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## **Parametric Portfolio Policies: An application for a Global Tactical Asset Allocation Model**

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## **Abstract**

Despite the extensive literature on the predictability of asset class returns and its economic significance, it is common for many asset managers to implement portfolio models built around active management within an asset class, while generally having passive allocations to each asset class based on the risk profile of the investor. We can exploit some of the predictability by using information on economic factors and momentum that explain broad asset class moves through a parametric portfolio approach introduced by Brandt, Santa-Clara and Valkanov (2009). I obtain significant improvements over fixed allocations and Markowitz optimal portfolios, even when applying significant restrictions.

**Key words:** Tactical Asset Allocation Portfolio

## 1. Introduction and Literature Review

Many fund managers and individual investors have faced the decision of how to allocate their wealth among various asset classes. In the case of individual investors setting up their own savings, most will be confronted by an asset manager that will recommend a fixed asset allocation among stocks, bonds and cash that will depend on their risk profile and investment horizon<sup>1</sup>. As for institutional investors, Anson (2004) describes how many institutional investors and pension funds perform strategic asset allocation<sup>2</sup> at the asset class level, typically reviewing their allocation every 3 to 5 years, while following a more active approach for securities within those asset classes. This tendency to either define a fixed asset mix or strategically re-define the asset allocation mix over large intervals can be understandable, given the size of many institutional funds, large asset allocation shifts on a regular basis may be costly and hard to put in practice.

Nevertheless, existing literature points out that almost all of the total return level of institutional funds is explained by the fund's asset allocation policy, e.g. Blake, Lehman and Timmerman (1999) and Ibbotson and Kaplan (2000). Ibbotson and Kaplan (2000) find that on average the asset allocation policy decision accounts for 100% and 99% of mutual funds and pension funds respectively, while even the best funds that manage to beat their benchmarks (5<sup>th</sup> percentile) have 82% and 86% of their total return explained by their policy decision.

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<sup>1</sup> A good example is Fidelity's website where in the first step of the portfolio construction, the investor is advised to choose an asset allocation mix and then proceed to with the fund selection.

<sup>2</sup> I define strategic and tactical asset allocation as in Dahlquist and Harvey (2001). While tactical allocation refers to changing asset allocation with a horizon of 1 to 3 months, strategic allocation refers to a more long term decision of 1 year or longer.

It is important to note that this analysis alone does not mean that frequently shifting a fund's asset allocation has the potential outcome of delivering higher returns, however, when combined with the fact that asset classes have different levels of risk premia which are time varying and to some extent predictable, might tilt a manager in favour of tactically managing their asset class exposure. In fact there is a wide range of literature that points out the varying risk premia on certain asset classes and their predictability. Fama and French (1989) describe how stock and bond risk premia are time varying and are related to the business cycle. More importantly, they identify two variables that anticipate changes in the risk premium, the default spread and the term spread. These findings are consistent with many others that also document the predictable returns in financial assets: Ferson and Harvey (1991), Ferson and Harvey (1993), Ait-Sahalia and Brandt (2001) and Goyal and Welch (2002) while at the same time pointing out other variables with forecasting powers such as short term interest rates and dividend price ratios. More recently, Ferreira and Santa-Clara (2011) develop a model to forecast equity returns by first predicting the individual components of stock returns. Very interestingly, they assess the economic importance of their forecasts on equity returns using them to dynamically allocate between stocks and cash reporting a maximum gain in terms of Sharpe ratio of 73% versus using historical mean returns. Furthermore, Keim and Stambaugh (1986) also show how equity as an asset class displays momentum, that is, equity prices are predictable through a trending variable (the log of the ratio of the current price and the historical average). Later, Faber (2007) and Moskowitz, Ooi and Pedersen (2012) show how they exploit can momentum across asset classes in order to obtain improved risk adjusted returns.

Despite the predictability to some extent of major asset class returns and the potential to benefit from this predictability to actively manage asset classes, many funds strategically define their asset allocation by performing minor adjustments at an infrequent basis. In the case of large pension and mutual funds this can be understandable for 2 reasons. First, it is important for some funds to have a portion of their assets allocated to low risk investments, for example bonds and cash, in order to meet short to medium term obligations such as guaranteeing income to pensioners as in the case of pension funds, however this problem can potentially be solved by only tactically managing a portion of the fund. The other problem is the trading costs and organizational problems that can arise from tactically managing asset classes. Imagine a large fund which has a 60% and 40% allocation to stocks and bonds respectively and wishes to change it the next month to 40% and 60%. All of a sudden teams managing the equity portfolios will have to liquidate a large portion of their positions, while those handling bond investments have 50% more assets to handle. The high trading costs that can arise from quickly liquidating and buying new securities, and the organizational problems associated with tactically managing asset class exposure, can be solved through the use of the wide range of available derivatives and futures contracts with broad indices as underlyers. Instead of shifting 20% of the fund's assets, it is possible to take a short position in equity futures and a long one in bond futures in order for the portfolio to reach a net exposure of 40% and 60% in stocks and bonds as initially intended. The costs that arise from this alternative should be much lower due to the high liquidity of broad index futures<sup>3</sup>. Despite this kind of solution bringing regulatory

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<sup>3</sup>For example, the average daily volume for the S&P E-mini futures contract in 2011 was \$134bn worth of contracts. Source: Bloomberg

challenges for some funds, it is nevertheless a possible solution that can be applied to at least a portion of a fund's portfolio.

With previous evidence that there are potential gains from attempting to time the market, I will proceed to develop a tactical asset allocation model based on the Brandt, Santa-Clara and Valkanov (2009) parametric portfolio policy approach. The next part of the paper will focus on the methods used to create a global tactical asset allocation model, followed by a discussion of the obtained results and ending with concluding remarks. I find that using this method significantly improves performance versus the benchmark portfolio as well as a Markowitz optimization approach.

## **2. Methodology**

### **2.1. Data**

The asset choice in a global tactical asset allocation model should include an asset choice representative of the investment universe, while at the same time, these assets should be liquid given the tactical nature of the model. Given this, the focus will be on broad asset classes: US Equity, US Government Bonds, and 4 groups of commodities. To represent these 6 groups, I use a S&P 500 Index Futures Rolling Strategy, a US Government Bond Futures Rolling Strategy<sup>4</sup> and the Goldman Sachs S&P Commodity Index for Precious Metals, Industrial Metals, Energy and Agricultural commodities respectively. I also use Fed Funds rate as the risk free rate, or the Cash asset. In this asset choice equity indices from other regions, such as Europe or Asia, are excluded due to their high correlation with US equity. Including highly correlated assets can at times can result in extreme weights being estimated in sample which can result in large

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<sup>4</sup> Rolling futures strategies available on Bloomberg as FRSIUSE <Index> for US Equity and FRSIUSB <Index> for US Bonds.

drawdowns out of sample. In this case, it is arguable that a broader and more encompassing index like the MSCI World would be more appropriate; however, in this case, the S&P 500 Index is preferable since it is more easily tradable through index futures than the MSCI World. The same issue is present in the choice of government bonds government bonds, where in this case, the same approach will be taken resulting in a focus on US government bonds.

Furthermore, I use additional data as information variables in order to model the asset allocation decision over time. From the existing literature, the default spread and the term spread are generally revealed as good predictors of equity and bond returns, variances and covariances as found in Ait-Sahalia and Brandt (2001). Bjornson and Carter (1997) also document the importance of these variables in predicting commodity returns. Additionally, allocating to asset classes based on momentum is documented in Faber (2007) and Moskowitz et al. (2012) with impressive results. For this reason, I will also use the 6 month return on each asset as a momentum variable.

For the term spread, I use the yield on 10 year rate on US government treasury notes minus the yield on 2 year notes. As for the default spread, I use the annual yields on Moody's Baa rated long term corporate bonds minus the yield on 10 year US government treasuries.

## **2.2. Model Choice and Basic Setup**

In order to exploit the possible gains from a global tactical asset allocation model, I use the parametric portfolio policy model developed by Brandt, Santa-Clara and Valkanov (2009). The basic idea behind this approach is to use information available on at time  $t$  such as the individual characteristics of each asset or broad market variables in

order to maximize our expected utility at time  $t+1$ . In this case, the investor problem is like in most tactical asset allocation models, to maximize a set of portfolio weights  $w_{i,t}$  in order to maximize the investor's expected utility conditional on portfolio's return  $r_{p,t+1}$ ,

$$\max_{\{w_{i,t}\}_{i=1}^N} E[u(r_{p,t+1})] = E \left[ u \left( \sum_{i=1}^N w_{i,t} r_{i,t+1} \right) \right] \quad (1)$$

$$= E \left[ u \left( rf + \sum_{i=1}^N w_{i,t} (r_{i,t+1} - rf) \right) \right] \quad (2)$$

The optimal portfolio weights are modelled as a function of each asset's individual characteristics such as the asset's return over the last 6 months, as well as broad market and macro-economic variables, such as changes in employment or the 12 month absolute change in the default spread level. The unconstrained portfolio problem can be written as in equation (2); note that in this case, by construction, portfolio weights will always add up to 1. Throughout most of the paper, weights for our 6 assets excluding cash will be modelled according to the following function:

$$\begin{aligned} w_{i,t} &= f(MOM_{i,t}, DEF_t, TERM_t, w_{bi}; b, c_i) \\ &= \bar{w}_{bi} + b \cdot MOM_{i,t} + c_i \cdot DEF_t + d_i \cdot TERM_t \end{aligned} \quad (3)$$

where  $MOM_{i,t}$  is the momentum variable, measured as 6 month return from  $t$  to  $t-6$ ,  $DEF_t$  is the default spread variable measured as the 12 month absolute change in the default spread level and  $TERM_t$  is the term spread variable measured as the 12 month absolute change in the term spread level. Parameters  $b$  and  $c_i$  are defined in sample at time  $t$  in order to maximize the investor's expected utility conditional on the portfolio return at  $t+1$ . In this model I re-estimate these parameters yearly using all available data



up to the current point in time. Finally,  $\overline{w_{bt}}$  defines the benchmark weight of each asset. I use an allocation of 47% to US Equity, 33% to US Bonds and 20% split evenly among the 4 commodity baskets<sup>5</sup>. Instead of directly estimating the weight for each asset that maximizes our objective function (or in other words using the optimal mean variance portfolio verified ex ante), weights are estimated as a function dependant on market and individual asset information allowing the model to trade based on significant market information.

I choose this model over estimating expected returns, variances and covariances of all or some of the assets of the portfolio and then using a Markowitz (1952) or a Black and Litterman (1991) approach to optimize the portfolio. The reason for is clearly explained in Aït-Sahalia and Brandt (2001), who also decide to skip the estimation of the asset moments and directly use the predictor variables to determine the optimal weights. The reason for this is that despite the statistically significant predictability of the moments of asset returns, forecasts are still very noisy, with Aït-Sahalia and Brandt (2001) mentioning that an  $R^2$  of 10% is usually hailed in the literature on predicting returns. By skipping this step, we can reduce some of the noise leading up to the weight estimation process. For comparison, I provide the results from estimating the Markowitz mean variance tangency portfolio ex-ante using the historical means and covariance matrix, as well as the optimal portfolio as a mix between the risk free asset and the tangency portfolio, given a quadratic utility function.

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<sup>5</sup> Based on the average asset allocation across pension funds worldwide, from the Towers Watson 2011 Global Pension Asset Study.

## 2.3 Model Refinements

### 2.3.1 Objective Function

Defining the objective utility function is an important part of the setup in order to obtain reasonable out of sample estimates. In this type of setup, where we apply optimally defined parameters for a specific sample and then apply them outside of that sample, there is a high probability that these parameters will be over fitted to the sample period. This can result in parameters that yield extreme weights, extreme returns and/or an excessively active strategy, which can be disastrous when applied out of sample.

There are several ways which can help mitigate this problem. In this case, the first step was to use a concave function, which incorporates the notion of a risk averse investor, that will penalize extreme returns. The function below is used as in Brandt et al. (2009):

$$U(r_p) = \frac{(1 + r_p)^{1-\gamma}}{1 - \gamma} \quad (4)$$

where  $\gamma$  is the level of risk aversion, with risk aversion increasing with  $\gamma$ . Other options for this could be the use of a common measure used to measure risk adjusted performance such as the Sharpe ratio, however the function in equation (3) allows us to control the level of risk aversion, which helps to prevent the problem of having extreme and over fitted outputs. Also, to prevent the estimation of parameters that result in an excessively active model, we can incorporate transaction costs, where I assume a 0.2% fixed transaction cost  $s_{i,t}$ , by subtracting them from the portfolio return,  $r_p$ , in equation (4):

$$TC = \sum_{i=1}^N s_{i,t} |w_{i,t}(1 + r_{i,t}) - w_{i,t-1}|. \quad (5)$$

Brandt et al. (2009), assume a fixed cost of 0.5% as one of their option. Given that such a cost relates to individual securities, I assume a lower cost given the increased liquidity of the instruments traded.

### 2.3.2 Truncating Variables

In order to avoid extreme weights out of sample, I also make a minor adjustment to the  $MOM_{i,t}$ ,  $DEF_t$  and  $TERM_t$  variables. I truncate these variables to remain within two standard deviations of their in-sample mean, in order to prevent for example, the case where one of the assets has a large drop in its price resulting in very negative  $MOM_{i,t}$  variable for that asset that could translate into an extreme weight.

## 3. Results and Discussion

The results obtained are favourable to the parametric portfolio approach. I start with a base case where I run the unconstrained optimization problem from equation (2) net of transaction costs in order to reduce excessive trading that can result in extreme positions. I then proceed to remove them to perform this initial analysis. In Table 1, we can see that even in the worst case, tactically managing portfolio asset allocation more than doubles the Sharpe ratio versus the Benchmark portfolio.

Besides risk adjusted performance, we can also witness the benefits of using this model versus a fixed weight approach when analysing the maximum drawdown<sup>6</sup>. Even in the scenario where we assume more risk, where the risk aversion coefficient is smaller, parametric portfolio policy (PPP) model manages to achieve a smaller

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<sup>6</sup> Measured as the difference between the highest peak and the lowest trough after that.

drawdown than the benchmark portfolio. In fact, the PPP model is positively skewed whereas the benchmark has a negative skew and has “fatter” tails (see Appendix A).

**Table 1 – Base Case, Unconstrained Model, Net of Transaction Costs**

	Parametric Portfolio Policy Portfolio OOS			Benchmark Portfolio
	$\gamma=10$	$\gamma=25$	$\gamma=40$	
Annualized Return	14.2%	9.5%	8.3%	5.2%
Annualized Standard Deviation	15.6%	9.9%	9.3%	9.3%
Sharpe Ratio	0.76	0.73	0.65	0.32
Maximum Drawdown	-27.9%	-20.6%	-18.9%	-30.9%
Annualized Alpha vs. Benchmark	11.4%***	5.2%**	3.7%**	-

Results for Out Of Sample (OOS) period range from January 2001 until December 2011. Parameters are estimated at the end of each year with all available information available up until that point. Results are gross of transaction costs for all portfolios.

\*, \*\*, \*\*\* - Significant at a 95%, 97.5% and 99% level respectively.

In order to make the analysis more realistic, it is important to analyse the trading cost involved in pursuing an active strategy. Where in the case of the benchmark portfolio, we can ignore the costs of rebalancing the fixed portfolio back to the target weights, for the PPP model, we can expect trading costs to have an impact on performance (see Table 2).

In fact, the impact of trading costs is most noticeable on the portfolio with the lowest risk aversion coefficient, which is significantly more active than its counterparts. The portfolio with risk aversion coefficient of 10, has trading costs that reach 1.2% annually while its counterparts lose 0.5% and 0.3% annually.

**Table 2 – Unconstrained Model, Including Transaction Costs**

	Parametric Portfolio Policy Portfolio OOS			Benchmark Portfolio
	$\gamma=10$	$\gamma=25$	$\gamma=40$	
Annualized Return	13.0%	9.0%	8.0%	5.2%
Annualized Standard Deviation	15.6%	9.9%	9.4%	9.3%
Sharpe Ratio	0.69	0.68	0.60	0.32
Maximum Drawdown	-29.8%	-21.5%	-19.6%	-30.9%
Annualized Alpha vs. Benchmark	10.3%**	4.7%**	3.3%*	-

Results for OOS period range from January 2001 until December 2011. Parameters are estimated at the end of each year with all available information available up until that point. All portfolios are net of 0.2% transaction costs except for the benchmark portfolio.

\*, \*\*, \*\*\* - Significant at a 95%, 97.5% and 99% level respectively.

I also take into account the restrictions that some investors might face, namely in terms of borrowing and short selling restrictions. In order to do this, I perform the following transformation on the unconstrained weights (in order):

$$w_{i,t}^+ = \text{Max}\{0, w_{i,t}^u\} \quad (6)$$

$$\begin{cases} w_{i,t}^* = w_{i,t}^+ & , \sum w_{i,t}^+ \leq 1 \\ w_{i,t}^* = \frac{w_{i,t}^+}{\sum w_{i,t}^+} & , \sum w_{i,t}^+ > 1 \end{cases} \quad (7)$$

**Table 3 – Constrained Model (Restricted Borrowing - B and Short Selling - SS)**

	Parametric Portfolio Policy Portfolio OOS		Benchmark Portfolio
	No SS, $\gamma=25$	No B &SS, $\gamma=25$	
Annualized Return	8.2%	7.1%	5.2%
Annualized Standard Deviation	10.3%	8.7%	9.3%
Sharpe Ratio	0.57	0.55	0.32
Maximum Drawdown	-22.1%	-20.2%	-30.9%
Annualized Alpha vs. Benchmark	3.3%*	2.4%	-

Results for OOS period range from January 2001 until December 2011. Parameters are estimated at the end of each year with all available information available up until that point. After running the unconstrained model, asset weights are restricted to being above 0% and net asset exposure is restricted to stay below 100%.

\*, \*\*, \*\*\* - Significant at a 95%, 97.5% and 99% level respectively.

In Table 3, we can see the results of imposing restrictions on the model. In the first case, “No SS”, only equation (6) is applied on the unrestricted weights, which results in no short selling any of the assets except for borrowing the risk free rate. In the next column, by applying equation (7), I also restrict borrowing the risk free rate, however, the investor may allocate to the risk free asset. Despite the two restrictions, results are still more favourable towards a more active approach. In both cases, there is a significant improvement in terms of risk adjusted returns of Sharpe ratio and alpha, as well as higher annualized returns and a lower maximum drawdown in both cases. In addition to these restrictions, it is also important to note that these 2 portfolios are both net of transaction costs (see appendix B for differences between the constrained and unconstrained models).

Finally I compare the previous results against the more traditional approach of the Markowitz (1952) mean variance optimization approach (see Appendix C for details on

how this is done). From Table 4, we can quickly grasp that the PPP approach yields superior results to this one.

**Table 4 – Markowitz Portfolios**

	Markowitz Optimal Portfolios			Tangency Portfolio
	$\lambda=1$	$\lambda=5$	$\lambda=10$	
Annualized Return	5.6%	8.5%	5.6%	4.2%
Annualized Standard Deviation	74.5%	14.9%	7.4%	11.6%
Sharpe Ratio	0.04	0.41	0.45	0.16
Maximum Drawdown	-89.8%	-22.4%	-11.0%	-28.5%
Annualized Alpha vs. Benchmark	35.6%	7.1%	3.6%	2.6%

Results for OOS period range from January 2001 until December 2011. Tangency and Optimal portfolios are re-estimated every month using all previously available historical data since December 1991. Optimal Portfolios are a dynamic combination of the tangency portfolio and the risk free asset based on the risk aversion coefficient  $\lambda$ .

\*, \*\*, \*\*\* - Significant at a 95%, 97.5% and 99% level respectively.

Here I am simply using the historical distribution of returns as to determine our optimal asset mix, whereas in the PPP method, it is possible to take advantage of variables which have been documented to partially anticipate asset returns. The best unrestricted PPP portfolio outperforms the Sharpe ratio of the best Markowitz portfolio by 69% while the most restricted model still manages a respectable 22% increase in terms of Sharpe ratio.

#### **4. Conclusion**

The results I present, demonstrate the potential added value towards actively managing the asset allocation decision. While it improves practically all risk and return metrics versus a static benchmark portfolio, it also manages to outperform a constantly

updated mean variance optimized portfolio. Given the increasing availability of passively managed ETF's, Index Futures and others, this type of strategy can be effectively be pursued by a wide range of investors.

It is also interesting to consider the relationship between the results obtained here and Fama's (1970) efficient market hypothesis. Some could consider that tactical asset allocation models contradict this paradigm; however there are other ways to look at it. It is accepted in the literature that investors require some form of predictable rate of return for bearing risks such as investing in the equity market, and that this rate of return is time varying and in part predictable. Black (2000) even points out how Fama and French (1989) acknowledge this. One way to look at this kind of tactical asset allocation model, is to think of it as a model that manages to capture time varying risk premia in different asset classes, by increasing and decreasing exposure to the asset classes with highest and lowest risk premia. In fact, this could potentially justify the choice of actively managing asset class exposure. If according to efficient market hypothesis, one cannot effectively select specific securities, such as individual stocks, with publicly available information, a dynamic asset allocation among passively managed broad based indices such as the S&P 500 could prove effective in enhancing returns for the investor.



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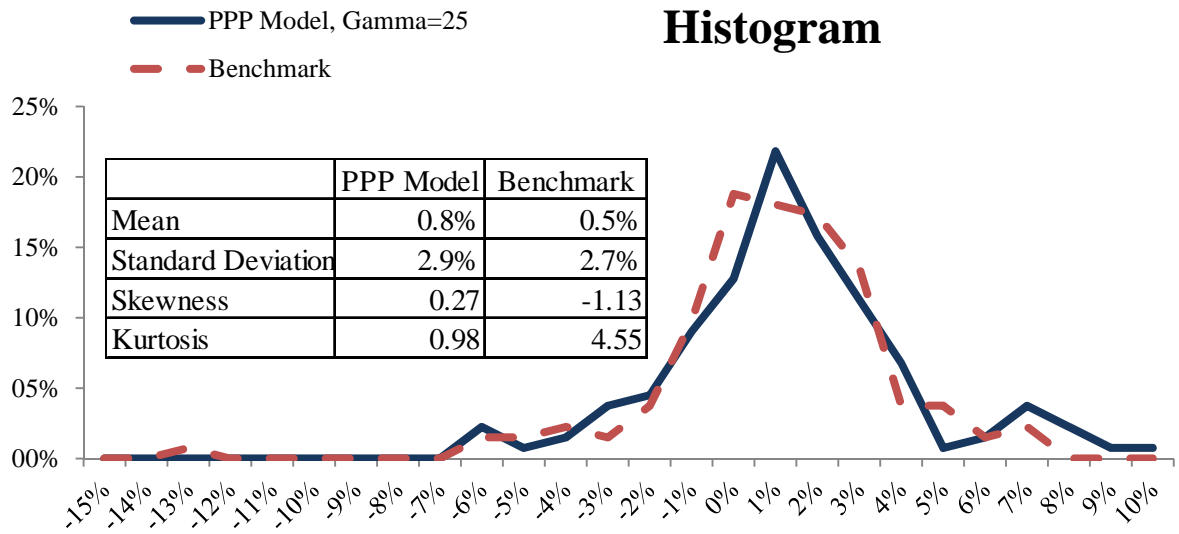
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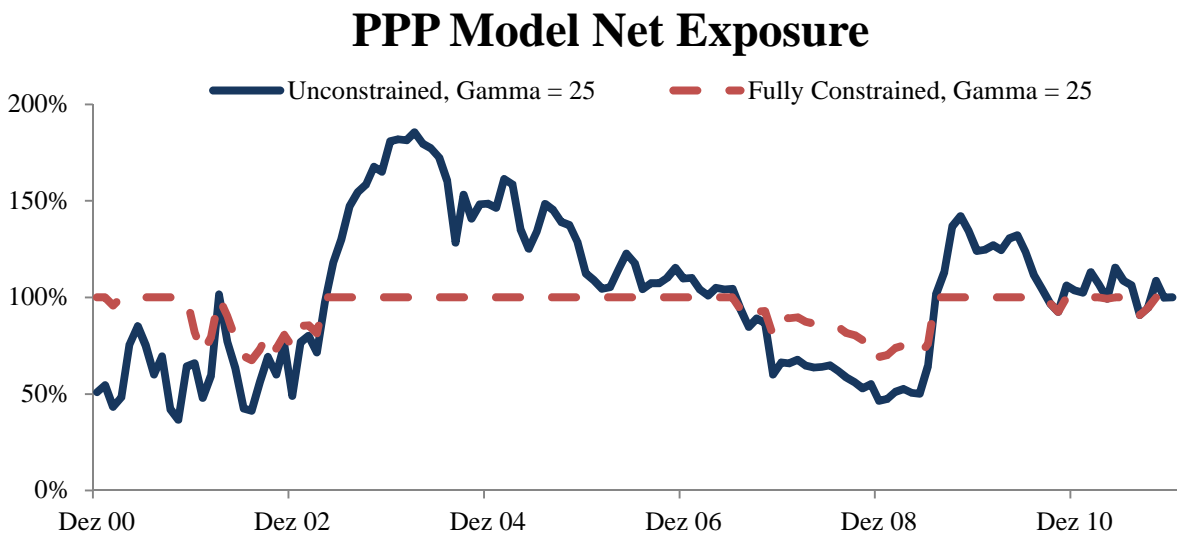
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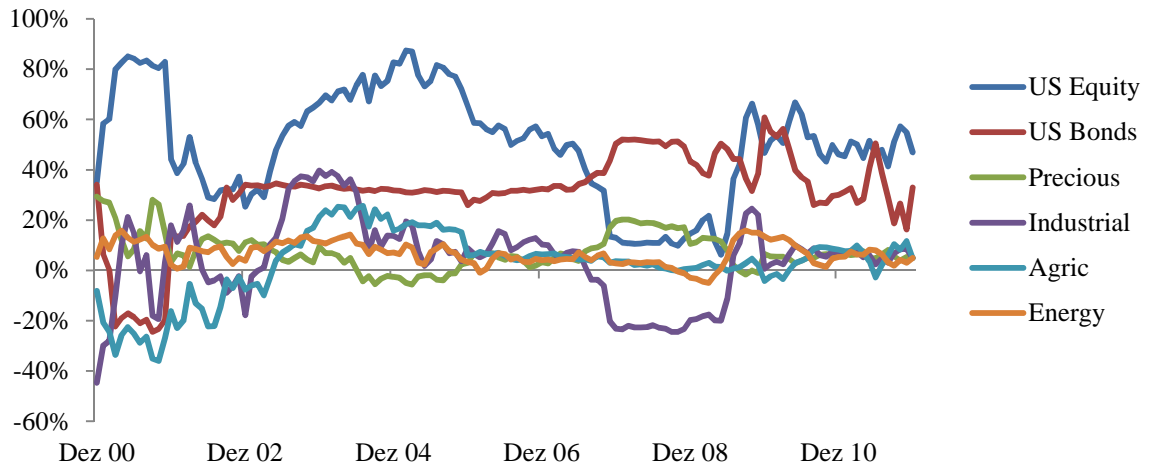
## Appendix A – Return Distributions of the PPP Model and the Benchmark



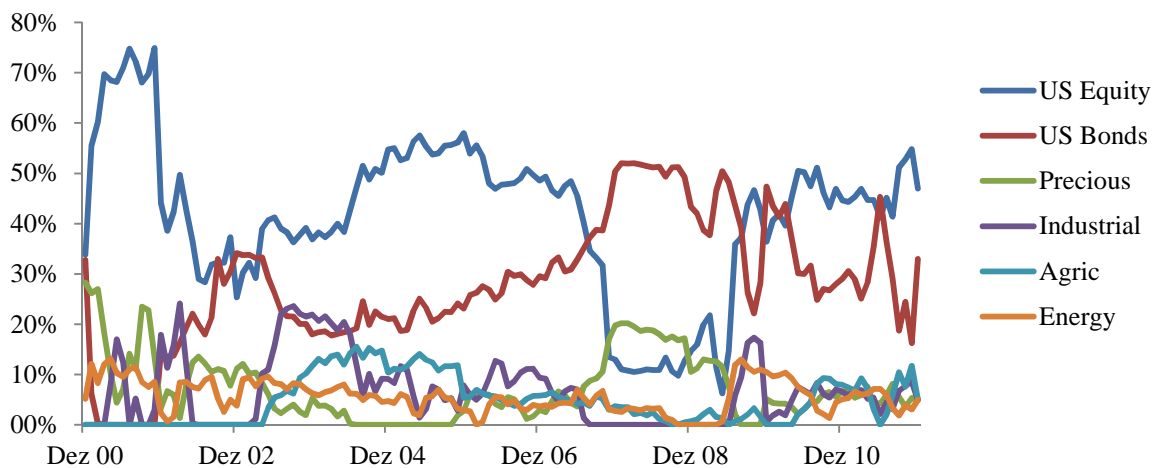
## Appendix B – PPP Constrained and Unconstrained Weights and Net Exposure



## Unconstrained Weights, $\gamma=25$



## Fully Constrained Weights, $g=25$



### Appendix C – Markowitz Portfolio Simulation

In order to perform this simulation, I use the same six assets and the risk free rate. The first step in this approach introduced by Markowitz (1952), is to calculate determine the tangency or efficient portfolio through,

$$w_P = \Omega^{-1} \cdot E(r)^7 \quad (8)$$

<sup>7</sup>  $\Omega$  = Sample covariance matrix;  $E(r)$ = vector of expected returns;  $w_P$ = vector of efficient weights

and scale the weights so they add up to 1. We then have to choose between an optimal mix of this tangency portfolio and the risk free asset based on our risk preferences  $\lambda$ . The first order condition solves the problem of how much to allocate to the tangency portfolio, using the common quadratic utility is:

$$w^* = \frac{E(r_P) - rf}{\lambda \sigma_p^2} \quad (9)$$

In the simulations above, I solve this problem every month using the historical moments of each asset.

#### **Appendix D – Parametric Portfolio Policy In Sample Results (Dec 1991 – Dec 2000)**

**Table 5 – Base Case, Unconstrained Model, Net of Transaction Costs, In-Sample**

	Parametric Portfolio Policy Portfolio OOS			Benchmark Portfolio
	$\gamma=10$	$\gamma=25$	$\gamma=40$	
Annualized Return	27.4%	16.7%	13.9%	11.4%
Annualized Standard Deviation	12.3%	7.6%	6.9%	7.4%
Sharpe Ratio	1.81	1.51	1.26	0.84
Maximum Drawdown	-13.6%	-6.5%	-5.7%	-8.0%
Annualized Alpha vs. Benchmark	14.5%***	5.5%***	3.2%***	-

Results for the In Sample (OOS) period range from December 1991 until December 2000. Results are gross of transaction costs for all portfolios.

\*, \*\*, \*\*\* - Significant at a 95%, 97.5% and 99% level respectively.

**Note:** It is expectable for such impressive results in-sample since the parameters this case are estimated in order to maximize the objective function for this specific sample.

Nevertheless, it is important to estimate robust in-sample parameters so that they can be reliable out of sample.