

Monitoring Liquidity Risk at BPI GA

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As a broad concept liquidity is the ability to readily access funds when needed at a minimal costs. In fact, there is little else of greater importance to ensure the smooth functioning of capital markets than sufficient liquidity. The credit crunch of 2008 was in essence a liquidity crisis. Some markets ceased trading, increasing investor panic and putting further pressure on financial markets. In the wake of these events, financial institutions and supervisory authorities have become more focused on liquidity risk measures promoting greater transparency and a risk culture inside the organizations.

A major concern of regulators are the implications for the banking industry brought by liquidity risk. Hence, regulation has tightened and more limits have been imposed to control the risk. The Risk Management Division at BPI Gestão de Activos (GA) is particularly concerned in having the right tools to actively assess liquidity risk. Providing valuable insights into the notion of liquidity to portfolio managers makes them aware of possible losses that might arise if the institution cannot meet its obligations, and consequently their counterparts can breach the contracts.

CMVM impositions¹



Article 14 of the Regulation 02/2015:

Recognize the individual liquidity risk of collective investment in transferable securities, in particular for:

- i) The assets comprising the portfolios such as the percentage of the issue owned by the body, the average transaction volume of the asset, the difference between buying and selling prices;
- ii) The subscription movements, transfer and fund redemptions.

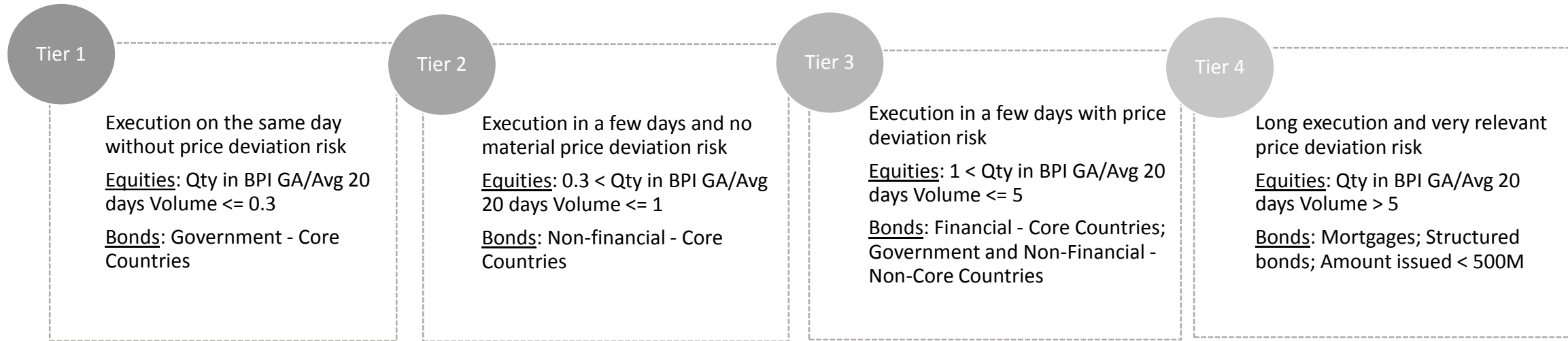
Article 23 of the Regulation 02/2015:

In addition, should be performed resistance tests to assess its liquidity risk, using:

- a) Scenario analysis;
- b) Periodic tests to evaluate the strength of the liquidity risk measures (backtesting);
- c) Frequent estimations and acceptable levels of loss;
- d) Counterparty risk mitigation policies.

¹Source: CMVM website (<http://www.cmvm.pt>)

Monitoring of liquidity risk at BPI Asset Management is currently based on a broad classification of assets in four categories:



However, BPI GA felt that that this approach is based on criteria somehow subjective, especially for debt securities, hence it has felt the need to evolve towards a more rigorous system.

This thesis intends to incorporate in the bank a procedure for the evaluation and monitoring of liquidity risk in different portfolios, as well as understand how they behave in stress scenarios. In order to develop this, it was selected a restricted group of portfolios, but enough to cover the most representative asset classes in BPI GA's portfolios.

	Portfolio	Asset class
EQUITY	<ul style="list-style-type: none"> BPI África BPI Portugal 	African Equity Portuguese Equity
FIXED INCOME	<ul style="list-style-type: none"> BPI Euro Taxa Fixa BPI Liquidez 	Eurozone Sovereign Bonds Short-term Corporate and Sovereign Bonds
MULTI-ASSET	<ul style="list-style-type: none"> BPI Global 	Several

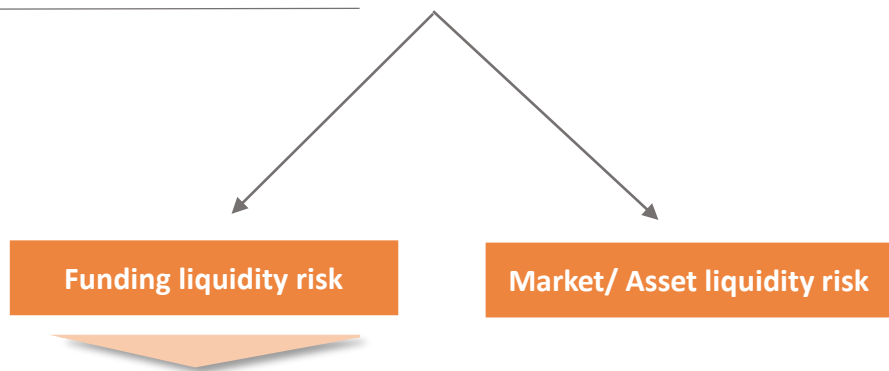
Besides the liquidity risk there are other types of risk faced by portfolio management such as market risk, credit risk or operational risk. However, our project is focused in developing a monitoring liquidity risk system for BPI GA.

In order to better understand and manage liquidity risk is important to first define it.

Concept

From the asset owner perspective, liquidity is the degree to which an asset or security can be bought or sold in the market without affecting its price within a given timeframe. Market liquidity risk is the risk of losing a certain amount of money when liquidating one or more positions in a portfolio to meet other obligations. The loss is generated by the difference between the price at which the financial asset is marked and the price at which it can be sold.

Types of liquidity risk¹



Arises from funding needs the institution may face, due to liabilities, fund redemptions, etc.

However, we will restrict our analysis to asset liquidity risk since funding needs are hard to observe.

Dimensions of asset liquidity risk

- 1 **Tightness:**
 - Costs when buying and reselling the same asset (round-turn transaction)
 - Transaction costs, deviation from mid price (bid-ask-spread)
- 2 **Depth**
 - Order size sensitivity of the asset price
- 3 **Resiliency**
 - Time the price needs to go back to the old level after an external shock
- 4 **Immediacy**
 - Time from order to execution

¹Source: Jorion, P. (2007) Value at Risk– The New Benchmark for Managing Financial Risk (3rd ed, pg.339 - 343). McGraw-Hill.

In the literature there is not a common approach to measure bond and stock liquidity. We have compared several commonly employed liquidity measures, in order to decide the one that best fits BPI GA needs with the available data resources.

Moreover, it has to be a model that enables us to aggregate in a single measure market liquidity for stocks and bonds at the same time, as well as one that allow for comparisons among different portfolios held by BPI GA.

Below, there are described briefly some models that have been considered to measure liquidity in bonds, equities, and methods that are currently used in the industry

Equities and bonds

Amihud's Model (price impact measure)

Proposes an illiquidity ratio that gives the effect on return of a given trading volume. High illiquidity ratios imply more illiquid stocks.¹

$$ILLIQ_{i,t} = \frac{|r_{i,t}|}{vol_{i,t}}$$

Data: Bloomberg pricing quotes; trading volumes

Advantages: easy to aggregate; large quantity held by an institution of a given stock means an higher cost of liquidation; easy to access data

Disadvantages: not applicable for other types of assets where data is scarce; difficult for stress testing since there are not parameters to manipulate; periods of high volatility usually imply low liquidity but higher volumes, the model will assume the increased volume as a fall in liquidity risk

¹Amihud, Y., (2000) Illiquidity and Stock Returns: Cross-Section and Time-Series. NYU Working Paper

²Kyle, A. S., (1985) Continuous Auctions and Insider Trading. *Econometrica*, 53(6), 1315–1335

³Meucci, A., (2012) Fully Integrated Liquidity and Market Risk Model. *Financial Analysts Journal*

Kyle's λ Model (price impact measure)

Depends on intraday trading data, the formula below gives the price change for each traded volume, separated from the effect of the bid-ask cost (given by the term ψ).²

$$\Delta P_t = \alpha + \lambda Q_t + \psi(D_t - D_{t-1})$$

Data: intraday trading prices and volumes; trader side (buy-side / sell-side)

Advantages: cost measure easy to aggregate; capture and distinguish clearly the price impact from the bid-ask cost of each trade

Disadvantages: difficult to access intraday data and trader side; requires a considerable computational effort

Meucci Model

Total P&L = mark-to-market P&L + adjusted liquidity cost

The amount to liquidate and the time to liquidation it is based on risk drivers and market conditions. Then, the adjusted VaR/CVaR is calculated and a liquidity score assigned.³

Data: risk drivers; trading volumes

Advantages: aggregated measure to assess the liquidity cost of a portfolio; possible to include trading strategies for liquidation and optimization; easy for stress-testing

Disadvantages: difficult to perceive how the risk drivers are chosen and the methodology behind; lack of transparency in the bond market to access data; significant computational effort

Literature review

Equities and bonds

LVAR (Jorion) (transaction cost measure)

Gives us the immediate liquidation cost of the portfolio, assuming that the spread is not dependent of the trading volumes. Moreover, an extension of VaR is allowed including the liquidation cost.¹

$$S = \frac{P(ask) - P(bid)}{P(mid)}$$

$$LVaR = VaR + \frac{1}{2} WS, \text{ if spread is fixed}$$

Data: Bloomberg bid/ask prices

Advantages: notion of liquidity cost; easy to aggregate if quoted data is available for all assets

Disadvantages: only applicable for stocks; ignores the price impact; stocks infrequently traded have low volatility, invalidating in a certain way the LVAR; difficult for stress testing

Adjusted Schultz Model (tightness model)

Checks the price difference between trades. The effective bid-ask spread is captured on α_1 , using only one dummy variable on whether the trade was buy-side or sell-side originated.²

$$\Delta_i = \alpha_0 + \alpha_1 D_i^{Buy} + \varepsilon_i$$

Data: Intraday trades (TRACE)

Advantages: gives a strong notion of the true transaction costs faced by agents

Disadvantages: very difficult, if not impossible, to obtain transaction-level data on bonds - if prices are difficult to obtain, the originating side of the trade is even harder; the model is mostly based on TRACE, which does not give a full view of the bond market transactions because only US bonds are covered

ETF vs NAV Model

Assuming that ETFs are liquid, then the price difference between the ETF and the value of basket it replicates – NAV – can only be due to liquidity differences between both. Finding ETFs for subsets of bonds, allows inferring liquidity by their characteristics.³

$$ILLIQ = -10.000 \times \ln \left[\frac{NAV}{NAV + |ETF - NAV|} \right]$$

Data: Bloomberg ETF and NAV prices, ETF composition

Advantages: large number of different ETFs allows extending the model to several assets; the differential between the ETF and its NAV may be time-varying and allow stress-testing; data is relatively easy to obtain

Disadvantages: assuming that ETFs are liquid may not be a safe assumption; does not give a notion of cost; may require huge amounts of data; the idea is not yet academically validated

Price dispersion

The further the transaction price is from the asset consensus price, the more illiquid the asset must be – if there is a consensus price, agents would only accept selling/buying at a price below/above it if the cost of finding a better price is too high, which is a strong definition of liquidity. The model is just a volume-weighted standard deviation of the transaction prices using the consensus price as the average⁴

Data: intraday transactions prices and quantities, consensus price

Advantages: easily allows measuring liquidity by bond characteristic and stress testing; standard deviations, can be used to build bid-ask spreads (tightness) based on confidence intervals

Disadvantages: intraday transaction prices and quantities are virtually impossible to obtain; many bonds are far from having a consensus price (subjective input)

^{1,2,3,4}All references are included in the references page

Literature review

Industry Models

LiquidityMetrics and Bloomberg

Both have developed complex data intensive algorithms that generate liquidity models comprising tightness, depth and resiliency dimensions for a wide range of asset classes that (supposedly) allow building trustworthy data for assets where it is not available.¹

Advantages: models give a full notion of liquidity in all its dimensions; allow automation of liquidity measurement and management techniques

Disadvantages: the models use proprietary algorithms comprising enormous amounts of data, many of them proprietary (Liquidity Observatory and Bloomberg proprietary data, respectively), making their replication extremely complicated; the quality of the output, in particular for assets that belong to opaque markets, is unknown and, therefore, questionable

Barclays Liquidity Cost Score bond model

For quoted bonds:

$$LCS = OASD_{i,td} \times (bid\ spread_{i,t} - ask\ spread_{i,t})$$

or,

$$LCS = \frac{(Offer\ price_{i,t} - Bid\ Price_{i,t})}{Bid\ Price_{i,t}}$$

For non-quoted bonds, linear estimation:

$$LCS = f(\text{sector, age, OAS, amt outstanding, trd volume, benchmark status, ...})$$

The idea is, using market realizable bid-ask information, based on information supplied by Barclays' traders, to generate a bid-ask spread that allows ranking and comparing bond liquidity. For bonds where such data is not available, liquidity is approximated by their characteristics based on the LCS of the quoted ones.²

Advantages: it is a model used by a well-known institution; the measure that comes out of the model is easily extendable to other asset classes, in particular equities, where data is easily obtainable; historical data on LCS allow easy stress testing

Disadvantages: does not consider other dimensions of liquidity, in particular the price impact (which is assumed by the model to be correlated with tightness); data is obtained using Barclays trading information, which has a much larger scale than BPI's trading

^{1,2} References for the models are included in the references page

Model – introduction

The portfolios that are evaluated by this report are **BPI África** and **BPI Portugal**, portfolios that are mostly composed by equities, **BPI Euro Taxa Fixa**, containing mostly bonds, **BPI Liquidez**, a portfolio that combines short term assets with bonds, and **BPI Global**, which is mostly a combination of equities and bonds.

The focus of this project is on improving the current liquidity assessment of equities and bonds, because those are the assets that represent the majority of assets in the five target portfolios and, in particular for bonds, those that urge the most an improvement in the assessment methodology. All other asset classes remain with the same liquidity assessment criteria implemented before by BPI.

For the two asset classes covered by the proposed liquidity model, liquidity is being assessed at two dimensions: **tightness** and **depth**. Resiliency and immediacy are dimensions that, despite being relevant, are very data intensive and require complex and heavy computations. They were not considered in this model.

Overview

Stocks are, in most cases, exchange listed, meaning that at any point in time every market participant is able to know the price at which he is able to buy or sell any amount of stocks. This implies that the levels of transparency in equity markets are, generally, very high.

Accurate data, both current and historical, is easily available on any finance terminal, such as Bloomberg.

The order book, being common to all investors and gathering all buy and sell orders placed on the exchange, should provide an accurate notion of how much it costs to perform a roundtrip transaction.

Theory

Higher demand and supply should imply more bid and ask quotes and smaller gaps between the best available buy and sell prices. In theory:

- The more liquid an asset is, the tighter the bid-ask spread should be.
- More liquid assets should have more bid and ask quotes for prices further from the best trading prices, making it cheaper to buy and sell large quantities.

The relative bid-ask spread (BAS) and Amihud's Price Impact ratio measure, respectively, each stock's **cost tightness** and **price impact**.

Model – Equities

Equities

Bonds

Aggregating
portfolio
liquidity

Methodology

Bid-ask Spread

Using Bloomberg data, the monthly relative bid-ask spread for each stock has been measured by averaging the daily relative bid-ask spreads of that month, where, for any day t :

$$\text{Daily Spread}_t = \frac{\text{Closing ask price}_t - \text{Closing bid price}_t}{\text{Closing mid price}_t}$$

$$\text{Mid price}_t = \frac{\text{Closing ask price}_t + \text{Closing bid price}_t}{2}$$

When data on either the bid price or the ask price is not available, the spread is not computed. Overall, for the equities that have been on the five portfolios under analysis between January 2008 and October 2015 (1552 equities) there are 2.886.655 daily spread observations.

A first filter is applied to remove erratic Bloomberg data. Daily spreads where the following conditions were met were removed from the sample:

- There is no corresponding trading volume data (or it is 0) – suggests that bid/ask quotation are static rather than realizable prices
- The daily bid price is higher than the ask price – which is an illogical condition: if that was the case, a trade should have occurred
- The daily bid price is less than half the ask price – when it happens, it is mostly due to either a sudden unreasonable jump in the ask price or fall in the bid price, as exemplified by the table

From the original 2.886.655 daily spread observations, 67.868 were removed.

OIBR US Equity

	Ask Price	Bid Price
05/03/2007	23,6415	22,0339
06/03/2007	24,1616	21,3089
07/03/2007	22,8534	21,2301
08/03/2007	24,6187	21,6871
09/03/2007	3152,198	0,0158
12/03/2007	25,7062	22,7746
13/03/2007	23,5784	23,5154
14/03/2007	24,6817	24,6502

Model – Equities

Removing outliers

Having these three conditions met, there are still situations where there seems to be erroneous data – such as sporadic and unrealistic high daily spreads, generally implying an erratic bid price, or occasional one-day spread drops, which are cases where the spread seems to be consistent over a time interval but, for one single day, it drops significantly to bounce back again on the next day, which usually implies an erratic ask price. To remove these outliers, a 3-step approach was used for each stock individually and for every month:

1. Computing the 95% percentile of the daily spreads for the last year;
2. Computing the average and standard deviation of daily spreads of that year for values below the previously computed 95% percentile if there is data for at least 50% of the year (or, if the stock has been listed less than 1 year ago, data for 50% of the working days between the listing date and the end of the month);
3. Removing daily spreads when the daily spread is more than two standard deviations away from its average.

By not using the highest 5% spreads in computing average and standard deviation, it is ensured that, in step 3, if the value is indeed an outlier it is removed – unusually high spreads will not increase Step 2's average and standard deviation in such a way that they are not considered outliers in Step 3.

This process removes 562.672 daily spreads from the 2.818.787 that passed the first filter.

Monthly spreads

As portfolio liquidity will be assessed on a monthly basis, monthly spreads are computed by averaging the daily spreads of each month. The only rule that applies here is that there must be at least 12 daily spreads data (after the previous filters) for the monthly spread to be computed. If a month has scarce trading or few accurate data, it is better to assume it has no bid-ask spread than to obtain an inaccurate one. Equities that do not have at least 12 accurate daily data points each month should be more illiquid than equities which do have them.



Outliers - 3-Step approach		
Date	Daily Spread	Filtered Spread
01/04/2010	0,15%	-
02/04/2010	0,20%	-
05/04/2010	0,18%	-
...		
25/03/2011	0,16%	0,16%
26/03/2011	0,07%	-
29/03/2011	0,15%	0,15%
30/03/2011	1,95%	-
31/03/2011	0,18%	0,18%
μ (bottom 95%)	0,20%	
σ (bottom 95%)	0,05%	

0,07% is lower than the criterion
 $0,2\% - 2 \times 0,05\% = 0,10\%$

1,95% is higher than the criterion
 $0,2\% + 2 \times 0,05\% = 0,30\%$

If there was no data for at least 50% of the working days between 01/04/2010 and 31/03/2011, then average and standard deviations would not be computed and all March's spreads would be removed

Amihud's Price Impact ratio

Equities

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review

Besides the direct cost of trading relatively small amounts of stock, there is also a cost associated with trading larger numbers of shares. The larger the amount BPI owns on a certain stock, the more likely it is that the currently available best bid and ask prices will not hold when liquidating the asset, i.e.: the probability of exhausting the demand (or supply) increases.

For that purpose, Amihud's Price Impact ratio (ILLIQ), which measures the change in price for each transacted share, was computed on a daily basis. Using Bloomberg data, for any day t :

Model

$$Daily\ ILLIQ_t = \frac{|r|_t}{Daily\ trading\ volume_t}$$

$$|r|_t = \left| \ln \left(\frac{Closing\ bid\ price_t}{Closing\ bid\ price_{t-1}} \right) \right|$$

Historical
results

Values for ILLIQ were not computed when the daily volume is 0, as the formula would result in an error.

Also, using a similar approach as in the bid-ask spread, Daily ILLIQs which are more than two standard deviations away from the average (using 1 year of data) are removed from the sample. These cases usually arise from days with particularly low trading volumes which increase significantly the ratio.

Monthly ILLIQs, as in the bid-ask spread, are computed by averaging the Daily ILLIQs as long as there are at least 12 Daily ILLIQ observations.

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Model – Equities

Inferring price impact

Equities

Bonds

Aggregating
portfolio
liquidity

If the ILLIQ measures the price impact of each share sold, then multiplying the amount owned by BPI GA by the ILLIQ should give the price impact of liquidating the position in the stock:

$$|\widehat{r}|_t = \text{Monthly ILLIQ}_t \times \text{Qty in BPI GA}_t$$

There are some differences in equity pricing between Bloomberg and BPI's database, Hexágono. Most equities are quoted at the same price as Bloomberg, but some seem to apply a "multiplier" – the quotations are 100 times lower than in Bloomberg. In this case, quantities reported by Hexágono were multiplied by 100 to compute $|\widehat{r}|_t$.

If there is not any direct match or 100x multiplier relationship between Hexágono and Bloomberg quotations, Amihud ILLIQs were not considered.

Assuming, as one should expect, that the price impact is positive when buying stock and negative when selling, a spread similar to the relative bid-ask spread previously explained can be built:

$$\begin{aligned} P_t^{Ask} &= P_t^{Last} \times e^{|\widehat{r}|_t} \\ P_t^{Bid} &= P_t^{Last} \times e^{-|\widehat{r}|_t} \end{aligned}$$

$$\text{Amihud Spread}_t = \frac{(P_t^{Ask} - P_t^{Bid})}{1/2 \times (P_t^{Ask} + P_t^{Bid})} = \frac{P_t^{Last} \times (e^{|\widehat{r}|_t} - e^{-|\widehat{r}|_t})}{1/2 \times P_t^{Last} \times (e^{|\widehat{r}|_t} + e^{-|\widehat{r}|_t})} \rightarrow \text{Amihud Spread}_t = \frac{e^{\text{Monthly ILLIQ}_t \times \text{Qty in BPI}_t} - e^{-\text{Monthly ILLIQ}_t \times \text{Qty in BPI}_t}}{1/2 \times (e^{\text{Monthly ILLIQ}_t \times \text{Qty in BPI}_t} + e^{-\text{Monthly ILLIQ}_t \times \text{Qty in BPI}_t})}$$

Model – Bonds



Overview

Information on bids/asks, order book and volumes is only available for liquid financial assets that trade on a daily basis in fairly transparent markets. In the case of bonds, the “transaction costs” are not as readily available as in the case of stocks since most bonds are transacted over-the-counter. In other words, financial information such as the trading volume required to calibrate the traditional price impact models of liquidity risk is not available for these securities, where a measure of this risk is mostly needed.

Methodology

Bond’s liquidity has been assessed in two different dimensions, first using the relative bid-ask spread – **tightness** – and then by a pure position risk alert - **depth**.

1. A bond’s relative bid-ask spread represents the round-trip cost, as a percentage of the bond’s price, of immediately executing a standard institutional transaction. So, according to this definition, a lower spread value denotes better liquidity. More formally, bid-ask spread can be computed as follows:

$$Daily\ Spread_t = \frac{Closing\ ask\ price_t - Closing\ bid\ price_t}{Closing\ mid\ price_t}$$

$$Mid\ price_t = \frac{Closing\ ask\ price_t + Closing\ bid\ price_t}{2}$$

2. The position risk alert for fixed income securities is the quantity held by BPI GA as a percentage of the amount issued for a given bond. This is the result of being traded in over-the-counter markets where transparency is not a dominant word. The lack of data, such as the trading volume, dictates the impossibility of measuring liquidity risk with price impact measures, such as the Amihud model for stocks. Therefore, the “price impact” for bonds can be expressed with the following formula:

$$Position\ risk_{bonds} = Quantity\ owned / Amount\ Issued$$

The intuition of this measure is to impose a position limit. If the percentage of a given bond owned by BPI as a whole is significant, there should be more difficulties in liquidating that bond.

Model – Bonds

Quoted bonds



Problem

BPI GA prices each security using the bid price from different Bloomberg pricing sources, named CBBT, TAS and BVAL.

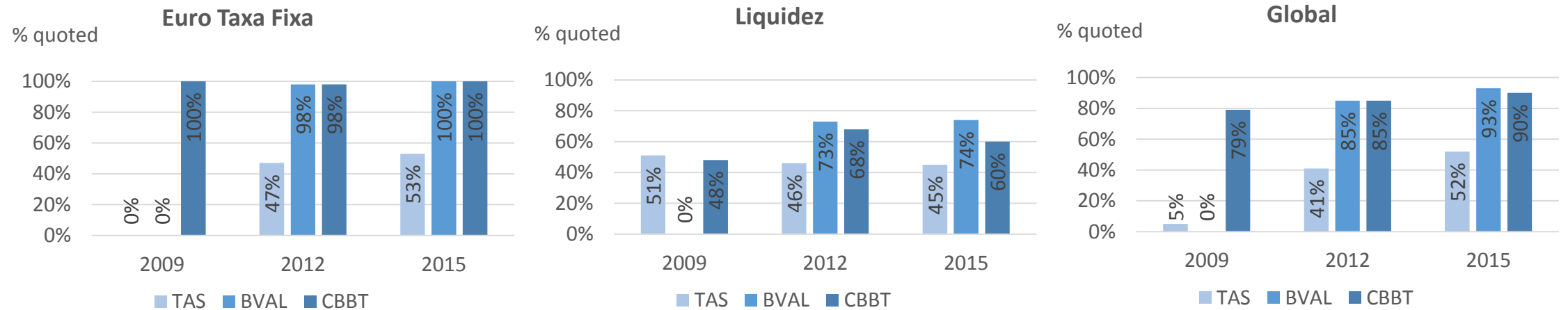
- BVAL ask prices are only available since 2010
- TAS has few quotes from 2013 onwards
- CBBT does not cover all the securities until 2010

Solution

Alternate the pricing sources when estimating the bid-ask spread gives the chance to cover a large number of bonds, which will be very useful in the historical analysis that we conduct later in this report.

We have chosen the pricing source of a specific bond in a month based on the minimum difference between the quoted bid price on Bloomberg from these three pricing sources and the price at which BPI has priced that security in their database – Hexágono.

The different pricing sources available in Bloomberg for fixed income securities are based on available market data. Bloomberg CBBT and BVAL’s methodology is based on Bloomberg’s real-time access to market observations from a wealth of contributed sources. This quantitative approach first corroborates market levels on actively traded bonds, using trades and indicative bid/asks on the target bond, then derives a comparable relative price on less liquid securities when direct market observations on the target bond are insufficient.



Note: We detected some inconsistencies between the bond pricing in BPI’s database (Hexágono) and Bloomberg. For inflation-linked bonds, BPI uses an index ratio adjustment to Bloomberg quotations. Therefore, we divided the prices at which BPI have priced inflation-linked bonds by their specific index ratio.

Model – Bonds

Adjustment factor



Data quality is key for good liquidity measures. Especially in OTC markets, where prices are not fairly transparent, it is important to be monitored. Some quotes are commitments to make a market, others are only indications and it may be difficult to execute at those quotes.

Therefore, we adjust these trader quotes wider by validating them through the effective BPI trading prices (available since 2011).

Bid and ask daily quotes for a certain bond are deviated if the effective price at which a bond is bought or sold falls outside the spread.

Since intraday data is not available, we have considered the ask price as the maximum between the current and the previous working day closing ask prices. Following the same logic, the bid is the minimum between the current and the previous working day closing bid prices.

The effective trading prices have been analyzed for three different pricing sources used by BPI to price their securities, TAS, BVAL and CBBT. However, quoted daily prices available in Bloomberg have to respect some conditions¹.

	# BPI trades	Quoted spreads		Deviated quotes	
		#	%	#	%
TAS	22193	8436	38%	3144	37%
BVAL	22193	21042	95%	4251	20%
CBBT	22193	20240	91%	3178	16%

The regressions for non-quoted bonds, later explained in this report, support the idea that bond ratings are highly correlated with bid-ask spreads. As a result, we have sorted the trades by the rating of the bond in order to assign an adjustment factor to each one of them, based on the average deviation from the effective trading price.

A rating scale from 1 to 5 is considered ²:

1 – AAA; 2 – AA; 3 – A; 4 – BBB; 5 – from BB to D

Rating	1	2	3	4	5
Trades within the spread	62%	67%	65%	60%	57%
Average spread	0,05%	0,13%	0,21%	0,36%	0,78%
Average deviation	0,05%	0,17%	0,29%	0,37%	2,65%

→ TAS

Rating	1	2	3	4	5
Trades within the spread	76%	79%	86%	81%	81%
Average spread	0,11%	0,16%	0,30%	0,31%	1,05%
Average deviation	0,02%	0,02%	0,03%	0,03%	0,20%

→ BVAL

Rating	1	2	3	4	5
Trades within the spread	71%	80%	94%	85%	91%
Average spread	0,08%	0,18%	0,45%	0,44%	1,55%
Average deviation	0,02%	0,02%	0,02%	0,02%	0,25%

→ CBBT

The tables above explain why the rating is the most consistent variable to conduct this analysis. The average spread and deviation increase smoothly as the rating increases. The daily spreads for the portfolios being analyzed are computed with the quote adjustment relative to the closest pricing source by rating, as the formula below suggests:

$$Adjusted\ Daily\ Spread_t = Daily\ Spread_t + 2 \times Avg\ deviation$$

Note: Non-rated bonds don't have adjustment factor, being considered as non-quoted. 17

¹The criteria respected will be explained in the next section.
²Composite ratings from S&P, Moody's and Bloomberg provided by BPI.

Model – Bonds



Outliers

The relative daily bid–ask spread is calculated through the formula mentioned before, imposing similar conditions as for stocks.

Criteria

Daily spreads

- Bid and ask prices for a given bond must exist at the same time
- Ask price \geq Bid price
- Bid price $> 10 \rightarrow$ prices below this level might refer to bonds in default which are most likely infrequently traded

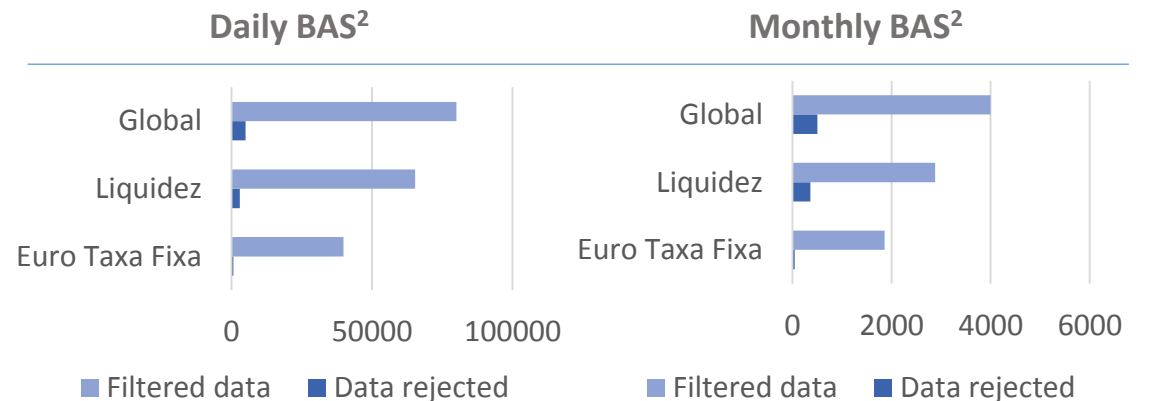
Monthly spreads

- Average of at least 12 daily spreads during that month

Rating

- Rating must be available for the quoted month in order to apply the adjustment factor¹

Daily spreads that do not respect these conditions for certain periods are considered outliers and removed from the sample to prevent from having unrealistic spreads, different from the effective ones.



If a bond has an insufficient number of observations for a month but has data available – rating, maturity and amount issued - its spread can be estimated through a **regression performed for non-quoted bonds**.

¹BPI has data available for ratings since January 2009. The rating criteria was not imposed for spreads computed in 2008, being the adjustment factor ignored for that year

²Observations from January of 2008 to October of 2015.

Model – Bonds

Non-quoted bonds



BAS of non-quoted bonds is estimated using the BAS of quoted bonds, priced with Bloomberg CBBT, which we considered to be most conservative pricing source, since the spreads are wider than in other pricing sources. Besides that, BPI GA considers CBBT as, generally, the most reliable pricing source to price fixed income securities of their portfolios.

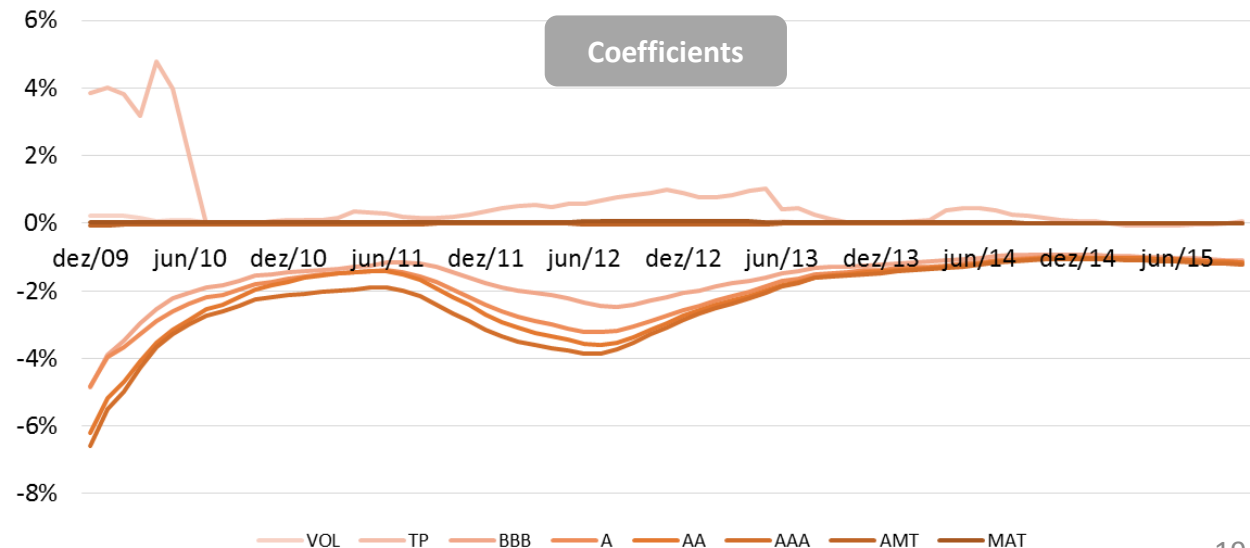
The following equation was used to perform regressions for quoted bonds:

$$BAS_{quoted,b,t} = \alpha + \beta_{mat}Mat + \beta_{amt}Amt + \beta_{BBB}BBB + \beta_A A + \beta_{AA}AA + \beta_{AAA} AAA + \beta_{tp}TP + \beta_{vol}VOL$$

Where: b - bond b; t - month t; MAT – time to maturity (years); AMT – amount issued (in Bn); BBB/A/AA/AAA – bond rating dummy; VOL – Eurozone volatility index (V2X Index); TP – term premium defined as the premium between the ten-year and the one-year European government bond yield (EUSR10Y Index – EUSR1Y Index)

In order to perform this analysis with an appropriate number of quoted bonds included in the sample we have used monthly spreads from other bonds traded by BPI GA in addition to the ones which belong to the portfolios we are analyzing. The criteria to calculate these spreads were the same explained previously. Again, if a bond is occasionally traded or if its prices are only indications, it is better to estimate a more accurate spread than considering those unrealistic prices.

Ratings are grouped within a 5 groups, already mentioned before, given that BBB-rated bonds to AAA-rated bonds represent most of the securities in BPI portfolios. Thus, four dummy variables were defined for the rating of the bond: BBB, A, AA and AAA. Furthermore, regressions were run only for bonds which have data available on the 3 characteristics used to predict the quoted BAS - maturity, amount issued and rating.



Note: Monthly regressions start being performed only in Jan-2009 due to unavailability of ratings. Therefore, in 2008, spreads for non-quoted bonds cannot be estimated.

Non-quoted bonds

Equities

Bonds

Aggregating
portfolio
liquidity

Results

- Typically, the time to maturity and the amount issued are very stable during the sample periods. Liquidity cost increases with the first and falls as the amount outstanding rises.
- All rating scales are negatively correlated with the spread – rating deterioration dictates higher liquidity costs. Throughout the period analyzed, as expected, the monthly coefficients for AAA-rated bonds are always below the coefficients for lower ratings.
- The term premium generally increases with the cost of liquidity, with few exceptions for months in which the variable is not significant.
- Lastly, the volatility index is also positively correlated with the spread.

All these variables are proved to be statistically significant and constant most of the times, with some exceptions for the macroeconomic variables in certain months.¹ The adjusted coefficient of determination is, on average, 39.6%, corroborating the idea that these independent variables are a good explanation of the quoted bid-ask spread.

After that, we have performed a backtesting analysis to see how the spread behaved since 2010, applying the coefficients estimated through the regressions for each month. We found some negative BAS values: however, these cases correspond to bonds with rating AAA or AA, the most liquid ones. Following this intuition, we assumed a 0% spread for these bonds since they are expected to be very liquid.

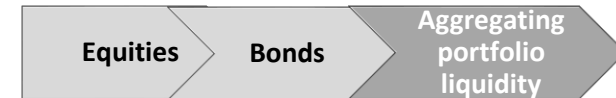
Limitations

- One year moving averages have been used because of the small amount of bonds included in the sample in 2009 and 2010. BPI GA only has data on trades since 2011, thus limiting our sample before 2011 to bonds that used to belong to one of the five portfolios and bonds that, being traded by BPI after 2011, had already been issued before.
- Using one year moving average implies losing one year of data, meaning that the regression analysis starts only in December 2009. This may create a break in the series of historical spreads.
- The parameters are stable over time using moving averages, however, it might be smoothing to much the effect of temporary fluctuations in certain months to reduce the noise.
- Other macroeconomic variables have been tested, such as the default premium and the EUR/USD exchange rate, not giving good results.
- A rating scale from 1 to 12 has been tested instead of dummy variables but linearity between ratings and spreads is not a reasonable assumption here.
- Regarding the risk country and sector, which are the characteristics used by BPI to allocate bonds to tiers referred earlier in this report, we could not find in them any consistent statistical significance.

¹See Appendix A1 and A2: Coefficients and T-student tests for significance levels.

Model – Portfolio aggregation

Most equities and bonds, after having passed through the model process, have a relative bid-ask spread allocated. These spreads are in fact comparable. They represent, approximately – and only approximately because they are relative to the mid price, which is not the pricing used by BPI to value their assets – how much in percentage it would cost to instantaneously buy and sell the asset. They are, in other words, comparable measures of tightness. All other asset classes have only one measure for liquidity, which is the currently BPI allocated tier, from 1 to 4. As such, they are not directly comparable with this model’s equity and bond liquidity measures.



Liquidity cost aggregation

Problem

Different assets have different liquidity cost measures:

Bid-ask spread:

- Equities
- Bonds

BPI Tiers (1 to 4):

- All other assets

While equities and bonds have direct liquidity cost comparability, other assets don't.

Solution

Temporarily, while other assets don't have any liquidation cost allocated, the solution found was grouping both equities and bonds into 4 tiers according to their spread levels.

While this will result in an output very similar to the one BPI currently has, using bid-ask spreads will create a significant difference in the way tiers are built:

- Equities are now grouped according to stock-specific liquidity cost and less dependent on amount owned by BPI
- Bonds' bid-ask spreads are mostly based on Bloomberg data
 - The tightness of spreads is linked with availability of supply and demand

Ideally, all assets should have a direct cost measure similar to the bid-ask spread.

Price deviation risk

Problem

Price deviation risk computed for equities and bonds aren't comparable measures:

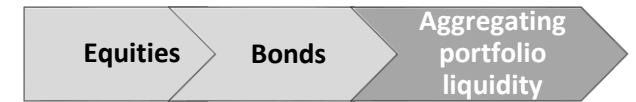
- Equities - Amihud's price impact ratio
- Bonds - Bond's position risk ratio

Solution

Each measure was grouped into 4 tiers and treated as auxiliary measures for the bid-ask spread.

High tiers imply higher risk of bid-ask spread inaccuracy due to the size of the position held by BPI GA.

Model – Portfolio aggregation



Calibrating tier thresholds

The main challenge of grouping equities and bonds into 4 tiers according to their bid-ask spread is deciding which spread thresholds must be imposed for each tier.

Problem

There is no golden rule to help in determining what are the best bid-ask spread thresholds for each tier, especially considering that these tiers will be used to compare equities and bonds with different asset classes and different liquidity assessment criteria.

Using BAS distributions on BPI GA's portfolios wouldn't be very useful either:

- BPI's sample would be biased towards the assets that BPI owns the most
- Even with a distribution, it wouldn't be obvious which criteria would allow matching equities and bonds with other tiers

Solution

The threshold decision should be an *ad hoc* criteria.

One possible way to achieve an informed *ad hoc* criteria would be asking traders, who have higher sensibility to asset liquidity, to create four baskets of equities and bonds according to their perception of liquidity levels. Then, by averaging the BAS in each basket, it would be possible to generate better informed thresholds.

The price deviation tiers' thresholds could be also based on baskets created based on traders' perceptions of which assets have higher position size risk.

Decision

The thresholds for each tier are an input to the model, meaning that the chosen thresholds can be easily changed at any point in time.

For the purpose of this project, the following thresholds have been chosen:

Tiers	BAS	Amihud	Bond Position Risk
1	[0 ; 0.15%[[0% ; 0.15%[[0% ; 0.2%[
2	[0.15% ; 0.5%[[0.15% ; 1%[[0.2% ; 0.5%[
3	[0.5% ; 1.5%[[1% ; 4%[[0.5% ; 2%[
4	[1.5% ; +∞[[4% ; +∞[[2% ; +∞[

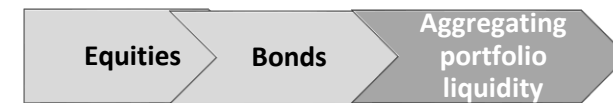
Logically, lower tiers suggest better liquidity profiles.

These thresholds are based on our personal judgement of what seems to be the most reasonable figures for each tier, given the values that the model calculates.

If using the suggested approach based on traders' feedback, it is possible to obtain *better-informed* thresholds.

Model – Portfolio aggregation

Aggregating Bid-ask spreads



Whenever a portfolio is composed predominantly by equities and/or bonds, the aggregate bid-ask spread should, by itself, be a solid measure of portfolio liquidity, being comparable across portfolios and across time. The aggregate spread is just the average of the equities' and bonds' bid-ask spreads, weighted by the portfolio value of the asset. Being "i" an asset which contains a bid-ask spread:

$$Portfolio\ aggregate\ spread = \frac{\sum_i (BAS_i \times Portfolio\ weight_i)}{\sum_i Portfolio\ weight_i}$$

It is a measure loses its relevance when portfolios contain other asset classes where liquidity is not measured in terms of spread. The higher the portfolio weight of assets which are neither equities nor bonds is, the poorer the representativeness of the aggregate spread on the portfolio overall liquidity.

Valuing the bid-ask spread

Jorion¹ suggests a more intuitive approach based on the relative BAS consisting of adding a liquidity cost term, L_1 , to VaR. Being W_i the value in € of asset i in the portfolio, then:

$$Portfolio\ L_1 = \sum_i \left(W_i \times \frac{1}{2} BAS_i \right)$$

This formula yields the amount (in euros) that would cost to fully liquidate all the assets in the portfolio. It assumes, however, that assets are valued at the mid price, which is not the case in BPI: bonds are valued at bid price and equities at closing price.

- Equities: the closing price is assumed to be a good proxy of the mid price and the formula for L_1 is directly applicable.
- Bonds: the formula for L_1 is transformed so that it may use bid prices and, as shown in appendix B, hold equivalence to the formula when using mid prices.

Adjusting portfolio value

Asset	BPI pricing	Pricing after liquidity costs	Difference
Equities	Last price	Bid price	≈ L1
Bonds	Bid price	Bid price - Adjustment factor	Adjustment factor

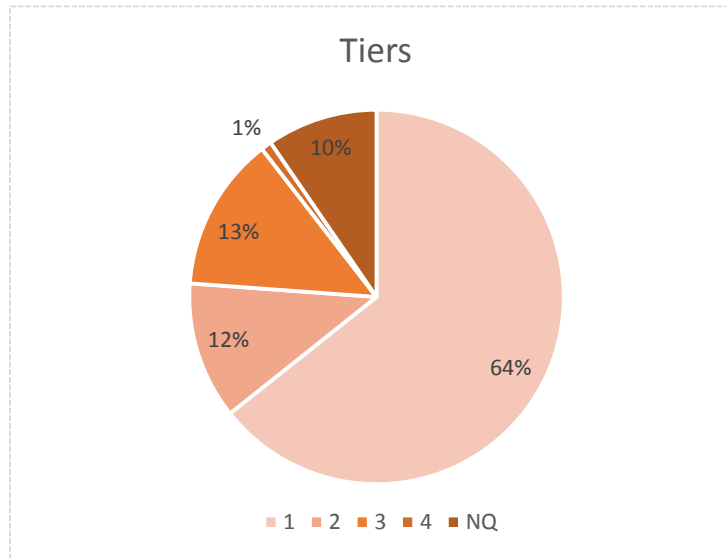
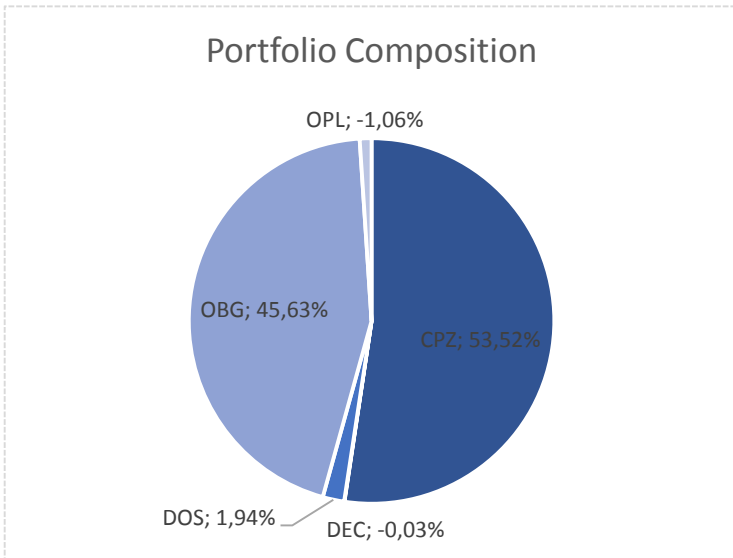
The portfolio value loss after adjusting for liquidity costs was computed as the sum of equities' L_1 , as it was assumed that the realizable price at liquidation is the bid price and not the last price, and the sum of each bond's W_i times the adjustment factor, to correct for the risk of inaccuracy on the quoted bid price.

1 – Jorion, P. (2001). "Liquidity risk", in "Value at Risk: The New Benchmark for Managing Financial Risk" 2nd ed., McGraw-Hil, 339-357

Model – Output

All the portfolio aggregating part of the model is coded under VBA, which requires only two spreadsheets directly obtainable from BPI’s Hexágono database (to obtain the portfolio composition at a certain end of month) and one or two auxiliary spreadsheets, depending on whether the portfolio contains equities, bonds or both, which gather and process Bloomberg data.

Using as an example BPI Liquidez portfolio as of October 2015, running the VBA code instantly gives the following graphical outputs¹:



October 2015	
Aggregate Spread	0.21%
Aggregated Assets	36.06%
Value of assets with BAS	157 704 723.65 €
Jorion L ₁	167 262.34 €
Original portfolio value	437 382 186.87 €
Value of assets with BAS	157 704 723.65 €
Liquidation costs	66 876.73 €

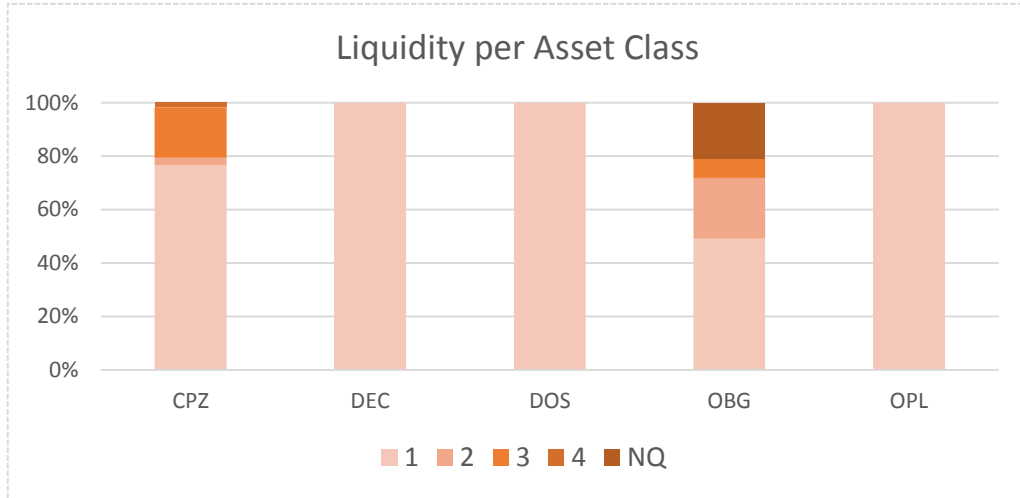
Understanding the portfolio composition is important to check whether the model is suitable or not. If the sum of weights on bonds (“OBG”) and equities (“ACC”) does not represent the majority of the portfolio, then all aggregating BAS measures won’t have enough power to represent the portfolio as a whole.

This groups all assets into unique tiers. The classification of equities and bonds depends on the BAS thresholds defined before running the VBA code, as previously explained. Those that, either for lacking data or for being outliers, do not have BAS, are classified as non quoted (NQ). Other asset classes are classified according to BPI’s previous criteria.

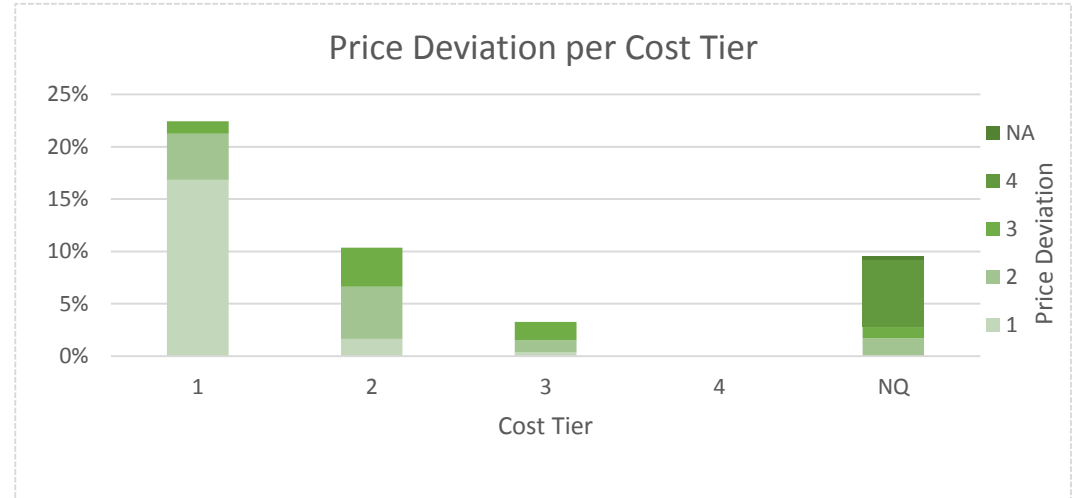
The aggregate spread figure is the one that enables comparison across portfolios and across time. It is also very important to perceive the aggregated assets as a measure of model power – in this case, only 36.06% of the portfolio’s value contains assets which have a BAS, so the model is significantly weak to evaluate the portfolio as a whole. The € figures are important to quantify the BAS in monetary terms.

1 – The output also contains a sheet with asset-by-asset data and tables used to build the graphs, which are not displayed here

Model – Output



It may be relevant to understand with detail where is the most illiquid part of the portfolio coming from. Even though the VBA code generates a sheet with data for each asset individually, having one output decomposing the liquidity of each asset class makes it easier to spot problematic assets.



For equities and bonds, the model also contains price deviation risk tiers, depending on the values of Amihud ratio and Bond position risk ratio and the pre-defined thresholds. This output displays the price deviation tier for each liquidity cost tier. The sum of the five columns is equal to the weight of bonds and equities in the portfolio.

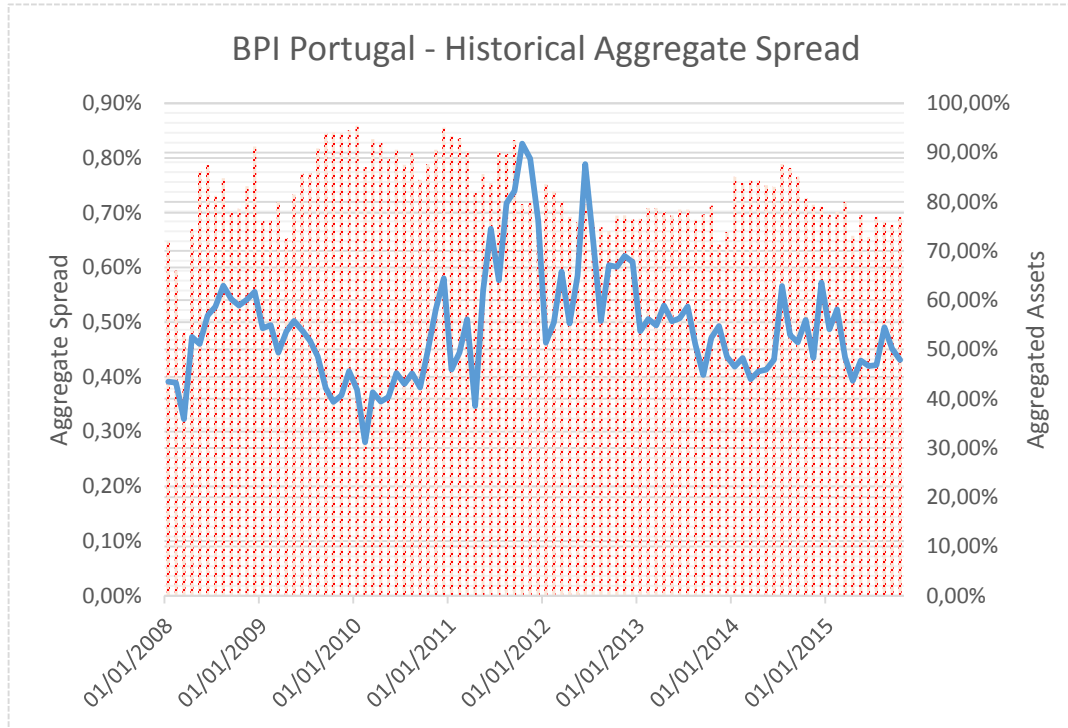
The VBA code also has the option of generating historical values of aggregate spreads and aggregated assets from January 2008 onwards¹. This feature, in particular for larger portfolios such as BPI Global, requires some computational power and may take a few hours to conclude, but it is not a feature that is supposed to be run on a regular basis.

The output of this is a time-series plot that makes it possible to spot changes in portfolio liquidity profile, either due to changes in its assets liquidity *per se* or due to changes in the composition of the portfolio.

The next section of the report contains the historical output for the five portfolios analyzed in this report.

¹ – For portfolios containing bonds, there are two breaks in the series – one in January 2009, when bond rating data is available and spread factor adjustments start being used; other in December 2009, when regressions for non-quoted bonds were first applied

Historical results – BPI Portugal

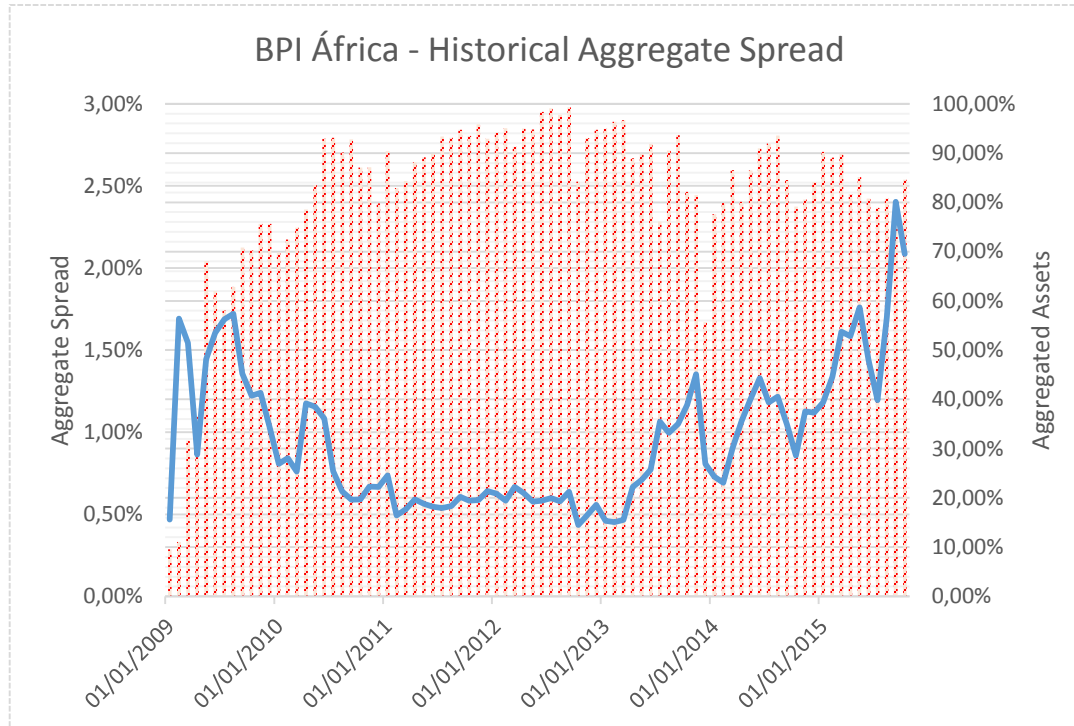


The blue line contains the historical values of the aggregate spread, while the red columns measure the sum of the portfolio weights of the assets which are used to generate the aggregate spread – i.e.: the weight on the portfolio of assets which have a BAS.

Being an equity-based portfolio, BPI Portugal contains mostly stocks listed in PSI20, along with other stocks that, being listed, are not in the PSI20 and, as such, not as liquid.

If compared to core equity exchanges, such as London Stock Exchange, Börse Berlin or Euronext Paris, the Lisbon Stock Exchange doesn't provide the same degree of liquidity. However, liquidity levels are still fairly reasonable and stable: as it can be seen from the graph the average spread ranges from around 0.3% to 0.8%.

The “peaks” occurred in 2011-2012, a critical period for the Portuguese economy where the sovereign crisis was probably a contributor to generate uncertainty and discourage investment, thus removing some market liquidity.



BPI África is a portfolio made almost entirely by African equities. And African stock markets are, generally, extremely illiquid markets.

It is reasonable to ignore the portfolio analysis for the period between 2009 and 2010. Since the portfolio had just recently been created, the number of assets under management is not large enough to conclude about the liquidity profile of the portfolio. The level of aggregated assets, which was constantly below 60% in that period, supports disregarding that data.

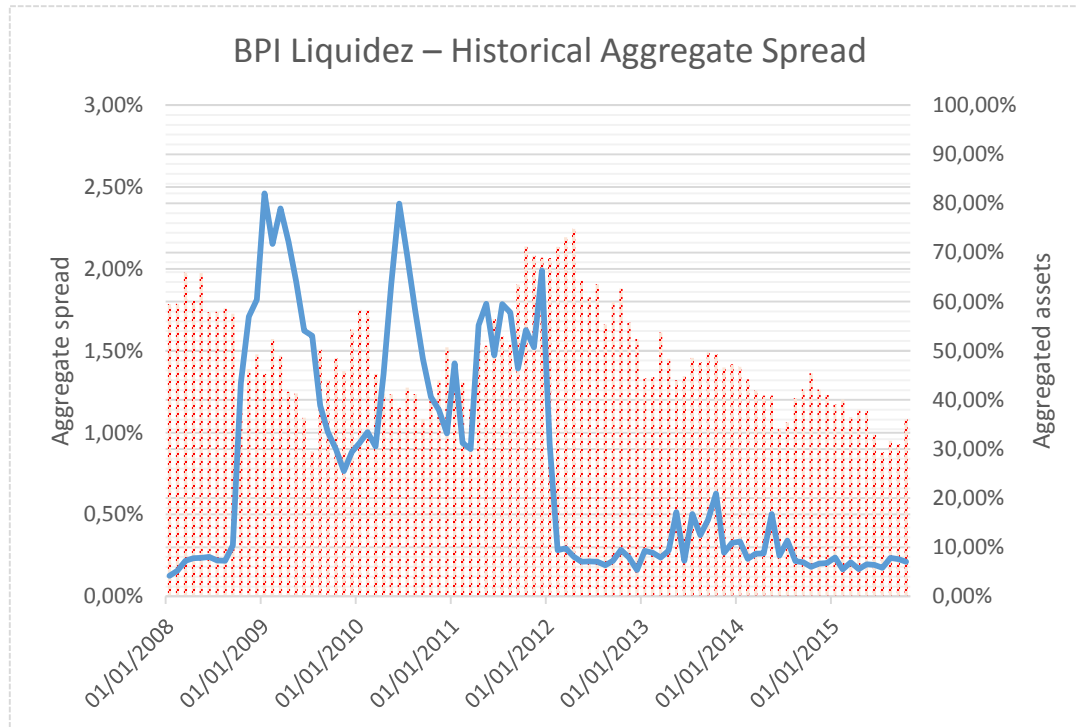
It may be surprising that the liquidity levels shown by the aggregate spread between 2010 and 2013 are similar or even lower than those presented in BPI Portugal on the same period. For that, it is important to clarify that a large weight of the equities that belong to BPI África are stocks listed in South Africa’s stock exchanges, where liquidity is much better than in the rest of Africa.

Even though South African equities seem to dominate the portfolio during the whole period, it seems that they have been losing weight against Nigerian and Egyptian equities which, despite being among the least illiquid in Africa, still present high illiquidity levels, way different from those of South Africa. This probably explains the rally of the aggregate spread from 2013 onwards.

Many shocks seem to be disrupting the recent increasing spread trend, but in such illiquid markets, where sometimes accurate data is scarce, they are mostly due to assets changing from non-quoted to quoted status, or vice-versa.

Historical results – BPI Liquidez

This is clearly, from the 5 portfolios, the one that is expected to give the most deceptive results: most of its assets are short term assets, which are not captured when measuring the aggregate spread. The overall liquidity level of the portfolio should be better than what is shown in this picture.



For the bonds that compose the portfolio it is still possible to check that there is a significant liquidity shock in October 2008, the month after the bankruptcy of Lehman Brothers – the spread increased from 0,31% to 1,3%, and it would remain at relatively high values in the following months (peaking at 2,46% in January 2009).

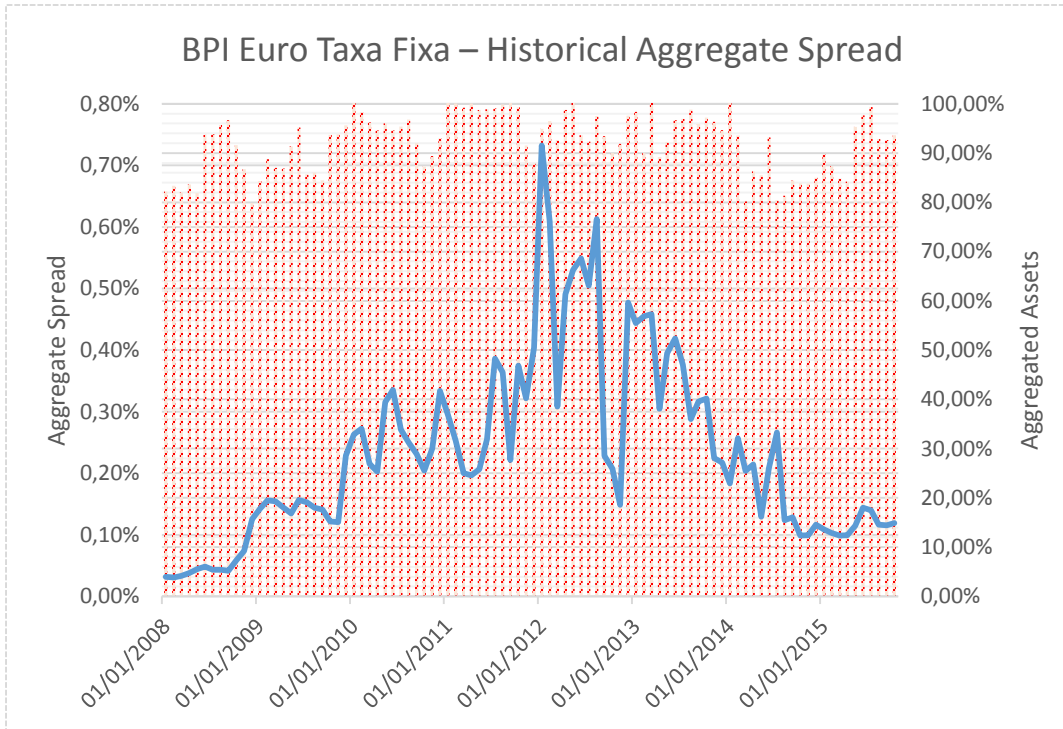
The assets used to create the spread fall, in that month from a weight of 57.52% to 37,72% exactly in October 2008, suggesting that there were probably important changes in the composition of the portfolio as a response to the crisis. That confirmed to be the case – probably under the need to meet its obligations, BPI has fully liquidated one of its most liquid assets: a bond from Netherland which was worth 18.52% of the Portfolio in October and had a spread of 0.05%.

The peak achieved in January 2009 is mostly due to a break in the series, where spread factor adjustments were first implemented – and those are significantly high for TAS quoted bonds, which is the pricing source of many bonds in this portfolio.

The euro sovereign crisis that intensified in early 2010's is also noticeable in the liquidity deterioration of this fraction of the portfolio – from April to June, a position worth 4.13% of the portfolio in a German bond with low spread was fully liquidated, while the average spread for existing bonds, in particular bonds from Portuguese banks (Montepio and BPI) increased due to the sovereign crisis pressure.

The sudden improvement in liquidity between December 2011 and February 2012 is due to a portfolio restructuring regarding bonds – in December, there were 28 bonds in the portfolio and, in the two month period, 16 were liquidated and 20 new were purchased. The bond segments of the portfolio are, therefore, very different.

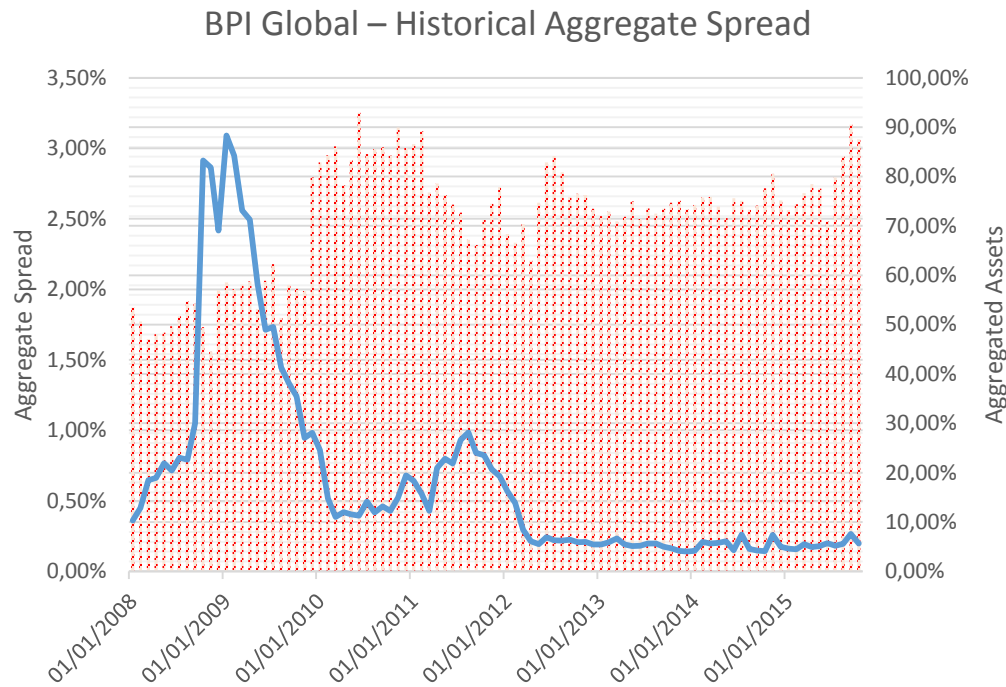
Historical results – BPI Euro Taxa Fixa



At first, it may seem that the graph of BPI Euro Taxa Fixa is the most volatile one regarding the aggregate spread. However, it is important to bear in mind the scale – its maximum, unlike in BPI Global and BPI Liquidez, does not move away from 0.7%, making it actually more stable.

The sudden peak in January 2012 occurred because the weight in Portuguese bonds increased from 2% to around 6.3% with the addition of a new Portuguese bond in the portfolio. At the time, Portugal was at a critical point of its sovereign crisis (with long-term sovereign bond interest rates reaching its maximum), being that reflected in the bid-ask spread of the bonds as well: the two Portuguese bonds that were in the portfolio had bid-ask spreads above 7%.

The “V” shape of the spread between August and December 2012 is mostly due to changes in portfolio composition: From August to September, two bonds with the second and third highest BAS of the portfolio (3.6% and 3.1%), which were worth around 7% of the portfolio, were liquidated, while five bonds, all with BAS below 0.6%, were added to the portfolio, being worth around 10% of the total portfolio value. Between November and December, 10 existing bonds, worth more than 40% of the portfolio value, were liquidated, all with BAS below 0.5%, and 6 new bonds were added – out of those 6, two were Portuguese sovereigns, both with BAS above 2%, which combined had a 14% weight in the portfolio.



Despite having a low number of assets – below 60% – contributing to the aggregate spread value before the regression is applicable, in December 2010, it is still possible to find a strong peak in the spread in October 2008, one month after the bankruptcy of Lehman Brothers. As a consequence of the economic struggle, many companies faced financial distress, and that is reflected mostly in corporate bond spreads, which increased quite significantly. For this month interval only:

- 11 bonds had BAS increases over 10%
- 37 bonds and 1 equity had BAS increases higher than 1%

Despite containing equities, which tend to be more liquid than bonds, this illiquidity measure results in higher illiquidity score for BPI Global than, for example, for BPI Liquidez and BPI Euro Taxa Fixa for troublesome periods. The reason is that, from the three portfolios, it is the one that is more exposed to corporate bonds, which have generally poorer ratings than sovereign bonds and are more prone to scarcity of liquidity.

There is also, apparently, a bounce in January 2009 in the liquidity levels, but that is not really the case: the reason is the spread adjustment series break – bond spreads are adjusted upwards to take into account pricing inaccuracy risk, thus increasing the BAS as well.

The remaining values of the plot are consistent with economic events:

- Some liquidity problems arise during the 2010-2012 European sovereign crisis, in particular because the portfolio is slightly biased towards Portuguese assets
- Liquidity levels have been good and stable from 2013 onwards

One important advantage of using the BAS instead of the currently static liquidity tiers imposed by BPI is that the BAS generates a time-varying series for each asset. If a sovereign bond will always be a sovereign bond independently of the liquidity level of the market, the BAS of that same sovereign bond should fluctuate in time according to market conditions.

The BAS, therefore, may be used for stress testing as long as one can predict what would the BAS be under specific market conditions.

While this project does not cover stress testing, some guidelines on how to approach this topic may be recommended:

Simulating BAS

As Bangia *et al.* (1999)¹ suggest, regarding the BAS, “*spread distributions are very far from normal*”. BAS are capped at a minimum of 0% and present very high Skewness and Kurtosis, and less liquid assets’ BAS tend to deviate more from the normal distribution.

The objective of stress testing BAS is similar to the objective of computing VaR – the goal is knowing, in the worst case scenario, how much would it cost to liquidate the portfolio. As such, the process to use to stress test liquidity may be similar to the one used to stress test VaR. From the three VaR methods, only the historical method seems to be applicable to the BAS:

Historical method	Variance-Covariance method	Monte Carlo method
Requires only historical BAS data	Requires BAS or BAS “returns” to follow a normal distribution	Requires being able to model future BAS values based on a random variable

The main problem of the historical method is that, in some cases, sufficient BAS data isn’t available to generate a large enough number of scenarios or to represent the portfolio as a whole.

1 – Bangia, A., Diebold, F.X., Schuermann, T, and Stroughair, J. (2001), "Modeling Liquidity Risk, With Implications for Traditional Market Risk Measurement and Management," in S. Figlewski and R. Levich (eds.), "Risk Management: The State of the Art" . Amsterdam: Kluwer Academic Publishers, 2002, 1-13

Equities

- Data – many infrequently traded stocks have few spread consistency, even within the same month
 - Keeping data regardless of data inconsistency creates inaccurate spreads
 - Filtering for potential outliers will make equities shift from quoted to non-quoted somewhat often
- Amihud price impact – the transformation of the ratio into a spread generates values that tend to be either close to 0% or unrealistically high. It is very dependent on the quantities owned by BPA GA
 - Inaccurate for small quantities (because of the BAS) and for high quantities (very high spreads)

Bonds

- Data – being OTC traded, there is no transparency; Bloomberg only supplies indicative pricing sources, and the BAS accuracy varies by source
- Spread adjustment factors are based on descriptive statistics
- The model does not adjust for the fact what non-quoted bonds should have an illiquidity premium versus quoted bonds
- The regression for non-quoted bonds requires all inputs and, due to the small and potentially biased sample (made of bonds traded by BPI), isn't as consistent as it could be under a larger sample

Portfolio aggregation

- Tiers – bonds and equities are grouped by BAS, while other assets are grouped through different measures; aggregation has some degree of subjectivity
- Aggregate spread
 - There are breaks in the series in January 2009 and December 2010 due to the lack of bond rating data
 - Weak power for portfolios containing small weights on equities and bonds

Stress testing

- Distribution of Spreads
 - No regular distribution seems to suit the BAS values
 - Feasible stress-testing is based on historical method
- Historical method
 - Assumes the past will hold in the future
 - When data on BAS is missing, the method won't represent the portfolio as a whole

Defining liquidity is, by itself, a difficult task. Measuring it is one of the biggest challenges asset management is facing at the moment. While advanced mathematics, combining all dimensions of liquidity, may be applicable to equity markets, they are not easy to implement in OTC markets, where data isn't accessible.

What this model proposes is a simple numerical way to measure liquidity, which is based on the bid-ask spread:



While quantifying liquidity is important to create a sense of objectivity in evaluating the portfolios, this is a topic that, at least nowadays, requires a lot of trading sensibility. The purpose of this project is not to replace that sensibility, but to create tools that make assessing the liquidity of the portfolios easier while giving an approximation of the cost that would arise if, for some reason, the portfolio manager had to liquidate all the assets.

Recommendations

Equities

Consider using a method to take into account stocks which are currently considered outliers. Changing frequently an equity status from quoted to non-quoted may create instability in the model output, particularly because outliers tend to be the equities that have the highest spreads; Disregarding outliers may also understate the true liquidity level of the portfolio.

Bonds

Consider improving the non-quoted regression – the current regression isn't exactly a time-series, so ARMA models aren't applicable here, but introducing a term in the regression which uses previous month spread may improve the results, being the downside the problem that non-quoted bonds may never have a previous spread. Enlarging the sample size should also make the regression more accurate. Also, if there is an efficient to directly obtain historical pricing sources, the adjustments may be more accurate than they currently are.

Portfolio aggregation

The ideal situation would be having an approximated liquidation cost value for all asset classes. For many of them it is very difficult to obtain data, but for some assets – eventually derivatives and futures – there may be some market information that allows estimating a liquidation cost which will improve the coverage of the model.

Stress testing

This should be the main priority. If a solution to parametrize and simulate the bid-ask spreads isn't found, at least a process similar to historical VaR should be applied, especially taking into account that CMVM regulation requires scenario analysis for liquidity risk.

Appendix A1

Date	Coefficients										
	VOL	TP	BBB	A	AA	AAA	AMT	MAT	ALPHA		
31/12/2009	0,22%	3,86%	-4,84%	-4,82%	-6,19%	-6,59%	-0,07%	0,03%	-10,53%		
29/01/2010	0,21%	4,00%	-3,87%	-3,95%	-5,17%	-5,50%	-0,06%	0,03%	-11,84%		
26/02/2010	0,21%	3,82%	-3,47%	-3,67%	-4,69%	-4,97%	-0,05%	0,03%	-11,61%		
31/03/2010	0,14%	3,19%	-2,95%	-3,28%	-4,08%	-4,27%	-0,04%	0,02%	-8,37%		
30/04/2010	0,07%	4,78%	-2,55%	-2,90%	-3,54%	-3,68%	-0,04%	0,02%	-11,63%		
31/05/2010	0,09%	3,99%	-2,21%	-2,60%	-3,14%	-3,28%	-0,03%	0,02%	-10,21%		
30/06/2010	0,08%	2,01%	-2,07%	-2,40%	-2,85%	-2,99%	-0,03%	0,02%	-4,28%		
30/07/2010	0,04%	-0,05%	-1,88%	-2,19%	-2,53%	-2,72%	-0,03%	0,02%	2,44%		
31/08/2010	0,04%	-0,02%	-1,84%	-2,12%	-2,40%	-2,62%	-0,03%	0,02%	2,30%		
30/09/2010	0,04%	-0,03%	-1,71%	-1,97%	-2,18%	-2,44%	-0,03%	0,02%	2,03%		
29/10/2010	0,04%	0,00%	-1,56%	-1,80%	-1,95%	-2,25%	-0,03%	0,02%	1,72%		
30/11/2010	0,04%	0,05%	-1,52%	-1,72%	-1,84%	-2,18%	-0,03%	0,02%	1,59%		
31/12/2010	0,04%	0,08%	-1,46%	-1,65%	-1,73%	-2,13%	-0,03%	0,02%	1,59%		
31/01/2011	0,03%	0,09%	-1,42%	-1,58%	-1,62%	-2,08%	-0,03%	0,02%	1,59%		
28/02/2011	0,03%	0,11%	-1,39%	-1,52%	-1,53%	-2,02%	-0,03%	0,02%	1,47%		
31/03/2011	0,03%	0,17%	-1,36%	-1,48%	-1,46%	-1,98%	-0,03%	0,02%	1,29%		
29/04/2011	0,03%	0,34%	-1,28%	-1,44%	-1,44%	-1,96%	-0,03%	0,02%	0,89%		
31/05/2011	0,03%	0,33%	-1,25%	-1,41%	-1,41%	-1,91%	-0,03%	0,02%	0,88%		
30/06/2011	0,03%	0,29%	-1,17%	-1,39%	-1,41%	-1,90%	-0,02%	0,02%	1,03%		
29/07/2011	0,03%	0,18%	-1,14%	-1,45%	-1,51%	-1,99%	-0,02%	0,02%	1,29%		
31/08/2011	0,05%	0,15%	-1,19%	-1,56%	-1,68%	-2,16%	-0,02%	0,02%	1,07%		
30/09/2011	0,04%	0,17%	-1,30%	-1,74%	-1,92%	-2,40%	-0,02%	0,02%	1,37%		
31/10/2011	0,04%	0,18%	-1,46%	-1,96%	-2,18%	-2,66%	-0,02%	0,02%	1,47%		
30/11/2011	0,04%	0,24%	-1,60%	-2,18%	-2,41%	-2,89%	-0,02%	0,03%	1,48%		
30/12/2011	0,04%	0,36%	-1,77%	-2,41%	-2,69%	-3,14%	-0,02%	0,03%	1,37%		
31/01/2012	0,04%	0,45%	-1,89%	-2,59%	-2,91%	-3,33%	-0,02%	0,03%	1,36%		
29/02/2012	0,04%	0,51%	-1,98%	-2,76%	-3,09%	-3,49%	-0,01%	0,03%	1,41%		
30/03/2012	0,04%	0,54%	-2,06%	-2,89%	-3,24%	-3,61%	-0,01%	0,04%	1,49%		
30/04/2012	0,04%	0,49%	-2,13%	-2,99%	-3,35%	-3,69%	-0,02%	0,04%	1,66%		
31/05/2012	0,04%	0,57%	-2,21%	-3,10%	-3,46%	-3,78%	-0,02%	0,05%	1,65%		
29/06/2012	0,04%	0,58%	-2,34%	-3,20%	-3,56%	-3,84%	-0,02%	0,05%	1,67%		
31/07/2012	0,04%	0,67%	-2,44%	-3,23%	-3,60%	-3,84%	-0,02%	0,05%	1,39%		
31/08/2012	0,04%	0,78%	-2,47%	-3,19%	-3,55%	-3,74%	-0,02%	0,06%	1,09%		
28/09/2012	0,04%	0,84%	-2,40%	-3,06%	-3,36%	-3,52%	-0,02%	0,06%	0,75%		
31/10/2012	0,04%	0,89%	-2,29%	-2,89%	-3,16%	-3,29%	-0,02%	0,06%	0,46%		
30/11/2012	0,05%	0,98%	-2,19%	-2,75%	-2,95%	-3,09%	-0,02%	0,06%	0,02%		
31/12/2012	0,04%	0,90%	-2,07%	-2,57%	-2,74%	-2,85%	-0,02%	0,06%	0,12%		
31/01/2013	0,04%	0,77%	-1,98%	-2,43%	-2,58%	-2,68%	-0,02%	0,06%	0,33%		
28/02/2013	0,04%	0,76%	-1,88%	-2,29%	-2,42%	-2,51%	-0,02%	0,06%	0,17%		
29/03/2013	0,04%	0,83%	-1,78%	-2,15%	-2,27%	-2,37%	-0,02%	0,05%	-0,07%		
30/04/2013	0,04%	0,98%	-1,70%	-2,02%	-2,14%	-2,23%	-0,02%	0,05%	-0,40%		
31/05/2013	0,04%	1,02%	-1,60%	-1,87%	-1,99%	-2,06%	-0,02%	0,04%	-0,53%		
28/06/2013	0,05%	0,42%	-1,49%	-1,72%	-1,83%	-1,88%	-0,02%	0,03%	0,32%		
31/07/2013	0,03%	0,44%	-1,43%	-1,63%	-1,73%	-1,76%	-0,02%	0,03%	0,61%		
30/08/2013	0,00%	0,24%	-1,33%	-1,52%	-1,61%	-1,62%	-0,02%	0,02%	1,50%		
30/09/2013	-0,01%	0,14%	-1,30%	-1,47%	-1,55%	-1,58%	-0,02%	0,02%	1,78%		
31/10/2013	0,00%	0,04%	-1,27%	-1,44%	-1,51%	-1,55%	-0,02%	0,02%	1,81%		
29/11/2013	0,01%	0,01%	-1,24%	-1,40%	-1,46%	-1,50%	-0,02%	0,02%	1,57%		
31/12/2013	0,01%	-0,02%	-1,22%	-1,37%	-1,43%	-1,47%	-0,02%	0,02%	1,59%		
31/01/2014	0,01%	0,03%	-1,18%	-1,33%	-1,38%	-1,42%	-0,01%	0,02%	1,44%		
28/02/2014	0,01%	0,06%	-1,16%	-1,30%	-1,35%	-1,39%	-0,01%	0,02%	1,30%		
31/03/2014	0,01%	0,11%	-1,13%	-1,28%	-1,33%	-1,36%	-0,01%	0,02%	1,18%		
30/04/2014	0,01%	0,38%	-1,10%	-1,24%	-1,29%	-1,32%	-0,01%	0,02%	0,66%		
30/05/2014	0,01%	0,45%	-1,07%	-1,20%	-1,24%	-1,28%	-0,01%	0,02%	0,52%		
30/06/2014	0,01%	0,43%	-1,02%	-1,15%	-1,18%	-1,22%	-0,01%	0,01%	0,55%		
31/07/2014	0,01%	0,38%	-0,97%	-1,09%	-1,12%	-1,16%	-0,01%	0,01%	0,60%		
29/08/2014	0,01%	0,26%	-0,95%	-1,06%	-1,08%	-1,12%	-0,01%	0,01%	0,81%		
30/09/2014	0,00%	0,21%	-0,93%	-1,03%	-1,05%	-1,09%	-0,01%	0,01%	0,92%		
31/10/2014	0,01%	0,17%	-0,93%	-1,01%	-1,04%	-1,07%	-0,01%	0,01%	0,87%		
28/11/2014	0,01%	0,11%	-0,93%	-1,01%	-1,03%	-1,06%	-0,01%	0,01%	0,87%		
31/12/2014	0,01%	0,07%	-0,94%	-1,01%	-1,03%	-1,07%	-0,01%	0,01%	0,92%		
30/01/2015	0,01%	0,07%	-0,95%	-1,02%	-1,04%	-1,07%	-0,01%	0,01%	0,92%		
27/02/2015	0,01%	-0,01%	-0,96%	-1,02%	-1,03%	-1,08%	-0,01%	0,01%	1,12%		
31/03/2015	0,01%	-0,07%	-0,97%	-1,02%	-1,03%	-1,08%	-0,01%	0,01%	1,23%		
30/04/2015	0,00%	-0,07%	-0,98%	-1,04%	-1,05%	-1,10%	-0,01%	0,01%	1,27%		
29/05/2015	0,00%	-0,07%	-1,00%	-1,06%	-1,06%	-1,12%	-0,01%	0,01%	1,29%		
30/06/2015	0,00%	-0,07%	-1,02%	-1,08%	-1,08%	-1,14%	-0,01%	0,01%	1,31%		
31/07/2015	0,00%	-0,03%	-1,04%	-1,10%	-1,10%	-1,16%	-0,01%	0,01%	1,33%		
31/08/2015	0,00%	-0,02%	-1,06%	-1,12%	-1,11%	-1,18%	-0,01%	0,01%	1,33%		
30/09/2015	0,00%	0,01%	-1,08%	-1,14%	-1,13%	-1,21%	-0,01%	0,01%	1,34%		
30/10/2015	0,00%	0,06%	-1,09%	-1,15%	-1,15%	-1,22%	-0,01%	0,01%	1,37%		

Appendix A2

Date	T-student tests											R2		
	VOL	TP	BBB	A	AA	AAA	AMT	MAT	ALPHA					
31/12/2009	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,23%	0,00%	30,82%
29/01/2010	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,05%	0,00%	28,58%
26/02/2010	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,01%	0,00%	27,08%
31/03/2010	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,01%	0,00%	23,84%
30/04/2010	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,01%	0,00%	23,31%
31/05/2010	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,01%	0,00%	21,48%
30/06/2010	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,04%	20,89%
30/07/2010	0,00%	77,70%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	21,68%
31/08/2010	0,00%	81,11%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	21,61%
30/09/2010	0,00%	69,57%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	23,15%
29/10/2010	0,00%	95,52%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	27,09%
30/11/2010	0,00%	28,53%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	26,59%
31/12/2010	0,00%	10,15%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	26,48%
31/01/2011	0,00%	7,74%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	25,80%
28/02/2011	0,00%	4,94%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	25,33%
31/03/2011	0,00%	0,66%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	25,30%
29/04/2011	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	25,74%
31/05/2011	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	25,31%
30/06/2011	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	24,58%
29/07/2011	0,00%	0,34%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	25,03%
31/08/2011	0,00%	2,12%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	27,20%
30/09/2011	0,00%	1,99%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	28,36%
31/10/2011	0,00%	2,21%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	29,14%
30/11/2011	0,00%	0,07%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	31,09%
30/12/2011	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	32,24%
31/01/2012	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	33,27%
29/02/2012	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	34,13%
30/03/2012	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	36,32%
30/04/2012	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	37,94%
31/05/2012	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	39,70%
29/06/2012	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	41,38%
31/07/2012	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	42,70%
31/08/2012	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	43,75%
28/09/2012	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	44,27%
31/10/2012	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,37%	45,11%
30/11/2012	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	91,76%	45,81%
31/12/2012	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	39,45%	47,50%
31/01/2013	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	1,25%	49,10%
28/02/2013	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	16,92%	51,04%
29/03/2013	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	58,40%	49,65%
30/04/2013	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,39%	48,36%
31/05/2013	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,01%	46,81%
28/06/2013	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	3,86%	44,70%
31/07/2013	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,01%	43,04%
30/08/2013	77,46%	0,11%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	42,50%
30/09/2013	5,20%	4,18%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	42,66%
31/10/2013	41,85%	58,27%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	42,32%
29/11/2013	0,71%	87,11%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	41,94%
31/12/2013	1,01%	79,47%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	40,67%
31/01/2014	0,43%	64,99%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	40,36%
28/02/2014	0,06%	25,89%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	39,82%
31/03/2014	0,04%	5,78%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	37,83%
30/04/2014	0,47%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	37,86%
30/05/2014	1,70%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	38,39%
30/06/2014	5,48%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	40,34%
31/07/2014	2,80%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	43,77%
29/08/2014	13,09%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	45,01%
30/09/2014	47,02%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	47,44%
31/10/2014	0,30%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	49,73%
28/11/2014	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	51,60%
31/12/2014	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	53,52%
30/01/2015	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	55,07%
27/02/2015	0,00%	48,44%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	56,68%
31/03/2015	0,05%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	63,47%
30/04/2015	0,42%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	62,74%
29/05/2015	0,63%	0,01%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	62,40%
30/06/2015	0,01%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	62,16%
31/07/2015	0,51%	7,75%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	61,54%
30/08/2015	0,31%	34,87%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	60,69%
30/09/2015	1,91%	79,63%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	60,03%
30/10/2015	81,27%	3,06%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	59,49%

Under the relative bid-ask spread, it is possible to state that:

$$\text{Bid Price} = \text{Mid Price} \left(1 - \frac{BAS}{2} \right)$$

Then:

$$\text{Mid Price} = \frac{\text{Bid Price}}{1 - BAS/2}$$

Being asset i 's L_1 given by the following formula, where W_i is the wealth invested in the asset if evaluated at the mid price:

$$\begin{aligned} \text{Asset } L_1 &= W_i \times \frac{1}{2} BAS_i = \frac{1}{2} \times BAS_i \times Q_i \times \text{Mid Price}_i = \\ &= \frac{1}{2} \times BAS_i \times Q_i \times \frac{\text{Bid Price}}{1 - BAS_i/2} = \\ &= \frac{BAS_i \times \text{Asset Wealth evaluated at bid price}}{2 - BAS_i} \end{aligned}$$

Which is the formula used to compute L_1 for bonds using directly BPI's valuation.

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