A Work Project presented as part of the requirements for the Award of a Masters Degree in

Finance from NOVA School of Business and Economics

Directed Research Internship

Structured Products in the Current Low Interest Rate Environment

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Abstract

This study presents an analysis on the Portuguese market of structured products and the features used to mitigate the problem of the current low interest rates. Given the possible combinations, it proposes a specific product that enables an annualized yield of 4.5% to the investor if his expectations materialize during the next 2 years, while providing the issuer with a 2% upfront commercial margin.

It also addresses some of the risks for the issuer of such product and the magnitude of their impact in the hedging process.

Keywords: Structured Products, Auto-Callable, Worst of Digital

A Project carried out under the supervision of Professor Afonso Eça

January 2015

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Introduction

Structured products were created to enable financial institutions address the different needs of their clients. Due to their customizable characteristics, these products can be designed either for hedging purposes or for speculation, taking always into account the risk profile of each investor.

A structured financial product can be seen as a bundle of different financial assets which put together create a desired payoff. Financial institutions sell these to their clients and, simultaneously, hedge themselves through a back-to-back transaction, which means buying the same product from another issuer. However, when they believe it is more efficient, they can replicate these products in the market by hedging each source of uncertainty. In either way, the profit of the institution is independent of the product's payoff and the price of the underlying.

One example of a structured product is a capital guaranteed investment that at maturity pays the notional back plus a return that is dependent on the value of the underlying. This kind of structured product can for example be replicated by buying a zero-coupon bond with the same maturity and notional and using the remaining funds (funding) to buy derivatives on the underlying asset.

However, recently, a problem arose in the Portuguese market of capital guaranteed structured products. When the interest rates fell to historical lows, offering attractive products started being difficult because funding ceased to cover the price of the required derivatives.

So, how can financial institutions enhance the yield of its structured products in the current low interest rate environment? There are limited ways to achieve this, thus the purpose of this study will be to analyze the options available in the market and propose a specific structured product that overcomes the problem in the most effective way, while respecting the commercial constrains. Afterwards, it will address the main issues of issuing such product, addressing the risks for the issuer and the impact of each variable considered when quoting and hedging this product.

The Portuguese Market of Structured Products

The purpose of this study is to propose a retail structured product to sell to clients that is still attractive even during the current low interest rate period. In order to choose the best structure that, simultaneously, takes into account the profile of the targeted investors and the current problem of low funding, it is important to understand how each Portuguese financial institution is overcoming the problem and analyze each option.

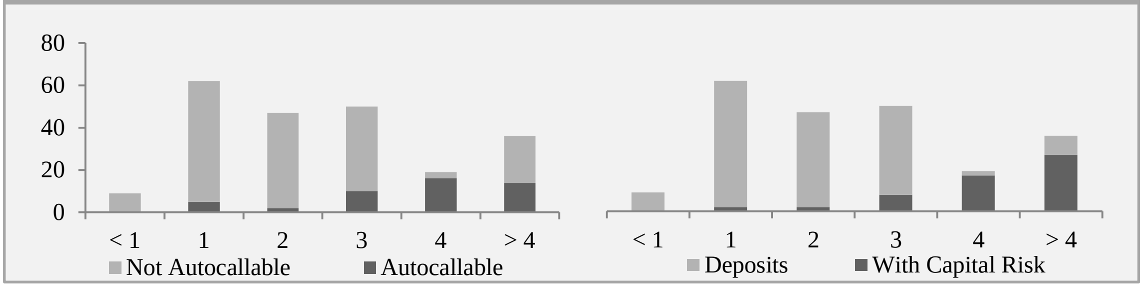
To do it, this study compiled all the retail structured products offered in Portugal, which are registered either with CMVM or Bank of Portugal since the beginning of 2014 (a total of 220 products), and extensively analyzed the characteristics embedded in each one that enabled the creation of a product with the required level of attractiveness. Subsequently, and after weighting the advantages and disadvantages of each option, it presents a final structure with the characteristics that best respond to the problem.

1. Features used in the Portuguese Market

The right structure is the key piece to increase commercial attractiveness while constrained by the low interest rates. Through the analysis of the Portuguese market, one can conclude that, in order to overcome the problem of lack of funding, an issuer can either: increase the maturity, add capital risk, cap the gain, make the payoff in the form of a conditional coupon or use other features to decrease the cost of the structure.

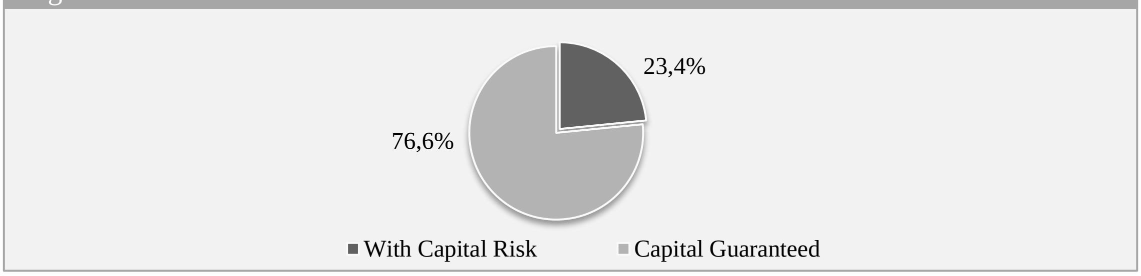
Regarding maturity, figure 1 illustrates that most products (159) have maturities from 12 to 36 months because investors dislike having their capital trapped for long maturities and because timeframes longer than 3 years might exceed investors' macro expectations' scope. A common solution to mitigate these constraints is the inclusion of an auto-callable feature that introduces the possibility of an earlier maturity.

Figure 1 – Maturity Distribution



Adding capital risk can be accomplished by embedding into the structure a short derivative (e.g. a put option). This way, the investor acts like the underwriter of that component, using the proceeds to finance the rest of the structure. One problem with adding capital risk results from the fact that structured products are a particularly good asset for very risk-averse investors to buy in order to get exposure to financial markets without endangering their initial capital. Besides lowering the number of potential investors, having capital risk on a product in Portugal makes it unable to have the denomination of a deposit and therefore, stops being guaranteed by the Deposit Guarantee Fund, which, given the current Portuguese environment, might weigh on the investors' decisions. As can be perceived in figure 1, products with capital risk usually have higher maturities because they are usually bought by the same, less risk-averse, investors.

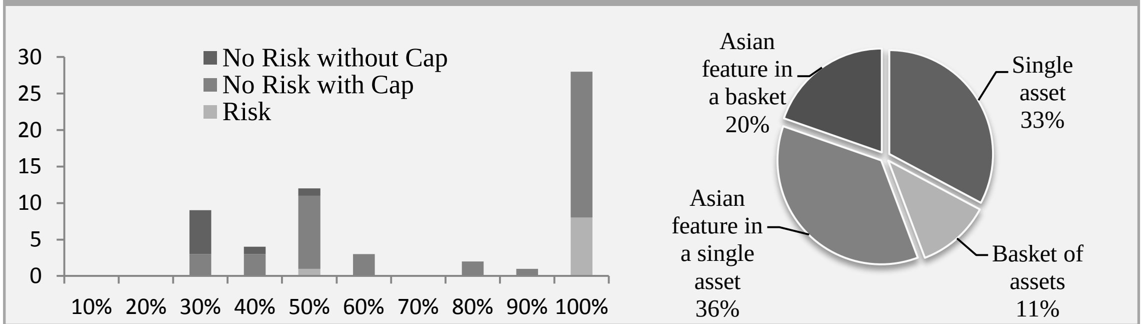
Figure 2 - With or Without Risk



Some features can be added in order to create cheaper structures. These decrease the value for the investor but try to maintain an attractive yield if market expectations are met. Figure 3 illustrates some of the features that are added to a product in order to increase the participation in the upside (for non-digital products). This can be done by choosing underlyings with low volatilities, or choosing stocks with high dividend yield or capping the gain at a certain level.

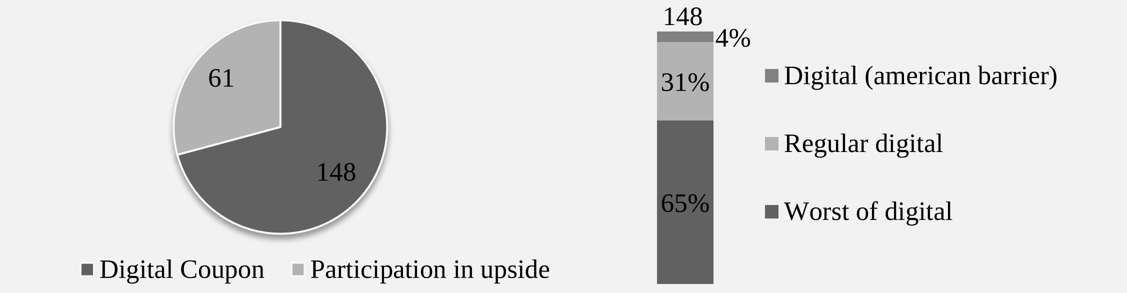
Regarding the underlying's volatility, instead of directly choosing an asset with low volatility, the issuer can choose a basket of low correlated assets or add an Asian feature. The Asian feature, in practice, is the same as having a lower volatility because it reduces the impact of a sudden price deviation.

Figure 3 – Participation



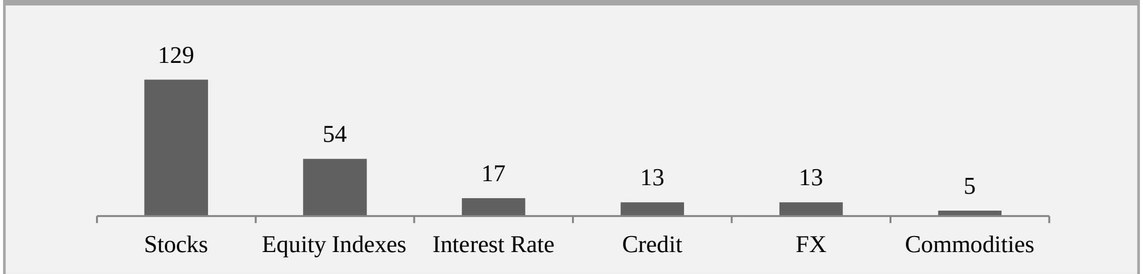
One other alternative to increase the yield with the same funding is by replacing the participation in the upside for a conditional fixed coupon. This way, the issuer can increase the value of the coupon by lowering the mathematical probability of the product actually paying it. Figure 4 proves that currently this option is being extensively used. There are two effective ways to decrease this probability: one is to add a *worst of* feature in a basket with low correlated assets, which will increase the probability of one asset not following the other's tendency and underperform, while the other option is to make the conditional coupon subject to a knock-out American barrier. The latter is used, for example, on range products, where the probability of underlying's price never breaching the barriers during the product's life is substantially lower than the probability of only requiring to be inside the range at maturity.

Figure 4 – Digital Options



Regarding the chosen underlying's asset class, this might have an impact on the cost of the structure but then, its most important purpose is to increase the commercial attractiveness of the product by being well-known and observable for the common investor.

Figure 5 – Underlying' asset class



2. The Proposed Structure

After this extensive analysis of the already issued retail products, it is possible to understand that the popularity of digital payoff is explained by their effectiveness in overcoming the low interest rates problem. Designing a product paying large coupons (even if with low mathematical probabilities of paying it) is the best option to enhance the yield obtained from correct market expectations.

The possibility of not being capital guaranteed will be disregarded here, since it would considerably decrease the targeted clients interested in this product. As can be perceived by the previous analysis, the common Portuguese investor might search for structured products as an alternative to a low paying term deposit. However, he will still look for the main characteristic of these popular deposits, which is having guaranteed capital (which are even guaranteed by the Deposit Guarantee Fund).

On the other hand, in capital guaranteed products, using conditional coupons instead of participations in the upside is not enough to finance high coupons; hence the structured product must have a longer maturity. To keep from reducing the targeted investors, this product will have a maturity of 2 year with an auto-callable feature every semester. While the maturity of two years still allows for market expectations for that timeframe, the auto-callable feature lowers the expected maturity. Additionally, an early maturity is good for both the investor and the issuer since the first will retrieve its investment earlier with the desired return and the latter will gain from the increasingly better commercial relationship with their clients. Moreover, a *worst of digital* in a basket of assets is the most effective structure to improve the control over the trade-off between a high coupon and the probability of paying it. In this type of product, the issuer can choose the number and the type of assets to include in the basket, giving him some control over the initial probability of payment. Choosing low correlated assets or assets paying high dividend yields are examples of methods to influence this probability. Ultimately, the product will have the issuer's desired combination between the coupon's amount and probability from the combinations allowed by this mathematical trade-off.

Table 1 - Summary of the Proposed Structure

Underlying:	Basket of Assets
Maturiry	2 Years
Type of Payoff:	Conditional Digital Coupon
Risk:	Capital Guaranteed
Features included:	Worst of
	Autocallable Every Six months

3. The Chosen Underlying

Regarding the best asset class, this depends mostly on the targeted investors and their specific needs. Given the objective of increasing the number of potential interested investors, an easily understandable asset class must be chosen in order to mitigate the already high complexity of such products. Therefore, the proposed asset class to include as underlying is equity because it is easy to commercialize given its high level of simplicity, liquidity and transparency.

It is in this stage of choosing the stocks that the issuer has the responsibility to add value to the product that is not captured by the options models valuation. By making a top-down analysis, starting with the development of macro expectations and then an individual research on each company, the issuer increases the actual probability of payment by using its expertise and knowledge to choose a promising industry and solid single stocks. Thus, the resulting difference between the model probability of payment and the actual probability will add value to the product and to the client without jeopardizing the issuer's commercial margin.

At the time of this report, it was in the author's opinion that there was unpriced value in the European utility sector. Given Europe's borderline deflation and shy recovery, the expectation of further expansionary policies from the European Central Bank (ECB) made utilities a promising sector. Utility companies are characterized by high levels of regulation and low cash flow volatility, which, combined with the necessary amount of non-current fixed assets, lead to high levels of leverage. Thus, this asset class works as a proxy to bonds since lower interest rates will boost earnings by decreasing the huge interest weight. This characteristic makes this sector the perfect choice in the current environment of low interest rates and therefore, is consistent with the objective of creating an attractive product for low interest rate periods.

Furthermore, this sector is known for its high dividend yields, which, besides increasing its demand in low interest rate periods, will improve the possible offered quote for the product. Additionally, given the exposure to country-specific regulatory changes, these companies might present low correlations if their activities are based in different countries.

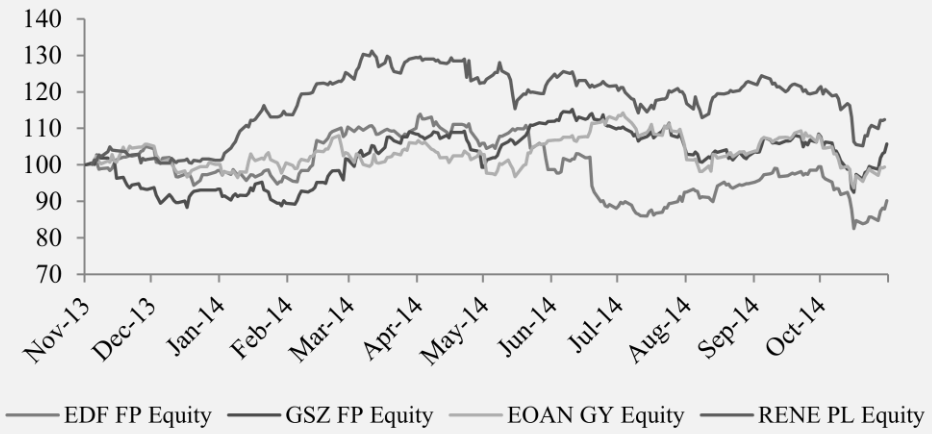
Regarding the single stocks chosen, a study on the existing European utility companies was performed and 4 companies were chosen based on demonstrated stability, solid operations and undervalued stock prices. Additionally, these companies had some priced negative news-flow that was believed to be exaggerated, such as the impact of the French Government desire to reduce nuclear power dependency to 50%, without having any viable economic alternative and the Belgium nuclear tax that might be considered illegal by the European Court of Justice. A summary of the 4 stocks and their 52 weeks performance are presented in table 2 and figure 6, respectively.

Table 2 – Stocks Chosen

i	Company	Bloomberg Ticker	Country	Market Cap.*	P/E 2014E	Div. Yield	Regulated Assets
1	Electricite de France	EDF FP Equity	France	47.915	10,6	5,7%	15%
2	GDF Suez	GSZ FP Equity	France	47.262	13,6	6,6%	30%
3	E.ON	EOAN GY Equity	Germany	28.935	14,3	4,7%	31%
4	REN	RENE PL Equity	Portugal	1.410	10,7	7,4%	100%

* Market capitalization values are presented in € Million

Figure 6 – Stock performance



Auto-Callable Worst of Digital

The product chosen is defined as a capital guaranteed Auto-callable (even though normally these have capital risk) on a basket composed by 4 stocks. It has a maximum maturity of 2 years with a possible conditional early maturity every six months. The auto-callable barrier coincides with the coupon barrier, thus this product will terminate earlier in the event a coupon is paid. The product is an Auto-callable with memory, which means the coupon offered increases linearly with the number of semesters in which there was no payment, thus maintaining a constant annualized rate independent of the payment period. Appendix 1 illustrates this payoff through a timeline.

The pricing of such product is not straightforward given the discontinuous payoff and unknown maturity. Deng, Mallett and McCann (2011) suggest a pricing formula based on the Partial Differential Equation approach but it is only applied when the underlying is a single asset. Since this product is a *worst of* on a basket of stocks, this option cannot be applied.

The approach used in this study was suggested by Boyle (1977) and requires the simulation of *n* sample price paths several times in the risk-neutral world (Monte Carlo simulations) and the computation of the discounted payoff of the option for each path. By relying on the law of large numbers, the average of these simulations will tend to the true value of option. This method will be described in depth in the methodology.

The issuance of such product has some risks associated, which are listed in Fower and Outgenza (2004) and will be further addressed in the section of other risks to the issuer.

1. Methodology

Given the unknown maturity and the path dependency of this product there is no closed-form solution to price it and, therefore, the only way to do it is through the already mentioned Monte-Carlo approach. This study followed the suggestions in Hull (2011) to execute this approach. This method required the simulation of 100.000 possible paths or each asset price of the underlying basket in a risk neutral framework. The equation 1[1] is used to simulate the price for a Δt length of time,

$$S_i(t + \Delta t) = S_i(t) \exp\left[\left(rf - dy_i - \frac{{\sigma_i}^2}{2}\right)\Delta t + \varepsilon_i\right] \qquad (1)$$

To generate ε_i, independent samples $(x_i, x_{i+1}, \cdots, x_k, \cdots, x_n)$ are generated from n independent standard normal distributions (where n is the number of assets in the basket) and are then subject to a procedure known as the *Cholesky decomposition*, which makes the n series of ε correlated with each other according to their implied correlations. This procedure is done by multiplying the vector x $(x_i, x_{i+1}, \cdots, x_k, \cdots, x_n)$ by matrix A^T, which is computed from the implied covariance matrix (Ω) through the equation,

$$\Omega = A \cdot A^T \qquad (2)$$

After simulating the 100.000 different scenarios, the payoff and maturity of the product in each one of these must be computed. The discounted average of the resulting payoffs is the value of the product. The quote of the product, in this case the coupon, is defined so that the value of the product matches the amount paid by the client less the issuer's commercial margin.

The reasoning behind this commercial margin is that the financial institution must charge something for the service it is providing and for the risks and costs it is incurring. This service includes, not only the advice regarding market expectations or concerning the appropriate product for each risk profile, but also the commercial

[1] where $S_i(t)$ is the price of asset *i* at time t, *rf* is the Euribor, dy_i is the implied dividend yield of asset *i*, σ_i is the implied volatility of asset *i* and ε_i is a random variable

relations with the product's issuers. It also must charge the operational costs, the risks that the issuer incurs in the issuance of such products and the costs with the management of the dynamic hedging process necessary afterwards.

1.1. Inputs

Estimating the inputs required might be straightforward or not, depending on the derivatives available in the market for the chosen stocks. Normally, the inputs that might be implied in market prices are volatilities and dividends, which can be extrapolated from options or dividend futures. When not available, volatilities can be estimated through the existing models and the dividends through the company's dividend policy, history, operation results and the analysis of comparables. Appendix 2 and 3 summarize the inputs used in the simulations for the different tenors and stocks.

The challenge surfaces with the estimation of the future covariance matrix to use in the pricing, which is not observable in market prices. Therefore, an issuer could choose to ask for a quote on a correlation swap with a counterparty but, besides sometimes being impossible, these might include a considerable premium due to the lack of liquidity of these contracts.

So, a practical way to do this is by estimating the future correlation through the historical correlation. However, as it will be addressed in the risk section, these inputs are very volatile and have a significant weight in the product's quote. Therefore, given the normal difference between implied volatility and realized volatility, implied correlations will also have a premium relative to the historical correlation, which in this product will be set at 0.2, in order to make a conservative quoting and allow for a certain level of protection against adverse variations of this input.

2. The Quote

In order to price this product and provide a corresponding quote, it is necessary to understand the possible scenarios and their corresponding probability.

There are 5 possible scenarios to the outcome of this product. The Monte Carlo simulation enables the estimation of the probability of each one. These estimated probabilities are the key output of the Monte Carlo simulation because with them the issuer can easily price all simple variation of the simulated product.

Table 5 describes the five different possible scenarios and the corresponding probabilities computed through the 100.000 simulations. The funding used here is the deposit rates offered by *Banco de Investimento Global* (Banco BiG) for the different tenors plus a small spread due to the fact that structured products usually have large notionals, therefore attaining better rates.

Table 5 – Possible Scenarios

Possible scenarios	**Maturity**	**Payoff to the investor**	**Funding**	**Discount factor**	**Probability**
Pays in the first semester	6 Months	Coupon	0,800%	99,91%	14,51%
Pays in the second semester	12 Months	2 × Coupon	1,700%	99,66%	6,69%
Pays in the third semester	18 Months	3 × Coupon	2,700%	99,61%	3,24%
Pays in the fourth semester	24 Months	4 × Coupon	3,800%	99,55%	2,44%
Does not pay	24 Months	Guaranteed min. coupon	3,800%	99,55%	73,12%

With this information, through equation 3, the different possible quotes can be computed for the issuer's desired commercial margin. In this equation, *i* designates each of the 5 possible scenarios described above with each corresponding probability ($Prob_i$), funding (F_i) and discount factor (df_i).

$$\sum_{i=1}^{5}[Prob_i(F_i - Payoff_i) \times df_i] = Commercial\ Margin \qquad (3)$$

Regarding the desired commercial margin, this mostly depends on each bank's position in the trade-off between the product's competitiveness and the contract's profit margin.

Even though each issuer has a reference margin, the latest structured products offered by the competitors were priced, in order to understand which margin leads to a competitive product in the Portuguese market. A total of 22 products were priced from 9 different banks, however, it is important to underline that the margins reached are not each bank's commercial margins but the margins that Banco BiG would get if it issued each exact product. The difference comes from the fact that each bank has its inputs, especially regarding their funding term structure that, for the purpose of comparing the competitiveness between products, must be assumed the same for the different banks.

Appendix 4 presents the results from this product analysis and shows that, lately, the average total commercial margin of the products offered in Portugal is 2.42% or 1.02% per year. Hence, the proposed product will have a target commercial margin per year of 1%. This margin would make this product more competitive than the products of 5 of the 9 banks analyzed.

Finally, to quote this structured product, an analysis of the possible combinations between the conditional coupon and the guaranteed minimum coupon must be performed in order to know which combination could reach the desired commercial margin. Table 6 summarizes these combinations, which were computed according with equation 3.

Table 6 - Possible quotes

Conditional Coupon	Guaranteed Coupon	Commercial Margin	Annual Commercial Margin
1,50%	0,00%	2,39%	1,20%
2,00%	0,00%	2,16%	1,08%
2,25%	**0,00%**	**2,04%**	**1,02%**
2,50%	0,00%	1,92%	0,96%
3,00%	0,00%	1,68%	0,84%
2,00%	1,00%	1,43%	0,71%
2,25%	1,00%	1,31%	0,65%
2,50%	1,00%	1,19%	0,60%

Thus, the final proposed product will pay a conditional coupon of 2.25% with no minimum guaranteed coupon, which translates into a maximum annualized rate of 4.50% and a minimum of 0%. It will provide the issuer with an upfront commercial margin of 2.04% at the time of the issuance.

3. Hedging Process

After selling this product, unless a back-to-back transaction takes place, the issuer will need to manage his consequent position periodically. Such management will be performed according to the sensitivity of the product to the various sources of uncertainty, depending on its importance. These are represented through the different sensitivity calculations, known as the Greeks.

Given the discontinuous payoff, a critical analysis to some of these Greek is required because these might result in absurd or frequently changing values that would be inefficient to hedge.

The Greeks were computed by the finite differencing method with the same 100.000 Monte-Carlo simulations. Equation 4 exemplifies this calculation for the *delta* of the product according to the suggestion of Taleb (1996), which states that in discrete changes, the impact of an increase in the spot price might not be the same as a decrease of the same magnitude. In this case, the ΔS_i used was 0.01 × S_i. The resulting *delta* represents the number of stocks required to replicate the issuer's position.

$$Delta_i = \frac{Payoff(S_i + \Delta S_i) - Payoff(S_i - \Delta S_i)}{2\Delta S_i} \qquad (4)$$

The *Deltas* are the main measures computed and require an immediate offset. Besides representing the main driver for the value of the product, their correct hedging aligns the incentives between clients and issuer, while protecting the issuer's commercial margin.

Gamma, which quantifies the sensitivity of *Delta* in EUR to a 1% change in the underlying price, was computed according to the equation[2],

$$Gamma_i = \frac{Delta_i^*(S_i + \Delta S_i) - Delta_i^*(S_i - \Delta S_i)}{2} \qquad (5)$$

Which is, approximately,

$$Gamma_i = \frac{Payoff(S_i + 2\Delta S_i) + Payoff(S_i - 2\Delta S_i) - 2 \times Payoff(S_i)}{4 \times 0.01} \qquad (6)$$

Since the issuer undertakes the mandatory dynamic hedging, supposedly, the *Gammas* of the product could be left hedged. However, in practice, because it is impossible to perform continuous trading, there is a *gap risk* that will be addressed in the other risks section.

Furthermore, a Greek which is not often computed but that in this kind of products is very important is the Correlation Vega. This sensitivity calculation translates the impact of a change in the correlation between the different pairs of assets in the basket, which given the worst of feature, changes significantly the probability of paying the coupon, and is calculated according with the equation,

$$Correlation\,Vega_{i,j} = \frac{Payoff\left(\rho_{ij} + \Delta\rho_{ij}\right) - Payoff(\rho_{ij} - \Delta\rho_{ij})}{2 \times 10} \qquad (8)$$

By defining $\Delta\rho_{ij} = 0.1$, the Correlation Vega measures the impact of a correlation increase of one percentage point, between asset *i* and asset *j*, in the value of the product.

Regarding Theta, since this Greek accounts for the impact of the passage of time, there is no use in measuring the impact of more time-to-maturity, only less. However, this variation does not represent any uncertainty given the fact that the passage of time is

[2] $Delta_i^*$ represents the delta of each stock denominated in EUR

certain. Therefore, even though its value can change considerably, the computation of this Greek is merely informative, given that the lack of uncertainty makes hedging the impact of the passage of time an unnecessary labor.

The Greeks of this product at the time of the issuance are described in table 7. All calculations were computed for a 1.000.000 € of notional.

Table 7 - Greeks

Stocks	**Delta**	**Gamma**	**Vega**		Overall Product Greeks	
EDF	-36.899	1.430	93		**Theta**	-67
GSZ	-17.414	2.167	42		**Rho**	-180
EOAN	-15.200	1.836	40		**Vega**	277
RENE	-44.666	2.204	103		**Correlation Vega**	-281

Pairs:	EDF / GSZ	EDF / EOAN	EDF / RENE	GSZ / EOAN	GSZ / RENE	EOAN / RENE
Correlation Vega	-43	-28	-63	-70	-44	-36

* All values are in EUR

Thus, if no other hedge carried out besides a dynamic delta hedge, such position for the issuer will mean a value decrease of 67€ per day, a decrease of 281€ for each percentage point increase in the correlation between all assets and an estimated loss of 180€ for every 10 basis points increase of the interest rates. Alternatively, this product contributes for the issuer being long on volatility (he gains 277€ if all assets' volatility increases by one percentage point), which is an uncommon position for issuers.

4. Other Risks to the Issuer

The possibility of having the price of the *worst of* stock (the stock with the worst performance since the initial date) near its strike price close to an auto-callable date is the main risk of issuing this kind of product.

When this situation happens, due to the discontinuous payoff, the delta of this stock explodes to values which might not be feasible or efficient to hedge. Additionally, this value changes drastically with small changes in the underlying, increasing substantially the Gap risk, and sometimes varies with magnitudes where the trading volumes of that stock do not allow for the necessary hedging (liquidity constraints).

Besides these problems, this product has a correlation risk exposure that might sometimes be underestimated by traders and lead to unpleasant surprises.

4.1. Unhedgeable *Deltas* due to liquidity constraints

Table 8 presents each stock's *Delta* and *Gamma* for a scenario where all stock prices are 15% higher their initial price except for REN, which would be at-the-money. This measures are computed for this scenario, two days before the second auto-callable date (therefore in one year from now).

Table 8 - At-The-Money Greeks near an Auto-Callable Date

Stock	in EUR		Number of Shares		Average Daily volume
	Delta*	**Gamma***	**Delta**	**Gamma**	
EDF	-17.164	2.501	-658	103	1.368.345
GSZ	-7.929	1.184	-378	60	5.798.758
EOAN	-5.964	813	-403	59	8.894.507
RENE	-1.096.621	38.862	-449.435	20.424	345.239

According to these values, the issuer will require a position of 1.096.621€ in REN for every 1.000.000 € of notional placed, representing a position close to 110% of the notional in one stock. Depending on the stock's liquidity, it might be impossible to buy such quantities in the necessary period since, for example, REN's last month's daily average volume is 345.239 shares. This means that for a product with 3 million euros of notional, a 2% price decrease in REN's shares in this scenario would oblige the issuer to buy almost 35% of the daily volume, according to the *gamma* computed. This could have an impact in the stock price, which could lead to the actual hedging process to sustain the price level at-the money until the product's auto-callable date. Afterwards, the issuer would need to sell this considerable position, which might be difficult to accomplish in a short time period without incurring losses.

Even if REN's stock price ends up slightly lower than the strike price in this second auto-callable date, its next day *delta* would drop to -144.347 (14,4% of the notional), which would mean that the issuer would need to sell quickly 390.159 shares for every million euros of notional of a stock that has an average daily volume of 345.239 shares.

Another situation that could happen when the hedging process has an impact on the market prices due to low liquidity is unexpectedly higher *deltas*. A scenario where this would happen is under similar circumstances to the ones described before but this time with REN's price 2% lower than its strike price. In such a scenario, REN's *delta* and *gamma*, resulting from this position, would be -456.425 € and -326.645€, respectively. Therefore, a 1% increase in the stock price would compel the issuer to buy 133.195 shares of REN, representing again almost 40% of the daily volume. A situation such as this would put a significant pressure in the stock price, driving its value even further up, therefore increasing considerably the *delta* of this stock.

Normally, both these situations could be mitigated by buying Calls on the at-the-money stock in order to leverage the position and, in the case of the second scenario, decrease the *gamma* risk. However, this might be impossible to accomplish for some stocks like REN, which do not have listed options.

4.2. Gap Risk

The gap risk exists due to the possible jumps in the underlying asset price at the opening of the market or between rebalancing of *deltas*. This gap risk is especially important in this product given the huge dynamics of *delta* when an auto-callable date is near. In such a situation, a jump in the price of the underlying will change considerably the product's deltas and lead to profits / losses from the hedging process very different from the changes in the products value for the investor.

To exemplify the magnitude that this gap risk can reach, it is necessary to retrieve the first situation where only REN was at-the-money two days before the auto-callable date. Again, the issuer is holding a 1.096.621€ position in REN (for a €1 million of notional) when REN's price either opens 5% lower or it suddenly decreases 5% without the issuer having the opportunity to rebalancing his position in between. In this situation, the product value for the issuer will increase 24.444 € but it will lose 54.831 € in REN's position, therefore losing 30.387 € in the hedging process.

Thus, the *Gamma* might sometimes be left unhedged but issuers must understand when its magnitude makes its offset mandatory.

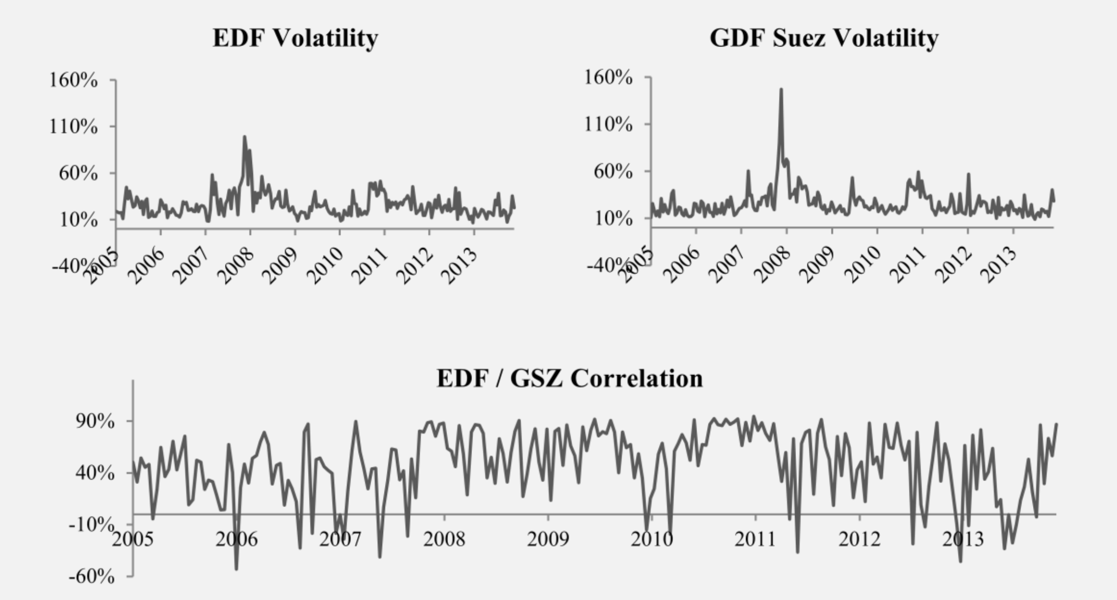
4.3. Vega vs. Correlation Vega

Normally, when issuing options, one of the most important inputs is the implied volatility and so this variable is closely followed by derivatives traders. However, in this product, it is arguable which is more important: volatility or correlation. In case of a Capital Guaranteed Auto-callable each stock's volatility loses gradually its importance to the implied correlations with each asset added to the basket.

The Vega and Correlation Vega values computed show that accurate correlations' estimates are at least as important as implied volatilities. Moreover, the similar magnitude between these values also proves that, in order to improve the quote of the project, choosing low correlated assets for the basket will be as significant as choosing high volatility stocks.

Furthermore, even though they present similar magnitudes, the volatility of the volatilities is lower than the volatility of the correlations. Figure 7 presents this with the computation of the series of correlation and volatilities for non-overlapping windows of 10 days for the stock price of EDF and GDF Suez. The difference between the standard deviation of the correlation series is twice the value of the standard deviation of the volatility series; therefore, the higher uncertainty in correlation makes the Correlation Vega especially important.

Figure 7 – Volatility vs. Correlation



<u>Conclusion</u>

The present study suggests capital guaranteed *Auto-Callables* as the products that maximize commercial attractiveness in the current low interest rate environment. This kind of product may be used to effectively enhance yield according to the market expectations of the issuers and clients by using a cheap structure.

To issue such a product, the issuer at the initial date would unavoidably be required to buy the computed *deltas* of each stock according with the notional placed, while rebalancing this value periodically with a dynamic hedging process. The remaining parameters may be left unhedged except for *gamma* when the product is at-the-money near an auto-callable date, which leaves the issuer very exposed to market prices variations, even if *delta* is hedged.

The main risks for the investors who buy this product are: market risk that is limited to the conditional coupons; liquidity risk, due to the fact that a deposit cannot be sold in the market; credit risk that in this case is subject to Banco BiG's creditworthiness; and tax risk because the tax framework may change prior to maturity. It is also subject to interest rate risk since its increase would lead to a lower present value of the coupons and their decrease would make an early maturity less attractive since reinvestments at the same rates would be difficult to accomplish.

When issuing this product, issuers must also understand the risks associated. These are, mainly: the gap risk that exists because it is impossible to perform continuous trading, and there are some situations where discrete trading might lead to considerable losses for the issuer; and the possibility of having *delta* positions which are impossible or inefficient to hedge due to liquidity constraints, in which case the hedging process can even drive market prices against the issuer.

Thus, after acknowledging the risks associated, this report suggests the product summarized in table 9 as a product that efficiently overcomes the problem of low interest rates.

Table 9 - Proposed Product Summary

Underlying:	Basket of 4 European Utility Stocks: - Electricite de France - GDF Suez - E.ON - REN
Type:	Auto-Callable
Risk:	Capital Guaranteed
Coupon:	2,25% (for each semester)
Memory:	Yes
Maturiry	2 Years
Early maturity:	Auto-callable every Six Months
Commercial Margin	2% Upfront (1% per year)

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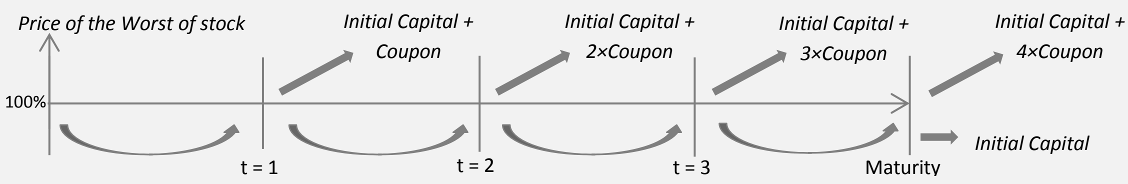
Appendix

Appendix 1 – Auto-callable Payoff



Appendix 2 – Current Euribor and Implied Volatility

Tenor	Euribor	Implied Volatility			
		EDF	GSZ	EOAN	RENE
15-05-2015	0,18%	22,3%	22,9%	23,6%	19,0%
17-11-2015	0,34%	21,5%	22,5%	23,5%	19,2%
16-05-2016	0,26%	21,3%	22,7%	22,9%	19,4%
16-11-2016	0,22%	21,1%	23,1%	22,3%	19,7%

Appendix 3 – Dividend Yield

Semester	Estimated Annualized Dividend Yield			
	EDF	GSZ	EOAN	RENE
I	5,1%	5,5%	0,0%	10,7%
II	5,0%	5,4%	6,8%	0,0%
III	5,1%	5,5%	0,0%	10,6%
IV	5,2%	5,5%	6,9%	0,0%

Appendix 4 – Competitors products commercial margins

Products	Bank	Margin	Annual Margin	Type
Product 1	Bank 1	2,34%	1,17%	Worst of Digital
Product 2	Bank 1	0,65%	0,65%	Worst of Digital
Product 3	Bank 1	2,96%	0,99%	Worst of Auto-callable
Product 4	Bank 1	1,00%	1,00%	Worst of Digital
Product 5	Bank 1	1,68%	1,12%	Worst of Digital
Product 6	Bank 1	2,29%	1,15%	Worst of Digital
Product 7	Bank 1	1,74%	1,16%	Worst of Digital
Product 8	Bank 1	2,84%	0,95%	Worst of Auto-callable
Product 9	Bank 2	5,91%	1,18%	Call Spread
Product 10	Bank 2	1,15%	1,15%	Digital
Product 11	Bank 3	5,19%	1,30%	Worst of Auto-callable
Product 12	Bank 3	6,52%	1,63%	Worst of Auto-callable
Product 13	Bank 4	0,78%	0,78%	Digital
Product 14	Bank 4	1,89%	0,75%	Call Spread
Product 15	Bank 4	0,99%	0,99%	Worst of Digital
Product 16	Bank 5	3,48%	1,12%	Several Asian Worst of Digital
Product 17	Bank 6	1,53%	1,02%	Worst of Digital
Product 18	Bank 6	1,31%	0,87%	Worst of Digital
Product 19	Bank 7	3,46%	0,69%	Worst of Digital
Product 20	Bank 8	1,10%	1,10%	Worst of Digital
Product 21	Bank 8	3,73%	1,24%	Asian Call
Product 22	Bank 9	0,69%	0,35%	Call Spread
Average		**2,42%**	**1,02%**	