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An RBC Model with a rich fiscal sector

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Abstract

Contributing to the general understanding of fiscal policy effectiveness, this study consists in the reformulation and estimation of the DSGE model developed in Azevedo and Ercolani (2012), to measure the potential relations between the private sector and the consumption and investment components of government expenditures. The estimation results show that public consumption and capital have both a substitutability effect on private factors. For the study of the dynamic effects, the model is augmented with strict fiscal rules, whose imposition creates a "crowding-out" effect of the simulated fiscal policy shocks on government consumption and investment.

Keywords: Public Spending Externalities, Public Investment Externalities, Fiscal Policy, Bayesian Estimation.

1 Introduction

The large-scale fiscal stimulus packages recently applied to overcome the worldwide 2008 financial crisis triggered a general concern with fiscal policy effectiveness.

Focusing on the investigation of the several channels through which government spending and investment can affect the private sector, this study contributes to the vivid discussion by revisiting the concept of *public externalities*. Considering the existence of a substitution or complementarity relationship between public purchases and private factors, government spending and investment decisions can have collateral effects on the household's marginal utility of consumption and the production function of final goods. In this context, the core of this work lies in qualifying the nature of such relationships and observe how they influence the impact of fiscal policy.

For a wide scope of empirical literature on this subject the Ricardian equivalence holds and thus, whether the government funds public expenditures through collecting taxes or issuing debt is irrelevant from the household's point of view. Growing apart from such framework, in the present model the

setting for income taxes and lump-sum transfers from the government allow to observe the costs of debt financing. How such alternative fiscal regime will coexist with the externalities channels in the model economy is yet another key question this work tries to address.

We follow closely the model proposed in Azevedo and Ercolani (2012) to evaluate the nature of the relationship between private and public consumption and investment. The model developed by these authors will help us to understand the research question because it allows government consumption to directly affect the marginal utility of consumption, through a Constant Elasticity of Substitution (CES) consumption aggregator, and public capital to shift the productivity of private factors. We then propose an extension of the model at two distinct levels. In the light of our main objectives, instead of considering public capital productivity [1], a CES capital aggregator is added to production function of final goods. Furthermore, following Traum and Yang (2010), two stabilizers of public debt, in the form of income taxes and lump-sum transfers, are imposed to the fiscal policy sector.

Representing the externalities created by public consumption and capital on the private sector, the CES aggregators combine the two types of productive inputs, allowing to observe relationship between both. Moreover, the new fiscal rules enrich the analysis by changing the transmission mechanism of fiscal shocks with fiscal variables responding to the level of the public debt.

Overall, our main interest rises from the fact that despite the extensive theoretical debate, few are the ones who effectively quantify these relationships and observe their role in a model economy with strict fiscal rules and a general production function. To answer our first purpose we estimate the model through Bayesian methods, without fiscal rules. With the results obtained, we report several impulse response functions and two dynamic multipliers related to government consumption an investment shocks, with and without the augmentations described.

Using U.S. data from 1969 to 2008, the estimation results support a substitutability effect between public and private consumption and, to a lesser extent, between public and private capital. Although opposite to the evidence found in Coenen, Straub and Trabandt (2012) for an open economy, both studies reflect the possibility of public and private capital having a weaker relationship than consump-

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[1]Simultaneously with strong evidence of substitutability between public and private consumption, the results in Azevedo and Ercolani (2012) achieved not so convincing evidence for the the effect of public investment on private sector productivity, motivating the adoption of a different reasoning to approach such relationship.

tion factors. When looking at the output dynamic multipliers, the value measured from the model with the substitution effects decreases the impact of a public consumption shock from 0.95 to 0.07, whereas with a shock in government investment the value of the output multiplier goes from 1.09 to 0.90, when the substitution effect of public capital is considered. In turn, the imposition of the fiscal rules implies that both shocks have a significant negative effect on economy, due to the distortionary properties of income taxation.

The remaining of the paper is organized as follows. Section 2 makes a brief review of literature on the topic of interest. Section 3 describes the theoretical model of analysis, while section 4 presents the quantitative analysis. Model dynamics are explained in section 5 and section 6 concludes, presenting conclusions and final remarks.

2 Literature Review

The potential of expansionary policy to foster aggregate economic variables as output, consumption or employment is an issue widely present in economic literature. Addressing this question, this paper contributes to two main fields of fiscal policy analysis. First, it explores the existence of cause-effect relationships between either public and private capital or consumption and, secondly, observes the implications of assuming Non-Ricardian Households, through the imposition of strong fiscal rules.

Empirical results on the relationship between public and private consumption hold evidence on both substitutability and complementarity hypothesis. A great set of earlier studies departs from a partial equilibrium model, using Euler equations to estimate the value for the elasticity of substitution. For example, Karras (1994) addresses different components of consumption, arriving to a general evidence of complementarity between public and private consumption. In contrast, Graham (1993) performs an extension to the model in Aschauer (1985), showing a crowding out effect of public consumption on the private sector, under the assumption of permanent income. Diverging from these studies, our analysis estimates the elasticity of substitution of both factors within a general equilibrium model, like Bouakez and Rebei (2007). The authors use the maximum likelihood estimation method, finding that a government spending shock leads to a persistent increase in the level of private consumption.

On the relationship between public and private capital, in Aschauer (1988) the author investigates how public capital can have both a crowding in and out effect on private investment. For different pe-

riods of the U.S. history, the analysis finds a substitutability effect between public and private capital, as the national rate of capital accumulation rises with an increase in public investment. However, narrowing the study, he also observes that public infrastructure capital fosters the productivity of private capital stocks, thus raising private investment. In fact, both effects seem to have an important role in economy, although the thesis of complementarity is slightly preferred among academics. Particularly, some important contributors to this literature include studies on developing countries like Greene and Villanueva (1991) or Blejer and Khan (1984).

Leaning now onto the last focus of our study, under Ricardian equivalence dynamics of fiscal financing assume a passive role (see Leeper (1991)). In these cases, for market clearing purposes, government debt is omitted from the analysis through the adjustment of lump-sump taxes. As an alternative to such absence of public debt issuing theory, the fiscal section of the model economy in the present study is grounded on the work of Traum and Yang (2010). Although with different purposes, the data and assumptions of both studies are quite similar and so will be the fiscal rules. For the Euro Area, Coenen, Straub and Trabandt (2012), while assessing the impact of the European Economics Recovery Plan on the Euro Area GDP, estimates the model with fiscal rules and the aforementioned externality channels. Our approach proposes a slight different study in the sense that they calibrate the share of public capital on the CES aggregator to 90%, whereas we estimate the model unrestricting the share of public capital and calibrating fiscal rules.

3 The Model

The model economy is an extension of the RBC model in Azevedo and Ercolani (2012), with a more general production function and a more complex fiscal block, where income taxes and lump-sum transfers from the government respond to the public debt level.

The Baseline model, which will be later subject to estimation, is achieved after replacing the private capital factor in the production function, $K_{j,t}$, by effective capital, $\tilde{K}_t$, discarding public capital productivity, K_t^G. Furthermore, the augmentation proposed is complete when labor and capital taxes are replaced by an income tax, τ_i, and the lump-sum transfers from the government, T_t, are reformulated.

3.1   Households

The model is composed by households that, facing an inter-temporal budget constraint, will choose the level of consumption C_t and work L_t that maximize their lifetime utility. C_t enters the utility function through a CES aggregator of private and public consumption, G_t, specified as follows:

$$\tilde{C}_t = \left[\phi(C_t)^{\frac{v-1}{v}} + (1-\phi)G_t^{\frac{v-1}{v}}\right]^{\frac{v}{v-1}}, \tag{1}$$

where $v \in (0; \infty)$ is the elasticity of substitution between C_t and G_t and the weight of private consumption in the effective consumption aggregator is given by ϕ.

The households, deriving utility from effective consumption and desutility from working, define their lifetime expected utility as:

$$E_0\left\{\sum_{t=0}^{\infty}\beta^k e^{\varepsilon_t^b}\left[\frac{\left(\tilde{C}_t - h\tilde{C}_{t-1}^A\right)^{1-\sigma_c}}{1-\sigma_c} - \chi\frac{1}{1+\sigma_L}(L_t)^{1+\sigma_L}\right]\right\}, \tag{2}$$

with $\beta \in (0,1)$ as the subjective discount factor and and χ a positive number. ε_t^b represents the preference shock, assumed to follow a first-order autoregressive process with an i.i.d.-normal error term: $\varepsilon_t^b = \rho_b\varepsilon_{t-1}^b + \eta_t^b$. The aggregate level of effective consumption at time $t-1, \tilde{C}_{t-1}^A$, introduces an external habit formation degree $h \in (0; 1)$. The parameter σ_c denotes the degree of relative risk aversion and σ_L is the inverse of the Frisch elasticity of labor supply. The existence of a steady-state growth path is assured by assuming complete separability between consumption and labour, given the neoclassical production function, but requiring setting σ_c equal to one.

Each household sets the levels of consumption, labor supply, next period's physical capital stock, K_{t+1}, level of investment, I_t, and installed capital stock utilization intensity, u_t, subject to taxation on labor (τ^w), consumption (τ^c) and capital (τ^k), expressed in marginal rates. They participate in a market of state-contingent securities, paying Z_{t+1}^h at $t+1$ if state h realizes, at the cost $E_t\left[\frac{1}{1+r_{t,t+1}}Z_{t+1}^h\right]$, where $1/(1+r_{t,t+1})$ is the stochastic discount factor. The budget constraint (expressed in real terms) is represented as follows:

$$(1+\tau^c)C_t + I_t + E_t\left[\frac{1}{1+r_{t,t+1}}Z_{t+1}^h\right] = \tag{3}$$

$$Z_t^h + (1-\tau^w)W_tL_t + (1-\tau^k)\left[r_t^k u_t - a(u_t)\right]K_t + DIV_t - T_t,$$

where r_t^k is the net return on capital, W_tL_t is labor income and $a(u_t) = \gamma_1(u_t-1) + \frac{\gamma_2}{2}(u_t-1)^2$ represents

6

the cost of using capital at intensity u_t. DIV_t are the dividends paid, while T_t are lump-sum transfers from the government.

As capital enters the decision process with one period discrepancy, the link between capital stock for two consecutive periods is given by:

$$K_{t+1} = (1 - \delta_k) K_t + I_t \left[1 - S \left(e^{\varepsilon_t^I} \frac{I_t}{I_{t-1}} \right) \right],$$

(4)

where δ_k is the depreciation rate and investment adjustments costs are set in the function $S(.)$, adopted from Christiano, Eichenbaum and Evans (2005). $S(.) = \frac{\kappa}{2} \left(e^{\varepsilon_t^I} \frac{I_t}{I_{t-1}} - e^{\bar{\gamma}} \right)^2$, where ε_t^I is a shock to the investment cost function that follows a first-order autoregressive process with an i.i.d.-normal error term and $\bar{\gamma}$ is the steady-state growth rate of productivity.

3.2   Government and Fiscal Policy

For each period, public consumption G_t and investment I_t^g represent a determined fraction of output, $G_t = \xi_t^g Y_t$ and $I_t^g = \xi_t^{ig} Y_t$, where ξ_t^g and ξ_t^{ig} follow:

$$\xi_t^g = \exp(\varepsilon_t^g + ss^g)/(1 + \exp(\varepsilon_t^g + ss^g)) \quad ; \quad \xi_t^{ig} = \exp(\varepsilon_t^{ig} + ss^{ig})/(1 + \exp(\varepsilon_t^{ig} + ss^{ig})).$$

In turn, real government expenditure and investment shocks, ε_t^g and ε_t^{ig} respectively, are exogenous and stochastic univariate first-order autoregressive processes:

$$\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \eta_t^g$$

(5)

$$\varepsilon_t^{ig} = \rho_{ig} \varepsilon i_{t-1}^{ig} + \eta_t^{ig},$$

(6)

where η_t^g and η_t^{ig} are normal i.i.d. and mutually independent with mean zero. ξ_t^g and ξ_t^{ig} are defined in order to ss^g and ss^{ig}, so that their steady-state levels are fixed :

$$ss^g = \log(\xi^{g,ss}/(1 - \xi^{g,ss})) \quad ; \quad ss^{ig} = \log(\xi^{ig,ss}/(1 - \xi^{ig,ss})).$$

Given that in this model public investment is split into defense and non-defense items:

$$I_t^{g,def} = \xi_t^{ig,def} Y_t$$

$$\xi_t^{g,def} = \exp(\varepsilon_t^{g,def} + ss^{g,def})/(1 + \exp(\varepsilon_t^{g,def} + ss^{g,def}))$$

$$\varepsilon_t^{ig,def} = \rho_{ig,def} \varepsilon i_{t-1}^{ig,def} + \eta_t^{ig,def},$$

(7)

where defense investment is $I_t^{g,def}$, $ss^{g,def} = \log(\xi^{g,def,ss}/(1 - \xi^{g,def,ss}))$ and $\eta_t^{ig,def}$ is a normal i.i.d. defense investment shock with mean zero. Note that, with this specification, defense items are not embedded in public capital I_t^g.

Since the paths of ξ_t^g, ξ_t^{ig} and $\xi_t^{ig,def}$ are considered exogenous while the paths for G_t, I_t^g or $I_t^{g,def}$ are not, facing a drop on the total factor productivity and the output level, government consumption and investment are expected to fall, without intervention of automatic stabilizers.

Whenever the balanced budget hypothesis holds, the following expression defines the government's budget constraint:

$$\tau^c C_t + \tau^w W_t L_t + \tau^k \left[r_t^k u_t - a(u_t) \right] K_t + T_t = G_t + I_t^g + I_t^{g,def}. \tag{8}$$

With respect to the extension of the fiscal policy section, following Traum and Yang (2010), the extended model sets income tax, τ_t^i, and lump-sum transfers from the government, T_t, to depend on past levels of public debt, B_t. The equations for both fiscal rules, measured in deviations from the steady-state (denoted with hats), are:

$$\widehat{\tau}_t^i = \rho_\tau \widehat{\tau}_{t-1}^i + (1 - \rho_\tau)\gamma_\tau \widehat{br}_{t-4} + \varepsilon_t^\tau \tag{9}$$

$$\widehat{T}_t = \rho_z \widehat{T}_{t-1} + (1 - \rho_z)\gamma_z \widehat{br}_{t-1} + \varepsilon_t^z \tag{10}$$

where $\widehat{br}_t$ is the debt-to-output ratio, $\widehat{B_t/Y_t}$, and ε_t^τ and ε_t^z are, respectively, the exogenous shocks of income tax and lump-sum transfers, both following an i.i.d. normal distribution.

As a result, the new budget constraints for the government and households are respectively defined as:

$$\tau^c C_t + \tau_t^i (W_t L_t + \left[r_t^k u_t - a(u_t) \right] K_t) + T_t + (1 + r_t^b) B_{t-1} = B_t + G_t + I_t^g + I_t^{g,def} \tag{11}$$

and

$$(1 + \tau^c)C_t + I_t + E_t \left[\frac{1}{1 + r_{t,t+1}} Z_{t+1}^h \right] + B_t = Z_t^h + (1 - \tau_t^i)(W_t L_t + \left[r_t^k u_t - a(u_t) \right] K_t) + DIV_t - (1 + r_t^b)B_{t-1} - T_t, \tag{12}$$

where r_t^b represents the interest the government needs to pay relative to the debt owed in the previous period, B_{t-1}.

3.3 Firms and labor market

Each firm j, under a monopolistic competitive set, produces a single variety of final goods $Y_{j,t}$ with the same elasticity of substitution across all goods varieties. Independently from the firm, households

allocate their consumption equally between all goods. In this framework, we can abandon index j and consider a representative firm that produces a final good Y_t, using effective capital $\tilde{K}_t$, labor L_t and the fraction of public capital that does not produce any externality, represented by the defense public investment $I_t^{g,def}$. From the solution of the profit maximization problem, competitive firms set price level $P_{j,t}$ as a markup $\lambda_{p,ss}$ to the marginal cost. The production function is then given by[2]:

$$Y_{j,t} = \max(A_t \tilde{K}_t^{\alpha} L_{j,t}^{1-\alpha} - A_t \Phi, 0), \tag{13}$$

where Φ is the production fixed cost and A_t is a productivity shock. The process for $\ln(A_t)$ has a unit-root and evolves according to:

$$\ln(A_t) = \overline{\gamma} + \ln(A_{t-1}) + \varepsilon_t^a, \tag{14}$$

where $\overline{\gamma}$ is the steady-state growth rate of productivity and $\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \eta_t^a$, where η_t^a is an i.i.d.-normal sequence.

The effective capital CES aggregator, $\tilde{K}_t$ adopted from Coenen, Straub and Trabandt (2012) is adjusted to enter the final goods production function of firms, allowing to measure the elasticity of substitution between public and private investment, v_k:

$$\tilde{K}_t = \left[\phi_k (K_t)^{\frac{v_k-1}{v_k}} + (1-\phi_k)(K_t^G)^{\frac{v_k-1}{v_k}} \right]^{\frac{v_k}{v_k-1}}, \tag{15}$$

where ϕ_k is the share of private capital stock in effective physical capital and K_t^G denotes the "productivity" of public capital. Relatively to the elasticity of substitution, when $v_k 1$ private and public capital are substitutes, for values lower than 1, as it tends to zero the complementarity effect emerges and if $v_k = 1$ any cause-effect relationship is observed.

The public capital and the productivity of defense capital are assumed to evolve respectively according to:

$$K_{t+1}^G = (1 - \delta_{Kg})K_t^G + \xi_t^{ig}, \tag{16}$$

and

$$K_{t+1}^{G,def} = (1 - \delta_{Kg,def})K_t^{G,def} + \xi_t^{ig,def} \tag{17}$$

where δ_{Kg} is the depreciation rate.

With respect to the Labor Market, households answer the demand that allows setting the wage rate

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[2]The specification is the same as in the model of Azevedo and Ercolani (2012), but in our model K_t^G enters the production function through the CES aggregator and since we discard the productivity of public capital $K_t^{G,def}$ is irrelevant to the Production function and can be omitted.

$W_{i,t}$ that maximizes their utility, according to labor demand, L_t, and aggregate nominal wage, W_t. The wage setting defines $W_{i,t}$ as a markup $\lambda_{w,t}$, over the marginal rate of substitution between consumption and leisure. $\lambda_{w,t}$ is stochastic and exogenous and follows a first order autoregressive process.

3.4 Remaining Considerations

In a standard Real Business Cycle model, prices have perfect flexibility, automatically adjusting to assure market clearing conditions under perfect competition. In equilibrium, supply will equal demand, for every market. Labour demanded by firms equals differentiated labour services supplied by households, at the aggregate wage rate W_t , capital services demanded by firms equals capital supplied by households and the final goods supply equals demand by households and the government:

$$Y_t = C_t + I_t + a(u_t)K_t + G_t + I_t^g + I_t^{g,def}. \tag{18}$$

3.4.1 The Solution

The process for the solution of the steady-state followed the standard procedure of DSGE models solution. Considering identical agents, the first order conditions associated with the households' and firms' problems are derived and combined with market clearing conditions and exogenous processes. The variables subject to generational growth C_t, I_t, K_t, W_t, Tax_t, T_t, G_t, I_t^g, $I_t^{g,def}$ and B_t need to be stationarized by the level of technology A_t. The same treatment is required for the Lagrange multipliers associated with the budget constraint and the capital accumulation equation, respectively Λ_t and Q_t (Tobin's q). The conditions which solve the equilibrium for the present model can be found in the Appendix C. For estimation purposes, apart from variables ξ_t^g , ξ_t^{ig} and $\xi_t^{ig,def}$, the model equations are log-linearized around the steady state.

4   Identification, Calibration and Estimation

With U.S. quarterly data from 1969Q1 to 2008Q3, the baseline model is identified and estimated by Bayesian methods, using the statistical software platform DYNARE of Matlab.

From the acknowledgment of unidentification issues on the estimation of DSGE models, our analysis attains the identification sufficient conditions established in Iskrev (2010), leaving the analysis on the strength of identification for later improvements to the study.

For an RBC model, the parameters identification can be established by the moment or the in-

formation based approach, but only the former is used in the identification package developed for DYNARE[3].

Without an exhaustive description of the concept, consider a parameter space $\Theta \subset \mathbb{R}^k$, defined as the set of all theoretically admissible values of the deep parameters vector θ. Given the observed data, it is possible to define $m_T : [\mu', \sigma'_T]'$, a $(T-1)\ell^2 + \ell(\ell+3)/2-$dimensional vector that collects the parameters which determine the first two moments of the data, μ' and σ'_T. It follows that m_T is a function of θ, assuming that to each admissible value of θ corresponds a unique value of τ, the vector of the reduced form parameters. In case the model's random vector of structural shocks, u_t, is Gaussian and no further assumptions on structural shocks are established, the restrictions on m_T have all the useful information for the estimation of θ. The parameter is identified if that information is sufficient according to the following theorem:

Theorem 1 *Considering m_T as a continuously differentiable function of the vector of deep parameters θ, θ_0 is locally identifiable if the Jacobian Matrix $J(T) = \frac{\delta m_q}{\delta \theta'}$ has full column rank at θ_0, for $q \leq T$.*

When $q = T$, this conditions is only sufficient for identification if u_t is normally distributed. In addition is also required that the number of deep parameters does not exceed the dimension of m_t.

Because the distribution of the data depends on θ through τ the program yet triggers the *Jacobian*[4] of the transformation from θ to τ, $J_2(T)$. Point θ_0 is locally identifiable if the rank of $J_2(T) = \frac{\delta \tau}{\delta \theta'}$ is equal to the dimension of θ. Note that, in practice, deep parameters are unknown. To overcome this issue the program relies on the prior distributions set by the analyst, assuming the mean values as the deep parameters of the model.

With both rank conditions verified, this section proceeds with the report of the Bayesian estimation[5] results and calibrated parameters. For estimation purposes, seven observable macroeconomic time-series are considered, mapped from the data through measurement equations in log-differences,

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[3]The information matrix is used for the sensitivity analysis, which we do not address in our study

[4]Denoted by H in DYNARE.

[5]Dynare uses a Bayesian MH-MCMC algorithm, performing different iterations chains. The initial parameters values and the prior distributions allow for the program to run the different chains starting at different points in the parameter space and find the mode and mean around which each parameter is centered.

except for consumption and investment, which are specified in levels. Data description and measurement equations are described in Appendix D.

Bayesian estimation combines the prior probabilities of the parameters with the likelihood function, which when maximized finds the values of the parameters that more probably generated the data, given the prior values defined. This posterior distribution is obtained as:

$$p(\theta|Y^T) = \frac{L(Y^T|\theta)p(\theta)}{\in L(Y^T|\theta)p(\theta)d\theta},$$

the quotient between the likelihood $L(Y^T|\theta)$ and its marginal value, of sample Y, with T observations and where $p(\theta)$ is the prior probability of vector θ.

Calibrations and priors imposed to the model's parameters are shown in Table I. The calibrations enter the estimation process as very strict priors. Generally, these parameters directly affect the steady-state, whereas the ones defining the dynamics of the model are preferably estimated. v_k and v are re-parameterized to $v_k = e^{vi_k}$ and $v = e^{v_b} \in(-\infty; +\infty)$, normally distributed with mean -1 and standard deviation 10. For values less than unity, as v_k tends to zero, public and private capital become more complements. They are considered substitutes for the opposite space and not related if $v_k = 1$. The share of private capital on the final goods production function, ϕ_k is uniformly distributed in the range [0,1]. Finally, we want to make clear the *Fiscal Strategy* model is not subject to estimation and thus, tax values are fixed. Nevertheless, on behalf of the remaining empirical evidence we need to specify the calibrations applied to the components of the fiscal rules, ρ_z, ρ_t, γ_z, $gamma_t$ and the steady state levels of br_t and tau_t^i, br_{ss}[6] and tau_{ss}^i.

The estimation results for three variations of the baseline model are presented in Table II [7]. Firstly, ϕ_k and ϕ are both set to unity, showing a model economy where government decisions do not have collateral effects on private inputs. The next stage studies the externality channel on utility, only restricting ϕ_k. Finally, the model is estimated without the imposition of any restrictions.

The highest marginal data is achieved when the externality channel on the production function is closed, with the share of private consumption on utility estimated to be around 52%. As the result obtained in Azevedo and Ercolani (2012), government and private consumption are estimated to be

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[6]The calibrated value in each quarter corresponds to an annual public debt to output ratio around 50% in the steady state.

[7]For further results see Appendix B.

Table I: Calibrated Parameters

Parameter	Value	Justification
β	0.995	Real interest rate (yearly) $\approx 4\%$
δ_k	0.025	Depreciation rate (yearly) $= 10\%$
δ_{kg}	0.025	"
$\theta_{kg,def}$	0.0	Azevedo and Ercolani(2012)
τ^w	0.223	Leeper et al. (2009)
τ^k	0.184	Leeper et al. (2009)
τ^c	0.028	Leeper et al. (2009)
τ^i_{ss}	0.20	Traum and Yang(2010)
br_{ss}	1.98	Traum and Yang(2010)
λ_p	0.20	Christiano et al. (2005)
λ_w	0.05	Christiano et al. (2005)
ρ_z	0.99	Traum and Yang(2010)
ρ_t	0.98	"
γ_z	0.00	"
γ_t	0.01	"
χ	Varying	$s.t.\ n\_ss = 0.31$
γ_1	Varying	r^k_{ss}, eq'm. relation

strongly substitutes with a posterior mean for vi_b of 8.4. Relatively to vi_k, public and private investment seem also to be substitutes, but with a weaker relationship. However, the wide confidence interval[8] of the estimated parameter leaves some reservations regarding the strength of such effect and the accuracy of the estimation[9]. Besides, the share of private capital ϕ_k is estimated to be 85% with the unrestricted version, but the accuracy of the model slightly decreases[10], which shows preference for the model with ϕ_k equal to one. Curiously, the main results are opposite to those found in Coenen, Straub and Trabandt (2012) [11] for an open economy. Still, also in their study, the elasticity of substitution between public and private capital is lower than the one between public and private consumption. In between, Mazraani (2010) finds a substitution effect between private and public consumption and a

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[8]Appendix B.

[9]Identification issues seem to damage the estimation of this parameter. Further investigations should be done regarding its specification and contribution to the model.

[10]Following the work of Smets and Wouters (2003),the overall fit of DSGE models can be measured by its marginal data density, useful for comparison purposes.

[11]Please recall that the share of private investment is calibrated by the authors to 90%, the parameter ϕ_k in our model.

complementarity effect between public and private investment.

Table II Priors and Posteriors of selected parameters 1969Q1-2008Q3
Total Separability, *Utility Channel* and Unrestricted

| Parameter | PRIOR | | | POSTERIOR | | | | | |
| | | | | $\phi_k = \phi = 1$ | | $\phi_k = 1$ | | *Unres.* | |
	Distr.	Mean	St. Dev.	Mode	Mean	Mode	Mean	Mode	Mean
Utility function									
σ_L	*Normal*	2	0.5	0.63	0.77	0.70	0.92	0.74	0.84
v_b	*Normal*	−1.0	10	–	–	3.58	8.95	3.79	8.64
ϕ	*Uniform*	[0,1]		–	–	0.54	0.55	0.52	0.52
Production function									
Φ	*Normal*	0.05	0.02	0.01	0.02	0.01	0.02	0.02	0.02
α	*Normal*	0.3	0.02	0.33	0.33	0.32	0.32	0.33	0.34
vi_k	*Normal*	−1.0	10	–	–	–	–	2.24	7.07
ϕ_k	*Uniform*	[0,1]		–	–	–	–	0.88	0.85
Investment Adj. costs									
$\kappa/100$	*Normal*	4	0.5	4.65	4.68	4.64	4.64	4.67	4.65
γ_2	*Normal*	0.0685	0.002	0.036	0.037	0.033	0.034	0.03	0.03
Constant terms									
$\overline{\gamma}/100$	*Normal*	0.4	0.02	0.39	0.39	0.39	0.39	0.39	0.39
$\xi^{g,ss}$	*Normal*	0.16	0.01	0.17	0.17	0.17	0.17	0.17	0.17
$\xi^{ig,ss}$	*Normal*	0.025	0.01	0.026	0.026	0.026	0.027	0.026	0.026
$\xi^{g,def,ss}$	*Normal*	0.008	0.001	0.008	0.008	0.007	0.009	0.008	0.008
Laplace Log D Dens.				2573.14		**2581.41**		2580.59	
Log D Dens.				2573.62		**2581.41**		2580.59	

5   Model Dynamics

5.1   Dynamic Effects

The following section analyses impulse response functions to a shock both in government consumption and investment and tries to unveil the main implications behind the imposition of the externalities channels and fiscal rules. For the simulations we use the posterior mode of the parameters estimated for the unrestricted model[12], which generate the lines in the plots displayed from figure 1 to 3.2 in Appendix A. Specifically, figure 1 presents the impulse response functions of a one standard deviation increase in public consumption, figure 2 shows the impact of a shock with the same magnitude in public investment and in figures 3.1 and 3.2 are alternative versions of the augmented model,

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[12]Although the unrestricted version is not preferred, we opt to use this version since it allows to observe the effects of both externalities channels.

again subject to a shock in government spending.

Conditional on specific parameterization (summarized in table III) we define four different versions of the model: the *Total Separability* (i) , the *Utility Channel* (ii), the *Public externality* (iii) and the *Fiscal Strategy* (iv). The notation adopted gives intuition for the mechanisms observed in each version. Looking at the figures, the first version, with a dashed grey line, shows the reaction of a model without either substitution effects or fiscal rules ($\phi = \phi_k = 1$). Secondly, (ii) considers the public externality on private consumption and in (iii) we add the substitutability between public and private capital, which are respectively represented by the blue and red dashed lines. Finally, a solid black line is traced for the model with strict fiscal rules[13].

Moreover, in the final version –*Fiscal Strategy* – income tax, τ_t^i, depends on the level of public debt one year ago, B_{t-4}, and supports by itself the burden of public debt ($\gamma_z = 0$, as earlier shown in Table I). We further explore this specification by setting different timings for the government's reaction and by assuming lump sum transfers to share the funding of public debt with τ_t^i.[14]

Table III - Parameterization

I. BENCHMARK MODEL	
a. *Total separability*:	$\phi_k, \phi = 1$
b. *Utility channel*	$\phi_k = 1, \phi = 0.54$
	$\nu_b = 3.4$
c. *Public Externalities*	$\phi_k = 0.88, \phi = 0.52$
	$\nu_b = 3.79, \nu i_k = 2.24$
II. FISCAL STRATEGY MODEL	
a. *With lump-sum tax mechanism*:	$\gamma_z = 0.01$
b. *For different reaction periods*:	t−1, t−4, t−12

Regarding the shock in government consumption (figure 1), with the *Total Separability* model, output, the interest rate and the level of labor are upward driven, in contrast with private consumption, investment and wage level. In turn, when subject to the public externality, private consumption decreases further, because families suffer from the combination of a negative wealth and substitution effect. With respect to the remaining variables, this shock loses impact and the return to the steady is

---

[13]Note that although this version was not subject to estimation, this exercise still adopts the posterior mode estimates of the unrestricted model. We simply adjust the*Public externality* version with the imposition of the fiscal rules to get the *Fiscal Strategy* framework.

[14]The legend for the respective plots are in Appendix A, along with the figures.

considerably faster. Regarding the *Fiscal Strategy* model, the decrease of private investment is empha-
sized and the decrease in consumption gains persistence. The distortionary tax represents an opposing
force to the shock's expansionary effect and leads output to decrease soon after impact. In addition,
an increase on taxes also generates a disincentive to work, reducing the number of hours individuals
devote to labor.

Turning to the effect of a shock in government investment (figure 2), please recall that in the
context of the *Public Externalities* framework public capital competes with private capital. With
an increase of the former, the interest rate increases on impact and there is an incentive for private
agents to slowdown investment. However, in this case, the negative substitution effect is dominated
by a positive wealth effect and investment increases around 6 years after impact. In parallel with the
dynamics observed for the previous shock, in the long run, the imposition of fiscal rules represents a
deterioration of the living standards at al levels.

Focusing on the *Fiscal Strategy* Model, when government transfers share the funding of the public
debt level with tax (figure 3.1), the fiscal policy shock is less damaging. Income taxation, τ_i, reaches
a lower maximum level and the steady state is sooner re-attained. As a result, the output period of
adjustment is also shorter and debt reacts to a lesser extent, inclusively dropping with the course of
recovery.

Finally, we assume the government reacts to the debt level by increasing income taxes at the first
quarter $(t-1)$ or three years after impact $(t-12)$ (figure 3.2), in alternative to the baseline assumption
of one year $(t-4)$. Although the marginal effect is quite low, there is evidence that if government waits
longer before increasing taxes the increase will have to be stronger than if the reaction is immediate.

5.2   Government Consumption and Investment Multiplier

Resorting to the formulation in Mountford and Uhlig (2009), to quantify the influence of fiscal
policy, dynamic multipliers are computed by discounting the cumulated responses of output facing
one standard deviation policy shock. The value for the dynamic multiplier expressed for t quarters
after the shock is given by:

$$\varphi_t = \frac{\sum_{k=1}^{t}(1+r_{ss})^{-k}\Delta y_k}{\sum_{k=1}^{t}(1+r_{ss})^{-k}\Delta g_k}$$

16

where Δy_k represents the deviation of output from its steady-state at time k and Δg_k represents the marginal variations of government purchases measured as a fraction of the steady-state output. r_{ss} is the steady-state real interest rate. This multiplier measures the cost of returning to equilibrium, in output terms, inherent to an increase in government consumption/investment.

In tables IV and V are presented the results of the dynamic multipliers of output, in response to the government policy shocks, for different periods.

When any of the externalities mechanism is allowed, the shock of public consumption has a strong effect on output, with the respective impact multiplier reaching a maximum level of 0.95, slightly different from the 1.072 found with the baseline model in Straub and Tchakarov (2007). With public consumption affecting the utility function, the impact on output is much smaller due to the negative wealth and substitution effects on households consumption. Remarkably, the high estimated value of 48%[15] for the share of public capital on the effective consumption function causes such a low value of the multiplier with the model's preferred version.

After the the introduction of fiscal rules, the contractionary effect on output is evident in the long-run. Coenen, Straub and Trabandt (2012) uses the same formulation of the multiplier, but finds an effect of 1.02 on impact and 0.84 in the long-run, with consumption and labour taxes adjusting to the public debt value.

Finally, the output to government investment multiplier for the first quarter decreases from 1.09 to 0.90 when the substitution effect between public an private capital is considered.

All in all, the model dynamics show that a shock in public consumption is generally unproductive, opposite to the effect of an increase in public investment.

Table IV - Dynamic Multipliers, **Government Consumption Shock** on Output Y

Quarters	1	2	4	8	12	24	48	72	100
Fiscal Rules	0.09	0.09	0.08	0.07	0.06	0.05	0.03	0.01	−0.01
Public Externalities	0.07	0.06	0.06	0.05	0.05	0.04	0.04	0.04	0.04
Utility Channel	0.07	0.06	0.06	0.05	0.05	0.05	0.05	0.04	0.04
Total Separability	0.95	0.90	0.82	0.69	0.65	0.63	0.63	0.62	0.62

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[15]The share of public capital in the CES aggregator is given by $(1 - \phi_k)$.

Table V - Dynamic Multipliers of **Government Investment Shock** on Output Y

Quarters	1	2	4	8	12	24	48	72	100
Fiscal Rules	0.80	0.74	0.65	0.57	0.54	0.52	0.51	0.50	0.49
Public Externalities	0.90	0.82	0.72	0.62	0.59	0.59	0.62	0.63	0.63
Utility Channel	0.92	0.84	0.73	0.62	0.57	0.53	0.51	0.51	0.50
Total Separability	1.09	1.03	0.92	0.81	0.75	0.69	0.67	0.67	0.67

6   Concluding Remarks

Two main objectives have driven this study. On one hand, an attempt was made to complement the evidence found in Azevedo and Ercolani (2012) by establishing a more general production function. On the other, in order to observe how the imposed channels behave under a more realistic fiscal policy, the set up economy is extended with more complex fiscal rules.

The empirical evidence shows a substitution relationship between private and public capital. else constant, as public investment increases, firms seem to be discouraged to invest. However, the presence of this externality on the estimated model does not constitute the preferred version and the respective elasticity of substitution is estimated in a wide confidence interval. Given the lack of robustness in these results and the limited set of DSGE models applications to this study, the overall validity of our conclusion is questionable and there is a need for further investigation.

As for public and private consumption, results support the evidence found in Azevedo and Ercolani (2012), showing the model is able to fit the data with a strong substitutability effect even adopting a normal prior distribution centered in complementarity. In this framework, the substitutability effect reduces the capacity of a shock in government spending to foster the output level.

Regarding fiscal policy, when the income tax is used to stabilize public debt, affecting the families disposable income, economy contracts. Depending on the labor supply elasticity and the distortionary power of the income tax, output may be crowded out by an initially expansionary policy, which seems to be a better approximation to real business cycle dynamics. Intuitively, for later improvements to the present study we suggest the estimation of the model with the fiscal rules.

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Appendix A: Impulse Response Functions
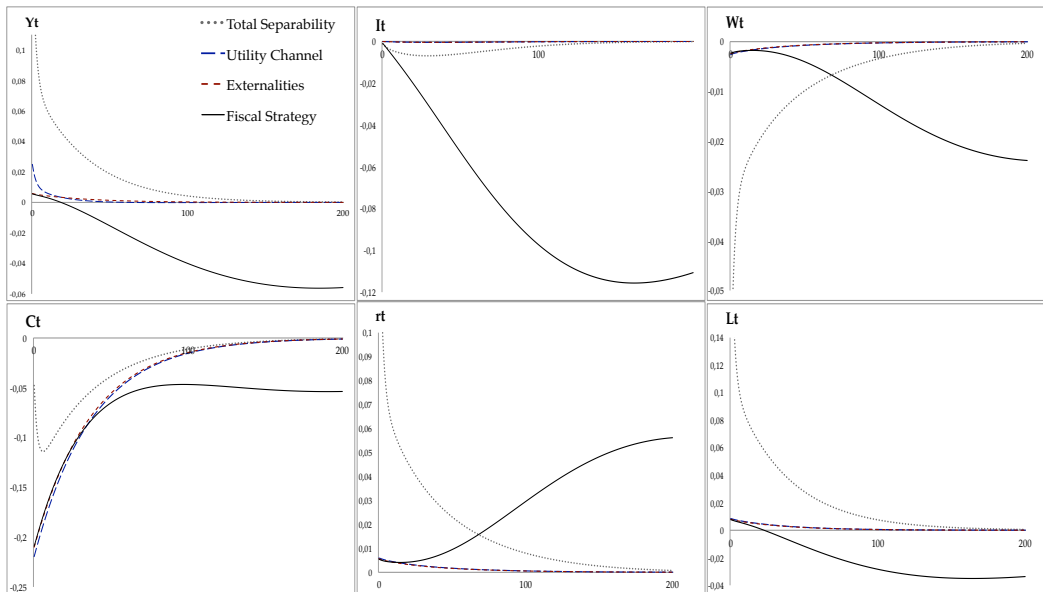
Figure 1. Effects on output (Y), consumption (C), investment (I), wages (W), hours (L) and return on capital (r_t), of a one standard deviation **government consumption shock**. The dashed grey line is the IRF obtained with the "Total Separability" model ($\phi = 1$, $\phi_k = 1$), the dashed blue line corresponds to "Utility Channel" model, the red line represents the "Public Externalities model" and finally the solid black line is the *Fiscal Strategy* response.
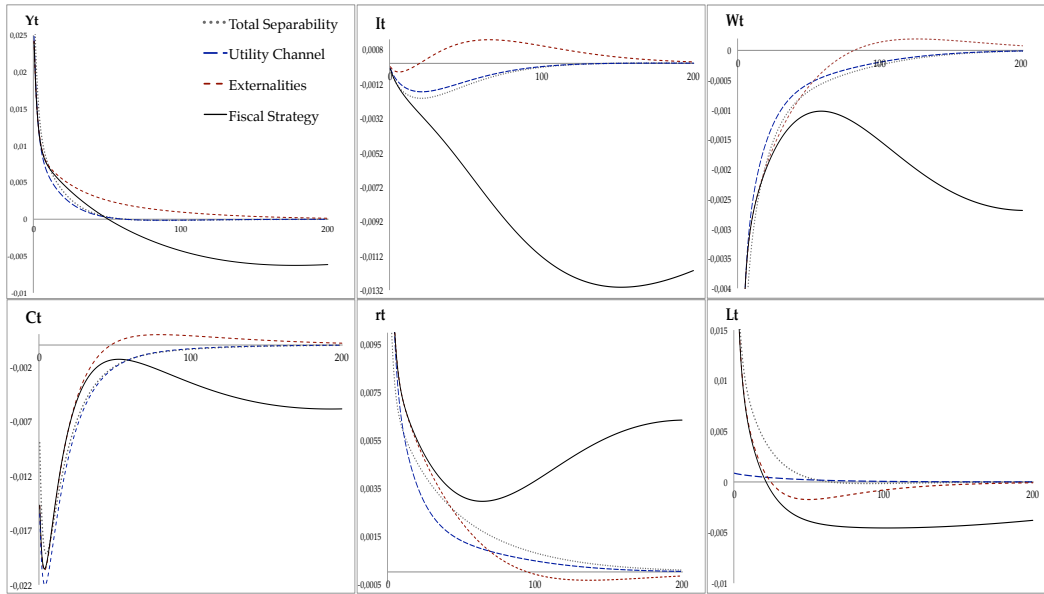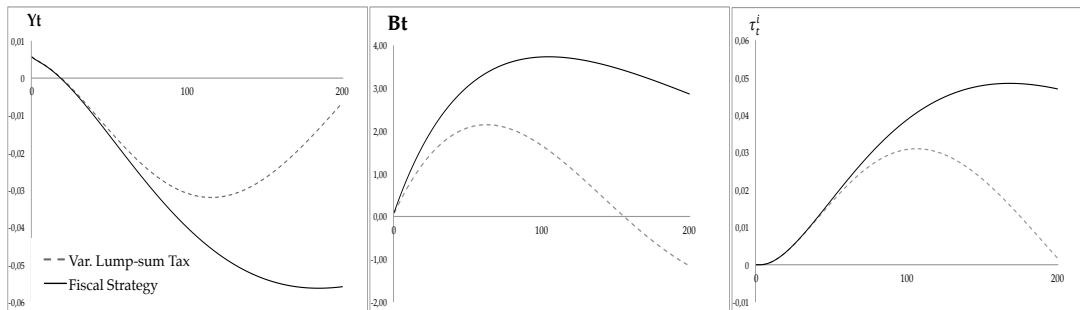
Figure 2. The *Fiscal Strategy* model: effects on output (Y), consumption (C), investment (I), wages (W), hours (L) and return on capital (r_t), of a one standard deviation **government investment shock**. The dashed grey line is the IRF obtained with the "Total Separability" model ($\phi = 1$, $\phi_k = 1$), the dashed blue line corresponds to "Utility Channel" model, the red line represents the "Public Externalities model" and finally the solid black line is the *Fiscal Strategy* response.

Impulse Response Functions (IRFs) of the *Fiscal Strategy* model:

Lump-Sum Tax Adjustment

Figure 3.1 Impulse Response Functions (IRFs) of the *Fiscal Strategy* model: effects on output (Y), public debt (B) and income tax (τ_i) of a one standard deviation **government consumption shock**. The dashed grey line is the IRF obtained with lump-sum tax responding to public debt and the solid black line corresponds to the original *Fiscal Strategy* model ($t = 4$).

Imediate and Late Fiscal Policy Reaction
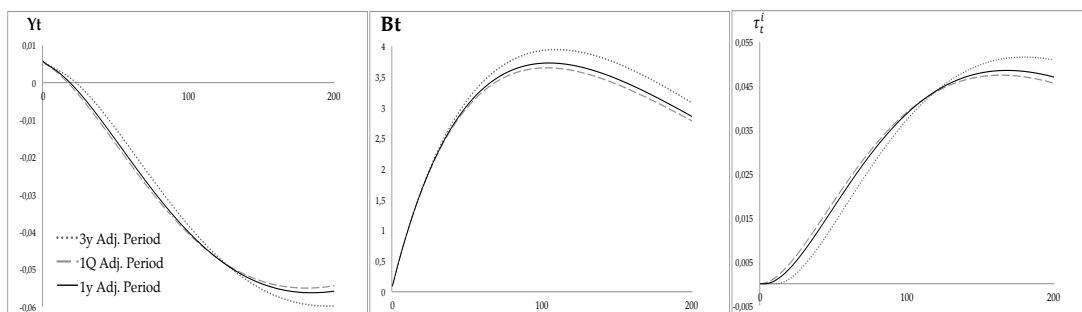
Figure 3.2 Impulse Response Functions (IRFs) of the *Fiscal Strategy* model: effects on output (Y), public debt (B) and income tax (τ_i) of a one standard deviation **government consumption shock**. The pointed grey line is the IRF obtained with a late tax increase ($t = 12$), the dashed line is the result when the reaction of the government is imediate ($t = 1$) and the solid black line corresponds to the original *Fiscal Strategy* model ($t = 4$).

Appendix B: Estimation Results

Table I - B Priors and Posteriors of selected parameters 1969Q1-2008Q3
Utility Channel specification vs. Unrestricted

Parameter	PRIOR			POSTERIOR							
				$\phi_k = 1$				Unrestricted			
	Distr.	Mean	St. Dev.	Mode	Mean	5%	95%	Mode	Mean	5%	95%
A. Utility function											
σ_L	Normal	2	0.5	0.70	0.92	0.43	1.41	0.74	0.84	0.46	1.24
v_b	Normal	−1	10	3.59	8.96	0.53	17.23	3.79	8.64	0.69	16.9
ϕ	Uniform	[0,1]		0.54	0.55	0.42	0.67	0.52	0.52	0.39	0.64
B. Production function											
Φ	Normal	0.05	0.02	0.009	0.02	0.00	0.039	0.007	0.022	0.00	0.04
α	Normal	0.3	0.02	0.32	0.32	0.28	0.34	0.33	0.33	0.31	0.35
vi_k	Normal	−1	10	–	–	–	–	2.24	7.07	−0.63	15.1
ϕ_k	Uniform	[0,1]		–	–	–	–	0.88	0.85	0.73	0.99
C. Investment Adj. costs											
$\kappa/100$	Normal	4	0.5	4.64	4.64	3.89	5.38	4.67	4.65	3.92	5.40
γ_2	Normal	0.0685	0.002	0.033	0.034	0.029	0.039	0.03	0.029	0.023	0.035
D. Constant terms											
$\overline{\gamma}/100$	Normal	0.4	0.02	0.39	0.39	0.36	0.43	0.39	0.39	0.36	0.43
$\xi^{g,ss}$	Normal	0.16	0.01	0.17	0.17	0.16	0.18	0.17	0.17	0.16	0.17
$\xi^{ig,ss}$	Normal	0.025	0.001	0.026	0.027	0.025	0.029	0.026	0.025	0.024	0.028
$\xi^{g,def,ss}$	Normal	0.008	0.001	0.008	0.008	0.007	0.009	0.008	0.008		

Table II - B Priors and Posteriors of Shocks parameters 1969Q1-2008Q3

Utility Channel specification vs. Unrestricted

Parameter	PRIOR			POSTERIOR							
				$\phi_k = 1$				Unrestricted			
	Distr.	Mean	St. Dev.	Mode	Mean	5%	95%	Mode	Mean	5%	95%
A. Autoregressive Parameters											
ρ_b	Beta	0.5	0.2		0.77	0.75	0.71	0.80	0.76	0.76	0.75	0.76
ρ_a	Beta	0.5	0.2		0.15	0.16	0.05	0.26	0.15	0.13	0.08	0.17
ρ_I	Beta	0.5	0.2		0.96	0.95	0.91	0.98	0.95	0.95	0.94	0.95
ρ_w	Beta	0.5	0.2		0.95	0.93	0.88	0.98	0.95	0.95	0.94	0.96
ρ_g	Beta	0.5	0.2		0.97	0.97	0.96	0.99	0.97	0.98	0.93	0.97
ρ_{ig}	Beta	0.5	0.2		0.94	0.90	0.91	0.98	0.95	0.94	0.93	0.95
$\rho_{ig,def}$	Beta	0.5	0.2	0.98	0.98	0.96	0.99	0.98	0.97	0.99	
B. Standard deviation of shocks											
σ_b	Inv Gamma	0.1	2.0		2.94	2.93	2.53	3.31	2.92	2.76	2.72	2.80
σ_a	Inv Gamma	0.1	2.0		0.016	0.017	0.015	0.019	0.017	0.016	0.016	0.017
σ_I	Inv Gamma	0.1	2.0		0.042	0.042	0.029	0.056	0.040	0.039	0.036	0.042
σ_w	Inv Gamma	0.1	2.0		0.026	0.034	0.020	0.050	0.025	0.026	0.025	0.027
σ_g	Inv Gamma	0.1	2.0		0.015	0.015	0.014	0.017	0.015	0.015	0.015	0.016
σ_{ig}	Inv Gamma	0.1	2.0		0.030	0.031	0.029	0.033	0.030	0.03	0.029	0.030
$\sigma_{ig,def}$	Inv Gamma	0.1	2.0		0.054	0.055	0.048	0.062	0.054	0.055	0.049	0.062

Appendix C: Equilibrium Conditions with Transformed Variables

Equilibrium conditions follow from the first order conditions (F.O.C.s) of households' and firms' problems while imposing symmetry, fiscal policy equations, market clearing conditions and processes for the exogenous processes. Variables C_t, I_t, K_t, W_t, Tax_t, T_t, G_t, I_t^g, $I_t^{g,def}$ and B_t are divided by the level of technology, A_t. Lower case letters indicate transformed variables. The same treatment is required for the Lagrange multipliers associated with the budget constraint and the capital accumulation equation, respectively λ_t and q_t (Tobin's q).

- *Consumption F.O.C.:*

$$(1 + \tau^c)\lambda_t \exp(-(\overline{\gamma} + \varepsilon_t^a)) = e^{\varepsilon_t^b}\left[\phi\left(\frac{\widetilde{c}_t}{c_t}\right)^{\frac{1}{v}}\left[\left(\widetilde{c}_t \exp(\overline{\gamma} + \varepsilon_t^a) - \gamma\widetilde{c}_{t-1}\right)^{-1}\right]\right] \qquad (19)$$

- *Aggregator (consumption):*

$$\widetilde{c}_t = \left[\phi(c_t)^{\frac{v-1}{v}} + (1 - \phi)(g_t)^{\frac{v-1}{v}}\right]^{\frac{v}{v-1}} \qquad (20)$$

- *Labor supply F.O.C.:*

$$\lambda_t = \frac{\chi L_t^{\sigma_n}}{(1 - \tau^w)w_t} \qquad (21)$$

- Risk free asset F.O.C.:

$$\beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\exp(-(\overline{\gamma} + \varepsilon_{t+1}^a))(1 + r_t)\right] = 1 \qquad (22)$$

- *Investment F.O.C.:*

$$\lambda_t = \lambda_t q_t E_t \left\{ \left[1 - \frac{\kappa}{2} \left(e^{\varepsilon_t^I} \frac{i_t \exp(\overline{\gamma} + \varepsilon_t^a)}{i_{t-1}} - e^{\overline{\gamma}} \right)^2 \right] - \frac{i_t}{i_{t-1}} \exp(\overline{\gamma} + \varepsilon_t^a) \kappa e^{\varepsilon_t^I} \left(e^{\varepsilon_t^I} \frac{i_t \exp(\overline{\gamma} + \varepsilon_t^a)}{i_{t-1}} - e^{\overline{\gamma}} \right) \right\} + \tag{23}$$

$$+ \beta E_t \left[\lambda_{t+1} q_{t+1} \exp(-(\overline{\gamma} + \varepsilon_{t+1}^a)) \left(e^{\varepsilon_{t+1}^I} \frac{i_{t+1} \exp(\overline{\gamma} + \varepsilon_{t+1}^a)}{i_t} \right)^2 \kappa e^{\varepsilon_{t+1}^I} \left(e^{\varepsilon_{t+1}^i} \frac{i_{t+1} \exp(\overline{\gamma} + \varepsilon_{t+1}^a)}{i_t} - e^{\overline{\gamma}} \right) \right]$$

- *Next period capital F.O.C.:*

$$\lambda_t q_t = E_t \left[\beta \lambda_{t+1} \exp(-(\overline{\gamma} + \varepsilon_{t+1}^a)) r_{t+1}^k u_{t+1} - a(u_{t+1})](1 - \tau^k) + q_{t+1}(1 - \delta) \right] \tag{24}$$

where $a(u_t) = \gamma_1 (u_t - 1) + \frac{\gamma_2}{2} (u_t - 1)^2$ represents the cost of using capital at intensity u_t.

- *Capital law of motion:*

$$k_{t+1} = (1 - \delta) k_t \exp(-(\overline{\gamma} + \varepsilon_t^a)) + i_t \left[1 - \frac{\kappa}{2} \left(e^{\varepsilon_t^I} \frac{i_t \exp(\overline{\gamma} + \varepsilon_t^a)}{i_{t-1}} - e^{\overline{\gamma}} \right)^2 \right] \tag{25}$$

- *Capacity utilization F.O.C.:*
$$r_t^k = a'(u_t) = \gamma_1 + \gamma_2 (u_t - 1) \tag{26}$$

- *MRS consumption/labor:*
$$mrs_t = e^{\varepsilon_t^b} \frac{\chi L_t^{\sigma_n}}{(1 + \tau^c)\lambda_t} \tag{27}$$

- *Wage markup:*
$$1 + \lambda_{w,t} = \frac{w_t(1 - \tau^w)}{mrs_t} \tag{28} \qquad 1 + \lambda_{w,t} = \frac{w_t(1 - \tau_t^i)}{mrs_t} \tag{29}$$

- *Production function:*
$$y_t = \exp(-\alpha(\overline{\gamma} + \varepsilon_t^a)) \widetilde{k}_t^\alpha (L_t)^{1-\alpha} - \Phi \tag{30}$$

- *Aggregator (capital):*
$$\widetilde{k}_t = \left[\phi (k_t)^{\frac{v_k-1}{v_k}} + (1 - \phi)(k_t^g)^{\frac{v_k-1}{v_k}} \right]^{\frac{v_k}{v_k-1}} \tag{31}$$

- *Factor demands:*
$$(1 - \alpha) \frac{y_t}{L_t(1 + \lambda_{p,ss})} = w_t \tag{32} \qquad \alpha \frac{y_t}{\widetilde{k}_t(1 + \lambda_{p,ss})} \exp(\overline{\gamma} + \varepsilon_t^a) \phi_k \left(\frac{\widetilde{k}_t}{k_t} \right)^{\frac{1}{v_k}} = r_t^k \tag{33}$$

- *Marginal cost:*
$$MC_t = \frac{\left(r_t^k \right)^\alpha w_t^{1-\alpha}}{\alpha^\alpha (1 - \alpha)^{1-\alpha} \phi_k \left(\frac{\widetilde{k}_t}{k_t} \right)^{\frac{1}{v_k}} (K_t^{G,def})^{\theta_{g,def}}} \tag{34}$$

- *Price markup:*
$$\frac{1}{MC_t} = 1 + \lambda_{p,ss} \tag{35}$$

- *Evolution of the productivity of public capital:*

$$K_{t+1}^{G} = (1 - \delta_{Kg})K_t^{G} + \xi_t^{ig} \tag{36}$$

where $\xi_t^{ig} = i_t^{g}/y_t = \exp(\varepsilon_t^{ig} + ss^{ig})/(1 + \exp(\varepsilon_t^{ig} + ss^{ig}))$ and $ss^{ig} = \log(\xi^{ig,ss}/(1 - \xi^{ig,ss}))$

$$K_{t+1}^{G,def} = (1 - \delta_{Kg,def})K_t^{G,def} + \xi_t^{ig,def} \tag{37}$$

where $\xi_t^{ig,def} = i_t^{g,def}/y_t = \exp(\varepsilon_t^{ig,def} + ss^{ig,def})/(1 + \exp(\varepsilon_t^{ig,def} + ss^{ig,def}))$ and $ss^{ig,def} = \log(\xi^{ig,def,ss}/(1 - \xi^{ig,def,ss}))$

- *Government consumption:*

$$\xi_t^{g} = g_t/y_t = \exp(\varepsilon_t^{g} + ss^{g})/(1 + \exp(\varepsilon_t^{g} + ss^{g})) \tag{38}$$

where $ss^{g} = \log(\xi^{g,ss}/(1 - \xi^{g,ss}))$

- *Fiscal Rules:*

$$\widehat{\tau}_t^{i} = \rho_\tau \widehat{\tau}_{t-1}^{i} + (1 - \rho_\tau)\gamma_\tau \widehat{br}_{t-4} + \varepsilon_t^{\tau} \tag{39}$$
$$\widehat{T}_t = \rho_z \widehat{T}_{t-1} + (1 - \rho_z)\gamma_z \widehat{br}_t - 1 + \varepsilon_t^{z}$$

- *Government budget:*

  - *Balanced:*

$$\tau^c c_t + \tau^w w_t L_t + \tau^k \left[r_t^k u_t - a(u_t) \right] k_t + t_t = \xi_t^{g} y_t + \xi_t^{ig} y_t + \xi_t^{ig,def} y_t \tag{40}$$

  - *Fiscal Strategy Model:*

$$\tau^c c_t + \tau_t^i \left(w_t L_t + \left[r_t^k u_t - a(u_t) \right] k_t \right) + t_t + (1 + r_t^b)b_{t-1} = b_t + \xi_t^{g} y_t + \xi_t^{ig} y_t \tag{41}$$

- *Shocks processes:*

$$\varepsilon_t^{b} = \rho_b \varepsilon_{t-1}^{b} + \eta_t^{b} \tag{42}$$
$$\varepsilon_t^{I} = \rho_i \varepsilon_{t-1}^{I} + \eta_t^{I}$$
$$\varepsilon_t^{a} = \rho_a \varepsilon_{t-1}^{a} + \eta_t^{a}$$
$$\varepsilon_t^{g} = \rho_g \varepsilon_{t-1}^{g} + \eta_t^{g}$$
$$\varepsilon_t^{ig} = \rho_{ig} \varepsilon i_{t-1}^{ig} + \eta_t^{ig}$$
$$\varepsilon_t^{\tau} = \rho_\tau \varepsilon i_{t-1}^{\tau} + \eta_t^{\tau}$$
$$\varepsilon_t^{T} = \rho_T \varepsilon i_{t-1}^{T} + \eta_t^{T}$$
$$\varepsilon_t^{ig,def} = \rho_{ig,def} \varepsilon i_{t-1}^{ig,def} + \eta_t^{ig,def}$$
$$\lambda_{w,t} = (1 - \rho_w)\lambda_{w,ss} + \rho_w \lambda_{w,t-1} + \eta_t^{w}$$

Appendix D: Data Description

Tables bellow summarize the data and notation for the observed variables. Azevedo and Ercolani (2012) followed Smets and Wouters (2007).

Table I - D Data Sources

Variable	Designation	Source	CODE
Gross Domestic Product (Nominal)	GDP	U.S. Dep. of Commerce - BEA	A191RC1
Personal Cons. Expenditures (Nominal)	C	U.S. Dep. of Commerce - BEA	DPCERC1
Personal Cons.Expenditures - Durables (Nominal)	Durables	U.S. Dep. of Commerce - BEA	DDURRC1
Private Fixed Domestic Investment (Nominal)	PFI	U.S. Dep. of Commerce - BEA	A007RC1
Federal Cons. Expenditures (Nominal)	G_Federal	U.S. Dep. of Commerce - BEA	A957RC1
State & Local Cons. Expenditures (Nominal)	G_StateLocal	U.S. Dep. of Commerce - BEA	A991RC1
State & Local Gross Investment (Nominal)	IG_StateLocal	U.S. Dep. of Commerce - BEA	A799RC1
Gross Domestic Product Deflator	GDPDEF	U.S. Dep. of Commerce - BEA	GDPDEF
Hourly Compensation, Non Farm Sector (Nominal)	Wages	Bureau of Labor Statistics	PRS85006103
Civilian noninstitutional population, 16 years and over	POPULATION	Bureau of Labor Statistics	LNU00000000Q

Table II -D - Observables for measurement equations

$$Y_t^{obs} = (\text{GDP/GDPDEF})/\text{POPULATION}$$
$$C_t^{obs} = ((\text{C-Durables})/\text{GDPDEF})/\text{POPULATION}$$
$$I_t^{obs} = ((\text{PFI+Durables})/\text{GDPDEF})/\text{POPULATION}$$
$$W_t^{obs} = (\text{Wages/GDPDEF})/\text{POPULATION}$$
$$\xi_t^{g,obs} = (\text{G\_Federal+G\_StateLocal})/\text{GDP}$$
$$\xi_t^{ig} = (\text{IG\_Federal+IG\_StateLocal})/\text{GDP}$$
$$\xi_t^{ig,def,obs} = \text{IG\_Defense}/\text{GDP}$$