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Harvest Control Rule vs Optimal Harvesting of an Age-structured Population:
The Case of the Ibero-Atlantic Sardine Fishery

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Abstract

In this study we apply an age-structured bio-economic model to the Ibero-Atlantic sardine stock, and compute an optimal harvesting strategy. We compare it with the ongoing harvest control rule, which overlooks economic incentives. We show that the optimal plan entails greater net returns from the fishery, though at a cost of reducing biomass below acceptable reference points. By incorporating precautionary constraints, we find that an optimal plan still yields higher economic returns, while better adhering to stock-recovery objectives.

Keywords: Fishery management; Harvest rule; Optimal harvesting; Age-structured model.

1 Introduction

Fisheries are an important source of income, employment and food provision.¹ Their sustainability, however, has been compromised time and again (FAO 2014). Many stocks are currently overexploited, and managers are looking for innovative ways to address this issue. In the design of policy options for a fishery, control rules are often used to dictate how harvesting is determined. These are frequently based on target reference points related to the size of the stock biomass. However, economic criteria appear to be absent from the process of determining allowable catch (Anderson 2013), despite it being essential not only for efficiency considerations, but also to ensure the compliance of the proposed regulations.

The Ibero-Atlantic sardine is one example of a species currently under pressure. Sardine is a pelagic fish distributed along the continental shelf of the Atlantic Ocean

¹ According to State of the World Fisheries 2014 (FAO 2014), more than 10% of the world's population depend on fisheries for their livelihoods.

(ICES Divisions VIIIc and IXa). In the last decades, catches have exhibited fluctuations, having peaked in 1981 with more than 200,000 tons of landings, but showing a general decrease ever since, with around 46,000 tons in 2013 (ICES 2014). Biomass also shows extensive variation. It is argued that in the early 90's the sardine population may have been more than two times its current size. This accentuated decrease is attributed to low recruitment success and overfishing (ICES 2013b). All catches are taken by Spanish and Portuguese fleets, but EU regulation does not establish Total Allowable Catches (TACs) for the fishery. Instead, a reference-point stock-rebuilding plan has been enforced by the two countries based on biological indicators. However, nowhere in the design of the harvest control rule (HCR) does there appear to be an estimation of the economic rents of alternative policies.

In this paper we apply an age-structured bio-economic model to the Ibero-Atlantic sardine stock to determine the harvesting policy that maximizes economic returns over time. The choice of an age-structured model hinges not only on the fact that technical regulation of fisheries is becoming more prevalent within a cohort-based framework², but more importantly, because the HCR for the sardine fishery was designed using age-structured information.

Our main objective is to assess quantitatively the performance of the currently implemented rule, comparing it with an optimal plan delivered by a model that explicitly incorporates economic considerations. We start by determining the equilibrium stock levels under both plans, as well as the economic returns associated with their transition paths. However, it will be shown that a profit maximizing trajectory leads to a non-precautionary stock level. As a consequence, we induce several stock rebuilding strategies and evaluate how they perform against the ongoing HCR. We find that there may be great losses in efficiency under the currently implemented rule, either from an economic or biological perspective. This result carries important management implications, considering the vulnerable state of the sardine stock, and the historically low levels of allowed catch for the fishery.

The paper is organized as follows. The next section briefly reviews the literature on bio-economic modeling in fisheries, while motivating our discussion in light of existing studies. Section 3 presents the age-structured bio-economic model. In Section 4 we provide a brief description of the Ibero-Atlantic sardine fishery, and calibrate the model to empirical data. Section 5 presents our results, and in Section 6 we conclude.

2 Literature Review

Economists have sought to create the appropriate environment to establish a clear link between economics and biological systems. Bio-economic modeling in fisheries is a mathematical representation that integrates biological components to represent the natural

² Mesh sizes or other gear selectivity measures, moratoria and area closure (FAO 2014).

resource dynamics, and economic components to represent resource users (Grafton *et al.* 2004, Clark 2010). Their objective is to determine the optimal level of resource extraction that maximizes economic profits, this way providing an important support for decision making.

Traditionally, bio-economic analyses are based on modeling fish populations as a uniform dynamic quantity - the resource biomass (Schaefer 1954, Smith 1969, Clark 2010). While these provide valuable insights over basic fishery economic principles, one crucial question is the level of biological detail that is needed to address specific problems, such as reproduction success or age-specific vulnerability to fishing. Age-structured models are increasingly used in the analysis of fisheries management, as they include greater detail regarding fish stock structure, individual weights, sizes, fecundity and mortality characteristics.

In age-structured bio-economic models the optimal harvest strategy specifies the number and age of harvested individuals (see Hilborn and Walters (1992), and Quinn and Deriso (1999) for a review of age-structured models). However, simplifying assumptions such as imperfectly selective gear, would generally lead to pulse fishing as an optimal strategy —a highly irregular policy of stock rehabilitation that may be ill advised, especially if vessel capital is imperfectly malleable (Clark *et al.* 1979). Nevertheless, some pioneering studies have embodied more realism, and thus complexity, in the multicohort dynamics, discussing smoother harvesting profiles in the event of risk aversion, adjustment costs, endogenous price and cost of catch (Hannesson 1975, 1988, Kennedy 1992), as well as stochastic recruitment to account for environmental uncertainty (Getz 1984).

Recently, some authors have also employed age-structured models in their studies (Bjørndal and Brasão 2006, Stage 2006, Tahvonen *et al.* 2013). Economic research in multicohort fisheries is almost exclusively based on case studies and numerical computations, that are not easily accessible as a management tool. However, Tahvonen (2008, 2009a,b) has developed analytical and numerical results on the optimal harvesting problem of an age-structured population, with a more general, and theoretical understanding of its optimization problem. The greatest features of Tahvonen's model are its simplicity, and flexibility to changes under different simplifying assumptions, such as the objective function specification, cohort dynamics, gear selectivity and other economic considerations.

One important contribution of bio-economic models is the estimation of the economic returns of different fishing policies. Sandberg (2005), for instance, although not determining an optimal harvest plan, compares a target escapement harvest rule with an already established reference-point rule for the Norwegian spring spawning herring, showing under which conditions one may yield higher economic returns than the other. Eikeset *et al.* (2013) by developing a bio-economic model for the Northeast Arctic cod fishery derive different HCRs, and conclude that the HCR that maximizes profits is the most precautionary among all. More recently, Yamazaki *et al.* (2015) report simulation results on how HCRs and no-take marine reserves perform in stock recovery plans, using

a lumped-parameter bio-economic model.

Our study can be included in the body of literature on the optimal harvesting strategy of an age-structured population. We adopt Tahvonen *et al.* (2013) model specification, but we modify some of its attributes to reflect the sardine population dynamics, such as the inclusion of intra-seasonal growth, and the specification of a different timing for the harvesting season to occur (Tahvonen 2009b). Additionally, and unlike other bio-economic studies, rather than simply comparing between alternative policies or formulating reference-point rules that are optimized for different objectives, we compute an optimal harvesting trajectory and compare it with a currently implemented harvest rule.

3 Model

We adopt the modeling framework developed in Tahvonen (2009b) and Tahvonen *et al.* (2013).

There are n age classes. The number of individuals in age class $s = 1, \dots, n$ in year $t = 0, 1, \dots$ is denoted by x_{st} (in 10^9 individuals). Let γ_s be the proportion of stock in the s -th age class which is sexually mature, and w_s the average weight-in-stock at spawning time. The spawning stock biomass, x_{0t} (1,000 tons), is given by

$$x_{0t} = \sum_{s=1}^n \gamma_s w_s x_{st} \quad (3.1)$$

We assume that spawning occurs in the beginning of each period, and only a fraction of the eggs will survive as recruits. Let ϕ denote the spawner-recruit relation, where the number of young fish (recruits) entering a population is related to the number of parent-fish (spawners). The next period number of recruits is given by

$$x_{1,t+1} = \phi(x_{0t}) \quad (3.2)$$

Let m_s denote the age-specific instantaneous natural mortality rate within each period. The fish population is also subject to fishing mortality, or harvesting, that we assume to happen after recruitment and at the middle of each period.³ If fishing mortality is zero, a fraction e^{-m_s} of an age class $s = 1, \dots, n$ will survive for the next period.⁴ After half a year, $e^{-\frac{m_s}{2}}$ corresponds to the fraction that is still alive. Given an effort level⁵ E_t , harvest from age class $s = 1, \dots, n$ is determined by the age-specific catch-effort

³ Sardines spawn in winter months which we assume to correspond to the beginning of each period, whereas catches are mostly taken in summer months. Hence our assumption of a different timing for the harvest season.

⁴ If we let m_s denote the instantaneous natural mortality (units t^{-1}), and treating mortality as negative population growth, the population numbers evolve according to $\frac{dx_s(t)}{dt} = -m_s x_s$, with $x_s(0) = x_{s0}$. The solution to this differential equation is $x_s(t) = x_{s0} e^{-m_s t}$. After one year, the size of age-class s is $x_s(1) = x_{s0} e^{-m_s}$. Thus the yearly natural survival fraction is e^{-m_s} .

⁵ Effort is commonly referred as an aggregate measure of several economic inputs devoted to fishing.

relationship $h_{st} = q_s E_t e^{-\frac{m_s}{2}} x_{st}$, where q_s are the age-specific catchabilities (Schaefer 1957). The stock updating equations can be defined as

$$x_{s+1,t+1} = e^{-\frac{m_s}{2}} (e^{-\frac{m_s}{2}} x_{st} - h_{st}), \quad s = 1, \dots, n-2 \quad (3.3)$$

$$x_{n,t+1} = e^{-\frac{m_{n-1}}{2}} (e^{-\frac{m_{n-1}}{2}} x_{n-1,t} - h_{n-1,t}) + e^{-\frac{m_n}{2}} (e^{-\frac{m_n}{2}} x_{nt} - h_{nt}) \quad (3.4)$$

where Eq.(3.4) for the oldest age class shows that fish in this cohort remain there, if they have survived natural and fishing mortality.

The total annual catch H_t (1,000 tons) can be obtained by summing harvest over all age classes, evaluated at their average weight-in-catch⁶ w_s^c , and is equal to

$$H_t = \sum_{s=1}^n w_s^c h_{st} = \sum_{s=1}^n w_s^c q_s E_t e^{-\frac{m_s}{2}} x_{st} \quad (3.5)$$

To accommodate H_t as the control variable, we can rewrite our stock updating equations explicitly as a function of total catch since by Eq.(3.5) we have

$$E_t = \frac{H_t}{\sum_{s=1}^n w_s^c q_s e^{-\frac{m_s}{2}} x_{st}} \quad (3.6)$$

Thus Eqs.(3.3) and (3.4) can be rewritten as

$$x_{s+1,t+1} = e^{-m_s} x_{st} - H_t G_{st}, \quad s = 1, \dots, n-2 \quad (3.7)$$

$$x_{n,t+1} = e^{-m_{n-1}} x_{n-1,t} + e^{-m_n} x_{nt} - H_t (G_{n-1,t} + G_{nt}) \quad (3.8)$$

where

$$G_{st} = \frac{e^{-m_s} q_s x_{st}}{\sum_{s=1}^n q_s w_s^c e^{-\frac{m_s}{2}} x_{st}}, \quad s = 1, \dots, n \quad (3.9)$$

are functions that transform total harvest, H_t , into the number of harvested individuals in a given age class s at time t .

Notice that the age-specific harvest strategies, h_{st} , are found by allocating total harvest, H_t , among all age classes using the catchability coefficients and stock numbers.⁷ It follows that harvest is non-selective, that is, different age-classes cannot be harvested independently. To make sure that the number of harvested fish does not exceed the number that exists in a given age-class, i.e., $h_{st} \leq e^{-\frac{m_s}{2}} x_{st}$, we place an additional set of restrictions that can be formulated as complementary constraints

$$h_{st} - H_t G_{st} e^{\frac{m_s}{2}} + y_{st}^1 = 0 \quad (3.10)$$

⁶ In contrast to weight-in-stock (w_s) which corresponds to weight at the time of spawning, weight-in-catch (w_s^c) corresponds to weight at the time of harvesting. These will be different as sardine weight increases substantially in summer months.

⁷ Since we modified our equations to accommodate H_t as the control variable, the age-specific production function will now be given by $h_{st} = H_t G_{st} e^{\frac{m_s}{2}}$, for $s = 1, \dots, n$.

$$h_{st} - e^{-\frac{m_s}{2}} x_{st} + y_{st}^2 = 0 \quad (3.11)$$

$$y_{st}^1 \geq 0, \quad y_{st}^2 \geq 0, \quad y_{st}^1 y_{st}^2 = 0 \quad (3.12)$$

where y_{st}^i for $i = 1, 2$, $s = 1, \dots, n$ and $t = 0, 1, \dots$ are slack variables.

Let π denote a concave and twice differentiable function for annual net revenues from the fishery that depends on total catch H_t . Let $d = \frac{1}{1+i}$ be the discount factor, with i as the interest rate. The problem consists of finding the harvest strategy that maximizes the present value of net economic returns, given the dynamics of an age-structured fish population. Thus we have a discrete-time optimal-control problem. Our optimization problem becomes

$$\max_{H_t} \sum_{t=0}^{\infty} d^t \pi(H_t) \quad (3.13)$$

subject to (3.1),(3.2),(3.7)-(3.12), an initial condition x_{s0} for each age class $s = 1, \dots, n$, $H_t \geq 0$ and $x_{st} \geq 0$, for each $s = 1, \dots, n$ and $t = 0, 1, \dots$

4 Ibero-Atlantic Sardine Fishery

Sardine (*Sardina Pilchardus*) is a pelagic fish that forms large schools distributed along the Atlantic coast, delimited in the north by southern Biscay, and by the Strait of Gibraltar in the south (ICES Divisions VIIIc and IXa). It is one of the most important species for the Portuguese fishing community and industry.

The International Council for the Exploration of the Sea (ICES) publishes data on sardine landings dating back from 1940. Catches have fluctuated considerably, showing a peak in 1981 with roughly 217,000 tons, and an overall downward trend ever since, reaching a minimum of approximately 46,000 tons in 2013. Also, from 1993 onwards, the combination of high fishing pressure and low recruitment resulted in a decrease of sardine biomass from over 900,000 tons, to less than 200,000 tons from 2011 onwards (Fig.1).

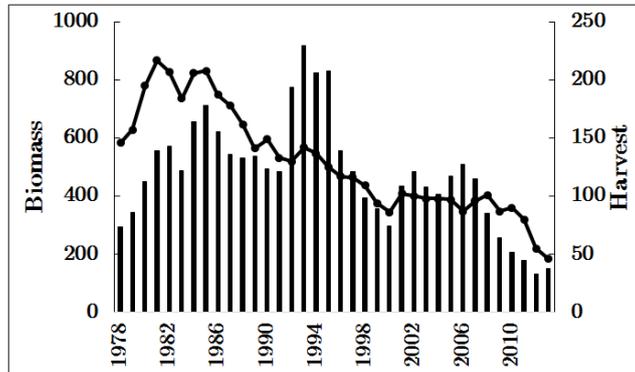


Figure 1: Sardine historical landings (line) and biomass (columns) (ICES 2014). Values in 1,000 tons

Over the last 35 years, sardine has shown extreme variation in recruitment, ranging from over 48 billion to less than 3 billion individuals (ICES 2014). Spawning takes places mostly in the winter months, and strong year classes are thought to be due to favorable environmental conditions (ICES 2013b). Another important biological factor is intra-seasonal growth. Sardine weight increases substantially in the summer months, the time at which most harvest takes place (INE 2008-2014).

The sardine population in ICES Divisions VIIIc and IXa is sufficiently discreet to be considered as a single stock for management purposes (ICES 2013b). Currently, there is no formal international TAC, but in order to ensure the recovery of the sardine stock, Portugal and Spain have developed a management plan that includes, among other measures, a limitation of total catches.⁸ The plan was developed in 2013 and took effect in 2014 (ICES 2013b). It consists of a reference-point strategy for the determination of harvest. The rule fixes a maximum TAC of 86,000 tons if biomass is greater than 368,400 tons.⁹ If below that threshold, catches are gradually reduced according to a predefined formula, $0.36(B - 135)$, where B is biomass in 1,000 tons. The fishery closes if sardine biomass is found below another reference point, 135,000 tons. Another indicator was set at 307,000 tons, the B_{lim} , above which biomass should lie for stock recovery purposes (Fig.2).

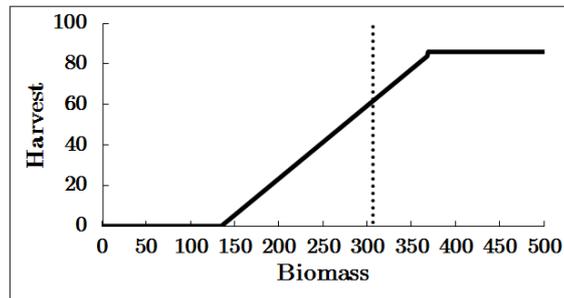


Figure 2: HCR (full line), and B_{lim} reference point (dotted line) (ICES 2013b). Values in 1,000 tons

4.1 Data and Calibration of the Model

The age-specific population parameters are taken from the 2014 ICES WGHANSA report (ICES 2014), and are listed in Table 1. Natural mortality rates (m_s) and maturities (γ_s) correspond to estimates reported by ICES. Input values for the numbers-at-age (x_{s0}) are those used in ICES (2013a) simulations. Catchabilities (q_s) are based on the average age-specific fishing mortalities from 1993 to 2013, where $q_s = 1$ for the age group with the highest fishing mortality, by normalization. Weights-in-stock (w_s) and catch (w_s^c) also correspond to their mean values from 1993 to 2013.

⁸ The quota amount is informally split between the two countries on a 70-30% ratio for Portugal and Spain, respectively.

⁹ The maximum catch was established so as to meet the demand requirements of the transforming industry.

Age-Class	Numbers (10^9)	Maturity (γ_s)	Natural Mortality (m_s)	Weight in Stock (w_s)	Weight in Catch (w_s^c)	Catchability (q_s)
1	6.247	0	0.8	0	0.024	0.130
2	1.652	1	0.5	0.024	0.043	0.371
3	0.636	1	0.4	0.044	0.059	0.697
4	0.276	1	0.3	0.057	0.068	1
5	0.222	1	0.3	0.065	0.074	1
6	0.076	1	0.3	0.070	0.079	1
7	0.234	1	0.3	0.079	0.1	0.325

Table 1: Population Parameters (ICES 2014)

Recruitment is assumed to follow the Ricker (1975) specification

$$\phi(x_{0t}) = ax_{0t}e^{-bx_{0t}} \quad (4.1)$$

where a is the productivity parameter, and b the (inverse) capacity parameter of the Ricker stock-recruitment relationship. Recruits were assumed to follow an autoregressive lag-4 lognormal error. The choice of an AR(4) error structure followed Santos *et al.* (2011) assertion of a cyclic 4-year recruitment periodicity, and later confirmed by inspecting autocorrelation of our model residuals. The stock-recruitment function parameters were obtained by fitting the Ricker model to data collected from ICES (2014) for 1993-2013.¹⁰ We used data on the number of recruits, and biomass of ages 2 – 7, as an indicator of spawning abundance (ICES 2013a). We linearized the Ricker model, and estimated the following equations

$$\ln\left(\frac{R_t}{x_{0t}}\right) = \ln(a) - b \cdot \ln(x_{0t}) + u_t \quad (4.2)$$

$$u_t = \rho u_{t-4} + v_t \quad (4.3)$$

where R_t refers to recruits in 10^9 individuals, x_{0t} is spawning biomass in 10^6 tons, and $\{v_t\}$ are independent normally distributed errors with standard deviation σ_v . We obtained estimates $\ln(\hat{a}) = 3.63$ (0.39), $\hat{b} = 1.61$ (0.85), $\hat{\rho} = 1.43$ (0.2), and $\hat{\sigma}_v = 0.46$ (0.08); standard errors in parentheses.

Because it is assumed that recruitment conditional on parental stock follows a log-normal distribution, Eq.(4.1) corresponds to the median recruit-production and not the mean, or expected recruitment. To allow for long-term trends over recruitment, we accounted for the difference between the median and the mean of a lognormal distribution from an AR(4) process by changing our productivity parameter a , such that $\ln(a') = \ln(a) + \frac{\sigma_v^2}{2(1-\rho^2)}$ (Appendix A). Thus we calibrated the deterministic recruitment function with the corrected estimates $\hat{a}' = 42.9$ and $\hat{b} = 1.61$.

Sardines are mostly caught by purse-seiners (ICES 2014). To estimate the harvesting cost function, we used the available data on variable fishing cost for the Portuguese

¹⁰Based on recruitment series for the sardine stock, ICES (2013a) proposes a separation of the stock in two productivity regimes, before and after 1993. It is argued that the mean productivity (recruits per spawner) of the period after 1993 is a good indicator of future stock productivity.

purse-seiner fleets from STECF (2014) for 2008-2012.^{11,12} In support of the schooling behavior of the sardine species, costs were found to be independent of stock biomass. Thus we estimated the non-linear cost function, $C(H_t) = \beta_0 H_t^{\beta_1}$, by means of the OLS regression

$$\ln(c_t) = \ln(\beta_0) + \beta_1 \ln(l_t) + \varepsilon_t \quad (4.4)$$

where c_t is the sum of variable costs in €million, and l_t is sardine landings in 1,000 tons by purse-seiners in Portugal (INE 2008-2012). The data used is listed in Table 2. We obtained estimates $\ln(\hat{\beta}_0) = -0.9$ (0.64), and $\hat{\beta}_1 = 1.1$ (0.17); standard errors in parentheses. The harvesting cost function is then given by $C(H_t) = 0.4H_t^{1.1}$.

Price is assumed to be independent of age, or size, and equal to the mean price $p = 0.8\text{€}/\text{kg}$ (INE 2008-2012).¹³ Finally, the net revenue function, $\pi(H_t) = pH_t - C(H_t)$, is given by

$$\pi(H_t) = 0.8H_t - 0.4H_t^{1.1} \quad (4.5)$$

Year	Total Landings	Sardine Landings	Sardine Landings/Total	Costs	Price
2008	74.78	56.05	0.75	39.26	0.69
2009	57.96	45.66	0.79	34.38	0.76
2010	67.49	48.86	0.72	32.64	0.68
2011	71.86	45.29	0.63	27.42	0.78
2012	66.59	27.97	0.42	17.78	1.30

Table 2: Purse-seiners total landings and sardine landings in Portugal (1,000 tons) from INE (2008-2012), costs (€million), as the sum of energy, crew, repair and other variable costs from STECF (2014), and price of sardine (€/kg) from INE (2008-2012). Costs and prices in real value, base 2012

5 Results

The objective is to make an investigation of different management plans applied to the sardine stock, comparing the ongoing HCR (Fig.2) to an optimal plan derived from our bio-economic model. The comparison will be based on the calculation of the fisheries' net present value (NPV) from 2014 to 2030, at a constant 5% interest rate. To determine the optimal development plan, we solve the dynamic optimization problem numerically using the Knitro optimization software with AMPL (Byrd *et al.* 2006). The AMPL code is provided in Appendix C.

¹¹ The Scientific, Technical and Economic Committee for Fisheries (STECF) is the entity responsible for publishing information on the structure and economic performance of EU Member States fishing fleets.

¹² There was only one data point available for Spanish purse-seiners costs, thus we assumed Portuguese purse-seiners to be representative of the entire fleet.

¹³ Prices show little variation in the period considered for economic calibration. The price figures in Table 2 correspond to the price of the first sale of sardine landings happening in fish markets by means of a descending price auction.

5.1 Steady-state Analysis

Before comparing the results, it is informative to make a steady-state analysis of the sardine fishery, since it is possible to represent the age-structured equilibrium as in the biomass model (Tahvonen 2009b). If we let x_s , $s = 1, \dots, n$, denote the equilibrium number of individuals, and take effort E as constant, we can obtain the equilibrium yield-biomass relationship from the age-structured model using Eqs.(3.1)-(3.5), and population data from Table 1. Notice that the outcome is not purely derived from biological factors, as it is partly determined by fishing technology as well (Appendix B).

Fig.3 illustrates the parabola relating equilibrium yield and biomass. Our calculations led us to a pristine equilibrium of approximately 700,000 tons of sardine stock, and a maximum sustainable yield (MSY) of about 73,000 tons (with an associated 303,000 tons of biomass). In this deterministic setting, any point along this curve corresponds to a combination of harvest and biomass that can be sustained as a long-run equilibrium. Thus any trajectory, either the one prescribed by the HCR or our optimal plan, is expected to converge to a point along this curve.

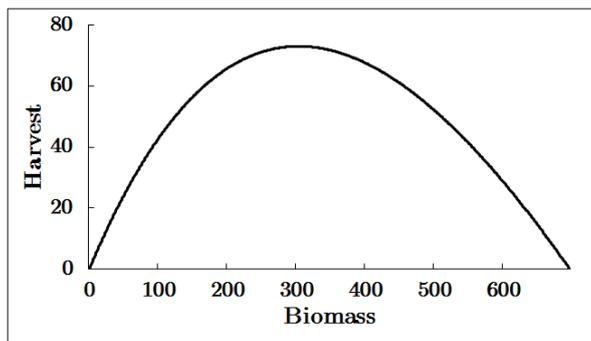


Figure 3: Equilibrium yield-biomass curve. Values in 1,000 tons

5.2 Temporal Dynamics

The yearly development of harvest under the HCR is illustrated in Fig.4B. Harvest in 2014 and 2015 was set at 20,520 tons and 19,095, respectively, representing the quota amounts established for those years. From 2016 onwards, harvest was calculated using the reference-point rule, as a function of our biomass estimates. We can see that both biomass and harvest increase smoothly, until they reach their equilibrium values of roughly 336,200 tons of biomass and 72,400 tons of harvest by 2060 (not shown).

The optimal development, considering our bio-economic model, gives a smooth increase of harvest and biomass (Fig.4C). A 100-period run of our model¹⁴, shows that the steady-state level of harvest is reached at around 70,700 tons, with an associated biomass of approximately 243,400 tons.

¹⁴Since it is not possible to perform a numerical simulation over an infinite-horizon, we made multiple runs with different horizons to make sure that our choice of a time-window did not affect the results.

Notice that long-run harvest under the HCR is maintained at a similar level as that of the optimal plan (at approximately 70,000 tons). However, the optimal plan reduces the stock to a lower equilibrium level by *overharvesting* in the initial periods. This is illustrated in Fig.5. We see that both development paths converge to a point in the equilibrium yield-biomass curve.¹⁵ However, while harvest converges to similar amounts in both trajectories, they will lie in opposite sides of the equilibrium curve. The optimal plan converges to an equilibrium where biomass is below 303,000 tons, the biomass level that sustains the MSY, whereas the equilibrium biomass under the HCR is more precautionary, with more than 330,000 tons.

If we compute the net present value of the two plans for 2014-2030, we obtain an estimated €111.73 million for the HCR, against €126.33 million for the optimal plan, thus amounting to a 13.07% higher discounted value for the sardine fishery.

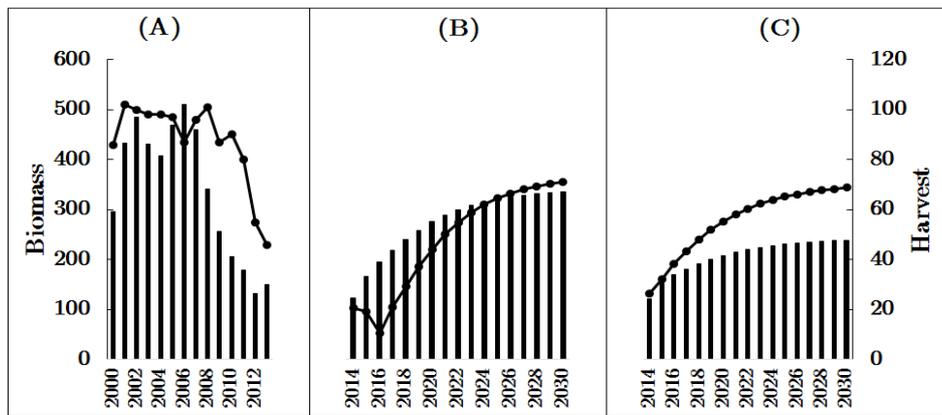


Figure 4: Sardine biomass (columns) and harvest (lines). (A) Historical development from 2000 to 2013 (ICES 2014). (B) Harvest and biomass stock development under the ongoing HCR. (C) Optimal development of sardine stock and catch. Values in 1,000 tons

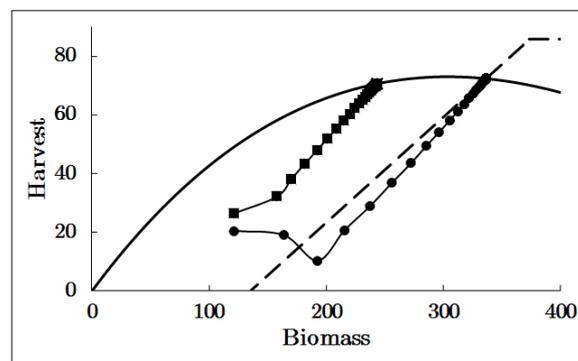


Figure 5: Equilibrium yield-biomass curve (full line), HCR (dashed line), optimal development (squares) and development under the HCR (circles). Values in 1,000 tons

¹⁵Notice that the HCR path converges to the point at which the HCR intercepts the equilibrium yield-biomass curve. The convergence path lies beneath the HCR dashed line since harvest is calculated as a function of biomass in the preceding period.

5.2.1 Precautionary Considerations

In the optimal plan, the sardine population stabilizes below the 307,000 tons B_{lim} reference point, at around 243,400 tons. So low an amount of biomass can be considered non-precautionary. We accommodated ecological concerns by including extra constraints. On one case we guaranteed that sardine biomass had to lie above 307,000 tons by the same period at which the HCR does, i.e., by 2023. On another, we imposed that the HCR equilibrium biomass of 336,200 tons was attained under our optimal plan, yet much sooner, by 2030.¹⁶

Fig.6B illustrates the case where the 307,000 tons lower-limit objective has to be satisfied. Our results show that it would be optimal to increase harvest until 2020, following a similar path to that of the unconstrained problem. Harvest would then decrease, to allow biomass to increase to 307,000 tons in 2023. By then, harvest would stabilize at around 73,000 tons¹⁷, enough to sustain 307,000 tons of biomass from 2023 onwards.

With this precautionary constraint, discounted profits over 2014–2030 would amount to €116.63 million, yielding a 4.4% higher NPV than with the currently implemented HCR in Fig.6A.

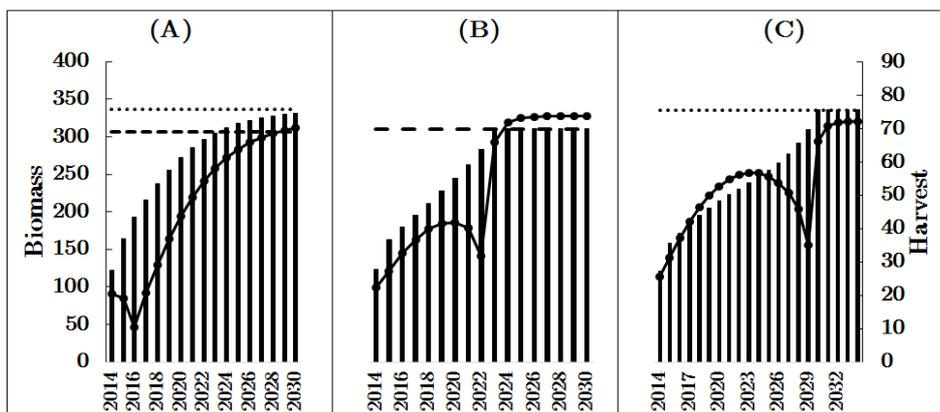


Figure 6: Sardine biomass (columns) and harvest (full lines). (A) Biomass and harvest under the HCR. (B) Optimal development of sardine catch and biomass with precautionary constraint $B \geq 307$ from 2023 onwards, where B is biomass in 1,000 tons. (C) Optimal development of sardine with precautionary constraint $B \geq 336.2$, from 2030 onwards. The horizontal dashed line marks the 307,000 tons B_{lim} reference point, and the dotted line marks the 336,200 tons equilibrium biomass under the HCR. Values in 1,000 tons

By setting a more stringent precautionary measure, in which it is established that by 2030 the sardine stock should not be below 336,200 tons, our results indicate that it is optimal to increase harvesting until 2023, and allow biomass to increase at a decreasing rate. By then, harvest decreases so that biomass can gradually increase (at an increasing

¹⁶The final period we have considered for discounted benefits comparisons.

¹⁷This harvest amount is similar to the MSY, since 307,000 tons is close to the biomass level that sustains the MSY, i.e., 303,000 tons.

rate), and attain 336,200 tons by 2030 (Fig.6C). From then onwards, harvest would be maintained at roughly 72,400 tons, the equilibrium equivalent to the HCR. This strategy still entails higher discounted economic benefits, around €4.43 million more, than the ongoing plan for the sardine fishery.

5.2.2 Stock Rebuilding Strategies

It is possible to generate several other stock rebuilding strategies. By forcing our regulator to comply with different levels of biomass from 2030 onwards, and calculating the NPV for 2014 – 2030 of each trajectory, we can draw a collection of points relating different biomass targets with the discounted returns of their transition paths. Our results are illustrated in Fig.7. We see that the greater the target level of biomass, the greater the reduction in harvest (to allow biomass to increase by more). Thus the smaller the discounted benefits of their adjustment paths.

These points can be compared with the equilibrium level of biomass and associated NPV for 2014 – 2030 under the HCR (Fig.7). The fact that it lies below the negatively sloped curve demonstrates that the ongoing plan prescribes an *underutilization* of the resource both in terms of economic returns and biological indicators. For the same target level of biomass as the equilibrium equivalent under the HCR, we could realize higher profits (€4.43 million more). We could also be more conservationist by allowing approximately 40,000 tons more biomass, when maintaining the same level of discounted benefits as the HCR.

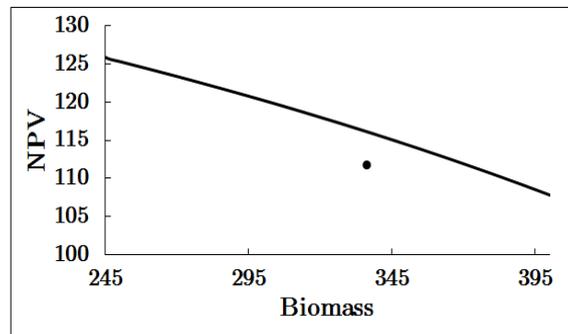


Figure 7: Target level of biomass from 2030 onwards, and associated NPV for 2014-2030 (line). Equilibrium level of biomass and NPV under the HCR (dot). Biomass in 1,000 tons, NPV in €million

This analysis can be complemented by considering how fast we can force a stock rebuilding strategy to attain the equilibrium levels of biomass under the HCR. Table 3 lists the NPV for 2014 – 2030 associated with complying with the 336,200 tons biomass level in 8, 9,..., or 16 years. Since the rule entails a NPV of €111.73 million, we could attain the HCR equilibrium biomass much sooner, by 2024, while still realizing higher economic returns (€376 thousand more). Since a positive interest rate places more weight over near-term profits, any sooner than 2024 would require a bigger reduction in harvest in the initial years and, consequently, a bigger reduction in discounted benefits.

<i>Year</i>	2022	2023	2024	2025	2026	2027	2028	2029	2030
NPV_{14-30}	109.59	111	112.10	113.01	113.79	114.46	115.06	115.61	116.16

Table 3: NPV for 2014 – 2030 (€million) associated with complying with 336,200 tons of biomass in different years

5.3 Sensitivity Analysis

5.3.1 Price

The price was set constant and equal to the average real price, given that over the period considered for the economic calibration of our model prices showed little variation. More recently, however, prices have behaved differently. There has been an increase in price, now that harvest has reached historically low amounts. In fact, over only three years, from 2011 to 2013, it has increased about 83%, from 0.8 to 1.4€/kg (INE 2000 – 2014). However, historical records did not prove helpful for estimating a price elasticity of demand.¹⁸

In turn, we increased the price exogenously to capture some of its implications in terms of net discounted benefits. Fig.8B illustrates the effect of increasing the price from 0.8 to 1.5€/kg on the NPV for 2014 – 2030. Not surprisingly, discounted profits increase as the price level increases. We can also see that the more the price increases, the greater the NPV of the optimal plan relative to the HCR (from 13.07% more when $p = 0.8$ €/kg, to 15.38% when $p = 1.5$ €/kg).

It is interesting to note that increasing the price does not affect the long-run levels of biomass and harvest under the optimal plan, i.e., 243,400 and 70,700 tons, respectively. Only the transition paths change. The higher the price, the more rapidly are the steady-state levels of biomass and harvest attained. In Fig.8A we can see that increasing the price from 0.8 to 1.5€/kg would entail a lower harvest amount in the initial periods (in 2014, with 0.8 and 1.5€/kg, catch would amount to 26,500 tons and 6,400 tons, respectively). But also that harvest would converge to its long-run equilibrium much faster (by 2020, harvest would amount to 55,300 tons when $p = 0.8$ €/kg, and 64,100 tons when $p = 1.5$ €/kg). This happens because when the price increases, so does the resource rent per unit of harvest. Thus the net gain from a marginal increase in the stock level increases as well. Since, for the same rate of interest, the profitability of the

¹⁸The estimation of a constant elasticity demand function $p(H) = kH^{-\frac{1}{\eta}}$ led to an optimal path that coincided with the static solution in every period. Using data on sardine prices and landings from INE (2000 – 2014), we estimated $k = 22.3(11.73)$ and $\eta = 1.15(0.19)$; standard errors in parentheses. The equilibrium instantaneous profit function, $\pi(H) = p(H)H - c(H)$, when evaluated at the sustainable harvest amounts, would have a static maximum at $\pi'(H) = 0$. With our calibration, this would entail 7,014 tons of harvest and 675,000 tons of biomass. In the dynamic optimization problem, the equilibrium level of harvest would be set at that amount from $t = 0$ onwards, while biomass would increase to 675,000 tons according to the population dynamics. Under this formulation, the regulator would maintain the maximum sustainable level of profits at all periods, since the optimal rate of extraction would not be constrained by the biological dynamics of the species.

fishery is higher, the more there is to gain to allow the stock to rebuild faster.¹⁹

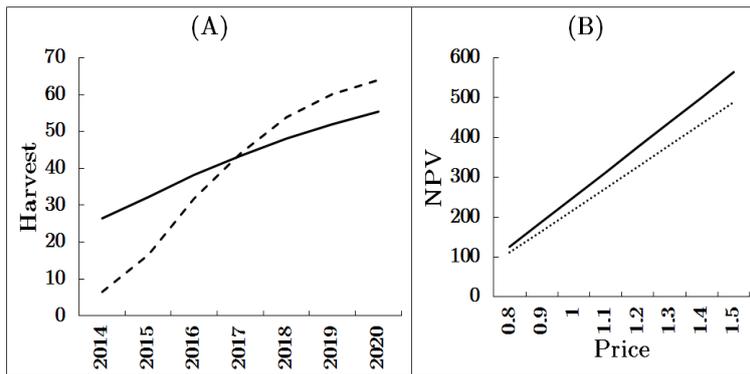


Figure 8: Impact of varying prices between 0.8 and 1.5€/kg. (A) Harvest trajectories for 2014 – 2020 with $p = 0.8$ €/kg (full line) and $p = 1.5$ €/kg (dashed line). (B) NPV for 2014-2030 under the HCR (dotted line) and the optimal plan (full line). Harvest in 1,000 tons, NPV in €million

5.3.2 Recruitment

The recruitment estimates are a key element in our previous results. However, large fluctuations in recruitment are typical of small pelagic species, and do not relate clearly to the abundance of parent-stock (Santos *et al.* 2011). Traditional approaches to population dynamics of small pelagic fishes based on invariant carrying capacity have been noted to be inappropriate, considering the marked fluctuations of their population numbers, and their high dependency on environmental and climatic factors (Solari *et al.* 2010). To understand the impact of different recruitment productivity regimes we considered three scenarios, the reference case as defined in the previous sections, and two others in which we allow for a low and high productivity regime.²⁰ Fig.9 illustrates the equilibrium yield-biomass curves under these three scenarios. Notice that the carrying capacity would change considerably, amounting to approximately 456,000 and 938,000 tons under the low and high productivity regimes, respectively. The MSY would also be significantly different, 28,000 tons under the low recruitment scenario, and 147,000 tons under high recruitment.

In the reference case, our simulations indicate that harvest and biomass would increase until they reach their equilibrium levels. Such trajectory contrasts with the marked decrease in biomass and harvest from 2007 to 2013 (Fig.4A&C). If we assume the low stock productivity scenario, harvest and biomass stabilize at around the same amounts as those in the latter years. In Fig.10C we see that biomass would be maintained at 140,500 tons and harvest would stabilize at approximately 25,600 tons. Under the HCR, biomass

¹⁹If we had a downward-sloping demand curve, we would expect a delay of the moment at which the fishery attains its long-run equilibrium, as we would benefit from a higher price when harvest is lower.

²⁰The low and high regimes were obtained by subtracting or adding one standard deviation from the productivity parameter a estimate, respectively. That is, $a'_L = 29.1$ and $a'_H = 63.2$.

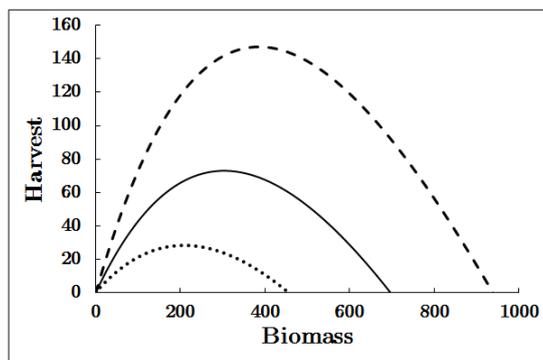


Figure 9: Equilibrium yield-biomass curves under the reference case (full line), high recruitment (dashed line), and low recruitment (dotted line). Values in 1,000 tons

would not increase above the B_{lim} reference point, as it would converge to 213,700 tons, while harvest would amount to 28,300 tons (Fig.10B). Under this scenario, our plan would still realize €9.79 million more than the contrasting HCR, but €45.72 million less than the reference optimal plan.²¹

These results indicate that there is a considerable impact due to recruitment uncertainty. Especially since, if low recruitment is to be maintained, harvest would have to lie at a remarkably low level, quite below its average records in the past decade (85,000 tons (ICES 2014)).

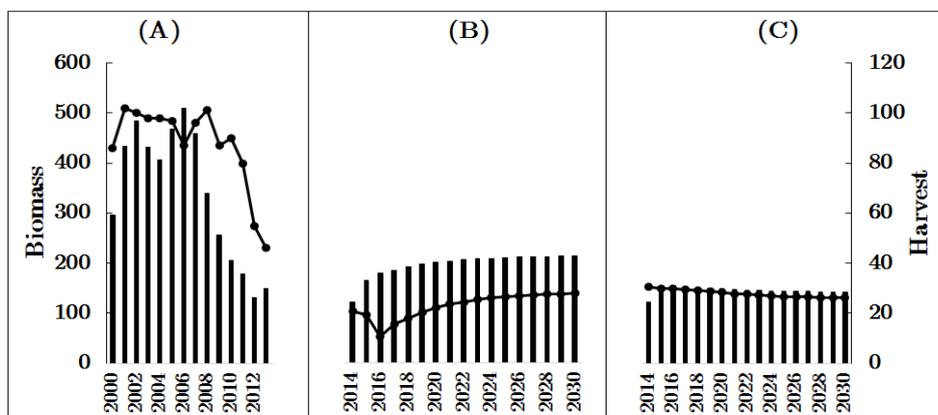


Figure 10: Sardine biomass (columns) and harvest (lines) under low productivity regime. (A) Historical development from 2000 to 2013 (ICES 2014). (B) Harvest and biomass stock development under the HCR. (C) Optimal development of sardine stock and catch. Values in 1,000 tons

5.3.3 Interest rate

The optimal long-run harvest and biomass levels, as well as the state of the fishery in transition, may crucially depend on the discount factor. Figs.11B&D show the effect of varying the rate of interest in the reference case and in the low productivity scenario.

²¹ Under high recruitment, biomass and harvest would follow a similar increasing pattern as that of the reference case, although much more accentuated. The optimal plan would still realize 36.23% higher discounted benefits than the HCR.

In both cases, we see that increasing the interest rate would lead to a lower equilibrium level of biomass. This happens because a higher preference over profits at an earlier date would entail a higher fishing effort during the initial years.

In the reference case, varying the interest rate between 0 and 30%, would lead a 1 percentage point increase in the rate of interest to a 4% average decrease in long-run biomass. Also, if we increase it to 30% or above, the population would be driven to extinction (Fig.11B). When comparing the discounted net benefits between our optimal plan and the HCR, the NPV of the optimal reference case would be from 10 to €19 million higher than the HCR (Fig.11A).

In the low recruitment regime, depletion of the sardine stock would be optimal under a lower rate of interest, around 16% (Fig.11D). Additionally, except for the case at which interest is equal to 0%, our optimal plan would realize from 1 to €17 million more than the HCR, when varying the rate of interest between 0 and 20%.

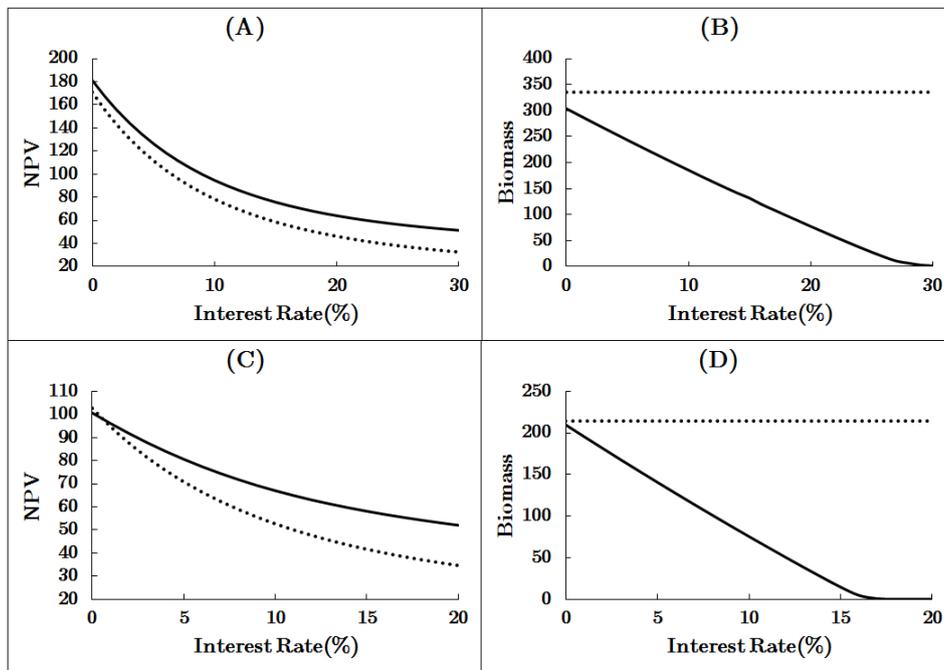


Figure 11: Impact of varying the interest rate on NPV for 2014 – 2030 and on the equilibrium biomass level under the reference case (A)&(B), and under low recruitment (C)&(D). Full lines correspond to the optimal plan, dotted lines to the HCR. Biomass in 1,000 tons, NPV in €million

6 Concluding Remarks

Reference-point strategies may be unsatisfactory since they tend to neglect other aspects of fishery management such as economic incentives (Clark 2010). Our comparisons based on discounted net returns between the HCR and an optimal plan for the sardine fishery suggest that the currently implemented scheme may entail great losses in economic value and possibly lead to non-compliance.

Maximizing net economic returns, however, leads to a harvesting pattern that is not compatible with what ICES considers to be precautionary, since biomass would fall below the B_{lim} reference point. Nevertheless, these results can be offset by accommodating different precautionary constraints. Our findings show that several stock rebuilding strategies can still realize higher profits under an optimal plan. We also found that an optimal harvest trajectory may not only be superior in economic terms but also be more conservationist. Our results revealed that we could allow for a greater long-run biomass level, with higher economic returns over the adjustment period. This leads us to conclude that the current HCR can be improved, as there exist efficiency gains that are yet to be captured.

It is the case, however, that we made some simplifying assumptions regarding economic and population dynamics. We assumed sardine recruitment to be deterministic, even though in small pelagic fishes recruit production shows great volatility. It is also argued that low recruitment success has been one of the most important factors affecting the decline in sardine numbers (ICES 2013a). In any case, a simulation with different productivity regimes revealed that under low recruitment, both the HCR and an optimal plan would involve a severe decrease in the fishery's economic returns and equilibrium population numbers. However, the harvest trajectory under this scenario would agree with the amounts observed in the most recent years. These have led to an unusual increase in the price level, which we assumed to remain constant. A price response to a reduced stock would most likely interact with harvesting strategies, as a higher price may provide an incentive to increase fishing effort further. Studying the implications of a stock-dependent price under a prolonged regime of impaired recruitment is beyond the scope of this paper, but a possible matter of future research.

Our modeling framework is favorable to several developments. By changing its attributes, we can evaluate the impact of different regulatory instruments, or include more realism in the population dynamics. An important extension is the inclusion of multiple species and their interactions into our age-structured model. Of particular note being the chub mackerel species. Chub mackerel is mostly caught as by-catch by purse-seiners that target sardine. In recent years, its landings have increased, given a decrease in sardine abundance. Such an extension would allow us to answer questions on fleet and market dynamics (Costa Duarte 1992, Skonhøft *et al.* 2012), as well as to relax some simplifying assumptions on economic parameters such as constant prices. There also seems to exist an interaction between the two species at the larval/juvenile stages, as sardine recruitment may be impaired because of competition for food (Martins *et al.* 2013).

Another possible expansion is to follow a multiobjective optimization approach to analyze the trade-offs between biological and economic goals. As previously mentioned, fishery policies are almost exclusively centered on single objectives, whose focus falls mainly on conservation. In contrast, economists have favored the objective of maximizing the present value of economic returns in their analyses. However, either of these

two independent approaches is incompatible with a more integrated consideration over societal goals for fisheries management (FAO 2014). Our analysis, despite having incorporated precautionary constraints to account for ecological objectives, does not capture the idea of a regulator who wishes to manage a fishery bearing in mind conflicting interests. A multiobjective optimization would allow for the determination of a policy frontier revealing the efficient utilization of the resource in terms of conservation and economic benefits (Sylvia and Enríquez 1994) and, in particular, answer the need to employ ecosystem-based approaches to fisheries management.

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A Stock Recruitment

The lognormal distribution is a common assumption for the random variability associated with observed recruitment

$$R \sim \phi(x_0)\varepsilon \quad (\text{A.1})$$

where ε is lognormally distributed, x_0 is spawning abundance and $\phi(\cdot)$ is the functional form of the stock-recruitment curve.

We can write $\varepsilon \equiv e^u$, with u being normally distributed. Thus expression (A.1) can be rewritten as

$$R \sim \phi(x_0)e^u \quad (\text{A.2})$$

where it is assumed that u has mean 0 and variance σ_u^2 .

It can be shown that $E[e^u] = e^{\frac{\sigma_u^2}{2}}$. The arithmetic mean number of recruits is then given by

$$AM[R] = \phi(x_0)e^{\frac{\sigma_u^2}{2}} \quad (\text{A.3})$$

although it is the case that the geometric mean (or median in lognormally distributed variables) is

$$GM[R] = \phi(x_0) \quad (\text{A.4})$$

When fitting the Ricker model $\phi(x_{0t}) = ax_0e^{-bx_0}$ to observed data using Eq.(4.2), we obtain the geometric mean estimates for a and b . While these provide the most probable value for recruitment in any year for the observed spawning abundance, they do not provide the long-term arithmetic average value of recruitments (Ricker 1975). Following Hilborn (1985), to analyze long-term trends we modified our productivity parameter a , such that

$$E[R] = ax_0e^{-bx_0}E[e^u] = ax_0e^{-bx_0}e^{\frac{\sigma_u^2}{2}} = a'x_0e^{-bx_0} \quad (\text{A.5})$$

where $a' = ae^{\frac{\sigma_u^2}{2}}$.

We further assumed that u followed an AR(4) (Santos *et al.* 2011), that is, $u_t = \rho u_{t-4} + v_t$, where $\{v_t\}$ are independent normally distributed errors with standard deviation σ_v , and ρ is the autoregressive parameter. Thus variance σ_u^2 will be given by $\sigma_u^2 = \frac{\sigma_v^2}{2(1-\rho^2)}$, which will then be substituted into a' , meaning

$$a' = ae^{\frac{\sigma_v^2}{2(1-\rho^2)}} \quad (\text{A.6})$$

B Equilibrium Yield-biomass Curve

Following Tahvonen (2008, 2009b), and under the assumption that the age-class distribution represents an equilibrium for a given biomass level B , the growth function from the biomass model can be obtained by solving numerically Eqs.(3.1)-(3.5), combined with total biomass $B_t = \sum_{s=1}^n w_s x_{st}$.

If we let x_s , $s = 1, \dots, n$, denote the equilibrium number of individuals, and take effort E as constant, we obtain the following nonlinear equation system (B.1)-(B.5)

$$x_1 = \phi \left(\sum_{s=1}^n \gamma_s w_s x_s \right) \quad (\text{B.1})$$

$$x_{s+1} = e^{-m_s} (1 - q_s E) x_s, \quad s = 1, \dots, n-2 \quad (\text{B.2})$$

$$x_n = e^{-m_{n-1}} (1 - q_{n-1} E) x_{n-1} + e^{-m_n} (1 - q_n E) x_n \quad (\text{B.3})$$

$$B = \sum_{s=1}^n w_s x_s \quad (\text{B.4})$$

$$H = E \sum_{s=1}^n w_s^c q_s e^{-\frac{m_s}{2}} x_s \quad (\text{B.5})$$

If we define $\mu_s = e^{-m_s} (1 - q_s E)$ and $\mu_{n-1} = \frac{e^{-m_{n-1}} (1 - q_{n-1} E)}{(1 - e^{-m_n} (1 - q_n E))}$, we can rewrite Eqs.(B.2) and (B.3) as

$$x_{s+1} = \mu_s x_s, \quad s = 1, \dots, n-1 \quad (\text{B.6})$$

making it possible to write the equilibrium number of individuals in each age-class for $s = 2, \dots, n$ as a function of x_1

$$x_s = \psi_s x_1 \quad (\text{B.7})$$

for $\psi_s = \prod_{i=1}^{s-1} \mu_i$, $s = 2, \dots, n$. Finally, the equilibrium number of individuals in age-class 1 will be given by

$$x_1 = \phi \left(\sum_{s=1}^n \gamma_s w_s \psi_s x_1 \right) \quad (\text{B.8})$$

where $\psi_1 \equiv 1$. Given that we assumed the Ricker (1975) specification, and setting $\sum_{s=1}^n \gamma_s w_s \psi_s = \Phi$, write Eq.(B.8) as

$$x_1 = a\Phi x_1 e^{-b\Phi x_1} \quad (\text{B.9})$$

where a and b are parameters of the Ricker recruitment function. The above equation can be solved for x_1 , that is

$$x_1 = \frac{\ln(a\Phi)}{b\Phi} \quad (\text{B.10})$$

And now, since by Eq.(B.4) $B = \sum_{s=1}^n w_s x_s = x_1 \sum_{s=1}^n w_s \psi_s$, it is possible to vary the level of biomass between $[0, B_K]$, where B_K is the carrying capacity (when $E = 0$), and solve for the equilibrium effort and sustainable level of harvest $H = x_1 E \sum_{s=1}^n w_s^c q_s \psi_s e^{-\frac{m_s}{2}}$.

C AMPL Code

```
# sard.mod file

param T;                               #time horizon (years)
param n;                               #number of age classes
param r;                               #annual interest rate
param w {s in 1..n};                   #weight in stock; unit kg per individual in age class
param c {s in 1..n};                   #weight in catch;
param g {s in 1..n};                   #maturity
param q {s in 1..n};                   #catchability coefficients
param m {s in 1..n};                   #natural mortality
param x0 {s in 1..n};                  #initial state, number of individuals; unit 10^9

var H {t in 0..T-1} >=0;                #total harvest; unit 10^3 tons; weight in catch
var x {s in 1..n, t in 0..T} >= 0;      #number of individuals; unit 10^9
var B {t in 0..T-1} = sum{s in 1..n} w[s]*x[s,t]*1000; #biomass; unit 10^3 tons
var Xo {t in 0..T-1} = sum{s in 1..n} w[s]*g[s]*x[s,t]*1000; #spawning stock; unit 10^3 tons
var G {s in 1..n, t in 0..T};           #transformation function;

var h {s in 1..n, t in 0..T-1} >=0;     #harvested individuals; unit 10^9
var y_1 {s in 1..n, t in 0..T-1} >=0;   #slack variable 1
var y_2 {s in 1..n, t in 0..T-1} >=0;   #slack variable 2

maximize objective_function: sum{t in 0..T-1} ((1/(1+r))^t)*(0.8*H[t]-0.4*H[t]^1.1);

subject to constraint1 {t in 0..T-1}: x[1,t+1]=42.9*(Xo[t]/1000)*exp(-1.61*(Xo[t]/1000));
subject to constraint2 {s in 1..n-2, t in 0..T-1}: x[s+1,t+1]=exp(-m[s])*x[s,t]-H[t]*(G[s,t]/1000);
subject to constraint3 {t in 0..T-1}: x[n,t+1]=exp(-m[n-1])*x[n-1,t]+exp(-m[n])*x[n,t]-H[t]*(G[n-1,t]/1000+G[n,t]/1000);
subject to constraint4 {t in 0..T, s in 1..n}: G[s,t]=exp(-m[s])*q[s]*x[s,t]/(sum{i in 1..n} c[i]*q[i]*x[i,t]*exp(-m[i]/2));
subject to initial_condition {s in 1..n}: x[s,0] = x0[s];

# complementary constraints

subject to constraint5 {s in 1..n, t in 0..T-1}: h[s,t]-H[t]*(G[s,t]/1000)*exp(m[s]/2)+y_1[s,t]=0;
subject to constraint6 {s in 1..n, t in 0..T-1}: h[s,t]-exp(-m[s]/2)*x[s,t]+y_2[s,t]=0;
subject to constraint7 {s in 1..n, t in 0..T-1}: 0 <= y_1[s,t] complements y_2[s,t] >= 0;

-----

# sard.dat file

param T := 200;
param n := 7;
```

```
param r := 0.05;
param w:=
  1 0.000
  2 0.024
  3 0.044
  4 0.057
  5 0.065
  6 0.070
  7 0.079;
```

```
param c:=
  1 0.024
  2 0.043
  3 0.059
  4 0.068
  5 0.074
  6 0.079
  7 0.100;
```

```
param q:=
  1 0.130
  2 0.371
  3 0.697
  4 1
  5 1
  6 1
  7 0.325;
```

```
param g:=
  1 0
  2 1
  3 1
  4 1
  5 1
  6 1
  7 1;
```

```
param m:=
  1 0.80
  2 0.50
  3 0.40
  4 0.30
  5 0.30
  6 0.30
  7 0.30;
```

```
param x0:=
  1 6.247
  2 1.652
  3 0.636
  4 0.276
  5 0.222
  6 0.076
  7 0.234;
```

```
# sard.run file
```

```
reset;
```

```
model sard.mod.txt
```

```
data sard.dat.txt
```

```
option solver knitro;
```

```
solve;
```

```
display H;
```

```
display B;
```

```
display x;
```

```
#table sardH OUT:[T] H;
```

```
#write table sardH;
```

```
#table sardB OUT:[T] B;
```

```
#write table sardB;
```