These include the assumption that the impact of educational inputs is age-invariant (its impact depends solely on the time the input was implied and the time span since its application, not the age it was applied) and additive separable with an equal rate geometric decay. However as the literature points out, even under these assumptions there is strong evidence that favours the unbiasedness of teacher Value-Added estimations.

[^9]:    ${ }^{14}$ The model was also estimated separately for each type of teacher. The results where equivalent to the ones presented.

[^10]:    ${ }^{15}$ The proportions for Mathematics and the proportion of students having score 1 in the Portuguese Language exam were excluded due to being highly multicollinearity with the included proportions for Portuguese Language

[^11]:    ${ }^{16}$ The same estimation procedure was applied for models with no restrictions on the minimum number of observations per teacher, and a minimum of 30,60 and 90 students per teacher as well as without cluster-robust SE. In all of these cases the results were equivalent to the ones presented here.

[^12]:    ${ }^{17}$ Also when using model selection indictors a model that includes school fixed effects does not outperform a model without. The adjusted $R^{2}$ falls from 0.467 in the base model to 0.229 in the model that includes school fixed effects.

[^13]:    ${ }^{18}$ These values serve as an upper-bound of the explanatory power of teacher-effects and correspond to the $\mathrm{R}^{2}$ of an equation containing solely teacher fixed effects. The lower bound is obtained by the absolute variation in $\mathrm{R}^{2}$ when we exclude the teacher fixed effects from a base model containing student and school characteristics. These values are $5.1 \%$ for Portuguese and $6.4 \%$ for Mathematics
    ${ }^{19}$ Using model (1) as our base of analysis we can see that the cohort 1113 have on average less 7.754 points on the $6^{\text {th }}$ grade exam score than the students in the cohort 1012, the base cohort. But students in the cohort 1315 have on average 9.134 more points in the $6^{\text {th }}$ grade exam score than the average student in the base cohort. It is unlikely that such a variation is explained by overall student improvement in the 2 years of difference between these cohorts.
    ${ }^{20}$ See "Decreto- Lei $n^{\circ} 20 / 2006$ " (Law n${ }^{\circ} 20 / 2006$ ) and "Decreto-Lei $n^{\circ} 83 \mathrm{~A} / 2014$ " (Law n${ }^{\circ} 83 \mathrm{~A} / 2014$ ) for legal descriptions of the reforms.

[^14]:    ${ }^{21}$ This value serves as an upper-bound indicator for the amount of variation explained by the referred variables. It corresponds to a $\mathrm{R}^{2}$ from regressing student scores solely on socioeconomic variables. The lower bounds are $3.4 \%$ for Portuguese Language and $5.04 \%$ for Mathematics and are obtained as the absolute variation in the $\mathrm{R}^{2}$ that occurs when individual characteristics are excluded from the base model.

[^15]:    ${ }^{22}$ The standard deviation in Mean Peer Effectiveness is 2.5, since the coefficient associated with it is 0.772 an improvement in 1 SD in mean peer effectiveness increases the teacher value added by 1.93 points which is equivalent to an average improvement of 0.34 SD in Teacher Effectiveness when we analyse both classes combined. Separately it corresponds to an improvement in 0.49 SD for Portuguese Language teachers and 0.28 SD for Mathematics teachers.

