Quantitative Equity Portfolio Optimization Model

Financial Consulting Project

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Executive Summary

Project Objectives

General goal: **Audit and enhance the capabilities of the quantitative equity portfolio optimization model** of BPI´s asset management department which is based on an expected return model, a risk input and an optimization routine.

Specific goals:
- Using BPI's current methodology based on the approach of Haugen and Baker (1995), **improve the rank accuracy and profitability of the expected return model.**
- Investigate different methods to **estimate the risk input** for the optimization procedure **more efficiently** than by employing the Sample Covariance Matrix (SCM).
- Implement several techniques to **improve** the optimal portfolio performance against the benchmark index, the S&P500, more successfully than via the BPI's optimization model.

Issues

1. **Multicollinearity** between characteristics

2. **High estimation error** in the SCM, ill conditioned: that is, difficult to invert in the optimization procedure

3. **Unstable results** in the portfolio optimization

Results and Recommendations

1. Do a factor reduction of the BPI's model. We suggest a new expected return model, the **NOVA Model**, comprising the following firm characteristics: **Accruals-to-Assets (Industry Standardized), Book Yield, Earnings Yield and Market Capitalization**. Our vector has an annual expected return of 20,21% in the first decile (vs. 16,7% BPI's); in terms of decile rank accuracy, our R^2 for the whole period (March 1992 - August 2011) is 46,9% (vs. 43,9% BPI) and the return slope through deciles is -1,7 (vs. -1,4 BPI); the NOVA Model displays greater diversification power.

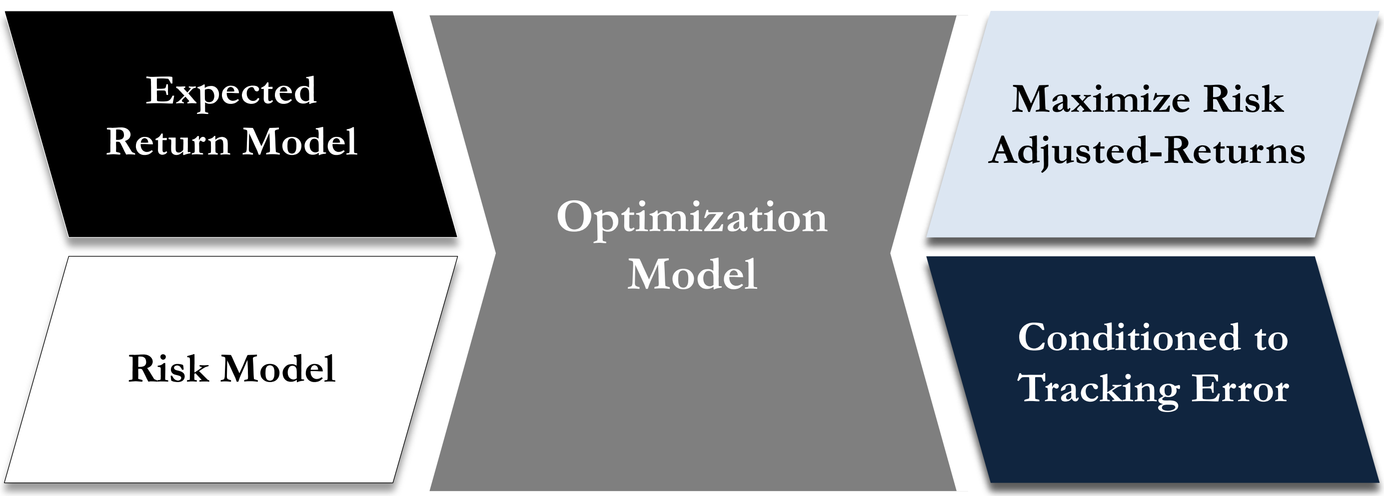
2. We conclude that the SCM is a poorer estimator of risk than other approaches; namely, the **Shrinkage** method developed by Ledoit and Wolf (2001), the **Fama-French 3-Factor Model with and without Momentum** and the **Fama-MacBeth approach** using the vector: **Earnings Yield, Market Cap, 1-Month Momentum, Accruals-to-Assets (Industry Standardized)** and the **average of all the Solvency variables**.

3. We test several optimization processes (with restrictions), **Markowitz, Mean-Variance Tracking Error (MVTE), Black-Litterman, Genetic Algorithm and Parametric Portfolio Policies**, and obtained good out-of-sample results. Different models are more suitable depending on the Managers' preferences. To Managers evaluated against a benchmark, we find that the **MVTE** method is the most appropriate (highest Information Ratio: 2,01); to Managers pursuing highest risk-return adjusted performances, the **Markowitz** model yields the best result (Sharpe Ratio: 1,26).

Project's Purpose

- The main intention of this business project is to **audit and improve the capabilities of the quantitative equity portfolio optimization model** of the BPI´s asset management division.
- This model is grounded on several empirical studies of portfolio selection and optimization.
- Our ultimate objective is to **build a monthly portfolio composed of stocks present in the S&P 500**. The optimal portfolio choice will be rebalanced on a monthly basis.

Portfolio Construction Framework



Document Structure

Expected Return Model	**Base Case: Haugen and Baker approach** ▪ Firms' Characteristics reduction and suitability ▪ Multifactor Expected Return Models comparison ▪ Test for different Factor's Estimation period ▪ Performance Accuracy and Diversification against BPI's Model
Risk Model	**Base Case: Sample Covariance Matrix** ▪ Shrinkage – Ledoit and Wolf's approach ▪ Fama-French 3 Factor model ▪ Fama-French with momentum ▪ Fama-McBeth with BARRA's methodology
Optimization Model	**Base Case: Genetic Algorithm** ▪ Markowitz with constraints ▪ Mean-Variance Tracking Error – Richard Roll's approach ▪ Black-Litterman method ▪ Parametric Portfolio Policies – M. Brandt, P. Santa-Clara and R. Valkanov ▪ Optimal Portfolio Allocation Analysis

Document Structure

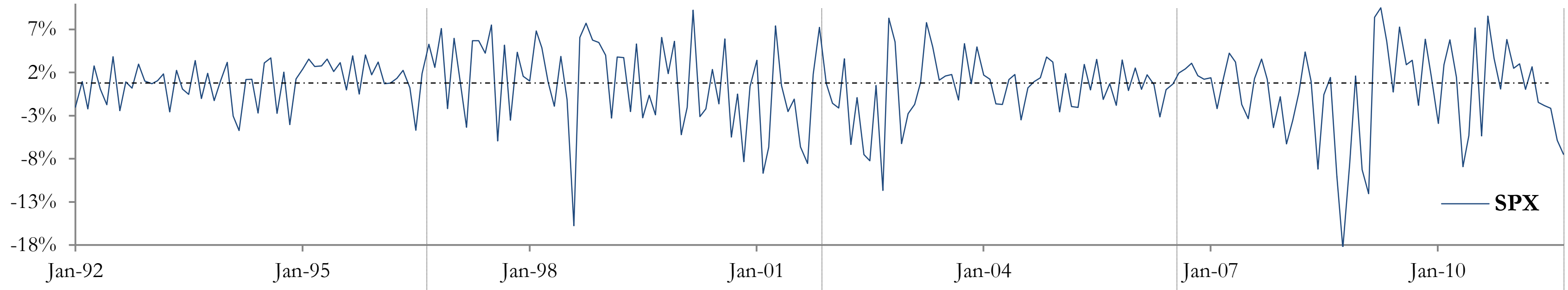
Our approach

Audit, correct misspecifications and improve the existing expected return model	• Grasp the features of each firm characteristic included in BPI's model • Drop redundant or incoherent variables • Include new variables and transformations of existing characteristics • Alternative approaches to characteristics' standardization (e.g. industry standardization) • Run Fama-McBeth regressions to infer variables' statistical significance • Examine the results obtained recurring to several measures and compare them against the BPI model
Enhance the capabilities of the risk model	• Estimate the Sample Covariance Matrix (SCM) recurring to several risk models • Use the Shrinkage method to reduce the estimation error of the SCM • Employ multi-factor models: Fama-French, Fama-McBeth • Assess the stability and precision quality of the covariance matrices attained using bootstrapping and out-of-the sample analysis
Attain stable results when solving portfolio choice problem	• Apply the Markowitz optimization procedure with constraints on short-selling and weights on specific stocks • Solve the optimization problem using the Mean-Variance Tracking Error technique • Use the Black-Litterman approach based on both market implied efficiency and expected model results • Employ the Parametric Portfolio Policies methodology • Assess the accuracy of the portfolio weights reached out-of-sample and compare the portfolio returns against the S&P 500 and the BPI's model • Development of several allocation analysis: Sector Allocation, Style and Brinson

Data Depiction

S&P 500 Sub-Period Performance/Analysis

In order to get a sense on the relevance of the results reached throughout our analysis we give a glance on the historical surroundings and market evolution during our study period.



1992-1996

Annual Return: 11,49%
Annual Vol : 8,62%
Sharpe Ratio: 0,85

Highlights:

During this period the stock market portrayed very low volatility as no major events were registered.

1997-2002

Annual Return: 2,87%
Annual Vol : 18,77%
Sharpe Ratio: -0,07

Highlights:

During this period the stock market was characterized by very high volatility and major market boosts. The higher level of volatility was essentially linked to the Asian crisis of 1997 and Dot.com bubble of the early 2000's.

2003-2006

Annual Return: 11,93%
Annual Vol : 8,23%
Sharpe Ratio: 1,15

Highlights:

In the period between 2004 and 2006 the market started to stabilize and recovered from the Dot-com. The S&P 500 and other major exchanges achieved very stable results across the period.

2007-Present

Annual Return: -4,76%
Annual Vol : 19,53%
Sharpe Ratio: -0,31

Highlights:

Since early 2007, the financial markets industry has been yielding catastrophic performances all over the world. The credit crisis of 2008 as well as the sovereign debt crisis that we are currently living in, brought a lot of tension to the financial world

EXPECTED RETURN MODEL

$E(r)$

1. Multi-Factor Models
2. Documented Anomalies
3. Firm's Characteristics Reduction
4. Multi-Factor Expected Return Model
5. Expected Return Models Analysis

1 Multi-Factor Models

Brief Description

What is it?

MFMs employ common factors to estimate the return sensitivity in relation to each of these factors. The basis of this model is that similar stocks or portfolios should have similar returns/ underlying factors.

Generic Formula

$$r_i = \alpha_i + X_{i,1}F_1 + X_{i,2}F_2 + \ldots + X_{i,n}F_n + \varepsilon_i$$

where:
- r_i is the returns of security I
- $X_{1,2,3\ldots n}$ are the characteristics used
- $F_{1,2,3\ldots n}$ are the estimated factors used
- ε_i is the error term
- α_i is the intercept

Multi-Factor Models (MFM)

Why to use them?

They provide an holistic view on the breakdown of the risk exposures of a stock/ portfolio when compared with single factor models as the CAPM. MFM are time responsive to changes in factors.

Shortcomings

- The decision of how many and which factors to include is not trivial.
- Are based on historical data; therefore, may not be accurate in the future.

What factors are normally used?

Factors that affect a large number of stocks so as to isolate the idiosyncratic risk (e.g. **returns of portfolios**, macroeconomic factors, statistical factors, fundamental factors).

Examples

Arbitrage Price Theory, Fama-French, etc.

There are some portfolios built upon strategies' rationales that yield high abnormal returns. These phenomena are the so-called "anomalies" that affect stocks transversely. A closer view is given on some **documented anomalies** as they will influence the factors chosen in our MFM.

Growth vs Value Stocks

The value effect is documented by Basu (1983), Keim (1983), Fama and French (1992) among others, and indicates that high book-to-market ratio stocks outperform the low book-to-market ratio stocks. In the same way, investors attain greater returns by acquiring stocks traded at low prices compared to their earnings or sales.

Trading Strategy

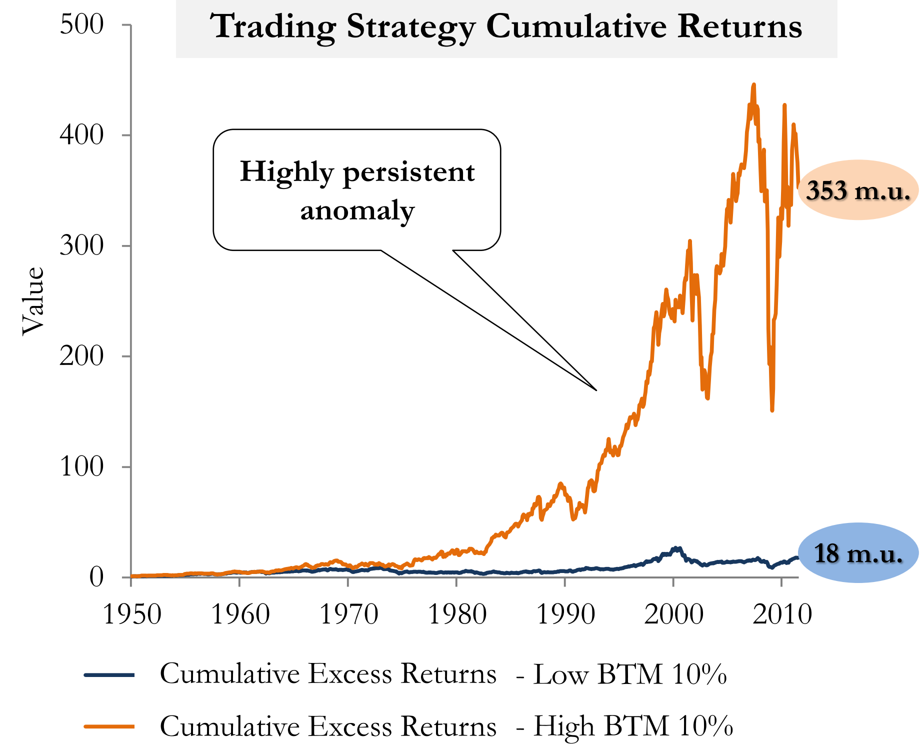
- Build two portfolios: one portfolio contains high BTM companies (top 10%), and the other consists of the 10% smallest BTM companies.
- Buy high BTM portfolio + Short low BTM portfolio

Descriptive Statistics[1]

Annualized Expected Return	5,33%
Annualized Standard Deviation	15,65%
Sharpe Ratio	**0,34**

- The expected return model should include firm characteristics that try to capture the value effect; for instance: **B/P, P/E, P/Sales, Return on Assets**

Trading Strategy Cumulative Returns



[1] Source: Kenneth R. French website; the data has monthly frequency and ranges from January 1950 to July 2011

Large vs Small Caps

Banz (1981) finds that the market capitalization adds to the explanation of the cross-section of returns provided by the market factor. He discovers that there is consistent premium offered by the smaller cap firms, that is, average returns on small caps are too high given their betas, and average returns on large caps are too low.

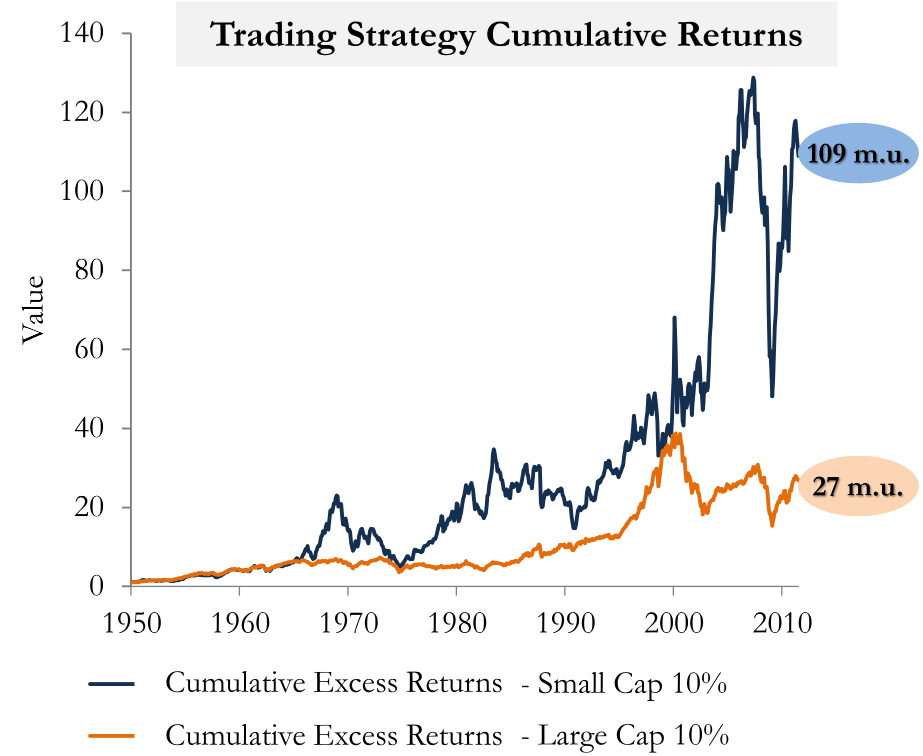
Trading Strategy

- Build two portfolios: the first portfolio comprises small market cap companies (bottom 10%), and the other consists of the 10% largest market cap companies.
- Buy small caps portfolio + Short large caps portfolio

Descriptive Statistics[1]

Annualized Expected Return	3,49%
Annualized Standard Deviation	15,97%
Sharpe Ratio	**0,22**

- The expected return model should include firm characteristics that try to capture the size effect; for instance: **Market Capitalization, Total Assets**



[1] Source: Kenneth R. French website; the data has monthly frequency and ranges from January 1950 to July 2011

Past Winners vs Past Losers

The momentum anomaly was firstly documented by Jegadeesh and Titman (1993). They showed that stocks that have outperformed in the past tend to continue to perform well over the succeeding period; likewise, stocks that have performed worse in the past are likely to keep that trend.

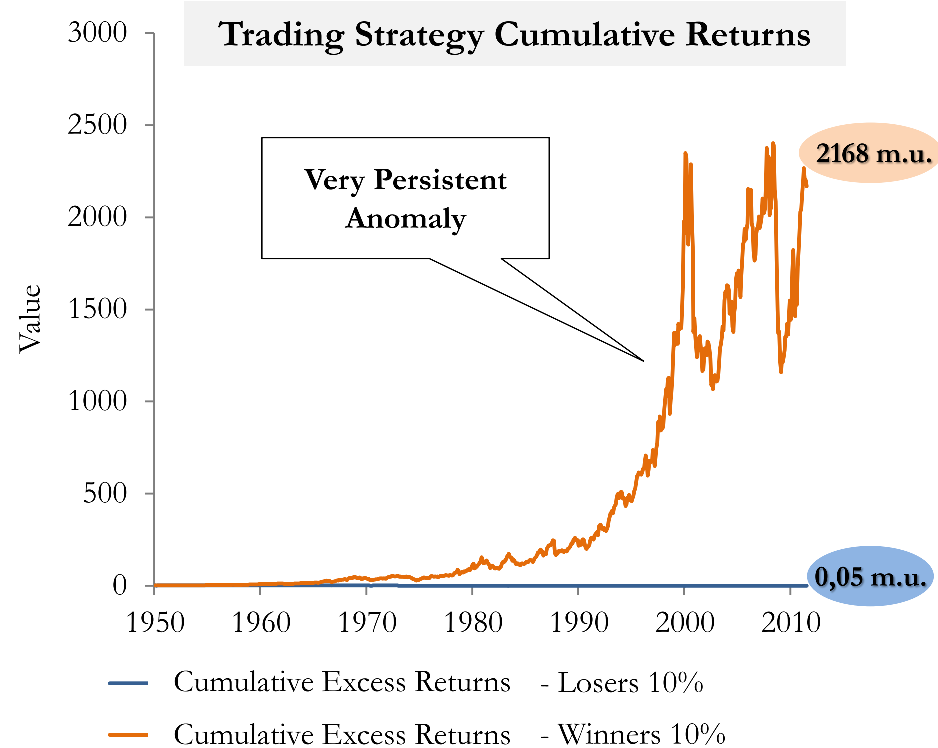
Trading Strategy

- Construct two portfolios: one portfolio contains the best performers over the last 12 month (top 10%), and the other consists of the 10% firms with worse performance.
- Buy "Winners" portfolio + Short "Losers" portfolio

Descriptive Statistics[1]

Annualized Expected Return	16,15%
Annualized Standard Deviation	22,57%
Sharpe Ratio	**0,72**

- The expected return model should include firm variables that aim to capture the momentum effect; for instance: **1/3/6/12/24 Months Momentum**



[1] Source: Kenneth R. French website; the data has monthly frequency and ranges from January 1950 to July 2011

Earnings Quality

Firms with high (low) accruals earn subsequent negative (positive) abnormal returns

High vs Low Accrual Firms

- Managers can handle accruals so as to meet earnings targets put forward by analysts in order to avoid price depreciations. In this way it is believed that firms with greater accruals have not so realistic earnings and that is a negative sign for the markets which penalize these type of companies.
- Sloan (1996) advocates firms with low (high) total accruals earn positive (negative) future abnormal returns. A simple strategy that can be used to profit from this anomaly is **to go long on firms with low accruals and short on firms with high accruals** (of course that accruals have to be adjusted to size). In the first-year after the strategy has been implemented the **annual return** Sloan found for the zero cost portfolio is **10,4%**, while in the second it is **4,8%**.
- Proxies of the earnings quality of a firm: **Accruals-to Assets/ Net Operating Assets (NOA)/ Change in NOA**

Earnings Surprises

Firms that announce earnings that are not in line with market expectations exhibit a drift in the stock price

Post Earnings Announcements Drift

- Ball and Brown (1968) identified the post-earnings announcement drift which consists of an upward drift displayed by companies that announced unexpectedly positive earnings and the opposite for firms with non-anticipated negative results.
- A variable that is widely used to capture this anomaly is the **Standardized Unexpected Earnings (SUE)** which is the difference between the announced earnings and the investors' consensus ex-ante divided by the volatility on earnings growth. Strategies relying on SUE provide **yearly returns around 5,5%.**
- Santa-Clara *et. all* (2007) suggest an alternative measure, **the Earnings Announcement Return (EAR)**: it is the excess stock return in relation to a portfolio of firms with similar risk exposures around the time of the announcement. A strategy that shorts firms with the lowest EARs and goes long on firms with the greatest EAR attain an **average return of 6,3% per year**.

See other documented anomalies in Appendix 1.

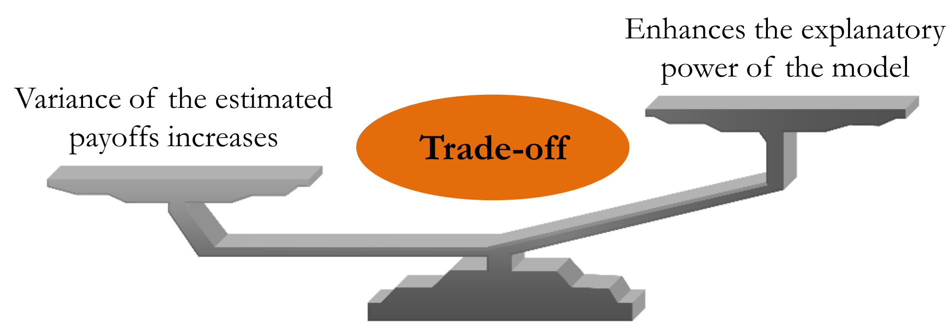
Firms' Characteristics Reduction: Theoretical Grounds

Why factor reduction? | Methodology

Assessment of characteristics' statistical and economical significance

The expected return model employed in BPI's optimization procedure uses **41 firm's characteristics** as explanatory variables for the cross-section of S&P 500 firm's returns[1].

More characteristics



Need to find the **right balance** in terms of the **number of variables** to be included in the expected return model

- One important aspect to bear in mind is that **if too many variables are removed** and still the model yields good results when backtested, one may be committing **insight bias** since the future may be different from the past.
- **Parsimonious models** are better at estimating returns as the **treat of multicollinearity is controlled** and therefore spurious relations between the variables are avoided.

Key question

Are those characteristics significant enough, in economic and statistical terms, to be included in the expected return model?

- We will decide which variables should or should not be included in the expected return model based on...

- ❑ Histograms
- ❑ Significance Tests – T-Statistics
- ❑ Long-short and Long-only portfolios
- ❑ Correlation Matrices[2]
- ❑ Cluster Analysis
- ❑ Economic Rationale/ Anomalies Exploitation

[1]We use monthly data from January 1992 to September 2011 sourced by CRSP Point-in-time database (not subject to forward looking-bias).
We present the correlation among variables families in Appendix 2. | A brief description of all the variables is provided in Appendixes 3 to 10. | A cluster analysis of all variables is shown in App. 11.

Explanation of the Measures and Techniques used

Indicators/ Methods	Description
Correlation Matrices	• This simple approach basically computes the correlation between characteristics and puts that info into a matrix: $\begin{bmatrix} Corr(F_1,F_1) & Corr(F_1,F_2) & \dots & Corr(F_1,F_n) \\ Corr(F_2,F_1) & Corr(F_2,F_2) & & \\ \dots & & \dots & \\ Corr(F_k,F_1) & & & Corr(F_k,F_n) \end{bmatrix}$ where $Corr(F_k,F_n) = \frac{Cov(F_k,F_n)}{\sigma_k \sigma_n}$
Cluster Analysis	• Group firm characteristics with similar statistical features into clusters in order to see if some variables can be transformed into a new joint variable (through *Principal Components Analysis* or linear combinations). By creating these new variables we hope to reduce the multicollinearity between the characteristics.
Standardization	• Normal (used by BPI): ${X_i}^{NS} = \frac{X_i - median}{MAD}$; this method assigns the median to missing values. The idea is to make all the variables in the same units in order to be comparable, reduce the effect of outliers and make the distribution smoother. • By Industry: ${X_i}^{IS} = \frac{X_i - industry\ median}{MAD}$; additionally, this method provides a different view as aims to adjust for under/overvaluation of stocks comparing to their peers.

NS – Normal Standardization/ IS – Industry Standardization/ MAD – Median Absolute Deviation

Explanation of the Measures and Techniques used

Indicators/ Methods	Description
Histograms	• It is a graphical depiction showing the data distribution. We build histograms for the raw variables, for the characteristics standardized normally and by industry. Our aim is basically to grasp whether we can improve the distribution of the variable by using logarithmic transformations, different approaches to standardization, etc.
Significance Tests	• Use t-statistics $t = \frac{\widehat{\lambda_j}}{se(\widehat{\lambda_j})}$; where $\widehat{\lambda_j}$ is the average of estimated factors across time and $se(\widehat{\lambda_j})$ is the standard error of the estimated factor. A two-sided test is used; the null hypothesis, that the factor is not statistically significant $(H_0: \lambda_j = 0)$, is tested against the alternative hypothesis $(H_1: \lambda_j \neq 0)$. The significance level is 5%; the critical values are +-1,96. We computed t-statistics for three sub-sample periods.
Zero Investment and Long-only Portfolios	• First, rank firm's returns according to a specific characteristic. Then, pick the top and bottom 50 firms in the case of the long/short portfolio and just the top 50 stocks for the long-only strategy. The decision of going long or short will depend on the economic rationale implicit in the characteristics. Finally, we work out the *Sharpe Ratios* so as to grasp the profitability of each characteristic and *Portfolio Turnovers* to have an idea of the transaction costs involved.

Firms' Characteristics Reduction: Analysis by Variables Type

Variables Study

In-sample period: January 1992 to December 2009 - Annual data

Indicator / Characteristics	Histograms			Long Only						Long/ Short						t-statistics[1]							
				Raw		NS		IS		Raw		NS		IS		NS				IS			
	Raw	NS	IS	SR	PT	SR	PT	SR	PT	SR	PT	SR	PT	SR	PT	92 09	97 01	02 06	07 09	92 09	97 01	02 06	07 09
Earnings Yield	◑	●	●	0,72	210%	**0,72**	210%	0,72	222%	0,58	222%	0,61	223%	0,39	237%	**8,90**	**6,42**	**4,57**	1,40	**7,80**	**5,85**	**4,67**	1,34
Earnings Growth	●	●	●	0,28	243%	0,27	243%	0,27	262%	0,03	251%	0,02	251%	-0,01	270%	**-2,75**	**-1,73**	0,26	**-2,20**	**-2,62**	**-1,37**	0,20	**-2,53**
Book Yield	◑	●	◑	0,26	225%	**0,26**	225%	**0,35**	237%	0,18	211%	**0,18**	212%	**0,33**	244%	-0,17	0,33	0,34	-2,26	1,92	0,63	1,65	-0,24
Sales Yield	○	◑	◑	0,33	180%	0,33	180%	0,31	193%	0,30	191%	0,30	192%	0,28	215%	-1,66	-1,81	-0,47	-0,41	-3,87	-1,75	-1,75	-2,91
Sales Growth	○	◑	◑	0,15	260%	0,15	260%	0,20	292%	-0,24	248%	-0,24	249%	-0,22	273%	-0,99	-0,27	0,23	0,15	-1,71	-1,37	0,87	-0,35
Dividend Yield	◑	◑	◑	0,14	153%	0,14	153%	0,32	185%	-0,29	150%	-0,29	150%	0,17	196%	0,00	0,05	-1,38	0,17	-0,37	-0,83	-0,79	0,37
Cash Flow Yield	○	◑	◑	0,43	301%	**0,43**	301%	**0,48**	383%	0,56	325%	**0,56**	326%	**0,60**	362%	0,15	0,50	-0,90	0,14	0,21	-0,60	-0,37	0,79
Growth Rate	◑	◑	◑	0,44	160%	**0,44**	160%	0,43	191%	0,45	200%	0,45	200%	0,22	233%	**2,72**	**2,43**	-0,14	0,39	1,31	1,54	-0,17	-0,10
Accruals-to-Assets	◑	◑	●	0,37	335%	**0,37**	335%	**0,38**	378%	0,40	359%	**0,40**	359%	**0,41**	388%	-1,49	1,33	**-1,96**	-0,32	**-1,64**	-0,64	**-1,88**	0,33
Market Cap	●	●	●	0,34	181%	**0,34**	181%	**0,37**	198%	0,34	153%	**0,34**	153%	**0,40**	164%	-0,37	0,81	-2,46	-1,07	-0,24	0,78	-2,11	-1,01

● <4% of missing data/ <3% outliers | ◑ 4%-20% of missing data/ 3%-5% outliers | ○ >20% of missing data/ >5% outliers | NS - Normal Standardization, IS - Industry Standardization | SR - Sharpe Ratio, PT - Portfolio Turnover

Findings:

- Earnings Yield / Earnings Growth / Market Capitalization exhibit "healthy" features in terms of missing data, outliers and observations' distribution.
- Sharpe Ratios (SR) of long portfolios according different characteristics are higher than zero investment portfolios.
- Earnings Yield/ Cash Flow Yield/ Sustainable Growth Rate/ Market Capitalization/ Accruals-to-Assets display the best portfolio SR (0,4-0,7).
- Accruals-to-Assets/ Cash Flow Yield/ Book Yield and Market Cap have higher SR when standardized by industry than when are normally standardized.
- Earnings Yield / Sustainable Growth Rate / Earnings Growth/ Accruals-to-Asset present some significant t-statistics.

[1] t-statistics calculated from BPI's expected return model

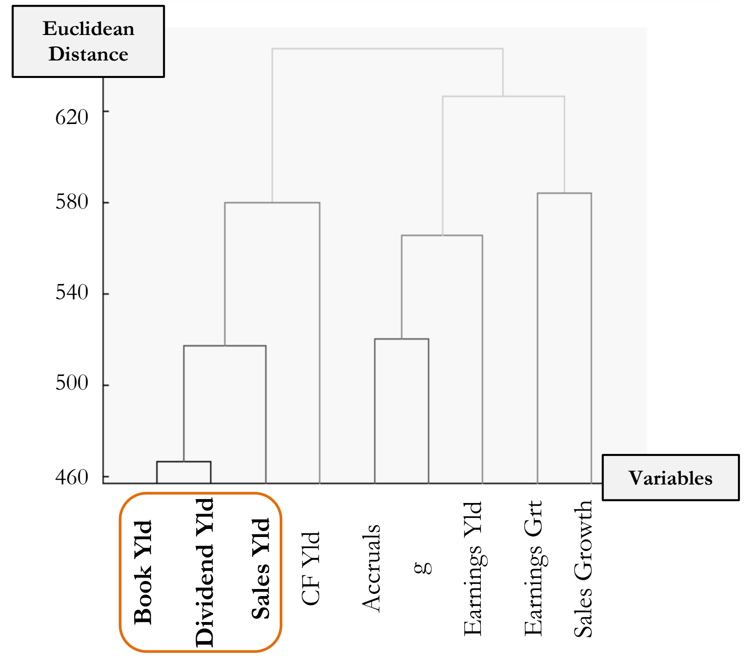
Note: Annualized Sharpe Ratio of the S&P 500 from 1992-2009: **0,44**

Variables Study

Correlation Matrix

	Accruals-to-Assets	Book Yield	Cash Flow Yield	Dividend Yield	Earnings Growth	Earnings Yield	g	Sales Growth	Sales Yield
Accruals-to-Assets	100%								
Book Yield	-11,2%	100%							
Cash Flow Yield	-56,9%	20,9%	100%						
Dividend Yield	-3,8%	25,9%	21,3%	100%					
Earnings Growth	4,5%	-3,4%	3,2%	-10,4%	100%				
Earnings Yield	30,4%	-1,4%	10,7%	11,3%	13,2%	100%			
g	21,9%	-41,5%	-9,1%	-34,2%	23,7%	26,2%	100%		
Sales Growth	2,0%	-4,0%	3,9%	-3,5%	**33,7%**	16,0%	6,5%	100%	
Sales Yield	-4,8%	**46,2%**	14,2%	9,1%	-3,7%	1,0%	-12,5%	0,0%	100%

Cluster Analysis



Findings:

- The correlation matrix generically indicates a weak relationship between all variables (the significant exceptions are Sales Yield/ Book Yield and Sales Growth/ Earnings Yield which display reasonable positive correlations which may undermine the model due to multicollinearity).
- The dendrogram (cluster analysis) shows that the variables are quite far away from each other and therefore no clear cluster can be defined; nonetheless, if there were a cluster to be defined, it would include Book Yield, Dividend Yield and Sales Yield.

Abbreviations: g: Sustainable Growth Rate

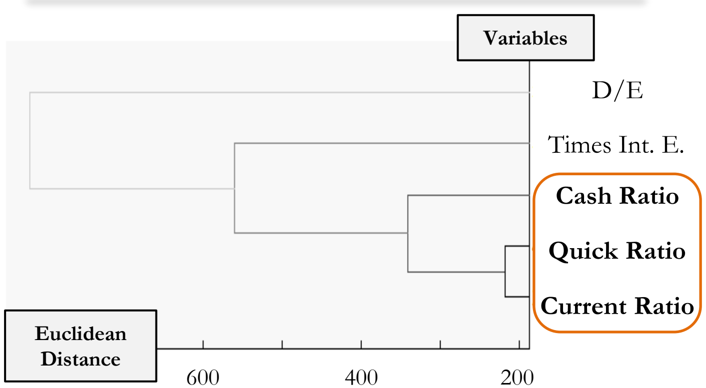
Variables Study

In-sample period: January 1992 to December 2009 - Annual data

Indicator / Characteristics	Histograms			Long Only						Long/ Short						t-statistics[1]							
				Raw		NS		IS		Raw		NS		IS		NS				IS			
	Raw	NS	IS	SR	PT	SR	PT	SR	PT	SR	PT	SR	PT	SR	PT	92 09	97 01	02 06	07 09	92 09	97 01	02 06	07 09
Current Ratio	○	◑	◑	0,15	168%	0,15	168%	0,22	183%	-0,08	167%	-0,08	166%	-0,02	197%	-1,84	-2,31	-0,99	1,05	-1,36	-1,39	-1,14	0,89
Quick Ratio	○	◑	◑	0,21	163%	0,21	163%	0,29	189%	0,03	174%	0,03	174%	0,04	206%	0,67	1,84	-0,65	-1,25	-0,01	0,37	-0,62	-0,91
Cash Ratio	○	○	◑	0,27	161%	0,29	168%	0,33	194%	0,16	196%	0,19	199%	0,17	207%	-0,09	-0,50	0,98	-0,01	0,90	0,89	1,52	-0,63
Debt-to-Equity	○	◑	◑	0,25	140%	0,25	140%	0,33	169%	0,02	140%	0,04	140%	-0,14	185%	-0,31	0,84	0,69	-2,52	0,81	0,88	0,14	-0,35
Times Interest Earned	○	○	◑	0,37	143%	0,37	143%	0,35	170%	0,01	173%	0,01	173%	-0,04	204%	-0,58	-0,35	-1,48	1,51	-0,98	-0,57	-1,17	1,03

● <4% of missing data/ <3% outliers | ◑ 4%-20% of missing data/ 3%-5% outliers | ○ >20% of missing data/ >5% outliers | NS - Normal Standardization, IS - Industry Standardization | SR - Sharpe Ratio, PT - Portfolio Turnover

Cluster Analysis



Correlation Matrix
(Solvency)

	Current Ratio	Quick Ratio	Cash Ratio
Current Ratio	100%		
Quick Ratio	86,7%	100%	
Cash Ratio	69,1%	81,9%	100%

Correlation(Debt-to-Equity, Times-Interest Earned) = -32,2%
(Financial Risk)

Findings:

- The characteristics present a considerable amount of missing data and poorly shaped distributions.
- The SRs of these families of characteristics are not that good (0,2-0,3).
- The t-statistics are not good overall.
- Huge correlation between the solvency characteristics (70%-87%).
- A cluster might be built using the three solvency variables as the distance between them is small.

[1] t-statistics calculated from BPI's expected return model

Note: Annualized Sharpe Ratio of the S&P 500 from 1992-2009: **0,44**

3 Firms' Characteristics Reduction: Analysis by Variables Type

Value | Solvency & Fin. Risk | **Operating Efficiency** | Operating Profitability | Technical

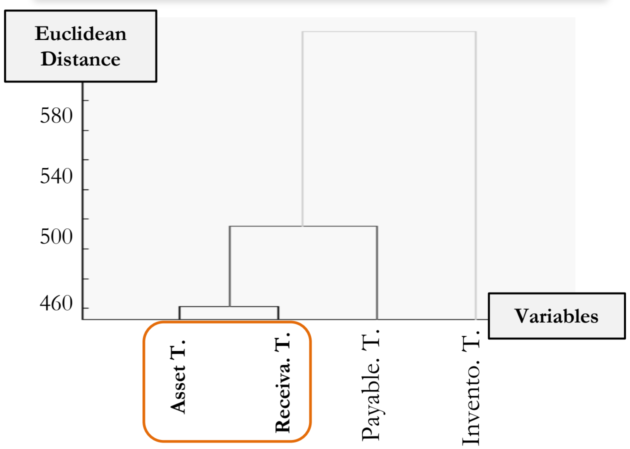
Variables Study

In-sample period: January 1992 to December 2009 - Annual data

Indicator / Characteristics	Histograms			Long Only						Long/ Short						t-statistics[1]							
				Raw		NS		IS		Raw		NS		IS		NS				IS			
	Raw	NS	IS	SR	PT	SR	PT	SR	PT	SR	PT	SR	PT	SR	PT	92 09	97 01	02 06	07 09	92 09	97 01	02 06	07 09
Asset Turnover	◑	◑	●	**0,38**	114%	**0,38**	114%	**0,30**	131%	0,14	113%	0,14	113%	0,03	151%	-0,89	**1,91**	-0,15	0,16	**4,69**	**3,12**	**1,65**	**3,16**
Inventory Turnover	○	○	◑	**0,34**	155%	**0,35**	156%	**0,31**	176%	0,16	156%	0,18	157%	-0,05	189%	-0,30	-0,11	-1,65	0,87	0,01	1,23	-1,98	0,67
Payables Turnover	○	○	◑	-	-	-	-	-	-	-	-	-	-	-	-	-1,15	-0,38	-1,10	-1,23	-1,26	-0,92	-1,49	-0,08
Receivables Turnover	○	◑	◑	**0,36**	138%	**0,36**	138%	**0,39**	178%	0,10	130%	0,10	131%	0,26	203%	1,42	0,67	0,76	1,16	-0,65	-0,67	-0,29	0,10

● <4% of missing data/ <3% outliers | ◑ 4%-20% of missing data/ 3%-5% outliers | ○ >20% of missing data/ >5% outliers | NS - Normal Standardization, IS - Industry Standardization | SR - Sharpe Ratio, PT - Portfolio Turnover

Cluster Analysis



Correlation Matrix

	Asset Turnover	Inventory Turnover	Payables Turnover	Receivables Turnover
Asset Turnover	100%			
Inventory Turnover	-0,5%	100%		
Payables Turnover	25,6%	10,9%	100%	
Receivables Turnover	53,5%	14,8%	30,4%	100%

Findings:

- The variables are somewhat profitable (SR range from 0,3 to 0,4).
- In terms of t-stats and variable distribution, these variables seem not to be relevant excluding asset turnover.
- Correlation points for a weak relationship between variables (except Asset Turn. and Receivables Turnover).

[1] t-statistics calculated from BPI's expected return model

Note: Annualized Sharpe Ratio of the S&P 500 from 1992-2009: **0,44**

Variables Study

In-sample period: January 1992 to December 2009 - Annual data

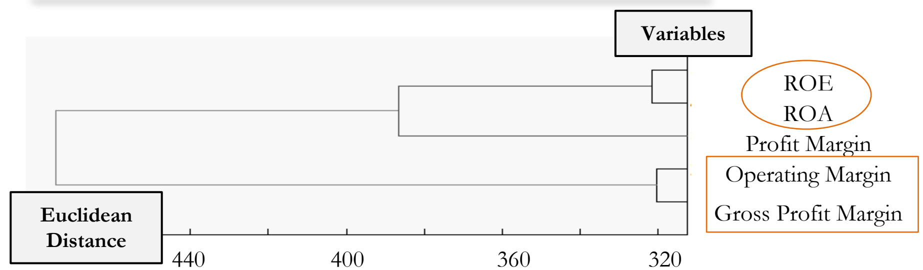
Indicator / Characteristics	Histograms			Long Only						Long/ Short						t-statistics[1]							
				Raw		NS		IS		Raw		NS		IS		NS				IS			
	Raw	NS	IS	SR	PT	SR	PT	SR	PT	SR	PT	SR	PT	SR	PT	92 09	97 01	02 06	07 09	92 09	97 01	02 06	07 09
Return on Assets	◐	◐	●	**0,42**	142%	**0,42**	142%	0,31	163%	**0,06**	186%	**0,06**	186%	-0,10	204%	**1,88**	1,27	**3,07**	-1,39	-0,33	-0,17	1,28	-1,44
Return on Equity	◐	●	●	**0,45**	138%	**0,45**	138%	0,40	184%	**0,02**	184%	**0,02**	184%	-0,06	215%	**-3,33**	**-2,70**	**-2,61**	0,43	-1,52	-1,72	-0,77	-0,07
Gross Profit Margin	◐	◐	◐	**0,46**	122%	**0,46**	122%	0,30	148%	**0,29**	147%	**0,29**	147%	0,10	166%	1,31	0,55	0,21	-0,07	1,51	0,77	0,17	0,63
Oper. Profit Margin	○	◐	◐	0,30	135%	0,30	135%	0,43	148%	-0,06	166%	-0,06	166%	0,28	181%	0,52	0,65	1,36	0,14	**2,20**	0,82	**2,58**	0,77
Profit Margin	◐	◐	●	0,31	143%	0,31	143%	0,29	160%	-0,07	188%	-0,07	188%	-0,11	200%	-2,79	-1,57	-2,19	0,18	-1,40	-0,01	-2,65	0,49

● <4% of missing data/ <3% outliers | ◐ 4%-20% of missing data/ 3%-5% outliers | ○ >20% of missing data/ >5% outliers | NS - Normal Standardization, IS - Industry Standardization | SR - Sharpe Ratio, PT - Portfolio Turnover

Correlation Matrix

	Return on Assets	Return on Equity	Gross Profit Margin	Oper. Profit Margin	Profit Margin
Return on Assets	100%				
Return on Equity	**77,9%**	100%			
Gross Profit Margin	30,2%	24,4%	100%		
Oper. Profit Margin	24,2%	24,8%	58,9%	100%	
Profit Margin	**69,5%**	**62,7%**	47,5%	**61,0%**	100%

Cluster Analysis



Findings:

- ROA, ROE and GPM show the best SR in this category.
- These variables are also rather significant and depict well behaved distributions.
- Correlations between ROE, ROA and Profit Mg are extremely high.

[1] t-statistics calculated from BPI's expected return model

Note: Annualized Sharpe Ratio of the S&P 500 from 1992-2009: **0,44**

Variables Study

In-sample period: January 1992 to December 2009 - Annual data

Indicator / Characteristics	Histograms			Long Only						Long/ Short						t-statistics[1]							
				Raw		NS		IS		Raw		NS		IS		NS				IS			
	Raw	NS	IS	SR	PT	SR	PT	SR	PT	SR	PT	SR	PT	SR	PT	92 09	97 01	02 06	07 09	92 09	97 01	02 06	07 09
12 Months High	◐	◐	◐	0,17	356%	0,17	356%	0,31	403%	0,17	591%	0,17	591%	0,40	551%	-1,77	-1,89	-0,95	0,15	-1,62	-1,56	-1,08	0,18
Momentum 1 Month	●	●	●	0,20	100%	0,20	100%	0,30	100%	0,08	928%	0,08	928%	0,17	936%	**-3,32**	**-2,73**	**-1,01**	-0,11	**-4,29**	**-3,57**	**-1,16**	**-0,31**
Momentum 3 Months	●	●	●	0,24	665%	0,24	665%	0,33	686%	0,01	670%	0,01	670%	0,22	685%	0,09	-1,29	0,55	1,20	0,49	-1,24	0,90	1,51
Momentum 6 Months	●	●	●	**0,37**	522%	**0,37**	522%	**0,33**	535%	0,02	513%	0,02	513%	-0,10	533%	-1,38	-0,25	0,84	-0,38	-0,87	-0,06	1,22	0,06
Momentum 1 Year	◐	●	●	**0,44**	397%	**0,44**	397%	**0,42**	416%	0,11	401%	0,11	401%	0,03	422%	**3,41**	**2,93**	**1,54**	-0,74	**3,21**	**2,12**	**1,99**	-0,56
Momentum 3 Years	◐	◐	◐	0,30	265%	0,30	265%	0,29	293%	0,15	277%	0,15	277%	0,13	295%	-0,58	-0,35	-1,48	1,51	-0,98	-0,57	-1,17	1,03
Momentum 5 Years	◐	◐	◐	0,28	234%	0,28	234%	0,31	260%	0,12	241%	0,12	241%	0,24	255%	-0,89	0,52	-0,82	-1,72	-0,94	0,57	-1,16	-1,51
Δ Shares Outstanding	◐	◐	◐	**0,50**	214%	**0,50**	214%	0,37	271%	0,25	221%	0,32	216%	0,11	256%	**-2,63**	**-1,50**	-0,71	-1,17	**-2,08**	**-1,89**	0,53	-1,36

● <4% of missing data/ <3% outliers | ◐ 4%-20% of missing data/ 3%-5% outliers | ○ >20% of missing data/ >5% outliers | NS - Normal Standardization, IS - Industry Standardization | SR - Sharpe Ratio, PT - Portfolio Turnover

Findings:

- All variables display sound histograms.
- Some momentums (1, 6 and 12 months) are statistically significant and provide reasonable SRs.
- Δ Shares Outstanding yields the 2nd greatest SR among all variables (0,5) and is also statistically significant.

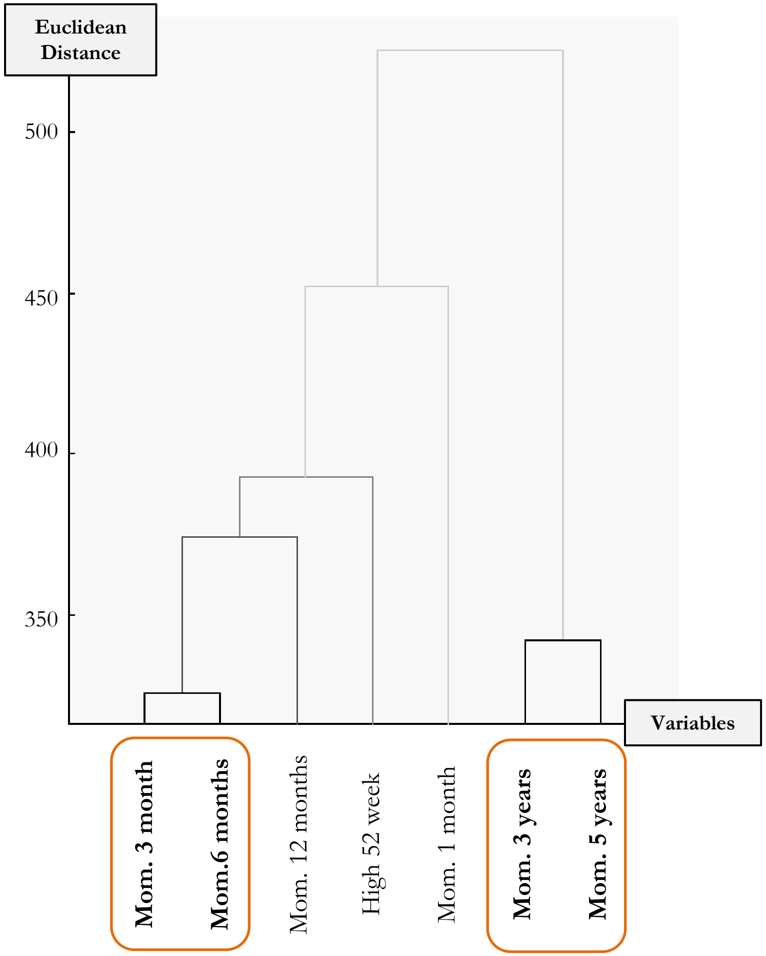
[1] t-statistics calculated from BPI's expected return model

Note: Annualized Sharpe Ratio of the S&P 500 from 1992-2009: **0,44**

Variables Study

Correlation Matrix	12 Months High	Momentum 1 Month	Momentum 3 Months	Momentum 6 Months	Momentum 1 Year	Momentum 3 Years	Momentum 5 Years
12 Months High	100%						
Momentum 1 Month	38,2%	100%					
Momentum 3 Months	54,2%	54,7%	100%				
Momentum 6 Months	**63,0%**	38,7%	**68,1%**	100%			
Momentum 1 Year	**61,7%**	28,0%	47,8%	**67,9%**	100%		
Momentum 3 Years	32,9%	14,2%	23,7%	33,4%	49,9%	100%	
Momentum 5 Years	22,1%	10,5%	17,7%	24,8%	35,9%	**70,0%**	100%

Cluster Analysis



Findings:

- The correlation matrix presents several pairs of variables with high correlation (more than 60%) and all the correlations are positive. Therefore, one should be careful including momentum variables as they are likely to be rather correlated and that may damage the model.
- The cluster analysis shows that the closest variables are Momentum 3 and 6 months and Momentum 3 and 5 years.

3 Synopsis on Variables Analysis

Main Conclusions	Action
The **Earnings Yield** is the characteristic with the **best mix of indicators**: extremely statistically significant over time, highly profitable (especially when normally standardized), not too much correlated with other variables. Reasonable to **capture the "value effect"**.	**Must be included in the model**
The family of characteristics "**Value**" is the stronger in terms of explanatory power, profitability and **low correlation** between value characteristics, thus the risk of multicollinearity is little.	**Ought to add several value variables to the model**
Solvency characteristics form the **closest cluster** amongst all the variables; moreover, the **correlation** between them is **very high**.	**Should create a joint variable**
Some variables as Accruals-to-Assets, Cash Flow Yield, Book Yield, Market Cap display **better results when standardized by industry than when normally standardized**.	**Include industry standardized characteristics**
The **Change in Shares Outstanding** is a very **profitable** and **significant** characteristic; moreover, it is a good **proxy** for the **"Net Equity Issuance anomaly"**.	**Sound variable to be included**
Apart from being a well behaved and profitable characteristic **Market Capitalization** is the only variable available to accurately **capture the "size effect"**.	**Should be added in the model**
Accruals-to-Assets is an appropriate variable (the only we have) to seizure the **earnings quality anomaly**; furthermore, it provides reasonable risk-adjusted returns.	**Should be added in the model**
Return on Assets and **Return on Equity** are highly profitable, highly significant and have well shaped distributions. Additionally, these characteristics are very correlated and close in terms of cluster analysis.	**Include a joint variable**
Operating Efficiency and **Financial Risk** characteristics do not display interesting results overall.	**Should not be added**
Momentum variables have some interesting features, namely, the best histograms of all the characteristics' families, some significant variables and decent Sharpe Ratios overall.	**Can include momentum variables**

4 Multi-Factor Expected Return Model

Top 5 Firm Characteristics' Vectors

	Characteristics Included	Rationale
Vector 1[1]	• *Earnings Yield* • *Accruals-to-Assets(IS)* • *Book Yield* • *Market Cap*	▪ Captures the "value effect", "size effect" and earnings quality anomaly ▪ Parsimonious model: factors are estimated with less uncertainty.
Vector 2	• *Earnings Yield* • *Average - ROE,ROA* • *Market Cap*	▪ Size effect, value effect ▪ Creation of a joint variable for ROA and ROE ▪ Parsimonious model
Vector 3	• *Earnings Yield* • *Accruals-to-Assets(IS)* • *Δ Shares Outstanding* • *Market Cap*	▪ Size effect, value effect, earnings quality, net equity issuance ▪ Parsimonious model
Vector 4	• *Earnings Yield* • *Accruals-to-Assets* • *Market Cap* • *Δ Shares Out* • *Average- MM1,MM60* • *Interaction – Solvency*[2] • *Average – Efficiency*[3] • *Average - ROE, ROA*	▪ Size effect, value effect, earnings quality, net equity issuance, momentum, solvency, turnover, operating profitability
Vector 5	• *Earnings Yield* • *Accruals-to-Assets(IS)* • *Market Cap* • *g* • *CFY(IS)* • *BY(IS)* • *ROE* • *ROA* • *GPM* • *MM1* • *MM12* • *Δ Shares Out*	▪ Size effect, value effect, earnings quality, net equity issuance, momentum ▪ Inclusion of significant operating profitability variables

[1]In this vector we opt by only using the Industry Standardization on the accruals variable since it is the only one that yields better results in terms of T-Stats when standardized by industry.
[2]Includes all Solvency variables: Cash Ratio, Quick Ratio and Current Ratio/ [3]Emcompasses all Operational Efficiency characteristics: Asset, Receivables, Payables and Inventory Turnovers
Abbreviations: IS – Industry Standardized/ g – Sustainable Growth Rate/ GPM – Gross Profit Margin/ MM1, MM12 and MM60 – Momentum 1, 12 and 60 months/ CFY – Cash Flow Yield

Cross-Sectional Prediction Method

This technique, used by Haugen and Baker (1995), estimates the factors for each firm-characteristic and subsequently predict monthly returns for each stock.

Steps	Procedure
1 **Cross Section Regression** 1st step Fama-MacBeth Regression (1973)	For each month, regress each stock return on the characteristics included in the 5 vectors so as to define each stock's factor using a Robust Regression[1]. $r_{j,t} = \sum_i \hat{F}_{i,t} * X_{j,i,t-1} + u_{j,t}$ where: $r_{j,t}$ is the return of stock *j* in month *t* $\hat{F}_{i,t}$ is the estimated factor *i* in month *t* $X_{j,i,t-1}$ is the characteristic associated to factor *i* for stock *j* at the end of month *t-1* $u_{j,t}$ is the unexplained component of return for stock *j* in month *t*
2 **Expected Return Estimation**	Compute the averages of the factors observed in the 12 months prior to the month for which expected return is estimated. $E(r_{j,t+1}) = \sum_i E(\hat{F}_{i,t}) * X_{j,i,t}$ where: $E(r_{j,t})$ is the expected rate of return of stock *j* in month *t* $E(\hat{F}_{i,t})$ is the expected factor *i* in month *t* (the arithmetic average of the estimated factor over the trailing 12 months) $X_{j,i,t}$ is the exposure to factor *i* for stock *j* based on the available information at the end of month *t-1*

[1] This regression method aims to minimize the deviation between the estimated stock returns using OLS and the observed returns through iterations.

4 Multi-Factor Expected Return Model

Selected Characteristics' Vectors → Returns Estimation Process → **Top 5 Vectors Backtest** → Factors Estimation Period

	Vectors	1	2	3	4	5
Precision	RMSE	0.1080	0.1080	0.1080	0.1081	0.1082
	Correct Signs	74,25%	74,29%	73,92%	73,68%	73,79%
	Score	●	●	◕	◕	◕
Rank	Slope	-1,8%	-1,7%	-1,4%	-1,5%	-1,6%
	R-square	47,1%	50,9%	39,7%	46,7%	49,6%
	Score	◕	●	◑	◑	◑
Profitability	Avg 1st Ret	22,0%	20,7%	21,1%	19,5%	20,2%
	Avg 10th Ret	3,5%	4,5%	5,7%	2,4%	2,6%
	1st-10th	18,5%	16,1%	15,4%	17,1%	17,6%
	Sharpe 1st	0,33	0,31	0,30	0,27	0,27
	Sharpe 10th	0,03	0,05	0,07	0,02	0,02
	Score	●	◑	◕	◔	◔
	Overall Classification	1st	2nd	3rd	5th	4th

After carrying out all the analysis quoted before, we conclude that the vector composed by **Accruals-to-Assets, Earnings Yield (Industry Standardized), Market Cap, Book Yield** is the best performer among all the tested variables' combinations.

This vector will be called the "NOVA Model".

This analysis is carried out in-sample (1992-2009).

4 Multi-Factor Expected Return Model

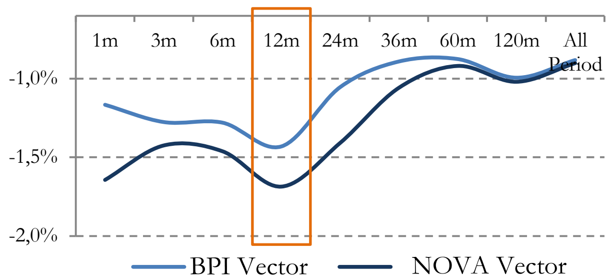
Selected Characteristics' Vectors → Returns Estimation Process → Top 5 Vectors Backtest → **Factors Estimation Period**

Short term estimation period to compute factor premium	Estimated factors are strongly influenced by economic cycle	Industries/ stocks' expected return are impacted by momentum	The "winners"/ "losers" tend to be stocks from the same industries	**Less diversification and low stock's rank accurateness**
Long term estimation period to calculate factor premium	Incorporates a longer time-horizon in the factors	Stocks' expected return barely influenced by cycles/ momentum	Divergence between model stock selection and cycle opportunities	**More diversification and low stock's rank accurateness**

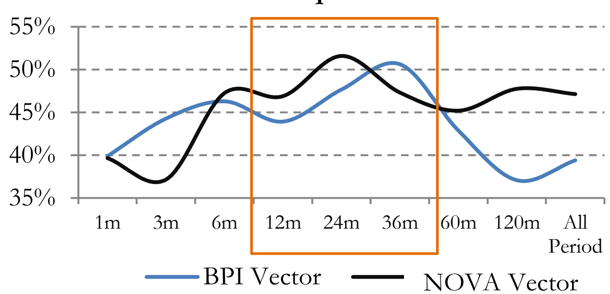
*The estimation period of **12 months** seems to be the one that better softens the trade-off between too much momentum and low diversification. This phenomena leads to constant exposure to variables momentum.*

Rank Analysis

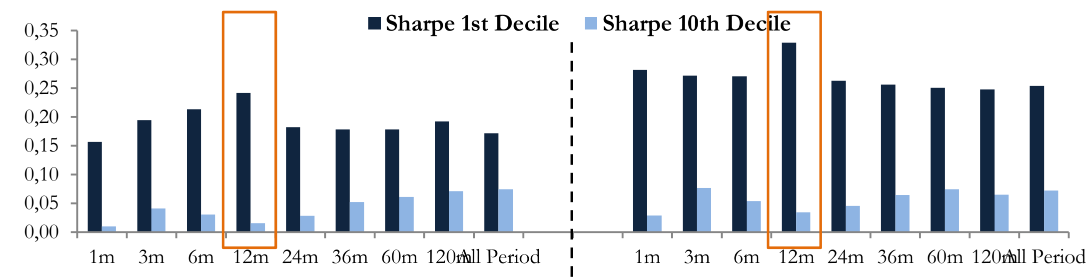
Slope

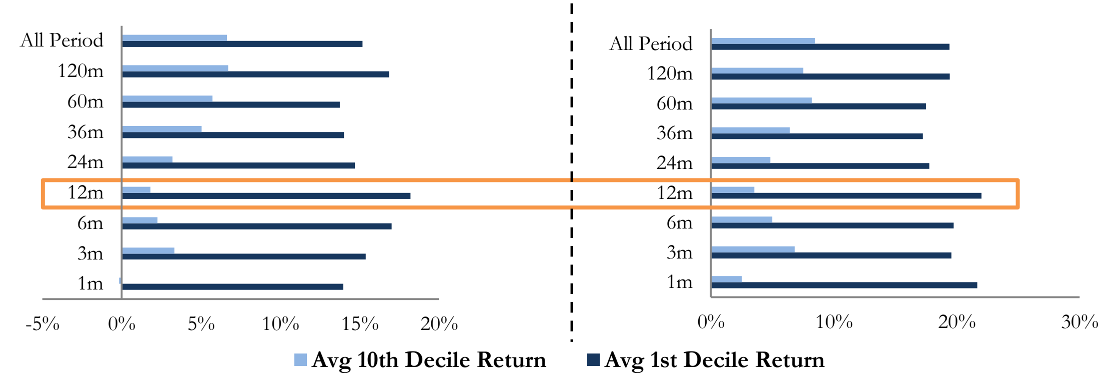


R-Squared



Profitability Analysis





5 Expected Return Models Analysis

NOVA Model vs. BPI's Factor Model

The **evaluation of the quality** of the model we have shaped, the NOVA Model, against the model employed by BPI will be carried out from **three different perspectives**. Despite executing a thorough analysis upon the model outcomes, we will highlight the analysis done on the **1st decile** as we conceived it as the most relevant taking into account the number of stocks in which the BPI model invests (roughly 50).

Performance

- Cumulative Returns
- Cum. Ret. 1st vs. 10th Ret.
- Risk/ Return trade-off
- Sharpe Ratio stability
- % Positive Months
- Maximum Return/ Maximum Drawdown
- CAGR

Accuracy

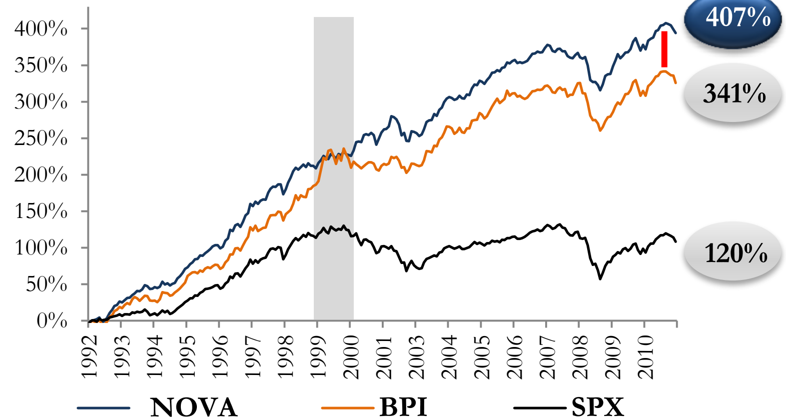
- Decile precision
- R^2 evolution
- Slope over time

Diversification

- Industry/ stock allocation over time
- A close-up on the Dotcom Crisis

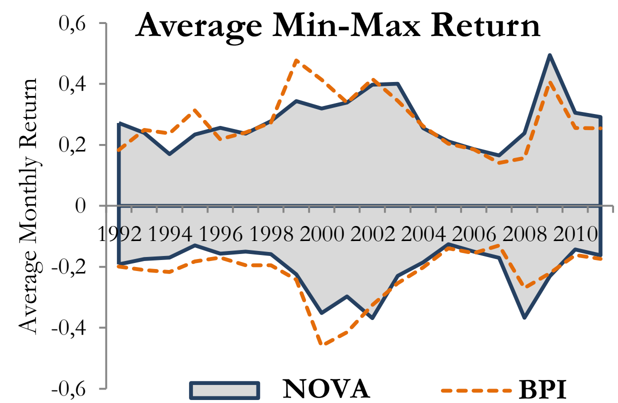
The NOVA Model consistently provides superior returns

1st Decile Cumulative Return

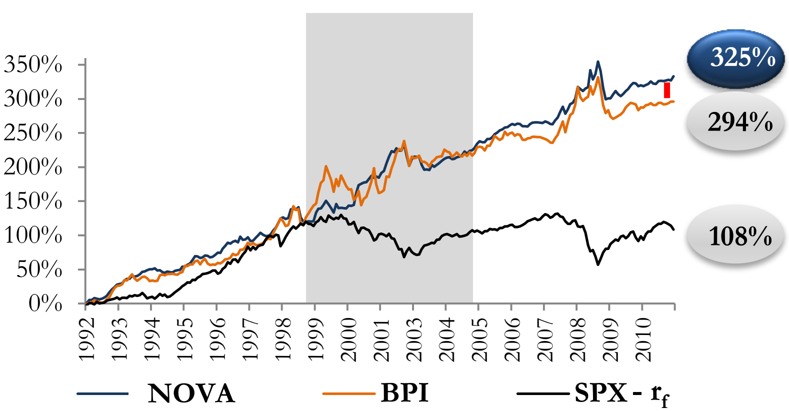


- The cumulative return for the 1st decile is **greater for the NOVA Model** (except in 1999).
- Our model is **therefore more precise in placing the best performing stocks** in the 1st decile than the BPI model.

Profitability Statistics	BPI	NOVA
Average Return (All Period)	16,7%	**20,2%**
Volatility (All Period)	**19,9%**	17,7%
Sharpe Ratio (All Period)	0,66	**0,94**
% Positive Months Return	61%	65%
CAGR	1,2%	1,6%



1st-10th Decile Cumulative Return



- The **NOVA Model** attains marginally **bigger cumulative returns** for the **difference** between the **1st and 10th deciles'** returns than BPI (this difference can be understood as a long/ short strategy on the 1st and 10th deciles).
- Our model is **slightly better** at **allocating** stocks to the **1st and 10th deciles** than BPI's.

- Our model shows **better profitability indicators** than BPI's overall: more return, less volatility, more consistency in positive returns, a greater Sharpe Ratio and the average return of the first decile for the NOVA Model is bigger (chart above).

Abbreviations: SPX – S&P 500 | See Appendix 12 and 13: performance of NOVA and BPI Models across deciles | See Appendix 14: 1st decile Sharpe ratios over time

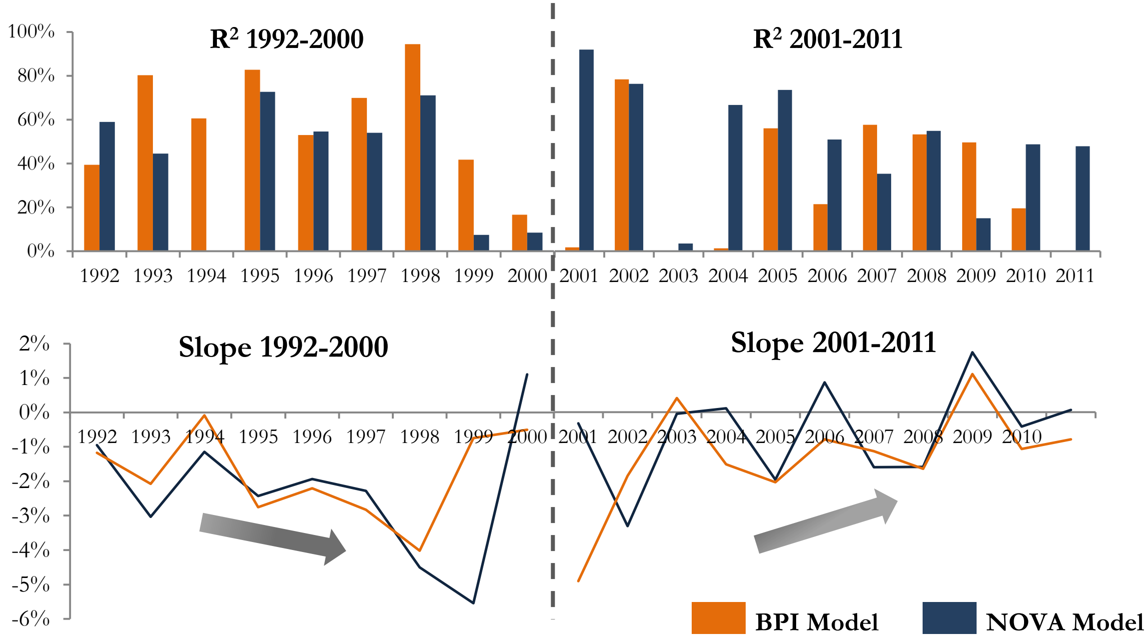
NOVA Model slightly dominates in R^2 and Slope

Indicators \ Vectors	BPI Model			NOVA Model		
	1992-2000	2001-2011	All Period	1992-2000	2001-2011	All Period
Average R^2	**59,9%**	30,9%	43,9%	41,4%	**51,4%**	46,9%
Slope	**-2,3%**	-0,7%	-1,4%	**-1,8%**	-1,6%	-1,7%

R^2 and Slope Evolution Breakdown

BPI Model is more accurate

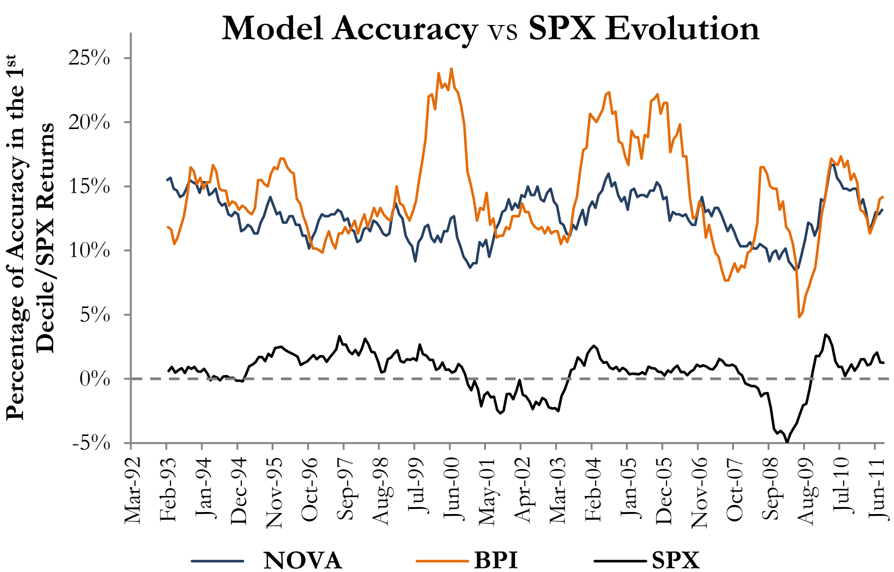
NOVA Model is more precise

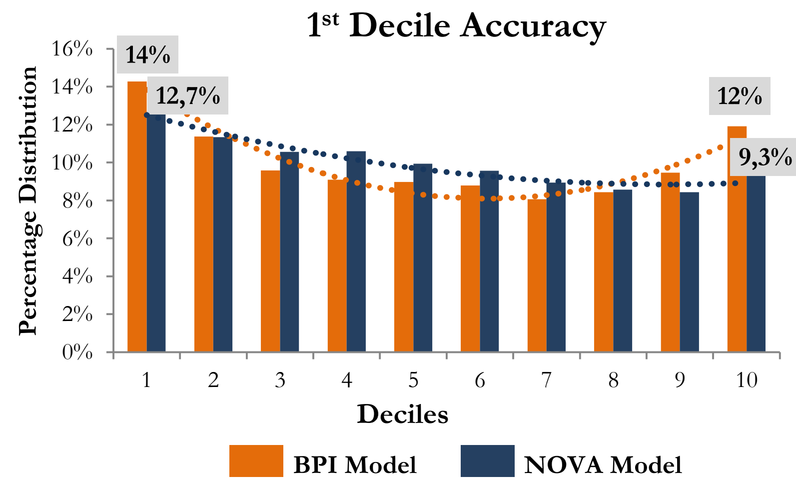


Conclusions:

- The R^2, which is the proportion of the variation in deciles' returns explained by the change in the deciles considered, is **greater in the NOVA Model** than it is for the BPI's.
- By doing the breakdown of the R^2 observed for the period 1992 to 2000 and from 2001 to 2011, one can clearly see that **before 2000** the **BPI model was more assertive** in allocating the stocks in the deciles than the NOVA Model. Conversely, from 2001 onwards our model outshines BPI's.
- The values obtained for the **average monthly slopes of both models are almost always negative**, meaning that the deciles' expected return decreases as we are moving from the 1st to the 10th decile.
- The slope of the deciles is **smaller for our model** than what it is for BPI's for **all period.**
- **Before 2000** the slopes obtained with the **BPI** model were **more negative** than the ones of our model. **After 2001** the **NOVA** Model displays **greater negative average s**lopes than BPI's mirroring that our model was better at establishing the differences in returns from the 1st to the 10th decile.

Rank Accuracy in the 1st Decile





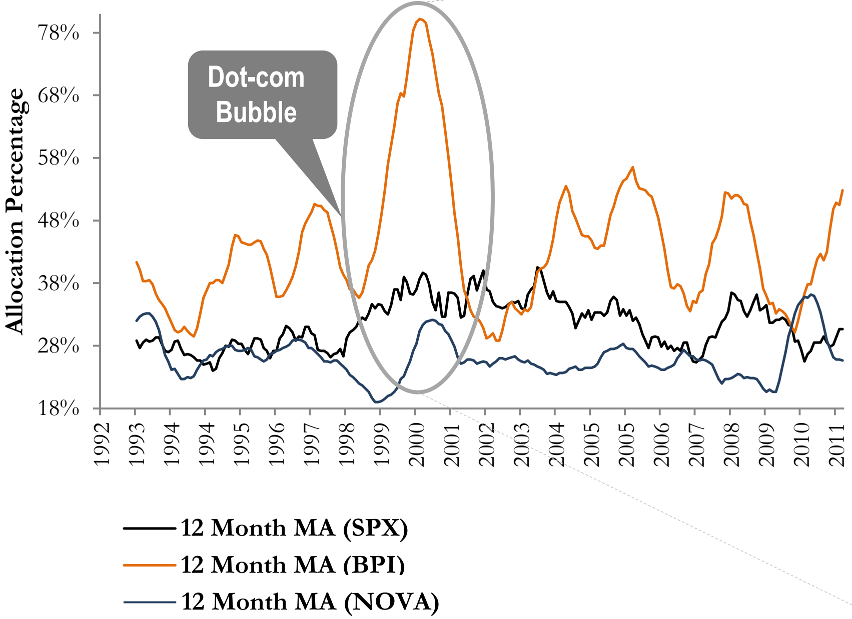
Explanation: *Accuracy is measured by the* ***percentage of stocks*** *indicated by the expected return model to be placed in the first decile,* ***that are indeed in the first decile*** *when returns are realized.*

- **BPI's** Multi-Factor Model **outperforms the NOVA Model** in terms of accuracy picking stocks for the first decile that rank in the top 50 in reality.
- The **accuracy increases with the market**: when the S&P 500 goes up, both models tend to be more precise picking the stocks on the first decile and vice-versa. This **correlation is stronger** with the **NOVA** Model.

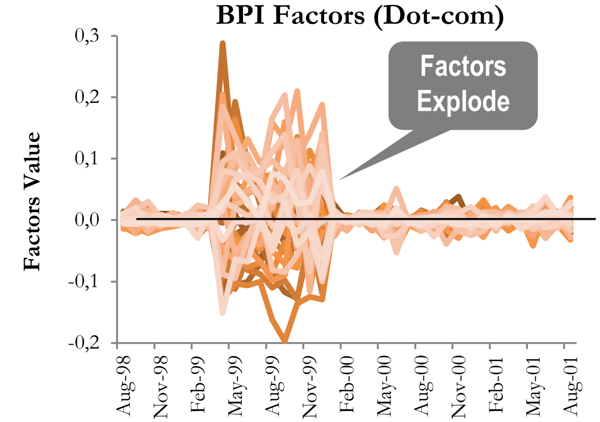
Explanation: *This graph shows the real distribution of the stocks across all deciles. Ex-ante, these stocks were pointed by the expected return models to be present in the 1st decile.*

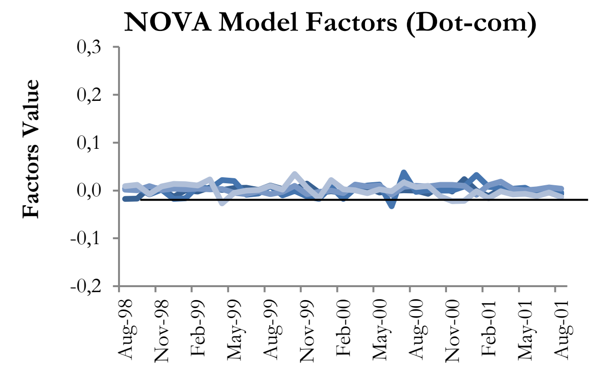
- 14% of the stocks that were indicated by the BPI Model to be in the 1st decile are actually there (vs. 12,7% NOVA).
- 12% of the stocks that were pointed by the BPI Model to be in the 1st decile are instead in the 10th decile (vs. 9,3% NOVA Model).
- **BPI's** multi-factor Model is **more precise** picking stocks that rank on the **first decile.**
- The **NOVA** Model **picks less stocks** that rank on the **last decile**.
- Depite choosing more stocks that really rank on the 1st decile, BPI also picks a significant percentage of those that rank on the last decile entailing a greater negative impact on the 1st decile realized return.

Maximum Industry Allocation



- BPI's Multi-Factor Model estimated factors are much **more instable** and extreme than the ones estimated by the NOVA Model.
- BPI's Model will tend to overestimate stock expected returns due to inclusion of several correlated characteristics.





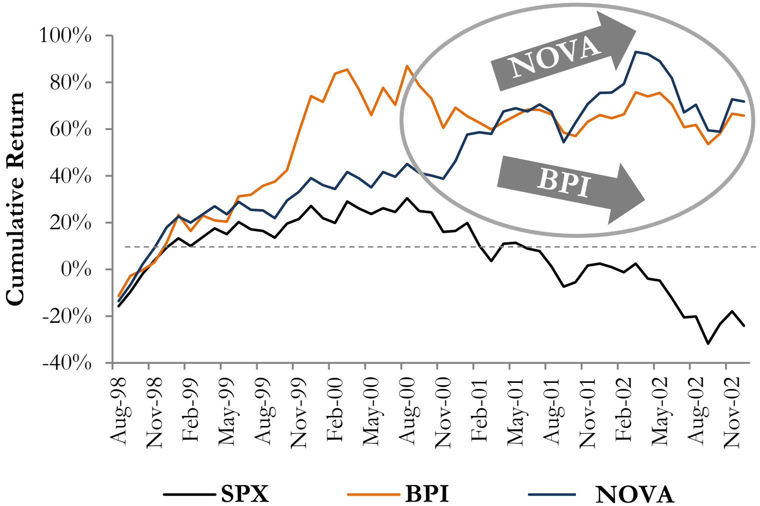
Close-up on the Dot-com Crisis

Conclusions:

- BPI's Model "overweights" certain industries along time. The stock allocation is usually **focused around 5 industries** (out of 10), with about 50% of the stocks picked in the first decile belonging to only one industry.
- The NOVA Model stands out for its **diversification** power. The stock allocation is typically concentrated around 9 industries, with about 25% of the stocks picked in the first decile belonging to only one industry.
- The S&P 500 usually has around 30% of the stocks of the first decile belonging to only one industry.

The NOVA Model has an outstanding Diversification Power

1st Decile Cumulative Return – Dot-com



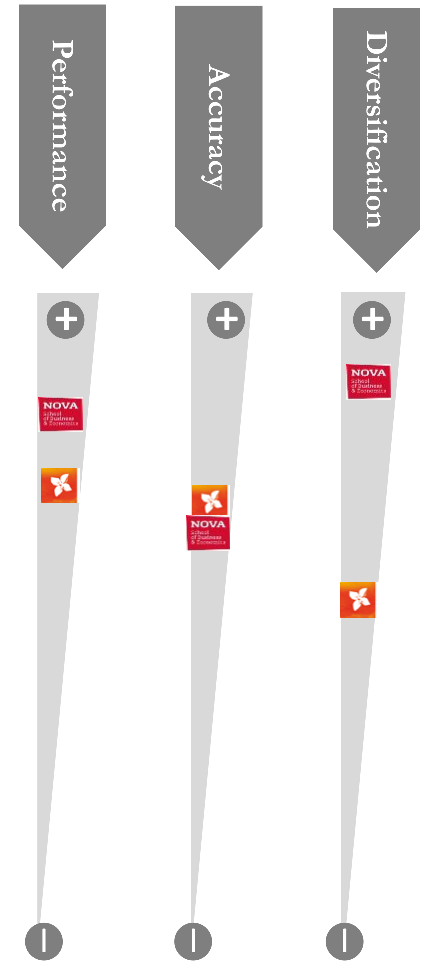
Average Allocation (Aug/98 - Aug/01)		
Industry	**BPI**	**NOVA**
Energy	3,5%	8,5%
Materials	1,0%	5,1%
Industry	1,6%	9,5%
Consumer Discretionary	5,5%	15,1%
Consumer Staples	2,2%	5,9%
Health Care	14,2%	12,5%
Financials	9,0%	12,3%
Info Techs	**46,1%**	**19,4%**
Utilities	9,7%	2,4%
Telecom	7,2%	9,3%
Total	**100%**	**100%**

Conclusions:

- The burst of the Dot-com bubble brings the cumulative return of the portfolio (formed with BPI's model picking) down with the trend of the market.
- A portfolio formed with the NOVA Model yields opposite returns upon the burst of the bubble, surpassing the value of the portfolio formed with BPI's model.
- The allocation of BPI's model overweighs the IT sector (table on the right), causing the model to fail stock picking predictions.

The effect of Diversification is Clear

Snapshot on Expected Return Models Results



Wrap-up on the Expected Return Model

Topics

- NOVA Model
- Optimal Estimation Period
- NOVA Model Performance
- NOVA Model Accuracy
- NOVA Model Diversification

Conclusions

After carrying out an holistic analysis on the documented anomalies/ economic meaning and variables' profitability/ statistical features, we concluded that the vector displaying the best combination of the indicators analyzed is the following:

Earnings Yield – Accruals-to-Assets (Ind. Std.) – Book Yield – Market Cap.

The estimation period that yields better results in terms of performance and ranking is **12 months**. Thus it is built upon **momentum**. By using this estimation period to estimate returns, both models, BPI's and ours, attain the lowest slope (good accuracy) and by far the best results in terms of profitability on a risk-ajusted basis.

For the period of 1992 to 2011, our model provides **bigger average realized returns** in the **first decile** than the model of BPI; moreover, it also has **less volatility**. The annualized Sharpe Ratio for the considered period will therefore be greater for the NOVA Model than what it is for BPI's (BPI: 0,66 vs. NOVA Model:0,94).

The NOVA Model model is **more accurate** than the BPI model. Despite the fact that in the sub-period 1992-2000 both the R^2 and the Slope pointed out for a slightly more precision of the BPI model; however, in the subsequent period (2001-2011) our model clearly surpasses the one of BPI at allocating stocks to deciles[1] .

The greatest **succeeding feature** accomplished by the NOVA Model when compared to BPI's expected return model is on the **diversification power** it possesses. Strengthening this statement is the fact that the BPI model usually concentrates its allocation on fewer industries than the NOVA Model model.

[1] The NOVA Model yields better results than BPI 75% of the periods ranging from 2001-2011both in terms of R^2 and Slope.

RISK MODEL

σ^2

1. Sample Covariance Matrix
2. Shrinkage - Ledoit and Wolf Approach
3. Multi-Factor Models
4. Bootstrapping
5. Out-of-Sample Volatility Prediction

Risk Estimation

Initial Remarks

Optimization's ultimate goal: Find the optimum portfolio

↓

Portfolio with the best combination of risk/return constrained to some restrictions

↓

Have to estimate the risk input for the optimization procedure

↓

Need to measure securities dispersion (σ) and comovement (ρ)

↓

Variance-covariance matrix gathers that information

↓

A proper covariance matrix needs to be estimated

↓

Variance-covariance Matrix estimation issues

1. Estimation Period
   Should not be a too long series of data to estimate risk as variance is time-varying, persistent and contercyclical. We will use the last 5 years of monthly data on the S&P500[1].

2. Number of assets should be smaller than the number of periods (N<T)
   When optimizing, the covariance matrix needs to be inverted but, in our case, we do not have enough periods to properly[2] estimate the covariance matrix. As N>T we may obtain misleading results as a consequence of estimation error linked to the existence of multicollinearity between the inverted covariance estimates.

[1] Due to the absence of data, the whole risk analysis will only employ 337 stocks present in the S&P 500 on August 2011.
[2] Portfolio optimization accuracy requires well conditioned risk inputs.

Sample Covariance Matrix (SCM)

The Base Case

What is it?

- The sample covariance is a square matrix whose *i, j* element is the covariance between pairs of the variables' observed values and whose *i, i* element is the variance of the observed values of one variable. It is easy to calculate and update.

$$S = \begin{pmatrix} s_1^2 & s_{12} & \cdots & s_{1p} \\ s_{21} & s_2^2 & \cdots & s_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ s_{p1} & s_{p2} & \cdots & s_p^2 \end{pmatrix}$$

Drawbacks

- The number of stocks for each covariance matrix should be at least equal to the number of periods; using montlhy data it is not possible to find such long history[1].

- Relies on historical covariances between individual stocks. Thus, in some periods, there may be high correlations between stocks as a result of the statistical relationship between them; however, these stocks may be from totally different industries ("statistical flaws").

a. High estimation error
b. Ill conditioned[2]

How to overcome/ minimize these problems?

Multi-Factor Models (MFM)

- Less estimation error than SCM as less parameters have to be estimated.
- Provide an holistic view on the breakdown of the risk exposures of a stock.
- Allow for standardization of the characteristics so than one attains less biasness on estimators, due to the presence of outliers and miscalculated data.
- Time-responsive to changes in the macro environment and also to individual firms' inherent features.

Shrinkage

This is a technique that aims to impose some structure on the sample covariance matrix, so as to better condition this matrix and therefore minimize the estimation error involved.

[1] Using lower frequency data would not be the solution considering that despite having more data, we will be exposed to computational and data gathering problems.
[2] The condition number of a matrix measures the sensitivity of the solution of a system of linear equations to errors in the data. If a matrix is ill conditioned the accuracy of the results from matrix inversion are going to be penalized.

Refining the Sample Covariance Matrix

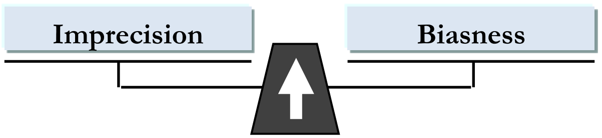
The main goal of this approach is to minimize the estimation error coming from inverting the covariance matrix. This method brings both **structure** and **better conditioning** to the covariance matrix.

The **shrinkage estimator** for the covariance matrix of stock return is defined as: $\hat{\boldsymbol{S}}^{Shrinkage} = \delta^* \boldsymbol{F} + (1-\delta^*)\boldsymbol{S}, \ \delta^* \in [0,1]$

where: δ^* is the optimal shrinkage intensity[1] *|* ***F*** *is the shrinkage target computed using stocks betas and sample variance of market returns |* ***S*** *is the sample covariance matrix*

F has a lot of **bias** coming from the structural assumption but **little estimation error**

S is an **unbiased estimator** however has **a lot of estimation error**



δ^* depends on the correlation between the estimation error on the ***S*** and on the shrinkage target (***F***). If there is a positive (negative) correlation, the benefit of combining the information is smaller (larger).

Statistical factor models and Ledoit-Wolf shrinkage are competing methods for estimating variance matrices of returns:

Pros

- ✓ Increased efficiency
- ✓ Well conditioned
- ✓ No need to specify an arbitrary multifactor structure

Cons

- The biasness of the shrinkage target may lead to inaccurate covariance estimates

[1] The analytical approach used to calculate the optimal shrinkage intensity is depicted in the Appendix 15. We obtained an optimal shrinkage intensity of **0,745**.

Description and Methodology

- **Fama-French (FF)**

The Fama-French model includes other factors aside from the market premium (used in the CAPM), particularly the firm size (SMB factor – small size minus big size firms) and book-to-market ratio (HML factor – high BTM minus low BTM):

$$r_{i,t} = \beta_{1,i}MKT_t + \beta_{2,i}SMB_t + \beta_{3,i}HML_t$$

- **FF plus Momentum (FFM)**

In this case we use the same model described above and add a momentum factor which tries to capture the premium associated with this "documented anomaly".

$$r_{i,t} = \beta_{1,i}MKT_t + \beta_{2,i}SMB_t + \beta_{3,i}HML_t + \beta_{4,i}MOM_t$$

Steps to estimate portfolio risk using FF/ FFM

0. **The Factors are available**
1. **Estimate Betas** *(Time Series Regression)*
2. **Estimate Factor Cov. Matrix** — **Σ**: matrix kxk where k is the number of factors
3. **Estimate Stock Cov. Matrix** — $\Omega = B^T \Sigma B + u$
4. **Estimate Portfolio Variance** — $\sigma^2_p = w^T \Omega w$

Where:

Σ is the covariance matrix of factors
B is the Betas vector
Ω is the stocks' covariance matrix
w is the vector of stocks' weights
u is the diagonal matrix of specific risk variances
$\sigma^2{}_p$ is the portfolio variance

Description and Methodology

- **Fama-McBeth (FMB)**

This approach uses firm specific characteristics (e.g. technical, value, solvency, operating profitability, industry, etc.). Based on BARRA studies we infer that industry is a crucial risk explanatory component; therefore, we applied it as an intrinsic factor instead of as a variable regressor:

$$r_{i,t} - r_{f,t} - r_{industry,t} = X_{1,t}F_{1,t} + X_{2,t}F_{2,t} + X_{3,}F_{3,t} + \dots$$

Selected Vector: **5 variables**

We tried different combinations of variables to estimate the characteristics' premiums, for instance:

Earnings Yield • Market Cap • MM1 • Accruals-to-Assets (Industry Standardized) • Average(Solvency)

Accruals-to-Assets • Debt-to-Equity • Earnings Yield • MM12 • Times Interest Earned • Average(Profitability)

Steps to estimate portfolio risk using FMB

0. **The Beta Characteristics are available**
1. **Estimate Factors** *(Cross-Sectional Regression)*
2. **Estimate Factor Cov. Matrix** — **Σ**: matrix kxk where k is the number of factors
3. **Estimate Stock Cov. Matrix** — $\boldsymbol{\Omega = X^T \Sigma X + u}$
4. **Estimate Portfolio Variance** — $\boldsymbol{\sigma^2{}_p = w^T \Omega w}$

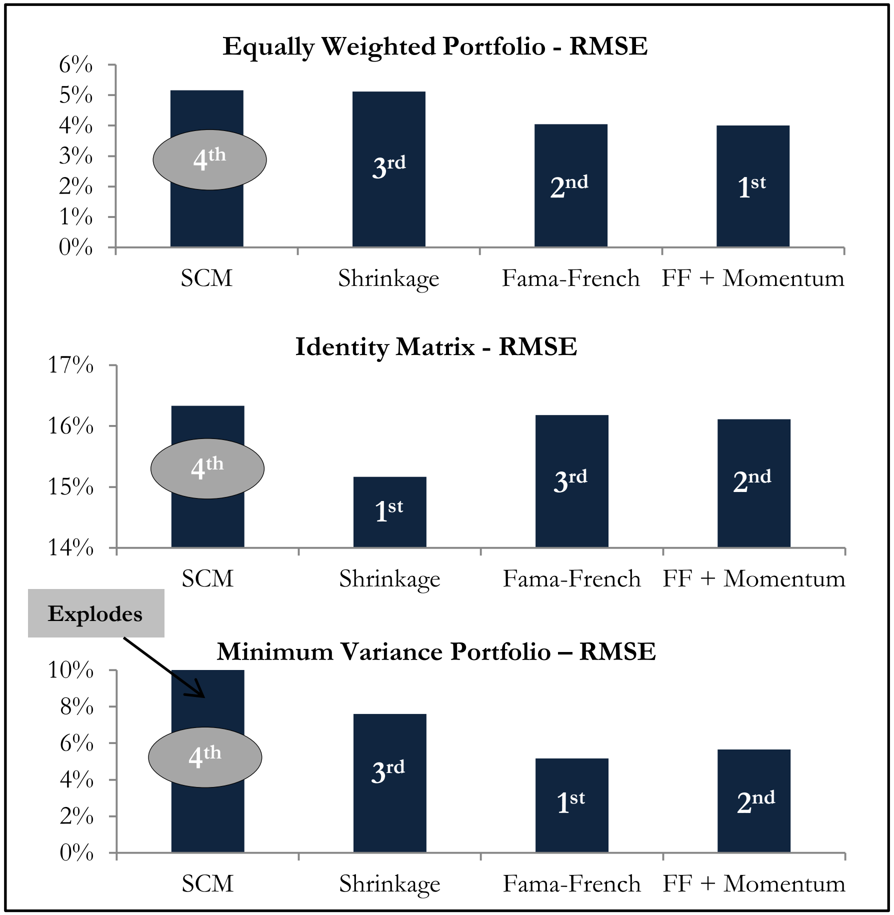
Where:

Σ is the covariance matrix of factors
X is the current months characteristics
Ω is the stocks' covariance matrix

w is the vector of stocks' weights
u is the diagonal matrix of specific risk variances
$\sigma^2{}_p$ is the portfolio variance

Accuracy test through resampling

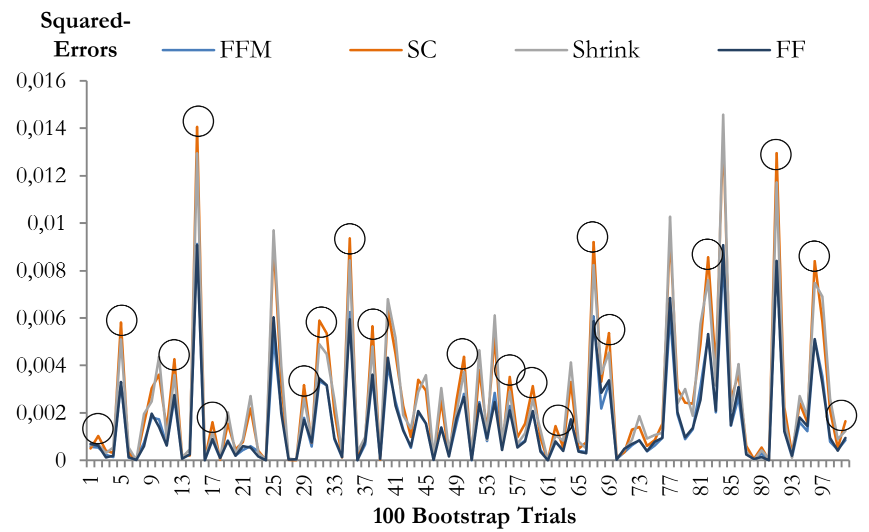
This method picks return observations from different periods to estimate the covariance matrix using shrinkage and multi-factor models. The precision of these estimates is tested against the realized volatility using basic market portfolios.



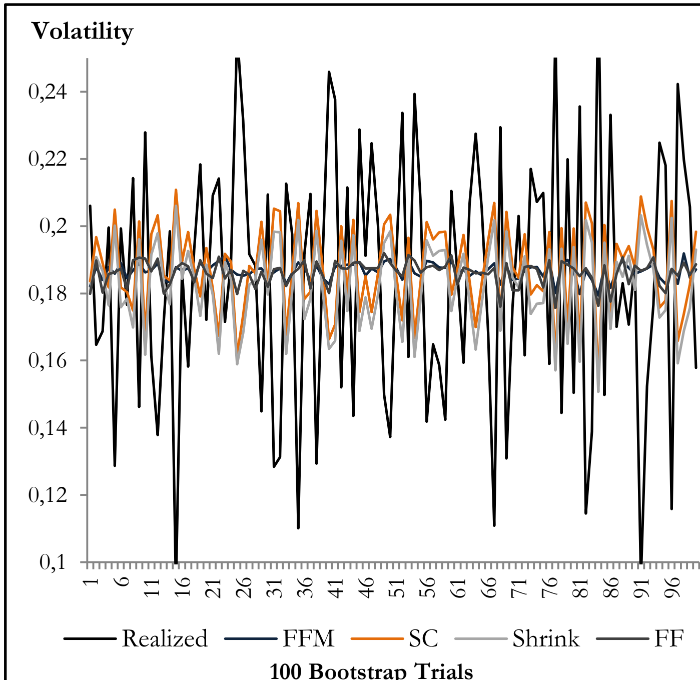
- As expected, the **Sample Covariance Matrix** as risk input yields the **poorest results** for all the portfolios built.
- When predicting volatility solely recurring to the identity matrix (**without accounting for diversification**) the method that displays best outcomes is **Shrinkage**. This is underpinning the idea that imposing structure to a covariance matrix entails sounder volatility estimates.
- Overall, the **Fama-French multifactor models** show better precision when estimating volatility as risk inputs.
- We **did not test the Fama McBeth** risk approach since the estimation process of this method does not allow for a Bootstrap test[1].

[1] This method only employs the previous month's characteristics to estimate the next month volatility.

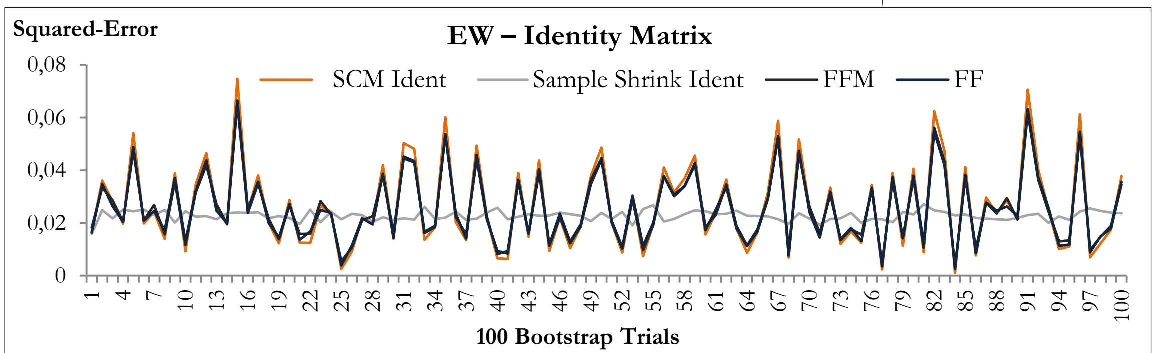
Equally Weighted Portfolio (EW) Risk Analysis



The **Sample Covariance Matrix** as risk input portrays considerably **high mean squared errors** when compared to the peers tested.



As we are estimating volatility truly based on historical observations **our estimations present a smoother path for all the trials in relation to the Realized Volatility**, with FF and FF + Momentum showing less volatility upon the set of portfolio volatility estimations.



By applying only the identity matrix to estimate risk for a EW portfolios we find that **Shrinkage** shows a very smooth path in terms of MSE estimates.

Robustness of Volatility Estimations

So as to assess the quality of our volatility forecasts, we established an in-sample period *(Jan 2005-Dez 2009)* and used the covariance matrices estimated to forecast the volatility of different portfolios out-of-sample *(Jan 2010-Aug2011)*.
A rolling window approach is used to estimate the variance of equally-weighted portfolios for each out-of-sample month.
The performance was compared to realized volatility for the same portfolios using the following realized volatility estimation equation: $\sigma^2{}_{Monthly} = \sqrt{\left(\frac{252}{N^r\ of\ Days\ in\ month\ t}\right) * \sum r^2{}_{Daily}}$

Model Rank (+) → (−)

Equally Weighted Portfolio

RMSE	FMB (5 Variables)	Shrinkage	FF	FFM	SCM
2010	7,10%	7,16%	7,20%	7,21%	7,23%
	1st	2nd	3rd	4th	**5th**
2011	12,28%	12,39%	12,46%	12,48%	12,50%
	1st	2nd	3rd	4th	**5th**
Δ2010-2011	*+5,18 pp*	*+5,23 pp*	*+5,26 pp*	*+5,27 pp*	*+5,27 pp*
OOS period	9,52%	9,60%	9,65%	9,67%	9,69%
	1st	2nd	3rd	4th	**5th**

Method Rank

- The results of **RMSE changed a lot from 2010 to 2011**, due to the **higher volatility in the market in 2011**.
- **Fama-MacBeth, Shrinkage and Fama-French** approaches are the risk models **that show less RMSE in 2010 and 2011**.
- The less sensitive model to the rise in markets' volatility is the **Fama-MacBeth**.

Robustness of Volatility Estimations

Consumer Discretionary Sector Equally Weighted Portfolio

RMSE	FMB (5 Variables)	Shrinkage	FF	FFM	SCM
2010	8,91%	8,96%	9,24%	9,58%	10,12%
	1st	2nd	3rd	2nd	**5th**
2011	13,67%	13,69%	13,99%	14,29%	14,79%
	1st	2nd	3rd	4th	**5th**
Δ2010-2011	*+4,76 pp*	*+4,73 pp*	*+4,76 pp*	*+4,71 pp*	*+4,67 pp*
OOS period	11,07%	11,10%	11,38%	11,69%	12,20%
	1st	2nd	3rd	4th	**5th**

Minimum Variance Portfolio (MVP)

RMSE	Shrinkage	FMB (5 Variables)	FFM	FF	SCM
2010	4,76%	4,13%	6,32%	6,33%	-
	2nd	**1st**	3rd	4th	**5th**
2011	3,12%	5,41%	5,34%	5,18%	-
	1st	4th	3rd	2nd	**5th**
Δ2010-2011	*-1,63 pp*	*+1,28 pp*	*-1,15 pp*	*-0,98 pp*	-
OOS period	4,18%	4,69%	5,94%	5,90%	-
	1st	2nd	4th	3rd	**5th**

- We tried other portfolios besides the simple equally weighted portfolio since when the number of stocks is large the portfolio volatility will converge to the average covariance and thus will yield similar value for the variance.
- Using a equally weighted portfolio for the Consumer Discretionary Sector we find that the best models to estimate risk are the Fama-McBeth and Shrinkage approaches. Using the MVP we see that the same two methods provide most accurate estimations.
- There are no results for the SCM in the MVP analysis since the presence of high estimation error entailed extreme outcomes when inverting this matrix to calculate the MVP.
- In both portfolios the SCM is the worse risk estimator.

Method Rank

Wrap-up on Risk Models

Topics

SCM is the poorest risk estimator

Fama-MacBeth out-of-sample rubostness

Risk models performance in mercurial markets

Fit between Risk, Return and Optimization?

Conclusions

Both the bootstrap method and out-of-sample prediction corroborate our initial guess that the **Sample Covariance Matrix alone is a bad risk estimat**or. Using the Root Mean Squared Error as measure for exactitude, these analyses placed the SCM as the worst risk model in all types of portfolios tested.

Among all the techniques used, the **Fama-MacBeth model using Accruals (Industry Standardized), Earnings Yield, Market Cap, Momentum 1M, Average Solvency Variables** is the most exact method out-of-sample. Moreover, it is also the least responsive method to sharp increases in volatility.

In periods of great volatility in the market, **all risk estimator models provide poor outcomes**. This phenomenon is linked to the fact that these models rely considerably on historical stocks' behavior.

As a final comment, we must stress that it is **not possible to look for the Risk Model on a standalone basis**, that is, without taking into account the fit between this model, the Expected Return Model and the Optimization Model. Hence one needs to put all models together in the optimization process.

OPTIMIZATION MODEL

$$\max_{\{w_i\}} \frac{E(r) - r_f}{\sigma}$$

1. Unconstrained Optimization – Constraints Role
2. Optimization Models – Theoretical Grounds
3. Optimization Models – Performance and Statistics
4. Best Performers’ Analysis
5. Optimal Portfolio Allocation Analysis

Optimization Framework

Portfolio Choice Process

Now that we have studied ways to expand the capabilities of the Expected Return Model and Risk Model which are inputs for the optimization process, we are ready to put all the pieces together and reach a final solution for the portfolio choice problem.

Optimization Inputs

Expected Return Vector
- NOVA Model
- BPI Model

Covariance Matrix
- Sample Covariance Matrix
- Shrinkage
- Fama-French
- Fama-French + Momentum
- Fama-McBeth

Objective Function
- Mean-Variance
- Max Expected Return
- Mean-Variance Tracking Error

Constraints
- No short sale allowed
- Maximum weight per stock
- Sector deviation from S&P

Optimization Models
- Markowitz
- Black-Litterman
- Mean-Variance - Tracking Error
- Genetic Algorithm

Optimal Portfolio Choice

Our goal will be to obtain the portfolio weights that maximize the portfolio returns subjected to a certain level of restrictions.

Important Note: We know that the best solutions found for the Risk and Expected Return Models to improve the inputs' performance **do not necessarily entail better optimization outcomes** when putting everything together in the optimization procedure. The reason for this is that there may be some kind of **incompatibility between the inputs and constraints imposed** in the optimization leading to poorer results.

1 Unconstrained Optimization

Constraints Role[1]

Indicator \ Inputs	Markowitz					Black-Litterman				
	SCM	Shrink.	FF	FFM	FMB	SCM	Shrink.	FF	FFM	FMB
Annualized Return	8,1E+16	2280%	2871%	3030%	4688%	2,5E+06	289%	362%	402%	139%
Portfolio Volatility	5,0E+16	1318%	2704%	2835%	4059%	1,6E+06	168%	342%	275%	137%
Max Return	5,E+16	1268%	1853%	2321%	2135%	1,5E+06	159%	233%	224%	93%
Min Drawdown	-9,E+15	-523%	-1734%	-1548%	-2019%	-5E+05	-69%	-221%	-128%	-62%
Max Weight	8,5E+16	1748%	1939%	2030%	2243%	4,7E+05	221%	244%	278%	209%
Min Weight	-7,9E+16	-791%	-959%	-1123%	-1224%	-4,8E+05	-98%	-119%	-118%	-78%
∑ (W<0)	-2,3E+18	-28747%	-28985%	-31168%	-34675%	-1,5E+07	-3557%	-3599%	-3740%	-3487%
Tracking Error	5,0E+16	1316%	2698%	2830%	4061%	4,7E+05	48%	97%	78%	38%
Sharpe Ratio	1,63	1,73	1,06	1,07	1,16	1,57	1,73	1,06	1,46	1,01
Information Ratio	1,63	1,73	1,06	1,07	1,15	1,57	1,71	1,06	1,45	1,02

Pros of setting constraints:

- Extreme weights on stocks are avoided
- Enhanced diversification of the optimal portfolio
- Greater stability of portfolio weights
- Tracking error reduction
- Reasonable results

Risk Model	Covariance Conditioning Level	Evaluation
Sample Covariance Matrix	1,40E+24	○
Shrinkage	2,20E+07	◑
Fama-French	1,95E+07	◑
Fama-French plus Momentum	1,94E+07	◑
Fama-MacBeth	1,39E+05	◕

As N > T, we will not be able to attain a well-conditioned cov. Matrix.

Conclusions:

- Results for SCM are, clearly, the most unstable and irrational.
- The stability of the outcomes is a direct consequence of the different covariance matrices, as an inversion of those matrices is needed to produce a solution to the weight allocation.
- The quality of the inversion is linked to the conditioning level of the variables, as ill conditioned covariance matrices are more likely to produce inaccurate results.
- Unconstrained optimization is of prohibitive use, as, for example, no one could be faced against a monthly return of -9E+15, with extreme weight allocation to the stocks belonging to a portfolio.
- As BL starts with the implied weights in the market, the outcomes are much more stable.

Note: E=10^x (e.g. -9E+15 = -9*10^15)

[1]In this slide we want to stress the relevance of using constraints in portfolio optimization routines. To do this we show results from Markowitz and Black-Litterman models that we will study later.

Markowitz Groundbreaking Contribution

The Markowitz problem (1952) is a typical portfolio optimization process in which the investor picks stocks in a way that minimizes the portfolio risk for a certain level of expected return.

Intuition

- Markowitz claims that a rational investor will make its portfolio choice by **maximizing its expected utility for a given level of risk**. By solving the mean-variance optimization problem one will obtain the efficient frontier which represents the best allocation of wealth incorporating investor's preferences as well as their expectation of return and risk.

Classic Mean-Variance Problem

$$\min_{\{w\}} w^T \Sigma w$$ *Portfolio Variance*

$$\text{s.t. } w^T \mu = \bar{\mu}$$ *Target Portfolio Expected Return*

$$w^T \iota = \mathbf{1}$$ *Weights Adding-up Constraint* (without unlimited borrowing)

Drawbacks (without constraints)

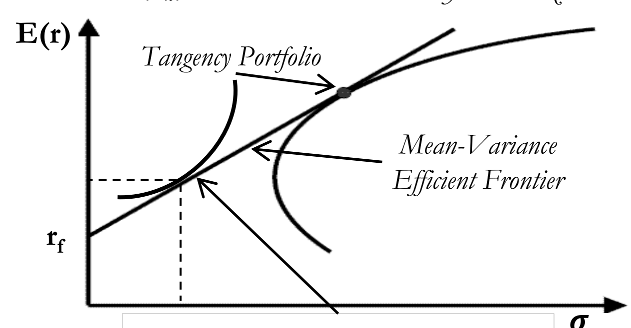
1. Extreme weights
2. Unsteady input sensitivity
3. Estimation error maximization
4. Unintuitive results

Optimization Process

1st step: $\max_{\{w_k\}} \frac{E(r_p)-r_f}{\sigma}$ *Sharpe Ratio Maximization*

Obtain the tangency portfolio and combine it with the risk-free taking into account the investor's level of risk aversion (we assumed γ=4)

2nd step: $\max_{\{w_k\}} E(r_p) - \frac{\gamma}{2}\sigma_p^2$ *Mean-Variance Utility Maximization*



Optimal Portfolio
(will depend on the risk aversion coefficient)

Minimizing the Volatility of Tracking Error

Richard Roll (1992) among others put forward an alternative methodology rooted on the Mean-Variance paradigm of Markowitz. The basic goal of this technique is to attain a certain return performance above the benchmark whilst minimizing the tracking error volatility.

Intuition

- Portfolio managers are judged by their relative performance against a specified benchmark. This method is well suited to **structure active management conduct** as it allows the portfolio managers to set performance objectives and evaluate them against indexing strategies.

Mean-Variance Tracking Error Problem

$$\text{Min } x^T \Sigma x \quad \text{— } \textit{Tracking Error Variance Minimization}$$

$$\text{s.t. } x^T 1 = 0 \quad \text{— } \textit{Self-Financing Constraint}$$

$$x^T R = G \quad \text{— } \textit{Target Expected Performance}$$

where:

Σ: Variance-Covariance Matrix

x: Vector of weights' difference between the managed portfolio and the benchmark $(w_p - w_b)$

R: Expected Return Vector

G: Gain over the benchmark's return

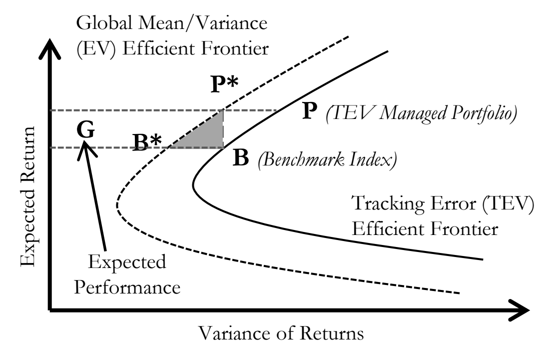
Optimization Process

$$\underset{\{w_k\}}{\text{Max}}\ w_p R - \gamma TE^2 \quad \text{— } \textit{Expected Return Maximization with Tracking Error penalty}$$

$$\text{where } TE^2 = x^T \Sigma x$$

Additionally, we also take into account the investor's level of risk aversion to the level of tracking error variance (we assumed γ=4).

Problem's Illustration



The figure shows an inefficient benchmark portfolio (**B**), which is generally what one faces in reality. Solving the mean-variance problem to find an optimal portfolio in the efficient frontier (**P***) with **G** expected performance above **B**, one is deviating from the benchmark. By minimizing the tracking error volatility one is sacrificing risk-return efficient combinations to obtain a portfolio **P** on the TEV.

Adding Views to Expected Returns

The Black-Litterman model (1990) starts by establishing portfolio weights equal to the equilibrium asset allocation; then changes them by incorporating the Manager's opinion with a certain confidence level. Finally, this model computes the desired mean-variance efficient allocation.

Intuition behind Black-Litterman	▪ The Black-Litterman Model depends on **investor's views** on expected returns to produce mean-variance efficient portfolios. This method relies on the **market efficiency** hypothesis and therefore any investor allocation should be proportional to the market values of the assets available in a benchmark. To this initial approach each investor adds is unique alpha views to define the final portfolio allocation.
Types of Investor Views	▪ **Absolute View** (e.g. "the Financial Sector will have an absolute excess return of X%") ▪ **Relative View** (e.g. "the Healthcare Sector and Utilities Sector will outperform the market by Y%")
Formula Explanation	$E(r)=[(\tau\Sigma)^{-1} + P'\Omega^{-1} P]^{-1}[(\tau\Sigma)^{-1} \Pi + P' \Omega^{-1} Q]$ where: - E(r) is the new Combined Return Vector (Nx1) - τ is a scalar - Σ is the covariance matrix of excess returns (NxN) - P is a matrix that identifies the assets involved in the views(KxN) - Ω is a diagonal covariance matrix of error terms from the expressed views representing the uncertainty in each view(KxK) - Π is the Implied Equilibrium Return Vector(Nx1) - Q is the View Vector(Kx1)
Advantages	▪ More diversified portfolios (vs highly concentrated portfolios) ▪ Less input sensitivity (as it is based on investors insights) ▪ Less estimation error (spreads the errors throughout expected returns)

A technique that mirrors the process of Natural Selection[1]

This optimization procedure intends to generate solutions based on the evolution through selection of the fittest individuals, in our case, portfolios. The great benefit of this stochastic process is that it can scan a vast range of solutions of a complex problem. A major drawback of this process is the instability of results as it can get stuck in a local optimum.

Initialization

- **Initial population - 10 portfolios:** the optimal portfolio from Markowitz, one equally weighted portfolio, one value weighted portfolio and 7 random portfolios with a maximum weight per stock of 2%.

1 Reproduction

- Do **combinations of the 10 portfolios in groups of 2** (45 portfolios' combinations in total). We employ the **weights average** to give birth to a new generation.
Another example would be to pick 50% of fathers' weights and 50% of mothers' weights.

2 Selection *(based on a Score Function)*

- We choose the **10 portfolios** that have the best results in a **Score Function**. We use the following score functions: 1 – **Mean-variance** function; 2 - **Mean-variance** function with **penalty for sector deviation** (10% per percentage points if the portfolio deviates more than 5% from the S&P 500); 3 - **Maximize return.**

3 Mutation

- We arbitrarily **pick one of the 10 chosen** combined portfolios and apply a random "mutation".
- A "mutation" is a **change in the weights of the chosen portfolio.** We do 3 mutations: 1 – Randomly change 5 weights of that portfolio; 2 – Substitute the chosen vector by a new random vector; 3 - Insert a view of the Expected Return model.

4 Gathering and Regenerate

- **Gather** the "mutated" portfolios with the other 9 combined portfolios.
- **Feed again the genetic algorithm** with these 10 transformed portfolios.

Termination

- After a few thousand iterations the Score Function value starts to stabilize.
- **Termination condition:** if in 1000 sequential iterations the Score Function value does rise by more than 0,001 the process stops and a final solution is reached

[1]As BPI did not disclose its genetic algorithm model to serve as the base case for our optimization analysis, we created our own algorithm to grasp what could be the pros and cons of this approach.

3 Optimization Models: Performance and Statistics

Markowitz | Mean-Variance Tracking Error | Black-Litterman | Genetic Algorithm

Summary Table

Out-of-sample period: Jan 2010 to August 2011 - Annual data

Indicator / Inputs	S&P	1st Decile[1]		BPI Model					NOVA Model				
		BPI	NOVA	SCM	Shrinkage	FF	FFM	FMB	SCM	Shrinkage	FF	FFM	FMB
Annualized Return	6,60%	9,30%	**17,36%**	8,43%	**7,09%**	8,02%	7,79%	9,51%	12,65%	12,58%	11,49%	11,58%	12,27%
Active Return	--	2,70%	**10,76%**	1,83%	**0,49%**	1,42%	1,19%	2,91%	6,05%	5,98%	4,89%	4,98%	5,67%
Portfolio Volatility	16,23%	21,24%	**21,39%**	11,50%	12,20%	10,73%	10,85%	16,40%	10,31%	**9,95%**	10,10%	10,10%	13,59%
Max Return	8,76%	11,81%	**12,51%**	6,50%	7,12%	6,12%	6,29%	8,80%	6,52%	5,87%	6,32%	6,42%	7,64%
Max Drawdown	-8,20%	**-10,98%**	-9,20%	-5,64%	-5,88%	-5,72%	-5,71%	-7,95%	-5,32%	-5,07%	-5,42%	-5,27%	-5,63%
Portfolio Beta	--	1,21	**1,25**	0,67	0,72	0,62	0,62	0,97	0,59	0,58	**0,57**	0,58	0,80
Tracking Error	--	8,87%	8,40%	6,89%	5,96%	7,62%	7,60%	**4,96%**	8,27%	8,23%	8,47%	8,31%	6,09%
Sharpe Ratio	0,41	0,44	0,81	0,73	0,58	0,75	0,72	0,58	1,23	1,26	1,14	1,15	0,90
Information Ratio	--	0,30	1,28	0,27	0,08	0,19	0,16	0,59	0,73	0,73	0,58	0,60	0,93

Findings:

- The annualized returns yielded by the expected return models only (1st Decile in the Table) are superlative in relation to the results of Markowitz portfolios; nonetheless, the effect of adding a risk input is clear as the **volatilities from the Markowitz portfolios are roughly half of those from the expected return models alone.** Following the same line of reasoning, the portfolios obtained using the Markowitz procedure display less extreme Maximum Return and Maximum Drawdown than the 1st Decile portfolios.
- Concerning systematic risk, measured by the Beta, the **least market correlated portfolios** are those that are built using **the NOVA Model** in the optimization process less correlated than the ones that use the BPI vector; lastly, the 1st Decile equally-weighted portfolios that have betas around 1,2.
- **BPI portfolios depict lower Tracking Errors (TE) than NOVA portfolios** (the lowest TE is attained using the **FMB risk model**).
- The NOVA Model obtains **better results in terms of Sharpe Ratio and Information Ratio** than the BPI Model, regardless of the risk input used.

[1]This is an equally-weighted portfolio composed by the stocks that are placed in the first decile by the expected return models (NOVA and BPI).
Abbreviations: SCM – Sample Covariance Matrix/ FF - Fama-French risk model/ FFM - Fama-French plus Momentum risk model/ FMB – Fama-McBeth risk model

Summary Table

Out-of-sample period: Jan 2010 to August 2011 - Annual data

Indicator \ Inputs	S&P	1st Decile[1]		BPI Model					NOVA Model				
		BPI	NOVA	SCM	Shrinkage	FF	FFM	FMB	SCM	Shrinkage	FF	FFM	FMB
Annualized Return	6,60%	9,30%	**17,36%**	7,50%	7,81%	7,56%	7,36%	9,67%	15,81%	**17,75%**	17,06%	16,77%	16,67%
Active Return	--	2,70%	10,76%	0,90%	1,21%	0,96%	0,76%	3,07%	9,21%	11,15%	10,46%	10,17%	10,07%
Portfolio Volatility	16,23%	21,24%	21,39%	18,62%	18,93%	18,94%	18,97%	21,05%	17,70%	18,18%	18,09%	17,99%	20,84%
Max Return	8,76%	11,81%	12,51%	10,20%	10,76%	10,67%	10,64%	11,66%	10,29%	10,70%	10,31%	10,32%	11,41%
Max Drawdown	-8,20%	-10,98%	-9,20%	-8,26%	-8,44%	-8,56%	-8,65%	-10,80%	-7,30%	-6,76%	-7,00%	-6,94%	-9,32%
Portfolio Beta	--	1,21	1,25	1,13	1,15	1,15	1,15	**1,21**	1,06	1,09	1,09	1,08	**1,23**
Tracking Error	--	8,87%	8,40%	**3,71%**	**4,01%**	**4,04%**	**4,06%**	**8,43%**	5,29%	5,56%	5,36%	5,21%	7,83%
Sharpe Ratio	0,41	0,44	0,81	0,40	0,41	0,40	0,39	0,46	0,89	0,98	0,94	0,93	0,80
Information Ratio	--	0,30	1,28	0,24	0,30	0,24	0,19	0,36	**1,74**	**2,01**	**1,95**	**1,95**	**1,29**

Findings:

- The portfolios that use the **NOVA Model** as input for the Mean-Variance Tracking Error **have annualized returns approximately two times bigger** than the ones that use the **BPI Model** as expected return vector (except in the case of FMB risk model); using the Shrinkage method as risk input and the NOVA Model we reached an annualized return even greater than the 1st Decile equally weighted portfolio using our vector (17,75 vs. 17,36%).
- Despite having greater annualized returns, the **portfolios using the NOVA Model as input have smaller volatilities than BPI portfolios** irrespective of the risk input used. The logical implication of this finding coupled with the previous one is that **Sharpe Ratios will be greater for NOVA portfolios**.
- All the **portfolio Betas are greater than 1** meaning that this optimization model produces **cyclical portfolios**; regardless of the expected return model used the portfolios with the greatest Betas are those that employ the FMB model as risk input.
- Despite having **greater TEs** (BPI aver. TE: 4,85% vs. NOVA aver. TE 5,85%), the **NOVA Model shows much bigger Information Ratios** as a consequence of a **better stock selection than BPI** which is translated in greater annualized returns that will entail a **bigger alpha**.

[1]This is an equally-weighted portfolio composed by the stocks that are placed in the first decile by the expected return models (NOVA and BPI).

Summary Table

Out-of-sample period: Jan 2010 to August 2011 - Annual data

Indicator \ Inputs	S&P	1st Decile[1]		BPI Model					NOVA Model				
		BPI	NOVA	SCM	Shrinkage	FF	FFM	FMB	SCM	Shrinkage	FF	FFM	FMB
Annualized Return	6,60%	9,30%	17,36%	10,39%	10,39%	9,17%	8,89%	9,59%	15,32%	16,38%	16,74%	15,11%	**17,22%**
Active Return	--	2,70%	10,76%	3,79%	3,79%	2,57%	2,29%	2,99%	8,71%	9,78%	10,14%	8,51%	10,62%
Portfolio Volatility	16,23%	21,24%	21,39%	17,13%	17,13%	17,24%	16,66%	20,45%	14,51%	13,87%	14,32%	13,38%	17,97%
Max Return	8,76%	11,81%	**12,51%**	9,06%	9,06%	8,67%	8,64%	10,49%	8,43%	7,75%	8,64%	7,83%	9,14%
Max Drawdown	-8,20%	**-10,98%**	-9,20%	-7,99%	-7,99%	-8,16%	-7,89%	-10,49%	-6,09%	-6,30%	-6,37%	-6,16%	-8,11%
Portfolio Beta	--	1,21	1,25	0,99	0,99	0,99	0,96	1,17	0,86	0,82	0,85	0,79	1,06
Tracking Error	--	8,87%	8,40%	6,25%	6,25%	6,47%	6,29%	8,26%	5,62%	5,84%	5,73%	6,19%	6,44%
Sharpe Ratio	0,41	0,44	0,81	0,61	0,61	0,53	0,53	0,47	1,06	1,18	1,17	1,13	0,96
Information Ratio	--	0,30	1,28	0,61	0,61	0,40	0,36	0,36	**1,55**	**1,67**	**1,77**	**1,37**	**1,65**

Findings:

- Using the Black-Litterman, **BPI portfolios' annualized returns are lower than those obtained by NOVA portfolios**. The highest annualized return across all optimized portfolios is reached (17,22%) using the **Fama-McBeth risk model** combined with the NOVA Model as optimization inputs.
- The **Sharpe Ratios of the NOVA portfolios are bigger than those which utilize BPI's expected return vector** due to greater returns (as mentioned above) and lower volatilities; moreover, **all the volatilities from BPI portfolios are greater than the S&P 500 volatility**.
- **The differentials between maximum return and minimum drawdown** are **greatest for the 1st Decile** equally weighted portfolios, than for the BPI portfolios and **the lowest discrepancies are verified in the NOVA model optimized portfolios**. Underlying this conclusion is the **augmented risk** of both **BPI and 1st Decile portfolios** when compared to the **NOVA Model** (Betas corroborate this: **BPI's aver. Beta: 1,02 vs. NOVA's aver. Beta: 0,87**).
- **NOVA portfolios' TE is lower than the BPI portfolios** regardless of the risk input used; this fact, plus **greater active returns of NOVA portfolios** in relation to portfolios using BPI vector in the Black-Litterman optimization results in much **bigger Information Ratios for NOVA portfolios**.

[1]This is an equally-weighted portfolio composed by the stocks that are placed in the first decile by the expected return models (NOVA and BPI).

Summary Table

Out-of-sample period: Jan 2010 to August 2011 - Annual data

Indicator \ Inputs	S&P	1st Decile[1]		BPI Model					NOVA Model				
		BPI	NOVA	SCM	Shrinkage	FF	FFM	FMB	SCM	Shrinkage	FF	FFM	FMB
Annualized Return	6,60%	9,30%	17,36%	10,63%	9,18%	10,11%	10,73%	10,38%	**18,64%**	**19,38%**	**18,22%**	**18,32%**	**18,80%**
Active Return	--	2,70%	10,76%	4,03%	2,58%	3,50%	4,12%	3,78%	12,04%	12,92%	11,62%	11,72%	12,20%
Portfolio Volatility	16,23%	21,24%	21,39%	**19,69%**	**20,29%**	**19,70%**	**20,13%**	**20,96%**	17,03%	16,78%	16,88%	17,34%	20,25%
Max Return	8,76%	11,81%	12,51%	10,90%	11,38%	11,03%	11,30%	11,62%	10,24%	10,13%	10,61%	10,54%	11,69%
Max Drawdown	-8,20%	-10,98%	-9,20%	-10,25%	-10,57%	-10,16%	-10,11%	-10,90%	-7,25%	-7,21%	-7,60%	-7,74%	-8,25%
Portfolio Beta	--	1,21	1,25	1,11	1,15	1,12	1,14	1,18	**0,98**	**0,98**	**0,98**	**0,99**	1,19
Tracking Error	--	8,87%	8,40%	8,26%	8,57%	8,11%	8,56%	9,22%	7,05%	6,78%	6,86%	7,35%	7,72%
Sharpe Ratio	0,41	0,44	0,81	0,54	0,45	0,51	0,53	0,50	**1,09**	**1,15**	**1,08**	**1,06**	**0,93**
Information Ratio	--	0,99	1,28	0,49	0,30	0,43	0,48	0,41	**1,71**	**1,88**	**1,69**	**1,60**	**1,58**

Findings:

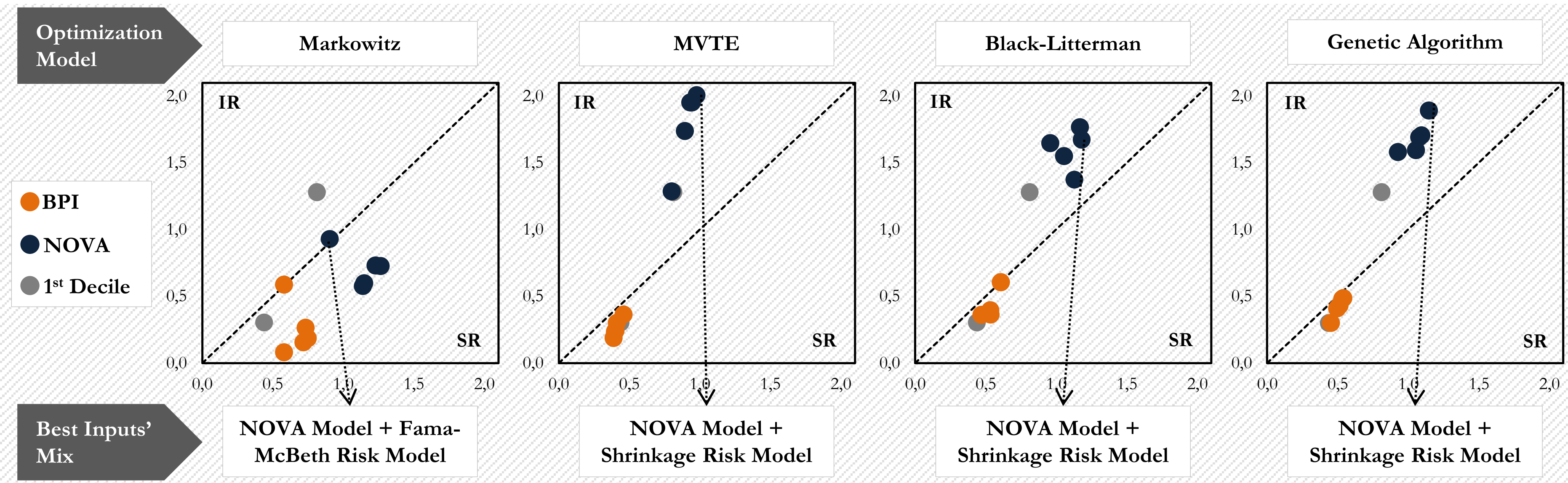
- The Genetic Algorithm (based on a mean-variance objective function) yields portfolios with **sound annualized returns**, especially using the NOVA Model as expected return input.
- The **volatility magnitude is high overall** (BPI portfolios have volatilities around 20% and NOVA 17%).
- Despite the high portfolio volatility, **the NOVA Model portf**olios are still able to attain **exceptional Sharpe Ratios (around 1)** due to the very positive contribution of the annualized return. Dissimilarly, the BPI portfolios' Sharpe Ratios are much lower as the volatility is huge and there was no correspondent rise in returns to compensate.
- All portfolio **Betas are roughly 1** or a little higher. The **NOVA portfolios** are **more conservative** than BPI's as their betas are smaller (except for FMB).
- The **Information Ratios (IR) are very decent for NOVA portfolios** as a consequence of the significant rise in the active return. The **IRs are way more smaller for the BPI portfolios** due to much lower active returns and higher TE.

[1]This is an equally-weighted portfolio composed by the stocks that are placed in the first decile by the expected return models (NOVA and BPI).

4 Best Performers' Analysis

Combinations of Performance Measures for each Optimization

The main idea of this analysis is to give a flavor about the Manager's performance that might be evaluated by the Information Ratio. This can be an issue to certain Clients as they may prefer the Sharpe Ratio as metric for portfolio's risk-adjusted profitability measure.



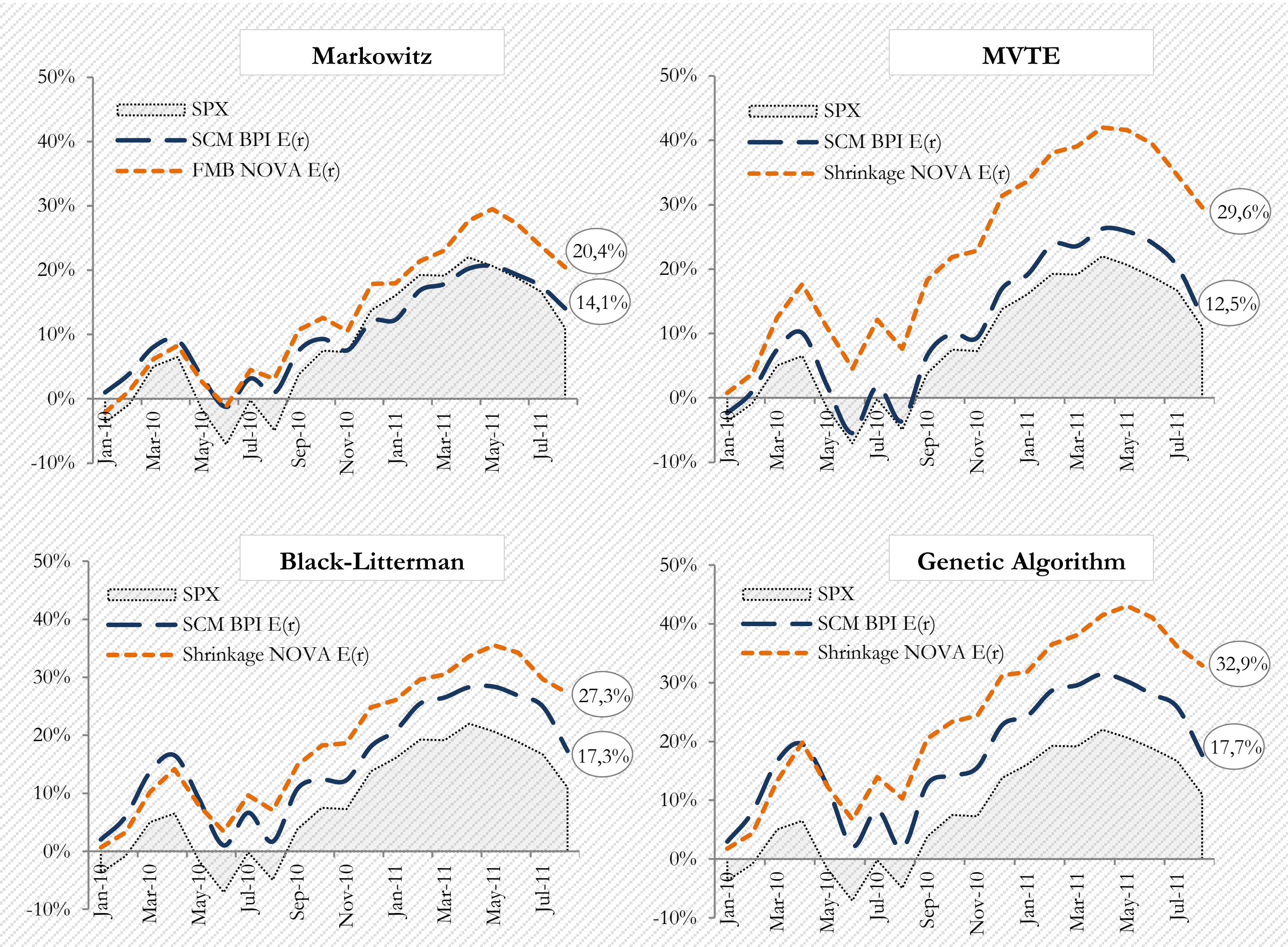
Findings:

- Using the Markowitz procedure, the best combination between optimization inputs in terms of SR and IR is the NOVA Model plus the Fama-McBeth Risk Model. In the remaining models analyzed, the best combination of inputs is always the NOVA Model pooled with the Shrinkage Model.
- As expected the MVTE objective function penalizes the SR in relation to the Markowitz approach and at the same time shows a clear shift towards greater Information Ratios.
- Black-Litterman optimization procedure portrays good combinations between IR and SR. The IR are quite significant as this model is grounded on the efficient market implied returns.
- By trying to mimic the Markowitz approach, this model presents very good results leveraged on both IR and SR. The IR benefits from high alphas, instead of low tracking error levels.

Abbreviations: SR – Sharpe Ratio/ IR – Information Ratio / MVTE – Mean Variance Tracking Error

4 Best Performers' Analysis

Portfolio Cumulative Returns OOS – 2010/11 – Best Inputs' Combination



Conclusions

- Overall, optimization procedures using both BPI and NOVA's expected return model inputs, yield cumulative returns above the S&P 500 Index, using different risk inputs.
- It is clear that, cumulative returns are bigger, using NOVA's expected return model and both the Fama-MacBeth and Shrinkage covariance as inputs, when compared to BPI's base case (BPI's expected return model and Sample covariance matrix).
- Performance differences are quite significant between BPI and NOVA's inputs' combinations, especially using the mean-variance tracking error and the genetic algorithm optimization procedures (these differences can go up to 17% and 15% in cumulative return, respectively).

Glimpse on Optimization Models

Main Conclusions

- The **diversification and selective power** surrounding NOVA's expected return vector allows for **better risk-return combinations** across all optimization procedures when using all different risk inputs.
- The Markowitz procedure yields the **highest Sharpe Ratios**. Despite being the model that provides the lowest returns, it has a powerful method of **combining stocks into low volatile portfolios** (portfolios built upon Markowitz optimization have the lowest risk).
- The Information Ratios obtained with the Mean-Variance Tracking Error (MVTE) optimization are greater than those of Markowitz thanks to two distinct effects. First, this approach uses an **objective function that penalizes for deviations from the benchmark** (tracking error) and therefore will decrease the IR denominator. Secondly, the numerator of the IR (alpha) will also increase since the MVTE function **maximizes return** and thereby it will bet on riskier stocks that provide greater returns in comparison to the benchmark (which in turn will punish the Sharpe Ratio).
- The Black-Litterman (BL) model yields **intermediate results** in terms of IR and SR in relation to the Markowitz and Mean-Variance Tracking Error Models.

  Sharpe Ratio: Markowitz > BL > MVTE

  Information Ratio: MVTE > BL > Markowitz

  Our explanation for this fact stems from the construction of the BL model as it uses a combination of the NOVA Model (50%) and the Market Implicit Return (50%) as expected return inputs. Hence, the annualized returns of the obtained **portfolio will not deviate from the S&P as much** as those from the Markowitz portfolio as it is somehow "forced" by the Market Implicit Returns to **converge to the benchmark**; indeed, the Information Ratios are greater for the BL than for the Markowitz model. On the other hand, since the **objective function punishes variance**, the portfolio volatility ends up being smaller than the one attained using the MVTE; thus, the Sharpe Ratios will be greater for the BL than using the MVTE.
- The Genetic Algorithm approach provides, despite some **instability in the results** (the results will vary depending upon the starting point of the iteration), the **highest values for returns**. Regardless of creating portfolios with high levels of risk and tracking error, both **Sharpe and Information Ratios are high** due to these "fat returns". It is worth stating that some mutations can (and do in fact) increase some weights allocated to some actions, which could be the reason behind those magnified returns
- Different optimization processes yield different results, as one is changing not only the structure of the procedure, but also the utility function to be maximized. In this sense, a careful evaluation of the procedure to use must be done, as one could be faced against a **client needs vs. investor objectives trade-off**. Specifically, if a client's needs are to be satisfied (maximum return with the lowest volatility possible), an active manager could, for instance, opt for a Markowitz or a Black-Litterman optimization as these are the ones that maximize the Sharpe Ratio. On the other hand, and if an investor is not to deviate much from a specified benchmark, a mean-variance tracking error optimization could be chosen to minimize that deviation.

5 Optimal Portfolio Allocation Analysis

Deeper Scrutiny of the Portfolio Choices advanced by each Method

After presenting the optimization models employed and their results using different mixes of expected return and risk inputs, we will provide a closer view on the **composition, dynamics and style of the portfolio choices yielded by the different models**. These analysis will be carried out for the models that showed a best performance across all the different optimization models (**combination Information Ratio/ Sharpe Ratio**).

Industry Allocation and Contribution

- Portfolio Industry Allocation
- Portfolio Industry Return
- Benchmark Allocation Differential
- Benchmark Return Differential

Style Analysis

- Value and Size as capital gain sources
- Combination of Size and Value features
- Dissection of Portfolio Style composition over time

Brinson Analysis

- Portfolio performance breakdown into Selection, Allocation and Interaction
- Dynamic and sub-period analysis of Brinson measures
- Stock Picking vs. Allocation

5 Optimal Portfolio Allocation Analysis

Theoretical Grounds

Analysis Type	Description
Industry Allocation and Contribution	• This analysis aims at describing the way each portfolio's stocks are structured in terms of industry allocation and industry contribution. The core of this analysis is to scrutinize how much of the portfolio invests in each sector and how much it will be the gain/ loss when compared to the benchmark by doing this specific allocation.
Style Analysis	• The Style Analysis aims to dissect the Portfolio's composition in order to measure the asset allocation skill of Portfolio Managers. The fundamental is to determine what is the style pursued by the Manager and what is the outcome of hunting that style. We opt by performing this analysis on two well-known sources of return: size and value.
Brinson Analysis	• The purpose of the Brinson Analysis is to grasp where do the Portfolio gains come from. Selection represents the capacity of a Manager to pick the right stocks within a segment; on the other hand, Allocation stands for the Managers' skill to spot the best performing sectors/ asset classes/ regions/ clusters against the benchmark; Interaction gains, as the name indicates, are originated from the ability to underweight or overweight specific stocks depending on the allocation in the predefined sector.

Portfolio Inputs

Optimization Model: Markowitz

Expected Return Input: NOVA Model

Risk Input: Fama-McBeth

Average Portfolio Allocation by Industry | Average Portfolio Return per Industry

Average Portfolio Allocation	Industry	Average Portfolio Return
35,7%	Consumer Staples	0,8%
23,3%	Health Care	0,5%
19,7%	Utilities	0,9%
9,1%	Telecom	0,6%
5,1%	High Tech	3,4%
4,2%	Energy	1,5%
1,4%	Consumer Discretionary	1,8%
0,6%	Financials	0,9%
0,5%	Industrials	0,8%
0,3%	Materials	1,0%

Findings

- The Markowitz portfolio has a clear preference for non-cyclical sectors with significant weights in Consumer Staples, Health Care and Utilities.
- Despite of this preference the higher average returns belong to sectors with large betas such as High Tech, Energy and Consumer Discretionary.

Industries' Allocation Differential to the S&P 500 | Excess Return to the S&P 500 due to Industries' Differential Allocation

Allocation Differential	Industry	Excess Return
24,9%	Consumer Staples	0,5%
15,2%	Utilities	0,3%
12,6%	Health Care	0,1%
6,4%	Telecom	-0,6%
-3,5%	Materials	-0,1%
-6,0%	Energy	0,7%
-10,0%	Industrials	0,4%
-11,1%	Consumer Discretionary	0,7%
-12,6%	High Tech	2,5%
-16,0%	Financials	1,4%

Findings

- The Markowitz portfolio presents huge deviations from the Benchmark sector weights allocation.
- The dynamics of active allocation against the benchmark weights depicts a clear preference upon defensive vs. cyclical sectors.
- High Techs and Financials yield the best performances on an active average sector return basis.

5 Optimal Portfolio Analysis

Industry Allocation and Contribution | Style Analysis | Brinson Analysis

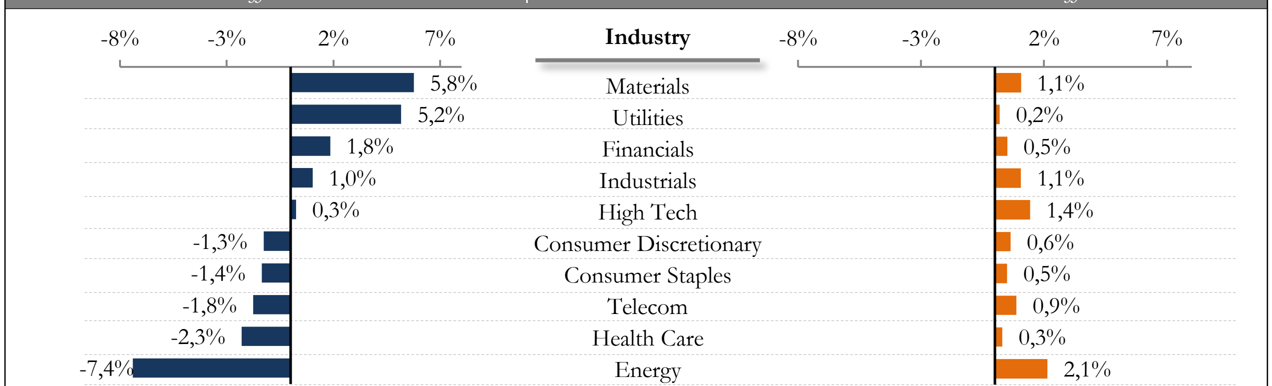
Portfolio Inputs — **Optimization Model: MVTE** — **Expected Return Input: NOVA Model** — **Risk Input: Shrinkage**

Average Portfolio Allocation by Industry | Average Portfolio Return per Industry

Findings

- The MVTE Portfolio sector allocation displays a clear tendency on cyclical industries (Financials/High Techs).
- The highest average portfolio returns highlight Energy and Consumer Discretionary as the best sector performers.

Industries' Allocation Differential to the S&P 500 | Excess Return to the S&P 500 due to Industries' Differential Allocation



Findings

- In terms of active allocation against the benchmark, Materials and Utilities are the main yielders of overweight positions .
- Probably due to volatility issues the Energy sector is constantly underweighted by the portfolio against the benchmark. On the other hand, on the basis of sector active average performance, Energy spots in the top. Thus, the trade off return vs. volatility is captured by our model by building positions upon the balance of both features.

5 Optimal Portfolio Analysis

Industry Allocation and Contribution | Style Analysis | Brinson Analysis

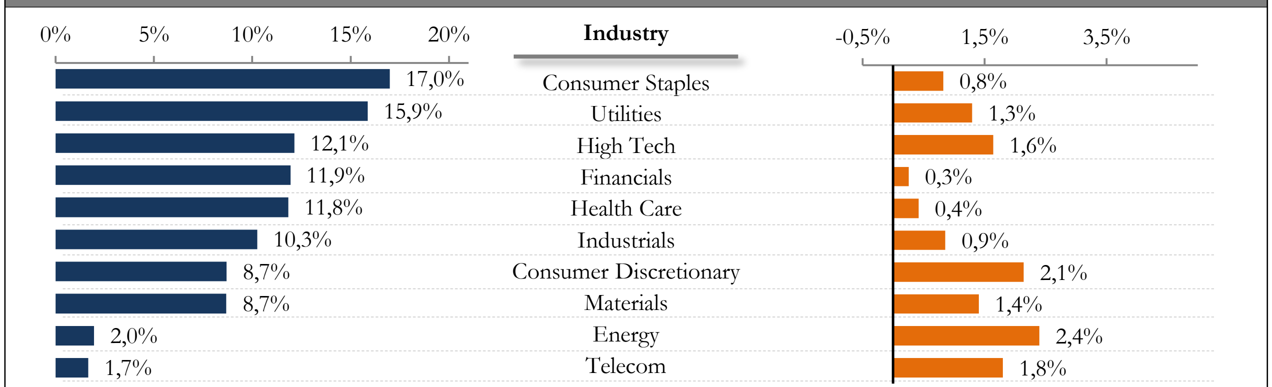
Portfolio Inputs

Optimization Model: Black-L.

Expected Return Input: NOVA Model

Risk Input: Shrinkage

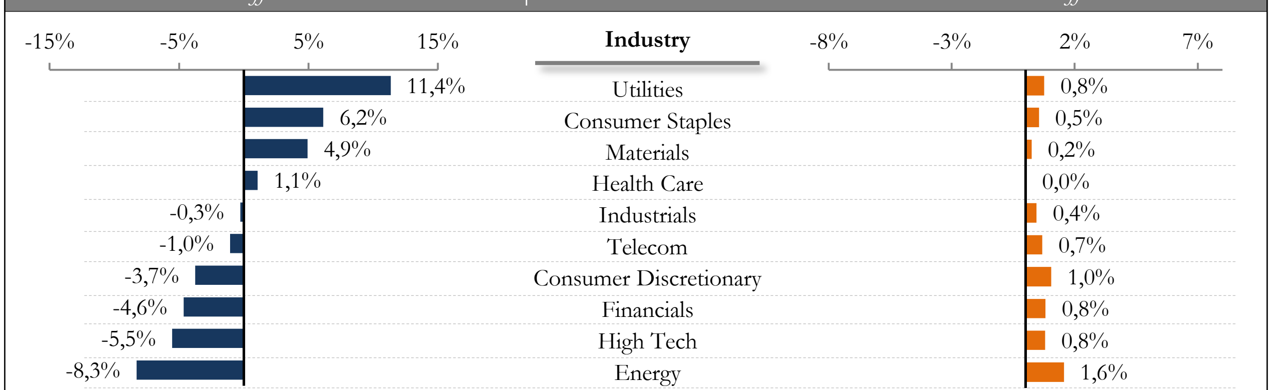
Average Portfolio Allocation by Industry | Average Portfolio Return per Industry



Findings

- The highest portion of sector allocation imposed by BL portfolio employs both defensive and cyclical industries, Consumer Staples/Utilities and High Techs/Financials respectively.
- The Energy sector yields the best average return across the different sectors.
- The average return attained by the portfolio suggests a considerable quality on the selection/stock picking factor.

Industries' Allocation Differential to the S&P 500 | Excess Return to the S&P 500 due to Industries' Differential Allocation



Findings

- The portfolio allocates a considerable portion to the Utilities and Consumer Staples sector when compared to the benchmark industry weights. On the other hand High Techs and Energy are the sectors underweighted by the portfolio in a benchmark comparison basis.
- The average return increment on the excess return feature of the portfolio against the benchmark seems to be higher in the allocation extremes.

5 Optimal Portfolio Analysis

Industry Allocation and Contribution | Style Analysis | Brinson Analysis

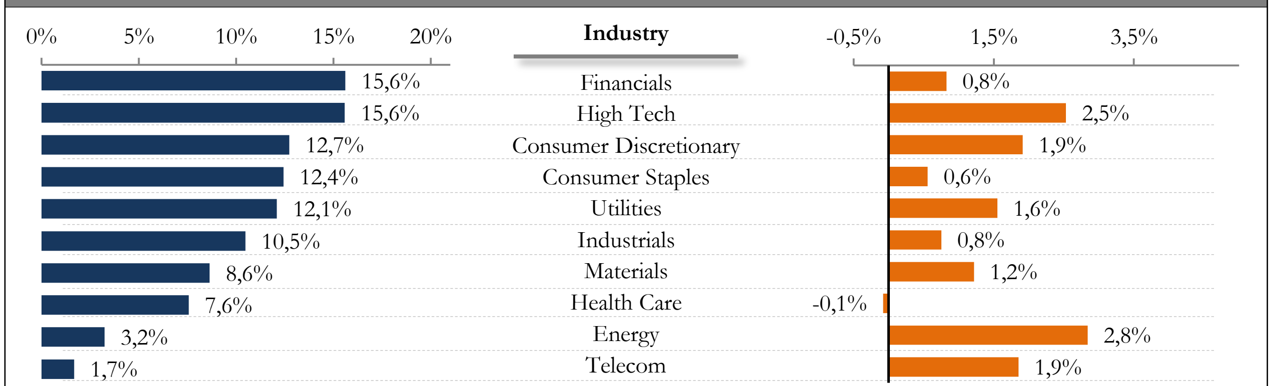
Portfolio Inputs

Optimization Model: Genetic A.

Expected Return Input: NOVA Model

Risk Input: Shrinkage

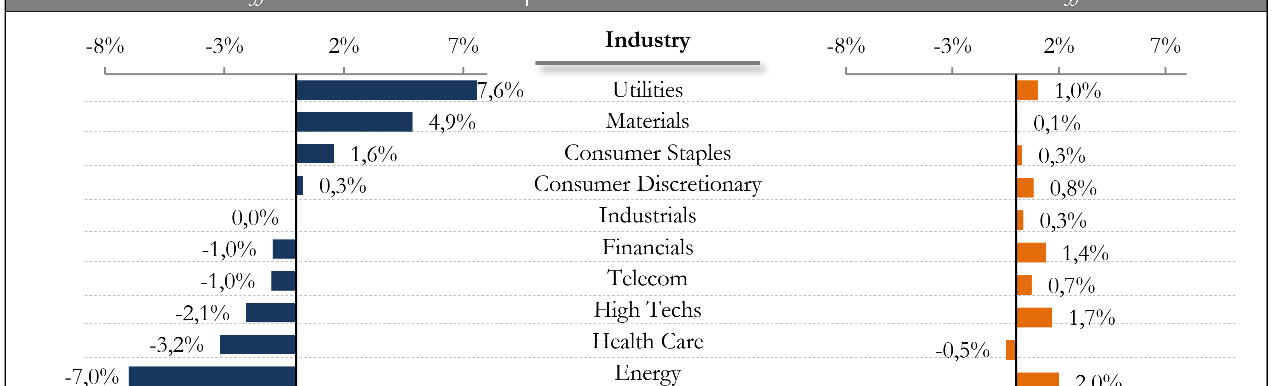
Average Portfolio Allocation by Industry | Average Portfolio Return per Industry



Findings

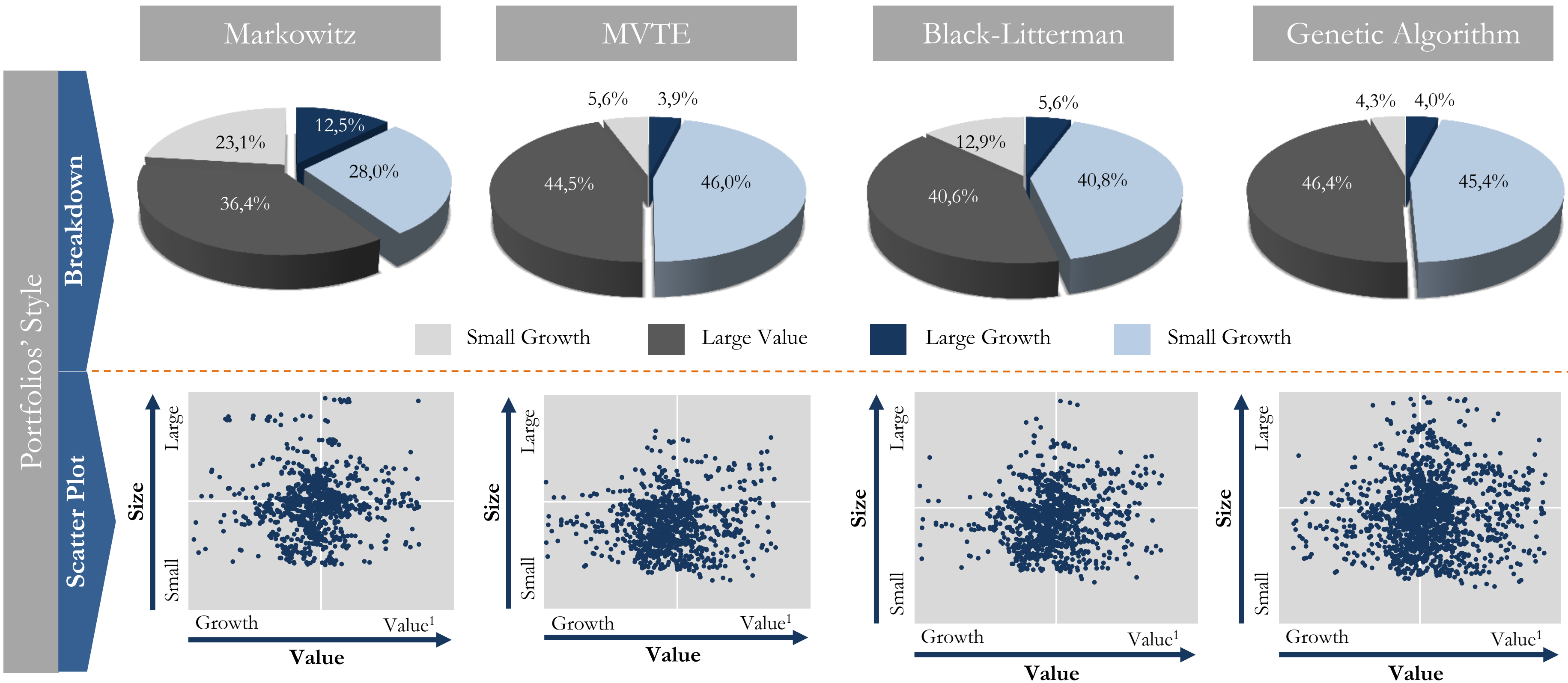
- The Genetic portfolio seems to have a preference for more cyclical and volatile sectors, such as Financials and High Techs.
- The average sector portfolio returns seem to be greater when compared to other portfolios tested, highlighting both High Techs and Energy as best Performers
- The Healthcare sector employs negative average returns, that may be due to poor stock picking allocation.

Industries' Allocation Differential to the S&P 500 | Excess Return to the S&P 500 due to Industries' Differential Allocation



Findings

- When compared to the benchmark industry allocation, the portfolio opts by overweighting the defensive sectors.
- The neutral and underweighting positions held by the portfolio against the benchmark weights suggests a lower presence in volatile sectors in comparison terms, though obtaining pretty excessive average returns.

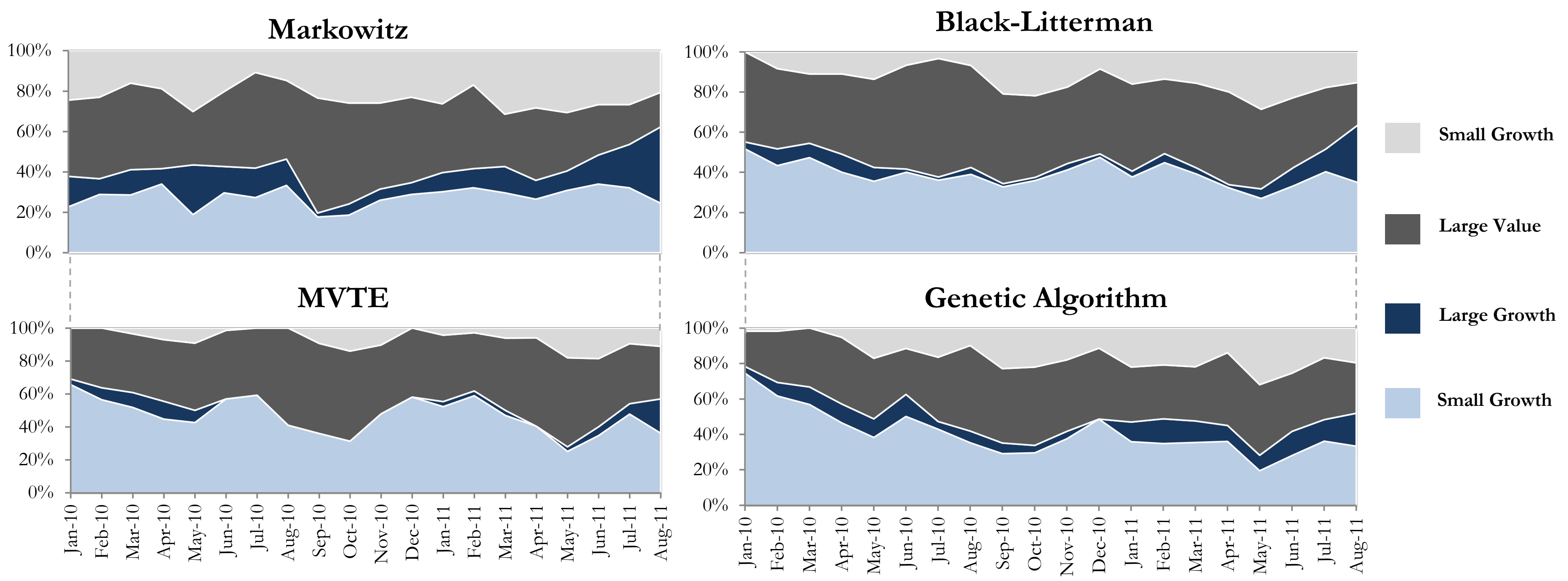


Findings

- Across all the different portfolios presented above we are able to highlight a clear preference for Small Stocks.
- The Markowitz is the model that portrays the most different structure for style allocation. By using a simple mean variance utility function this model captures the diversification effect in a more clear manner, where large represents 35,6%, small 64,4%, value 59,5% and growth 40,5%.
- The scatter plot analysis give a clear insight on the dispersion level of each portfolio stock combination of Earnings Yield (value) and Market Cap(size).
- Despite similar style relative allocations, the MVTE and the Genetic portfolios' depict a considerable dispersion difference on the scatter plot.

[1]This means the stock has a high Book-to-Market or high Earnings-Yield.

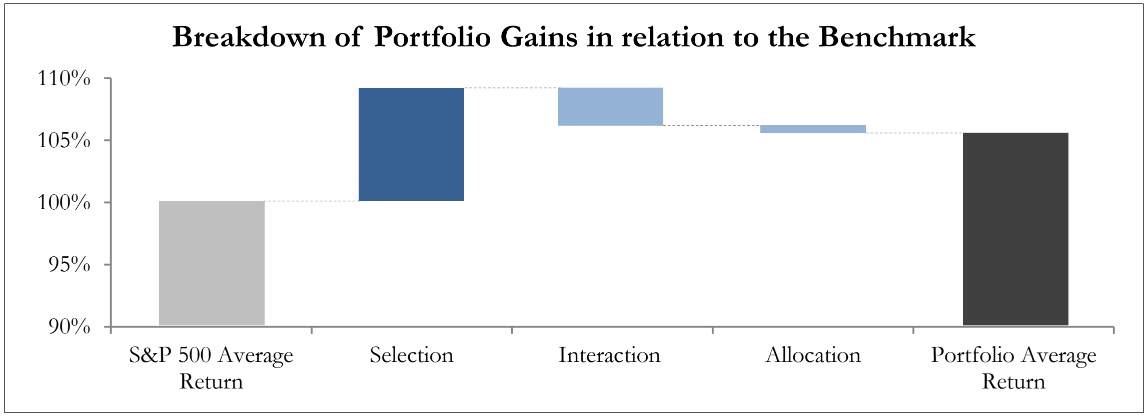
Evolution of the portfolio's composition in terms of Size and Value relative weights



Findings

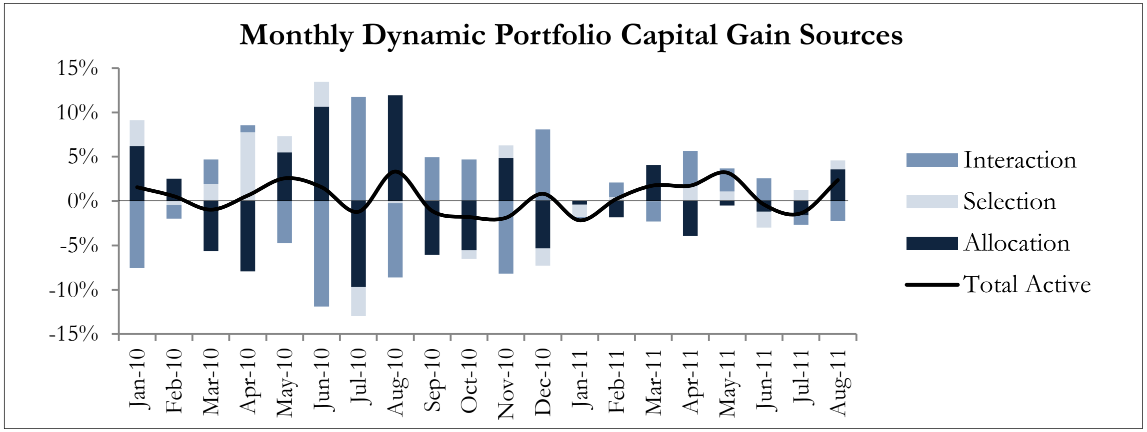
- It is easily perceivable that all portfolios face a shift towards Large Growth stocks in the last months. This phenomenon can be linked to the high volatility implied in the market during this period. Thus, the model will increase the allocation in Large Growth stocks in order to decrease volatility exposure.
- The Markowitz Portfolio evolution over time in terms of size, and value, seems to present a very smooth pattern.
- The Genetic portfolio style effects evolution since 2010, seems to be considerably volatile when compared to the other portfolio models.

Portfolio Inputs — **Optimization Model:** Markowitz — **Expected Return Input:** NOVA Model — **Risk Input:** Fama-McBeth



Findings

- The selection effect portrays a huge portion of the active portfolio return.
- The Allocation effect gives a slightly negative contribution to the active return feature.
- The Interaction component yields the poorest results when compared to the other effects.



Findings

- The active return over time presents a very smooth pattern.
- The selection component over the different monthly periods is less volatile than the other effects. However, this effect tend to present positive returns.
- Allocation and interaction has the opposite contribution to the active return constituent.

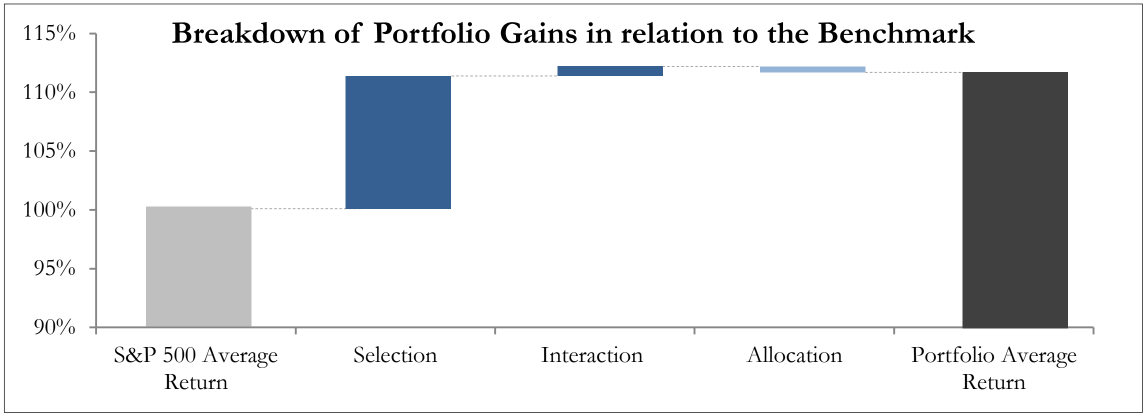
5 Optimal Portfolio Analysis

Portfolio Inputs

Optimization Model: MVTE

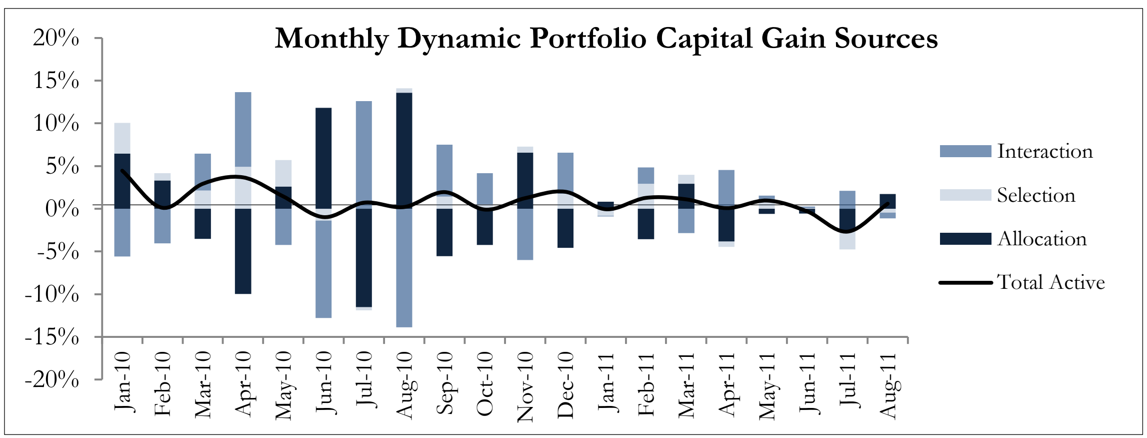
Expected Return Input: NOVA Model

Risk Input: Shrinkage



Findings

- The selection feature is the responsible for the active portion of the portfolio return against the benchmark.
- The effect of allocation and interaction on the portfolio excess returns seem to have an opposite effect of the similar magnitude.



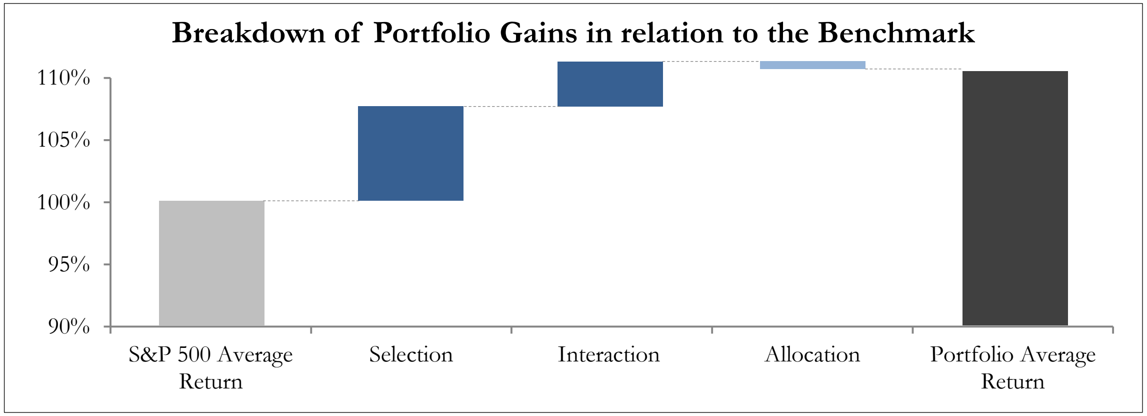
Findings

- The selection component over the different monthly periods is less volatile than the other effects.
- The total active return is much smoother, which is linked to the lower tracking error figure.

5 Optimal Portfolio Analysis

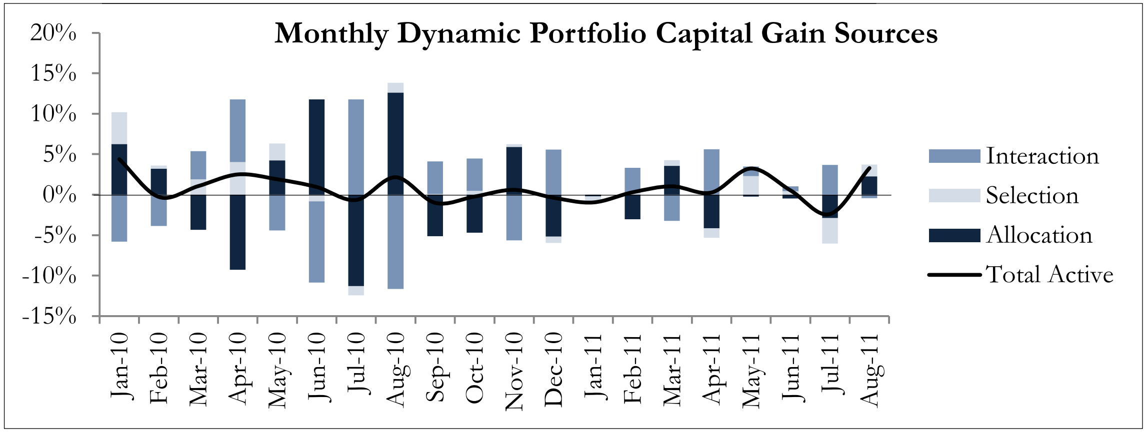
Industry Allocation and Contribution | Style Analysis | **Brinson Analysis**

Portfolio Inputs — **Optimization Model:** Black-L. — **Expected Return Input:** NOVA Model — **Risk Input:** Shrinkage



Findings

- The selection effect is the one that contributes the most to the active portfolio return.
- The Interaction component is also a good performer in terms of active portfolio management.



Findings

- The active return line is very smooth, as the BL model does a good job tracking the market.
- Despite interaction having a positive average active contribution it seems to be one of the most volatile effects.

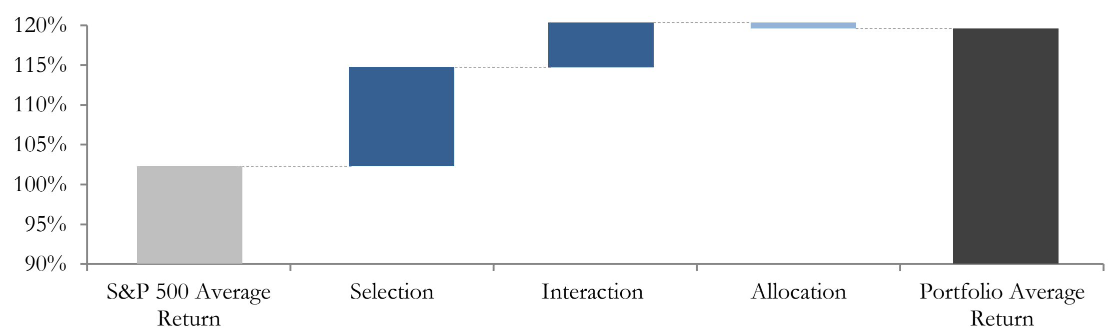
Portfolio Inputs

Optimization Model: Genetic A.

Expected Return Input: NOVA Model

Risk Input: Shrinkage

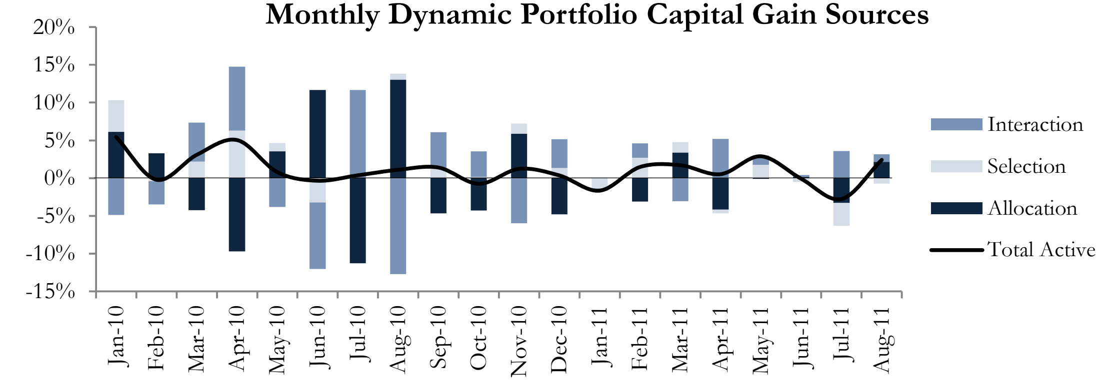
Breakdown of Portfolio Gains in relation to the Benchmark



Findings

- The selection effect is very high in the case of the genetic portfolio, which is linked to a higher preference for higher returns and consequent volatility.
- The interaction effect displays a positive return contribution to the active return.

Monthly Dynamic Portfolio Capital Gain Sources



Findings

- Across periods the selection effect is less volatile than the other effects. However, this effect provides constant positive returns.
- Despite presenting some positive returns, the allocation effect presents on average a negative contribution to the portfolio active return.

Synopsis on Optimal Portfolio Analysis

Main Conclusions

- In terms of industry allocation we can conclude that the Markowitz portfolio clearly prefers to invest in defensive industries in detriment of cyclical Industries, by overweighting and underweighting positions respectively against the benchmark. As protection against volatility the portfolio puts extreme weights on defensives when compared to the benchmark sector allocation. The returns will definitely be more stable, but low at the same time. The extreme industry active weights will contribute to an increase of the TE figure.
- Besides Markowitz the effect of industry allocation across all the different portfolios is very similar in terms of sector dynamics and diversification. As the appetite for volatility and boosting return changes, the allocation can shift towards cyclical or defensive sectors (example: Energy as the most volatile sector - historical 5 years volatility of 22,6% - will be constant underweighted by all different portfolios). Independent on the sector weights differential against the benchmark, all the portfolios yield sector excess returns against the benchmark index. This effect is mainly explained by the persistency of high stock picking skills.
- In the industry allocation analysis is also important to highlight the active allocation of the MVTE model, as it portrays considerably low excess sector allocations when compared to the benchmark. This effect will lead to a lower TE value.
- We also opt by breaking down the portfolio allocation into combinations of two major market features: value and size. Across all different portfolios presented above we are able to highlight a clear preference for Small Stocks. Implied on the size market anomaly and considering that it emerges as one of the most relevant characteristics of our expected return model, might be a explanation to the excess exposure to small stocks in order to capture their return boost tendency. This happens mainly because of the momentum effect present on the model factors
- One of the most relevant highlights of the style analysis, was to understand the dynamic and fast adaptation imposed by our portfolio models. In the last OOS periods the portfolios faced a shift towards Large Growth stocks. This phenomenon might be associated to the high volatility implied in the market during this period. Thus, the model will increase the allocation in Large Growth stocks in order to decrease volatility exposure.
- Looking at the Brinson analysis it is easily perceivable that the stock picking (selection) qualities of the portfolios represent always the biggest portion of the active return.
- We can imply that the markowitz portfolio is the one that yields the highest dispersion in terms of the effects that characterize active returns. The Markowitz portfolio depicts a negative interaction effect, as the portfolio underweighted the sector with good selection. On the other hand portfolios formed with all the other optimization procedures exhibit positive interaction, which comes from the fact that overweighting is done to those sectors with good selection.

RECOMMENDATIONS

Final Considerations

BPI Expected Return Model

- *Too many Variables*
- *Lack of diversification*
- *Unstable Factors*

- The main issue is related to the number of variables present in the multi-factor model.. Serious reduction on the number of characteristics is recommended;
- The reduction process must be grounded on economic/fundamental and statistical judgment;
- Opt by choosing variables that are clearly linked to common risk factors across stocks;
- At the end less variables will diminishes the existence of spurious relations, which brings stability to the estimate d factors and consequently model accuracy;
- On our analysis we presented the NOVA model, a solution that mitigates some of the bpi's model drawbacks.

Risk Methodology

- *Several assets to consider*
- *Too much parameters to estimate*
- *Large estimation error*
- *SCM as poor risk estimator*

- We provide a series of alternative approaches to estimate volatility, which incorporate benefits in terms of estimation accuracy and computational burden;
- 1st Approach: Shrinkage; employs structure to the covariance matrix leading to estimation error reduction;
- 2nd Approach: Factor models; Estimation based on different information and less parameters , reduces estimation error;
- Our work shows that both approaches provide better input for risk estimation;
- We highlight shrinkage as one of the most stable solution in alternative to the SCM .

Final Considerations

Optimization Model

- *Unstable results*
- *Strong Restrictions – Cap the output*
- *Computational burden*

- Different optimization approaches might be used depending on the targets and objectives of the investment manager
- We underline the BL and MVTE as very pleasant methodologies in terms of performance for investor s evaluated against a benchmark basis;
- We advise the use of the Genetic Algorithm and the Markowitz approach for risk adjusted return seekers;
- In terms of the risk input to be used in the optimization process we believe that the shrinkage covariance matrix would be the most suitable.

Further Developments

- Backtest the portfolio performances for longer time periods;
- Test different variables on the Expected return model (such as the inclusion of the variance of the residuals);
- Balanced estimation inputs for the optimization process will allow to the relaxing of restrictions on the optimization routine, which may give room to better results;
- Explore the Fama-Mac Beth methodology, by clearly analyzing accurate risk factors;
- The Parametric Portfolio Policies routine should be considered as an easy and more flexible approach when compared to traditional optimization processes. In the next set of slides we give a glance on this.

Modern Portfolio Theory Approach

Markowitz Solution $w \; \alpha \; \Sigma^{-1}\mu$

where:
w are the portfolio optimal weights
Σ is the covariance matrix
μ is the expected return vector

Input

μ

- Very hard to estimate
- Unconditional, based on historic means
- Subject to great estimation error and variation

Σ^{-1}

- Large number of parameters to estimate
- Hard to achieve well-conditioned Matrix
- Hard to incorporate time-varying volatility.

Output

W

- Naïve implementation yields very extreme weights
- Optimal solution is very sensitive to small changes in the inputs
- Non-unique solution

General comment:

- Although the standard Markowitz approach is backed up by an elegant and well accepted conceptual framework the mathematical sophistication of the optimization algorithm is far greater than the level of information in the input forecasts. The mean-variance optimization operates in such manner that it magnifies the errors associated with the input estimates. Given that, since for a problem with N stocks we have to model N first moments and $\frac{(N^2-N)}{2}$ second moments of returns, the naive solution of the MV approach will yield very poor results.
- There are several fixes for the error maximizing issue; such as imposing constraints in the optimization problem or use different estimation methods like shrinking the covariance matrix. However these procedures will always present important tradeoffs like loss of information and limitation of possible optimal limitations.

Parametric Portfolio Policies

Breaking with the Modern Portfolio Theory approach

Intuition	▪ This model purposes a different approach to optimize portfolios with large number of assets that model directly the portfolio weight in each asset as a function of the asset's characteristics. The coefficients of the function are found by optimizing the investor's average utility of the portfolio return over a certain sample period.
Function	$w_{i,t} = \overline{w}_{i,t} + \frac{1}{N_t}\theta^T \hat{x}_{i,t}$ This particular parameterization captures the idea of an active portfolio management relative to a performance benchmark..
Function Terms	$\hat{x}_{i,t}$ Standardized stock characteristics with zero mean and standard deviation of one. • Guarantees stationary through time. • Deviations across stocks will sum to zero. θ^T vector of coefficients to be estimated • Constant across assets and through time • Portfolio weight in each stock depends only on the stock's characteristics and not on the stock's historic returns . $\frac{1}{N_t}$ Normalization term • Turns the weight function applicable to an arbitrary and time-varying number of stocks. Without this term a change in the number of stocks would affect the cross-sectional distribution of the characteristics.
Objective Function	▪ One of the advantages of this method is the flexibility to adapt to investor preferences; one can use different utility functions to estimate the parameters that define the portfolio policy. In this illustration example we will use a Mean-Variance utility function.

Parametric Portfolio Policies

Applying PPP methodology

Sample estimation period: Jan 1992 – Dez 2009

Characteristics

$\hat{x}_{i,t}$

- For comparison proposes we use the same characteristics present in the NOVA model: *Accruals Book Yield Earnings Yield and Market Cap*. Since the estimation methods are quite different this combination is not necessarily the best fit for the policy. For example with these characteristics the policy will ignore momentum anomaly, while the NOVA model incorporates it in the factors estimation period

Characteristics	Accruals	Book-to-Market	Earnings Yield	MarketCap
	-0,33	-0,53	2,68	-2,82

Out-of-Sample Period:: Jan 1992 – Dez 2009

Inputs / Indicator	S&P	PPP	
		Hedged	Combined
Annualized Return	6,6%	11,0%	17,6%
Active Return	--	4,4%	11,0%
Portfolio Volatility	16,2%	6,4%	15,5%
Max Return	8,8%	4,8%	12,2%
Min Drawdown	-8,2%	-2,0%	-4,5%
Portfolio Beta	--	-0,12	0,88
Tracking Error	--	19,21%	6,42%
Sharpe Ratio	0,41	1,72	**1,14**
Information Ratio	--	0,23	**1,72**

Comments:

- ❑ Since the benchmark used is the value-weighted market, the parameterization function problem can be interpreted as an investor that holds the market while investing in long-short hedge fund with weights that add up to zero, hence the combined return will be the return of the benchmark plus the return of the hedged portfolio.
- ❑ From the table above we can see the contribution of the investment policy defined in the sample period, this policy yields an portfolio with outstanding low volatility while achieving a good average returns.