A Work Project, presented as part of the requirements for the Award of a Master Degree in Finance from the NOVA – School of Business and Economics.

Testing for tail breaks in currency returns

Mariana Cartaxo Lima #990

A Project carried out on the Master in Finance Program, under the supervision of:

Professor Paulo M. M. Rodrigues

January, 2016

Testing for tail breaks in currency returns

Abstract

In this work project we study the tail properties of currency returns and analyze whether changes in the tail indices of these series have occurred over time as a consequence of turbulent periods. Our analysis is based on the methods introduced by Quintos, Fan and Phillips (2001), Candelon and Straetmans (2006, 2013), and their extensions. Specifically, considering a sample of daily data from December 31, 1993 to February 13, 2015 we apply the recursive test in calendar time (forward test) and in reverse calendar time (backward test) and indeed detect falls and rises in the tail indices, signifying increases and decreases in the probability of extreme events.

Keywords: Heavy Tails, Tail Index, Tail Breaks, Exchange Rates

1. Introduction

Unusual large movements in economic and financial time series seem to occur more often than expected by normality, and since in practice the exact distribution is unknown, accurate insights on the tail behavior of the data are imperative. In a heavy-tailed distribution the likelihood of observing significant deviations from the mean is much higher than in the case of the normal distribution. Today it is generally accepted that financial returns are, indeed, heavy-tailed.

Extremal phenomena analysis typically concentrates on estimating the tail index α of the return distribution. The tail index characterizes the rate at which probability mass falls away in the tail of a distribution; a relatively low tail index corresponds to a relatively high probability of extreme events. However, although this parameter is frequently assumed to be constant over time, the question arises whether this assumption is realistic. In fact, traditional regression-based risk proxies like standard errors, CAPM-βs or factor model loadings are time-varying; see e.g. Ross et al. (2005), and Straetmans and Candelon (2013).

Testing for instability in unconditional tail risk measures, like the tail index, is relevant not only from a statistical perspective but also from an economic point of view; see e.g. Straetmans and Candelon (2013). From a statistical perspective, the applicability of extreme value theory (EVT) or GARCH models depends on the stationarity assumption for the unconditional tail. Moreover, a non-constant tail index implies a violation of covariance stationarity, which may invalidate standard regression based statistical inference. From an economic point of view, for risk managers and financial regulators the ability to accurately assess the tail distribution translates into ability to manage extreme financial risks effectively and to properly monitor overall financial stability. Indeed, it is important for the calculation of the unconditional Value-at-Risk (VaR) very far into the distributional tail (e.g. for stress testing purposes), for safety first portfolio selection for pension funds (Jansen et al., 2000) or for the assessment of trading limits for unhedged forex positions in commercial banks (see Danielsson and de Vries, 1997).

Recent years have been characterized by significant instabilities in financial markets worldwide. Episodes like the October 1987 stock market crash, the 1997 Asian financial crisis (which extended to Russia in 1998 and Argentina in 2001), the hedge fund crisis in 1998 or the 2007 global financial and economic crisis highlight the pertinence of outlier activity and consequently the importance of its focus on empirical analysis. The question one wants to answer is whether it is valid to assume that a time invariant tail of an unconditional distribution can explain both highly volatile periods and periods of market stability.

Along with financial markets in general, foreign exchange markets in particular have been characterized as well by turbulence and volatility, with extreme variations marking most exchange rates. Economic crises, speculative attacks, bailouts, stabilization efforts, regime reforms and regulatory changes, etc., are potential causes of outliers in currency returns; see e.g. Ibragimov et al. (2013). Large fluctuations in exchange rates carry significant real repercussions for international trade, foreign investment, asset prices, and a broad range of other economic and financial variables. Due to the intrinsic importance of exchange rate management, our study is an attempt to investigate if there are structural shifts capable of generating changes in the tail index of currency returns.

The rest of the work is structured as follows. Section 2 contains an overview of important contributions on the study of tail behavior, section 3 characterizes the methodology followed throughout this work by reviewing the relevant estimation and testing procedures, section 4 describes our extensive empirical investigation regarding the tail properties of exchange rate returns for several currencies, and, finally, section 5 concludes.

2. Literature Review

Recognizing that the distributions of many variables of interest deviate from the Gaussian distribution paradigm, there have been numerous contributions from the many areas of application of heavy-tailed distributions, namely in computer science and telecommunications, in economics and finance and in insurance. Specifically in economics and finance, this stream of literature on the analysis of heavy tails is pioneered by Mandelbrot (1963) and Fama (1965).

Over the decades heavy tails have been identified in most financial asset classes. Although it is commonly assumed that the tail index is constant in the whole observed time domain, recent literature has found empirical evidence against this assumption, even more in finance whose time series suffer from the changes of their underlying models due to changes of monetary policies and critical social events; see e.g. Quintos, Fan and Phillips [QFP] (2001), and Straetmans and Candelon (2006, 2013), and Kim and Lee (2009). About exchange rate data in particular, empirical studies conclude that, as well as exhibiting extreme outlier behavior, changes in tail behavior should be also taken into consideration when covering different exchange regime periods; see e.g. Koedijk et al. (1990), Hols and de Vries (1991), Koedijk et al. (1992), and Loretan and Phillips (1994), and Dacorogna et al. (2001).

Assuming a single exogenous or known breakdate, Phillips and Loretan (1990) and Koedijk, et al. (1990) seminal papers introduce tests for the null hypothesis that the tail index is constant over time. In these works, for exchange rate data from the US, Japan and several Western European countries, it is found that the tail index of the distribution changed over time. Pagan and Schwert (1990), among other studies, implement parameter constancy tests with stricter moment conditions (i.e. finite fourth moments) and reach similar conclusions, i.e., reject the null of constant tail thick.

The tests constructed by Phillips and Loretan (1990) and Koedijk et al. (1990) rely on Hill's (1975) conditional maximum likelihood estimator of the tail index of a distribution. Proof has been given that the Hill estimator performs well in simulations (Kearns and Pagan, 1997) and is more robust for tail slope estimation than other estimators that use the full sample and, consequently, look at the center of the distribution as well (DuMouchel, 1983). In addition, Hill's estimator and tests built based on it do not require finite fourth moments.

Relying on Hill's index estimator as well, but overcoming the assumption of exogenously selected breakpoints, QFP (2001) develop new test procedures for the constancy of the tail behavior over time which assume that the breakdate is endogenous or unknown, i.e., detect both breakpoints and corresponding break dates in the tail index. Their analysis conclude that Asian stock market returns display a time-varying tail index, and structural break points identified by recursive testing coincide with suspected break points resulting from previously known shifts in institutional arrangements.

Applying the recursive test methodology of QFP (2001), Werner and Upper (2004), Galbraith and Zernov (2004) and Candelon and Straetmans (2006) likewise test for tail stability in bund Future returns, US stock market returns and Asian currency returns, respectively, and they all demonstrate that the corresponding tail index exhibits a structural break point.

Moreover, focusing on the case of multiple structural changes occurring at unknown breakdates, Candelon and Straetmans (2006) extend the QFP (2001) analysis arguing that the single break point recursive testing procedure can be used in a sequential way in order to test for more than one break in the tail behavior and consequently detect gradual increases and decreases in the tail index. Indeed, they were able to reveal multiple structural break points in emerging currency tails.

3. Methodology

3.1. Tail Index Estimation – The Hill Estimator

Following QFP (2001) and Straetmans and Candelon (2013), we express estimation and testing procedures, without loss of generality, in terms of the right tail. Therefore, considering a distribution function F, the respective survival function is:

$$\bar{F}(x) \coloneqq 1 - F(x) = P\{X \geq x\}. \tag{1}$$

Under general conditions, we can approximate the survival function of heavy tailed distributions by the second order Taylor expansion for large x as:

$$1 - F(x) \coloneqq ax^{-\alpha}(1 + bx^{-\beta} + o(x^{-\beta})), \tag{2}$$

with $a > 0$, $\alpha > 0$, $b \epsilon$ R and $\beta > 0$; see e.g. Haan and Stadtmüller (1996) and Straetmans and Candelon (2013). The parameter α is connected with the range of extreme values, i.e., the larger the value of α the further out outliers are from the mean. The parameters β and b manage the second order behavior and indicate the deviation from pure Pareto behavior in the tail. For instance, for $\beta = 0$ the survival function is $P\{X \geq x\} \approx ax^{-\alpha}(1 + b \ln x)$ and when additionally $b = 0$ the tail particularizes to an exact Pareto, i.e.,

$$P\{X \geq x\} \approx ax^{-\alpha}. \tag{3}$$

The regular variation property implies that the (appropriately scaled) upper extremal returns lie in the maximum domain of attraction of the Type II extreme value ("Frechet") distribution. Moreover, the regular variation property also implies that all distributional moments higher than α, i.e., $E[X^r]$, $r > \alpha$, are unbounded, evidencing "fat tails". Therefore, the tail index α informs about the speed at which the tail probability decays if x is increased. The smaller the value of α, the slower the probability decay and the higher the probability mass in the tail of X, *ceteris paribus* the level of x.

In order to test for tail index breaks we will use the QFP test statistics. These test statistics use Hill's (1975) estimator for α as an input. Hence, considering a sequence of random variables $\{X_t\}_{t=1}^{T}$ and the respective order statistics, $X_{1,T} \geq X_{2,T} \geq \cdots \geq X_{T,T}$, the maximum likelihood estimator of α proposed by Hill is:

$$\alpha(m_T) = \left(\frac{1}{m_T} \sum_{j=1}^{m_T} \ln X_{j,\mathrm{T}} - \ln X_{m_T,\mathrm{T}} \right)^{-1} \tag{4}$$

where $m_T = [kT]$ is the number of highest order statistics used in the estimation of α and $k \in (0,1)$. Hall (1982) showed for $m_T/T \to 0$ as $m_T, T \to \infty$ that $\sqrt{m_T}\left(\frac{\hat{\alpha}(m_T)}{\alpha} - 1\right)$ is asymptotically normal distributed. The choice of the m_T largest order statistics that are accounted for in the computation of the tail index is a crucial point of research since the convergence in distribution of the Hill statistic critically depends on the rate at which the nuisance parameter m_T grows with the total sample size. Several approaches have been proposed; see e.g. Danielsson, Haan, Peng and de Vries (2001) and Nguyen and Samorodnitsky (2012).

Nonetheless, it has also been shown that the optimal selection rule for m_T depends on the tail properties (Hall, 1982). Because of this inherent circularity in tail slope estimation, DuMouchel (1983) recommended the simple rule that m_T be chosen as a fixed fraction of the sample size. This rule has been demonstrated to perform well in simulations and is commonly used by practitioners, though some caution needs to be taken since DuMouchel's rule can lead to the wrong test size for standard tests of structural change. When the fixed fraction is such that the corresponding m_T grows too fast with the sample size, then all the tests considered in QFP diverge under the null (see QFP, 2001).

3.2. Testing for Breaks in the Tail Index – The QFP Tail Break Tests Statistics

In this work we bear in mind the contributions of QFP (2001) and Candelon and Straetmans (2006) for testing tail stability when there are one or more unknown breakpoints, respectively. QFP consider the null hypothesis that the tail index α is constant over time. That is, defining α_t as the tail index estimate of the distribution of X_t, and $t \coloneqq [rT]$ for $r \in R_\epsilon \coloneqq [\epsilon, 1-\epsilon]$, such that R_ϵ is a pre-specified compact subset of (0,1) for some small $\epsilon > 0$, the null hypothesis of a time invariant α then takes the form:

$$H_0\colon \alpha_{[Tr]} = \alpha\,, \forall r \in R_\epsilon \coloneqq [\epsilon, 1-\epsilon] \subset [0,1] \qquad (5)$$

with the two-sided alternative H_A: $\alpha_{[Tr]} \neq \alpha$, for some $r \in R_\epsilon$, and where $[\cdot]$ represents the integer value of the argument. Sets like R_ϵ are frequently used in the construction of parameter constancy tests (see e.g. Hawkins (1987) and Andrews (1993)) and represent some constant fraction of the total sample size, while being bounded away from zero and unity.

Once the choice of R_ϵ considered for the computation of the test statistics will impact on the limit distributions (critical values), we employed a small correction on the statistics such that $t^* \coloneqq [r^*T]$, with $r^* \in [0,1]$, as recommended by Nicolau and Rodrigues (Forthcoming). Thus, the tests for tail change proposed in QFP are constructed from the quantities:

$$Y_T(t) \coloneqq \left(\frac{(t-[\,r^*T])m_t}{T-\,[\,r^*T]}\right)^{1/2}\left(\frac{\hat{\alpha}_t}{\hat{\alpha}_T}-1\right); \tag{6}$$

$$Y_T^R(t) \coloneqq \left(\frac{(T-t-[\,r^*T])m_t}{T-\,[\,r^*T]}\right)^{1/2}\left(\frac{\hat{\alpha}_t^R}{\hat{\alpha}_T}-1\right); \tag{7}$$

$$V_T(t) \coloneqq \left(\frac{w_t^* m_{w_t^*}}{T-\,[\,r^*T]}\right)^{1/2}\left(\frac{\hat{\alpha}_t^*}{\hat{\alpha}_T}-1\right); \tag{8}$$

$$Z_T(t) \coloneqq \left(\frac{(t-[\,r^*T])m_t}{T-\,[\,r^*T]}\right)^{1/2}\left(\frac{\hat{\alpha}_t}{\hat{\alpha}_{2t}}-1\right) \tag{9}$$

where $\hat{\alpha}_t$ is estimated from the subsample $[1,\dots,t]$; $\hat{\alpha}_{2t}$ is the post-break estimator which corresponds to the reverse estimator with sample size $w_t = w_{2t} = T - t$ and $\hat{\alpha}_T$ is the full sample estimator.

As specified in QFP, the recursive test – either in calendar time (forward test) in (6) or in reverse calendar time (backward test) in (7) – is based on the fluctuations test of Ploberger-Kramer-Kontrus (1989), since $\left(\frac{\hat{\alpha}_t}{\hat{\alpha}_T}-1\right) = \hat{\alpha}_T^{-1}(\hat{\alpha}_t - \hat{\alpha}_T)$. The same applies to the rolling test in (8), which corresponds to the recursive test rolled through the full sample. The sequential test in (9) measures the fluctuation of the recursive estimator against the reverse recursive estimator as opposed to the full sample estimator.

Accordingly, the tests we will consider are:

$$Q_S^{rec} \coloneqq \sup_{r \epsilon R_\epsilon} [\eta Y_T([rT])]^2 \overset{d}{\to} \sup_{r \epsilon R_\pi} \left[\widetilde{W}(r)\right]^2 \tag{10}$$

$$Q_S^{R,rec} \coloneqq \sup_{r \epsilon R_\pi} [\eta Y_T^R([rT])]^2 \overset{d}{\to} \sup_{r \epsilon R_\pi} \left[\widetilde{W}(r)\right]^2 \tag{11}$$

$$Q_S^{rol} \coloneqq \sup_{r \epsilon R_\pi} [\eta V_T([rT])]^2 \overset{d}{\to} \sup_{r \epsilon R_\pi} \left[\widetilde{W}(r, \gamma_0)\right]^2 \tag{12}$$

$$Q_S^{seq} \coloneqq \sup_{r \epsilon R_\pi} [\eta Z_T([rT])]^2 \overset{d}{\to} \sup_{r \epsilon R_\pi} \left[\widetilde{W}^{\#}(r)\right]^2 \tag{13}$$

where $W(r)$ denotes a standard Wiener process, $W(r, \gamma_0) = W(r) - W(s)$ with $s = r - \gamma_0$, $\widetilde{W}(r) = W(r) - rW(1)$, $\widetilde{W}(r, \gamma_0) = W(r, \gamma_0) - (r - s)W(1{,}1)$ and $\widetilde{W}^{\#}(r) = W(r) - \left(\frac{r}{1-r}\right) W(1 - r)$. Furthermore, η is a nuisance parameter which is the serial dependence parameter in case $\{X_t\}_{t=1}^T$ follows an ARMA or GARCH process; see Hsing (1991) and QFP (2001). Hence, consistency and asymptotic normality is not affected by serial dependence as long as the statistics are scaled by a consistent estimate of η.

Overall the outcomes of Candelon and Straetmans (2006) analysis on size-corrected power and the ability to detect breaks support that the recursive version of the stability test is to be preferred provided that the sample is sufficiently large (at least 2000 observations). Even though they alerted to the recursive test's inability to detect breaks when $\alpha_1 < \alpha_2$, we can perform the recursive test both in calendar time (forward recursive test) as well as by inverting the sample (backward or reverse recursive test) so as to easily resolve this apparent lack of power for one type of α-jump. Indeed, when present in the data, a fall in the tail index should then be signaled by the forward version of the recursive test whereas a rise should be detected by the backward version of the recursive test.

This will also be the strategy in our work. We will concentrate on recursive stability tests – both forward tests and backward tests. The corresponding asymptotic critical values of the tests are those tabulated in QFP (2001).

For testing tail stability when there are possibly multiple unknown breakpoints and consequently gradual increases and decreases in the tail index, we follow Candelon and Straetmans (2006) extension of the QFP recursive testing procedure. Hence, the approach consists in applying the recursive test in rounds: if a full sample break is detected, the single break recursive test is repeated over the two subsamples determined by the initial break date and so forth. This multistage implementation of the single breaks test can continue as long as the parameter constancy hypothesis is rejected and the subsamples are not too small. Nonetheless, given our comparatively large data set and so as to avoid over-partitioning, we repeat the recursive test only until a fourth significant breakpoint is signed, what means we find a maximum of five subsamples and five corresponding values of α for each currency series.

4. Empirical Analysis

4.1 Data

In order to study the tail properties of currency returns and particularly whether changes in the tail indices of these series have occurred, our dataset consists of the daily exchange rate returns series of an array of 22 currencies, all expressed against the US dollar (USD): the euro (EUR), the Japanese yen (JPY), the Chinese yuan renminbi (CNY), the Canadian dollar (CAD), the Swiss franc (CHF), the Australian dollar (AUD), the Brazilian real (BRL), the South African rand (ZAR), the South Korean won (KRW), the Swedish krona (SEK), the Norwegian krone (NOK), the Mexican peso (MXN), the Singapore dollar (SGD), the Pound sterling (GBP), the Hong Kong dollar (HKD), the Polish zloty (PLN), the Danish krone (DKK), the Argentine peso (ARS), the New Zealand dollar (NZD), the Chilean peso (CLP), the Ukraine hryvnia (UAH) and the Colombian peso (COP). We extracted all data from Thomson Datastream and expressed currency returns as log price differences between daily closes. Although a few series exhibit different starting points depending on data availability, our sample broadly covers the period from December 31, 1993 to February 13, 2015.

Indeed, we have a large time window (around 5511 observations for each series) which covers both the mature and the emerging currency blocks[1] and considers all continents: 1 African currency, 1 North American currency and 5 Latin American currencies, 5 Asian currencies, 8 European currencies as well as 2 currencies from the Oceania region. Hence, it certainly comprises worldwide noteworthy episodes for studying tail activity since possible sources of tail index changes include any structural shifts, among which, e.g., changing trading systems, financial regulatory reform and financial liberalization, changes in monetary and exchange rate policies, or even financial and economic turmoil – like the recent financial crisis.

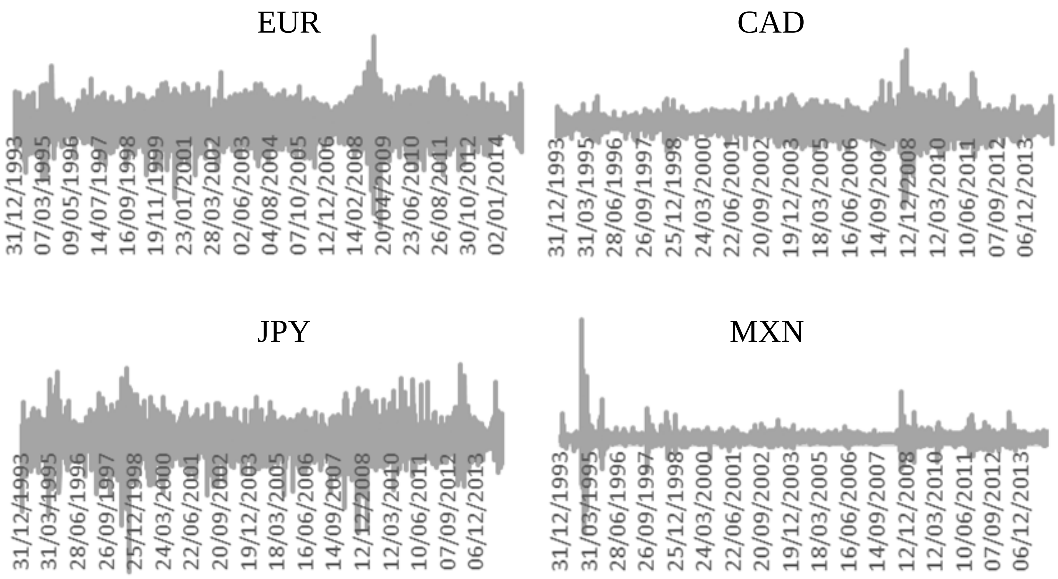

Figure 1: Exchange rate returns

Figure 1 graphs the time series of exchange rate returns for the euro, the Canadian dollar, the Japanese yen and the Mexican peso and seems to indicate that all four currencies exhibit a change in the degree of extreme movements over time. To be precise, it is clear from these graphs that there was a change in the degree of extreme movement in all the series around 2008, the time of the global financial crisis. Additionally, a change in the degree of extreme movement

[1] The classification is based on Morgan Stanley's Markets classification http://www.msci.com/products/indexes/market_classification.html. Note that care needs to be taken with this classification given that over the sample period considered some of the countries may have switched groups. Further, in our analysis, frontier markets are included in the emerging group.

is also pronounced in the Mexican peso series around 1994, coinciding with the Tequila crisis, and in the Japanese yen series around 1998, reflecting the Asian crisis. One possible cause of such variations in the tail behavior of a series is that the tail thickness of the underlying return distribution is not constant over time. A fall (rise) in α would imply an increase (a reduction) of extreme market movements.

4.2. Descriptive Statistics

In Table 1 we present some descriptive statistics of the exchange rate returns so as to have a better understanding of the behavior of each of the currencies under analysis. More concretely, for each currency we display the annualized mean return, the annualized standard deviation, the skewness and the excess kurtosis.

The annualized mean return is positive for all emerging currencies, which indicates the average depreciation or weakening of these currencies comparatively to the US dollar, while for most mature currencies, except for JPY, SEK, NOK and HKD, this statistic is negative, which reflects the average appreciation or strengthening of these currencies also against the US dollar. The annualized standard deviation sheds light on the dispersion of each set of data from its mean. The larger the annualized standard deviation, the wider the range between the lowest and highest values, i.e., the more significant the outliers. Here we observe that on average this statistic is larger for emerging currencies than for developed currencies, revealing the potentially higher risk profile.

Skewness can be interpreted as a measure of the behavior of extreme returns, once positive (negative) skewness indicates a distribution with an asymmetric tail extending towards more positive (negative) values. Following Bulmer's (1979) rule of thumb, since for most currencies this statistic is between −½ and +½, we can say that they exhibit an approximately symmetric distribution, what is consistent with symmetry in volatility of foreign exchange markets reported in the literature (see e.g. Ibragimov et al., 2013). It may be further noted that this statistic seems to be negative (and lower than −½) for CHF, KRW and HKD, suggesting that crashes cause an asymmetric return distribution, while for CNY, AUD, MXN, ARS, CLP, UAH (all emerging currencies, except AUD) it is found to be positive (and higher than +½). This latter type of asymmetry in exchange rate returns is the opposite of the asymmetric behavior characteristically observed in financial markets and may depict regulatory interventions in these currency markets (Ibragimov et al., 2013).

Excess kurtosis measures the heaviness of the tails of a distribution, being positive (negative) if the tails are heavier (lighter) comparatively to the normal distribution. We uniformly detect that this statistic is large, indicating that for these currency returns extremely large movements are more likely than for normally distributed returns.

4.3. Tail Breaks

In Tables 2 and 3 we report the results for right and left tail breaks. We provide in addition the Hill estimator of the index for the full sample.

Here it is worth clarifying that, since daily logarithmic exchange rate returns are calculated as $r_t = \ln P_t - \ln P_{t-1}$, where P_t and P_{t-1} are the price of the domestic currency per unit of the US dollar at time t and time $t-1$, respectively, a rise in r implies a depreciation (or devaluation) while a drop in r suggests an appreciation (or valuation). Thus, the right (left) tail refers to extreme currency depreciation (appreciation) events and structural breaks found in the right (left) tail indicate that the likelihood of such sharp drops (rises) in currency prices is nonconstant over time.

First and foremost, regarding the full sample Hill estimates, we observe that both the right and the left tail indices of exchange rates of emerging currencies are on average smaller than those of developed currencies, reflecting the potentially higher risk profile and consequently more pronounced heavy-tailedness properties of exchange rates of emerging countries compared to those of developed countries. Moreover, in all emerging currencies the tail indices of exchange rates are slightly larger in the left tail than in the right tail, i.e., upward moves appear to be relatively more heavy-tailed than downward moves; whereas in most developed currencies, except for CAD, AUD, NOK, SGD and NZD, the tail indices of exchange rates are slightly higher in the right tail than in the left tail, meaning they have somewhat more pronounced heavy-tailedness in downward moves than in upward movements; or even in some cases both tail indices are approximately equal. These findings are in line with the analysis of skewness and with the aforementioned literature in the sense that: first, some corroborate the consensus that in forex markets volatility tends to be symmetric with respect to positive and negative shocks; second, some seem to follow the stylized statistical asymmetry that in financial markets one typically observes large drawdowns but not equally large upward movements; third, some face the opposite type of asymmetry, which is generally justified by regulatory interventions.

However, apart from EUR, JPY and DKK in the right tail and CHF and GBP in the left tail whose test statistics do not display significant breaks, most striking in the results is indeed the widespread nonconstancy of α over time. Effectively, monetary history tells us that large swings in foreign exchange markets happen quite frequently. From the turbulent episodes during the 90s of the 1994-95 Latin American Tequila crisis, the Asian financial crisis in 1997-98, the Russian financial crisis of 1998, or the 1998–2002 Argentina Great depression, to the more recent 2008-09 global financial crisis and the 2010 Eurozone debt crisis, most emerging as well as developed economies have experienced currency crashes. Also institutional interventions reforms and regime switches, more regularly in emerging countries, have given rise to sharp fluctuations in currency markets.

Hence, let us look in more detail at the results for right and left tail breaks as well as the corresponding tail index estimates for the subsamples. To begin with, we may notice that some coincident periods in which breaks are more frequent across currencies can be highlighted and, in addition, it seems these coincident periods are a common pattern within both the right and the left tail. More specifically, and having in mind that our data set extends from 1994 until 2015, three waves of structural breaks in tail indices pop out, respectively in or about 1999, 2006 and finally 2010.

Our sample begins in a particularly turbulent period that covers the Latin American Tequila crisis in 1994-95 (which sparked in Mexico and spread to other economies in the region, particularly Argentina) and the Asian and Russian financial crises in 1997-98 (which impacted other major emerging economies, like Brazil). The consequences of these crises were so devastating that they triggered shifts in the macroeconomic policies of countries in these regions, namely in what concerns their exchange rate regimes; see e.g. Frankel et al. (2001); and Frenkel and Rapetti (2011). For example, after the Tequila crisis, Mexico adopted a floating exchange rate regime (Frenkel and Rapetti, 2011), and a similar policy was followed by Brazil, Colombia and Chile. By 1999, however, analysts saw signs that the economies of Asia were beginning to recover (Pempel, 1999) and working toward financial stability.

Accordingly, over this period our results in Tables 2 and 3 show relatively smaller tail indices, with the smallest belonging to Asian and Latin America countries, which translates the upswings in extreme volatility for most currencies. Furthermore, the evident wave of significant breaks around 1999 (detected by backward testing) and the subsequent overall rise in tail indices might be correlated with the widespread attempts in most Latin and Asian countries to recover from the crises. Still in 1999 the euro was launched. This event might explain the fact of European currencies and the Euro itself being included in the wave of significant breaks in or about 1999.

As previously mentioned, from 1999 onwards the tail indices have increased, suggesting a potential risk reduction in currency returns, which is validated by monetary history. In fact, the 2000-2006 period was generally prosperous and without crisis both for developed and emerging countries. The exception was the massive default of Argentina´s external debt, the subsequent abandoning of its currency board and the devaluation of its currency in early 2002 (Nicolau and Rodrigues, 2015).

However, following the housing bubble burst starting in 2006 and the collapse of Lehman Brothers two years later, the recent financial and economic crisis was originated in the United States and quickly spread globally. Interestingly, a second wave of significant breaks around 2006 (detected by forward testing) and a consequent common drop in tail indices, which suggests a higher extreme risk profile of developed and emerging currencies owing to the global crisis, is visible from our results.

Despite evolving into the worst crisis since the Great Depression of the 1930s, financial contagion was short and by 2009 many emerging countries had recovered access to the international financial system at low interest rates. To this early response certainly contributed the switch to flexible managed floating regimes made by emerging economies (see e.g. Reinhart and Rogoff, 2004 (updated country chronologies)) and also the accumulation of foreign exchange reserves. In 2010 the global economy grew by about 3.6 per cent, with Asia leading the gradual recovery while Europe lagged behind (United Nations Department of Economic and Social Affairs, 2011). Still in Europe a sovereign debt crisis and a resulting crisis of confidence developed, particularly tense for Eurozone members Greece, Ireland and Portugal. Once more, our results accurately mirror the chronological background since a further wave of significant breaks in or about 2010 can be caught, followed by an increase in tail indices, symptomatic of a reduction in extreme volatility for most currencies. Moreover, the Hill estimates clearly reveal a U-shaped pattern in the tail index over the period corresponding to the global crisis (2006-2010), with drops in α usually preceding the rises in α, what is consistent with previous literature concerning the second half of the 1990s and the Asian crisis (Candelon and Straetmans, 2006).

Finally, it is interesting to observe that, albeit the recurrent fluctuation, the tail index estimates tend to increase toward the end of the sample period, specifically for most emerging currencies, reflecting the phenomenon of gradually decreasing financial turbulence over time.

5. Conclusion

Exchange rate is perhaps one of the most important macroeconomic variables that link the economy of one country with the rest of the world. When it changes, it affects almost all other sectors and many other macro variables.

Having in mind the empirical stylized fact that exchange rate returns exhibit more probability mass in the tails than the normal distribution (heavy tails), in this work project we formally investigate whether there is temporal stability of the parameter governing the respective tail decline. To lead this investigation, we base ourselves on the methods introduced by QFP (2001), Candelon and Straetmans (2006, 2013), and their extensions, for detecting multiple structural breaks in the tails of currency returns distributions. Using the Hill's index estimator as an input, the tests are endogenous once they yield an estimate of the breakpoint date upon prior detection of a statistically significant break.

Indeed, we perform the recursive version of the stability test, since this version outperforms its counterparts both in power and in estimation of the breakpoint for decreases in α, and afterwards we further apply the same test yet over the inverted sample in terms of calendar time, so as to overcome the fact that the recursive approach cannot detect rises in α. Moreover, we apply the procedure in multiple stages, which allows the detection of multiple breakpoints: in case a significant breakpoint is signed, the recursive test is repeated over a partitioning of subsamples. By implementing this multistage version of the recursive and reverse recursive approach to a large set of 22 currency returns, including all continents and both developed as well as emerging markets, we are able to identify multiple statistically significant breaks. Furthermore, the detected breaks generally were found to be meaningful in the sense that they match well with suspected breakpoints arising from documented shifts in monetary regimes and exchange rate policies, as well as known periods of economic and financial turmoil, such as the 1998 Asian and Russian crisis or the recent global crisis.

Nonetheless, some shortcomings in our study should also be taken into account, particularly those arising from data availability or reliability in a few series from emerging currencies. Still, a deeper analysis is possible and would certainly enrich the results. Indeed, given our wide panel data and due to time and over-partitioning constraints, we stopped the breakpoint detection when a fourth significant break was found. It would be interesting to allow for more stages and continue until the parameter constancy hypothesis was not rejected or the subsamples were too small. Finally, as a way of making the approach less repetitive, improvements in the contributions of QFP (2001) and Candelon and Straetmans (2006, 2013) would be welcome.

All in all, our finding of breaks in the tail behavior of currency returns may constitute relevant information for both governments and investors. In fact, not only do they signal the potential effectiveness (or failure) of exchange rate policies to policymakers but they also highlight the need for care in applying risk management tools to currency traders. Specifically, our results have crucial implications for value-at-risk models that parameterize the tails of returns resorting to financial models that, despite taking into account time varying mean and volatility, assume tail thickness constancy over time.

References

1. Andrews, D. W. K. 1993. "Tests for Parameter Stability and Structural Change with Unknown Change Point." *Econometrica*, 59: 817–858.
2. Bulmer, M. G. 1979. *Principles of Statistics*. New York: Dover.
3. Candelon, B., and S. Straetmans. 2006. "Testing for multiple regimes in the tail behavior of emerging currency returns." *Journal of International Money and Finance*, 25: 1187-1205.
4. Dacorogna, M. M., U. A. Muller, O. V. Pictet, and C. G. de Vries. 2001. "Extremal Forex Returns in Extremely Large Data Sets." *Extremes*, 4(2): 105-127.
5. Danielsson, J., and C. G. de Vries. 1997. "Tail index and quantile estimation with very high frequency data." *Journal of Empirical Finance*, 4(2-3): 241-257.
6. Danielsson, J., L. de Haan, L. Peng, and C. G. de Vries. 2001. "Using a Bootstrap Method to Choose the Sample Fraction in Tail Index Estimation." *Journal of Multivariate Analysis*, 76: 226-248.
7. DuMouchel, W. M. 1983. "Estimating the Stable Index α in Order to Measure Tail Thickness: A Critique." *Annals of Statistics*, 11: 1019-1031.
8. Fama, Eugene F. 1965. "The behaviour of stock market prices." *Journal of Business*, 38: 34 - 105.
9. Frankel, J. A., E. Fajnzylber, S.L. Schmukler, and L. Serven. 2001. "Verifying exchange rate regimes." *Journal of Development Economics*, 66: 351 - 386.
10. Frenkel, R., and Rapetti, M. 2011. "Exchange Rate Regimes in Latin America" In Handbook of Latin American Economics, ed. José Antonio Ocampo and Jaime Ros. Oxford University Press.
11. Galbraith, John W., and Serguei Zernov. 2004. "Circuit Breakers and the Tail Index of Equity Returns". *Journal of Financial Econometrics*, 2 (1): 109-129.

12. Haan, L. de, and U. Stadtmüller. 1996. “Generalized Regular Variation of Second Order.” *Journal of the Australian Mathematical Society*, 61: 381-395.
13. Hall, P. 1982. “On some simple estimates of an exponent of regular variation.” *Journal of the Royal Statistical Society*, 42: 37-42.
14. Hawkins, D. L. 1987. “A Test for a Change Point in a Parametric Model Based on a Maximal Wald-Type Statistic.” *Sankhya*, 49: 368–376.
15. Hill, Bruce M. 1975. “A simple general approach to inference about the tail of a distribution.” *Annals of Statistics*, 3: 1163-1174.
16. Hols, M. C. A. B., and de C. G. de Vries. 1991. “The Limiting Distribution of Extremal Exchange Rate Returns.” *Journal of Applied Econometrics*, 6: 287-302.
17. Hsing, T. 1991. “On Tail Index Estimation Using Dependent Data.” *Annals of Statistics*, 19(3): 1547–1569.
18. Ibragimov, M., R. Ibragimov, and P. Kattuman. 2013. “Emerging markets and heavy tails.” *Journal of Banking & Finance*, 37: 2546–2559.
19. Jansen D.W., K. G. Koedijk, and C. G. de Vries. 2000. “Portfolio selection with limited downside risk.” *Journal of Empirical Finance*, 7(3-4): 247-269.
20. Kearns, P., and A. Pagan. 1997. “Estimating the Density Tail Index for Financial Time Series.” *Review of Economics and Statistics*, 79: 171-175.
21. Kim, M., and S. Lee. 2009. “Test for tail index change in stationary time series with Pareto-type marginal distribution.” *Bernoulli*, 15: 325-356.
22. Koedijk, K. G., M. M. A. Schafgans, and C. G. de Vries. 1990. “The tail index of exchange rate returns.” *Journal of International Economics*, 29: 93-108.
23. Koedijk, K. G., P. A. Stork, and C.G. de Vries. 1992. “Differences between Foreign Exchange Rate Regimes: The Views From the Tails.” *Journal of International Money and Finance*, 11 (5): 462-73.

24. Loretan, M., and P. C. B. Phillips. 1994. "Testing the Covariance Stationarity of Heavy-Tailed Time Series: An Overview of the Theory with Applications to Several Financial Datasets." *Journal of Empirical Finance*, 1: 211-48.

25. Mandelbrot, B. 1963. "The variation of certain speculative prices." *Journal of Business*, 36: 394-419.

26. Nguyen, T., and G. Samorodnitsky. 2012. "Tail Inference: where does the tail begin?" *Extremes*, 15 (4): 437-461.

27. Nicolau, J., and P. M. M. Rodrigues. 2015. "A new regression-based tail index estimator: an application to exchange rates". Banco de Portugal Working Paper no. 14.

28. Nicolau, J., and P. M. M. Rodrigues. Forthcoming. "Testing for Tail Breaks". Banco de Portugal Working Paper.

29. Pagan, A. R., and G. W. Schwert. 1990. "Testing for Covariance Stationarity in Stock Market Data." *Economics* Letters, 33: 165-170.

30. Pempel, T. J. 1999. *The Politics of the Asian Economic Crisis*. Ithaca: Cornell University Press.

31. Phillips, P. C. B., and M. Loretan. 1990. "Testing Covariance Stationarity under Moment Condition Failure with an Application to Common Stock Returns." Cowles Foundation Discussion Paper no. 947.

32. Ploberger, W., W. Kramer, and K. Kontrus. 1989. "A New Test for Structural Stability in the Linear Regression Model." *Journal of Econometrics*, 40: 307–318.

33. Quintos, C., Z. Fan, and P.C.B. Phillips. 2001. "Structural change tests in tail behavior and the Asian crisis." *The Review of Economic Studies*, 68(3): 633-663.

34. Reinhart, C. M., and K. S. Rogoff. 2004. "The Modern History of Exchange Rate Arrangements: A Reinterpretation." *Quarterly Journal of Economics*, 119(1): 1-48.

35. Ross, S. A., R.W. Westerfield, and J. Jaffe. 2005. *Corporate Finance*. New York: McGraw-Hill.

36. Straetmans, S. and B. Candelon. 2013. "Long-term asset tail risks in developed and emerging markets." *Journal of Banking and Finance*: 37: 1832-1844.

37. United Nations Department of Economic and Social Affairs. 2011. "The global economic crisis: causes and transmission." In *The Global Social Crisis*. New York: United Nations.

38. Werner, T., and C. Upper. 2004. "Time variation in the tail behaviour of Bund futures returns." *Journal of Futures Markets*, 24 (4): 387-398.

Appendices

Table 1: Descriptive Statistics

	Annualized mean return	Annualized standard deviation	Skewness	Excess kurtosis
EUR	-0,0006	0,0950	-0,1829	2,6079
JPY	0,0028	0,1086	-0,4559	5,1374
CNY	0,0033	0,0875	72,3632	5326,9902
CAD	-0,0028	0,0826	-0,0947	7,0611
CHF	-0,0214	0,1107	-0,7898	20,3327
AUD	-0,0062	0,1222	0,7510	13,3537
BRL	0,0485	0,1509	0,4653	20,2708
ZAR	0,0564	0,1496	0,3153	7,2390
KRW	0,0140	0,1385	-0,7621	105,6427
SEK	0,0003	0,1131	-0,1824	3,6441
NOK	0,0003	0,1134	-0,0250	5,3289
MXN	0,0716	0,1441	2,8163	94,0719
SGD	-0,0079	0,0566	-0,4040	11,2526
GBP	-0,0018	0,0860	0,0413	4,4530
HKD	0,0002	0,0044	-2,7225	65,0179
PLN	0,0194	0,1266	0,1846	5,7646
DKK	-0,0018	0,0958	-0,1945	2,5615
ARS	0,0989	0,1276	18,1269	697,8446
NZD	-0,0132	0,1245	0,3712	6,1464
CLP	0,0168	0,0917	0,5800	7,7019
UAH	0,1159	0,1769	10,9556	414,4240
COL	0,0496	0,0965	0,3806	10,0118

Table 2: Right Tail Breaks

EUR	2,68	04/01/1994 2,68										13/02/2015
JPY	2,66	04/01/1994 2,66										13/02/2015
CNY	1,12	04/01/1994 0,90		11/02/1999 1,49		12/05/2003 3,36	24/05/2005 1,62			09/12/2011 1,99		13/02/2015
CAD	2,22	04/01/1994 2,56					27/05/2004 2,89	30/03/2006 2,33			01/08/2013 3,45	13/02/2015
CHF	2,83	04/01/1994 2,53		11/05/1998 3,09			09/07/2004 4,41	24/08/2006 2,59				13/02/2015
AUD	2,26	04/01/1994 2,39					19/03/2004 2,77	24/11/2005 2,06			10/01/2012 2,86	13/02/2015
BRL	1,86	05/07/1994 1,23		31/03/1998 2,09		03/06/2002 1,87				08/11/2010 2,72		13/02/2015
ZAR	2,11	04/01/1994 1,60		07/03/2000 4,67	15/12/2000 2,31							13/02/2015
KRW	1,63	10/01/1994 1,08		04/10/1999 2,40	24/09/2001 2,16			05/05/2006 1,63			08/03/2013 2,97	13/02/2015
SEK	2,61	04/01/1994 2,73		25/08/1998 3,05			18/07/2005 3,63	09/08/2006 2,33		01/04/2010 2,55		13/02/2015
NOK	2,47	04/01/1994 2,60					11/06/2004 3,53	07/04/2006 2,26				13/02/2015
MXN	1,75	10/01/1994 1,08	06/02/1997 1,53	02/03/1999 2,46	08/10/2001 2,00					07/04/2011 2,61		13/02/2015
SGD	2,06	04/01/1994 1,80		05/08/1999 3,32	01/03/2000 3,20	23/11/2001 2,63	22/12/2005 2,20					13/02/2015
GBP	2,76	04/01/1994 2,86			12/05/2000 3,18		19/03/2004 3,31	01/06/2006 2,30		05/11/2010 3,37		13/02/2015
HKD	1,54	04/01/1994 1,72			06/10/2000 1,13			06/02/2006 2,17	11/05/2009 1,86		22/01/2013 1,85	13/02/2015
PLN	2,03	05/01/1995 2,13				08/11/2002 2,49	13/06/2005 2,44				01/05/2012 2,58	13/02/2015
DKK	2,75	04/01/1994 2,75										13/02/2015
ARS	0,99	10/01/1994 0,55			19/01/2000 1,06		06/08/2004 2,03	31/08/2007 1,70			22/11/2012 1,45	13/02/2015
NZD	2,27	04/01/1994 2,22						05/04/2006 2,32		06/04/2010 3,43	07/11/2011 2,88	13/02/2015
CLP	2,11	04/01/1994 2,15			27/08/2001 2,67		02/07/2004 2,20					13/02/2015
UAH	0,88			03/12/1998 1,56	30/03/2000 0,96	30/07/2001 0,67	25/07/2003 0,87					13/02/2015
COL	1,99	04/01/1994 2,02						27/06/2006 2,02		23/07/2010 2,65	15/03/2011 2,63	13/02/2015

Note: For each series, we report the tail index estimate for the full sample. In case of significant breaks in the tail index, we report the corresponding break dates (dd/mm/yy) and the Hill estimates for the subsamples determined by the break.

Table 3: Left Tail Breaks

EUR	2,56	04/01/1994 2,37		20/12/1999 3,05		14/11/2003 4,39	31/12/2004 2,37			30/08/2010 2,51		13/02/2015
JPY	2,42	04/01/1994 2,14		16/12/1999 2,89	07/12/2001 2,60							13/02/2015
CNY	1,39	04/01/1994 1,34		16/04/1998 2,86		03/12/2002 1,17		27/11/2006 2,97		17/03/2010 2,31		13/02/2015
CAD	2,38	04/01/1994 2,09			23/02/2000 2,75	19/01/2004 3,60	21/02/2005 3,29	23/02/2007 2,35				13/02/2015
CHF	2,62	04/01/1994 2,62										13/02/2015
AUD	2,5	04/01/1994 2,53			29/11/2001 3,23		07/07/2005 2,02		17/03/2010 3,76	25/10/2010 2,84		13/02/2015
BRL	2,15	05/07/1994 3,22	13/12/1994 0,92	30/09/1999 2,25			19/10/2005 3,09	23/10/2007 2,23				13/02/2015
ZAR	2,46	04/01/1994 1,69		05/04/2000 2,44	14/09/2001 2,87		16/05/2005 3,34	05/03/2007 2,68				13/02/2015
KRW	1,83	10/01/1994 1,06		18/06/1999 2,50		22/01/2002 3,03		29/03/2006 1,73		18/11/2011 3,11		13/02/2015
SEK	2,58	04/01/1994 2,52			13/12/2000 3,57		22/06/2005 2,20		31/05/2010 4,57	03/05/2011 2,56		13/02/2015
NOK	2,61	04/01/1994 2,58				02/05/2002 3,51	20/09/2004 3,96	21/04/2006 2,70				13/02/2015
MXN	2,07	10/01/1994 1,14	09/10/1996 2,07	02/03/1999 3,20			27/04/2006 3,55	28/05/2007 2,23				13/02/2015
SGD	2,16	04/01/1994 2,04	04/10/1996 1,48	31/12/1999 2,38		25/02/2002 2,50						13/02/2015
GBP	2,63	04/01/1994 2,63										13/02/2015
HKD	1,41	04/01/1994 1,60			06/10/2000 0,94		19/09/2005 2,06	09/01/2007 1,73				13/02/2015
PLN	2,35	05/01/1995 1,75		08/03/1999 3,01			25/07/2005 2,65		14/02/2008 2,86	09/06/2011 2,38		13/02/2015
DKK	2,56	04/01/1994 2,61							02/01/2009 3,14		13/08/2012 2,53	13/02/2015
ARS	1,08	10/01/1994 2,57	12/07/1995 5,31	25/05/1998 6,46	20/01/1999 1,35			22/02/2007 1,74				13/02/2015
NZD	2,59	04/01/1994 2,10		27/07/1999 2,82		10/06/2002 4,02	04/03/2005 2,61			28/06/2011 2,94		13/02/2015
CLP	2,41	04/01/1994 1,86		10/02/1999 2,72	29/11/2000 2,82			25/04/2006 2,38		04/11/2010 3,15		13/02/2015
UAH	1,08		03/12/1998 2,83	03/09/1999 1,07			04/08/2005 2,70	19/12/2006 1,09		28/11/2011 1,04		13/02/2015
COL	2,07	04/01/1994 1,95						07/11/2006 2,47	19/06/2009 3,17	24/12/2010 2,73		13/02/2015

Note: For each series, we report the tail index estimate for the full sample. In case of significant breaks in the tail index, we report the corresponding break dates (dd/mm/yy) and the Hill estimates for the subsamples determined by the break.