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OPTIMAL CHOICE BETWEEN EVEN- AND UNEVEN-AGED FORESTRY: THE CASE OF NON-INDUSTRIAL PRIVATE FOREST OWNERS

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Abstract

An infinite-horizon discrete time model with multiple size-class structures using a transition matrix is built to assess optimal harvesting schedules in the context of Non-Industrial Private Forest (NIPF) owners. Three model specifications accounting for forest income, financial return on an asset and amenity valuations are considered. Numerical simulations suggest uneven-aged forest management where a rational forest owner adapts her or his forest policy by influencing the regeneration of trees or adjusting consumption dynamics depending on subjective time preference and market return rate dynamics on the financial asset. Moreover she or he does not value significantly non-market benefits captured by amenity valuations relatively to forest income.

Keywords: uneven-aged management, optimal harvesting schedule, financial asset, amenity valuation, size-structured model.

1 Introduction

Forest land has a wide range of applications and impacts on daily lives of economic agents. On the one hand, it provides market goods such as timber. On the other hand, it is a major source of non-timber products, often non-market goods. It is estimated that forest products contribute about 1 % of world gross domestic product (GDP) through wood production and non-wood products. In addition, forests offer vital habitat for key species and are relevant for water and soil protection as well as landscape quality. Growing importance has been attributed to forests as a major source of carbon storage. Another widely valuable use of forests is related to recreational activities, such as sightseeing and hiking.

When considering forest assets, harvesting decisions determine both the economic welfare from forestry and the state of nature preservation for large geographic areas. These developments are crucial to understand the consequences of land market liberalization, the changes in forest owners' income level or even the changes in their average age. Past evidence suggests that some forest owners believe timber assets are much more secure than financial assets. In several countries, the forest sector contributes to a large share of a country's GDP and foreign trade.

One of the early contributions to Forest Economics dates back to Faustmann (1849), who considered a forest owner and a plot of land with trees of equal age used at its highest and best use for timber production. He showed that the optimal rotation age[1] of any stand could be determined as the one that maximizes the net present value of the land. This model was a first attempt to study management practices of a standing forest and relied on assumptions such as perfect capital markets, forest rotation period as being the only variable to be optimized and profit as the sole goal to achieve. In particular, the model includes as special cases the maximization of average timber volume on a given forest site over time [Maximum Sustained Yield (MSY)] and the maximization of average annual net revenues. According to the MSY approach, a forest should be harvested when its average growth (mean annual increment) is equal to its marginal growth (current annual increment). Later, Hartman accounted for the non-market value of a standing forest (i.e., benefits that are not internalized in forest land markets), namely amenity value, shedding new light on optimal rotation age. These include ecosystem services, landscape aesthetics, recreational services, hunting, among others. Hartman (1976) viewed amenity services as depending solely on stand's age by introducing a quasi-linear specification of timber revenues and amenity valuation. However, the Hartman model only considered a single stand and thus ignored the interdependencies of multiple age-classes. These models do not rely on age-class structures of stands.

Over the past decades, active research has been conducted on two different ways of forest management: even- and uneven-aged management. The former considers a landowner who manages multiple stands but each distinct stand has trees of equal age. The latter assumes a landowner managing a forest with trees of different ages, heights and diameters, where all grow together on the same unit of land. These are competing alternatives in explaining forest management practices. From an economic stand point, the management system should be determined endogenously by means of an optimization model, accounting for both economic and biological factors.

Forestry and forest industries are a major source of income and employment for NIPF owners. As stated by Baardsen et al. (2008), these operate in incomplete markets where their subjective preferences and idiosyncratic characteristics influence their harvesting decisions, commonly modeled using utility-maximization frameworks. When making decisions, these agents consider whether or not to harvest and the harvesting level. A review and synthesis by Abt et al. (2005) identified four categories of forest management determinants: (i) market drivers, which include price changes; (ii) policy variables (e.g., local programs designed to change land allocation to forestry); (iii) owner characteristics (mainly NIPF owners' preferences); (iv) plot/resource conditions (e.g., soil quality, slope of land). Kuuluvainen and Tahvonen (1999) found that non-forest income had a negative impact on harvesting. They inferred that wealthier forest owners could afford more financial losses than owners with lower non-timber income, in order to enjoy non-timber benefits. Other studies [e.g., Kuuluvainen et al. (1996)] have found non-significant

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[1] In Forest Economics literature it stands for the rotation periods of timber, i.e., the harvest time.

impact of forest owner's exogenous income on harvesting. According to Tahvonen et al. (2001), forest economists believe NIPF owners rely on harvesting revenues as a mean of direct financing their consumption expenditures. In addition, these agents control a large share of the forest land in many countries and this input is valuable to assess future forest landscapes and develop sound forest policies. As argued by Beach et al. (2003) around 69% of forest land in the south of US is controlled by NIPF owners compared to 58% in the US overall. Hence the highlighting of these economic agents for timber supply and wood products based. Moreover, these economic agents have a strong presence in the Nordic countries, Portugal and in the south of US, thus, it is worth studying how they manage forest land, since intervention in forest land markets does not fall under government control.

The first work developed on modeling NIPF owners' behavior was due to Irving Fisher. As pointed out by Amacher et al. (2009) and Bolkesjø et al. (2007), the well-known Fisherian Separation Theorem stated the separability of consumption decisions from harvesting decisions. This formulation relied in strong assumptions such as perfect capital markets which later research proved it to be incomplete and rather limited.

As Amacher et al. (2003) point out, previous work modeling NIPF owners' decisions has been focused on how harvesting decisions are influenced by market characteristics, forest owner preferences and type or timber characteristics [see Greene and Blatner (1986), Royer (1987), Romm et al. (1987), and Dennis (1989, 1990), Birch (1992), Hyde and Newman (1991), and Kuuluvainen et al. (1996) for further discussion]. Conway et. al (2003) show how bequest motives, debt and participation in non-economic activities, and harvesting decisions are interrelated and dependent on landowner preferences, market and land characteristics. The same author also drew attention on two types of forest owners: absentee owners (who do not live on the property) and resident owners (who do). He pointed out on the negative impact on both harvesting and non-timber activities from absentee owners. This was explained by absentee owners having perhaps less information regarding harvesting than resident owners. Hultkrantz (1992) has also studied the possibility of forest owners to make bequest plans for future generations. Additionally, Fina et al. (2001) showed that individuals with higher debt were willing to accept lower timber prices, in order to meet financial obligations. Other models have dealt with uncertainty in the analysis of forestry decisions. These models denoted by anticipative optimization models, have an attempt to capture the degree of risk-aversion of the forest owner, the position in capital markets and the relative risk of investments in and outside forestry, as suggested by Andersson et al. (2010). The two-periods optimization framework has been augmented by a biomass harvesting model, where the forest owner maximizes the production of biomass. Tahvonen (1998) has accounted for the *in situ* value of forests but in a single stand framework.

Kuuluvainen and Uusivuori (2005) developed an infinite-time horizon discrete model for consumption and harvesting behavior of NIPF owners who manage a multiple age-class forest, and who value both consumption derived form harvested trees and amenities derived from standing trees. Tahvonen (2009) and Lähde et al. (2010) developed a model on optimal choice between even- and uneven-aged management of a forest based on a size-structured transition matrix. Moreover, Lähde et al. (2009) provide a discussion for even- vs. uneven-aged forest management and Aakala et al. (2012) review the main studies on the topic.

The present study builds a theoretical model on the decisions taken by NIPF owners regarding the optimal rotation framework of forests and infer about even- and uneven-aged forestry for the Nordic spruce, a trees specie with high economic value throughout northern Europe. NIPF owners are assumed to manage a plot of land for timber and amenity value. They are also endowed with a financial asset that provides an exogenous market return over time. The paper introduces a new model built upon two other ones developed respectively by Kuuluvainen and Uusivuori (2005), and Tahvonen (2009) and thus it brings a new contribution to Forest Economics literature by studying even- vs. uneven-aged forest management in the context of NIPF owners. Therefore, it disregards the optimal management of a single stand and considers instead a multiple size-class structured model.

The remainder of the paper is organized as follows. Section 2 introduces the theoretical model and the optimization procedure. Section 3 presents numerical simulations emerging from the empirical specification. Section 4 concludes and section 5 identifies important issues for future research.

2 Theoretical Model

The land owner manages a forest consisting of n stands representing n size-classes over an infinite-time horizon. In its general specification, the decision-maker derives utility from both periodic consumption and amenity valuation of the standing forest according to an additively separable utility function.

The optimization problem of the forest owner consists of a maximization of the present value of utility derived simultaneously from consumption and non-market benefits by choosing a_t and $h_{s,t,}$, that is, the amount of a financial asset to consume and the per hectare number of harvested trees from each size-class at a given period, respectively, subject to constraints $(2a) - (2b)$ presented below:

$$(1) \qquad V\left(a_t, h_{s,t}\right) = \underset{\{a_t,\, h_{s,t},\, t\geq 0;\, s=1,\ldots,n\}}{max} \sum_{t=0}^{+\infty} b^t \left[U\left(C_t\right) + M\left(A_t\right)\right]$$

$$(2a) \qquad C_0 = \sum_{s=1}^{n} \left[p_s q_s - c g_0 - C\left(Q_0\right)\right] + a_0 - a_1$$

$$(2b) \qquad C_t = \sum_{s=1}^{n} \left[p_s q_s - c g_t - C\left(Q_t\right)\right] + a_t\left(1+r\right) - a_{t+1},\ t \geq 1,$$

where $V\left(a_t, h_{s,t}\right)$ denotes the functional objective, $U\left(C_t\right)$ is a twice differentiable and strictly concave utility function, C_t denotes consumption in each time period, b is a discount factor defined as $b = \frac{1}{1+\rho}$, where ρ is the annual subjective rate of time preference. s denotes each size-class[2], $M\left(A_t\right)$ accounts for amenities, where $A_t = \sum_{s \geq \underline{s}}^{n} q_s x_{s,t}$, $s = \underline{s}, ..., n$ and $\underline{s}$ represents a pre-specified value, or $A_t = q_n x_{n,t}$. In other words, one can account for the timber volume of standing trees above a given size-class threshold $(\underline{s})$ or only for the timber volume of

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[2] Trees are distributed across size-classes based on diameter.

standing trees of the largest size-class. Moreover, $M(A_t)$ is assumed to be a strictly increasing utility function and the forest owner is endowed with an amount of a financial asset, a_0, at the beginning of the time period ($t = 0$).

Furthermore, p_s is the per ton timber price of size-class s trees, q_s is the per hectare timber volume of size-class s trees, c is a non-negative constant denoting the unit cost of artificial regeneration, while g_t is the number of planted trees in each period. $C(Q_t)$ is an harvesting cost function defined as $C(Q_t) = \xi_1 Q_t^{\xi_2}$, where $\xi_1 \geq 0$ and $\xi_2 \geq 1$.

In addition, r is the annual market return rate on the financial asset for $t \geq 1$ and Q_t denotes total harvest volume at a given period defined as follows:

$$Q_t = \sum_{s=1}^{n} q_s h_{s,t} \tag{3}$$

The dynamics of tree size-classes can be written as

$$\mathbf{x}_{t+1} = \mathbf{G}_t \mathbf{x}_t - \mathbf{l}_t \tag{4}$$

where $\mathbf{x}_{t+1}$ is the number of standing trees in the next period. $\mathbf{G}_t$ is an $n \times n$ transition matrix, $\mathbf{x}_t$ is an n-dimensional vector for the number of trees in different size-classes defined as $\mathbf{x}_t = \sum_{s=1}^{n} x_{s,t}$, and $\mathbf{l}_t$ is an n-dimensional vector representing regeneration and harvest of trees in each size-class as follows:

$$\mathbf{l}_t = [-\phi_t + h_{1,t}, h_{2,t}, ..., h_{n-1,t}, h_{n,t}]', \tag{5}$$

where ϕ_t denotes regeneration or ingrowth of trees to the smallest size-class.

The transition matrix can be defined as follows:

$$\mathbf{G}_t = \begin{bmatrix} \beta_1(\mathbf{x}_t) & 0 & \cdots & 0 & 0 \\ \alpha_1(\mathbf{x}_t) & \beta_2(\mathbf{x}_t) & \cdots & 0 & 0 \\ 0 & \alpha_2(\mathbf{x}_t) & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \beta_{n-1}(\mathbf{x}_t) & 0 \\ 0 & 0 & \cdots & \alpha_{n-1}(\mathbf{x}_t) & \beta_n(\mathbf{x}_t) \end{bmatrix} \tag{6}$$

where $\alpha_s(\mathbf{x}_t) \leq 1$, $s = 1, ..., n-1$ is the share of trees that move to the next size-class for period $t+1$, $\beta_s(\mathbf{x}_t)$ is the share of trees that remain at their present size-class for period $t+1$, $\sigma_s(\mathbf{x}_t) = 1 - \alpha_s(\mathbf{x}_t) - \beta_s(\mathbf{x}_t) \geq 0$, $s = 1, ..., n-1$ is the share of trees that die in each size-class, implying that $\sigma_n(\mathbf{x}_t) = 1 - \beta_n(\mathbf{x}_t) \geq 0$, given $\alpha_n(\mathbf{x}_t) = 0$.

The following system of difference equations illustrates the dynamics of the transition of trees:

$$x_{1,t+1} = \phi_t + \beta_1(\mathbf{x}_t) x_{1,t} - h_{1,t} \tag{7a}$$

$$x_{s+1,t+1} = \alpha_s(\mathbf{x}_t) x_{s,t} + \beta_{s+1}(\mathbf{x}_t) x_{s+1,t} - h_{s+1,t}, \ s = 1, ..., n-2 \tag{7b}$$

$$x_{n,t+1} = \alpha_{n-1}(\mathbf{x}_t) x_{n-1,t} + \beta_n(\mathbf{x}_t) x_{n,t} - h_{n,t} \tag{7c}$$

Thus, the number of trees in size-class $s+1$ in the beginning of the next period equals the number of trees that will move from size-class s plus the number of trees of size-class $s+1$ that will remain in this size-class, minus the number of harvested trees from size-class $s+1$.

Regeneration or ingrowth of trees is the process by which forest lands are restocked by trees that develop from seeds that fall and germinate *in situ.* Following Tahvonen (2009), two functional forms for the regeneration are specified as follows:

(8a) $\phi_t = \theta_1 y_t e^{\frac{-y_t}{\theta_2}}$, where $\theta_1 \geq 0$ and $\theta_2 > 0$ and

(8b) $\phi_t = \sum_{s=1}^{n} \eta_s h_{s,t}$[3], where η_s denotes the number of seedlings per harvested tree.

Moreover,

(9) $y_t = \sum_{s=1}^{n} x_{s,t} \pi \left(\frac{d_s}{2}\right)^2$ stands for the basal area of the tree, i.e., its cross-sectional area of tree stems measured at breast height and summed over all trees in the stand and d_s represents the tree diameter in size-class s.

The elements of the transition matrix (6) in (7a) $-$ (7c) are given by:

(10) $\alpha_s\left(y_t\right) = 1 - e^{\frac{-\gamma s1}{1+\gamma s2 y_t}}$

where $\gamma s1$ and $\gamma s2$ are positive constants. In addition,

(11) $\beta_s\left(y_t\right) = \tau_s\left[1 - \alpha_s\left(y_t\right)\right], \ s = 1, ..., n$

where $0 \leq \tau_s \leq 1$, $s = 1, ..., n$. If $\tau_s < 1$, a fraction of trees dies. It follows that mortality, if it exists, becomes dependent on stand structure and equals $\left(1 - \tau_s\right)\left[1 - \alpha_s\left(y_t\right)\right]$.

Finally, the following non-negativity constraints have to be satisfied:

(12a) $\mathbf{x}_t \geq 0$

(12b) $\mathbf{h}_t = \sum_{s=1}^{n} h_{s,t} \geq 0$

(12c) $g_t \geq 0$, for $t \geq 0$

(13) $\mathbf{x}_0 = \underline{\mathbf{x}_0}$ corresponds to the initial state of the forest.

(14) $a_0 = \underline{a_0}$ denotes the endowment of the financial asset.

2.1 Optimization Procedure

The dynamic optimization problem consists of maximizing the functional objective in (1) subject to constraints (2a)$-$(2b) and (12a)$-$(14), and taking into account the definitions (3)$-$(11). In Mathematical jargon, the problem is a discrete-time non-linear programming problem, in which the decision variables assume non-negative integer values.

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[3] This specification for regeneration was initially proposed by Usher (1966).

Therefore, one cannot derive the first-order conditions given such specification. The model is solved numerically by direct substitution of the laws of motion and decision variables in (1), using Knitro 9.0.1 optimization software that implements state-of-the-art interior-point and active-set methods.[4] Convergence paths toward the steady-state are then showed using an iterative algorithm. The infinite-horizon solutions are approximated by applying finite-time horizons.

3 Numerical Analysis

This section presents numerical simulations for the model introduced in (1) − (14). The analysis follows Tahvonen (2009) where the example of the Norway spruce, a trees specie with high economic value in Scandinavia is taken. It takes into account potential key-drivers of the optimal rotation framework. On the biological side, important factors are identified such as the regeneration forms of trees and the effects of density dependence. On the economic side, parameters of the model such as the market interest rate, the subjective rate of time preference and the initial state of the forest are considered. A benchmark model is considered and sensitivity analyses are performed.

Two initial states of the forest, $x_{s,0}$, are studied. Specifically, a scenario of low endowment of forest resources is analysed, where there are only 10 trees in the size-class with the lowest economic value (size-class 1) and 10 trees in the size-class with the highest economic value (size-class 10), and there are no trees in the size-classes in between. By contrast, a normal forest[5] structure is considered, where trees are uniformly distributed across size-classes at a level of 40, thus, it represents a higher endowment of forest resources for the NIPF owner.

A finite-horizon of 200 periods is taken. Furthermore, there are 10 size-classes and the forest owner is endowed with an initial amount a_0 of a financial asset. Natural regeneration and stumpage prices are assumed such that logging costs are nil. In other words, NIPF owners sell their timber to a third party responsible for harvesting the stands, which is a common practice adopted by these type of agents. In the model, m denotes the number of timber types and j_s its respective weighted price. The volume of timber for size-class s trees is denoted by q_s as before and the tree diameter by d_s. The number of seedlings per harvested tree is denoted by η_s, as stated before.[6]

Linear specifications for the share of trees that move to the next size-class, $\alpha_{s,t}$, and for the share of trees that stay at its current size, $\beta_{s,t}$, are included in the model. A logarithmic[7] specification for the utility the forest owner derives from periodic consumption is considered.

Finally, when the amenity valuation is introduced in the optimization problem, an alternative for the functional objective in (1) specified as a Cobb-Douglas utility function is considered.

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[4] The computations treat $h_{s,t}$ as a continuous variable, in the sense it can assume positive non-integer values.

[5] In Forest Economics a normal forest means that forest land is evenly distributed across all size-classes.

[6] New seedlings emerge over the rotation.

[7] A logarithmic specification is suitable since consumption is assumed to be greater than one.

Table 1 summarizes the parameter values for the numerical analysis and functions, which follow Tahvonen (2009). The parameter values for the tree diameter and volume are roughly in line with the data for the Norway spruce.

Parameter | Function	Value | Functional Form
T	200
n	10
r	$i)$ 0.01 $ii)$ 0.03
ρ	$i)$ 0.01 $ii)$ 0.03
a_0	1000
m	10
j_s	0, 0, 2, 3, 4, 10.7, 20.1, 29.2, 42.4, 45
q_s	0, 0, 0.03, 0.0745, 0.1742, 0.2928, 0.4856, 0.7019, 0.9671, 1.2192
d_s	2, 6, 10, 14, 18, 22, 26, 30, 34, 38
$x_{s,0}$	$i)$ 10, 0, 0, 0, 0, 0, 0, 0, 0, 10 $ii)$ 40, 40, 40, 40, 40, 40, 40, 40, 40, 40
η_s	0, 0, 0, 1, 2, 3, 5, 10, 15, 20
ϕ_t	$i) 20 y_t e^{\frac{-y_t}{10}} \ ii) \sum_{s=1}^{n} \eta_s h_{s,t}$
$\alpha_{s,t}$	$1 - \frac{y_t}{50}$
$\beta_{s,t}$	$0.85\left(1 - \alpha_{s,t}\right)$
$U\left(C_t\right)$	$log\left(C_t\right)$
$M\left(A_t\right)$	$i)\ log\left(A_t\right) = log\left(\sum_{s \geq \underline{s}}^{n} q_s x_{s,t}\right)\ ii)\ log\left(A_t\right) = log\left(q_n x_{n,t}\right)$
$W\left(C_t, A_t\right)$	$C_t^{\lambda} A_t^{1-\lambda}$, $\lambda = 0.5$

Table 1: Parameter values for numerical simulations and model specifications

The numerical analysis is divided into three steps. First, two functional forms for the utility function are considered with no financial asset nor amenity, i.e., the forest owner derives utility from consumption financed solely from timber revenue. Second, an exogenous financial asset, a_t, is introduced in the model so that the forest owner can benefit from both timber revenue and wealth from a non-forest asset. Third, amenity valuations are introduced, jointly with the two previous specifications.[8] Economic analyses are provided at each stage and policy implications regarding forest management for a NIPF owner are derived. The results are compared with those from Kuuluvainen and Uusivuori (2005) and Tahvonen (2009).

3.1 Benchmark model with a logarithmic objective function

This subsection considers a utility-maximizer forest owner who derives utility exclusively from harvesting revenue of her or his standing forest. In other words, the functional objective consists of the logarithm of timber revenues without amenities nor financial asset, i.e., consumption decisions are derived solely from forest income.

The following table presents a baseline scenario for a set of parameters and model specifications.

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[8] Note that the theoretical model in Section 2 is presented in its broader generalization where all these three cases are accounted for.

Parameter | Function	Value | Functional Form
ρ	0.01
$x_{s,0}$	10, 0, 0, 0, 0, 0, 0, 0, 0, 10
ϕ_t	$20 y_t e^{\frac{-y_t}{10}}$
$U(C_t)$	$log(C_t)$

Table 2: Illustration of baseline scenario I

With regeneration that depends on the basal area and the initial state of the forest in Table 2, the optimal solution converges towards a steady-state where a NIPF owner harvests her or his standing forest when trees reach the dimension of size-class 9 and harvesting takes place every year. Figure 1 below depicts the harvesting schedule. This state is also characterized by a stationary basal area of 14 m^2 and smooth cutting of trees. Under this scenario only around 60 trees reach size-class 9 and the forest yields a timber revenue of approximately €1838, where around 43 m^3 of timber are cut from size-class 9 in the steady-state. In addition, around 72 % of trees move to the next size-class for the next period and only 24 % stay at its current size. Hence, around 4 % of the trees die every period. In what concerns natural regeneration and since it depends on the basal area, it increases until it becomes stable at the steady-state level.

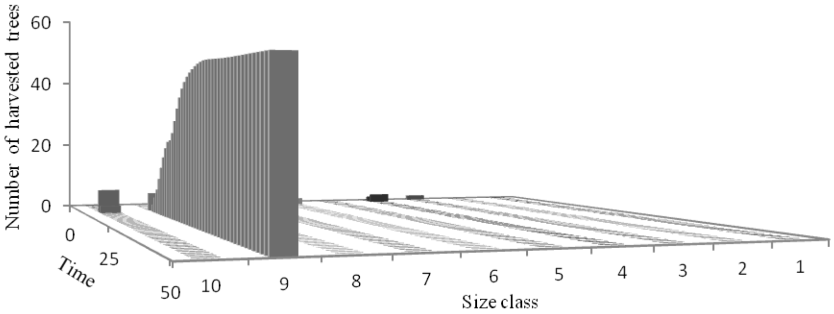

Figure 1: Harvesting schedule

Table 3 below illustrates a new scenario where a new functional form for the regeneration form is considered.

Parameter | Function	Value | Functional Form
ρ	0.01
$x_{s,0}$	10, 0, 0, 0, 0, 0, 0, 0, 0, 10
ϕ_t	$\sum_{s=1}^{n} \eta_s h_{s,t}$
$U(C_t)$	$log(C_t)$

Table 3: Illustration of scenario II

When accounting for regeneration in gaps and considering the initial state of the forest in Table 3, a new steady-state is achieved. In this case, trees are harvested simultaneously when they reach size-classes 3 and 7. Size-class

7 trees are the ones that yield higher timber volume and around 111 trees are harvested, whereas in size-class 3 around 291 trees are cut in the steady-state. Note that size-class 3 is the first class of trees with economic value, given the values for weighted timber type prices in Table 1, that is, $j_3 = €2$. Timber revenue is stationary at a level of around €2823. Furthermore, the optimal basal area reaches the value of 20 m^2 and when harvested the forest yields around 62 m^3 of timber per period. Regarding the transition of trees, around 59 % of the trees move to the next size-class in the next period and 24 % stay at its current size, in the steady-state. Consequently, 17 % of the trees die in each period. Natural regeneration increases towards the steady-state level.

By contrast, in Tahvonen (2009) where the problem consists of a profit-maximization based on a linear function, all the cycles become smooth and the management practice remains unchanged. In other words, uneven-aged management is observed. Therefore, the forest equilibrium is no longer a stationary-cycle but a stable steady-state. When harvesting occurs at size-class 7, harvesting cycles no longer exist, since the NIPF owner aims to smooth consumption over time. Figure 2 depicts the harvesting schedule under this scenario, where thinning[9] from below is concentrated in size-class 3, i.e., harvesting of trees from the lower end of the size-class distribution occurs.

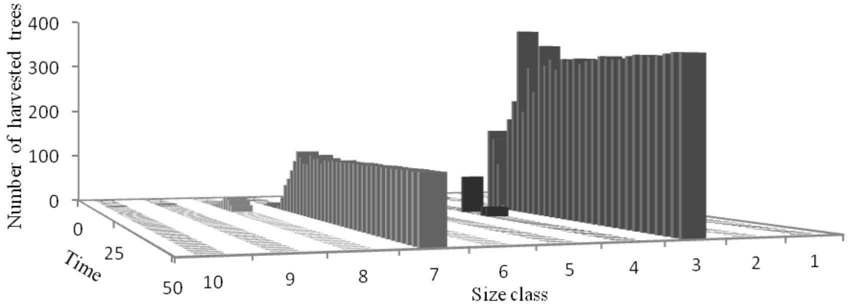

Figure 2: Harvesting schedule

3.1.1 Sensitivity analysis for the discount rate

The following table introduces a scenario where sensitivity to the discount rate is analysed.

Parameter | Function	Value | Functional Form
ρ	0.03
$x_{s,0}$	10, 0, 0, 0, 0, 0, 0, 0, 0, 10
ϕ_t	$20 y_t e^{\frac{-y_t}{10}}$
$U(C_t)$	$log(C_t)$

Table 4: Illustration of scenario III

With an increase in the subjective rate of time preference, ρ, the NIPF owner is more impatient and values

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[9] Thinning refers to a regime where the forest owner selects trees for harvesting during a rotation, thereby creating improved growing conditions for the remaining stand of trees. Cao et al. (2006), Hyytiäinen et al. (2006) and Chrimes et al. (2007) discuss deeply the implications of thinning in the optimal harvesting schedule.

relatively more the present than the future. Note that the subjective rate of time preference reflects impatience for future consumption, while the market interest rate is the payoff of delaying consumption. In this case and under regeneration that depends on the basal area, trees are harvested when they reach size-class 9, yielding 42 m^3 and €1838 periodically. The basal area is stationary at the level of 14 m^2 and 72 % of the trees move to the next size-class for the next period, while around 24 % stay at its current size. The regeneration of trees increases toward the steady-state level. The optimal solution is the same as the one presented before in scenario I. This result is consistent with the findings in Tahvonen (2009), where under an increase in the subjective rate of time preference, it is optimal to apply uneven-aged forest management.

Table 5 accounts for a regeneration form that depends on the gaps left by harvested trees.

Parameter | Function	Value | Functional Form
ρ	0.03
$x_{s,0}$	10, 0, 0, 0, 0, 0, 0, 0, 0, 10
ϕ_t	$\sum_{s=1}^{n} \eta_s h_{s,t}$
$U(C_t)$	$log(C_t)$

Table 5: Illustration of scenario IV

Based on a similar scenario as the previous one but now accounting for a regeneration form that depends on the gaps left by harvested trees, one may conclude that trees are harvested when they reach both size-classes 3 and 7, yielding 63 m^3 of timber and €2820 of revenue. Furthermore, the basal area reaches a level of around 20 m^2 in the steady-state and 60 % of the trees reach the next size-class, whereas 34 % remain at its current size. Additionally, regeneration increases over time until it remains stable in the optimal equilibrium. Thus, uneven-aged management still occurs and thinning from below is concentrated on size-class 3 as in Tahvonen (2009).

3.1.2 Sensitivity analysis for the initial state of the forest

In this part, a different initial state of the forest is considered.

Parameter | Function	Value | Functional Form
ρ	0.01
$x_{s,0}$	40, 40, 40, 40, 40, 40, 40, 40, 40, 40
ϕ_t	$20 y_t e^{\frac{-y_t}{10}}$
$U(C_t)$	$log(C_t)$

Table 6: Illustration of scenario V

When considered the case of a normal forest structure and regeneration dependent on the basal area, the convergence to the steady-state remains unchanged, comparatively to the initial state for $x_{s,0}$ in the baseline scenario I. Natural regeneration becomes almost steady since the initial time span given the homogeneity of trees over size-classes. Even though the forest owner has a higher initial endowment of forest resources equally distributed over size-classes, it does not change the harvesting behavior over time because, by influencing regeneration and ingrowth of trees, harvesting still occurs when trees reach size-class 9 and it does not pay off to harvest trees from lower size-classes.

Table 7 below shows the definition of regeneration that is dependent on the gaps left by harvested trees in Table 1.

Parameter | Function	Value | Functional Form
ρ	0.01
$x_{s,0}$	40, 40, 40, 40, 40, 40, 40, 40, 40, 40
ϕ_t	$\sum_{s=1}^{n} \eta_s h_{s,t}$
$U(C_t)$	$log(C_t)$

Table 7: Illustration of scenario VI

In the case where regeneration depends on the gaps left by harvested trees, harvesting still occurs when trees reach size-classes 3 and 7. Minor cycles occur at the beginning of the time span. Timber revenue remains stationary at the level of €2820, around 59 % of the trees move to the next size-class and around 34 % remain at its current size. Natural regeneration is volatile initially as a result of cycles in the harvesting regime. Thus, when harvesting at size-class 3 new seedlings germinate *in situ* and the forest owner is able to influence the regeneration of trees. The outcome remains unchanged despite the higher endowment of forest resources. Thinning from below at size-class 3 occurs as in Tahvonen (2009), which allows the forest owner to harvest her or his forest when trees reach size-class 7. The optimal solution is a stable steady-state, despite minor cycles in the variables of interest in the beginning of the time horizon.

Discussion of the results

All these solutions are characterized by uneven-aged forest management where trees of different sizes and diameters coexist in the same unit of land. In general, linear transition specifications imply uneven-aged management regimes and the results are in line with the findings in Tahvonen (2009) and in Tahvonen (2011).

Harvesting of trees takes place over the same size-classes as in Tahvonen (2009). As showed above, if regeneration in gaps is considered, the forest owner will harvest the trees from size-classes 3 and 7 and a larger number of trees is harvested when they reach size-class 3. This is because size-class 3 is the smallest one with economic value at which regeneration is non-existent ($\eta_3 = 0$). In this sense, she or he is interested in controlling regeneration in order to increase future forest income flows and this is the reason why thinning from below occurs in this size-class. In other words, a rational and forward-looking forest owner can make a certain amount of timber revenue when trees reach this size-class and at the same time harvesting works as regeneration contributing to ingrowth of trees. Even though the percentage of trees that die in every time period is larger under regeneration in gaps, higher timber revenue is obtained under this form because the forest owner harvests trees in two size-classes over time, influencing namely the regeneration.

By simply including a logarithmic specification for the objective function, harvesting cycles are eliminated since the NIPF owner aims at smoothing periodic consumption.

3.2 Augmented model with an exogenous asset endowment

This subsection accounts for the introduction of an exogenous financial asset. Thus, the decision-maker no longer decides solely on the harvesting level, but also considers the amount of a financial asset. To put it differently, the NIPF owner can use both harvesting income and wealth from a financial asset that evolves at a market interest rate to make consumption decisions. Hence, the utility-maximizer NIPF owner derives utility from periodic consumption specified in logarithmic form, taking into account constraints $(2a) - (2b)$ and $(12a) - (14)$. She or he is endowed with an amount a_0 of the financial asset at the beginning of the time horizon. Financial assets may accumulate over time, implying that the forest owner economic condition need not to be the same after each harvest.

Table 8 below shows the parameters values and specifications to one of the baseline scenarios analysed in this model specification.

Parameter | Function	Value | Functional Form
ρ	0.01
r	0.01
$x_{s,0}$	10, 0, 0, 0, 0, 0, 0, 0, 0, 10
ϕ_t	$20y_t e^{\frac{-y_t}{10}}$
$U(C_t)$	$log(C_t)$

Table 8: Illustration of the baseline scenario II

When regeneration depends on the basal area and considering that ρ and r are the same as before, she or he will harvest the forest when trees reach size-class 9 and timber revenue converges to the level of €1838. In contrast, in the initial time periods harvesting revenue is smaller given that trees yield less volume and, therefore, the NIPF owner consumes the income from the asset to offset the low level of revenue. Despite that $\rho = r$, harvesting will not be constant initially because the optimal cutting policy depends on the initial state of the forest. This is in line with the life cycle theory, since the forest owner uses the asset in periods of lower timber revenue to smooth consumption over time. The yield is stationary at around 42 m^3 and the basal area remains constant at 14 m^2 in the steady-state. In addition, around 72 % of trees move to the next size-class every period in the steady-state, while 24 % remain at its current size. Natural regeneration increases toward the steady-state level.

Even though the forest owner can benefit from financial asset income, it does not change the harvesting schedule, since trees are still cut when they reach size-class 9. This is consistent with the conclusions in Tahvonen (2009).

Table 9 accounts for a different regeneration function.

Parameter | Function	Value | Functional Form
ρ	0.01
r	0.01
$x_{s,0}$	10, 0, 0, 0, 0, 0, 0, 0, 0, 10
ϕ_t	$\sum_{s=1}^{n} \eta_s h_{s,t}$
$U(C_t)$	$log(C_t)$

Table 9: Illustration of scenario VIII

When regeneration depends on the gaps left by harvested trees and considering cases, the NIPF owner harvests trees from size-classes 3 and 7. Under this scenario, optimal timber revenue exhibits a cyclical pattern. This is in part explained by cyclical harvesting, mainly from size-class 3, which gives rise to a cyclical timber volume and number of standing trees. Consequently, given the form of regeneration that depends on the gaps left by harvested trees, a cyclical harvesting schedule arises. The basal area has a value of 14 m^2 and 59 % of the trees move to the next size class, while around 35 % do not grow in equilibrium. Furthermore, the NIPF owner has access to higher timber revenue and, hence, to higher periodic consumption. Given the cyclical behavior in the basal area, the regeneration form evolves accordingly. However, the optimal solution is still characterized by a stable steady-state, consistent with the findings in Tahvonen (2009).

3.2.1 Sensitivity analysis for the discount rates

Table 10 below shows an increase in the subjective rate of time preference relatively to the market return rate on the financial asset.

Parameter | Function	Value | Functional Form
ρ	0.03
r	0.01
$x_{s,0}$	10, 0, 0, 0, 0, 0, 0, 0, 0, 10
ϕ_t	$20 y_t e^{\frac{-y_t}{10}}$
$U(C_t)$	$log(C_t)$

Table 10: Illustration of scenario IX

With an increase in the subjective rate of time preference and assuming a regeneration form that depends on the basal area, the forest owner harvests trees when these reach size-class 9. Furthermore, she or he exhausts the asset income in the first periods as current utility levels are relatively more valued. Therefore, there is no accumulation of the financial asset income to future periods, implying that consumption will be derived solely from timber revenue in the steady-state. Figure 3 below depicts the financial asset income path. In this case, the opportunity cost of postponing consumption is higher than the market interest rate that is the return she or he would obtain from postponing consumption. She or he enjoys a steady periodic consumption level after a given period, above which convergence to the stable steady-state occurs.

Timber revenue is stationary at €1838 and around 43 trees are harvested in each period, yielding around 42

m^3 of timber periodically. The basal area remains constant at 14 m^2. Around 72 % of the trees move to the next size-class and 24 % remain at its current size in equilibrium. In this case, natural regeneration increases until it reaches the optimal value.

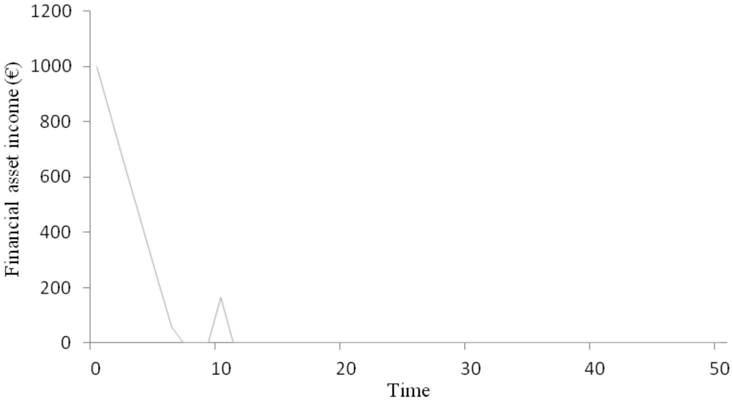

Figure 3: Financial asset income schedule

Table 11 depicts a scenario where the market interest rate is higher than the subjective rate of time preference.

Parameter | Function	Value | Functional Form
ρ	0.01
r	0.03
$x_{s,0}$	10, 0, 0, 0, 0, 0, 0, 0, 0, 10
ϕ_t	$20y_te^{\frac{-y_t}{10}}$
$U(C_t)$	$log(C_t)$

Table 11: Illustration of scenario X

When setting $r > \rho$, even though timber revenue converges to a steady-state at around €1838 given regeneration dependent on the basal area, periodic consumption shows an increasing fashion over time. This is because the return rate on the asset is higher and, therefore, the NIPF owner can enjoy higher periodic consumption because she or he is richer, as it pays off to postpone consumption. Around 43 trees are harvested when they reach size-class 9, yielding around 42 m^3 of timber periodically. The basal area is constant at the level of 14 m^2. In addition, around 72 % of the trees move to the next size-class, while around 24 % remain at its current size and natural regeneration increases until its stable equilibrium. Figure 4 depicts the financial asset income time path.

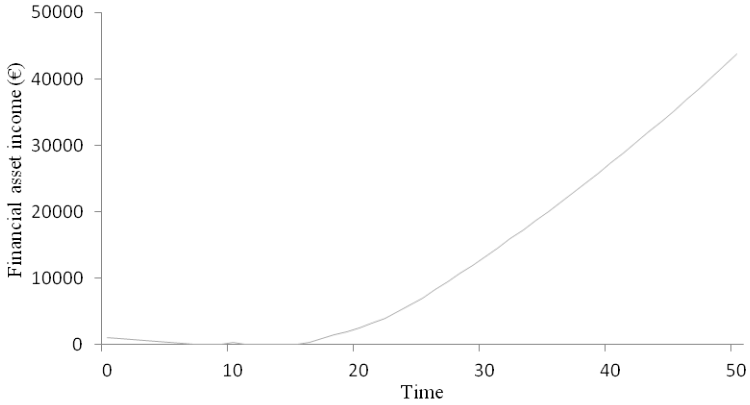

Figure 4: Financial asset income schedule

3.2.2 Sensitivity analysis for the initial state of the forest

The next cases include a different state of the forest at the beginning of the time horizon with 40 trees uniformly distributed over size-classes, i.e., the case of a normal forest. Table 12 below characterizes one of these scenarios.

Parameter | Function	Value | Functional Form
ρ	0.01
r	0.01
$x_{s,0}$	40, 40, 40, 40, 40, 40, 40, 40, 40, 40
ϕ_t	$20y_te^{\frac{-y_t}{10}}$
$U\left(C_t\right)$	$log\left(C_t\right)$

Table 12: Illustration of scenario XI

Under scenario XI, the forest owner decides upon harvesting when trees reach size-class 9. In this case, numerical simulations suggest timber yield undershoots toward its equilibrium value, 42 m^3, because harvesting initially decreases. The basal area undershoots and then adjusts toward the equilibrium value of 14 m^2. Timber revenue is stationary around €1838 after a given period, while consumption is steady in the initial periods and then adjusts toward the steady-state value at around €1838. Even though the asset income initially increases, it is suddenly exhausted. Around 72 % of the trees move to the next size-class and 24 % stay at its current size. Moreover, natural regeneration remains almost steady since the very beginning of the time horizon. This can be explained by the uniform distribution of trees over size-classes, which allow the NIPF owner to have a forest land where trees grow continuously.

Table 13 depicts a baseline scenario. The next three scenarios introduce a regeneration function that depends on the gaps left by harvested trees and considers the case of a normal forest.

Parameter | Function	Value | Functional Form
ρ	0.01
r	0.01
$x_{s,0}$	40, 40, 40, 40, 40, 40, 40, 40, 40, 40
ϕ_t	$\sum_{s=1}^{n} \eta_s h_{s,t}$
$U(C_t)$	$log(C_t)$

Table 13: Illustration of baseline scenario III

With equal subjective rate of time preference and market return on the financial asset, trees are harvested when they reach size-classes 3 and 7. Harvesting is cyclical over time, which gives rise to fluctuations in the timber yield and, consequently, on timber revenue. This also gives rise to fluctuations in the number of standing trees and, thus, on fluctuations in the basal area. Periodic consumption is constant at the level of €2824 in the steady-state. It is derived mainly from timber revenue, because the asset income is initially exhausted and then fluctuates but the variations are not of high magnitude. Around 59 % of the trees move to the next size-class while 34 % do not grow. Since the optimal harvesting policy is characterized by cycles, natural regeneration has also a cyclical behavior over time.

As in Tahvonen (2009), trees are cut when they reach size-classes 3 and 78, where thinning from below takes place at size-class 3, giving rise to natural regeneration.

Table 14 accounts for an increase in the subjective rate of time preference, relatively to the market return rate.

Parameter | Function	Value | Functional Form
ρ	0.03
r	0.01
$x_{s,0}$	40, 40, 40, 40, 40, 40, 40, 40, 40, 40
ϕ_t	$\sum_{s=1}^{n} \eta_s h_{s,t}$
$U(C_t)$	$log(C_t)$

Table 14: Illustration of scenario XIII

When the subjective rate of time preference is higher than the interest rate, trees are harvested when they reach size-classes 3 and 7. Even though there are cycles in the initial periods, the optimal solution converges toward a stable steady-state with smooth cutting. Therefore, the initial fluctuations in harvesting will influence the behavior of the other variables that depend on harvesting. Consumption decisions are derived solely from timber revenue, given that the NIPF owner exhausts the asset income in the initial periods, because of her or his high valuation of the present. In other words, impatience for future consumption increases, that is, it is more costly to postpone consumption. Due to the existence of cycles in the initial time horizon, natural regeneration follows this cyclical behavior.

Table 15 accounts for a market interest higher than the subjective rate of time preference, which reflects a higher opportunity cost of postponing consumption.

Parameter | Function	Value | Functional Form
ρ	0.01
r	0.03
$x_{s,0}$	40, 40, 40, 40, 40, 40, 40, 40, 40, 40
ϕ_t	$\sum_{s=1}^{n} \eta_s h_{s,t}$
$U(C_t)$	$log(C_t)$

Table 15: Illustration of scenario XIV

Under this scenario, trees are harvested when they reach size-classes 3 and 7. Cyclical harvesting takes place in these size-classes, which gives rise to fluctuations in timber yield, basal area and timber revenues. Hence, natural regeneration is volatile. The consumption pattern increases over time since the asset becomes more valuable as time goes by, as it pays off to delay consumption. Thus, the NIPF owner can enjoy greater periodic utility and has higher welfare. It is worth noting that for low subjective time preference, financial assets accumulate and the rational forest owner increases her or his consumption level.

Discussion of the results

All these cases illustrate uneven-aged forest management where harvesting occurs in different size-classes and trees are cut with different dimensions over time. By introducing a financial asset in the model, the forest owner is allowed to change her or his consumption schedule, based on her or his impatience for delaying consumption and the return on the financial asset. Alternatively, by accounting for a non-forest asset, the forest owner has an additional tool that can use to adjust consumption decisions. Depending on the regeneration of trees, the model predicts thinning from below may occur and that changes the harvesting schedule over time, according to the findings in Tahvonen (2009).

3.3 Augmented model with an amenity valuation: preliminary results

In this extension, the impacts of amenity valuation on the decision-making of a NIPF owner and, consequently, on the forest equilibrium are studied. This is motivated by the fact that the value of amenities is unknown, otherwise it would appear in the forest owner utility function $U(C_t)$. In this sense, it is assumed the forest owner makes value comparisons between timber income and the *in situ* value of her or his forest, by trading-off and choosing the alternative that maximizes her or his welfare.

3.3.1 Sensitivity analysis for the regeneration form

In this section, several scenarios are analysed. The baseline scenario is represented below in Table 16.

Parameter | Function	Value | Functional Form
ρ	0.01
r	0.01
$x_{s,0}$	10, 0, 0, 0, 0, 0, 0, 0, 0, 10
ϕ_t	$20y_te^{\frac{-y_t}{10}}$
$U\left(C_t\right)+M\left(A_t\right)$	$log\left(C_t\right)+log\left(\sum_{s\geq \underline{s}}^{n}q_sx_{s,t}\right)$

Table 16: Illustration of baseline scenario IV

This scenario corresponds to the case with equal ρ and r, the regeneration form depends on the basal area, the initial state of the forest is not the one of a normal forest structure and amenities comprise the volume of all standing trees. In this case, trees are harvested when they reach size-classes 9 and 10, resulting in a periodic constant timber yield of around 41 m^3 since the beginning of the time span. The basal area is stationary at around 16 m^2 and amenity valuation converges toward a steady-state. Timber revenue is constant at the level of €1624 and periodic consumption is steady since the early beginning at the level of €1631. Around 69 % of the trees move to the next size-class, while 27 % remain at its current size. Regeneration is constant over time and emerges from the constant harvest policy when trees reach both size-classes 9 and 10.

A new scenario is depicted in Table 17.

Parameter | Function	Value | Functional Form
ρ	0.01
r	0.01
$x_{s,0}$	10, 0, 0, 0, 0, 0, 0, 0, 0, 10
ϕ_t	$\sum_{s=1}^{n}\eta_sh_{s,t}$
$U\left(C_t\right)+M\left(A_t\right)$	$log\left(C_t\right)+log\left(\sum_{s\geq \underline{s}}^{n}q_sx_{s,t}\right)$

Table 17: Illustration of scenario XVI

Considering a similar case than the previous one but now accounting for a regeneration form that depends on the gaps left by harvested trees, harvesting occurs when trees reach size-classes 3 and 9. The majority of harvesting occurs in size-class 3. At the beginning of the period, numerical simulations suggest that harvesting is slightly cyclical, giving rise to cyclical timber yield in the beginning, before convergence toward the steady-state at the level of 49 m^3. The basal area is stationary at around 27 m^2. The amenity valuation assumes minor cycles until it becomes roughly constant. Timber revenue fluctuates in the initial periods but becomes stationary at around €2396. Consumption is also smooth over time. Under this scenario, 47 % of the trees move to the next size-class while 45 % of the trees stay at its current size in every period. Regeneration of trees varies in the initial periods and then becomes constant at around 607 trees.

By comparison, it should be noted that regeneration in gaps yields higher welfare for the NIPF owner in these two scenarios. Not only the amenity valuation is higher, but also timber revenue is higher. This is explained by the regeneration created when harvesting occurs at size-class 3 and trees are thinned from below.

3.3.2 Sensitivity analysis for the amenity valuation

The next two cases account for a change in the amenity valuation, $M(A_t)$. Specifically, Table 18 accounts for an amenity valuation based on the standing volume of the last size-class trees.

Parameter | Function	Value | Functional Form
ρ	0.01
r	0.01
$x_{s,0}$	10, 0, 0, 0, 0, 0, 0, 0, 0, 10
ϕ_t	$20y_te^{\frac{-y_t}{10}}$
$U(C_t)+M(A_t)$	$log(C_t)+log(q_nx_{n,t})$

Table 18: Illustration of scenario XVII

When regeneration depends on the basal area and amenities are valued solely from the timber volume of trees from the last size-class, harvesting occurs when trees reach size-class 9 and it gives rise to a timber yield of around 21 m^3, with basal area stationary at 17 m^2. The amenity valuation is constant after a given period and the same happens with timber revenue and consumption. The steady-state is also characterized by 69 % of trees moving to the next size-class and around 28 % staying at its current size. Natural regeneration is constant over time.

3.3.3 Sensitivity analysis for the discount rates

The following two cases address an increase in the subjective rate of time preference, so that it is higher than the market interest rate. In Table 19 the regeneration function depends on the basal area, whereas in Table 20 it is dependent on the gaps left by harvested trees.

Parameter | Function	Value | Functional Form
ρ	0.03
r	0.01
$x_{s,0}$	10, 0, 0, 0, 0, 0, 0, 0, 0, 10
ϕ_t	$20y_te^{\frac{-y_t}{10}}$
$U(C_t)+M(A_t)$	$log(C_t)+log\left(\sum_{s\geq \underline{s}}^{n} q_sx_{s,t}\right)$

Table 19: Illustration of scenario XVIII

When regeneration depends on the basal area, trees are harvested when these reach size-classes 9 and 10, timber yield is stationary at the level of 41 m^3, the basal area reaches the steady state at around 15 m^2, while the amenity valuation is constant. Periodic consumption is constant at the level of €1650 and it is derived solely from timber revenue, since the asset income is exhausted in the initial time periods. In the steady-state, around 69 % of the trees move to the next size-class, while 26 % stay at its current size. Natural regeneration is stationary at the level of 66 trees. Through comparison of scenarios XVII and XVIII, this result is in line with the conclusions in Kuuluvainen and Uusivuori (2005), given that when the subjective rate of time preference is higher than market return rate on the financial asset, the forest owner reacts by having lower utility from amenities of the standing timber.

Parameter | Function	Value | Functional Form
ρ	0.03
r	0.01
$x_{s,0}$	10, 0, 0, 0, 0, 0, 0, 0, 0, 10
ϕ_t	$\sum_{s=1}^{n} \eta_s h_{s,t}$
$U(C_t) + M(A_t)$	$log(C_t) + log\left(\sum_{s \geq \underline{s}}^{n} q_s x_{s,t}\right)$

Table 20: Illustration of Scenario XIX

By considering a similar scenario as the previous one but accounting for a regeneration form that depends on the gaps left by harvested trees, harvesting occurs when trees reach size-classes 3 and 9, resulting in a periodic timber yield of around 50 m^3, timber revenue of around €2430 and constant consumption. Around 47 % of the trees move to the next size-class, while around 45 % remain at its current size. The optimal value for regeneration is 66 trees.

It is worth noting that once more the regeneration form is determinant for the level of timber revenue. By direct comparison, in these two last scenarios, regeneration in gaps yields higher welfare. However, this result is not in line with the findings in Kuuluvainen and Uusivuori (2005), who conclude that, given $\rho > r$, the forest owner values less of amenities of standing trees, because the amenity value in scenario XIX is lower than in scenario XVI.

Tables 21 and 22 introduce scenarios for an increase in the market interest rate, so that it is higher than the subjective rate of time preference, holding the same specifications as in the two previous cases but accounting for different regeneration functions.

Parameter | Function	Value | Functional Form
ρ	0.01
r	0.03
$x_{s,0}$	10, 0, 0, 0, 0, 0, 0, 0, 0, 10
ϕ_t	$20 y_t e^{\frac{-y_t}{10}}$
$U(C_t) + M(A_t)$	$log(C_t) + log\left(\sum_{s \geq \underline{s}}^{n} q_s x_{s,t}\right)$

Table 21: Illustration of scenario XX

Thus, when regeneration depends on the basal area, harvesting occurs mainly at size-class 9 (even though for a short period minor harvesting occurs when trees reach size-class 10), the timber yield is stationary at around 42 m^3, the amenity valuation becomes steady, timber revenues reach the level of €1838 and since the financial asset income increases over time, it allows the NIPF owner to enjoy an increasing periodic consumption. The increasing consumption path arises because the marginal costs of postponing consumption are lower than the market return rate, supporting the findings in Kuuluvainen and Uusivuori (2005). In equilibrium, around 71 % of the trees move to the next size-class, while around 24 % stay at its current size. The regeneration is stable at the level of 69 trees.

Parameter | Function	Value | Functional Form
ρ	0.01
r	0.03
$x_{s,0}$	10, 0, 0, 0, 0, 0, 0, 0, 0, 10
ϕ_t	$\sum_{s=1}^{n} \eta_s h_{s,t}$
$U(C_t) + M(A_t)$	$log(C_t) + log\left(\sum_{s \geq \underline{s}}^{n} q_s x_{s,t}\right)$

Table 22: Illustration of scenario XXI

In the case of regeneration that depends on the gaps left by harvested trees, these are harvested when they reach size-classes 3 and 9, timber yield is stationary at the level of 50 m^3 and the basal area is stationary at 26 m^2. The amenity valuation is stable and both timber revenue and consumption are stationary at the level of around €2428, since the NIPF owner exhausts the asset income in the beginning of the time period. Furthermore, around 47 % of the trees move to the next size-class, while around 45 % stay at its current size. Regeneration is stable at the level of 613 trees. The results are in line with the findings in Kuuluvainen and Uusivuori (2005).

By comparison of these two last cases, the regeneration form that depends on the gaps left by harvested trees[10] yields higher welfare.

3.3.4 Sensitivity analysis for the functional form of utility

The final scenario to be analysed is the one that involves a Cobb-Douglas specification for the functional objective, which is depicted below in Table 23.

Parameter | Function	Value | Functional Form
ρ	0.01
r	0.01
$x_{s,0}$	10, 0, 0, 0, 0, 0, 0, 0, 0, 10
ϕ_t	$20y_t e^{\frac{-y_t}{10}}$
$W(C_t, A_t)$	$C_t^{\lambda} A_t^{1-\lambda}$, $\lambda = 0.5$

Table 23: Illustration of scenario XXII

A scenario with $\rho = r$, regeneration that depends on the basal area and initial state of non-normal forest is studied. In this case, trees are harvested when they reach size-class 9. Timber yield and basal area are constant at 42 m^3 and at 14 m^2, respectively. The amenity valuation schedule is depicted in Figure 5 below. Periodic timber revenue is €1838, while consumption is valued at €1845. The consumption schedule is illustrated in Figure 6. Around 72 % of the trees move to the next size-class, while 24 % remain at its current size. Finally, natural regeneration is constant at the level of 69 trees, over time. In all the cases, the results suggest uneven-aged forest management.

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[10] The modeling of the regeneration form that depends on the gaps left by harvested tree may seem too restrictive to represent a long-run forest equilibrium when a stand is not harvested. In this sense, one could include natural mortality of large trees and delayed regeneration in the gaps left by dead trees as suggested by Tahvonen (2009). However, the results would remain unchanged, since in almost all the cases previously studied harvesting takes place before trees reach the largest size-class.

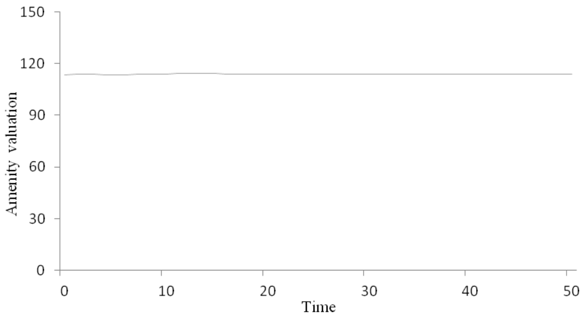

Figure 5: Amenity valuation schedule

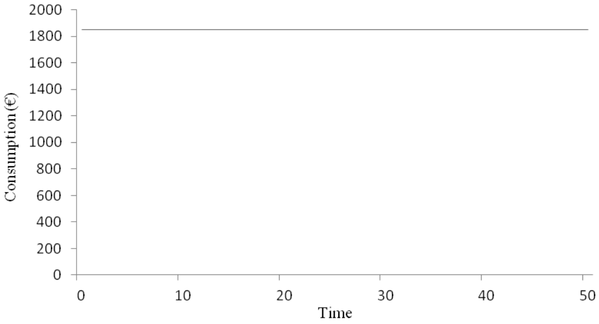

Figure 6: Consumption schedule

Discussion of the results

Numerical simulations suggest that when a forest owner values amenity benefits from her or his standing forest, it leads to higher heterogeneity in harvesting policy and in the size-classes harvesting takes place. Furthermore, when considering amenity values, it makes the decision problem more complex than in the classical rotation framework of MSY or the Faustmann solution. In addition, the forest owner enjoys greater amenity utility when the regeneration form is dependent on the gaps left by harvested trees. Moreover, in all the previous scenarios analysed, the NIPF owner values relatively more harvesting benefits than non-market benefits measured by an amenity valuation. This is because most NIPF owners finance their daily expenditures through harvesting income, hence the higher valuation of forest income, rather than amenities, such as landscape aesthetics.

The results also suggest that with *in situ* valuation functions, the optimal solution converges towards forests with increasing heterogeneity in age-structure, as pointed out by Salo and Tahvonen (1999), given that harvesting occurs in different size-classes.

4 Conclusions

A new forest economic optimization framework based on a transition matrix model for trees explains how NIPF owners managing a multiple forest size-class structure take decisions regarding optimal forestry, accounting for both economic and non-market benefits of standing forests and determining the optimal forest management regime endogenously. An infinite-horizon discrete time utility model is developed, where sensitivity analyses for key-drivers such as the subjective rate of time preference, the market return on a financial asset, the initial state of the forest and the regeneration forms of trees influencing the optimal solution are considered.

Numerical simulations suggest uneven-aged forest management occurs in all the three model specifications addressed. In its simpler formulation, a NIPF owner values harvesting revenues solely or values consumption from simultaneously harvesting revenue and a financial asset that evolves at a market return rate. The analysis also widens to include a specification where a forest owner values consumption derived from harvesting revenue, exogenous asset and also amenity benefits.

When the first specification is addressed, optimal forestry is consistent with the findings in Tahvonen (2009). By controlling for the regeneration form and harvesting from lower size-classes, the forest owner obtains higher harvesting revenue, which allows her or him to enjoy higher welfare. Considering also a financial asset, the forest owner makes use of the asset income to smooth consumption in the periods harvesting revenues are lower. Her or his subjective time preference will determine whether or not the financial income will be exhausted in the initial time span or will provide her or him with greater periodic flows over time, allowing her or him to enjoy an increasing periodic consumption. Therefore, the financial asset proves to be an additional tool she or he can benefit from to adjust the consumption schedule. When amenities are introduced in the model, higher heterogeneity arises, namely in the harvesting schedule. The forest owner values forest income relatively more than the benefits derived from standing forests, which is explained by the fact that most forest owners rely on harvesting income to finance their consumption decisions.

In this paper, it is showed that a multiple-period generalization of the two-period size-class model will not make the model equivalent to the Faustmann model and its extensions. This model contributes to Forest Economics literature by applying to NIPF owners a multiple size-class structure and accounting for the complexities of amenity valuation of a standing forest, by introducing endogenously the management form.

To sum up, given the importance of these economic agents in the forest sector in major countries, the model sheds new light in understanding their behavior and how that is crucial to determine timber markets developments.

5 Future Research

In the theoretical model proposed to design NIPF owner's behavior, an infinite-time horizon was assumed in its theoretical specification. However, a finite-time horizon was considered. Typically, a forest owner holds a forest property for 30 to 40 years and inheritance is a common practice of forest land transfer. This means that one can extend the model to see how bequest motives influence forest management decisions. The relevance of this topic is attributed to the role bequest motives have in the long-run optimal forest stock and amenities in an economy which itself is crucial for policymakers concerned about sustaining forest cover over time.

Another possible channel is to see to which extent government fiscal policies influence forest owner bequests and affect even- vs. uneven-aged forestry. In this sense, heavy inheritance taxes are likely to make monetary bequests relatively more profitable and this may lead forest owners to harvest forests when they inherit them. An additional relevant analysis is to widen the model where shading[11] specifications can be addressed and see the implications in the optimal forest regime. Further analyses in the third specification of the model will be developed in a near future.

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[11] Shading affects inter-trees competition and consequently impacts on trees transition and growth.

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