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EXPLOITING THE COINTEGRATION BETWEEN VIX AND CDS IN A

CREDIT MARKET TIMING MODEL

ANDREA RICCIARDI N° 976

A Project carried out on the Finance course, under the supervision of:

Paulo Rodrigues

NOVA – School of Business and Economics

Paul Beekhuizen & Patrick Houweling

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Abstract

We investigate the cointegration between VIX and CDS indices, and the possibility of exploiting it in an existing credit market timing investment model. We find cointegration over most of the sample period and the leadership of VIX over the CDS in the price discovery process. We present two methods for including cointegration into the model. Both strategies improve the in-sample and out-of-sample model performances, even though out-of-sample results are weaker. We find that in-sample better performances are explained by a stronger cointegration, concluding that in the presence of cointegration our strategies can be profitable in an investment model that considers transaction costs.

Keywords: Cointegration, VIX, Credit Default Swaps, Pairs Trading.

1. Introduction

This thesis is the result of a six months internship at the Quantitative Research department of Robeco Asset Management. I focused on the already existing Credit Default Swaps (CDS) Indices[1] market timing model, with the specific task of improving it. The model is called iBeta and it is the performance driver of the Robeco Quant High Yield Fund, in which around €170 mln are currently invested.

In this work, we study the cointegration between VIX and CDS indices, aiming at using it as a signal through a variable in the model. The VIX is the implied volatility index extracted from different options on the S&P500, and it is often refer to as the fear index. It is reasonable to

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[1] CDS indices are baskets of single-name CDSs. See (Markit, 2014) for more details.

believe in a strong relationship between the VIX, measure of the global market risk, and the CDS indices, measure of the global credit risk. We find cointegration between VIX and CDS indices, and we construct two cointegration variables which, when added to the iBeta model, improve the model performance over the full-sample period.

1.1. The current iBeta model

The model consists of a number of themes aiming at forecasting the spread direction[2]. Such themes are grouped into: *Equity*, which gathers information from the equity market and the VIX; *Short-Term Trend*, indication of the short-term spread momentum; *Long-Term Trend*, used to capture business cycle variations; *Seasonal*, that applies the "*Sell in May and Go Away*" strategy. The outcome of each theme is translated into a score with mean 0 and variance 1 (the *z-scores*), and such scores are combined into a model score. In order to avoid exaggerated contribution from any variable, all the scores are capped at ±1. A long position is taken when the model score is positive, and a short position is taken when the model score is negative. The investment strategy is weekly.

1.2. Data

The iBeta model universes are four: CDX Investment Grade (USIG), CDX High Yield (USHY), iTraxx Main (EUIG), iTraxx Crossover (EUHY)[3]. The fund invests in 5-years maturity CDS, the most liquid contracts, and currently just in the HY markets. However, we are interested in a profitable strategy for both IG and HY, mainly for two reasons: first, the portfolio manager shares the same vision for the determinants moving the IG and HY markets, making the IG universe an important robustness check; second, in the future the iBeta model might be extended to the IG universe.

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[2] See (Houweling, Beekhuizen, Kyosev, & Van Zundert, 2014) for more details on the iBeta model.
[3] We use roll-adjusted versions of the spreads which take into account the changes in spread stemming from the semestral rolling of the indices.

The sample starting date is 23-Jun-2004, date where all the indices returns are available. Throughout this work, we split the sample into two sub-periods: an *in-sample* period [23-Jun-2004, 23-Jun-2009], over which the model is calibrated, and an *out-of-sample* period [24-Jun-2009, 13-Jul-2015], over which the model is eventually tested. The motivation behind this sample division will be clear later on.

Clearly, we need some performance measure to evaluate the strategy. Since this work aims at having a direct impact on the iBeta model, transaction costs must be taken into account.[4] Our main performance measures will be the annualized Net Performance and the annualized Net IR, defined as the Net Performance over its volatility, and both of these measures will be presented for IG and HY markets separately. We also look at the Turnover, defined as

$$T = 52 \cdot mean\left(mean\left(\left|Signal_t^i \times PositionSize_t^i - Signal_{t-1}^i \times PositionSize_{t-1}^i\right|\right)\right),$$

thus the annualized mean over markets and time of the positions' difference in the market.

It is also useful to look at the single variable performances. Recalling that each variable is translated into scores between -1 and 1, we define the *z-performance* as the annualized average of the scores times return. Such measure can give an initial insight on the contribution of the single variable. Our single variables evaluation performances will be *z-perfomances*, *z-volatility* and *z-IR*, for IG and HY.

The rest of the thesis is organized as follows. Section 2 gives an overview of the existing literature. Section 3 describes the methodology that will be used throughout this work. In Section 4 we report in-sample and out-of-sample results, eventually discussing the differences. Section 5 presents some ideas for future researches and Section 6 concludes. Numbered graphs, tables and an additional derivation are reported in Appendix A. Other graphs and tables are reported in Appendix B.

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[4] Robeco has its own transaction costs model whose details go beyond the purpose of this work.

2. Literature Review

The relationship between credit risk and equity volatility has been widely studied in the literature, starting from (Merton, 1974) structural credit model. When studying indices instead of single credit spreads, the asset volatility is replaced by VIX, recognized measure for the market fear, as in (Collin-Dufresne & Goldstein, 2001) and (Shaefer & Strebulaev, 2008). However, even though the close relationship between VIX and CDS is widely known, their cointegration investigation is almost not present in the literature. As far as we know, (Figuerola-Ferretti & Paraskevopoulos, 2013) is the only paper studying the cointegration between VIX and CDS. They find cointegration between VIX and iTraxx markets, and they propose a pairs trading strategy in the VIX futures and 5 years iTraxx that brings to abnormal positive returns.

This thesis adds value to (Figuerola-Ferretti & Paraskevopoulos, 2013) in several ways. First, in the out-of-sample robustness check, they simply recalculate the cointegration relationship using future information they did not have at that point in time. Our out-of-sample is meant as a pure out-of-sample test: we perform weekly rolling regressions using just information we had at that point in time, thus calculating every week the cointegration relathionship. Second, even though (Figuerola-Ferretti & Paraskevopoulos, 2013) acknowledge the Gonzalo-Granger measure as a measure of VIX leadership in the price discovery process, they do not take advantage of it in the trading strategy. Instead, our cointegration variables will include such information, crucial to understand the contribution of each series to the equilibrium readjustment. Third, we extend the data sample from 2004-2011 to 2004-2015.

In the literature, cointegration pairs are usually exploited with pairs trading strategies. Since our model can invest just in one of the two assets, the CDS, we develop a unilateral pairs trading strategy, method present in the "hedge fund literature" thanks to (Altucher, 2004), but

not in the academic's one. We follow the investment model presented by (Caldeira & Moura, 2012), who apply a pairs trading strategy on the cointegration pairs identified in the Brazilian stock market. Unlike most of the literature, where investors exploit cointegration within a high-frequency framework, such as (Miao, 2014) and (Hanson & Hall, 2012), we will exploit it with a long term variable.

3. Methodology

In this section, we illustrate the methodology for our work. We start by testing for stationarity, since non-stationarity is an essential requirement for cointegration. Then we perform the cointegration Johansen test and we introduce the Gonzalo-Granger measure, indicator of the VIX leadership over the CDS in the price discovery process. Finally, we construct two cointegration variables: a *discrete* one, and a *continuous* one.

3.1. Non-stationarity

Before testing for cointegration, we must assure that we are dealing with non-stationary time series. Practitioners often consider VIX as mean-reverting. However, VIX is simply the implied volatility extracted from the nearby S&P 500 index options, using a wide range of strikes. Thus, the statistical properties of VIX stem from the distribution of weighted average option prices. (Figuerola-Ferretti & Paraskevopoulos, 2013) empirically show that the VIX is not mean-reverting. We test this hypothesis by performing the Augmented Dickey-Fuller (ADF) test with drift for unit roots and we find stationarity over the full-sample. However, for our cointegration analysis we will use a 5-years rolling window: by using such window, the VIX turns out to be non-stationary.

3.2. Cointegration

After testing for unit roots, we test for cointegration between CDS indices spreads and VIX approaching the Johansen test. Let s_t^i denote the CDS spread at time t for market i, and let v_t

be the VIX spot at time t. We wonder if there exists a non-trivial vector $[\gamma_0^i, \gamma_1^i]$ such that the process $\{s_t^i - \gamma_0^i - \gamma_1^i v_t\}_t$ is stationary. The vector $[1, -\gamma_0^i, -\gamma_1^i]$ is called the *cointegration vector* and the process $\{z_t^i\}$ defined as

$$z_t^i := s_t^i - \gamma_0^i - \gamma_1^i v_t \tag{1}$$

is called the *cointegration relationship*. If such vector exists, the spread can be replicated by borrowing/investing γ_0 in the risk-free asset and by buying/selling γ_1 units of the asset v.

3.3. VECM and Gonzalo-Granger measure

Cointegration is a statistical property that is widely exploited in pairs trading strategies. In such a framework, when process (1) widens, we can short the "winner" and long the "loser", confident in a reversion to the long-run equilibrium. Since our model does not invest in one of the two assets – the VIX –, our strategy will follow the pairs trading strategy but unilaterally, meaning that we open positions just on the spread.

A legitimate objection to this kind of strategy relies on the fact that pairs trading works because a pairs of assets is indeed traded. Cointegration suggests that the gap between the two assets will eventually go back to its equilibrium, but it gives no indication about which asset will contribute the most to the resettlement of such relationship. Therefore, by investing in just one asset, one could argue that a priori we lose half of the strategy's power. (Figuerola-Ferretti & Paraskevopoulos, 2013) show that the CDSs do all the work in terms of equilibrium readjustment. They explain such predominance with a higher number of participants (thus, higher liquidity) in the VIX futures market.

We follow the model developed by (Figuerola-Ferretti & Paraskevopoulos, 2013) describing the interaction between trades in the CDS and VIX market, and we test for VIX predominance in our data sample. Such model leads to a Vector Error Correction Model (VECM) framework such as:

$$\begin{pmatrix} \Delta s_t^i \\ \Delta v_t^i \end{pmatrix} = \begin{pmatrix} \alpha_1^i \\ \alpha_2^i \end{pmatrix} z_{t-1}^i + \begin{pmatrix} u_t^{s,i} \\ u_t^{v,i} \end{pmatrix} \tag{2}$$

where α_1^i and α_2^i are known as *adjustment coefficients* and $u_t^{s,i}$ and $u_t^{v,i}$ are the error terms. If the two coefficients are both statistically significant, they must have opposite sign, since a deviation from equilibrium will be readjusted with opposite movements of spread and VIX. Moreover, a α_2^i not significantly different from zero indicates that VIX does not adjust to the spread, meaning that VIX dominates the CDS spreads in the price discovery process. Such result would justify our unilateral pairs trading strategy.

Following (Blanco, Brennan, & Marsh, 2005), we can introduce a measure of VIX leadership in the price discovery process, inspired by (Gonzalo & Granger, 1995):

$$GG_{VIX}^i = \frac{-\alpha_1^i}{\alpha_2^i - \alpha_1^i} \tag{3}$$

The Gonzalo-Granger measure is useful when both coefficients are statistically significant and have opposite sign. In such a case, $GG_{VIX}^i \in [0,1]$, and a value close to 1 indicates the VIX leadership, whereas a value close to 0 indicates that the spread dominates the VIX.

3.4. Trading strategy implementation

In this section, we dig into the construction of our two cointegration variables.

We choose a window of D days over which the cointegration relationship is calibrated. Per each day, we compute the parameters γ_0^i and γ_1^i in (1) using the data of the past D days, ending up having time-dependent parameters $\gamma_0^{i,D}(t)$ and $\gamma_1^{i,D}(t)$ and as a consequence a time-dependent cointegration relationship $z^{i,D}(t)$. Parameters are estimated with the Johansen method, and lags are chosen following the AIC criterion. Acknowledging the regression estimates sensitivity to outliers, we set an interval within which the parameters should reasonably lay. If the new estimates overstep the interval, we look backwards using the

closest parameters satisfying the constraints. In order to make $z^{i,D}(t)$ comparable across variables, we divide it by the moving standard deviation $\sigma_z^{i,D}(t)$. Notice that we do not have to subtract it by the mean, since by construction process (1) is stationary with constant mean 0. Recalling that the investment strategy is weekly, in order to avoid any possible day-of-the-week effect we take the average of $z^{i,D}(t)$ over the last 5 (working) days, ending up with a stationary process $\tilde{z}^{i,D}(t)$.

We then give two alternative ways of building the *Cointegration VIX (CV)* variable. The first method tackles the problem in a way similar to the pairs trading literature, as for instance (Gatev, Goetzmann, & Rouwenhorst, 2006) and we call it $CV_{discrete}$. With the second method we build a more continuous variable, similarly to most of the variables at Robeco Quantitative Strategies, and we call it $CV_{continuous}$. The two variables are described in the next sections. In any case, given the nature of the variable, we add it to the *Equity* basket. In the *Equity* basket there is already a variable, called *VIX Trend*, which looks at the trend information from the VIX. Thus, we checked for correlation between such variable and our cointegration variables. The variables show low-mutual correlation, as reported in Appendix B.

3.4.1. Discrete variable

When spread and VIX depart too much from each other, we bet the spread will move towards the replication strategy $\gamma_0^{i,D}(t) + \gamma_1^{i,D}(t)v(t)$. The deviation from each other is measured by $\tilde{z}^{i,D}(t)$. Recalling that $\tilde{z}$ is stationary with mean 0 and unit variance, the implementation rule is the following (see Figure 1 for a graphical description):

Let $k > 0$. Go long whenever $\tilde{z}^{i,D} > k$ and offset the position when $\tilde{z}^{i,D} < 0$; go short whenever $\tilde{z}^{i,D} < -k$ and offset the position when $\tilde{z}^{i,D} > 0$. Moreover, offset the position if cointegration has not been found in the past D days. (4)

Let V be the variable described by (4). So far we defined the direction of V, but not the position size. We set the variable to zero when the signal is neutral, and to $\pm M$ when we open a long or short position. M must be chosen in such a way that the variable weight (defined as the average of the scores' absolute values) in the basket is the same as the other two variables (the *Equity Trend* and the *VIX Trend*). We calibrated the value with Monte Carlo simulations, resulting in $M = 2.4$. See Appendix A for the derivation.

In (4), k is a parameter indicating how far we have to be from equilibrium before switching on the signal. After in-sample calibrations, we chose $k = 1.5$.

Notice that when we do not find cointegration in the past D days we set the variable to zero. This is motivated mainly by two reasons. First, if we did not find cointegration this means that we did not find any pair (γ_0, γ_1) such that process (1) is stationary, resulting in biased (spurious) coefficients estimates from equation (2) and biased Gonzalo-Granger measures. Second, this is the method generally used in the pairs trading literature.

The variable will be composed of two factors:

$$CV^{i,D}_{discrete}(t) = V^{i,D}(t) \cdot GG^{i,D}(t) \,, \tag{5}$$

where $GG^{i,D}(t)$ is the Gonzalo-Granger measure at time t for market i defined by (3), and $V^{i,D}(t)$ is the variable described by rule (4). We decided to include the Gonzalo-Granger measure in the variable since it is crucial information for our unilateral pairs trading strategy. The bigger the measure, the more chances we have that the spread will follow the VIX, instead of the other way around. Finally, the variable is capped at ± 1.

3.4.2. Continuous variable

The variable constructed above is discrete, in the sense that it assumes either 0 or $M \times GG^i_t$. We can also define a continuous variable using basically the process $\tilde{z}^{i,D}(t)$. However, all the

variables in the model are capped at ±1. This means that, assuming $k > 1$, the variable at k and 1 would have the same score 1, given as a consequence the same weight to $\tilde{z}^{i,D} = 1$ and to $\tilde{z}^{i,D} = k$, critical value in the previous variable. Since we are trying to exploit the same effect from both variables, for consistency we divide the process by k, such that the variable will have score 1 just from k onwards. Lastly, we multiply the variable by the Gonzalo-Granger measure:

$$CV_{continuous}^{i,D}(t) = \min\left(\max\left(\frac{\tilde{z}^{i,D}(t)}{k} \cdot GG^{i,D}(t), -1\right), 1\right) \tag{6}$$

An insightful scheme of the construction of the two variables is reported in Appendix B.

4. Results

In this section, we present the main in-sample and out-of-sample results. We find stronger performances in the in-sample period with respect the out-of-sample, and we eventually give an explanation for such better performances.

4.1. In-sample

4.1.1. Non-Stationarity

Confirming (Figuerola-Ferretti & Paraskevopoulos, 2013) results, we find VIX and spreads to be non-stationary within the in-sample period. However, if we take the full-sample period, the ADF test rejects the null hypothesis of a unit root for the VIX series. In order to better understand what is going on, we plot the ADF statistics over time in Figure 2. At each point in time the ADF statistic is calculated looking backwards from 1998 until the current time. Figure 2, while showing the non-stationarity of the spreads, confirms our suspicions: for most of the sample, the null hypothesis of unit root in the VIX is rejected. Recalling that non-stationarity is a necessary property for cointegration, this thesis should end here.

However, our variable is not constructed by considering the series from the beginning of the sample period, instead it looks backwards just D days. Therefore we are not interested in non-stationarity over the full sample, but in non-stationarity in a moving window of D days. Thus, we need to calculate the ADF statistics by looking backwards D days instead of looking from the beginning of the sample. We choose 5 years as a moving window[5] and we plot the ADF statistics over time in Figure 3. We can see how things get significantly better. Even though there are still some periods over which the series seems stationary, the ADF test generally does not reject the null of a unit root, opening the door for cointegration. We can conclude that the VIX displays persistence change over time.

4.1.2. Cointegration

A preliminary cointegration analysis can be made by performing the cointegration tests over the in-sample period. Table 1 and Table 2 report the test results and the coefficient estimates. Both tests show strong cointegration between VIX and CDSs. Table 1 shows that with the Johansen estimates we replicate the spreads by borrowing the risk free asset and investing in γ_1 units of VIX. Table 2 reports the adjustment coefficients for each market and the respective Gonzalo-Granger measure, defined by (3). As underlined in Section 3.3, a Gonzalo-Granger measure close to one indicates a strong leadership of VIX in the price discovery process. For the HY markets α_2 is not even significant, saying that the CDSs do all the adjustment towards the equilibrium, and this is a very promising sign for our strategy. For the USIG market the results are slightly less strong, but they still show a strong predominance of VIX over the CDSs in the price discovery.

By looking at Figure 5, we can have a visual idea of how the discrete variable works.[6] Figure 5 refers to the Johansen parameters estimates reported in Table 1. Clearly, such parameters

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[5] The window has been calibrated in-sample by looking at a wide range from 80 days to 5 years.
[6] For the other markets, see Appendix B.

are estimated by looking forward at the whole in-sample period, therefore, at each point in time, Figure 5 uses information we did not have at that moment. The yellow stripes indicate a bearish signal from the variable (that is, betting that the spread increases), the blue stripes a bullish signal, and the white stripes a neutral signal. The only difference between the discrete and the continuous variables occurs when the discrete variable gives a neutral signal. In such circumstances, the continuous variable gives a signal based on whether the spread is above or below the replication strategy. However, the distance between these two processes is not enough to switch on the discrete variable. See Appendix B for the continuous variable graphs.

4.1.3. Backtest

Let's now move to the performances section. Even though a good performance is not a sufficient condition for stating a variable as a good one for our model (in fact we need also economical meaning, robustness, etc.), it is definitely a necessary one.

We start with the in-sample backtest, which of course could not have been a doable investment strategy at that time, since for it we use information we still did not have. As mentioned in Section 1.2, we start by studying the impact of the single variable. Results are shown in Table 3. Here, performances are meant as *z-performances*. See Section 1.2 for the definition. Considering that the other variables in the iBeta model have a *z-IR* around 0.5 (see Appendix B), both variables look very promising.

From the model point of view, our benchmark will be the current iBeta model. The most important statistics will be Net Performance and Net IR, but also Turnover will be carefully watched (see Section 1.2 for their definition). Table 4 reports the in-sample model performances. The cointegration variables improve significantly the performances. A performance of 0.31 indicates an annual average return of 31 basis points. Even though the discrete variable is clearly the best, the continuous variable still brings a big improvement to the iBeta model, especially for the IG markets. Figure 4 shows the in-sample cumulative

signal performances of the iBeta model and the model with the discrete cointegration variable. We can notice that the variable's greatest contribution happens to be at the beginning of the subprime crisis, meaning that the VIX spotted the crisis before the CDSs did.

4.2. Out-of-sample

The out-of-sample backtest is a fundamental test to prevent data mining and to assess the quality and robustness of the variables, because here we are using just information we had at that point in time, making of this strategy an investable strategy that we could have used from 2009 to 2015. Recall that every week we are looking 5 years backwards, calculate the cointegration parameters, and add the cointegration variable to the *Equity* basket.

Again, we start by looking at the single variables performances, reported in Table 5. Comparing the above results with Table 3, the variables seem clearly not as strong as in the in-sample period, therefore we do not expect an outstanding performance at a model level. However, the variable's contribution is still overall positive. It is interesting to look at how often the variable is set to zero due to non-cointegration. By taking the average over the markets, we see that this happens 731 times in the out-of-sample period, resulting in a fraction of 11% of the total observations, but 717 out of 731 times the non-cointegration comes from the EUIG market. Thus, the GG measure for the EUIG market turns out to be the most volatile and less reliable one, as we can see from the plot of the GG measure in Appendix B.

Table 6 shows the models' performances. Results are not as strong as in-sample. However, whereas results for HY markets do not basically change, IG performance and IG IR double, meaning that the variable still adds value to the model.

4.3. Difference between in-sample and out-of-sample

Even though the out-of-sample performances are good, they are far from the outstanding in-sample results. We test three possible reasons for this:

1. VIX is stationary in the out-of-sample period;

2. cointegration is as strong as in the in-sample period, but it is not exploitable in an investment strategy that does not use a forward-looking window;

3. cointegration in the out-of-sample period is not as strong as in the in-sample period.

In this section, we show that the third explanation holds. Thus, our strategies can be profitable without "knowing the future", and results are impressive when cointegration is strong, and less impressive, but still positive, when cointegration is weak.

4.3.1. Test for VIX non-stationarity in the out-of-sample period

We first check whether the VIX non-stationarity still holds in the out-of-sample period. By plotting the ADF statistic over time in Figure 6, we can see that, even though the test rejects the null hypothesis of a unit root in the first half of 2014, the process seems generally non-stationary, similarly to the in-sample results plotted in Figure 3. Figure 6 confirms the VIX as a non-stationary series in the short term. Thus, the difference in results between in-sample and out-of-sample is not due to a stationarity of the VIX.

4.3.2. Test for strategy's feasibility without a forward-looking window

In order to test reason (2), we can perform an "a posteriori" analysis by computing the cointegration parameters for the out-of-sample period in the same way we computed them for the in-sample. Namely, we perform the Johansen test over the whole out-of-sample period, ending up with just one constant pair of cointegration parameters per market, γ_0^i and γ_1^i, which will be used for constructing the cointegration variables. We then backtest this model (*fitted* model) in the out-of-sample period.

Results are shown in Table 7, where are reported the outperformances of the model with respect the iBeta in the in-sample and the out-of-sample period. The third column refers to the outperformance of the fitted model just described. Values in Table 7 are obtained by

subtracting the statistics of the current iBeta model from the statistics of the model with the $CV_{discrete}$ variable. By comparing columns 2 and 3 of Table 7, we see that the improvements coming from the fitted model are of the same out-of-sample model's order. As a consequence, even with a forward looking window performances would not have changed. Thus, we do not need a forward looking window to exploit the cointegration.

On the other hand, we are going to show that a strategy with weekly rolling regression would still have been profitable during the in-sample period. In order to test it, we need to reduce our moving window. By choosing 3 years, we can estimate the model in the new out-of-sample period (within the original in-sample) from 2007 to 2009. Results are shown in Table 8. Improvements get closer to the original in-sample improvements. IG market performances slightly worsen, but the HY market performances and IRs increase by one third, basically the same improvement reported in Table 4 for the in-sample back tests. One could argue that Table 8 could be explained by a better performance of the variable with 3 years moving window. Such argument is refuted by Table 9, where the models with 3 and 5 years moving window are backtested for the full sample period. The 5-years moving window model works better than the 3-years one, although results are similar.

Summarizing, we do not need a forward looking window to exploit the cointegration, and we can build a profitable investment strategy exploiting cointegration without "knowing the future". As a result, argument (2) is discarded.

4.3.3. Test for cointegration in the out-of-sample period

Finally, we test for argument (3) by looking at the cointegration in the out-of-sample period. Johansen statistics for the in-sample and out-of-sample periods are reported in Table 10.[7] Whereas the IG markets stay more or less at the same levels (EUIG's weaker cointegration is

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[7] See Appendix B for a plot of the cointegration statistics over time.

balanced by the USIG's stronger cointegration), the HY markets show much higher statistics in the in-sample period. This stronger cointegration results in a better performance for HY in the in-sample period, as shown by the first two columns of Table 7. For the IG markets, the outperformance of the cointegration variable is lower in the out-of-sample period, but not dramatically lower. For the HY markets, the net performance goes from a +0.50 in the in-sample period to a poor +0.01 in the out-of-sample period.

As a consequence, argument (3) holds. Therefore, a trading strategy exploiting the cointegration between VIX and CDS spreads is feasible, and results depend on the strength of the cointegration.

5. Future Research

In our model, the bet sizes of the cointegration variables do not take into account how far the assets are from equilibrium. When the mispricing is strong, the position size will always be M for the discrete variable, and 1 for the continuous one (times the GG measure), resulting in a variable whose nature is static from the distance-from-equilibrium point of view. However, it might be interesting to use dynamic bet sizes. (Jurek & Yang, 2006) show the existence of a critical level of mispricing beyond which an optimal allocation requires a reduction in the bet size. When applied to Siamese twin shares, such dynamic bet sizes result in a significant improvement in the Sharpe ratio relative to a simple threshold rule like ours one.

Since the iBeta model invests just on CDSs, we developed a unilateral pairs trading strategy. Although such choice is justified by the GG measure (3), that for most of the sample indicates the VIX leadership over the CDSs in the price discovery process, the results of a pure pairs trading strategy that takes positions in both CDS indices and VIX futures would be insightful, in order to understand the full power of the strategy.

It might also be useful to build a basket of cointegration variables with different look back horizons. In our model, we chose a 5-years moving window, after an in-sample calibration. However, extracting information from different horizons could add value to the variable, especially in terms of robustness.

A further interesting follow-up for the model would be the cointegration analysis among the spreads over region and/or credit rating. Trends information from different markets is already present in the iBeta model but, as we have seen throughout this work, if the series are cointegrated we can obtain additional signals not captured by the trend variables.

6. Conclusions

This thesis investigates the cointegration between VIX and CDS indices, aiming at improving the current Robeco CDS market timing model by adding a *Cointegration* variable. After testing for non-stationarity (VIX shows persistence change over time), we find cointegration over most of the sample period (2004-2015). We make use of the VECM (2) to define the Gonzalo-Granger measure (3). Such measure mostly assumes values close to one (its average over time and markets is 0.86, as reported in Appendix A), meaning that VIX leads CDSs in the price discovery process. This result justifies the use of a unilateral pairs trading strategy. We then construct two cointegration variables aiming at exploiting the same effect, comparing eventually the results: a *discrete* variable and a *continuous* variable. We split the sample in an in-sample period, over which we use a forward looking window to calibrate the parameters, and an out-of-sample period, over which weekly regressions build the variable without using any future information. The variables improve the current model in both the in-sample and out-of-sample periods, after transaction costs. However, improvements are significantly lower in the out-of-sample period when compared to the in-sample ones. We prove that such difference in the performances is explained by a stronger cointegration during

the in-sample period, and not by impossibility in implementing the strategy without knowing the future. Concluding, the existing cointegration between VIX and CDS indices can be exploited in a profitable trading strategy after transaction costs, and profits increase with the strength of the cointegration.

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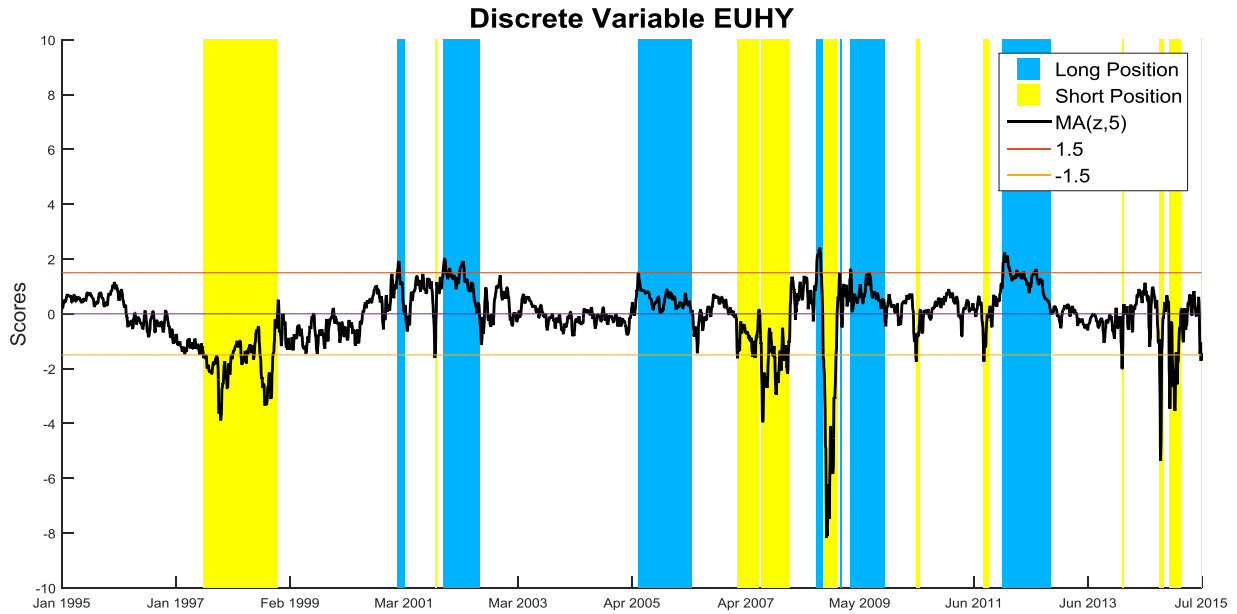
A.1. Figures

Figure 1: $\tilde{z}$ process underlying the discrete variable for the EUHY market. Yellow stripes indicate a short signal, blue stripes a long one.
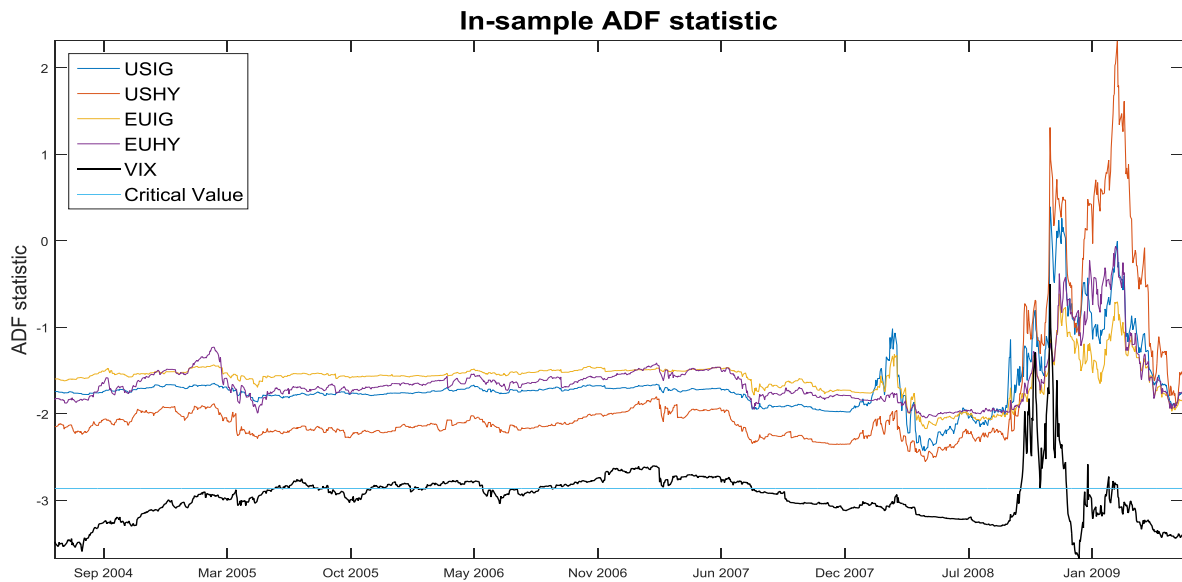

Figure 2: In-sample spreads and VIX ADF statistics. The statistics at time t refer to the period $[1998, t]$. A statistic below the critical value indicates the rejection of the null hypothesis of unit root in favor of the alternative of stationarity

Figure 3: In-sample VIX ADF statistic. The statistics at time t refer to the period $[t - 5 \text{ years}, t]$. A statistic below the critical value indicates the rejection of the null hypothesis of unit root in favor of the alternative of stationarity.
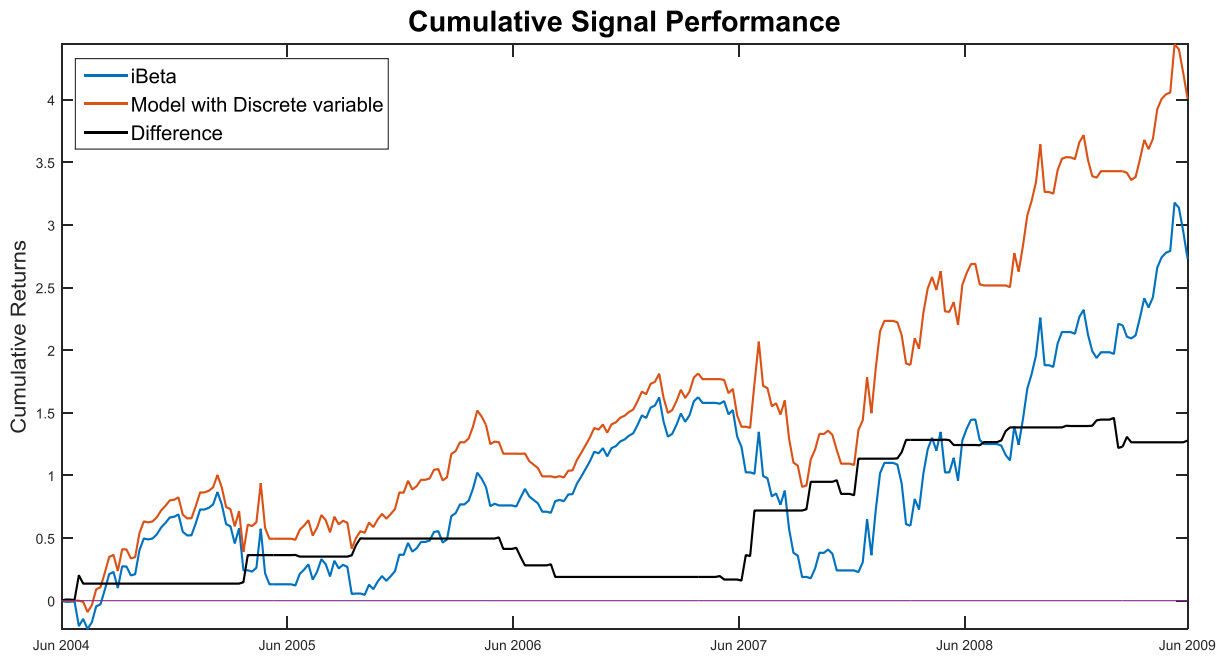

Figure 4: In-sample cumulative signal performance comparison between iBeta and iBeta with the discrete cointegration variable.

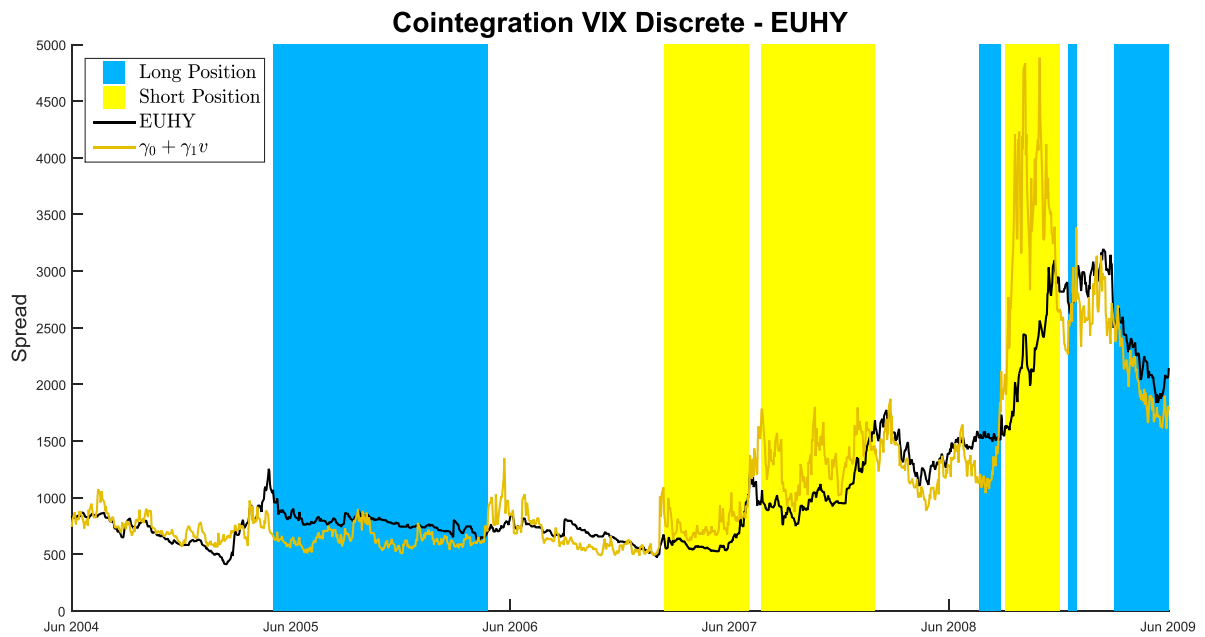Figure 5: Discrete variable signals for the EUHY market. In gold, $\gamma_0 + \gamma_1 v$ represents the spread replication stemming from the cointegration.
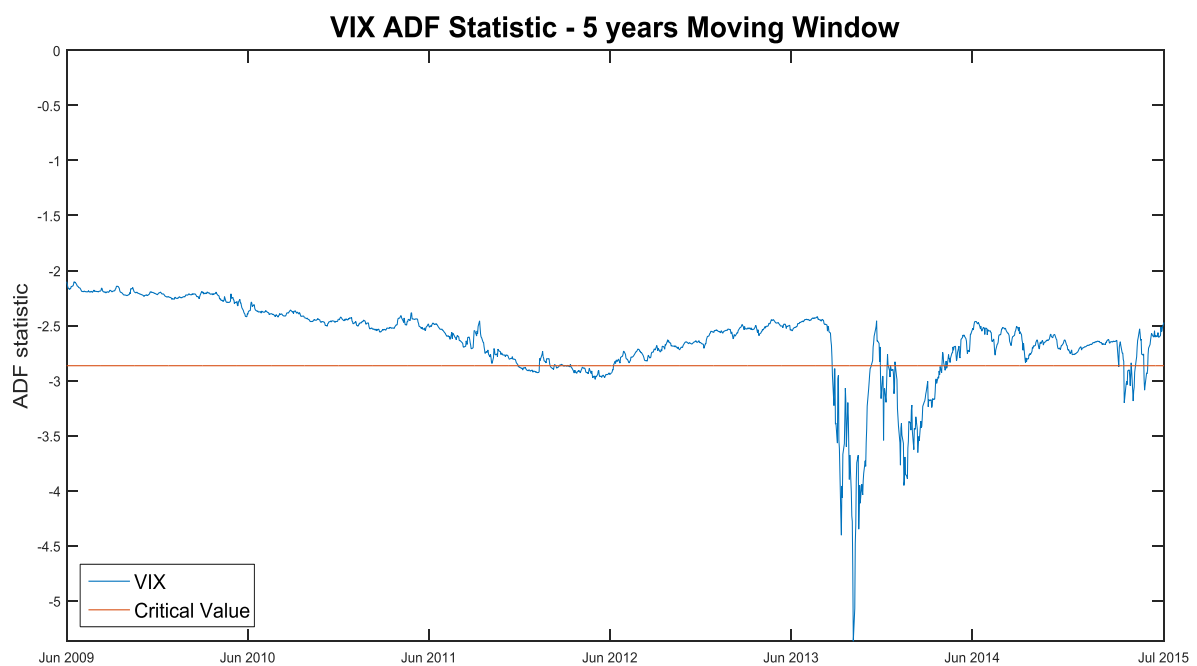

Figure 6: Out-of-sample VIX ADF statistic. The statistics at time t refer to the period $[t - 5\ \text{years}, t]$. A statistic below the critical value indicates the rejection of the null hypothesis of unit root in favor of the alternative of stationarity.

A.2. Tables

Table 1: Johansen cointegration results and coefficient estimates.

Assets	Johansen Test		
	Cointegrated	γ_0	γ_1
USIG, VIX	yes	-306.4	29.6
USHY, VIX	yes	-9320.5	742.4
EUIG, VIX	yes	-1138.9	91.58
EUHY, VIX	yes	-3257.7	323.44

Table 2: Adjustment coefficients and Gonzalo-Granger measure.

Assets	Adjustment Coefficients		GG_{VIX}
	α_1	α_2	
USIG, VIX	-0.0052*	0.0031*	0.61
USHY, VIX	-0.0091*	0.0004	0.95
EUIG, VIX	-0.0187*	0.0007*	0.96
EUHY, VIX	-0.0156*	0.0004	0.98

Table 3: In-sample single variables $z-$performances.

Variable	Perf IG	Perf HY	Vol IG	Vol HY	IR IG	IR HY
Cointegration VIX discrete	0.90	7.12	2.12	6.02	0.43	1.18
Cointegration VIX continuous	1.79	6.55	2.18	6.02	0.82	1.09

Table 4: In-sample models performances.

Model	Net Perf IG	Net Perf HY	Net IR IG	Net IR HY	Turnover IG	Turnover HY
iBeta	**0.31**	**0.78**	**0.80**	**0.55**	**3.45**	**3.38**
Discrete	0.38	1.21	1.05	0.84	2.64	2.90
Continuous	0.38	0.95	1.04	0.67	3.13	2.89

Table 5: Out-of-sample single variables $z-$performances.

Variable	Perf IG	Perf HY	Vol IG	Vol HY	IR IG	IR HY
Cointegration VIX discrete	-0.02	1.18	0.72	4.82	-0.03	0.25
Cointegration VIX continuous	0.17	1.09	0.87	4.62	0.20	0.24

Table 6: Out-of-sample model performances.

Model	Net Perf IG	Net Perf HY	Net IR IG	Net IR HY	Turnover IG	Turnover HY
iBeta	**0.06**	**1.07**	**0.20**	**0.73**	**4.43**	**3.44**
Discrete	0.12	1.08	0.40	0.74	3.68	3.05
Continuous	0.10	1.02	0.34	0.70	3.94	3.21

Table 7: iBeta model outperformance when including discrete variable.

Cointegration VIX discrete - iBeta	In-sample	Out-of-sample	Out-of-sample fitted
Net Performance IG	+ 0.10	+ 0.06	+ 0.07
Net Performance HY	+ 0.50	+ 0.01	- 0.01
Net IR IG	+ 0.31	+ 0.20	+ 0.22
Net IR HY	+ 0.33	+ 0.02	- 0.01

Table 8: Out-of-sample models results for cointegration variables with a lookback window of 3 years instead of 5.

3 years Moving Window, 2007-2009						
Model	Net Perf IG	Net Perf HY	Net IR IG	Net IR HY	Turnover IG	Turnover HY
iBeta	**0.55**	**0.65**	**0.95**	**0.37**	**5.11**	**5.14**
Cointegration VIX discrete	0.49	1.00	0.87	0.56	4.64	4.65
Cointegration VIX continuous	0.54	1.00	0.96	0.56	4.64	4.65

Table 9: Full-sample model results with 3 and 5 years moving window.

Full sample: 2004-2015					
Moving Window	Model	Net Perf IG	Net Perf HY	Net IR IG	Net IR HY
	iBeta	**0.18**	**0.97**	**0.52**	**0.67**
3	Cointegration VIX discrete	0.25	1.12	0.74	0.78
	Cointegration VIX continuous	0.22	0.96	0.64	0.67
5	Cointegration VIX discrete	0.25	1.17	0.74	0.81
	Cointegration VIX continuous	0.24	1.02	0.71	0.71

Table 10: Johansen statistics for the in-sample and out-of-sample periods.

Cointegration Johansen Statistic				
Period	USIG	USHY	EUIG	EUHY
In-sample	21.0942	23.1324	26.8138	31.555
Out-of-sample	25.2629	22.4168	23.4586	26.3029

A.3. $CV_{discrete}$ score computation: Monte Carlo simulations

We compute the value M in (4) via Monte Carlo simulations. The *CV* variable is added to the *Equity* basket, where other two variables are already present. Therefore we generate 100 000 vectors of two multivariate normal variables following the distribution $N_2(0, \Sigma)$, where Σ is the historical covariance matrix of the two variables *Equity Trend* and *VIX Trend*. We compute the historical correlation ρ for the full sample, getting a value of $\rho = 0.52$. After generating the multivariate vector, we cap each variable at ± 1 and we calculate the average of the absolute value of these two variables. By taking the average over the 100 000 simulations, we get the expected contribute of each variable to the *Equity* basket, that is $C = 0.63$. However, this is not the value of M, since $CV_{discrete}$ assumes also value 0. Therefore its expected contribution must be larger than M. Denoting by p the probability $P(CV_{discrete} = 0)$, CV expected contribution is

$$E|CV_{discrete}| = p \cdot 0 + (1 - p) \cdot M \cdot E(GG(t)).$$

We can compute the expected Gonzalo-Granger measure by taking its average over time and markets. The result is $E(GG(t)) = 0.86$. Since the variable has to give the same contribute as the other two, we set

$$0.86 \cdot M(1 - p) = C \quad \Rightarrow \quad M = \frac{C}{0.86 \cdot (1 - p)}$$

Thus we have to compute p. We can calibrate it by extracting the implied probability from the in-sample period, by looking at how many times the variable equals 0. The variable is calculated with the Johansen method with a window of 5 years. Our estimated probability is $p = 0.7$, giving a value of $M = 2.4$.