



**NOVA School of Business and Economics and Insper
Instituto de Ensino e Pesquisa**

Masters of Science in Economics

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**A DECISION-THEORETIC MODEL FOR M&A SALE
PROCESSES**

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2016**

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A decision-theoretic model for M&A sale processes

A dissertation presented as part of the requirements for titles of Master of Science in Economics from NOVA – School of Business and Economics and Master in Economics from Insper Instituto de Ensino e Pesquisa

A Dissertation carried out under the supervision of: José Heleno Faro, Andrea Maria Accioly Fonseca Minardi and Steffen H. Hoernig

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1. M&A. 2. Real Options

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Para a minha avó Maria Thereza

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ABSTRACT

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When potential buyers receive invitations from target companies to engage in competitive M&A sale processes they face a challenging decision. Considering significant due diligence investments, target value uncertainty and unclear competitive environment, should they accept such invites? The main purpose of this study is to formulate a decision rule for prospective acquirers to enter takeover contests according to these relevant factors. In addition, this dissertation explores the formation of optimal bids in the decisive stage of controlled sales (sealed-bid auctions) with uncertain presence of one competitor. A decision-theoretic model is designed where a potential buyer (Player A) is invited to participate in an M&A sale process. Its due diligence investments are modeled as the purchase of real options and the optimal bid value is calculated according to its expected payoff maximization. The participant has incomplete information regarding the existence of rivals and their strength and takes decisions that seek robustness with respect to misspecifications. Due diligence investments decision rules are established according to Player A's capabilities to create value through the acquisition, its beliefs regarding the potential rivalry, and required spending to analyze the target. Optimal bidding strategies ultimately depend on our participant's beliefs concerning the potential competition. Our findings show that an uncertain strong second bidder might prevent Player A to place a higher offer. This is an exciting result that prompts re-thinking on competition's threat. Sellers and their financial advisors should take these results into account when tailoring efficient sale processes, especially managing a proper perceived competitive environment.

Keywords: M&A; real options; sealed-bid auctions; due diligence

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1. INTRODUCTION

Firms often face competition for acquiring a target company. Rivalry is particularly challenging when sellers initiate the transaction approaching a selected group of buyers (controlled sale process). Potential acquirers are invited to perform an investigation and retrieve information on the target value. On the one hand, conducting this due diligence demands from prospective acquirers substantial expenditures with advisors, consultants, lawyers, and involves opportunity costs. On the other hand, carrying it out enhances the amount and quality of information available which contributes considerably to a successful bid. After the analysis, bidders might submit a final offer in a sealed-bid auction with unknown number of competitors.

Taking into account considerable due diligence costs, uncertain target value and unclear competitive environment, buyers face a puzzling decision to accept the invite. The leading purpose of this dissertation is to examine the influence of these factors on this decision. A second purpose is to explore the formation of optimal bids in sealed-bid auctions, the moment when due diligence investments are sunk costs, target value is known but competition uncertainty remains.

In this dissertation we analyze three research questions. Firstly, we examine whether a potential acquirer must enter into a bid contest investing in a superficial due diligence. Subsequently, we study if it should perform an in-depth examination on the target given the information revealed in the first inquiry. Finally, we investigate buyers' optimal bid value.

In an effort to explore these questions, we design a theoretical model where a potential buyer (Player A) is invited to an M&A sale process. Due diligence investments are evaluated as the purchase of real options whereas the bid value is treated in light of individual decision theory. Player A's decisions are analyzed according to its beliefs regarding the likelihood of existence and strength of rival bids. These beliefs represent, in essence, the threat of competition.

The significance of this dissertation lies on its unparalleled contribution to competition preceding binding offers. Moreover, a unique aspect of this research is building a theoretical framework for a problem regularly encountered by many buy-side professionals and yet scarcely studied. The results provide them an additional

decision-making tool. Ultimately, it also contributes valuable insights to vendors design more efficient controlled sales.

Regarding the organization of this study, Chapter 2 presents a review of the literature including a detailed explanation of M&A sale processes, a comprehensive description of the controlled sale - focus of our model - and a revision of previous studies that apply real options and game-theoretic approach to mergers and acquisitions.

Chapter 3 describes the theoretical methodology. The bidding contest is designed and solved through subgame perfection. We move backwards from the decision of the bid value to the decision to enter the process. Decision rules are established for each Player A's due diligence investments and a function to the optimal bid value is settled.

Chapter 4 presents the study's findings concerning the interpretation of the decision rules and competition impact on optimal bids. We interpret main results of our model in relation to the participant's beliefs.

The final chapter summarizes the research, discusses findings, explores implications for bidders and sellers, and recommends further research.

2. LITERATURE REVIEW

2.1. Description of M&A sale processes

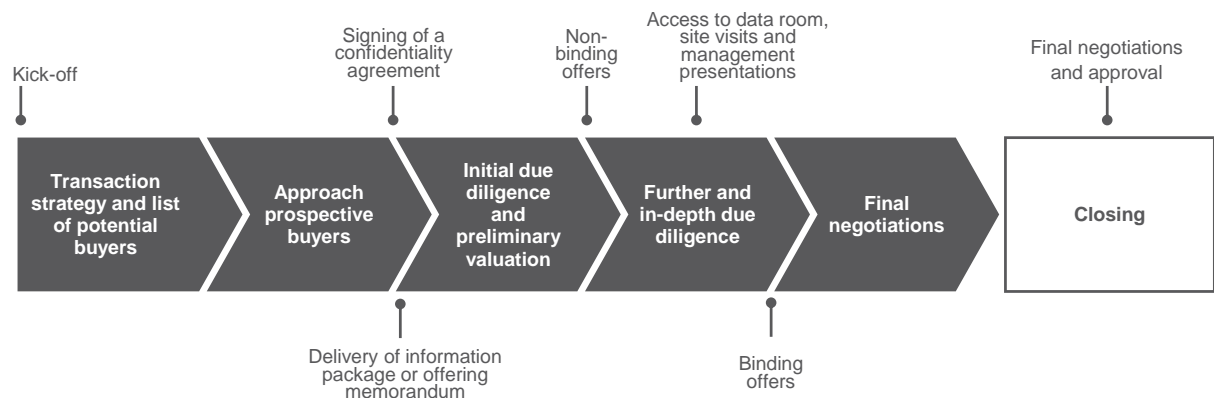
The sale of a company is an event of ultimate importance for its shareholders and managers. It is a complex and time-consuming undertaking that require months of analysis and negotiations. For that reason, once the decision to put a firm up for sale is taken, its shareholders often retain an advisor such as an investment bank to conduct the M&A sales process. Together, they tailor a specific process to fit the optimal mix of value maximization, timing, confidentiality and other relevant issues for the sellers (ROSENBAUM; PEARL, 2013).

One possible way is to conduct the process is a negotiated sale, approaching a single buyer and negotiating the best possible terms. A second possibility is a controlled sale (or target auction) where only a select group of potential bidders are addressed. Finally, the parties may consider a third method running a full-scale (broad) auction reaching out as many potential acquirers as possible (ROSENBAUM; PEARL, 2013; BOONE, MULHERIN, 2009). A typical standardized auction process follows detailed.

At the beginning advisor and target define a list of potential acquirers who are contacted by the advisor. A teaser, a document with a brief and no-name description about the target, is sent to prospective buyers. Those who express interest in the opportunity are asked to sign a confidentially agreement in exchange for detailed information about the investment through an offering memorandum in order to conduct their initial due diligence and undertake a formal valuation. In addition, potential bidders receive a bid procedure letter indicating the date by which they must submit their non-binding offer and main terms expected in the proposal. On the non-binding offer date, sell-side advisors receive the first bids which are then analyzed along with the selling-party. Together, they reduce the number of potential acquirers based on their indications of interest and invite the remaining participants to the next round. During this stage prospective buyers receive access to a data room with more in-depth information, perform site visits and are delivered management presentations in order to continue their due diligence. Once more, participants of the auction receive a bid procedure letter indicating the date by which they must submit their binding offer and main terms expected in the proposal. Upon the end of this round,

bidders must submit their offers which are again scrutinized by advisors and target. After this revision, the selling party and its advisor may negotiate with two interested parties and ask for best and final offers. Finally, they choose a winning bidder, usually the highest bid, and present it to the target’s board of directors and shareholders to final approval (BOONE, MULHERIN, 2009; HANSEN, 2001; ROSENBAUM; PEARL, 2013).

Figure 1. Detailed M&A controlled sale process



Source: Adapted from Rosembaum; Pearl (2013)

2.2. Real options and game-theoretic approach to mergers and acquisitions

An option is defined as the right, but not an obligation, to buy or sell a specific underlying asset by paying a prespecified price on or before a certain date. Its core value derives from the right to be exercised only if it is in the holder’s interest. It is appropriate for this dissertation to differ financial from real options: while the former defines rights on an underlying financial asset in a liquid market, the latter relates to business opportunities such as capital budgeting, investment decisions and transactions. Real options can be classified into three dimensions: proprietary or shared according to the existence of competition or the firm’s capacity to fully appropriate the option value; single or compound in line with the connection with other investment opportunities and; expiring or deferrable in relation to timing to decision. Incorporating the game theory framework allows for a competitive pricing perspective. An increasing number of researchers have demonstrated the importance of this combined structure to examine returns, occurrence and dynamics of mergers and acquisitions (TRIGEORDIS, 1996; BRACH, 2003).

Smith and Triantis (1994) demonstrate that a merger or acquisition, besides the generated synergies, brings a new portfolio of strategic options for the resulting firm. The acquirer's competitive position within its industry can be altered through growth, flexibility or divestiture options created or facilitated by the transaction. Incorporating these options into valuation techniques of the target is crucially important for capturing strategic benefits and accurately assessing its value.

Lambrecht (2004) studies the procyclicality of friendly mergers and hostile takeovers motivated by economics of scale through continuous-time real options techniques. The author models two companies with an option to merge whose payoff depends on their current equity value, benefits from their synergies and one-off costs such as fees and restructuring expenses. The optimal timing to merger balances their stochastic equity value, positive incentives of synergistic advantages and negative incentives of the permanent nature of the deal. The results of the model indicate that companies have incentives to merger that are positively related with economic activity when transactions are driven by economies of scale. Another interesting finding for this dissertation is that in comparison to friendly mergers that seek a global optimization, hostile takeovers happen at a higher level of economic activity. This results from the fact that when target companies set the terms for the transaction it imposes an additional bid premium to potential acquirers.

Smit (2001) proposes a valuation methodology based on real options and game theory frameworks to "buy and build" acquisition strategies. The "buy and build" strategy entails an equity investors acting as an industry consolidator that first acquires a platform company and then leverages its core competencies into add-on acquisitions. Therefore, managers should account for strategic and growth options embedded in earlier investments when valuing the target. They should also take competition into consideration, what can be done through game theory principles. In the real-options approach, acquisitions can be classified into two dimensions: simple or compound according its link with other synergistic deals and; proprietary or shared according to its degree of competitiveness. In that context, competitive bidding and auctions are therefore examples of shared simple options.

The study of competitive bidding as shared simple options has increased in recent years. Smit, van der Berg and De Maeseneire (2005) examine competitive bidding contests between asymmetric bidders and the value appropriation for the

acquirer. Competing players have the opportunity to buy a target company modeled as a real option acquired by the investment in due diligence (option premium) that reveals player's private valuation about the target (underlying value) and can be exercised making a bid (exercise price). It is modeled a two-stage bidding game between two players. In the first stage, player A has identified, conducted a costly due diligence and placed a bid for the target. Player B, after observing the offer, decides to enter or not in the contest. In the second stage, if player B decides not to enter, player A acquires the company with the opening bid whereas if player B enters, they compete through an English auction. The first insight of the model concerns the initial bid: when player A bids, two contrary singling effects are sent. On the one hand, given their level of similarity player B can infer its own value of the target and on the other hand the potential competitiveness of the deal. Regarding their strategies, player A can seek a pre-emptive bid such that the value of the option for player B is smaller than its due diligence cost. On the contrary, an accommodating bid will lead player B to participate in the auction. The model provides an interesting finding that value appropriation follows a U-shaped curve with the degree of relatedness. For high very low degrees of similarity there is an increase in value appropriation since pre-emptive bids are made easier. For intermediate levels, accommodating bids usually leads to a contest and value appropriation decreases.

Other researchers have provided new insights into acquisition strategies of financial buyers such as private equity firms. While financial buyers acquire companies exclusively as an investment, strategic buyers are those who seek acquisitions that will provide synergies and fit into their business plans (e.g. operating companies). In his doctoral thesis Van den Berg (2007) brings Smit, van der Berg and De Maeseneire (2005) model into the field of private equity firms' competition focusing on the degree of similarity between players. In line with the previous article, Van den Berg (2007, p.126) demonstrates that "whether the first bidder is able to offer the deterring opening bid or the lower accommodating bid depends on the degree of relatedness of its resources with the opponent's resources".

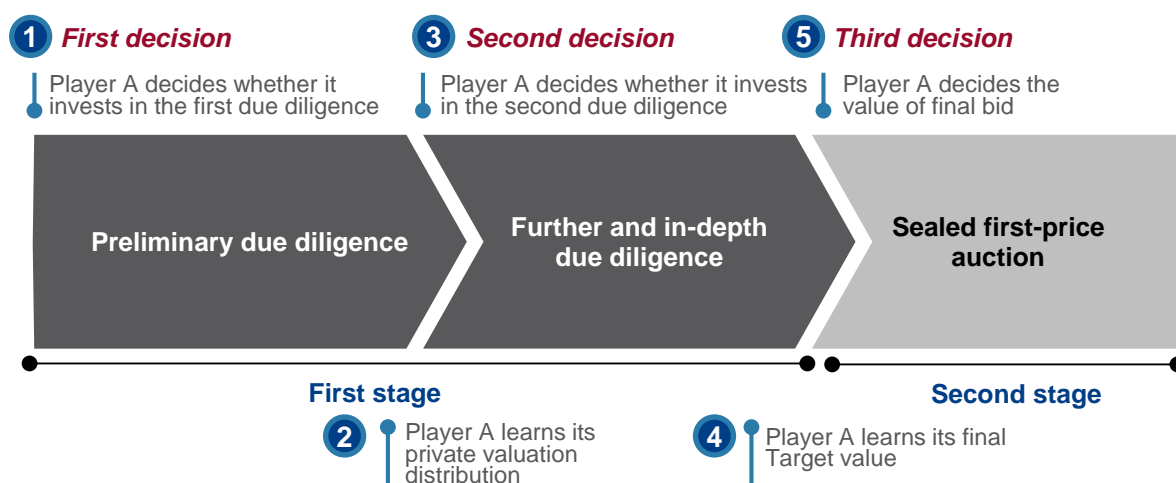
Following a similar approach, Dai, Yun et al. (2013) also investigate a sequential-entry takeover contest competition between similar bidders whose valuations of a target are likely correlated according to their degree of similarity.

Following the two-stage bidding game as in Smit, van der Berg and De Maeseneire (2005) and Van der Berg (2007) they examine the relationship between their level of similarity, likelihood of a multi-bidder process and expected final prices. Their innovative approach includes a laboratory experiment recreating the conditions of the game that provided three interesting results. First, multi-bidders contests are more likely between intermediately similar bidders while single-bidder contests are mostly a result of games with very similar and dissimilar players. Intermediately similar bidders seemed unable to preempt competition whereas very similar and dissimilar bidders could preempt bidder due to potential competition and low information value, respectively. Second, expected prices and the degree of similarity presented an inverted u-shaped relationship for both single and multi-bidders contests. When it comes to single-bidder contests, increasing preempt bids are required to discourage competition up to an inflection point where potential competition effect starts to dominate the information value and expected prices starts to decrease. Likewise, multi-bidder contests present rising prices as similarity increases, however, almost identical players seems to compete only for low valuation levels. Third, the similarity between players seemed to have a relevant connection with competition. For any level of similarity, the returns for the target are higher in single-bidder than in multi-bidder acquisitions since a bid premium is needed to deter competition.

3. A DECISION-THEORETIC MODEL FOR M&A CONTROLLED SALE PROCESSES

In this section we design a decision-theoretic model in which a firm (“Player A”) is invited by the shareholders of a company for sale (“Target”) to engage in an M&A controlled sale. This process comprises two stages: (i) sequential due diligences (preliminary and in-depth) and; (ii) sealed-bid auction. In the first phase potential acquirers retrieve information on the Target value. This inquiry requires from prospective buyers substantial expenditures such as fees for financial advisors, auditors, lawyers, and consultants. In addition, they incur in opportunity costs since resources are allocated to accomplish this particular deal. In the second phase the Target manages a sealed-bid auction. At this point, Player A faces uncertainty regarding the number of competitors in the auction.

Figure 2. Detailed steps of the M&A sale process model



We assume that prior to the opening stage sellers disclose their reserve price and share a superficial information memorandum. An initial expenditure is obligatory to scrutinize the data which reveals the participant’s private Target valuation distribution. Subsequently, our player decides whether it withdraws from the contest or invests in a deep and expensive investigation. Through this second study Player A assesses its final Target value hence finishing the first stage and proceeding to the auction.

Finally, Player A may submit a final offer in a sealed-bid auction. We assume that vendors seek to maximize their wealth and will opt for the highest bid conditioned on exceeding their reserve price. With support of game theory framework, we separately design each stage of the M&A process.

3.1. Designing the sequential due diligences stage

Following Smit and Moraitis (2014), we schematize the initial part of model in four dimensions: the players, the actions available to them, the timing of these actions, and the payoff structure associated to each possible outcome. Our first step is to characterize players as Player A and Nature whose role is to reveal the parameters of the private valuation distribution.

Next we define available information and actions. Before investing in the first due diligence Player A knows that this spending reveals the mean, \bar{v}_A , of its private Target valuation distribution, $f(v_A)$. The mean will be either v_A^H with probability q , which means high potential of value creation, or v_A^L with probability $1 - q$, representing low potential. We assume that a high potential of value creation leads our participant to proceed to the second due diligence whereas low potential leads it to withdraw from the contest. In addition, our player is informed about sellers' reservation price v_S , the standard deviation of its valuation distribution σ_A , and the support of the distribution $v_A \in [\bar{v}_A - \theta, \bar{v}_A + \theta]$ where θ is defined as a spread. That is, beforehand our participant is not certain about its ability to profit from transaction.

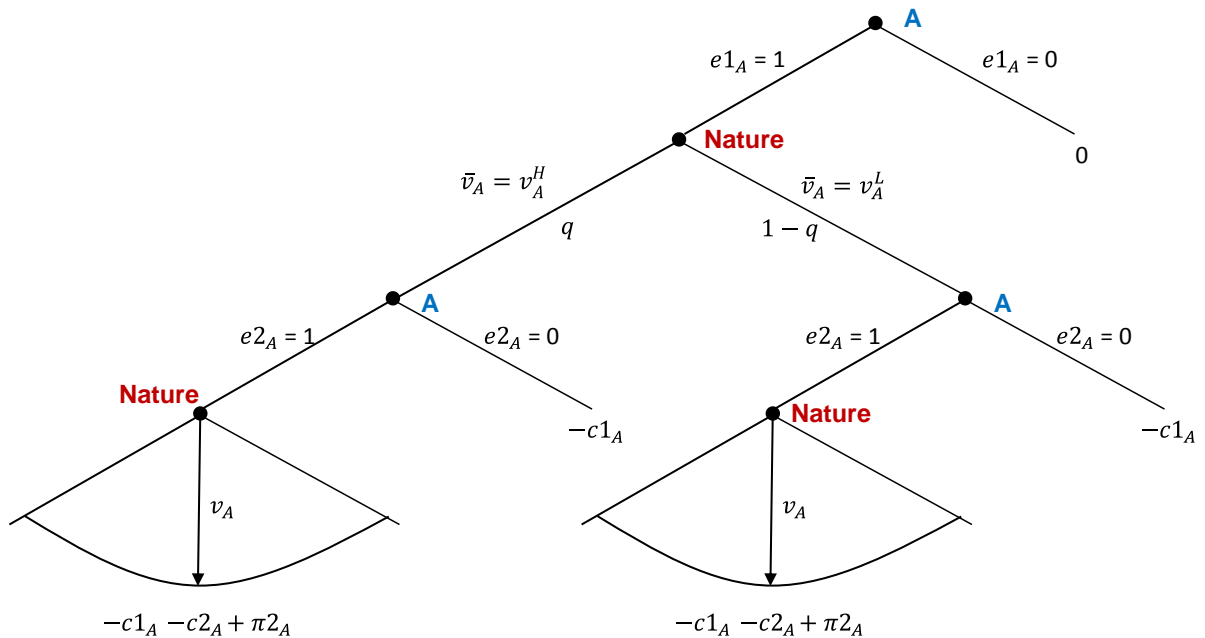
The primary stage requires two actions regarding each investment on analyzing Target's data. We denote the first action, decision to enter the first diligence by $e1_A \in \{0,1\}$ where $e1_A = 0$ indicates that participant A does not invest and $e1_A = 1$ indicates that it enters and spends $c1_A$ to accomplish the examination. Thus, it learns the mean of its private valuation distribution, \bar{v}_A , equal to v_A^H with probability q or v_A^L with probability $1 - q$ and consequently the support of the distribution. Next, we represent the second action or decision to enter the deep investigation by $e2_A \in \{0,1\}$ where $e2_A = 0$ indicates that player A does not enter and $e2_A = 1$ indicates that it enters and spends $c2_A$ to learn its final private valuation, v_A .

Our third step is defining the sequence of play as sequential movements. There are four sequential movements: Player A's first decision, Nature stochastically draws the mean of $f(v_A)$, Player A's second decision, and Nature stochastically draws v_A from the support of $f(v_A)$.

At last, we represent the payoff associated to each result to bidder A as π_A . If the acquirer does not enter the process its payoff is null and the investment opportunity is lost. If it performs the early due diligence and decides to withdraw from the process after learning its private distribution its payoff is negative with the initial spending. If our participant invests twice performing the full scrutiny it profits the total cost of the study augmented by the auction stage payoff, π_{2A} , which is detailed in the next section:

$$\pi_A = \begin{cases} 0, & \text{if } e_{1A} = 0 \\ -c_{1A}, & \text{if } e_{1A} = 1 \text{ and } e_{2A} = 0 \\ -c_{1A} - c_{2A} + \pi_{2A}, & \text{if } e_{1A} = 1 \text{ and } e_{2A} = 1 \end{cases}$$

Figure 3. First stage decision problem



3.2. Designing the sealed-bid auction stage

Once again we design the process in four dimensions: players, actions, timing of actions, and payoff structure. Firstly, players continue to be Player A and Nature.

Nature defines the existence of a second bidder B and, in that case, also generates its type. The choice for a single competitor is justified by the study of Boone and Mulherin (2011) which states that in 870 studied takeovers between 2003 and 2007 received on average 1.3 private binding offers. In essence, the second bid represents the threat of competition.

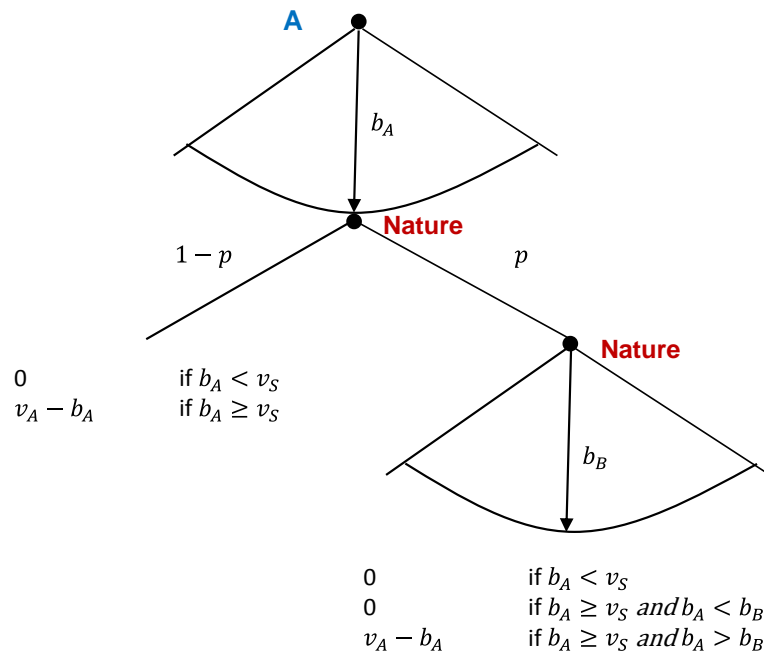
Secondly, we define our buyer third and conclusive action which is choosing its final bid value, b_A . Player A believes that there is a probability $p, p < 1$ that another rival offer would be placed. It believes this offer is stochastically drawn from a uniform distribution of opponent types denoted by $h(b_B)$ whose interval is $[v_S, v_S + \alpha * (v_A - v_S)]$. In other words, the minimum competing offer is, of course, seller's reserve price whereas the maximum contestant bid is a convex combination between v_A and v_S . The coefficient $\alpha, \alpha > 0$ represents the level of competitiveness or capacity of rivals to transfer synergy creation to sellers. Consequently, we can infer that there is a probability of losing the auction given by $p * \frac{\alpha * (v_A - v_S) + v_S - b_A}{\alpha * (v_A - v_S)}$. It is important to mention that all beliefs are formed prior to the first stage. The information set is constructed to indicate that firms are unsure about competitors' existence, valuation and costs. Put differently, our model is constructed to reflect that information is incomplete in controlled sale processes.

Afterwards, we design the timing of actions. Player A's third action is taken without knowledge about the results of Nature's movements. Player A decides the value of its final proposal and Nature simultaneously draws competitors' existence and type.

Lastly, we determine final payoffs given each possible outcome. Player A has joined a sealed-bid auction where the highest offer wins and pays the value of her bid. It means the value of proposals not only affects whether participants win or not but also how much they pay. At this point both investments in due diligence are sunk costs, private valuation is known and there is uncertainty respecting the rivalry. For this reasons our participant receives $v_A - b_A$ if it wins the auction and zero otherwise. We neglect the scenario where $b_A = b_B$ because it is a zero-probability event.

$$\pi_{2A} = \begin{cases} 0, & \text{if } b_A < v_S \\ 0, & \text{if } b_A \geq v_S \text{ and } b_A < b_B \\ v_A - b_A, & \text{if } b_A \geq v_S \text{ and } b_A > b_B \end{cases}$$

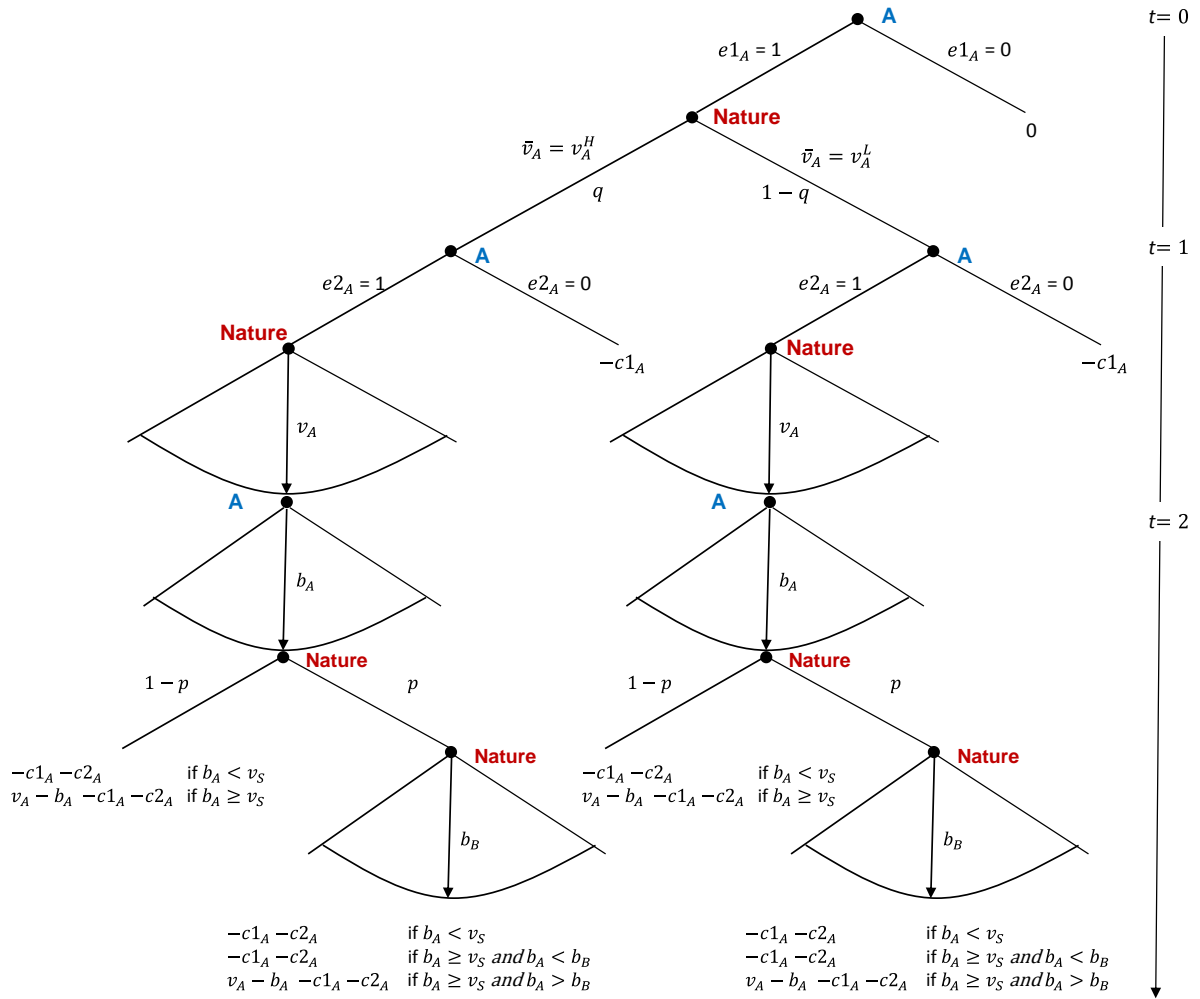
Figure 4. Second stage decision problem



Finally, we outline total payoffs and the full representation of the process:

$$\pi_A = \begin{cases} 0, & \text{if } e1_A = 0 \\ -c1_A, & \text{if } e1_A = 1 \text{ and } e2_A = 0 \\ -c1_A - c2_A, & \text{if } e1_A = 1, e2_A = 1, \text{ and } b_A < v_S \\ -c1_A - c2_A, & \text{if } e1_A = 1, e2_A = 1, b_A \geq v_S, \text{ and } b_A < b_B \\ v_A - b_A - c1_A - c2_A, & \text{if } e1_A = 1, e2_A = 1, b_A \geq v_S, \text{ and } b_A > b_B \end{cases}$$

Figure 5. M&A controlled sale process decision problem



3.3. Subgame perfection and optimal bid

We proceed to solve our model by predicting Player A's likely behavior according to its optimal decisions. Our examination begins looking forward and finding the subgame perfection at the auction. Thus, we reason backwards from our participant's last decision concerning the submission of an optimal offer at the sealed-bid auction.

Proposition 1. Given a private Target valuation equal or higher than sellers' reserve price, $v_A \geq v_S$, bidder A places an optimal bid $b_A^* \in [v_S, v_S + \alpha * (v_A - v_S)]$ that depends on the beliefs set α and p :

$$b_A^* = \begin{cases} v_S + \alpha * (v_A - v_S), & \text{if } \alpha \leq \frac{p}{1+p} \\ v_S + \frac{v_A - v_S}{2} * \frac{p - \alpha * (1-p)}{p}, & \text{if } \frac{p}{1+p} < \alpha < \frac{p}{1-p} \\ v_S, & \text{if } \alpha \geq \frac{p}{1-p} \end{cases}$$

Proof: Appendix A

Proposition 1 results from the analyses that follows. First of all, towards pursuing the best course of action we separate cases when the revealed private valuation falls behind sellers' reserve price from cases when it surpasses. It is clear from $v_A < v_S$ that Player A is better off bidding below v_S thus having a rejected offer, instead of bidding above and reaping a negative profit. In contrast, if $v_A \geq v_S$, our participant reaches a greater outcome by bidding $b_A \geq v_S$ with some probability of winning rather than offering below resulting in a null payoff. Obviously, Player A is always better off bidding below its valuation oppositely to bidding equal or above it.

Therefore, given $v_A \geq v_S$, our player should bid $v_A > b_A \geq v_S$. By now it has to choose an optimal value that counter-balances its absolute payoff and the probability of winning. That is, bidding incrementally below its private valuation increases Player A's net payoff margin when it wins which provides a marginal benefit. In contrast, at the same time it reduces the likelihood of having the highest offer which represents a marginal cost. With the purpose of calculating the optimal bid that counterweights these two effects we take the derivative of the auction expected payoff in relation to the bid value. We must remember that a second proposal occurs with probability p and non-competition happens with probability $1 - p$. We also impose lower and upper bounds given by the interval of competing rival bidders types, $h(b_B)$.

Finally, it is relevant to highlight that for a given v_A we can estimate Player A's payoff before the auction.

$$E[\pi_{2A} | v_A]_{t=2} = \begin{cases} (v_A - v_S) * (1 - \alpha), & \text{if } \alpha \leq \frac{p}{1+p} \\ (v_A - v_S) * \frac{(\alpha + p - \alpha p)^2}{4\alpha p}, & \text{if } \frac{p}{1+p} < \alpha < \frac{p}{1-p} \\ (v_A - v_S) * (1 - p), & \text{if } \alpha \geq \frac{p}{1-p} \end{cases}$$

Proof: Appendix A

3.4. Due diligence investments as the purchase of real options

By now we analyze decisions referring whether or not our participant enters each due diligence. In addition to solve our model with standard decision theory we will show that due diligence investments are similar to the purchase of real options providing an additional tool for buy-side professionals.

Table 1. Real options parameters

Due diligence	Option	Option premium	Underlying asset	Strike price
First or superficial	Option to buy the second due diligence	Cost of the first due diligence	Option to bid in the sealed-bid auction	Cost of the second due diligence
Second or in-depth	Option to bid in the sealed-bid auction	Cost of the second due diligence	Target value	Bid value

After computing the best course of action in the third decision node we move backwards analyzing the second node. At this point, Player A decides whether or not invest in a second due diligence, $e_{2A} \in \{0,1\}$. This decision resembles a real option with cost c_{2A} that is indispensable to compute the Target valuation. That is to say, acquirers buy the opportunity to place an offer exercising the option at the bid value. We seek to establish a decision rule for $e_{2A} = 1$ which happens if the expected payoff of the auction surpasses the cost of proceeding to it, $E[\pi_{2A}]_{t=1} > c_{2A}$. This also means that Player A is exercising the first option at an underlying value greater than the strike price.

Proposition 2. The decision to enter the second due diligence depends on Player A's probability adjusted value creation exceeding the correspondent investment adjusted by an optimal bidding factor

$$e2_A = 1 \text{ iff } \begin{cases} \varphi(\bar{v}_A) > \frac{c2_A}{1-\alpha}, \text{ if } \alpha \leq \frac{p}{1+p} \\ \varphi(\bar{v}_A) > \frac{c2_A * 4\alpha p}{(\alpha + p - \alpha p)^2}, \text{ if } \frac{p}{1+p} < \alpha < \frac{p}{1-p} \\ \varphi(\bar{v}_A) > \frac{c2_A}{1-p}, \text{ if } \alpha \geq \frac{p}{1-p} \end{cases}$$

Where $\varphi(\bar{v}_A) = \int_{v_S}^{\bar{v}_A + \theta} v_A f(v_A) dv_A - v_S * \int_{v_S}^{\bar{v}_A + \theta} f(v_A) dv_A$ represents the probability-adjusted value creation for Player A.

Proof: Appendix B

Lastly, we move to the first decision node where Player A decides whether or not enter the first due diligence, $e1_A \in \{0,1\}$. In other words, prospective buyers purchase the opportunity to perform the second due diligence at a cost $c2_A$.

Proposition 3. The decision to enter the first due diligence depends on Player A's probability adjusted value creation, given $\bar{v}_A = v_A^H$, exceeding the total due diligence investment adjusted by an optimal bidding factor

$$e1_A = 1 \text{ iff } \begin{cases} \varphi(\bar{v}_A | \bar{v}_A = v_A^H) > \frac{q * c2_A + c1_A}{q * (1-\alpha)}, \text{ if } \alpha \leq \frac{p}{1+p} \\ \varphi(\bar{v}_A | \bar{v}_A = v_A^H) > \frac{4\alpha p * (q * c2_A + c1_A)}{q * (\alpha + p - \alpha p)^2}, \text{ if } \frac{p}{1+p} < \alpha < \frac{p}{1-p} \\ \varphi(\bar{v}_A | \bar{v}_A = v_A^H) > \frac{q * c2_A + c1_A}{q * (1-p)}, \text{ if } \alpha \geq \frac{p}{1-p} \end{cases}$$

Proof: Appendix C

4. FINDINGS AND RESULTS

4.1. Decision rules for due diligence investments

Analyzing Propositions 2 and 3, we notice that the left side of both decision rules - represented by $\varphi(\bar{v}_A)$ - refers to the ability of Player A creating value through the acquisition for any optimal bidding strategies. The function $\varphi(\bar{v}_A)$ is composed positively by Player A's expected private Target valuation whereas sellers' reservation price appears negatively impacting the term. According to the equation, both terms are adjusted by the probability of value creation, that is, possible Target values above v_S in the support of $f(v_A)$. Altogether this is consistent with the fact that bidders only benefit from their valuation and make disbursements if the deal is worthy for them.

On the other hand, the right side of both decision rules depends on the cost of the due diligences adjusted by strategy-related factor. In the first rule, the primary due diligence investment is treated as an inherent cost of entering the M&A process while the second due diligence investment is adjusted by the probability of it being made. This happens when the second rule is satisfied. On the contrary, the second rule considers the first spending as a sunk cost and the subsequent as a necessary expenditure to move to the auction. Optimal bid factors that adjust the due diligence investments result from the substitution of the optimal bid function and the correspondent probability of winning in the expected payoff value as described in Appendix B. As expected, when the required investment increases firms are less likely to perform the study.

4.2. Analysis of the optimal bid

First of all, we restate the formulas of Proposition 1 to facilitate readers understanding and interpretation:

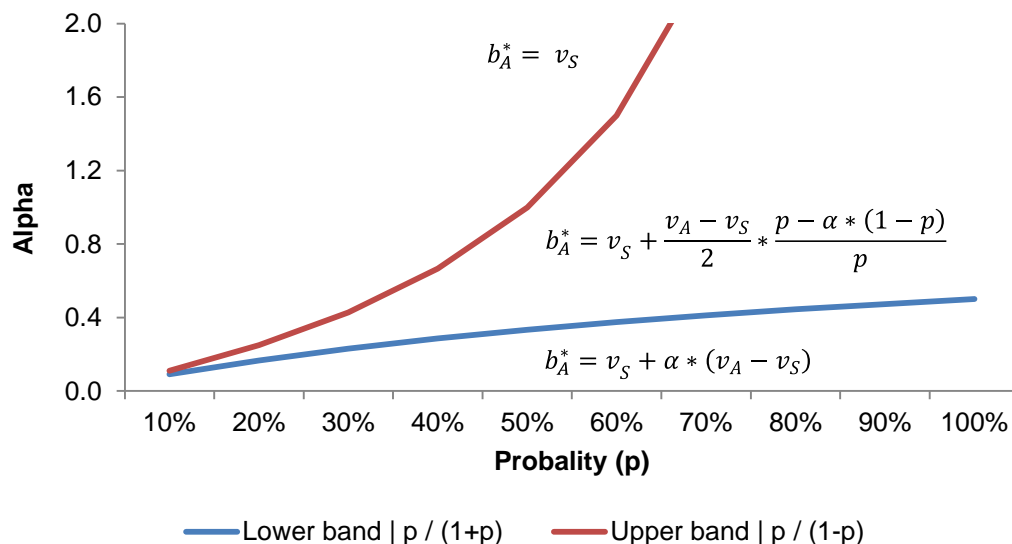
$$b_A^* = \begin{cases} v_S + \alpha * (v_A - v_S), & \text{if } \alpha \leq \frac{p}{1+p} \\ v_S + \frac{v_A - v_S}{2} * \frac{p - \alpha * (1-p)}{p}, & \text{if } \frac{p}{1+p} < \alpha < \frac{p}{1-p} \\ v_S, & \text{if } \alpha \geq \frac{p}{1-p} \end{cases}$$

In the first case, sets of α and p below the lower band in Figure 6, the best game plan is seeking to outbid any potential rival offer. To put it differently, our player tries to reduce to zero the probability of losing the dispute. Notably for this case, if the belief of having an opponent increases then the participant uses this approach for greater intensities of rivalry as captured by a positive slope in the lower band line.

In regards to the second case, beliefs sets above the lower band and below the upper band, b_A^* follows a specified equation. One of most significant discoveries is that if $p = 100\%$ then $b_A^* = \frac{v_A + v_S}{2}$. These offer insight into the fact that participants will utmost divide half of its value creation with the sellers. On the contrary, Player A reduces the value creation it was willing to transfer, $\frac{v_A - v_S}{2}$, by the factor $\frac{\alpha * (1-p)}{p}$.

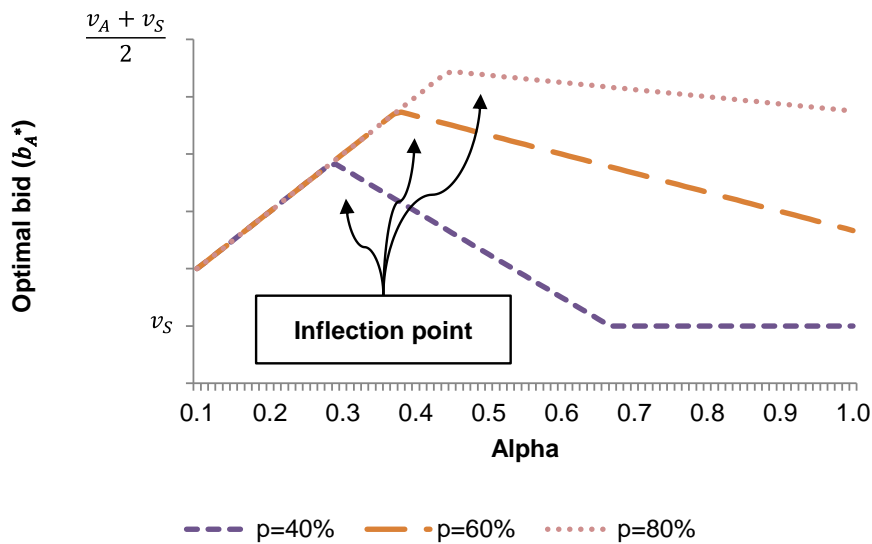
When it comes to the third case, sets above the upper band, b_A^* equals the minimum value sellers will accept. This conclusion implies that fierce competition might motive Player A to bid as low as possible and benefit from cases where contest does not exist. This is an important issue to selling parties when designing a tailored M&A sale process.

Figure 6. Optimal bidding functions given beliefs set



Another striking conclusion lies in a mutable impact of alpha on the optimal bid. At first, as alpha increases the optimal bid also grows. However, as beating competitors reaches a certain level of difficulty alpha's effect encounters an inflection point and bids move in the opposite direction. In addition, the inflection point takes a higher alpha to occur when the perceived probability of competition is greater. That is to say, if our participant is confident on the presence of a contest takeover then it continues increasing its proposal for higher expected levels of competition.

Figure 7. Optimal bid for increasing α levels

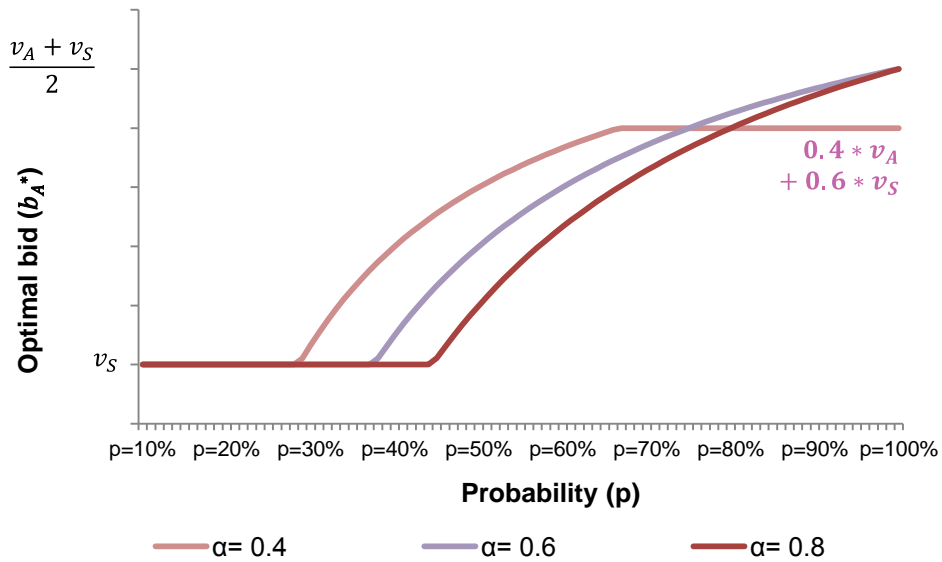


$$\frac{\partial b_A^*}{\partial \alpha} = \begin{cases} v_A - v_S, & \text{if } b_A^* = \alpha * (v_A - v_S) + v_S \\ -\frac{v_A - v_S}{2} * \frac{(1-p)}{p}, & \text{if } b_A^* = \frac{v_A + v_S}{2} - \frac{\alpha * (1-p)}{p} * \frac{v_A - v_S}{2} \\ 0, & \text{if } b_A^* = v_S \end{cases}$$

We can also interpret our findings from the point of view of the presence of a second bidder. As expected, low degrees of p lead to the best response of offering the lowest possible value vendors will accept. Nonetheless, as p rises Player A is encouraged to escalate its proposal. This is an exciting outcome which prompts rethinking on competition's threat leading to higher offers. In our study, because of competition uncertainty players only increase their offers at certain beliefs sets. Controversially, the capabilities of potential adversaries delay the moment when

players start to increase their offers, but on the other hand, the proposal escalates quickly for higher levels of competition.

Figure 8. Optimal bid for increasing p levels



$$\frac{\partial b_A^*}{\partial p} = \begin{cases} 0, & \text{if } b_A^* = \alpha * (v_A - v_S) + v_S \\ (v_A - v_S) * \frac{\alpha}{2p^2}, & \text{if } b_A^* = \frac{v_A + v_S}{2} - \frac{\alpha * (1 - p)}{p} * \frac{v_A - v_S}{2} \\ 0, & \text{if } b_A^* = v_S \end{cases}$$

5. CONCLUSIONS

In the previous section, we have reported the main outcomes of our model. Chapter V summarizes our study, discusses main results and their implications for practice, recommends further research, and concludes the study.

This dissertation has developed a theoretical two-stage model in order to analyze potential acquirers' participation in M&A sale processes. The first phase comprises two sequential investments in due diligence in order to assess the Target value whereas the second phase is a sealed-bid auction. We have studied the influence of significant due diligence investments, uncertainty regarding the target value during the due diligence stage and unclear competitive environment in their decision to engage in these auctions. In addition, with support of decision theory, we have also examined the formation of optimal bids in the last stage of model.

We have analyzed three research questions. First, we have studied whether a prospective buyer should enter into a bid contest financing a preliminary due diligence. Then, we have examined if it should perform an in-depth scrutiny on the Target given the information revealed in the first inquiry. Lastly, we have investigated buyers' optimal bid value.

When it comes to the first two research questions, we have created decision rules for participants' investments in each due diligence. Both decision rules depend on Player A's ability to create value compared to the required investments to assess the Target value. They are also related to the optimal bidding strategy function that we calculated according to the expected sealed-bid auction payoff.

Regarding the third research question, we have found that the optimal bid lies between sellers' reserve price and the maximum potential rival offer. The optimal value between these extremes depends on the beliefs concerning the probability of a rival bid and the strength of potential competitors. As expected, low expected probability levels of a second bid motivates Player A to offer sellers' reservation price and, at a certain point as p continues rises, Player A starts to escalate its proposal. In addition, this moment occurs at higher levels of p for higher levels of expected competition. This finding is an exciting result on competition's threat because not always intense competition leads to higher offers.

Another finding concerns the variable impact of an increasing alpha coefficient on our participant's bid. First, as alpha increases the optimal bid follows the same path. Nonetheless, as outbidding rivals becomes more difficult alpha's effect encounters an inflection point and the optimal bid starts to decrease. That is, the uncertainty regarding the existence of a rival bid might motivate Player A to decrease its offer if potential competitors are expected to have strong capabilities of value creation.

These results have far-reaching implications for M&A advisors interested in the dynamics between sellers and buyers. This study identified decision rules to enter M&A sale process and proceed to in-depth due diligences that can provide a financial framework benefiting buy side professionals. For selling-parties, this study offers insight into creating an appropriate competitive environment especially when the process has only one interest potential buyer in the due diligence stage.

The findings of this study, although significant, have some limitations. The main limitation is the assumption of a uniform distribution of competing bidders' type. The problem is that it strongly limits the robustness of your results. For example, we have calculated that participants will utmost divide half of its value creation with the sellers. This is not a robust finding if we do not assume uniform distributions for competing bidders' type. Another limitation is the focus on a situation of a single player against nature. One could argue that we should model an M&A sale processes as a game between participants. In that case we should compute Bayesian-Nash equilibrium for the game. Further research along these lines could use other distributions in order to calculate the optimal bid for Player A. Future research into this subject should also include the possibility of a second rival bid.

The findings of this dissertation expanded the literature of competition in mergers and acquisitions. Specifically, it focused on M&A sale processes when initiated by the selling party. This investigation modeled the formation of optimal bids in sealed-bid auction according to a bidder's belief regarding the probability of existence of a rival offer and its strength. It also computed two decision rules for prospective acquirers' decision to enter costly superficial and in-depth due diligences which combined fulfilled its two main purposes detailed in the first chapter.

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APPENDIX A. Derivative of Player A's expected payoff in relation to its optimal bid

Given a private valuation equal or higher than sellers' reserve price, $v_A \geq v_S$, expected payoff of the auction at $t = 2$:

$$\begin{aligned}
 E[\pi_{2A}|v_A]_{t=2} &= (v_A - b_A) * (1 - p) + (v_A - b_A) * p * P(b_A > b_B) \\
 &= (v_A - b_A) * [1 - p * (1 - P(b_A > b_B))] \\
 &= (v_A - b_A) * [1 - p * P(b_B > b_A)] \\
 &= (v_A - b_A) * [1 - p * \frac{\alpha * (v_A - v_S) + v_S - b_A}{\alpha * (v_A - v_S)}]
 \end{aligned}$$

We take the derivative of the expected payoff in relation to b_A and equal it to zero in order to find the optimal bid, b_A^* :

$$b_A^* = \frac{v_A + v_S}{2} - \frac{\alpha * (1 - p)}{p} * \frac{v_A - v_S}{2}$$

The first thing we need to remember is the optimal bid has a lower bound given by the seller's reserve price. Payoffs cannot be further maximized by bids below v_S . Thus, if $b_A^* \leq v_S$ then $b_A^* = v_S$, which occurs when:

$$\begin{aligned}
 \frac{v_A + v_S}{2} - \frac{\alpha * (1 - p)}{p} * \frac{v_A - v_S}{2} &\leq v_S \\
 \alpha &\geq \frac{p}{1 - p}
 \end{aligned}$$

On the other hand, the optimal bid has an upper bound given by the potential uppermost competing bid. Translating it to our model, the probability of winning, $P(b_A > b_B)$, cannot be further increased by offers above the expected highest possible rival bid. If $b_A^* \geq b_{B \max}$ then $b_A^* = b_{B \max}$, which happens when:

$$\frac{v_A + v_S}{2} - \frac{\alpha * (1 - p)}{p} * \frac{v_A - v_S}{2} \geq \alpha * (v_A - v_S) + v_S$$

$$\alpha \leq \frac{p}{1+p}$$

Finally, we rewrite $b_A^* = \frac{v_A+v_S}{2} - \frac{\alpha*(1-p)}{p} * \frac{v_A-v_S}{2}$ for cases where $\frac{p}{1+p} < \alpha < \frac{p}{1-p}$ as $b_A^* = v_S + \frac{v_A-v_S}{2} * \frac{p-\alpha*(1-p)}{p}$

Therefore, we can calculate auction's expected payoff given a known private valuation, v_A . A key point to this analysis is inferring $P(b_B > b_A^*)$. This probability does not depend on v_A and thus is not a random variable. It depends on the parameters set composed by α and p that result in the bidding strategy. For example, when Player A opts for offering the expected highest competing bid the correspondent probability of losing is null. On the other hand, when it bids sellers' reserve price it loses for any existing competitor.

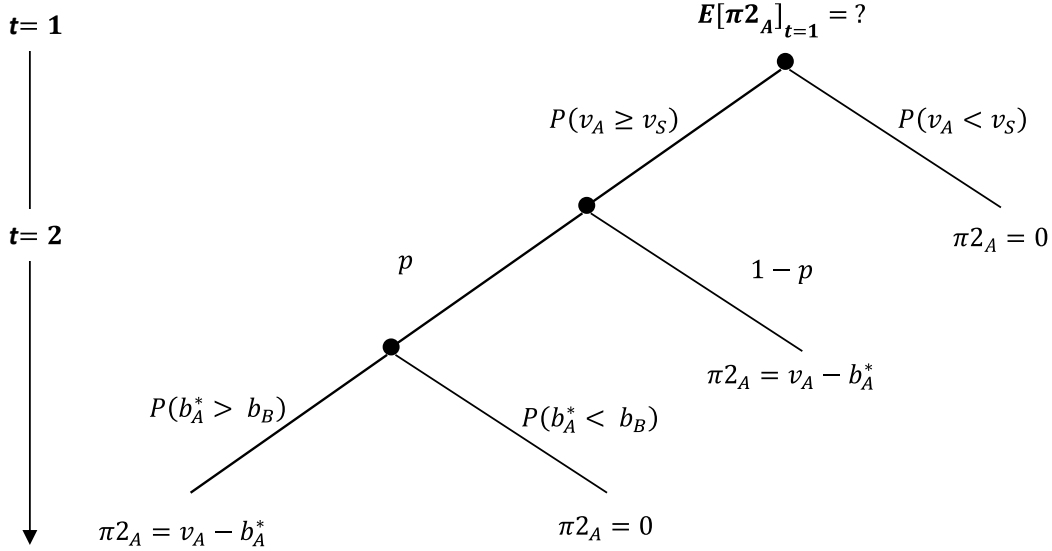
Table 2. Probability of losing the auction with competition

Optimal bid	$P(b_B > b_A^*)$
$b_A^* = \alpha * (v_A - v_S) + v_S$	0
$b_A^* = \frac{v_A + v_S}{2} - \frac{v_A - v_S}{2} * \frac{\alpha * (1 - p)}{p}$	$\frac{\alpha p - p + \alpha}{2\alpha p}$
$b_A^* = v_S$	1

$$E[\pi_{2A} | v_A]_{t=2} = \begin{cases} (v_A - v_S) * (1 - \alpha), & \text{if } \alpha \leq \frac{p}{1+p} \\ (v_A - v_S) * \frac{(\alpha + p - \alpha p)^2}{4\alpha p}, & \text{if } \frac{p}{1+p} < \alpha < \frac{p}{1-p} \\ (v_A - v_S) * (1 - p), & \text{if } \alpha \geq \frac{p}{1-p} \end{cases}$$

APPENDIX B. Expected auction payoff before the second due diligence and second decision rule

Figure 9. Expected payoff of the auction at $t = 1$



$$E[\pi_{2A}]_{t=1} = P(v_A \geq v_S) * [(1 - p) * (E[v_A | v_A \geq v_S] - b_A^*) + p * P(b_A^* > b_B) * (E[v_A | v_A \geq v_S] - b_A^*)]$$

$$E[\pi_{2A}]_{t=1} = P(v_A \geq v_S) * (E[v_A | v_A \geq v_S] - b_A^*) * (1 - p * P(b_B > b_A^*))$$

A key point to this analysis is inferring $P(b_B > b_A^*)$ that was calculated for each case in Appendix A. Substituting $P(b_B > b_A^*)$ and b_A^* in the expected payoff equation:

$$E[\pi_{2A}]_{t=1} = \begin{cases} \left(\int_{v_S}^{\bar{v}_A + \theta} v_A f(v_A) dv_A - v_S * \int_{v_S}^{\bar{v}_A + \theta} f(v_A) dv_A \right) * (1 - \alpha), & \text{if } \alpha \leq \frac{p}{1+p} \\ \left(\int_{v_S}^{\bar{v}_A + \theta} v_A f(v_A) dv_A - v_S * \int_{v_S}^{\bar{v}_A + \theta} f(v_A) dv_A \right) * \frac{(\alpha + p - \alpha p)^2}{4\alpha p}, & \text{if } \frac{p}{1+p} < \alpha < \frac{p}{1-p} \\ \left(\int_{v_S}^{\bar{v}_A + \theta} v_A f(v_A) dv_A - v_S * \int_{v_S}^{\bar{v}_A + \theta} f(v_A) dv_A \right) * (1 - p), & \text{if } \alpha \geq \frac{p}{1-p} \end{cases}$$

If we defined $\varphi(\bar{v}_A) = \int_{v_S}^{\bar{v}_A + \theta} v_A f(v_A) dv_A - v_S * \int_{v_S}^{\bar{v}_A + \theta} f(v_A) dv_A$ we reach:

$$E[\pi_{2A}]_{t=1} = \begin{cases} \varphi(\bar{v}_A) * (1 - \alpha), & \text{if } \alpha \leq \frac{p}{1+p} \\ \varphi(\bar{v}_A) * \frac{(\alpha + p - \alpha p)^2}{4\alpha p}, & \text{if } \frac{p}{1+p} < \alpha < \frac{p}{1-p} \\ \varphi(\bar{v}_A) * (1 - p), & \text{if } \alpha \geq \frac{p}{1-p} \end{cases}$$

Therefore, given that Player A decides by $e_{2A} = 1$ if $E[\pi_{2A}]_{t=1} > c_{2A}$:

$$e_{2A} = 1 \text{ iff } \begin{cases} \varphi(\bar{v}_A) > \frac{c_{2A}}{1-\alpha}, & \text{if } \alpha \leq \frac{p}{1+p} \\ \varphi(\bar{v}_A) > \frac{c_{2A} * 4\alpha p}{(\alpha + p - \alpha p)^2}, & \text{if } \frac{p}{1+p} < \alpha < \frac{p}{1-p} \\ \varphi(\bar{v}_A) > \frac{c_{2A}}{1-p}, & \text{if } \alpha \geq \frac{p}{1-p} \end{cases}$$

APPENDIX C. First due diligence investment decision rule

Our assumptions in section 3.1 imply that $E[\pi_{2_A} | \bar{v}_A = v_A^H]_{t=1} > c_{2_A}$ and $E[\pi_{2_A} | \bar{v}_A = v_A^L]_{t=1} < c_{2_A}$. Hence, Player A chooses $e_{2_A} = 1$ if $\bar{v}_A = v_A^H$ and $e_{2_A} = 0$ if $\bar{v}_A = v_A^L$. Thus, the expected payoff for $e_{1_A} = 1$ is $q * (E[\pi_{2_A} | \bar{v}_A = v_A^H]_{t=1} - c_{1_A} - c_{2_A}) + (1 - q) * (-c_{1_A})$.

$$e_{1_A} = 1 \text{ iff } \begin{cases} \varphi(\bar{v}_A | \bar{v}_A = v_A^H) > \frac{q * c_{2_A} + c_{1_A}}{q * (1 - \alpha)}, \text{ if } \alpha \leq \frac{p}{1 + p} \\ \varphi(\bar{v}_A | \bar{v}_A = v_A^H) > \frac{4\alpha p * (q * c_{2_A} + c_{1_A})}{q * (\alpha + p - \alpha p)^2}, \text{ if } \frac{p}{1 + p} < \alpha < \frac{p}{1 - p} \\ \varphi(\bar{v}_A | \bar{v}_A = v_A^H) > \frac{q * c_{2_A} + c_{1_A}}{q * (1 - p)}, \text{ if } \alpha \geq \frac{p}{1 - p} \end{cases}$$

APPENDIX D. Illustrative case

The purpose of this section is to demonstrate the applicability of our model. Player A is invited to participate in an M&A sale process whose sellers' reservation price $v_S = 100$. The first due diligence spending, $c1_A = 0.1$, reveals a uniform private Target valuation distribution, $f(v_A)$, with: (i) mean, \bar{v}_A , equal either to $v_A^H = 105$ with probability $q = 50\%$ or $v_A^L = 95$ with probability $1 - q = 50\%$; (ii) spread $\theta = 10$ and; (iii) support $v_A \in [\bar{v}_A - \theta, \bar{v}_A + \theta]$. The second due diligence investment, $c2_A = 1.0$, reveals Player A's private Target value, v_A , stochastically drawn from the support of $f(v_A)$.

When it comes to the expected competition in the sealed-bid auction, Player A believes that: (i) one rival bid will be submitted with probability $p = 75\%$ and no bid will be placed with probability $1 - p = 25\%$ and; (ii) the competing bid will be stochastically drawn from a uniform distribution of opponent types denoted by $h(b_B)$ whose interval is given by $[v_S, v_S + \alpha * (v_A - v_S)]$ with $\alpha = 0.75$.

Proceeding backwards in order to find the most likely outcome we analyze Player A's last decision choosing its bid value. As described in Section 4.2 the optimal course of action is to bid below v_S if $v_A < v_S$ and to place an optimal bid, b_A^* , according to the equation below as provided by Proposition 1 otherwise:

$$b_A^* = v_S + \frac{v_A - v_S}{2} * \frac{p - \alpha * (1 - p)}{p}$$

As Player A is aware about its bidding strategy we move to its second decision regarding investing in the in-depth due diligence. Proposition 2 provides a decision rule (stated below) that allows us infer the outcome. Therefore, we conclude that Player A invests in the second due diligence if the revealed \bar{v}_A equals v_A^H and does not if $\bar{v}_A = v_A^L$.

$$e2_A = 1 \text{ iff } \varphi(\bar{v}_A) > \frac{c2_A * 4\alpha p}{(\alpha + p - \alpha p)^2}$$

Finally, we analyze Player A's entrance in the M&A process. According to Proposition 3 stated below, we conclude that Player A invests in the first due diligence.

$$e_{1A} = 1 \text{ iff } \varphi(\bar{v}_A | \bar{v}_A = v_A^H) > \frac{4\alpha p * (q * c_{2A} + c_{1A})}{q * (\alpha + p - \alpha p)^2}$$

Figure 10. Solving an illustrative case

