Forthcoming in
Journal of Economic Dynamics and Control

Experimentation with Accumulation*

Maria A. Cunha-e-Sá   Vasco Santos

Universidade Nova de Lisboa, Faculdade de Economia
Campus de Campolide, PT–1099–032 Lisboa, Portugal

February 2007

Abstract

We study signal-dependent experimentation in the presence of accumulation and show that the passive-learner's action surprisingly coincides with the experimentor's when the unknown term is the one determining the decay rate of the stock, while they differ when the parameter being learned is the one measuring the accumulation rate. These results highlight the importance of the dynamic structure of the problem in signal-dependent experimentation. Moreover, they have important consequences for the pollution-accumulation debate currently in progress.

Keywords: Signal-dependent experimentation, accumulation, pollution.
JEL classification numbers: D83, Q20.

1 Introduction

RECENTLY, the experimentation literature has begun to study *signal-dependent* problems where today's signal, besides prompting the updating of beliefs, is also directly payoff relevant in the future.[1] We add to the signal-dependent experimentation literature by showing that with accumulation, the experimentation effect—defined as the difference between the experimentor's choice and the passive learner's—can be zero. This is so in problems with accumulation when the unknown parameter is the one affecting the decay rate of the stock.

These results highlight the importance of the dynamic structure in signal-dependent problems, as they contrast with Datta, Mirman and Schlee's (2002). These authors have shown that the experimentation effect is in general not zero in signal-dependent models. However, their dynamic structure cannot fully accommodate accumulation and decay, and the uncertainty surrounding them. It is exactly for the latter that we show experimentation to be necessarily nil.

One important application of our results involves the currently raging pollution-accumulation debate concerning the tradeoff between the benefits of extra output and consumption today versus the future social cost of the pollution thus caused. For instance, the effects of greenhouse gases (GHG) on climate

*Thanks are due to Pedro P. Barros, Maria M. Ducla-Soares, Iliyian Georgiev, Mário Páscoa, Volker Wieland, three anonymous referees and very especially Leonard J. Mirman for useful suggestions. The authors retain responsibility for any remaining errors.

[1] Most of the extant experimentation literature involves *signal-independent* problems, of which probably the best-known example is that of one or several firms altering their myopic quantity or price choice to learn the demand they face (see *inter alia* Alepuz and Urbano (1999), Mirman et al. (1977, 1993a, b, 1994)). Experimentation is also relevant in macroeconomics (see, among others, Balvers and Cosimano (1994), Bertocchi and Spagat (1998) and Wieland (2000b, 2006).)

change are of great relevance to the issue of whether or not to curb emissions (and thus consumption) now.

We model a regulatory agency mandated to control consumption within a dynamic model of a stock externality in which the regulator may learn over time the relation between consumption and the pollution stock.[2] This problem involves, by its very nature, uncertainty regarding the parameters associated with physical processes. Thus, the agency entertains different likely scenarios concerning the unknown variables by holding priors whose supports encompass the possible realizations that the unknown parameters may take. This uncertainty makes learning of paramount importance to the agency since the information available to it is constantly evolving.

We discuss three different and realistic ways in which the agency may learn. First, it may simply take a prior and stick to it without ever revising it in light of new evidence. We term it a non-learner. This depicts the case of an agency faced with very limited resources to perform its regulatory task. Second, the agency may choose consumption without realizing that its own choice influences the subsequent learning process. We call it a passive learner. This might be the case of an agency with moderate though enough resources to keep updating its knowledge of the physical phenomena at the root of pollution accumulation. A third and fully-sophisticated behavior would entail the agency setting consumption while also weighting the value of extra information generated by different consumption levels. We name it an experimentor. We compare optimal consumption decisions in these three cases.[3]

Thus, besides contributing to the theoretical experimentation literature, our paper also adds to the climate change literature. Several papers have examined the effect of learning on optimal decisions with irreversibilities, building on Arrow and Fisher (1974), Henry (1974) and Epstein (1980).[4] Kelly and Kolstad (1999) developed an experimentation model based on the unobservable relationship between GHG levels and global mean temperature change, computing the expected time involved in learning this relationship. Recently, Karp and Zhang (2006) have studied the impact on optimal emissions of exogenous, anticipated learning about the relation between environmental stocks and economic damages in the context of a linear-quadratic model. In this setup, anticipated exogenous learning always increases emissions for a given set of beliefs.

Finally, tangential to this paper, there exists an important computationally-oriented literature that quantifies the extent of active learning in economic models with Bayesian learning, building upon the engineering dual-control literature. In this regard, particular mention should be given to the contributions of Kiefer (1989), Wieland (2000a, b, 2006), and Beck and Wieland (2002) who have studied the likelihood of incomplete learning and the speed of convergence to the true parameter values. The theoretical problems underlying these computational models, such as the asymptotic properties of beliefs, the possibility of incomplete learning, and the existence and characterization of the

[2]One may question the realism of this setting insofar as one does not observe regulatory agencies directly regulating consumption. In reality, regulators curb emissions. This, in turn, reduces consumption. In order not to clutter the presentation, the model includes only consumption and pollution while omitting emissions.

[3]Beck and Wieland (2002) numerically compare the decisions of an experimentor and a passive learner in a different setup.

[4]See, among others, Manne and Richels (1992), Kolstad (1993, 1996a, b), Peck and Teisberg (1993), Ulph and Ulph (1997), Nordhaus and Popp (1997) and Kennedy (1999).

optimal policy and value function in a Bayesian learning framework, are addressed in Easley and Kiefer (1988), Kiefer and Nyarko (1989), Aghion, Bolton, Harris and Jullien (1991) and Nyarko (1991), among others.

Our main conclusions concerning the pollution-accumulation problem are that a passive-learner agency, despite being less informationally sophisticated than an experimentor, does as well as the latter when the decay rate is unknown, but less well if the marginal contribution of consumption to the pollution stock is unknown. Thus, the lack of informational sophistication involved in passively learning may or may not matter to welfare, depending on the parameter whose value is unknown. Moreover, it is shown that when there is accumulation, beliefs interact with the pollution stock, qualifying the "naïve" view that one should always consume more in order to learn more.[5] This, in turn, yields a new (informational) rationale for conservative emissions' policies.

The remainder of the paper is organized as follows. The model is presented in Section 2. In Sections 3 and 4, the agencies' problems are examined when uncertainty affects the decay rate and the marginal contribution of consumption to the stock, respectively. Brief conclusions are drawn in Section 5.

2 The Model

The general problem under scrutiny is[6]

$$\begin{aligned} \underset{c_0, c_1, \dots}{\text{Max}} \quad & \sum_{t=0}^{\infty} \delta^t E\left[U\left(c_t, p_t\right)\right] \\ \text{s.t.} \quad & p_{t+1} = \alpha p_t + \beta c_t + \epsilon_t \\ & p_0 \text{ given} \end{aligned} \tag{1}$$

where c_t is consumption in period t, p_t stands for the pollution stock in period t, α represents the pollution decay rate, and β the rate at which consumption adds to the stock of pollution. Finally, ϵ_t is a random variable capturing all determinants of pollution besides consumption.[7]

The rates at which the stock of pollution decays, α, and consumption adds to the pollution stock, β, are unknown, being instead parameters to be learned; yet, other factors of a random nature (climatological, say) captured by ϵ_t obscure this relationship.[8] The agency must set consumption in this, to our mind, quite informationally-realistic rendition of the pollution-accumulation process.

Further restrictions and assumptions are now added.

Assumption 1 $\delta \in (0, 1)$.

[5] As Kelly and Kolstad (1999), p. 493, write: "We could interpret our climate change policy as a grand experiment: by increasing GHG emissions we obtain information about how emissions influence the climate."

[6] From now on, we severely abuse notation by writing the pollution-accumulation problem as if α and β were known, thus avoiding a cumbersome writing of the whole problem.

[7] Below we assume (in Assumption 7) that the distribution of ϵ_t has full support, $f(\epsilon) > 0, \forall \epsilon \in (-\infty, +\infty)$, thus ensuring that Bayesian updating never leads to a degenerate posterior. However, this permits negative values of p_t. It will become clear that this modeling trick does not affect the main results of the paper. Moreover, by increasing the exogenous initial pollution stock, p_0, one can arbitrarily decrease the likelihood of negative future pollution stocks without any qualitative impact on the paper's conclusions. Thus, we suggest that the reader, while gathering the intuition of the main results, think of the pollution stock as being positive.

[8] Note that these two issues are conceptually distinct. While in one case there are parameters whose values are unknown, in the other there is stochastic uncertainty perturbing an otherwise deterministic relationship.

Assumption 2 $0 < \alpha \leq 1, 0 < \beta < +\infty$.

Assumption 3 $U(\cdot, \cdot)$ *is continuously differentiable in* c *and* p.

Assumption 4 $U_c > 0$.

Assumption 5 $U_p < 0$ *and* $U_{pp} < 0$.

Assumption 6 *The random variables* ϵ_t *have zero mean and are identically, independently distributed according to the continuously-differentiable density function* $f(\epsilon)$, *such that* $\frac{f'(\epsilon)}{f(\epsilon)}$ *is continuous and decreasing in* ϵ.

Assumption 7 $f(\epsilon) > 0, \forall \epsilon \in (-\infty, +\infty)$.

Assumption 1 states the discounting rate. Assumptions 2, 4 and 5 (first part) ensure that the economic problem is meaningful. Assumption 2's first part rules out instantaneous decay, thus ensuring that accumulation takes place, while the second part guarantees that consumption contributes positively to the pollution stock. Assumption 4 makes plain that consumption is an economic good whereas Assumption 5 (first part) shows that pollution is an economic "bad." These assumptions, together with restriction (1), imply that there is a tradeoff between current and future welfare. Assumptions 3, 5 (second part), 6, and 7 are technical. Assumption 3 permits differentiation in what follows. Assumption 5 (second part) allows for the computations in the paper. Assumption 6 requires $\frac{f'(\epsilon)}{f(\epsilon)}$ to be decreasing in ϵ. This amounts to imposing the strict monotone likelihood-ratio property (MLRP).[9] Assumption 7's rôle was already explained in a footnote. Finally, the existence and characterization of the optimal policy and value function in a Bayesian learning framework such as ours is outside the scope of this paper (see the Introduction).

The timing is as follows. The stock of pollution is known at the beginning of each period. Consumption is then chosen, pollution decay takes place, the random factors behind ϵ are felt, and a new pollution stock is observed. Bayesian updating of beliefs over the unknown parameters, α and β, takes place, and a new period begins. The non-learner agency sticks to its prior concerning the values of α and β. The passive-learner agency knows Bayes' rule, but does not take into account that it affects the rule's realization by its choice of consumption. The experimentor agency knows Bayes' rule and fully incorporates it in its decision process.

Since there are two parameters to be learned, we deal with each separately while assuming the other to be known. We thus avoid entangling the effects of two learning processes. We first treat the case where α is a parameter to be learned while the true value of β is known, and then study the opposite case.

When the pollution decay rate, α, is unknown, suppose that it can take values $\alpha_i, i = 1, \ldots, m$ such that $0 < \alpha_1 < \alpha_2 < \cdots < \alpha_m \leq 1$. Each α_i constitutes a scenario that the agency entertains concerning pollution decay

[9] To see this, consider any two values that α and β may take and denote them α' and α'' such that $\alpha' > \alpha''$, and β' and β'' such that $\beta' > \beta''$. As simple computations using (1) show, $\frac{f'(\epsilon)}{f(\epsilon)}$ decreasing is a necessary and sufficient condition for $\frac{\mathrm{d}\left[\frac{f(p_{t+1}-\alpha' p_t-\beta e_t)}{f(p_{t+1}-\alpha'' p_t-\beta e_t)}\right]}{\mathrm{d}p_{t+1}} > 0$ and $\frac{\mathrm{d}\left[\frac{f(p_{t+1}-\alpha p_t-\beta' e_t)}{f(p_{t+1}-\alpha p_t-\beta'' e_t)}\right]}{\mathrm{d}p_{t+1}} > 0$. In plain words, Assumption 6 implies that a bigger future pollution stock, p_{t+1}, is associated with a higher relative posterior probability of a worse state of Nature occurring. See Milgrom (1981).

and to which it attaches a probability.[10] Denote each period's prior by $\pi_i \equiv \Pr[\alpha = \alpha_i], i = 1, \ldots, m$ and write it succinctly as $\{\pi_i\}_{i=1}^m$.

Similarly, when the marginal contribution to the stock, β, is unknown and can take values $\beta_i, i = 1, \ldots, n$ such that $0 < \beta_1 < \beta_2 < \cdots < \beta_n < +\infty$, denote each period's prior by $\pi_i \equiv \Pr[\beta = \beta_i], i = 1, \ldots, n$ and write it concisely as $\{\pi_i\}_{i=1}^n$.[11]

Finally, because degenerate beliefs do not allow for experimentation or even learning, we make the following mild

Assumption 8 *Initial prior beliefs, denoted π_i^0, are non-degenerate: $\pi_i^0 < 1, \forall i = 1, 2, \ldots, m$ when α is unknown and $\pi_i^0 < 1, \forall i = 1, 2, \ldots, n$ when β is unknown.*[12]

2.1 Complete information

Suppose that the values of both α and β are known, in which case there is no scope for either learning or experimentation. Then, the value function for such a problem is

$$\begin{aligned} V(p) = \quad & \underset{c}{\text{Max}} \quad U(c,p) + \delta E_{\hat{p}}[V(\hat{p})] \\ & \text{s.t.} \quad \hat{p} = \alpha p + \beta c + \epsilon, \end{aligned}$$

where ϵ being a random variable explains the expectation over $\hat{p}$ regarding the future.

2.2 Experimentor agency

2.2.1 Unknown α

Suppose that β's value is known while α's is not, i.e., the agency entertains a prior on α which it updates by experimenting. From the prior $\{\pi_i\}_{i=1}^m$, we obtain the posterior

$$\hat{\pi}_i(\hat{p}, c) = \frac{\pi_i f(\hat{p} - \alpha_i p - \beta c)}{\sum_{j=1}^m \pi_j f\left(\hat{p} - \alpha_j p - \beta c\right)}, \qquad i = 1, 2, \ldots, m. \tag{2}$$

The value function for such a problem is[13]

$$\begin{aligned} V\left(p, \{\pi_i\}_{i=1}^{m-1}\right) = \quad & \underset{c}{\text{Max}} \quad \left\{U(c,p) + \delta E_{\hat{p}}\left[V\left(\hat{p}, \{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{m-1}\right)\right]\right\} \\ & \text{s.t.} \quad \hat{p} = \alpha p + \beta c + \epsilon \qquad (3) \\ = \quad & \underset{c}{\text{Max}} \quad \left\{U(c,p) + \delta \int V\left(\hat{p}, \{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{m-1}\right) h(\hat{p},c)\, \mathrm{d}\hat{p}\right\} \\ & \text{s.t.} \quad \hat{p} = \alpha p + \beta c + \epsilon, \end{aligned}$$

where

$$h(\hat{p}, c) = \sum_{i=1}^m \pi_i f(\hat{p} - \alpha_i p - \beta c),$$

[10] Thus, low values of α constitute good states of Nature, i.e., optimistic scenarios.

[11] Since we always assume one of the parameters to be known, using this simplified notation for beliefs, π_i, (instead of coining different symbols for beliefs about α and β) spares confusion.

[12] Assumption 8 does not exclude the possibility of one or several elements in the support of the distributions having zero probability in the initial prior. A version of the paper with only two possible realizations for α and β is available from the authors upon request.

[13] Only $m - 1$ probabilities are needed as arguments of the value function since both prior and posterior probabilities sum up to one. Obviously, the probability to be dropped can be arbitrarily chosen.

is the probability of $\hat{p}$, and $V\left(\hat{p}, \{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{m-1}\right)$ is a simplified notation for $V(\hat{p}, \hat{\pi}_1(\hat{p},c), \hat{\pi}_2(\hat{p},c), \ldots, \hat{\pi}_{m-1}(\hat{p},c))$.[14]

2.3 Unknown β

Suppose now that α's value is known while β's is not, i.e., the agency entertains a prior on β which it updates by experimenting. From the prior $\{\pi_i\}_{i=1}^{n}$ one obtains the posterior

$$\hat{\pi}_i(\hat{p},c) = \frac{\pi_i f(\hat{p} - \alpha p - \beta_i c)}{\sum_{j=1}^{m} \pi_j f\left(\hat{p} - \alpha p - \beta_j c\right)}, \qquad i = 1, 2, \ldots, n. \tag{4}$$

The value function for such a problem is

$$\begin{aligned} V\left(p, \{\pi_i\}_{i=1}^{n-1}\right) = \; & \operatorname*{Max}_{c} \quad \left\{U(c,p) + \delta E_{\hat{p}}\left[V\left(\hat{p}, \{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{n-1}\right)\right]\right\} \\ & \text{s.t.} \quad \hat{p} = \alpha p + \beta c + \epsilon \\ = \; & \operatorname*{Max}_{c} \quad \left\{U(c,p) + \delta \int V\left(\hat{p}, \{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{n-1}\right) h(\hat{p},c)\, \mathrm{d}\hat{p}\right\} \\ & \text{s.t.} \quad \hat{p} = \alpha p + \beta c + \epsilon, \end{aligned}$$

where

$$h(\hat{p},c) = \sum_{i=1}^{n} \pi_i f(\hat{p} - \alpha p - \beta_i c).$$

2.4 Passive-learner agency

The passive-learner's problem is formally described as the experimentor's but for one crucial informational aspect. Hence, rather than giving a formal description of the passive learner's problem similar to the one in the previous subsection, we instead explain the *difference* between the two problems.[15]

A passive-learner agency does not factor in the effect of its choice of consumption on posterior beliefs, but otherwise fully understands the dynamics of the problem, including that beliefs updating takes place. The passive learner's problem thus coincides with the experimentor's, except that the former does not factor in the impact of its choice of consumption, c, on the posterior distribution of beliefs, $\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{m}$. This raises the question of how the passive-learner's beliefs and their updating feature in the problem. The passive-learner optimizes based on future beliefs which are taken to be unaffected by its choice of consumption, c, yet those beliefs must result from the Bayesian updating described by (2) and (4) where c does appear.

Formally, take the objective function in (4) and write it as

$$V\left(p, \{\pi_i\}_{i=1}^{m-1}\right) = \operatorname*{Max}_{c} \left\{U(c,p) + \delta E_{\hat{p}}\left[V\left(\hat{p}, \{\hat{\pi}_i(\hat{p}, y = c)\}_{i=1}^{m-1}\right)\right]\right\}$$

The experimentor is aware that y is indeed c and thus fully uses Bayes' rule to compute the f.o.c. The passive learner instead takes y as being independent of c *when optimizing*, while using the latter to compute $\{\hat{\pi}_i(\hat{p}, y = c)\}_{i=1}^{m-1}$ according to Bayes' rule.[16]

[14] A simplified notation that we will use from now on.

[15] While restricting attention in the remainder of this section to the case where α is unknown. The opposite case follows straightforwardly *mutatis mutandis*.

[16] Note that $\{\hat{\pi}_i(\hat{p}, y = c)\}_{i=1}^{m-1}$ determines the values of the last $m-1$ arguments of $V\left(\hat{p}, \{\hat{\pi}_i(\hat{p}, y = c)\}_{i=1}^{m-1}\right)$. Hence, the passive learner must rationally expect the value function.

From a computational viewpoint, the difference between the experimentor's and passive learner's problems is that, when computing a f.o.c., the passive-learner agency does not take into account the derivative of each $\hat{\pi}_i(\cdot,\cdot)$ with respect to c whereas the (fully-rational) experimentor does.[17]

2.5 Non-learner agency

Take the non-learner agency's problem. Its value function is

$$\begin{aligned} V\left(p;\left\{\pi_i^0\right\}_{i=1}^{m-1}\right) &= \underset{c}{\text{Max}} \quad \left\{U(c,p) + \delta E_{\hat{p}}\left[V\left(\hat{p};\left\{\pi_i^0\right\}_{i=1}^{m-1}\right)\right]\right\} \\ &\quad\ \text{s.t.} \quad \hat{p} = \alpha p + \beta c + \epsilon \\ &= \underset{c}{\text{Max}} \quad \left\{U(c,p) + \delta \int V\left(\hat{p};\left\{\pi_i^0\right\}_{i=1}^{m-1}\right) h(\hat{p},c)\,\mathrm{d}\hat{p}\right\} \\ &\quad\ \text{s.t.} \quad \hat{p} = \alpha p + \beta c + \epsilon, \end{aligned}$$

where

$$h(\hat{p},c) = \sum_{i=1}^{m} \pi_i^0 f(\hat{p} - \alpha_i p - \beta c).$$

3 The Agencies' Problems: Unknown α

Let β be known and α the object of learning. We begin by formally comparing the experimentor's choice of consumption with the passive learner's and the non-learner's, explaining the results intuitively and relegating all proofs to an appendix.

Theorem 1 *A passive-learner agency chooses the same consumption level as an experimentor agency.*

We now compare the experimentor and non-learner agencies' choice of consumption.

Theorem 2 *Experimentor and passive-learner agencies may choose higher, lower or the same consumption level as a non-learner agency.*

These results are intuitive. Let us begin with Theorem 1. Note that a unitary increase in c shifts the distribution of $\hat{p}$ by the *same* amount, namely β, *whatever the realization of* α. To see this, take Figure 1 which considers two possible realizations of the unknown parameter and denotes them α' and α'' such that $\alpha' > \alpha''$. Then, note that the *relative* position of the distributions of $\hat{p}$ conditional on α' and α'' stays the same whatever the value chosen for the decision variable, c. Formally, $\frac{\mathrm{d}\hat{\pi}_i}{\mathrm{d}c} = 0, i = 1,2,\ldots,m$.[18] Plainly, whatever the consumption level chosen, nothing more is learned at the end of one period about the true value of the unknown parameter, α, besides the information generated by

[17] See equations (A.2) and (A.3) in Datta, Mirman and Schlee (2002), p. 604.

[18] Note that

$$\frac{\mathrm{d}\hat{\pi}_i}{\mathrm{d}c} = \frac{\partial\hat{\pi}_i}{\partial\hat{p}}\frac{\mathrm{d}\hat{p}}{\mathrm{d}c} + \frac{\partial\hat{\pi}_i}{\partial c}, \qquad i = 1,2,\ldots,m,$$

which, by (1) and (A.1), yields

$$\frac{\mathrm{d}\hat{\pi}_i}{\mathrm{d}c} = \beta\frac{\partial\hat{\pi}_i}{\partial\hat{p}} - \beta\frac{\partial\hat{\pi}_i}{\partial\hat{p}} = 0, \qquad i = 1,2,\ldots,m.$$

pollution decaying from p to αp. Hence, the experimentor agency—who considers the informational consequences of its choice—and the passive-learner—who does not—choose the same consumption level (Theorem 1). On the other hand, the non-learner ignores the information generated by pollution decay. This explains the difference in behavior between the non-learner and the other two agencies (Theorem 2).

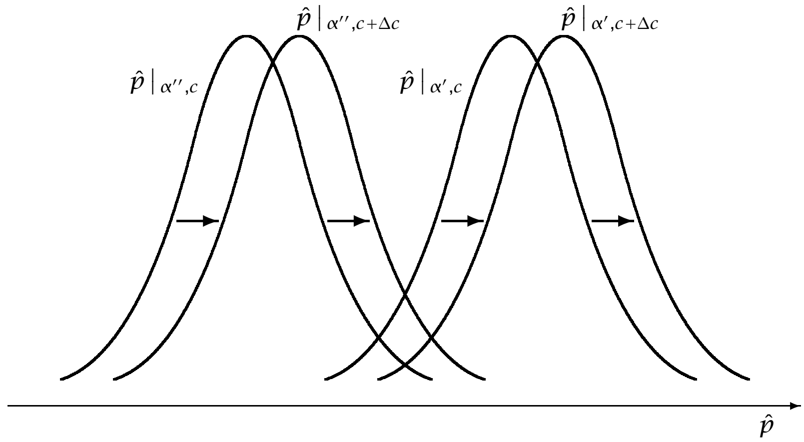

FIGURE 1. *Immediate informational effect of changes in c: α unknown* ($\Delta c > 0$).

Interestingly, setting a higher consumption today does yield extra information after two or more periods. Why? A unitary increase in current consumption implies an increase in tomorrow's pollution stock of β which, after one more period, will start decaying at a rate determined by the realization of the unknown parameter, α. Such an increase in p brought about by a previous increase in c shifts the distributions of $\hat{p}$ contingent on α' and α'' by different amounts, *driving them apart* (see Figure 2). Therefore, the more consumption an agency allows in one period, the more it learns about the true value of the unknown parameter two periods hence and beyond.

One may then find Theorem 1 paradoxical in view of this fact: why do *not* these delayed informational effects lead to a difference in the setting of optimal consumption between an experimentor agency and a passive-learner agency? To see why, note that the latter, while not taking into account the impact of extra consumption in one period on beliefs updating *at the end of that period*, fully understands the dynamics of the problem. Thus, it realizes the long-run impact of an extra unit of consumption on all subsequent beliefs updating once it translates into β extra units of pollution stock.[19] Therefore, all informational gains other than that occurring at the end of the current period are equally perceived by both the passive-learner and the experimentor agencies. Any difference in their behavior, were it to exist, would result from different assessments of the impact of their choice of consumption on immediate (end-of-period) beliefs updating. Yet, while the experimentor agency chooses *c know-*

[19]Formally, note that the passive-learner agency acts as if $\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^m$ did not depend on c only when optimizing, but otherwise reasons as the experimentor (recall subsection 2.4). Hence, once consumption feeds the pollution stock, the passive-learner agency gathers all the information generated by its decay.

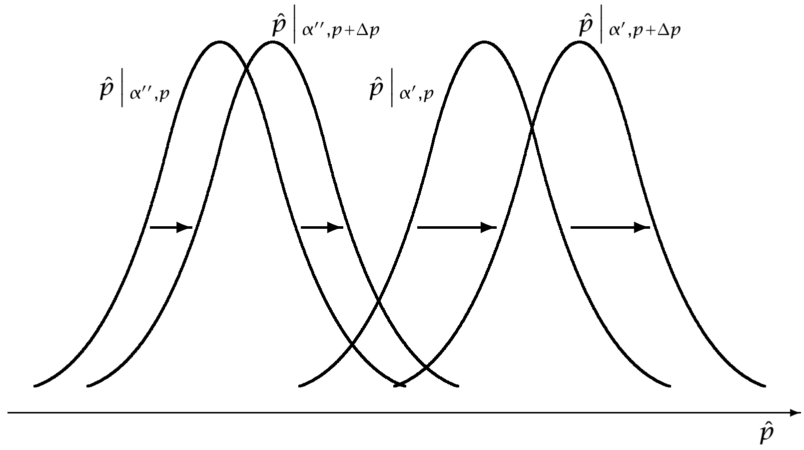

FIGURE 2. *Delayed informational effect of changes in c: α unknown* ($\Delta c > 0$).

ing that beliefs will not be affected by that choice, $\frac{\mathrm{d}\hat{\pi}_i}{\mathrm{d}c} = 0, i = 1, 2, \ldots, m$, the passive-learner agency chooses it *assuming* that they are not affected by c, i.e., $\frac{\mathrm{d}\hat{\pi}_i}{\mathrm{d}c} \equiv 0, i = 1, 2, \ldots, m$, an assumption that thus turns out to prove correct. *Ergo*, the same level of consumption is chosen by the experimentor and the passive-learner agencies.

Let us explain Theorem 2 now. Informational effects are taken into account by the experimentor and passive-learner agencies and ignored by the non-learner. Hence their different choice of consumption. How will an experimentor and a passive-learner agency set consumption when compared to a non-learning agency? At this point, one may be tempted to invoke Mirman et al.'s (1993a, p. 560) intuition—namely, that "... the firm will adjust quantity so as to increase the spread between mean demand curves," i.e., always adjust the decision variable so as to increase information—and conclude that consumption as set by the experimentor and the passive-learner agencies will necessarily be higher than the non-learner's in a quest for more information. As will be demonstrated, this is not necessarily the case.[20] The proof makes it plain that the result depends on the sign and magnitude of the terms involving the cross-derivatives of the value function with respect to the pollution stock and beliefs. This interaction between beliefs and the pollution stock may lead the experimentor and passive-learner agencies to choose less consumption than the non-learner agency even though more consumption, and the associated pollution, would yield more information.

4 The Agencies' Problems: Unknown β

Let α be known and β the object of learning. Interestingly, the results of the previous section do not carry over to the case where the marginal contribution of the flow to the stock is unknown. In fact,

[20]This is the major consequence of the signal-dependent nature of the pollution-accumulation problem as we formulate it. Mirman et al.'s (1993a) problem is signal independent. Hence, their conclusion.

Theorem 3 *An experimentor agency may choose a higher, lower or the same consumption level as a passive-learner agency.*

The reason for the striking difference between Theorems 1 and 3 is easy to understand by reference to the previous section's intuitions. Now, an additional unit of consumption *immediately* drives apart the distributions of pollution conditional on different realizations of β. Hence, beliefs are impacted differently by different choices of consumption. Therefore, informational gains are obtainable by taking into account the impact of c on beliefs updating, something that the experimentor does and the passive learner does not.

Why does not the experimentor agency necessarily set a higher consumption level in order to gather more information? Again, the explanation was already given at the very end of the previous section.

Taking a non-learner agency as the benchmark, we obtain the following

Theorem 4 *An experimentor agency may choose a higher, lower or the same consumption level as a non-learner agency.*

The intuition for this result is the same as the one in the previous section.

To summarize, when the marginal contribution of the flow to the stock is uncertain, the experimentor agency may choose a higher or lower consumption, or the same as the passive-learner and non-learner agencies, even though consuming more yields more information.

5 Conclusions

We study a signal-dependent experimentation problem involving accumulation. We show that if the decision variable is a flow and the parameter to be learned is the decay rate of the stock, the experimentation effect is nil, and passive learning encapsulates all the learning that may take place, whereas this result does not hold if the unknown parameter is the one measuring the marginal contribution of the flow to the stock. This uncovers the importance of the dynamic structure of signal-dependent problems.

We illustrate these points in the context of an informationally-realistic rendition of the pollution-accumulation problem by assuming that uncertainty surrounds two physical processes, the pollution-stock decay and the marginal rate at which consumption adds to the pollution stock. We conclude that if there is uncertainty concerning the decay rate of pollution while the rate at which consumption feeds the pollution stock is well-determined, then an agency that does not experiment but acts instead as a passive learner induces no welfare loss, whereas a non-learner will likely induce such a loss. In contrast, if uncertainty surrounds the marginal contribution of consumption to the pollution stock, both the passive-learner and the non-learner agency will likely induce a welfare loss. Moreover, it may be optimal for an experimentor agency to decrease consumption *vis-à-vis* a passive- or a non-learner agency, even though increasing consumption yields more information concerning those processes. This hinges on the impact that increased pollution resulting from increased consumption has on the value of extra information, and qualifies the naïve informational view that consumption should always be increased when one is in the position to learn more by emitting.

One can draw an analogy between our (signal-dependent) problem and the signal-independent one treated in Mirman *et al.* (1993a). They study how a monopolist distorts its myopic choice in order to learn about the unknown demand it faces: our α-case resembles their unknown-intercept case, whereas the β-case is reminiscent of the unknown-slope one.[21]

The three agencies depict three types of behavior resulting from different levels of sophistication in dealing with information. Only the experimentor can be described as fully optimal in dealing with information. The passive-learner agency's lack of sophistication becomes apparent if one thinks of it as erroneously computing a first-order condition. As to the non-learner agency, it correctly solves its mathematical problem but exhibits naïve understanding. To see this, suppose that it is endowed with an optimistic prior (i.e., a positively skewed π_0 entailing a high probability of α or β taking small values) and sees large levels of pollution steadily accumulating (i.e., a bad state of Nature involving a high value of α or β turns out to be true). This non-learner agency, by definition, sticks to its prior even though it observes dauntingly-increasing pollution stocks. Yet, because one can easily imagine situations where pollution-setting agencies may sub-optimally deal with information in a manner that our analysis captures, these theoretical results are of practical interest.

One may reasonably ask—as one referee did—which type of physical process is more likely to be unknown and subject to learning and experimentation over time. First, both types of processes, those captured by α as well as those modeled by β, are equally likely to involve uncertainty. For an example of an α-type parameter, take carbon dioxide's natural abatement, mainly an oceanic process. The depreciation rate, α, determines the time constant, $\frac{1}{\alpha}$, of CO_2's residence in the atmosphere. Nordhaus (1991) surveyed the literature and set $\alpha = 0.005$, corresponding to a time constant of 200 years. However, there is controversy concerning the residence time of CO_2 in the atmosphere as the literature that studies carbon sinks and CO_2 mitigation strategies makes plain (IPCC report (Watson et al., 2001), Moura-Costa and Wilson (2000)). For an example of a β-type parameter, consider the use of forests as a carbon sink. The amount of CO_2 that is released when the forest is harvested depends upon the use given to timber, as well as the amount of litter, branches, tops, stump and roots produced at harvest. With this is mind, Van Kooten, Binkley and Delcourt (1995) introduced a parameter, b, which measures the fraction of timber that goes into long-term storage in permanent structures. The precise value of b is unknown.[22] As to the relative likelihood of these two types of parameter uncertainty, one may only conjecture that since β-like parameters are directly associated with consumption whereas α-type parameters are associated with the pollution stock, the former may be easier to ascertain than the later insofar as consumption and emissions are easier to observe than the pollution stock. Moreover, current technology seems to be better at finely controlling emissions (e.g., scrubbers, car converters) than the pollution stock.

From a policy standpoint, we show that it can be socially rational to decrease consumption, even though this leads to less information gathering. This is a new reason for conservative emissions' policies, stemming as it does from

[21]We are grateful to Leonard J. Mirman for pointing out this analogy.

[22]Interestingly, b is not only a technology-driven parameter but an economic-driven one, too, since it depends on the relative prices of competing timber uses.

an informational argument, and should be contrasted to the better-known conservation argument associated with the accumulation effect of an economic "bad."

Appendix

Proof of Theorem 1 We begin by calculating $\frac{\partial \hat{\pi}_i}{\partial \hat{p}}$ and $\frac{\partial \hat{\pi}_i}{\partial c}$, $i = 1, 2, \ldots, m$, which will later prove useful. From (2), recall that

$$\hat{\pi}_i = \frac{\pi_i f\left(\hat{p} - \alpha_i p - \beta c\right)}{\sum_{j=1}^{m} \pi_j f\left(\hat{p} - \alpha_j p - \beta c\right)}, \qquad i = 1, 2, \ldots, m.$$

Thus,

$$\begin{aligned}\frac{\partial \hat{\pi}_i}{\partial \hat{p}} &= \frac{\pi_i f'\left(\hat{p} - \alpha_i p - \beta c\right) \sum_{j=1}^{m} \pi_j f\left(\hat{p} - \alpha_j p - \beta c\right)}{\left(\sum_{j=1}^{m} \pi_j f\left(\hat{p} - \alpha_j p - \beta c\right)\right)^2} - \\ &\quad - \frac{\pi_i f\left(\hat{p} - \alpha_i p - \beta c\right) \sum_{j=1}^{m} \pi_j f'\left(\hat{p} - \alpha_j p - \beta c\right)}{\left(\sum_{j=1}^{m} \pi_j f\left(\hat{p} - \alpha_j p - \beta c\right)\right)^2}, \qquad i = 1, 2, \ldots, m.\end{aligned}$$

Similarly,

$$\begin{aligned}\frac{\partial \hat{\pi}_i}{\partial c} &= -\frac{\pi_i \beta f'\left(\hat{p} - \alpha_i p - \beta c\right) \sum_{j=1}^{m} \pi_j f\left(\hat{p} - \alpha_j p - \beta c\right)}{\left(\sum_{j=1}^{m} \pi_j f\left(\hat{p} - \alpha_j p - \beta c\right)\right)^2} + \\ &\quad + \frac{\pi_i f\left(\hat{p} - \alpha_i p - \beta c\right) \beta \sum_{j=1}^{m} \pi_j f'\left(\hat{p} - \alpha_j p - \beta c\right)}{\left(\sum_{j=1}^{m} \pi_j f\left(\hat{p} - \alpha_j p - \beta c\right)\right)^2}, \qquad i = 1, 2, \ldots, m,\end{aligned}$$

which yields

$$\frac{\partial \hat{\pi}_i}{\partial c} = -\beta \frac{\partial \hat{\pi}_i}{\partial \hat{p}}, \qquad i = 1, 2, \ldots, m. \tag{A.1}$$

From section 2.2.1, recall that the value function of the problem of an experimentor agency is

$$\begin{aligned}V\left(p, \{\pi_i\}_{i=1}^{m-1}\right) = \; & \underset{c}{\text{Max}} \quad \left\{U\left(c, p\right) + \delta E_{\hat{p}}\left[V\left(\hat{p}, \{\hat{\pi}_i\left(\hat{p}, c\right)\}_{i=1}^{m-1}\right)\right]\right\} \\ & \text{s.t.} \quad \hat{p} = \alpha p + \beta c + \epsilon \\ = \; & \underset{c}{\text{Max}} \quad \left\{U\left(c, p\right) + \delta \int V\left(\hat{p}, \{\hat{\pi}_i\left(\hat{p}, c\right)\}_{i=1}^{m-1}\right) h\left(\hat{p}, c\right) \mathrm{d}\hat{p}\right\} \\ & \text{s.t.} \quad \hat{p} = \alpha p + \beta c + \epsilon,\end{aligned}$$

where

$$h\left(\hat{p}, c\right) = \sum_{i=1}^{m} \pi_i f\left(\hat{p} - \alpha_i p - \beta c\right),$$

The f.o.c. for an interior solution of this problem is such that

$$\frac{\partial U\left(c, p\right)}{\partial c} + \delta \frac{\partial E_{\hat{p}}\left[V\left(\hat{p}, \{\hat{\pi}_i\left(\hat{p}, c\right)\}_{i=1}^{m-1}\right)\right]}{\partial c} = 0.$$

The second term reflects both accumulation, which captures the welfare consequences of variations in the pollution stock brought about by changes in consumption, and experimentation, which concerns the influence of consumption on future beliefs. Formally,

$$\begin{aligned}
&\frac{\partial E_{\hat{p}}\left[V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{m-1}\right)\right]}{\partial c} \\
&= \frac{\partial \int V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{m-1}\right) h(\hat{p},c)\,\mathrm{d}\hat{p}}{\partial c} = \\
&= \int \left(\sum_{i=1}^{m-1} \frac{\partial V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{m-1}\right)}{\partial \hat{\pi}_i} \frac{\partial \hat{\pi}_i}{\partial c}\right) h(\hat{p},c)\,\mathrm{d}\hat{p} + \\
&+ \int V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{m-1}\right) \frac{\partial h(\hat{p},c)}{\partial c}\mathrm{d}\hat{p}.
\end{aligned}$$

Inserting (A.1), we obtain

$$\begin{aligned}
&\frac{\partial \int V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{m-1}\right) h(\hat{p},c)\,\mathrm{d}\hat{p}}{\partial c} = \\
&= -\beta \int \left(\sum_{i=1}^{m-1} \frac{\partial V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{m-1}\right)}{\partial \hat{\pi}_i} \frac{\partial \hat{\pi}_i}{\partial \hat{p}}\right) h(\hat{p},c)\,\mathrm{d}\hat{p} + \\
&+ \int V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{m-1}\right) \frac{\partial h(\hat{p},c)}{\partial c}\mathrm{d}\hat{p}. \qquad \text{(A.2)}
\end{aligned}$$

Rewriting the second term of the right-hand side, we have

$$\begin{aligned}
&\int V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{m-1}\right) \frac{\partial h(\hat{p},c)}{\partial c}\mathrm{d}\hat{p} = \\
&= -\beta \int V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{m-1}\right) \times \\
&\times \left(\sum_{i=1}^{m} \pi_i f'(\hat{p} - \alpha_i p - \beta c)\right) \mathrm{d}\hat{p}.
\end{aligned}$$

Integrating by parts, one obtains

$$\begin{aligned}
&\beta \int \left(\sum_{i=1}^{m-1} \frac{\partial V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{m-1}\right)}{\partial \hat{\pi}_i} \frac{\partial \hat{\pi}_i}{\partial \hat{p}}\right) \times \\
&\times \left(\sum_{i=1}^{m} \pi_i f(\hat{p} - \alpha_i p - \beta c)\right) \mathrm{d}\hat{p} + \\
&+\beta \int \frac{\partial V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{m-1}\right)}{\partial \hat{p}} \times \\
&\times \left(\sum_{i=1}^{m} \pi_i f(\hat{p} - \alpha_i p - \beta c)\right) \mathrm{d}\hat{p} = \\
&= \beta \int \left(\sum_{i=1}^{m-1} \frac{\partial V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{m-1}\right)}{\partial \hat{\pi}_i} \frac{\partial \hat{\pi}_i}{\partial \hat{p}}\right) h(\hat{p},c)\,\mathrm{d}\hat{p} + \\
&+\beta \int \frac{\partial V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{m-1}\right)}{\partial \hat{p}} h(\hat{p},c)\,\mathrm{d}\hat{p},
\end{aligned}$$

which, substituted in (A.2), yields

$$\begin{aligned}
&\frac{\partial \int V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{m-1}\right) h(\hat{p},c)\,\mathrm{d}\hat{p}}{\partial c} = \\
&= \beta \int \frac{\partial V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{m-1}\right)}{\partial \hat{p}} h(\hat{p},c)\,\mathrm{d}\hat{p}.
\end{aligned}$$

Thus, an experimentor agency evaluates the future welfare cost of pollution to be weighted against the utility of current consumption as

$$\frac{\partial E_{\hat{p}}\left[V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{m}\right)\right]}{\partial c} = \\ = \beta \int \frac{\partial V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{m}\right)}{\partial \hat{p}} h(\hat{p},c)\,\mathrm{d}\hat{p}.$$

Now, take the initial-period problem of the passive-learner agency. Its value function is

$$\begin{aligned} V\left(p,\{\pi_i\}_{i=1}^{m-1}\right) = & \operatorname*{Max}_{c} \quad \left\{U(c,p) + \delta E_{\hat{p}}\left[V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{m-1}\right)\right]\right\} \\ & \text{s.t.} \quad \hat{p} = \alpha p + \beta c + \epsilon \\ = & \operatorname*{Max}_{c} \quad \left\{U(c,p) + \delta \int V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{m-1}\right) h(\hat{p},c)\,\mathrm{d}\hat{p}\right\} \\ & \text{s.t.} \quad \hat{p} = \alpha p + \beta c + \epsilon, \end{aligned}$$

which thus equals that of the experimentor, except that the dependency of the posterior $\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{m}$ on consumption is ignored by the passive learner agency when optimizing. Differentiating the second term of the objective function with this in mind, one obtains

$$\begin{aligned} & \delta \frac{\partial E_{\hat{p}}\left[V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{m-1}\right)\right]}{\partial c} \\ = & \quad \delta \frac{\partial\left[\int V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{m-1}\right) h(\hat{p},c)\,\mathrm{d}\hat{p}\right]}{\partial c} = \\ = & \quad \delta \int V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{m-1}\right) \frac{\partial h(\hat{p},c)}{\partial c}\mathrm{d}\hat{p}. \end{aligned} \tag{A.3}$$

Rewriting the r.h.s. of the previous expression, we obtain

$$\begin{aligned} & \delta \int V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{m-1}\right) \frac{\partial h(\hat{p},c)}{\partial c}\mathrm{d}\hat{p} = \\ & \quad = -\delta\beta \int V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{m-1}\right) \times \\ & \qquad \times \left(\sum_{i=1}^{m} \pi_i f'(\hat{p} - \alpha_i p - \beta c)\right) \mathrm{d}\hat{p}. \end{aligned}$$

Integrating by parts while recalling that the dependency of the posterior on consumption is ignored by the passive learner, we get

$$\begin{aligned} & \delta \int V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{m-1}\right) \frac{\partial h(\hat{p},c)}{\partial c}\mathrm{d}\hat{p} = \\ = & \quad \delta\beta \int \frac{\partial V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{m-1}\right)}{\partial \hat{p}} \times \\ & \qquad \times \left(\sum_{i=1}^{m} \pi_i f(\hat{p} - \alpha_i p - \beta c)\right) \mathrm{d}\hat{p} = \\ = & \quad \delta\beta \int \frac{\partial V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{m-1}\right)}{\partial \hat{p}} h(\hat{p},c)\,\mathrm{d}\hat{p}, \end{aligned}$$

which, substituted in (A.3), yields

$$\delta\frac{\partial\left[\int V\left(\hat{p},\{\hat{\pi}_i\left(\hat{p},c\right)\}_{i=1}^{m-1}\right)h\left(\hat{p},c\right)\mathrm{d}\hat{p}\right]}{\partial c} = $$
$$= \delta\beta\int\frac{\partial V\left(\hat{p},\{\hat{\pi}_i\left(\hat{p},c\right)\}_{i=1}^{m-1}\right)}{\partial\hat{p}}h\left(\hat{p}\right)\mathrm{d}\hat{p}.$$

Therefore,

$$\delta\frac{\partial E_{\hat{p}}\left[V\left(\hat{p},\{\hat{\pi}_i\left(\hat{p},c\right)\}_{i=1}^{m-1}\right)\right]}{\partial c} =$$
$$= \delta\beta\int\frac{\partial V\left(\hat{p},\{\hat{\pi}_i\left(\hat{p},c\right)\}_{i=1}^{m-1}\right)}{\partial\hat{p}}h\left(\hat{p}\right)\mathrm{d}\hat{p}. \quad \text{(A.4)}$$

Hence, both agencies perceive the same tradeoff between present welfare-enhancing consumption and future welfare reductions due to increased pollution generated by current consumption. ■

Proof of Theorem 2 Now, take the problem of the non-learner agency. Its value function is

$$\begin{aligned} V\left(p;\left\{\pi_i^0\right\}_{i=1}^{m}\right) &= \underset{c}{\text{Max}} \quad \left\{U\left(c,p\right)+\delta E_{\hat{p}}\left[V\left(\hat{p};\left\{\pi_i^0\right\}_{i=1}^{m-1}\right)\right]\right\} \\ & \quad\ \text{s.t.} \quad \hat{p}=\alpha p+\beta c+\epsilon \\ &= \underset{c}{\text{Max}} \quad \left\{U\left(c,p\right)+\delta\int V\left(\hat{p};\left\{\pi_i^0\right\}_{i=1}^{m-1}\right)h\left(\hat{p},c\right)\mathrm{d}\hat{p}\right\} \\ & \quad\ \text{s.t.} \quad \hat{p}=\alpha p+\beta c+\epsilon, \end{aligned}$$

where

$$h\left(\hat{p},c\right) = \sum_{i=1}^{m}\pi_i^0 f\left(\hat{p}-\alpha_i p-\beta c\right).$$

Differentiating the second term of the objective function, one obtains

$$\begin{aligned} &\delta\frac{\partial E_{\hat{p}}\left[V\left(\hat{p};\left\{\pi_i^0\right\}_{i=1}^{m-1}\right)\right]}{\partial c} \\ &= \delta\frac{\partial\left[\int V\left(\hat{p};\left\{\pi_i^0\right\}_{i=1}^{m-1}\right)h\left(\hat{p},c\right)\mathrm{d}\hat{p}\right]}{\partial c} = \\ &= \delta\int V\left(\hat{p};\left\{\pi_i^0\right\}_{i=1}^{m-1}\right)\frac{\partial h\left(\hat{p},c\right)}{\partial c}\mathrm{d}\hat{p}. \end{aligned} \quad \text{(A.5)}$$

Rewriting the r.h.s. of the previous expression, we obtain

$$\begin{aligned} &\delta\int V\left(\hat{p};\left\{\pi_i^0\right\}_{i=1}^{m-1}\right)\frac{\partial h\left(\hat{p},c\right)}{\partial c}\mathrm{d}\hat{p} = \\ &= -\delta\beta\int V\left(\hat{p};\left\{\pi_i^0\right\}_{i=1}^{m-1}\right)\times \\ &\qquad\times\left(\sum_{i=1}^{m}\pi_i f'\left(\hat{p}-\alpha_i p-\beta c\right)\right)\mathrm{d}\hat{p}. \end{aligned}$$

Integrating by parts, we get

$$\begin{aligned}
\delta \int V\left(\hat{p};\left\{\pi_i^0\right\}_{i=1}^{m-1}\right)\frac{\partial h\left(\hat{p},c\right)}{\partial c}\mathrm{d}\hat{p} = & \\
= \quad \delta\beta \int \frac{\partial V\left(\hat{p};\left\{\pi_i^0\right\}_{i=1}^{m-1}\right)}{\partial \hat{p}} & \times \\
\times \left(\sum_{i=1}^{m}\pi_i f\left(\hat{p}-\alpha_i p-\beta c\right)\right)\mathrm{d}\hat{p} = & \\
= \quad \delta\beta \int \frac{\partial V\left(\hat{p};\left\{\pi_i^0\right\}_{i=1}^{m-1}\right)}{\partial \hat{p}} h\left(\hat{p},c\right)\mathrm{d}\hat{p}, &
\end{aligned}$$

which, substituted in (A.5), yields

$$\begin{aligned}
\delta\frac{\partial\left[\int V\left(\hat{p};\left\{\pi_i^0\right\}_{i=1}^{m-1}\right)h\left(\hat{p},c\right)\mathrm{d}\hat{p}\right]}{\partial c} = & \\
= \quad \delta\beta\int\frac{\partial V\left(\hat{p};\left\{\pi_i^0\right\}_{i=1}^{m-1}\right)}{\partial\hat{p}}h\left(\hat{p},c\right)\mathrm{d}\hat{p}. &
\end{aligned}$$

Thus,

$$\begin{aligned}
\delta\frac{\partial E_{\hat{p}}\left[V\left(\hat{p};\left\{\pi_i^0\right\}_{i=1}^{m-1}\right)\right]}{\partial c} & \\
= \quad \delta\beta\int\frac{\partial V\left(\hat{p};\left\{\pi_i^0\right\}_{i=1}^{m-1}\right)}{\partial\hat{p}}h\left(\hat{p},c\right)\mathrm{d}\hat{p}. & \qquad \text{(A.6)}
\end{aligned}$$

Direct comparison of (A.4) and (A.6) shows that their difference rests on the impact of beliefs updating on the future value of the game, an effect that cannot be signed. ■

Proof of Theorem 3 As before, we need to evaluate $\frac{\partial\hat{\pi}_i}{\partial\hat{p}}$ and $\frac{\partial\hat{\pi}_i}{\partial c}$, $i=1,2,\ldots,n$. From (4), recall that

$$\hat{\pi}_i = \frac{\pi_i f\left(\hat{p}-\alpha p-\beta_i c\right)}{\sum_{j=1}^{n}\pi_j f\left(\hat{p}-\alpha p-\beta_j c\right)}, \qquad i=1,2,\ldots,n,$$

from which we obtain

$$\begin{aligned}
\frac{\partial\hat{\pi}_i}{\partial\hat{p}} \quad = \quad & \frac{\pi_i f'\left(\hat{p}-\alpha p-\beta_i c\right)\sum_{j=1}^{n}\pi_j f\left(\hat{p}-\alpha p-\beta_j c\right)}{\left(\sum_{j=1}^{n}\pi_j f\left(\hat{p}-\alpha p-\beta_j c\right)\right)^2} - \\
& -\frac{\pi_i f\left(\hat{p}-\alpha p-\beta_i c\right)\sum_{j=1}^{n}\pi_j f'\left(\hat{p}-\alpha p-\beta_j c\right)}{\left(\sum_{j=1}^{n}\pi_j f\left(\hat{p}-\alpha p-\beta_j c\right)\right)^2}, \qquad \text{(A.7)} \\
& i=1,2,\ldots,n.
\end{aligned}$$

By the same token,

$$\begin{aligned}
\frac{\partial\hat{\pi}_i}{\partial c} \quad = \quad & -\frac{\pi_i\beta_i f'\left(\hat{p}-\alpha p-\beta_i c\right)\sum_{j=1}^{n}\pi_j f\left(\hat{p}-\alpha p-\beta_j c\right)}{\left(\sum_{j=1}^{n}\pi_j f\left(\hat{p}-\alpha p-\beta_j c\right)\right)^2} + \\
& +\frac{\pi_i f\left(\hat{p}-\alpha p-\beta_i c\right)\sum_{j=1}^{n}\pi_j\beta_j f'\left(\hat{p}-\alpha p-\beta_j c\right)}{\left(\sum_{j=1}^{n}\pi_j f\left(\hat{p}-\alpha p-\beta_j c\right)\right)^2}, \\
& i=1,2,\ldots,n.
\end{aligned}$$

which can be restated as

$$\begin{aligned}\frac{\partial \hat{\pi}_i}{\partial c} \quad &= \quad -\beta_i \frac{\partial \hat{\pi}_i}{\partial \hat{p}} - \frac{\pi_i \beta_i f\left(\hat{p} - \alpha p - \beta_i c\right) \sum_{j=1}^{n} \pi_j f'\left(\hat{p} - \alpha p - \beta_j c\right)}{\left(\sum_{j=1}^{n} \pi_j f\left(\hat{p} - \alpha p - \beta_j c\right)\right)^2} + \\ &+ \frac{\pi_i f\left(\hat{p} - \alpha p - \beta_i c\right) \sum_{j=1}^{n} \pi_j \beta_j f'\left(\hat{p} - \alpha p - \beta_j c\right)}{\left(\sum_{j=1}^{n} \pi_j f\left(\hat{p} - \alpha p - \beta_j c\right)\right)^2}, \\ & \qquad\qquad\qquad\qquad\qquad\qquad i = 1, 2, \ldots, n.\end{aligned}$$

Using (4), we finally obtain

$$\begin{aligned}\frac{\partial \hat{\pi}_i}{\partial c} \quad &= \quad -\beta_i \frac{\partial \hat{\pi}_i}{\partial \hat{p}} - \frac{\hat{\pi}_i \beta_i \sum_{j=1}^{n} \pi_j f'\left(\hat{p} - \alpha p - \beta_j c\right)}{\sum_{j=1}^{n} \pi_j f\left(\hat{p} - \alpha p - \beta_j c\right)} + \\ &+ \frac{\hat{\pi}_i \sum_{j=1}^{n} \pi_j \beta_j f'\left(\hat{p} - \alpha p - \beta_j c\right)}{\sum_{j=1}^{n} \pi_j f\left(\hat{p} - \alpha p - \beta_j c\right)}, \qquad i = 1, 2, \ldots, n.\end{aligned} \tag{A.8}$$

Take the value function of the experimentor agency's problem:

$$\begin{aligned}V\left(p, \{\pi_i\}_{i=1}^{n-1}\right) = \quad & \underset{c}{\text{Max}} \quad \left\{U\left(c, p\right) + \delta E_{\hat{p}}\left[V\left(\hat{p}, \{\hat{\pi}_i\left(\hat{p}, c\right)\}_{i=1}^{n-1}\right)\right]\right\} \\ & \text{s.t.} \quad \hat{p} = \alpha p + \beta c + \epsilon \\ = \quad & \underset{c}{\text{Max}} \quad \left\{U\left(c, p\right) + \delta \int V\left(\hat{p}, \{\hat{\pi}_i\left(\hat{p}, c\right)\}_{i=1}^{n-1}\right) h\left(\hat{p}, c\right) \mathrm{d}\hat{p}\right\} \\ & \text{s.t.} \quad \hat{p} = \alpha p + \beta c + \epsilon,\end{aligned}$$

where

$$h\left(\hat{p}, c\right) \quad = \quad \sum_{i=1}^{n} \pi_i f\left(\hat{p} - \alpha p - \beta_i c\right).$$

Differentiating the second term of the functional objective, one obtains

$$\begin{aligned}&\delta \frac{\partial E_{\hat{p}}\left[V\left(\hat{p}, \{\hat{\pi}_i\left(\hat{p}, c\right)\}_{i=1}^{n-1}\right)\right]}{\partial c} = \\ &= \quad \delta \int \left(\sum_{i=1}^{n-1} \frac{\partial V\left(\hat{p}, \{\hat{\pi}_i\left(\hat{p}, c\right)\}_{i=1}^{n-1}\right)}{\partial \hat{\pi}_i} \frac{\partial \hat{\pi}_i}{\partial c}\right) h\left(\hat{p}, c\right) \mathrm{d}\hat{p} + \\ &\quad + \delta \int V\left(\hat{p}, \{\hat{\pi}_i\left(\hat{p}, c\right)\}_{i=1}^{n-1}\right) \frac{\partial h\left(\hat{p}, c\right)}{\partial c} \mathrm{d}\hat{p}.\end{aligned} \tag{A.9}$$

Using (A.8) and integrating the second term above by parts, we obtain

$$
\begin{aligned}
-\delta \Bigg[& \int \left(\sum_{i=1}^{n-1} \beta_i \frac{\partial V\left(\hat{p}, \{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{n-1}\right)}{\partial \hat{\pi}_i} \frac{\partial \hat{\pi}_i}{\partial \hat{p}} \right) \times \\
& \quad \times \left(\sum_{i=1}^{n} \pi_i f\left(\hat{p} - \alpha p - \beta_i c\right) \right) \mathrm{d}\hat{p} + \\
& + \int \left(\sum_{i=1}^{n-1} \beta_i \frac{\partial V\left(\hat{p}, \{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{n-1}\right)}{\partial \hat{\pi}_i} \hat{\pi}_i \right) \times \\
& \quad \times \left(\sum_{i=1}^{n} \pi_i f'\left(\hat{p} - \alpha p - \beta_i c\right) \right) \mathrm{d}\hat{p} - \\
& - \int \left(\sum_{i=1}^{n-1} \frac{\partial V\left(\hat{p}, \{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{n-1}\right)}{\partial \hat{\pi}_i} \hat{\pi}_i \right) \times \\
& \quad \times \left(\sum_{i=1}^{n} \pi_i \beta_i f'\left(\hat{p} - \alpha p - \beta_i c\right) \right) \mathrm{d}\hat{p} - \\
& - \int \left(\sum_{i=1}^{n-1} \frac{\partial V\left(\hat{p}, \{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{n-1}\right)}{\partial \hat{\pi}_i} \frac{\partial \hat{\pi}_i}{\partial \hat{p}} \right) \times \\
& \quad \times \left(\sum_{i=1}^{n} \pi_i \beta_i f\left(\hat{p} - \alpha p - \beta_i c\right) \right) \mathrm{d}\hat{p} - \\
& - \int \frac{\partial V\left(\hat{p}, \{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{n-1}\right)}{\partial \hat{p}} \times \\
& \quad \times \left(\sum_{i=1}^{n} \pi_i \beta_i f\left(\hat{p} - \alpha p - \beta_i c\right) \right) \mathrm{d}\hat{p} \Bigg]. \qquad \text{(A.10)}
\end{aligned}
$$

We need to integrate the second and third terms above by parts. Let us begin with the second term:

$$
\underbrace{-\int \left(\sum_{i=1}^{n-1} \beta_i \frac{\partial V\left(\hat{p}, \{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{n-1}\right)}{\partial \hat{\pi}_i} \hat{\pi}_i \right)}_{u(\hat{p})} \underbrace{\left(\sum_{i=1}^{n} \pi_i f'\left(\hat{p} - \alpha p - \beta_i c\right) \right) \mathrm{d}\hat{p}}_{v'(\hat{p})\mathrm{d}\hat{p}}.
$$

We get

$$
\begin{aligned}
& \int \left(\sum_{i=1}^{n-1} \beta_i \frac{\partial^2 V\left(\hat{p}, \{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{n-1}\right)}{\partial \hat{\pi}_i^2} \frac{\partial \hat{\pi}_i}{\partial \hat{p}} \hat{\pi}_i + \right. \\
+ \quad & \sum_{i=1}^{n-1} \beta_i \frac{\partial^2 V\left(\hat{p}, \{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{n-1}\right)}{\partial \hat{\pi}_i \, \partial \hat{p}} \hat{\pi}_i + \\
+ \quad & \left. \sum_{i=1}^{n-1} \beta_i \frac{\partial V\left(\hat{p}, \{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{n-1}\right)}{\partial \hat{\pi}_i} \frac{\partial \hat{\pi}_i}{\partial \hat{p}} \right) v\left(\hat{p}\right) \mathrm{d}\hat{p},
\end{aligned}
$$

where $v\left(\hat{p}\right) = \sum_{i=1}^{n} \pi_i f\left(\hat{p} - \alpha p - \beta_i c\right)$.

Now, take the third term of (A.10),

$$
\underbrace{\int \left(\sum_{i=1}^{n-1} \frac{\partial V\left(\hat{p}, \{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{n-1}\right)}{\partial \hat{\pi}_i} \hat{\pi}_i \right)}_{u(\hat{p})} \underbrace{\left(\sum_{i=1}^{n} \pi_i \beta_i f'\left(\hat{p} - \alpha p - \beta_i c\right) \right) \mathrm{d}\hat{p}}_{v'(\hat{p})\mathrm{d}\hat{p}},
$$

and subject it to a similar integration, yielding

$$-\int\left(\sum_{i=1}^{n-1}\frac{\partial^2 V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{n-1}\right)}{\partial\hat{\pi}_i^2}\frac{\partial\hat{\pi}_i}{\partial\hat{p}}\hat{\pi}_i+\right.$$
$$+\sum_{i=1}^{n-1}\frac{\partial^2 V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{n-1}\right)}{\partial\hat{\pi}_i\,\partial\hat{p}}\hat{\pi}_i+$$
$$\left.+\sum_{i=1}^{n-1}\frac{\partial V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{n-1}\right)}{\partial\hat{\pi}_i}\frac{\partial\hat{\pi}_i}{\partial\hat{p}}\right)v(\hat{p})\,\mathrm{d}\hat{p},$$

where $v(\hat{p}) = \sum_{i=1}^{n}\pi_i\beta_i f(\hat{p}-\alpha p-\beta_i c)$. Replacing all these in (A.10), we finally obtain

$$\begin{aligned}
&\frac{\partial E_{\hat{p}}\left[V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{n-1}\right)\right]}{\partial c}=\\
&=\delta\left[\int\left(\sum_{i=1}^{n-1}\beta_i\frac{\partial^2 V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{n-1}\right)}{\partial\hat{\pi}_i^2}\frac{\partial\hat{\pi}_i}{\partial\hat{p}}\hat{\pi}_i\right)\times\right.\\
&\qquad\times\left(\sum_{i=1}^{n}\pi_i f(\hat{p}-\alpha p-\beta_i c)\right)\mathrm{d}\hat{p}+\\
&+\int\left(\sum_{i=1}^{n-1}\beta_i\frac{\partial^2 V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{n-1}\right)}{\partial\hat{\pi}_i\partial\hat{p}}\hat{\pi}_i\right)\times\\
&\qquad\times\left(\sum_{i=1}^{n}\pi_i f(\hat{p}-\alpha p-\beta_i c)\right)\mathrm{d}\hat{p}-\\
&-\int\left(\sum_{j=1}^{n-1}\frac{\partial^2 V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{n-1}\right)}{\partial\hat{\pi}_i^2}\frac{\partial\hat{\pi}_i}{\partial\hat{p}}\hat{\pi}_i\right)\times\\
&\qquad\times\left(\sum_{i=1}^{n}\pi_i\beta_i f(\hat{p}-\alpha p-\beta_i c)\right)\mathrm{d}\hat{p}-\\
&-\int\left(\sum_{i=1}^{n-1}\frac{\partial^2 V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{n-1}\right)}{\partial\hat{\pi}_i\partial\hat{p}}\hat{\pi}_i\right)\times\\
&\qquad\times\left(\sum_{i=1}^{n}\pi_i\beta_i f(\hat{p}-\alpha p-\beta_i c)\right)\mathrm{d}\hat{p}+\\
&+\int\frac{\partial V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{n-1}\right)}{\partial\hat{p}}\times\\
&\qquad\left.\times\left(\sum_{i=1}^{n}\pi_i\beta_i f(\hat{p}-\alpha p-\beta_i c)\right)\mathrm{d}\hat{p}\right].
\end{aligned}\tag{A.11}$$

The comparable expression for a passive-learner agency is as follows:

$$\begin{aligned}
&\frac{\partial E_{\hat{p}}\left[V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{n-1}\right)\right]}{\partial c}=\\
&=\quad\delta\int\frac{\partial V\left(\hat{p},\{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{n-1}\right)}{\partial\hat{p}}\times\\
&\qquad\times\left(\sum_{i=1}^{n}\pi_i\beta_i f(\hat{p}-\alpha p-\beta_i c)\right)\mathrm{d}\hat{p}.
\end{aligned}\tag{A.12}$$

The difference between (A.11) and (A.12) captures the experimentation effect. The comparison is made easier by simplifying the problem to only two states of Nature, $\left\{\overline{\beta},\underline{\beta}\right\}$. Without loss of generality, let $\overline{\beta}>\underline{\beta}$ and π and $\hat{\pi}$ be the prior and posterior probabilities associated with $\overline{\beta}$. In this case, (A.11) can be written as

$$\begin{aligned}
\delta\Bigg[&\int\overline{\beta}\frac{\partial^2 V\left(\hat{p},\hat{\pi}\left(\hat{p},c\right)\right)}{\partial\hat{\pi}^2}\frac{\partial\hat{\pi}}{\partial\hat{p}}\hat{\pi}\times\\
&\quad\times\left(\pi f\left(\hat{p}-\alpha p-\overline{\beta}c\right)+\left(1-\pi\right)f\left(\hat{p}-\alpha p-\underline{\beta}c\right)\right)\mathrm{d}\hat{p}+\\
&+\int\overline{\beta}\frac{\partial^2 V\left(\hat{p},\hat{\pi}\left(\hat{p},c\right)\right)}{\partial\hat{\pi}\partial\hat{p}}\hat{\pi}\times\\
&\quad\times\left(\pi f\left(\hat{p}-\alpha p-\overline{\beta}c\right)+\left(1-\pi\right)f\left(\hat{p}-\alpha p-\underline{\beta}c\right)\right)\mathrm{d}\hat{p}-\\
&-\int\frac{\partial^2 V\left(\hat{p},\hat{\pi}\left(\hat{p},c\right)\right)}{\partial\hat{\pi}^2}\frac{\partial\hat{\pi}}{\partial\hat{p}}\hat{\pi}\times\\
&\quad\times\left(\pi\overline{\beta}f\left(\hat{p}-\alpha p-\overline{\beta}c\right)+\left(1-\pi\right)\underline{\beta}f\left(\hat{p}-\alpha p-\underline{\beta}c\right)\right)\mathrm{d}\hat{p}-\\
&-\int\frac{\partial^2 V\left(\hat{p},\hat{\pi}\left(\hat{p},c\right)\right)}{\partial\hat{\pi}\partial\hat{p}}\hat{\pi}\times\\
&\quad\times\left(\pi\overline{\beta}f\left(\hat{p}-\alpha p-\overline{\beta}c\right)+\left(1-\pi\right)\underline{\beta}f\left(\hat{p}-\alpha p-\underline{\beta}c\right)\right)\mathrm{d}\hat{p}-\\
&+\int\frac{\partial V\left(\hat{p},\hat{\pi}\left(\hat{p},c\right)\right)}{\partial\hat{p}}\times\\
&\quad\times\left(\pi\overline{\beta}f\left(\hat{p}-\alpha p-\overline{\beta}c\right)+\left(1-\pi\right)\underline{\beta}f\left(\hat{p}-\alpha p-\underline{\beta}c\right)\right)\mathrm{d}\hat{p}\Bigg],
\end{aligned}$$

which collapses to

$$\begin{aligned}
\delta\Bigg[&\int\frac{\partial^2 V\left(\hat{p},\hat{\pi}\left(\hat{p},c\right)\right)}{\partial\hat{\pi}^2}\frac{\partial\hat{\pi}}{\partial\hat{p}}\hat{\pi}\times\\
&\quad\times\left(\overline{\beta}-\underline{\beta}\right)\left(1-\pi\right)f\left(\hat{p}-\alpha p-\underline{\beta}c\right)\mathrm{d}\hat{p}+\\
&+\int\frac{\partial^2 V\left(\hat{p},\hat{\pi}\left(\hat{p},c\right)\right)}{\partial\hat{\pi}\partial\hat{p}}\hat{\pi}\times\\
&\quad\times\left(\overline{\beta}-\underline{\beta}\right)\left(1-\pi\right)f\left(\hat{p}-\alpha p-\underline{\beta}c\right)\mathrm{d}\hat{p}+\\
&+\int\frac{\partial V\left(\hat{p},\hat{\pi}\left(\hat{p},c\right)\right)}{\partial\hat{p}}\times\\
&\quad\times\left(\pi\overline{\beta}f\left(\hat{p}-\alpha p-\overline{\beta}c\right)+\left(1-\pi\right)\underline{\beta}f\left(\hat{p}-\alpha p-\underline{\beta}c\right)\right)\mathrm{d}\hat{p}\Bigg].
\end{aligned}$$

By the same token, (A.12) yields

$$\begin{aligned}
\delta&\int\frac{\partial V\left(\hat{p},\hat{\pi}\left(\hat{p},c\right)\right)}{\partial\hat{p}}\times\\
&\quad\times\left(\pi\overline{\beta}f\left(\hat{p}-\alpha p-\overline{\beta}c\right)+\left(1-\pi\right)\underline{\beta}f\left(\hat{p}-\alpha p-\underline{\beta}c\right)\right)\mathrm{d}\hat{p}\Big].
\end{aligned}$$

Direct comparison of the last two expressions shows that the experimentor and the passive-learner agencies would choose the same consumption level iff

$$\begin{aligned}
&\int\frac{\partial^2 V\left(\hat{p},\hat{\pi}\left(\hat{p},c\right)\right)}{\partial\hat{\pi}^2}\frac{\partial\hat{\pi}}{\partial\hat{p}}\hat{\pi}\times\\
&\quad\times\left(\overline{\beta}-\underline{\beta}\right)\left(1-\pi\right)f\left(\hat{p}-\alpha p-\underline{\beta}c\right)\mathrm{d}\hat{p}+\\
&+\int\frac{\partial^2 V\left(\hat{p},\hat{\pi}\left(\hat{p},c\right)\right)}{\partial\hat{\pi}\partial\hat{p}}\hat{\pi}\times\\
&\quad\times\left(\overline{\beta}-\underline{\beta}\right)\left(1-\pi\right)f\left(\hat{p}-\alpha p-\underline{\beta}c\right)\mathrm{d}\hat{p}=0,
\end{aligned}$$

that is,

$$\left(\overline{\beta}-\underline{\beta}\right)\int\left(\frac{\partial^2 V\left(\hat{p},\hat{\pi}\left(\hat{p},c\right)\right)}{\partial\hat{\pi}^2}\frac{\partial\hat{\pi}}{\partial\hat{p}}+\frac{\partial^2 V\left(\hat{p},\hat{\pi}\left(\hat{p},c\right)\right)}{\partial\hat{\pi}\partial\hat{p}}\right)\times$$
$$\times\quad\hat{\pi}\left(1-\pi\right)f\left(\hat{p}-\alpha p-\underline{\beta}c\right)\mathrm{d}\hat{p}=0.$$

From (4), we have that $\hat{\pi}\left(1-\pi\right)f\left(\hat{p}-\alpha p-\underline{\beta}c\right)=\pi\left(1-\hat{\pi}\right)f\left(\hat{p}-\alpha p-\overline{\beta}c\right)$. Thus, we may alternatively write the expression above as follows

$$\left(\overline{\beta}-\underline{\beta}\right)\int\left(\frac{\partial^2 V\left(\hat{p},\hat{\pi}\left(\hat{p},c\right)\right)}{\partial\hat{\pi}^2}\frac{\partial\hat{\pi}}{\partial\hat{p}}+\frac{\partial^2 V\left(\hat{p},\hat{\pi}\left(\hat{p},c\right)\right)}{\partial\hat{\pi}\partial\hat{p}}\right)\times$$
$$\times\quad\pi\left(1-\hat{\pi}\right)f\left(\hat{p}-\alpha p-\overline{\beta}c\right)\mathrm{d}\hat{p}=0. \tag{A.13}$$

This expression measures the effect of consumption on expected utility through beliefs. Its first term is necessarily positive: Fusselman and Mirman (1993) have proven the convexity of the value function, i.e., that $\frac{\partial^2 V\left(\hat{p},\hat{\pi}\left(\hat{p},c\right)\right)}{\partial\hat{\pi}^2}>0$, implying that information is valuable.[23]

Also, $\frac{\partial\hat{\pi}}{\partial\hat{p}}>0$. To see it, note that (A.7) collapses to

$$\frac{\partial\hat{\pi}}{\partial\hat{p}}=\frac{\pi\left(1-\pi\right)\left[\overline{f}'\underline{f}-\overline{f}\underline{f}'\right]}{\pi\overline{f}+\left(1-\pi\right)\underline{f}},$$

where $\overline{f}=f\left(\hat{p}-\alpha p-\overline{\beta}c\right)$, $\underline{f}=f\left(\hat{p}-\alpha p-\underline{\beta}c\right)$, $\overline{f}'=f'\left(\hat{p}-\alpha p-\overline{\beta}c\right)$, and $\underline{f}'=f'\left(\hat{p}-\alpha p-\underline{\beta}c\right)$. Dividing the expression above by $\overline{f}\underline{f}$ does not change its sign and yields

$$\frac{\partial\hat{\pi}}{\partial\hat{p}}=\frac{\pi\left(1-\pi\right)\left[\frac{\overline{f}'}{\overline{f}}-\frac{\underline{f}'}{\underline{f}}\right]}{\pi\overline{f}+\left(1-\pi\right)\underline{f}}.$$

Assumption 6 (MLRP) implies that the term in square brackets is positive. Hence, $\frac{\partial\hat{\pi}}{\partial\hat{p}}>0$, implying that the whole first term of (A.13) is positive.

In the second term of (A.13), $\frac{\partial^2 V\left(\hat{p},\hat{\pi}\left(\hat{p},c\right)\right)}{\partial\hat{\pi}\partial\hat{p}}$ measures how increased future pollution affects the value of future information through future beliefs. Thus, it can be understood as a dynamic experimentation effect. This term cannot be signed. Hence, in general, (A.13) cannot be signed.

Finally, the future pollution stock, besides having an impact on the informational aspects of the problem—already discussed—also influences expected utility physically through

$$\delta\int\frac{\partial V\left(\hat{p},\hat{\pi}\left(\hat{p},c\right)\right)}{\partial\hat{p}}\times$$
$$\times\left(\pi\overline{\beta}f\left(\hat{p}-\alpha p-\overline{\beta}c\right)+\left(1-\pi\right)\underline{\beta}f\left(\hat{p}-\alpha p-\underline{\beta}c\right)\right)\mathrm{d}\hat{p}\Big]. \tag{A.14}$$

Insofar as (A.13) differs from zero, these terms also differ for a passive-learner agency and an experimentor and their difference also cannot be signed.

Ergo, the effect of experimenting vs. passively learning on an agency's behavior cannot be signed without specific information on the value function. ■

[23] $U_{pp}<0$ (Assumption 5) is needed for convexity. Computations are available from the authors upon request.

Proof of Theorem 4 Take the value function of an experimentor agency:

$$\begin{aligned} V\left(p, \{\pi_i\}_{i=1}^{n-1}\right) &= \underset{c}{\text{Max}} \quad \left\{U(c,p) + \delta E_{\hat{p}}\left[V\left(\hat{p}, \{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{n-1}\right)\right]\right\} \\ &\quad \text{s.t.} \quad \hat{p} = \alpha p + \beta c + \epsilon \\ &= \underset{c}{\text{Max}} \quad \left\{U(c,p) + \delta \int V\left(\hat{p}, \{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{n-1}\right) h(\hat{p},c)\, \mathrm{d}\hat{p}\right\} \\ &\quad \text{s.t.} \quad \hat{p} = \alpha p + \beta c + \epsilon, \end{aligned}$$

Differentiating the second term of the functional objective, one obtains

$$\begin{aligned} &\delta \frac{\partial E_{\hat{p}}\left[V\left(\hat{p}, \{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{n-1}\right)\right]}{\partial c} = \\ &= \delta \int \left(\sum_{i=1}^{n-1} \frac{\partial V\left(\hat{p}, \{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{n-1}\right)}{\partial \hat{\pi}_i} \frac{\partial \hat{\pi}_i}{\partial c}\right) h(\hat{p})\, \mathrm{d}\hat{p} + \\ &\quad + \delta \int V\left(\hat{p}, \{\hat{\pi}_i(\hat{p},c)\}_{i=1}^{n-1}\right) \frac{\partial h(\hat{p})}{\partial c} \mathrm{d}\hat{p}, \end{aligned}$$

which reduces to (A.11), as seen in the proof of the previous theorem. The comparable expression for a non-learner agency is as follows:

$$\begin{aligned} &\frac{\partial E_{\hat{p}}\left[V\left(\hat{p}; \{\pi_i\}_{i=1}^{n-1}\right)\right]}{\partial c} = \\ &= \delta \int \frac{\partial V\left(\hat{p}; \{\pi_i\}_{i=1}^{n-1}\right)}{\partial \hat{p}} \times \\ &\quad \times \left(\sum_{i=1}^{n} \pi_i \beta_i f(\hat{p} - \alpha p - \beta_i c)\right) \mathrm{d}\hat{p}. \end{aligned}$$

Now, besides the experimentation effect, already analyzed in the previous theorem, we have an additional difference: the experimentor does update its beliefs whereas the non-learner does not, a disparity that cannot be signed. ■

References

AGHION, P., BOLTON, P., HARRIS, C., JULLIEN, B., 1991. "Optimal Learning by Experimentation." *Review of Economic Studies* **58**, 621–654.

ALEPUZ, M.D., URBANO, A., 1999. "Duopoly Experimentation: Cournot Competition." *Mathematical Social Sciences* **37**, 165–188.

ARROW, K., FISHER, A., 1974. "Environmental Preservation, Uncertainty and Irreversibility." *Quarterly Journal of Economics* **88**, 312–319.

BALVERS, R. J., COSIMANO, T. F., 1994. "Inflation Variability and Gradualist Monetary Policy." *Review of Economic Studies* **61**, 721–738.

BECK, G., WIELAND, V., 2002. "Learning and Control in a Changing Economic Environment." *Journal of Economic Dynamics and Control* **26**, 1359–1378.

BERTOCCHI, G., SPAGAT, M., 1998. "Growth under Uncertainty with Experimentation." *Journal of Economic Dynamics and Control* **23**, 209–231.

DATTA, M., MIRMAN, L., SCHLEE, E., 2002. "Optimal Experimentation in Signal-Dependent Decision Problems." *International Economic Review*, **43**, 577–607.

EASLEY, D., KIEFER, N., 1988. "Controlling a Stochastic Process with Unknown Parameters." *Econometrica* **56**, 1045–1064.

EPSTEIN, L., 1980. "Decision-Making and the Temporal Resolution of Uncertainty." *International Economic Review* **21**, 269–284.

FUSSELMAN, J., MIRMAN, L., 1993. "Experimental Consumption for a General Class of Disturbance Densities," in *General Equilibrium, Growth, and Trade II*, New York: Academic Press.

GROSSMAN, S., R. KIHLSTROM AND L. MIRMAN, 1977. "A Bayesian Approach to the Production of Information and Learning by Doing." *Review of Economic Studies* **44**, 533–547.

HENRY, C., 1974. "Investment Decisions under Uncertainty: The Irreversibility Effect." *American Economic Review* **64**, 1006–1012.

KARP, L., ZHANG, J., 2006. "Regulation with Anticipated Learning About Environmental Damages." *Journal of Environmental Economics and Management* **51**, 259–279.

KELLY, D., KOLSTAD, C.D., 1999. "Bayesian Learning, Growth, and Pollution." *Journal of Economic Dynamics and Control* **23**, 491–518.

KENNEDY, P., 1999. "Learning About Environmental Damage: Implications for

Emissions Trading." *Canadian Journal of Economic* **32**, 1313–1327.

KIEFER, N., NYARKO, Y., 1989. "Optimal Control of an Unknown Linear Process with Learning." *International Economic Review* **30**, 571–586.

KIEFER, N., 1989. "A Value Function Arising in the Economics of Information." *Journal of Economic Dynamics and Control* **13**, 291–223.

KOLSTAD, C.D., 1993. "Looking versus Leaping: The Timing of CO_2 Control in the Face of Uncertainty and Learning." In Kaya, Y., Nakicenovic, N., Nordhaus, W., Toth, F. (Eds.), *Costs, Impacts and Benefits of CO_2 Mitigation.* Austria: IIASA, 63–82.

KOLSTAD, C.D., 1996a. "Fundamental Irreversibilities in Stock Externalities." *Journal of Public Economics* **60**, 221–223.

KOLSTAD, C.D., 1996b. "Learning and Stock Effects in Environmental Regulation: The Case of Greenhouse Gas Emissions." *Journal of Environmental Economics and Management* **31**, 1–18.

MANNE, A., RICHELS, R.G., 1992. *Buying Greenhouse Insurance: The Economic Costs of CO_2 Emission Limits.* Cambridge, MA: The MIT Press.

MILGROM, PAUL R., 1981. "Good News and Bad News: Representation Theorems and Applications," *Bell Journal of Economics* **12**, 380–391.

MIRMAN, L., SAMUELSON, L., URBANO, A., 1993a. "Monopoly Experimentation." *International Economic Review* **34**, 549–564.

MIRMAN, L., SAMUELSON, L., URBANO, A., 1993b. "Duopoly Signal Jamming." *Economic Theory* **3**, 129–150.

MIRMAN, L., SAMUELSON, L., SCHLEE, E., 1994. "Strategic Information Manipulation in Duopolies." *Journal of Economic Theory* **62**, 363–384.

MOURA-COSTA, P.H., WILSON, C., 2000. "An Equivalence Factor between CO_2 Avoided Emissions and Sequestration." *Mitigation Adoption Strategies for Global Change* **5**, 51–60.

NORDHAUS W., 1991. "To Slow or Not to Slow: The Economics of the Greenhouse Effect." *Economic Journal* **101**, 920–937.

NORDHAUS W., POPP, D., 1997. "What is the Value of Scientific Knowledge? An Application to Global Warming Using the PRICE Model." *Energy Journal* **18**, 1–45.

NYARKO, Y., 1991. "The Number of Equations Versus The Number of Unknowns: The Convergence of Bayesian Posterior Processes." *Journal of Economic Dynamics and Control* **15**, 687–713.

PECK, S.C., TEISBERG, T. J., 1993. “Global warming Uncertainties and The Value of Information: An Analysis Using CETA.” *Resources and Environmental Economics* **15**, 71–97.

ULPH, A., ULPH, D., 1997. “Global Warming, Irreversibility and Learning.” *Economic Journal* **107**, 636–650.

WATSON, R.T., NOBLE, I.R., BOLIN, B., RAVINDRANATH, N.H., VERARDO, D.J., DOKKEN, D.J., 2000. “Land Use, Land-Use Change and Forestry.” Special Report of the Intergovernmental Panel on Climate Change (IPCC). Cambridge, UK: Cambridge University Press.

WIELAND, V., 2006. “Monetary Policy and Uncertainty about the Natural Unemployment Rate: Brainard-Style Conservatism versus Experimental.” *Berkeley Electronic Press: Advances in Macroeconomics* **26**, 1–34.

WIELAND, V., 2000a. “Learning by Doing and the Value of Optimal Experimentation.” *Journal of Economic Dynamics and Control* **24**, 501–534.

WIELAND, V., 2000b. “Monetary Policy, Parameter Uncertainty and Optimal Learning.” *Journal of Monetary Economics* **46**, 199–228.