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***EVALUATION OF SPATIAL INTERPOLATION TECHNIQUES FOR MAPPING CLIMATE VARIABLES WITH LOW SAMPLE DENSITY***

***A case study using a new gridded dataset of Bangladesh***

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Dissertation submitted in partial fulfilment of the requirements
for the Degree of *Master of Science in Geospatial Technologies*

# EVALUATION OF SPATIAL INTERPOLATION TECHNIQUES FOR MAPPING CLIMATE VARIABLES WITH LOW SAMPLE DENSITY

## A case study using a new gridded dataset of Bangladesh

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# EVALUATION OF SPATIAL INTERPOLATION TECHNIQUES FOR MAPPING CLIMATE VARIABLES WITH LOW SAMPLE DENSITY

## A case study using a new gridded dataset of Bangladesh


ABSTRACT

This study explores and analyses the impact of sample density on the performances of the spatial interpolation techniques. It evaluates the performances of two alternative deterministic techniques – Thin Plate Spline and Inverse Distance Weighting, and two alternative stochastic techniques – Ordinary Kriging and Universal Kriging; to interpolate two climate indices - Annual Total Precipitation in Wet Days and the Yearly Maximum Value of the Daily Maximum Temperature, in a low sample density region - Bangladesh, for 60 years – 1948 to 2007. It implies the approach of Spatially Shifted Years to create mean variograms with respect to the low sample density. Seven different performance measurements - Mean Absolute Error, Root Mean Square Errors, Systematic Root Mean Square Errors, Unsystematic Root Mean Square Errors, Index of Agreement, Coefficient of Variation of Prediction and Confidence of Prediction, have been applied to evaluate the performance of the spatial interpolation techniques. The resulted performance measurements indicate that for most of the years the Universal Kriging method performs better to interpolate total precipitation, and the Ordinary Kriging method performs better to interpolate the maximum temperature. Though the difference surfaces indicate a very little difference in the estimating ability of the four spatial interpolation techniques, the residual plots refer to the differences in the interpolated surfaces by different techniques in terms of their over and under estimation. The results also indicate that the Inverse Distance Weighting method performs better for both indices, when the sample density is too low, but the performance is questioned by the inclusion of measurement errors in the

interpolated surfaces. All the error measurements show a decreasing trend with the increasing sample density, and the index of agreement and confidence of prediction show an increasing trend over years. Finally, the strong correlation between the Sample Coefficient of Variation and the performance measurements, implies that the more representative the samples are of the climate phenomenon, the more improved are the performances of the spatial interpolation techniques. The correlation between the sample coefficient of variation and the number of samples implies that the high representativity of the sample is attainable with an increased sample density.

# KEYWORDS

Spatial Interpolation

Low Sample Density

Climate Change Index

Performance Evaluation

# ACRONYMS

**AR** – Autoregressive

**ARMA** – Autoregressive Moving Average

**BMD** – Bangladesh Meteorological Department

**CP** – Confidence of Prediction

**CV** – Coefficient of Variation

**IDW** – Inverse Distance Weighting

**IOC** – Intergovernmental Oceanographic Commission

**IPCC** – Intergovernmental Panel on Climate Change

**MA** – Moving Average

**MAE** – Mean Absolute Error

**MGC** – Meteorological & Geo-Physical Centre

**OK** – Ordinary Kriging

**PRISM** – Partnership for Research Infrastructures in earth System Modelling

**RADAR** – Radio Detection and Ranging

**RMSE** – Root Mean Square Errors

**RMSEs** – Systematic Root Mean Square Errors

**RMSEu** – Unsystematic Root Mean Square Errors

**SWC** – Storm Warning Centre

**TPS** – Thin Plate Spline

**UK** – Universal Kriging

**WMO** – World Meteorological Organization

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# 1. INTRODUCTION

"*The choice of the appropriate methodology of interpolation of climatic data is crucial in order to obtain a correct representation of climatic fields*"

\- CRISCI, et al. 2006

The choice of appropriate spatial interpolation technique is a crucial research question since there is no single preferred technique; rather the choice depends on the interpolation performance in regards to the characteristics of the study area and data set. The question becomes more critical since sample density of the irregularly distributed space-time climate data has a significant effect on the spatial interpolation techniques in their performance. The chapter outlines the rationale of the sample density impact research in spatial interpolation performance analysis for the irregularly distributed space-time data in light of existing researches. It describes and sorts out the research objectives and propositions to carry out the entire research. It also structures the research manuscript from this starting chapter to the concluding chapter.

## 1.1 Background and rationale

Spatial interpolation techniques have been used for mapping the spatial patterns of climatic fields in several regions of the world, such as France (WEISSE, and BOIS, 2001), Germany (HABERLANDT, 2007), Great Britain (LLOYD, 2005), Italy (DIODATO, 2005), Mexico (BOER, et al., 2001; CARRERA-HERNÁNDEZ, and GASKIN, 2007), Portugal (DURÃO, et al., 2009; and GOOVAERTS, 2000), and the United States of America (KYRIAKIDIS et al., 2001). There have been a few studies conducted on the Bangladesh climate, based on the data from meteorological stations; but so far no study has been conducted in Bangladesh to analyze the spatial patterns of climate indices. This kind of study is very important since many climate indices representing a wide variety of Asian climate aspects are already in the phase of implementation. For example, SUHAILA and JEMAIN (2011) analyzed the spatial patterns of rainfall intensity and concentration indices over the Peninsular Malaysia. Moreover, global continuous surface models of climate response are no longer useful for practical reasons; international agencies, especially funding agencies are nowadays asking for regional datasets of climate change from the developing countries for the purpose of their recently taken funding scheme (UNFCC, 2012).

Spatial interpolation techniques have been profoundly used to quantify region-specific climate change based on historical data (DIRKS et al. 1998). But since there is no single preferred technique for spatial interpolation, there is no local accurate interpolated surface for mapping climate indices. Additionally, data unavailability and low sample density for spatial interpolation have made the problem more complicated for the developing countries. Recently it has been explored that for low-density datasets, complicated spatial interpolation techniques do not show a significantly greater predictive skill than simpler techniques (FRICH, et al., 2002; GOOVAERTS, 1998; and ISAAKS, and SRIVASTAVA, 1988). On the other hand, a high density climate dataset is not attainable for developing countries due to techno-economic reasons.

Therefore, selecting the locally appropriate interpolation technique is very important for mapping climate indices of Bangladesh in respect to very low density of sample. Yet the problem of low sample density has not been properly addressed by the scientific community. Though some of the authors have addressed the problem, the contribution is insignificant (ANDERSON, 1987; and CHOWDHURY, and DEBSHARMA, 1992). Consequently, these issues motivated the research on an evaluation of the available interpolation techniques based on the spatio-temporal characteristics of a climate dataset to analyze the climate variability phenomenon for a low sample density region - Bangladesh. Two climate indices have been selected, which are recommended by the Joint Project Commission for Climatology/Climate Variability and Predictability (CLIVAR) and Joint WMO/IOC Technical Commission for Oceanography and Marine Meteorology Expert Team on Climate Change Detection and Indices (PETERSON et al. 2001; ZHANG, 2009), namely PRCPTOT and TXx. The PRCPTOT characterizes the annual total precipitation in wet days, and the TXx corresponds to the yearly maximum value of the daily maximum temperature.

## 1.2 Research objectives

The following research objectives have been established:

*Exploration and Indices' Pattern Analysis:*

- To compile a rainfall and temperature dataset for Bangladesh.
- To compute annually defined climate indices, able to provide information on the climatic variability, from the dataset.

- Investigate the spatial and temporal variability of the climate indices.

*Uncertainty reduction in modelling and Interpolation:*

- Prepare continuous surfaces with two alternative deterministic spatial interpolation techniques - Thin Plate Spline & Inverse Distance Weighting.

- Improve and model the experimental variograms for stochastic interpolation by providing them with enough pairs of points to model the spatial dependence in response to low sample density.

- Prepare continuous surfaces with two alternative stochastic methods - Ordinary Kriging & Universal Kriging applying improved variograms.

*Performance Evaluation and Sample-density Impact Analysis:*

- Evaluate the interpolation cross-validation results by using suitable statistical performance measurements.

- Analyse the impact of low sample density on the performance measurements over time.

## 1.3 Research scopes and propositions

This research is aimed to evaluate the performance of spatial interpolation techniques applied two most suitable and applicable indices that describe climate variability in Bangladesh. The indices are calculated for each of the available years in dataset and interpolated surfaces are then created. Additionally, the research is aspired to improve the performance quality of the stochastic interpolation techniques. Most importantly, the research is aimed to analyze changes in the performance of spatial interpolation techniques with changing sample or spatial point density.

The following propositions are considered in the light of the described research scopes.

1. Sample or spatial point density does have a significant effect on the performance of spatial interpolation methods; the performance improves with the increase in sample density.

2. As a consequence of the dissimilar inherent methodology, different spatial interpolation techniques result in significantly different climate surfaces even though they utilize the same climate dataset; but the difference decreases with the increase in sample density.

## 1.4 Structure of the manuscript

The manuscript consists of six chapters – Introduction, Literature Review, Study Area, Dataset and Climate Change Indices, Methodology, Results and Discussion and Conclusion and Further Scopes (Figure 1.1). The first chapter, introduction, outlines and describes the research background, rationale, objectives and propositions. The second chapter summarizes the literature that has been studied and describes them in the light of spatial interpolation of climate variables and low sample density. The third chapter introduces the study area and describes its important features. Furthermore, it describes the dataset and climate change indices. It also illustrates the spatial and temporal trend of the calculated climate change indices over the study area and study period. The fourth chapter, methodology, explain see above the used methods for spatial interpolation techniques and their performance evaluation. The fifth chapter describes the results that have been obtained from the analysis implying the methodology, and shows the created interpolated surfaces, their differences and estimation ability. It also explains the trend of performance measurements over time with the increasing sample density. The final chapter summarizes the findings from the study and outlines the limitations and further scopes of the study.

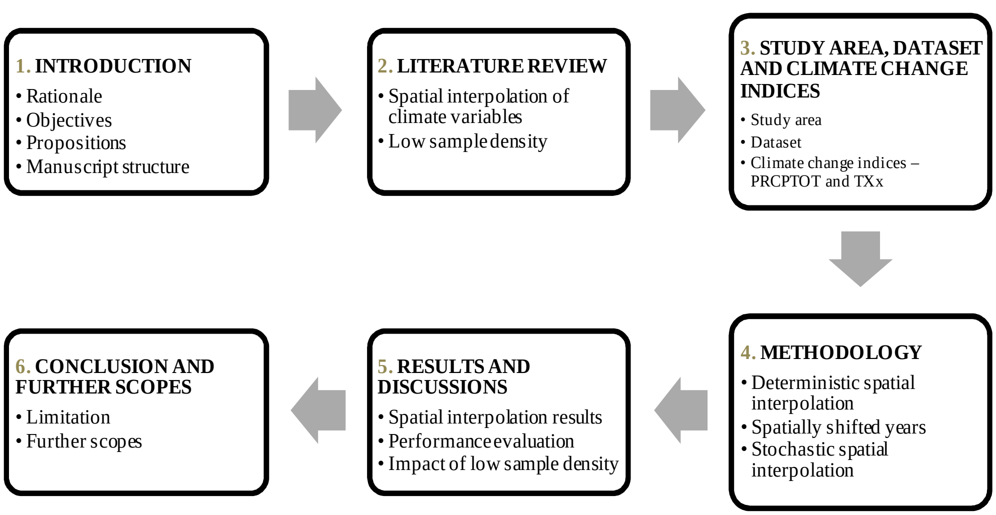



**Figure 1.1:** Structure of the manuscript.

The manuscript also contains the bibliographic reference and thirty annexes at the end. The bibliographic references list the literature that has been studied for the study and which the information has been extracted from. The annexes contain created interpolated surfaces by the four methods for the two climate change indices, their difference surfaces, the residual plots and the performance measurement tables.

## 1.5 Chapter conclusion

This chapter has discussed in detail the research background and rationale followed by the objectives and propositions. In a nutshell, the study is going to explore and analyze the impact of sample density on the performances of spatial interpolation techniques. Additionally, it is going to evaluate the performances of the four mentioned spatial interpolation techniques to interpolate the two climate change indices. The main goal of this study is to prove the propositions. The following chapter, literature review, is going describe the concepts of spatial interpolation of climate variables with low sample density; which will elaborate the concepts that have been mentioned in the objectives and propositions.

# 2. LITERATURE REVIEW

The inherent responsibility of the professionals who deal with climate and climate change is to provide insights regarding climate variables at any place at any time. The crucial part of this responsibility is to predict those variables at those places and times where observations of the climate elements do not exist (TVEITO, 2007). The problem has become even more critical when it was proved by the climatologists that the global metric is for climate change is no longer useful because climate effects are felt locally and they are region-specific (CHOWDHURY, and DEBSHARMA, 1992). From this point of view, special skills and knowledge are required to predict and result in the most reliable value for the desired climate information. As TVEITO (2007) presents, "traditionally this is done by using observed values at neighboring stations which are then adjusted for representativity, terrain and other effects affecting the local climatology. Such estimates have usually been carried out as single point calculations, often including subjective considerations based on local knowledge and experience. Most of these estimates will not be consistently derived and they are thereby not reproducible and cannot be regarded as homogenous. They are therefore of limited value, for example, for advanced climate analysis."

This chapter conceptualizes the application of spatial interpolation techniques to estimate the climate variables at not-sampled locations. It also describes what could be an ideal sample size for these spatial interpolations and how existing research has dealt with small sample size and low sample density in this respect.

## 2.1 Interpolating in space-time with climate variables

Spatial interpolation techniques, as geostatistical estimation techniques, with their inherent properties and applications, have successfully been implemented to combine different georeferenced climate variables and parameters in such a way that it is now possible to give consistently derived estimates at any place at any time (CHOU, 1997; GOOVAERTS, 1998; ISAAKS, and SRIVASTAVA, 1989; JOURNEL, and HUIJBREGTS, 1978; PHILLIPS, et al., 1992; and TABIOS, and SALAS, 1985). This is also because of the fact the interpolations techniques deal with the most important property of climate variables – they have a temporal extent along with a spatial extent (HUTCHINSON, 1995). CARUSO, and QUARTA (1998) have classified the techniques according to their fundamental hypotheses and mathematical properties, which are entitled as "deterministic method, statistical method, geostatistical method, stochastic simulation method, physical model simulation method and combined method”. The application and performance of the classified techniques are solely dependent on their research areas and algorithms and parameters used. Thus, obtaining a universally appropriate spatial interpolation technique for a particular application is impossible; rather locally an application oriented interpolation technique is obtainable (XIN, et al., 2003). Additionally, this locally appropriate spatial interpolation technique selection is subject to the qualitative and quantitative analysis of the local spatial data, their exploratory analysis and different stages of trial and errors with the techniques which is commonly recognized as cross-validation. More precisely, the result of the appropriate technique needs to be further examined for their accuracy (GOOVAERTS, 1997; and TVEITO, 2007).

Cressie (1991), Szentimrey (2002) and Szentimrey, et al. (2005) have suggested a range of mathematical statistical and geostatistical (stochastic) models of spatial interpolation in light of meteorological prediction. Among them, deterministic and stochastic methods have turned out to be the most simplistic and reliable methods for climate variability analysis. Recently it has been explored that for small sampled datasets, complicated kriging methods (stochastic) do not show significantly greater predictive skill than simpler techniques, such as the inverse square distance method (deterministic) (BHOWMIK, and CABRAL, 2011; and ISLAM, 2006).

## 2.2 Sample density and estimation uncertainty

Statisticians have been utilizing the concept of ‘Coefficient of Variation ($\boldsymbol{k}$)’ as a determinant of the sample size for statistical estimation with respect to the expected confidence level (BELLE, 2008) for a long time. As BELLE (2008) indicated, the coefficient of variation ($\boldsymbol{k}$) is a dimensionless number that quantifies the degree of variability in respect to the mean. The sample coefficient of variation is calculated using the following formula:

$$\boldsymbol{k} = {}^{\boldsymbol{s}}/_{\boldsymbol{M}} \qquad \textbf{(4.i)}$$

Where, $\boldsymbol{s}$ is the sample standard deviation, which is the calculated square root of the unbiased estimate of the variance, and $\boldsymbol{M}$ is the sample mean. The $\boldsymbol{k}$ value is sometimes multiplied by 100 so that the ratio of the standard deviation to the mean is expressed in terms of a percentage. Therefore, it is commonly accepted, if the coefficient of variation is high, the mean is not representative of the variable behavior. The threshold value has been set as 50% (AFONSO, and NUNES, 2011) which means if the coefficient of variation of a sample set is more than 50% then the statistical estimation using these samples will end up with high uncertainty in general which means the estimation is less accurate.

LYNCH, and KIM (2010) explain a way to prevent the curse of uncertainty due to the high coefficient of variation, which is to adjust the sample size. They describe the relationship among coefficient of variation, sample size and uncertainty with a mathematical function which illustrates that when the coefficient of variation is higher, the sample size should be high enough as well to reduce the uncertainly and obtain the accepted level of confidence. They, in conclusion, have provided a table (Table 2.1) showing the required number of samples for a certain level of coefficient of variation with corresponding expected uncertainty. The table clearly shows that if it is objected to estimate with 95% of confidence (which denotes that mean error of estimation should not be more than 5% of sample mean), for a coefficient of variation of 20%, 43 samples are required but on the other hand if the coefficient of variation is 80%, 693 samples are required for estimation.

KELLEY (2007) describes a similar concept like LYNCH, and KIM (2010), but in an elaborated and functional way. He introduces some important parameters – expected confidence interval width (E[w]), desired full confidence interval width ($\omega$) and desired degree of assurance ($\gamma$), to identify the level of confidence more precisely and then figures out the required number of samples for estimation with expected uncertainty through a mathematical function. As Table 2.2 illustrates, to estimate with 95% confidence where the desired full confidence interval width is 10% ($\omega$=0.10) and desired degree of assurance is 99% ($\gamma$=0.99); if the coefficient of variation is 20%, 62 is the required sample size and if the coefficient of variation is 50% then 401 samples are required to estimate with expected confidence level.

Thus the sample size plays a significant role in statistics and so in geostatistics, since estimating with reduced uncertainty is the explicit aim of any geostatistical analysis (ISAAKS, and SRIVASTAVA, 1989). This is even more critical in interpolation since it's required to take into account the relative distances of the samples along with their values; for there is a serious consequence of the property – "the global information carried by the stationary mean becomes preponderant in prediction as remote neighboring data bring less information about the unknown value at a distant location" (GOOVAERTS, 1997). This property along with the properties from LYNCH, and KIM (2010) and KELLEY (2007) clearly indicates the concept and problem of low sample density in spatial interpolation. Thus, if the sample size is smaller than the minimum requirement for a desired confidence with an appointed coefficient of variation and the area is in contrast too large to interpolate with inadequate samples, the interpolation results end up with a huge amount of unexpected uncertainty and thus hamper the interpolation quality.

Even without coefficient of variation, sample size always determines the risk of prediction (α value) for a constant variation in the samples, which is stated by Chebyshev's Rule (OSTLE, and MALONE, 1990). Sample size and α value are always inversely proportional, so a decrease in the sample size always increases the risk of prediction. And whenever the area of prediction increases the risk increases proportionally.

**Table 2.1:** Sample sizes and margin at different coefficient of variation described by LYNCH, and KIM (2010); N = sample size, z= score of divergence of the experimental result and cv = coefficient of variation.

| N=(z*z)*(cv*cv)/(%mean*%mean) | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Coefficient of Variation = .20 | | | Coefficient of Variation = .40 | | | Coefficient of Variation = .60 | | | Coefficient of Variation = .80 | | | Coefficient of Variation = 1.00 | | |
| 3% of Mean | 120.27 | 120.27 | 120.27 | 481.07 | 481.07 | 481.07 | 1,082.41 | 1,082.41 | 1,082.41 | 1,924.28 | 1,924.28 | 1,924.28 | 3,006.69 | 3,006.69 | 3,006.69 |
| 4% of Mean | 67.65 | 67.65 | 67.65 | 270.60 | 270.60 | 270.60 | 608.86 | 608.86 | 608.86 | 1,082.41 | 1,082.41 | 1,082.41 | 1,691.27 | 1,691.27 | 1,691.27 |
| 5% of Mean | 43.30 | 43.30 | 43.30 | 173.19 | 173.19 | 173.19 | 389.67 | 389.67 | 389.67 | 692.74 | 692.74 | 692.74 | 1,082.41 | 1,082.41 | 1,082.41 |

**Table 2.2:** Necessary sample size for 95% confidence intervals for the coefficient of variation in selected situations described by KELLEY (2007), with desired degree of assurance of achieving a confidence interval no wider than desired.

| | κ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| ω | 0.05 | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | 0.35 | 0.40 | 0.45 | 0.50 |
| | | | | | E[w] | | | | | |
| .010 | 199 | 790 | 1,812 | 3,325 | 5,408 | 8,166 | 11,724 | 16,233 | 21,866 | 28,819 |
| .025 | 37 | 131 | 295 | 537 | 871 | 1,312 | 1,882 | 2,603 | 3,505 | 4,618 |
| .050 | 13 | 37 | 78 | 139 | 222 | 333 | 475 | 656 | 882 | 1,160 |
| .075 | 8 | 18 | 36 | 63 | 100 | 149 | 213 | 296 | 396 | 520 |
| .100 | 6 | 12 | 22 | 38 | 59 | 87 | 123 | 168 | 225 | 294 |
| .125 | 5 | 9 | 16 | 26 | 40 | 58 | 81 | 110 | 146 | 191 |
| .150 | 5 | 8 | 13 | 20 | 29 | 42 | 58 | 78 | 104 | 135 |
| .175 | – | 7 | 10 | 16 | 23 | 32 | 44 | 59 | 78 | 101 |
| .200 | – | 6 | 9 | 13 | 18 | 26 | 35 | 47 | 61 | 79 |
| | | | | | $\gamma$ = .80 | | | | | |
| .010 | 215 | 823 | 1,866 | 3,401 | 5,511 | 8,300 | 11,896 | 16,450 | 22,136 | 29,150 |
| .025 | 42 | 144 | 316 | 567 | 911 | 1,366 | 1,950 | 2,690 | 3,613 | 4,751 |
| .050 | 16 | 43 | 88 | 153 | 242 | 360 | 510 | 700 | 936 | 1,227 |
| .075 | 9 | 22 | 43 | 73 | 113 | 167 | 238 | 325 | 433 | 565 |
| .100 | 7 | 14 | 27 | 45 | 69 | 100 | 140 | 190 | 253 | 328 |
| .125 | 6 | 11 | 19 | 31 | 47 | 68 | 94 | 127 | 169 | 219 |
| .150 | 5 | 9 | 15 | 24 | 35 | 50 | 69 | 93 | 122 | 158 |
| .175 | – | 8 | 12 | 19 | 28 | 39 | 54 | 72 | 94 | 121 |
| .200 | – | 7 | 11 | 16 | 23 | 32 | 43 | 57 | 75 | 97 |
| | | | | | $\gamma$ = .99 | | | | | |
| .010 | 246 | 886 | 1,964 | 3,540 | 5,698 | 8,544 | 12,207 | 16,841 | 22,621 | 29,745 |
| .025 | 55 | 170 | 357 | 625 | 989 | 1,466 | 2,079 | 2,852 | 3,814 | 4,996 |
| .050 | 22 | 57 | 110 | 184 | 283 | 413 | 578 | 785 | 1,042 | 1,357 |
| .075 | 13 | 31 | 58 | 94 | 142 | 205 | 286 | 385 | 507 | 656 |
| .100 | 9 | 22 | 39 | 62 | 92 | 131 | 179 | 239 | 311 | 401 |
| .125 | 8 | 17 | 30 | 46 | 67 | 93 | 127 | 168 | 219 | 281 |
| .150 | 7 | 14 | 23 | 36 | 52 | 72 | 98 | 129 | 167 | 212 |
| .175 | – | 12 | 20 | 30 | 43 | 59 | 79 | 104 | 134 | 170 |
| .200 | – | 10 | 17 | 26 | 37 | 50 | 67 | 87 | 112 | 142 |

GOULARD, and VOLTZ (1993) applied geostatistical interpolation methods to predict functions at non-sample sites assuming that the functions were only known at a small set of points. The idea has been extended by GIRALDO, et al. (2011) through overcoming the explicit assumptions on parametric modelling and a small number of observed points per function for penetration resistance of the soil. They applied a non-parametric fitting pre-process to the observed functions where the smoothing parameter is chosen by functional cross-validation. As such, smoothness improvement is a concern due to the existence of the low sample density problem and in the case where the density of samples is significantly low (even half or one third than the required sample size for expected confidence) the interpolation and smoothness improvement need careful analysis and determination of the variability function. The problem becomes even more critical when there is no possibility to get enough points or a superimposing layer of high resolution remotely sensed dataset, or other secondary data spatially correlated with the variable of interest which would allow using an alternative multivariate technique for estimation and smoothness improvement.

## 2.3 Interpolation in space-time with low sample density

In case of low sample density, spatial interpolations highly smooth the predictions, which is especially undesirable for climate variables since climate variability is not smooth neither perpetual. The smoothing basically depends on the local sample configuration, it is minimal close to the sample locations and increases as the location of estimation gets further away from the sample locations. Extensive smoothness of the interpolated surface justifies the problem of low sample density.

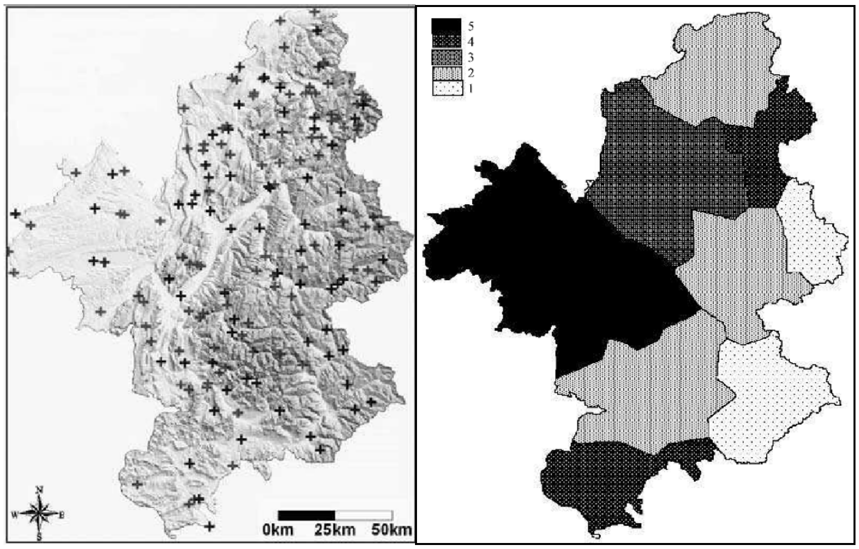
The problem of interpolation with low sample density has been realized by the geostatisticians in many cases (DIRKS, et al., 1998; HABERLANDT, 2007; PHILLIPS, et al., 1992; and TABIOS, and SALAS, 1985), but no one has actually dealt with it. All of them have adopted the classical approach of using auxiliary information in estimation, such as high resolution datasets. HABERLANDT (2007) superimposed 21 measurements stations for extreme precipitation with a high resolution RADAR dataset and overcame the problem of interpolation with small sample size.

The problem was actually addressed for the first time by DUMOLARD (2007) and TVEITO (2007), though their focus was basically on the irregular distribution of the samples that resulted in low sample density in some parts of their study areas. TVEITO (2007) eventually figured out that uncertainty of interpolation is a function of sample density and uncertainty increases with the decrease in sample density. In the spatial interpolation of temperature data, this author used well known as ‘residual kriging’ (detrended kriging) which consists of two components – a deterministic model and a stochastic residual model. Taking into account the variability in time, means of monthly, seasonal or annual temperatures were interpolated with different deterministic models for every month or season. Finally the deterministic model was regionalized by predicting the model parameters within a moving window and the remaining residual field was interpolated by applying stochastic kriging method. Interpolating precipitation was more complicated than temperature "since precipitation is non-continuous in space and time" (TVEITO, 2007).

DALY, et al. (1994) proposed the Precipitation-elevation Regression on Independent Slopes Model (PRISM), which is based on local climate-elevation regression functions. Long-term mean precipitation has been interpolated based on the principles of the PRISM-method which (SCHWARB, 2001) incorporates different terrain characteristics - slope, aspect, etc. with a linear regression approach enabling the use of topographic information at several spatial scales (DALY, et al. 2002; and 2006). SCHWARB (2001) combined radar information and in-situ observations to carry out further analysis.

On the other hand, DUMOLARD (2007) dealt with a typical problem of sparsely distributed samples. He figured out that a simple linear regression between latitudes and longitudes of the station locations for the 168 points gives a $R^2$ of 0.05 for an area of approximately 150,000 sq.km. And thus he found out that the probability $\alpha$ of a false rejection of independence between X and Y is 0.044. What he worked out was giving a weak $\alpha = 0.01$ risk, independence between latitude and longitude has been rejected but giving a larger but reasonable $\alpha = 0.05$ risk, it has been accepted. Finally, after introducing altitude and creating the samples' their 'influence buffer', he divided the entire study area into some clusters with homogenous distribution of sample density and interpolated each cluster separately and aggregating them to get the interpolation result of whole study area (Figure 2.1). Thus he achieved a global improved accuracy by combining the uncertainty of the local interpolation results since the cluster with higher sample density provided far lower uncertainty than the low density clusters. The combined result also gave reduced uncertainty than considering all samples of the study area as a whole.

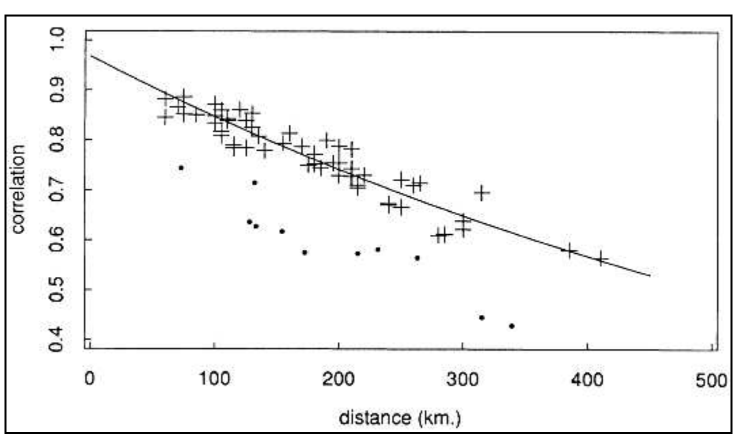
HASLETT, and RAFTERY (1989) compensated the fact of few spatial points in analysis with high density of temporal points. They use long term hourly records of wind speeds at the 12 synoptic meteorological stations on a simple and parsimonious approximating model which accounts for the main features of wind speeds in Ireland, namely seasonal effects, spatial correlation, short-memory temporal autocorrelation and long-memory temporal dependence. Based on the temporal autocorrelations of the station wind speed values and distance correlation analysis of the seasonal effect analysis, they decided to take one station (Rosslare) out of the variogram analysis since this station was acting as outlier (Figure 2.2). Resulted long-memory temporal dependence of the data was used in synthesizing deseasonalization, kriging, ARMA modelling and fractional differencing in a natural way.




**(a)** **(b)**

**Figure 2.1:** (a) Distribution of the samples and (b) clustering of the study area according to homogeneity of sample density in DUMOLARD (2007). The hierarchical legend means the sub-regions 1-with a low density of points and a concentrated pattern 2-with a low density of points 3-with a concentrated pattern 4-with correct density and pattern of points and 5-with a good sample (density + pattern).

For their case, simple kriging estimator performs well as a point estimator and ARMA modeling as good interval estimator after fitting both estimators in space and time. The cross-validation with the fitted models also resulted in significantly reduced errors which encouraged considering temporal variability and dependence in interpolating with low spatial density of samples. This means the low density in the spatial extent of the data and resulted uncertainty due to that can be minimized by using the high density in the temporal extent in creative ways.




**(a)**

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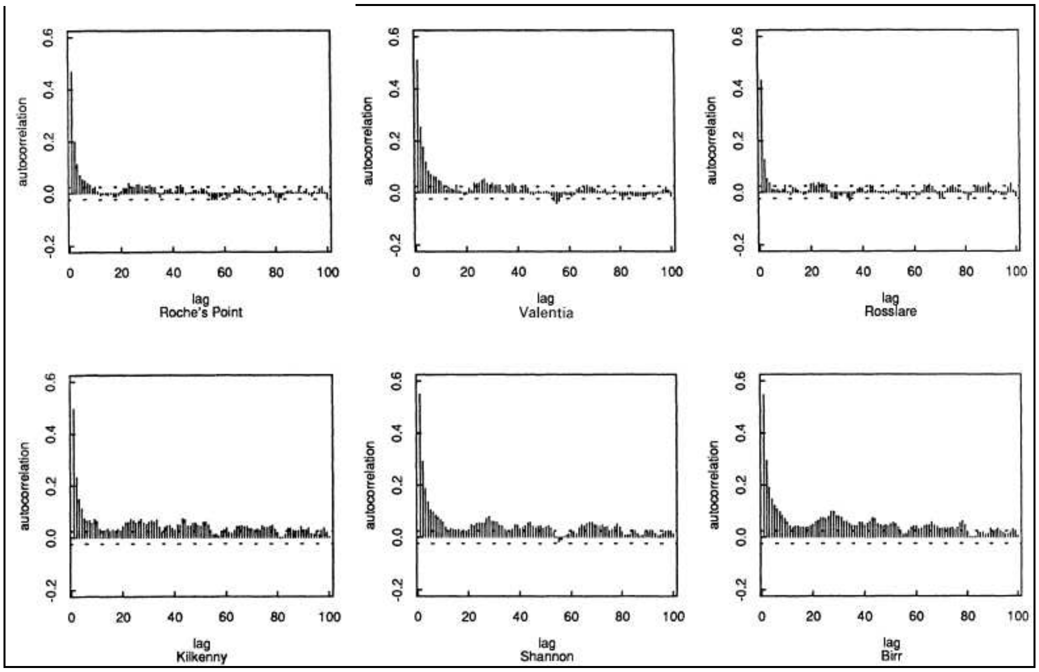



**(b)**

**Figure 2.2:** (a) Distance-correlation plot and (b) correlation functions of the velocity measurements for the first six stations in HASLETT, and RAFTERY (1989). Each cross at (a) corresponds to a pair of synoptic stations and the dots correspond to pairs which include Rosslare and show lower correlation than others. Rosslare also shows identical pattern in terms of autocorrelation at different time lags at (b).

RAUDYS, et. al (1991) analyzed typically the influence of both training and testing sample size on the design and performance of pattern recognition system. They clearly proved the existence of "curse of dimensionality" which implies that the classification accuracy obviously increases with the increase in the number of sample; thus a large training sample size is required for applications with large number of features and a complex classification rule, and a large test sample size is required to accurately evaluate a classifier with a lower error rate in cross-validation. It is true that classification and interpolation are two different concepts apart from the fact that they both need to train samples and evaluate their performance through cross validation. In interpolating continuous variables, leave-one-out cross validation is typically used.

But still the following approaches they suggested to increase the classification accuracy and to minimize estimation error from cross-validation seem even very useful for interpolation.

1. Increasing the features and thus artificially preparing a sample size sufficient to improve learning accuracy and estimation error.

2. From the finite number of training samples carefully choosing those samples which support to improve the design of spatio-temporal variability and discard outliers.

The second approach implies to take out samples acting as outliers during model preparation. In case of interpolating in space-time, more care is needed in this regard since full stations along with their complete time series might be needed to be taken out.

Finally, DOBESCH, et al. (2001) indicated some important properties of interpolation results using the sparsely distributed and low density samples. They claim that the map of local interpolated values of the variable will be smoothed if the variable is spatially continuous; but the representativeness of the sample locally can be smoothed and mapped if superimposed to the interpolated values and the global quality of the interpolation can be assessed through an analysis of variance. The authors suggested "test several methods, choose the right method, and correct use of the method and validation" as the sequence of approaches to carry out the interpolation in space-time with low sample density and reduced uncertainty.

## 2.4 Chapter conclusion

This chapter has presented a detailed overview of the preferred spatial interpolation techniques by the scientific community for mapping climate variables. It has also discussed the ideal size of sample for statistical estimation with acceptable accuracy. Furthermore, various approaches by the geostatisticians to deal with the sample density problem have been outlined. The next chapter, study area, dataset and climate change indices, will describe the study area and dataset; calculate the two climate change indices and analyse their behavior over space and time. The indices will be used as input of spatial interpolation in light of the experiences from the literature review.

# 3. STUDY AREA, DATASET AND CLIMATE CHANGE INDICES

The characteristics and important features of the study area and dataset are key issues to be considered in the choice of spatial interpolation techniques. There are specific spatial interpolation techniques, which are developed to specifically apply in case of certain features of the study area and dataset. This chapter outlines the important decision making features to choose appropriate spatial interpolation techniques to evaluate. In addition, it calculates and characterizes the two climate change indices that are used as input for the spatial interpolation of the climate phenomenon.

## 3.1 Study area - Bangladesh

Bangladesh, situated in south-east Asia, is one of the most vulnerable countries of the world regarding the adverse impacts of anthropogenic climate change (BRAUN, 2010; CAI, et al., 2010; CHOWDHURY, and DEBSHARMA, 1992; KLEIN, et al., 2006; and SHAHID, 2009) (Figure 3.1). The total area of the country is 147,570 square kilometer (BBS, 2009), approximately one fifth of which consists of low-lying coastal zones within one meter of the high water mark (IPCC, 2007).

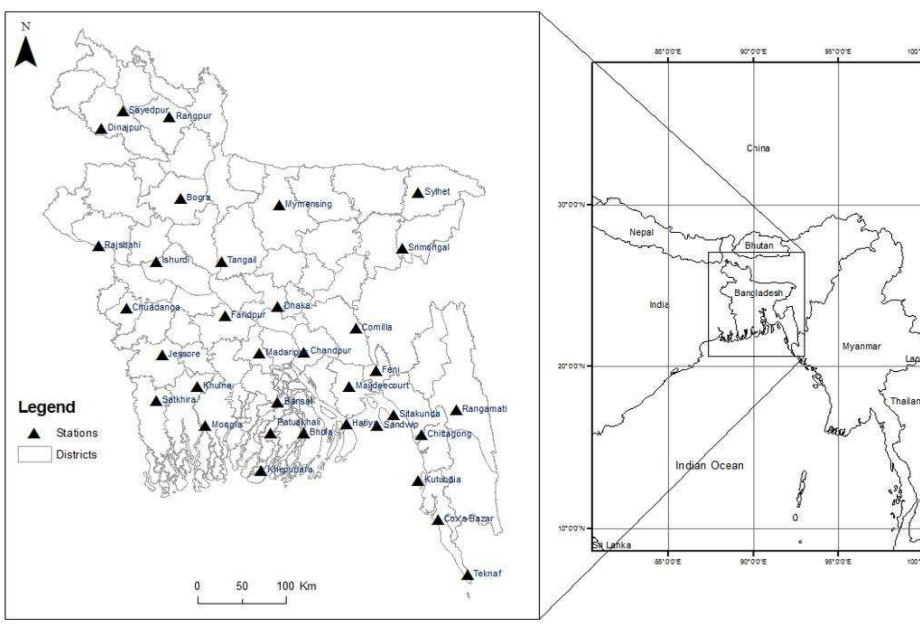



**Figure 3.1:** Study area – Bangladesh with world location and 34 meteorological stations to measure daily precipitation and temperature (BHOWMIK and CABRAL, 2011).

Threats of sea level rise, droughts, floods, and seasonal shifts due to global warming have been presented in many recent studies on the country. The rainfall regime of the country is highly variable in both time and space. The annual mean rainfall varies from 1400 mm in the west to more than 4300 mm in the east of the country (SHAHID, 2010). The mean annual temperature has increased during the period of 1895-1980 by $0.31^0$C (PARTHASARATHY, et al., 1987) and the annual maximum temperature is predicted to increase by $0.4^0$C and $0.73^0$C by the year of 2050 and 2100 respectively (KARMAKAR, and SHRESTHA, 2000; and MIA, 2003). The Bangladesh Meteorological Department (BMD) is the authorized government organization for all meteorological activities in the country. It maintains a network of surface and upper air observatories, radar and satellite stations, agro-meteorological observatories, geomagnetic and seismological observatories and meteorological telecommunication systems. The department has its headquarter in the capital Dhaka, with two regional centers – the Storm Warning Centre (SWC) in Dhaka and Meteorological & Geo-Physical Centre (M & GC) in Chittagong. It measures the daily precipitation and daily temperature with thirty-four meteorological stations situated in different locations all over the country (DMICCDMP, 2012) (Figure 3.1).

## 3.2 Dataset and materials

The dataset used in this study includes daily precipitation and temperature measurements from the meteorological stations of BMD for 60 years i.e. 1948-2007. The dataset is not available from the beginning of the study period for all stations; precipitation data from 8 stations and temperature data from 10 stations is available for 1948 and there is a gradual increase of precipitation data from 32 stations and temperature data from 34 stations by 2007 eventually. ‘Spacetime’, ‘intamap’, ‘fields’ and ‘gstat’ packages of the open source statistical software ‘R’ (ISMWUWW), 2012) and ArcGIS version 10.0 (Esri, 2012) by Esri are utilized in order to analyze and compute the data.

## 3.3 Climate change indices

Two climate change indices – PRCPTOT and TXx (PETERSON et al. 2001; PLUMMER, et al., 1999; Santos et al., 2011 and You et al. 2011) have been calculated from the available precipitation and temperature data for each year of 1948-2007 and for each station. These climate change indices are internationally recognized and have been used in different climate change studies of different regions of the world (IPCC, 2007).

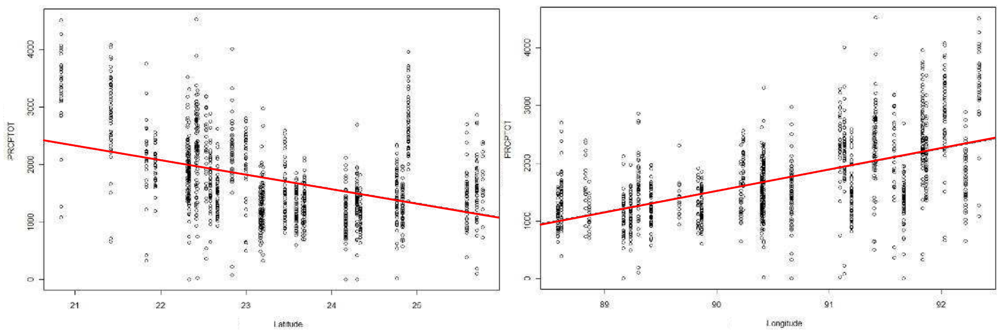
PRCPTOT refers to the annual total precipitation in wet days (Peterson et al. 2001; and You et al. 2011). Since Bangladesh has clearly defined wet days in the year, the weather phenomenon is known as 'Monsoon' and is present in June-September of every year (Alexander, 1999; Braun, 2010; MEF, 2008; IPCC, 2007; and WB, 2012), PRCPTOT is the most representative of change in precipitation. The formula for calculating PRCPTOT is: if $RR_{ij}$ is the daily precipitation amount on day $i$ in period $j$ and if $l$ represents the number of days in $j$, then (Peterson et al. 2001)

$$PRCPTOT_j = \sum_{i=1}^{l} RR_{ij} \quad \text{……………..……..………(3.i)}$$

TXx refers to the yearly maximum value of the daily maximum temperature (Peterson et al. 2001; and Plummer, et al., 1999). Previous studies have proven that the change in temperature of Bangladesh due to climate change is more recognizable from the change in maximum temperature (Braun, 2010; and Cai, et al., 2010). The formula for calculating TXx is: if TXx is the daily maximum temperatures in period $j$, then the maximum daily maximum temperature each year is (Peterson et al. 2001):

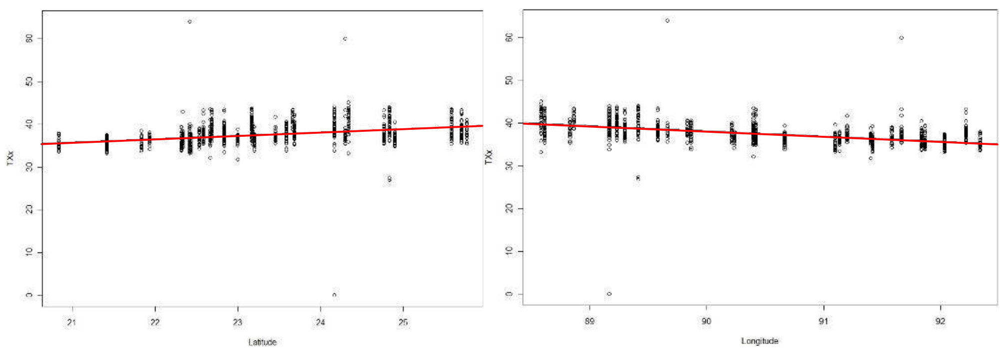
$$TXx_j = max(TXx_j) \quad \text{…………………………(3.ii)}$$

The calculated climate indices – PRCPTOT and TXx show spatial trends over the study area (Figures 3.2 and 3.3). The PRCPTOT values increase with an increase in longitude and decrease with an increase in latitude (Figure 3.2). This indicates that PRCPTOT shows a spatial trend from the northwest to the southeast direction i.e. higher monsoon precipitation is experienced in the southeast region of the country. The correlation of PRCPTOT with longitude (**0.55**) is higher than the correlation of PRCPTOT with latitude (**-0.42**), which indicates that the spatial trend is more dominant in west-east direction than north-south direction. On the other hand, the TXx values increase with an increase in latitude and decrease with an increase in longitude (Figure 3.3). This indicates that TXx shows a spatial trend from the southeast to the northwest direction, i.e. a higher yearly maximum of the daily maximum temperature is experienced in the northwest region of the country. The correlation of TXx with longitude (**-0.52**) is again higher than the correlation of TXx with latitude (**0.34**), which indicates that the spatial trend is more dominant in the west-east direction than the north-south direction. It is important, since Bangladesh is a flat country (Klein, et al., 2006), that the correlation of both PRCPTOT and TXx is insignificant with altitude and therefore height does not affect the spatial trend of the indices.




**(a)** **(b)**

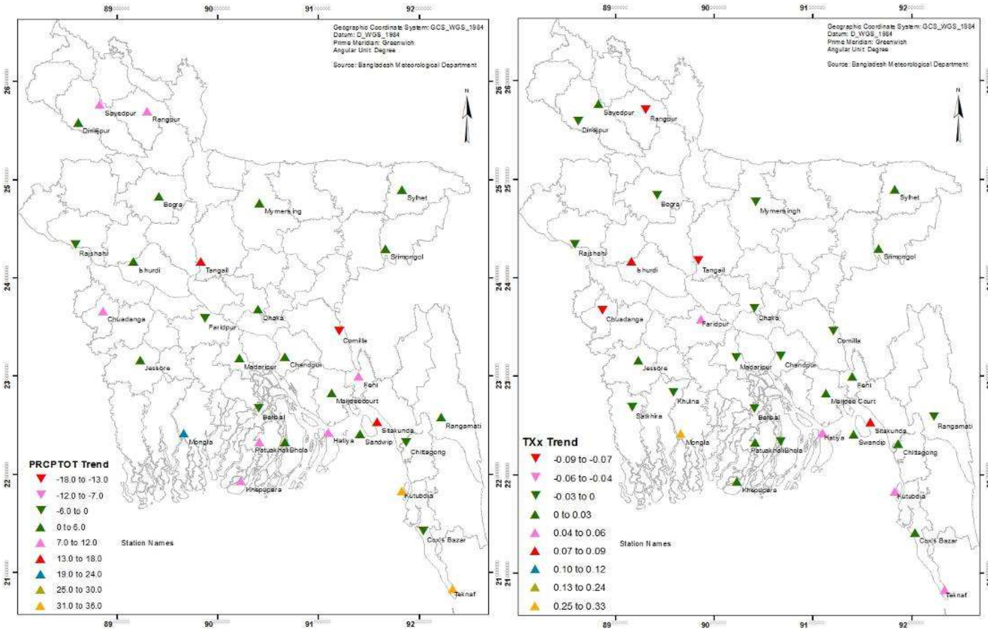
**Figure 3.2:** Spatial trend PRCPTOT to southeast direction for all years, (a) decreasing trend with increasing latitude and (b) increasing trend with increasing longitude.




**(a)** **(b)**

**Figure 3.3:** Spatial trend TXx to northwest direction - (a) increasing trend with increasing latitude and (b) decreasing trend with increasing longitude.

The results of the analysis of the general temporal trend of the calculated climate change indices are presented in Figure 3.4. For PRCPTOT, a range of -18 to 36 for the trend value has been obtained. Most of the stations show an increasing trend of PRCPTOT over time. Especially the stations in the southeast region of the country experience the highest increasing trend, where the highest values of PRCPTOT are also experienced. On the other hand, for TXx a range of -0.09 to 0.33 for the trend value has been obtained. Though most of the stations show an increasing trend of TXx, almost all the stations in the mid-region, including capital Dhaka, show a decreasing trend. The station at Rangpur district, which represents the warmest region of the country (BBS, 2009), shows a decreasing trend of TXx. This fact indicates clearly that the climate is shifting, which will result in the climate change consequences for the environment.




**(a)** **(b)**

**Figure 3.4:** Temporal trend of (a) PRCPTOT and (b) TXx in every station location.

Detailed temporal variability of PRCPTOT and TXx in every station is presented in Figures 3.5 and 3.6 respectively. It is difficult to predict any detailed temporal trend for the indices, but the variability at some station locations show a trend in the index behavior. It is obvious that the variability in the PRCPTOT index is more dominant than in the TXx index.

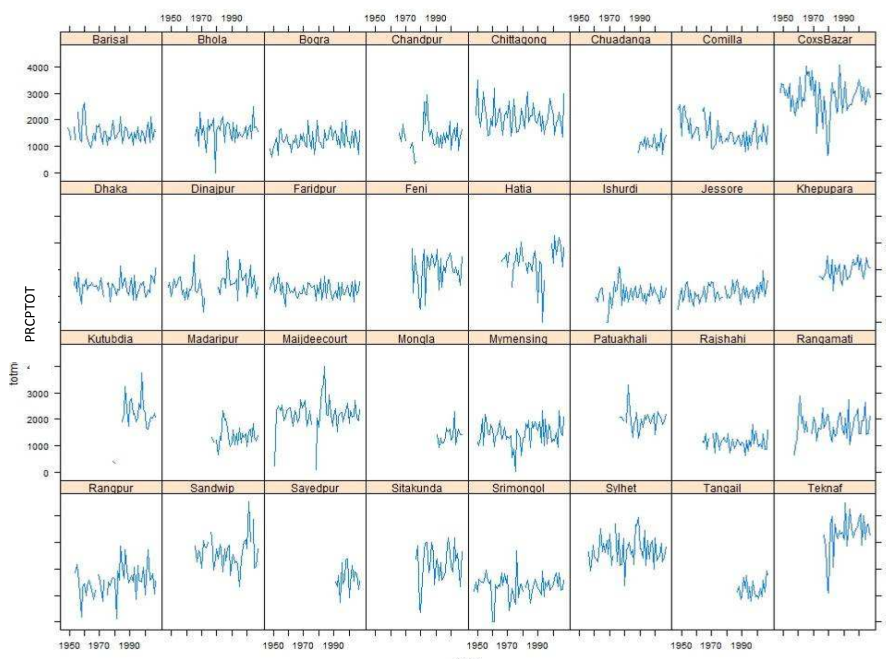



**Figure 3.5:** Temporal variability of PRCPTOT in every station location.

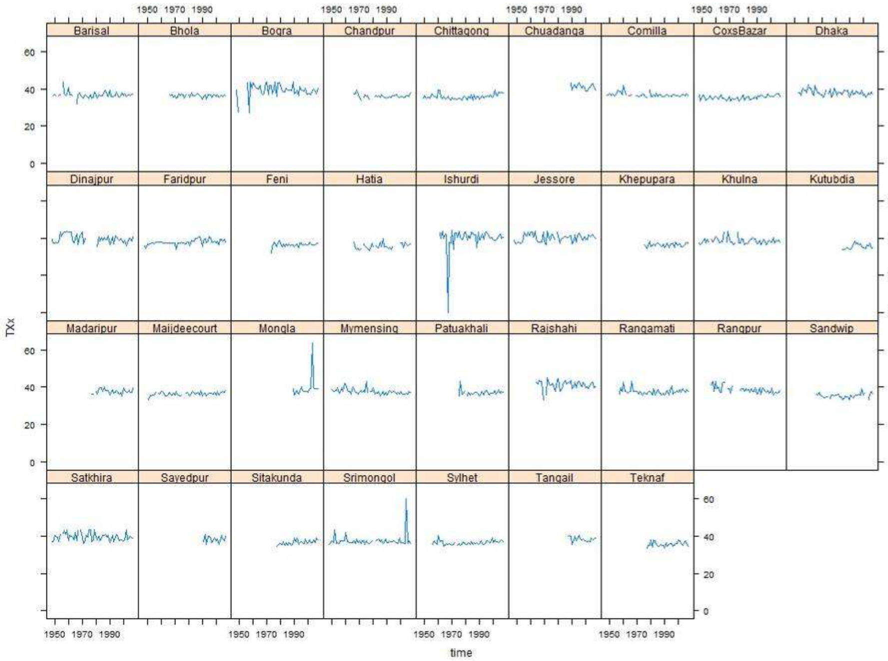



**Figure 3.6:** Temporal variability of TXx in every station location.

The sudden increase and decrease in the variability are caused by the inconvenient data quality, which could not be evaluated within this research scope. Figure 3.5 and 3.6 also represent a lot of missing indices in the time series, which are occurred by the missing data in the original dataset. The missing data has been considered as no data value in calculation of the indices and thus the produced missing indices in the time series do not take part in the spatial interpolation.

## 3.4 Low sample density problem to interpolate climate change indices

As described in chapter 2, like any statistical estimation, spatial interpolation requires a sufficient number and density of samples to obtain acceptable accuracy. In light of this discussion, interpolation of both PRCPTOT and TXx indices for Bangladesh experiences the low sample density problem. Considering 34 meteorological stations which are available in maximum in 2007 to interpolate TXx, each station is used to estimate the continuous surface for 4340 square kilometer area, which is very big for a station to estimate. Figure 3.7 represents the fact clearly; the size of the Voronoi polygons to estimate is very big. The $R^2$ value of simple linear regression between latitude and longitude of the stations is 0.339 which is good from the perspective described in DUMOLARD (2007), because it means that the stations are more or less evenly distributed.

But the risk of estimation α for TXx is 0.008 when the sample size is 34, but α decreases to 0.000006 if the sample size is increased to 100, when other parameters remain constant according to TVEITO (2007). This indicates that the number of stations is too little to interpolate with acceptable accuracy.

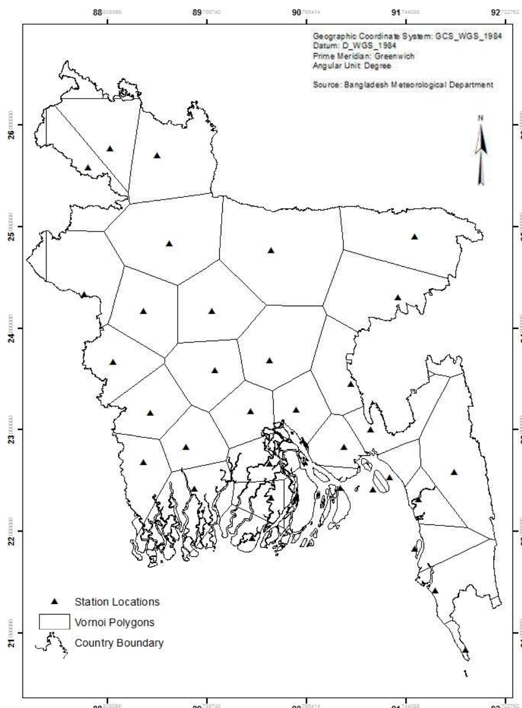



**Figure 3.7:** Vornoi polygons for prediction of continuous surface of TXx by each meteorological station.

Furthermore, interpolation methods will highly smooth the predictions in the presence of low sample density, which is undesirable for climate data. The global information carried by the stationary mean becomes preponderant in prediction as remote neighboring data bring less information about the unknown value at a distant location (GOOVAERTS, 1997). Consequently, the smoothing effect is minimal close to the data locations and increases as the location being estimated gets farther away from data locations. On the other hand, according to AFONSO, and NUNES (2011), if the coefficient of variation is high, the mean will not be representative of the attribute behavior. The coefficient of variation ($k$) of samples for PRCPTOT is 41% in average and ranges from 25% to 59%, while for TXx it is 6.2 % and ranges from 3.2% to 24%. Therefore, the mean will not be representative of the PRCPTOT behavior in many years; for TXx the attribute's variability is not so pronounced, but it might have the same problem as PRCPTOT in a few years. Moreover, according to these $k$ values, to interpolate and produce continuous surfaces with 95% confidence (where, mean error is not more than 5% of the sample mean), the required sample size for PRCPTOT is 173 on average which may vary from 43 to 390, and it is equal to 4 on average for TXx which may vary between 1 to 43 as described in LYNCH, and KIM (2010). As explained in KELLEY, (2007), to estimate with 95% confidence where the desired full confidence interval width is 10% ($\omega$=0.10) and desired degree of assurance is 99% ($\gamma$=0.99), the required sample size to interpolate PRCPTOT is 16,233 and to interpolate TXx is 199. For both cases, the number of samples or spatial points is too low to obtain acceptable accuracy.

## 3.5 Chapter conclusion

As presented in this chapter, the sample density of the study region is too low to provide acceptable accuracy in the spatial interpolation results. The calculated indices behavior over space and time and the features of the study area lead to the decision of evaluating possible deterministic and stochastic spatial interpolation techniques. As ATKINSON, and TATE, 2000; and ISAAKS, and SRIVASTAVA, 1988 discovered, that in case of uncertain sample distribution and size, deterministic methods should be the preferable options for spatial interpolation, two deterministic methods – Thin Plate Spline (TPS) and Inverse Distance Weighting (IDW) can be applied for interpolation of the indices. The two deterministic methods fit the interpolation models exactly through the measured points, but TPS performs some degree of smoothing and IDW performs with no smoothing. On the other hand, two stochastic methods can be applied after improving the variograms to fit the models – Ordinary Kriging (OK) and Universal Kriging (UK). The OK model can be applied taking the anisotropic behavior of the indices into account, while UK can be applied by taking the spatial trend of the indices into account. The following chapter, methodology, will describe the spatial interpolation models in detail, derived from the literature review and fitting to the dataset. It will also outline the methods for the performance evaluation of the methods.

# 4. METHODOLOGY

This chapter describes the detailed methodology of the four spatial interpolation techniques and their rationale of application for the study area and dataset. It elaborates the spatial interpolation models with mathematical equations fitting to the study area and climate change indices. It also outlines seven different performance measurements to evaluate the performances of these spatial interpolation techniques and their importance.

## 4.1 Spatial interpolation of climate change indices with low sample density

Spatial interpolation of climate change indices needs special spatialization since the indices contain both spatial and temporal information inherently (TVEITO, 2007). In practice, the spatial interpolation techniques that incorporate temporal information with spatial information in the modeling function are most appropriate for interpolating climate variables (HASLETT et al., 1989; and TRENBERTH et al., 2000). The structure of a basic spatial interpolation problem denotes that denotes that the dependent variable of interest $Z(s_0, t)$ is predicted as an output of the mathematical function of known predictors $(s_i, t)\ (i = 0, \ldots, M)$, where the location vectors $s$ are the elements of the given space domain $D$ and $t$ is time. The vector form of the predictors is $Z^{\dagger}T\ (t) = [Z(s_{\downarrow}1, t), \ldots \ldots, Z(s_{\downarrow}M, t)$ (SZENTIMREY et. al., 2007). The probability distribution of the climate variables sets up the appropriate interpolation formulae, which include some unknown interpolation parameters. These parameters can be obtained through known functions of certain statistical parameters. Modeling of climate variables with these statistical parameters assume that the expected values of the variables are changing in space and in time in a similar way (CHRISTAKOS, 2001; TVEITO, 2007). The spatial change in the variables indicates that the climate is different in the regions whereas temporal change is considered as the result of climate variability or of possible global climate change (HIJMANS et. al, 2005). As a result, expected values of climate variables can be obtained by the following linear model (CHRISTENSEN, 1990; PAPRITZ and STEIN, 1999):

$$E(Z(s_i, t)) = \mu(t) + E(s_i)\ (i = 0, \ldots, M) \quad \text{(4.i)}$$

Where, $\mu(t)$ is the temporal trend or the climate change signal, $E(s)$ is the spatial trend. Typically, there is only a single realization in time for the modeling of the statistical parameters in spatial interpolation. Therefore only the predictors $Z(s_i, t)\ (i = 0, \ldots\ldots, M)$ constitute the usable information or the sample for the modeling of variability over space (Szentimrey et. al., 2007).

At the linear model (4.i), the basic statistical parameters can be allocated into two categories - deterministic and stochastic parameters. Thus the spatial interpolation techniques can be divided into two groups – deterministic and stochastic spatial interpolation techniques (Christakos, 2001; and Szentimrey et. al., 2007).

### 4.1.1 Deterministic spatial interpolation techniques

Deterministic interpolation techniques create surfaces from the predictors by a mathematical function of the extent of similarity or the degree of smoothness (Webster and Oliver, 2001; Bhowmik and Cabral, 2011). In linear equation (4.i), the deterministic or local parameters are the expected values $(Z(s_i, t))\ (i - 0, \ldots\ldots, M)$. If $E(Z(t))$ denotes the vector of expected values of predictors, then the linear model for deterministic interpolation will be (Szentimrey et. al., 2007):

$$E(Z(t))^T = [E(Z(s_1,t)), \ldots\ \ldots, E(Z(s_M,t)) \qquad \textbf{(4.ii)}$$

The two climate change indices of the study can be modeled deterministically in the manner adopted by Hancock and Hutchinson (2005). The indices are considered as data observations $(z_i, x_{1i}, x_{2i}, \ldots\ldots, x_{di})$ measuring a dependent variable $z$ and predictor variables $x_1, x_2, \ldots\ldots, x_D$ which are included in a set of space domain $D$. These climate change indices are often well predicted using latitude, longitude and altitude. If $z$ has both continuous long range variation as well as discontinuous and random short range variation, then the data model can be expressed as:

$$z_i = g(x_1, x_2, \ldots\ldots, x_D) + \epsilon_i \quad (i = 1, \ldots\ldots, n) \qquad \textbf{(4.iii)}$$

Where, $n$ is the number of data observations, $g$ is a slowly varying continuous function and $\epsilon_i$ is the realization of a random variable $\epsilon$. The function $g$ represents the spatially continuous long range variation in the process measured by $z_i$. The errors of $\epsilon_i$ are assumed to be independent with mean zero and variance $\sigma^2$. This assumption is rooted in the measurement error and short range microscale variation that occurs over a range smaller than the resolution of the data set. The microscale variation may be spatially continuous, but the low spatial density of dataset (as discussed in literature review and study area, dataset and climate change indices chapters) is unable to represent it. That is why it is assumed as discontinuous noise of the data (TRENBERTH et al., 2000).

Therefore two deterministic approaches based on linear equation (4.ii) can be fitted to the data model of (4.iii). The Inverse Distance Weighting (IDW) approach predicts the dependent variable based on the extent of similarity, whereas the Thin Plate Spline (TPS) approach predicts it based on the degree of smoothing (JOURNEL and HUIJBREGTS, 1978).

#### *4.1.1.1 Inverse Distance Weighting*

The inverse distance weighting method predicts the process $g$ in (4.iii) by giving more weight to nearby measurements than to distant measurements. The analytical expression of the surface $f(x,y)$ can be expressed as (CARUSO and QUARTA, 1998):

$$f(x,y) = \frac{\sum_{i=1\ldots n} w(d_i) v_i}{\sum_{i=1\ldots n} w(d_i)} \qquad \textbf{(4.iv)}$$

where, $n$ is the number of measurements, $w_i$ is the measurement $i$ value, $d_i$ is the Euclidean distance with point $i$, and $w(d)$ is the weighting function. The weighting function $w(d)$ can be adjusted by the following formula:

$$w(d) = \begin{cases} \dfrac{1}{d_{min}^2}, & \text{if } d \le d_{min} \\ \dfrac{1}{d^2}, & \text{if } d_{min} < d < d_{max} \\ 0, & \text{if } d > d_{max} \end{cases} \qquad \textbf{(4.v)}$$

Where $d_{min}$ is minimum distance, $d_{max}$ is the maximum distance from the location being predicted. Index $d_{min}$ prevents infinite weight values for $d = 0$ . If no point falls into the circle of radius $d_{max}$ , average measurement value is taken (VICENTE-SERRANO et al., 2003).

Taking (4.iv) and (4.v) into account, the linear model 4(ii) can be written as following formula (SZENTIMREY et. al., 2007) for inverse distance weighting interpolation:

$$\hat{Z}_{\mathrm{IDW}}(s_0,t)=\frac{1}{\sum_{i=1}^{M}\lambda_i(s_0,t)}\sum_{i=1}^{M}\binom{M}{k}\lambda_i(s_0,t)\,Z(s_i,t)\text{, where, }\lambda_i(s_0,t)=\frac{1}{|s_0-s_i|^2} \quad \text{..(4.vi)}$$

*4.1.1.2 Thin Plate Spline*

The thin plate spline (TPS) method predicts the process $g$ in 4(iii) by a suitably continuous function $f$ that is able to separate the continuous signal $g$ from the discontinuous noise $\epsilon_i$ (HANCOCK and HUTCHINSON, 2005). This function can be estimated by minimizing

$$\frac{1}{n}\sum_{i=1}^{n}(z_i-f_i)^2+\lambda J_m^D(f) \quad \text{……………......…………(4.vii)}$$

over functions $f \in X$ , where $X$ is a space of functions whose partial derivatives of total order $m$ are in $L^2(E^d)$ (WAHBA, 1990; HANCOCK and HUTCHINSON, 2005). The $f_i$ are values of the fitted function at the $i$ th measurement, $\lambda$ is a fixed smoothing parameter, and $J_m^D(f)$ is a measure of the roughness of the function $f$ in terms of $m$ th order partial derivatives. The form of $J_m^D(f)$ depends on $m$ and the number of independent variables $D$ . For the typical value $m=2$ , $D=2$ , then $J_m^D(f)$ can be modeled as (CHRISTAKOS, 2001):

$$J_2^2(f)=\int_{-\infty}^{\infty} f_{x_1x_2}^2+2f_{x_2x_2}^2\,dx_1\,dx_2 \quad \text{……………........…(4.viii)}$$

Equation (4.viii) represents an exchange between fitting the data as closely as possible whilst maintaining a degree of smoothness (HANCOCK and HUTCHINSON, 2005). The smoothing parameter $\lambda$ controls the separation of long range and short range variation. If $\lambda=0$ , the function $f$ accurately interpolates the data, implying zero noise and when $\lambda$ is very large, the function approaches a hyperplane. The $\lambda$ corresponding to the spline function $f$ that best represents the underlying process $g$ can be predicted by minimizing the generalized cross validation (GCV) (CHRISTAKOS, 2001; and HANCOCK and HUTCHINSON, 2005) which is:

$$\frac{1}{n}\frac{(z-A(\lambda)z)^{T}(z-A(\lambda)z)}{\left(\frac{Tr\left(1-A(\lambda)\right)}{n}\right)^{2}} \quad \text{………...….........…........…(4.ix)}$$

$A(\lambda)$ is the matrix that transforms the vector of measurements into the vector of model predicted values. Therefore, the linear model (4.ii) for thin plate spline is:

$$\hat{Z}_{TPS}(s_0,t) = A(\lambda)Z(s_i,t) \quad \text{……….…......…..........…(4.x)}$$

Here, $\hat{Z}_{TPS}(s_0,t)$ is the vector of model predictedands and $A(\lambda)$ is thus termed as 'influence matrix' (HANCOCK and HUTCHINSON, 2005).

### 4.1.2 Variography of climate change indices

Covariance and correlation are two important measures of the similarity between two different spatio-temporal variables (CHRISTAKOS, 2001; HOULDING, 2000). Variography and the resulted variogram represent the measurement of spatial similarity in a similar fashion. They represent the correlation of measurement pairs of the same variable that are located in a certain distance from each other (BIVAND et al., 2008). The separation distance is known as 'lag', as used in temporal analysis. Thus the mathematical function of the semivariogram can be expressed as (JOURNEL and HUIJBREGTS, 1978):

$$\gamma\left(Z(s_i),Z(s_j)\right)=\frac{1}{2}\varepsilon\left[\{Z(s_i)-Z(s_j)\}^2\right] \quad \text{..............…..........…(4.xi)}$$

Where $Z(s_i)$ and $Z(s_j)$ are the dependent variables of interest at $i$ th and $j$ th locations, E is the statistical expectation operator. The important note is that the semivariogram, $\gamma()$ is a function of the separation between point vectors $Z(s_i)$ and $Z(s_j)$, and not a function of the specific location vector $Z(s_i)$ or $Z(s_j)$ (CHRISTAKOS, 2001). This mathematical definition is a useful abstraction and applied to the measured dependent variable by the formula for the experimental semivariogram:

$$\gamma\left(Z(s_i),Z(s_j)\right)=\frac{1}{2M\left(Z(s_i),Z(s_j)\right)}\sum_{(i,j)\in(Z(s))}(s_i-s_j)^2 \quad \text{............(4.xii)}$$

For the sake of simplicity, the expression variogram will be used instead of semivariogram. The variogram, as defined by geostatisticians, averages the squared differences of the variable and tends to filter the influence of a spatially varying mean (BIVAND et al., 2008). $\gamma\left(Z(s_i), Z(s_j)\right)$ is defined as the semivariance and this semivariogram can be applied whenever the first differences of the variable are second-order stationary and can be expressed as (SZENTIMREY et al., 2007):

$$\gamma\left(Z(s_i), Z(s_j)\right) = \gamma(s_i - s_j) \quad \text{(4.xiii)}$$

The form of stationarity in (4.xiii) is referred to as the intrinsic hypothesis, which is a weaker requirement than the second-order stationarity of the variable itself. Intrinsic stationarity means that the variogram varies only in function of distance, regardless of location. Eventually, the semivariogram can be defined in some cases where the covariance function cannot be defined, which is mostly the case for low sample density. In particular, the semivariance may keep increasing with increasing lag, rather than leveling off, corresponding to an infinite global variance. And, in such case, the covariance function is undefined (DEUTSCH, 2002).

A typical variogram has several components as presented in Figure 4.1.

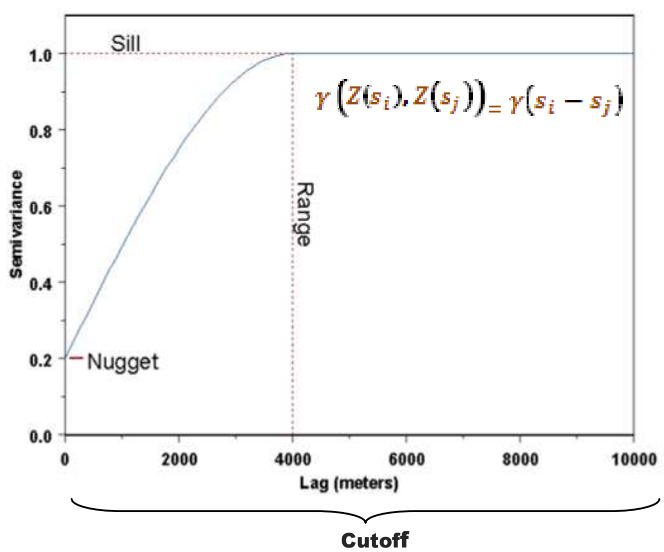



**Figure 4.1:** Components of a typical variogram (KELKAR and PEREZ, 2002).

The 'Sill' is the semivariance value at which the variogram levels off. The 'Range' is the lag distance at which the semivariogram reaches the sill value. Presumably, autocorrelation is essentially zero beyond the range. And, in theory, the semivariogram value at the origin (0 lag) should be zero. If it is significantly different from zero for lags very close to zero, then this semivariogram value is referred to as the 'Nugget'. The nugget represents variability at distances smaller than the typical sample spacing that may be caused by measurement error, which is the case for low sample density. ‘Cutoff’ is the maximum distance up to which pairs of points are considered to build the experimental variogram, and the width of the distance interval over which point pairs are averaged. This is a useful component for irregularly distributed measurements with low sample density, because in such case it is not expected to find many pairs of data values separated exactly by the lag distance for the whole study area. It is possible to set the extent of variography analysis to a certain limit of interest through the cutoff parameter and then analyze the semivariogram with proper lag distances (KELKAR and PEREZ, 2002; BIVAND et al., 2008).

#### *4.1.2.1 Variography with spatially shifted temporal measurement*

As discussed before, in case of low spatial density of measurement, the covariance function may not be defined and the number of pairs of measurements separated by the lag distance may not be enough to find in order to analyze the semivariogram. The temporal measurements in that case can be used to increase the number of pairs of measurements within the lag distance for semivariogram analysis. This approach is adopted from the pooled variogram analysis discussed by BIVAND et al. (2008) and also supported by the idea of ‘increasing features’ in analysis by (RAUDYS and JAIN 1991). It is named as ‘variography with spatially shifted temporal measurement’. In this analysis, the temporal measurements are distributed spatially to different sets of co-ordinates to prepare the sufficient sample size for variogram analysis (Figure 4.2). The sets are close enough to be considered as a whole study area for the creation of a variogram and simultaneously horizontal and vertical distances between every two sets are bigger than the maximum distance of measurements in the individual co-ordinates set. Given that the artificially created sets were too close to each other, one will get an uncontrolled temporal influence in the variogram. In pooled variogram analysis the temporal domain is included in a similar fashion by stacking the temporal measurements on top of each other. The vertical (temporal) distance from pooled measurements has to be rescaled in order to match the spatial distance in spatially shifted temporal measurement, which is the conversion from ‘temporal unit’ to ‘spatial unit’. Eventually, the limiting distance is given by the ‘cutoff’ parameter. The semivariogram function in such case can be expressed as:

$$\gamma\left(Z(Z(s_i),Z(s_j),t_i),Z(Z(s_i),Z(s_j),t_j)\right) = \frac{1}{2M\left(Z(Z(s_i),Z(s_j),t_i),Z(Z(s_i),Z(s_j),t_j)\right)} \sum_{(i,j)\in(Z(s,t))}\{((s_i-s_j)^2,t_i),((s_i-s_j)^2,t_j \text{..........(4.xiv)}$$

$Z(Z(s_i),Z(s_j),t_i),Z(Z(s_i),Z(s_j),t_j)$ is the vector of dependent variables in the temporal measurements of $t_i$ and $t_j$, which are considered as spatial measurements in different co-ordinates sets.

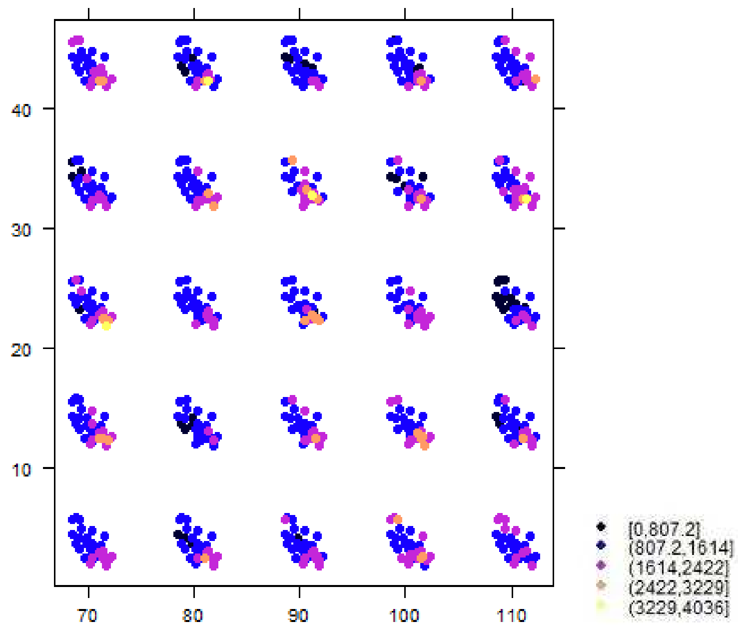
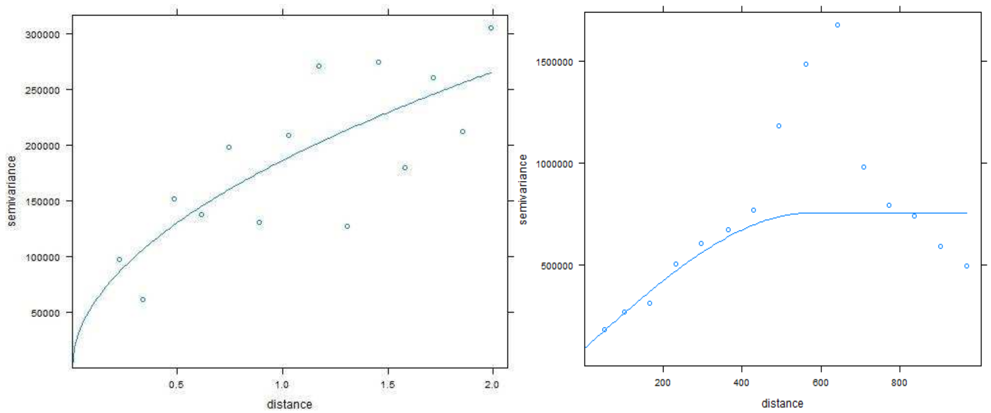



**Figure 4.2:** Spatially shifted temporal measurements (1956-1980) to different sets of co-ordinates of the index PRCPTOT.

The variography from spatially shifted temporal measurements improves the outcome in the variogram with reduced residuals. In such case the experimental variogram is far more structured to perform the variography analysis (Figure 4.3). The experimental variogram in Figure 4.3 (a) corresponds to the samples of the index PRCPTOT in 1956 over the study region, whereas the experimental variogram in Fig. 4.3(b) corresponds to the spatially shifted temporal measurements of the same index from 1956 to 1980 as shown in Figure 4.2. Thus the problem of low sample density can be diminished in the design of variography.




(a) (b)

**Figure 4.3:** Fitted variogram of the index PRCPTOT with (a) single temporal measurement at spatial points in 1956; and (b) spatially shifted temporal measurements (1956-1980).

### 4.1.3 Stochastic or geostatistical spatial interpolation

Stochastic or geostatistical spatial interpolation techniques exploit the statistical properties of the measured points and quantify the spatial autocorrelation among measured points. Furthermore, they account for the spatial configuration of the sample points around the prediction location (CHRISTAKOS, 2001; PAPRITZ and STEIN, 1999; and SEAMAN, 1983). Stochastic or geostatistical spatial interpolation is based on the covariance, or the variogram, between the dependent variables and the predictors. Important parameters (SZENTIMREY et. al., 2007) are:

- Dependent-predictors covariance vector expressed as $c$
- Predictors-predictors covariance matrix expressed as $C$
- Dependent-predictors variogram vector expressed as $\gamma$ and
- Predictors-predictors variogram matrix expressed as $\Gamma$

Thus the linear model of (4.i) between $Z(s_0,t)$ and predictors $Z(t)$ can be written for stochastic interpolation as:

$$\hat{Z}(s_0,t) = (\mu(t) + E(s_0)) + c^T C^{-1} (Z(t) - (\mu(t) l + E)) \text{ ..............(4.xv)}$$

Here, $E^T = [E(s_1), \ldots\ldots, E(s_M)]$ and $l^T = [1, \ldots\ldots 1]$ are identical. Obviously, the main problem lies in the estimation of the unknown climate change signal $\mu(t)$ in case the optimal linear interpolation model was applied (SZENTIMREY et. al., 2007). Equation (4.xv) can be simplified as follows:

$$\hat{Z}(s_0,t) = E(Z(s_0,t)) + c^T C^{-1}\left(Z(t) - E(Z(t))\right) \quad \textbf{(4.xvi)}$$

$\hat{Z}(s_0,t)$ is the best linear estimation that minimizes the mean-squared prediction error. Consequently, the linear model would be the optimal linear interpolation formula concerning the mean-squared prediction error (CHRISTENSEN, 1990; PAPRITZ and STEIN, 1999). However, with respect to the application, problems arise from the unknown statistical parameters $E(Z(s_0,t))$ $(i = 0, \ldots\ldots, M)$ and $c$, $C$. The covariance parameters can be replaced by variogram or semivariogram parameters $\gamma$ , $\Gamma$ (CARUSO and QUARTA, 1998).

Two stochastic approaches based on linear equation (4.xv) and (4.xvi) have been fitted. The first approach is Ordinary Kriging, which accounts for any spatial trend present in the attribute through the local kriging system, and also considers anisotropy. The final approach is Universal Kriging which explicitly models the trend component as a linear combination of functions of the spatial coordinates (VICENTE-SERRANO et al., 2003).

#### *4.1.3.1 Ordinary Kriging*

Ordinary kriging (OK) relies on the spatial correlation structure of the measurements to determine the weighting values. This is a more rigorous approach to modeling, as the correlation between measurement points determines the estimated value at an unsampled point (HOULDING, 2000). The assumed model for the expected values is $E(Z(s_i,t)) = \mu(t)$ $(i = 0, \ldots\ldots, M)$, thus there is no spatial trend. The generalized least-squares estimation for $\mu(t)$ by using only the predictors $Z(t)$ may be expressed in the form $\hat{\mu}._{gls}(t) =$ $(l^T C^{-1} l)^{-1} l^T C^{-1} Z(t)$ (SZENTIMREY et. al., 2007). By substituting the estimate $\hat{\mu}._{gls}(t)$ into the stochastic formula (4.xv), the ordinary kriging formula can be expressed as (WEBSTER and OLIVER, 2001):

$$\hat{Z}._{OK}(s_o,t) = \hat{\mu}._{gls}(t) + c^T C^{-1}(Z(t) - \hat{\mu}._{gls}(t)l) = \sum_{i=1}^{M} \lambda_i Z(s_i,t)$$

$$where \sum_{i=1}^{M} \lambda_i = 1 \quad \textbf{(4.xvii)}$$

The vector of weighting factors $\lambda^T = [\lambda_1, \ldots\ldots, \lambda_M]$ can be expressed as covariance form (SZENTIMREY et. al., 2007):

$$\lambda^T = \left(c^T + 1^T \frac{(1 - 1^T C^{-1} c)}{1^T C^{-1} 1}\right) C^{-1}$$ **..................................(4.xviii)**

Or, equivalently as variogram form:

$$\lambda^T = \left(\gamma^T + 1^T \frac{(1 - 1^T \Gamma^{-1} \gamma)}{1^T \Gamma^{-1} 1}\right) \Gamma^{-1}$$ **..................................(4.xix)**

### *4.1.3.2 Universal Kriging*

Universal Kriging (UK) is defined as kriging with changing mean where the trend is modeled as a function of coordinates. Thus there is an existing spatial trend in the model and therefore the universal kriging formula is the generalized case of the ordinary kriging formula (KASTELEC and KOŠMELJ, 2002). According to the model assumption, the universal kriging formula can be expressed as (SZENTIMREY et. al., 2007):

$$E(Z(s_i, t)) = \sum_{k=1}^{K} \beta_k(t) x_k(s_i) \; (i = 0, \ldots\ldots, M)$$ **................(4.xx)**

Which can be expressed in the vector form:

$$E(Z(s_o, t)) = x^T \beta(t), \qquad E(Z(t)) = X\beta(t)$$ **.......................(4.xxi)**

Here, $x$ , $X$ are given supplementary deterministic model variables. The generalized least-squares estimation for the coefficient vector $\beta(t)$ by using only the predictors $Z(t)$ can be expressed in the form $\hat{\beta}_{\cdot gls} = (X^T C^{-1} X)^{-1} X^T C^{-1} Z(t)$ (KASTELEC and KOŠMELJ, 2002; and SZENTIMREY et. al., 2007).

The spatial trend $E(s)$ can also be modeled by using only the predictors $Z(t)$. By substituting the estimates $x^T \hat{\beta}_{gls}(t)$, $X\hat{\beta}_{gls}(t)$ into the linear regression (CHRISTENSEN, 1990) formula (4.xvi), the universal kriging formula can be expressed as:

$$\hat{Z}_{\cdot UK}(s_o, t) = x^T \hat{\beta}_{gls}(t) + c^T C^{-1} \left(Z(t) - X\hat{\beta}_{gls}(t)\right)$$

$$= \sum_{i=1}^{M} \lambda_i Z(s_i, t), where \; \lambda^T X = x^T$$ **........................(4.xxii)**

The vector of weighting factors $\lambda^T = [\lambda_1, \ldots\ldots \lambda_M]$ can be expressed as covariance form:

$$\lambda^T = \{c + X(X^T C^{-1} X)^{-1}(x - X^T C^{-1} c)\}^T C^{-1} \text{....................} \mathbf{(4.xxiii)}$$

Or, equivalently as variogram form:

$$\lambda^T = \{\gamma + X(X^T \Gamma^{-1} X)^{-1}(x - X^T \Gamma^{-1} \gamma)\}^T \Gamma^{-1} \text{.....................} \mathbf{(4.xxiv)}$$

The unknown variogram values $\gamma$ , $\Gamma$ are modeled in the variography (BIVAND et al., 2008).

## 4.2 Evaluation of spatial interpolation techniques

The performance and effectiveness of each of the spatial interpolation techniques can be evaluated using the 'Fictitious-point method', which is popularly known as 'Cross-validation'. It was first proposed by SEAMAN (1983) and subsequently applied by geostatisticians in a whole range of studies for evaluating the performances of the spatial interpolation techniques (BHOWMIK and CABRAL, 2011, CARUSO and QUARTA, 1998; CHRISTAKOS, 2001; CHRISTENSEN, 1990; HANCOCK and HUTCHINSON, 2005; HASLETT et.al., 1989; HIJMANS et. al, 2005; PAPRITZ and STEIN, 1999; TRENBERTH et. al., 2000; and WEBSTER and OLIVER, 2001). Validation is a statistical method of evaluating and comparing learning algorithms by dividing data into two segments: one segment is used to learn or train a model and the other one is used to validate the model. In typical cross-validation, the training and validation sets must cross-over in successive rounds so that each data point has a chance of being validated against. The basic form of cross-validation is k-fold cross-validation (KOHAVI, 1995). In geostatistics, generally, the training set is the measured values of all sample points, which are validated using the appointed spatial interpolation technique. It is typically known as 'leave-one-out' cross-validation, since each sampled point is taken out successively and then estimated using the remaining measured points of the same sample set (BIVAND et al., 2008). An ideal cross-validation plot should give the points plotted along the $45^0$ line with the axes defined by the measured and predicted values. In practice, the predicted values differ from the observed values and several errors of prediction can be calculated from the residuals. These errors indicate the quality of prediction by a spatial interpolation technique (CHILES and DELFINER, 1999). Most importantly, the cross-validation results are used by the geostatisticians to show the ‘smoothness improvement’ in the ‘ratio of the variance of estimated values to the variance of observed values’ (HABERLANDT, 2007).

### 4.2.1 Willmott statistics

Willmott statistics use five ‘difference-measures’ of errors from cross-validation that are useful for evaluating the performance of the interpolation methods. The statistics are proposed by WILLMOTT (1984) based on the principle that the statistical measures for a particular spatial interpolation approach should not be over analysed. The five measures are: 1) Mean Absolute Error ($MAE$), 2) Root Mean Square Errors ($RMSE$), 3) Systematic Root Mean Square Errors ($RMSE_s$), 4) Unsystematic Root Mean Square Errors ($RMSE_u$), and 5) the Index of Agreement ($d$). Equations from Willmott statistics are given below:

$$MAE = N^{-1}\sum_{i=1}^{N}|P_i - O_i| \qquad \text{(4.xxv)}$$

$$RMSE = N^{-1}\sum_{i=1}^{N}[(P_i - O_i)^2]^{-\frac{1}{2}} \qquad \text{(4.xxvi)}$$

$$\hat{P} = a + bO_i \qquad \text{(4.xxvii)}$$

$$RMSE_s = N^{-1}\sum_{i=1}^{N}\left[\left(\hat{P}_{\cdot i} - O_i\right)^2\right]^{-\frac{1}{2}} \qquad \text{(4.xxviii)}$$

$$RMSE_u = N^{-1}\sum_{i=1}^{N}\left[\left(P_i - \hat{P}_{\cdot i}\right)^2\right]^{-\frac{1}{2}} \qquad \text{(4.xxix)}$$

$$PE = \sum_{i=1}^{N}(|P_i - O| + |O_i - O|)^2 \qquad \text{(4.xxx)}$$

$$d = 1 - \frac{N * RMSE^2}{PE} \qquad \text{(4.xxxi)}$$

Here, $O_i$ and $P_i$ are observed and predicted variable values at $i$ th location, respectively. The ordinary least-squares (OLS) simple linear regression coefficients of $a$ and $b$ are used to compute the difference measures - systematic and unsystematic root mean square errors ($RMSE_s$, $RMSE_u$). $MAE$ is sometimes preferred over the $RMSE$ as an evaluator for it is less sensitive to extreme values; however, $RMSE$ is the error measure commonly computed in geographic applications. The systematic $RMSE_s$ assesses whether the model errors are predictable, whereas the unsystematic $RMSE_u$ identifies those errors that are not predictable mathematically. The final error measure, $\mathbf{d}$, varies between 0.0 and 1.0. Therefore, the closer $\mathbf{d}$ is to 1.0 the better is the agreement between $O$ and $P$ with 1.0 conveying perfect agreement and 0.0 complete disagreements (BHOWMIK and CABRAL, 2011).

### 4.2.2 Confidence of prediction

The idea of confidence of prediction was introduced by CHILES and DELFINER (1999). This is the subtraction of the coefficient of variation of prediction from 100 (DIRKS et al., 1998). The coefficient of variation of prediction $\rho_f$ can be expressed as:

$$\rho_f = \frac{\mathrm{RMSE}}{\frac{1}{n}\sum_{i=1}^{n} O_i} \qquad \textbf{(4.xxxi)}$$

$\rho_f$ is expressed in percentage. It is the measure of spatial uncertainty of prediction (CHILES and DELFINER, 1999) and is also used to compare the performance of interpolation schemes for different integration times. Thus the formula for confidence of prediction can be expressed as:

$$\mathrm{CP} = 100 - \rho_f \qquad \textbf{(4.xxxii)}$$

## 4.3 Chapter conclusion

This chapter has described the applicability, inherent methods and importance of the four spatial interpolation techniques and seven performance measurements in respect to the study. It has also illustrated the procedure of spatially shifted years approach to minimize the low sample density impact in designing the variography for the stochastic spatial interpolation. The next chapter, results and discussion, will discuss and present the results that will be obtained applying the methods described in the methodology chapter.

# 5. RESULTS AND DISCUSSION

This chapter discusses and presents the results obtained applying the methodology described in the methodology chapter. It explores and analyzes the behavior and trend of performances of the spatial interpolation techniques with the change in sample density. It also validates the result with discussion in light of existing theories and reasoning.

## 5.1 Search Neighborhood

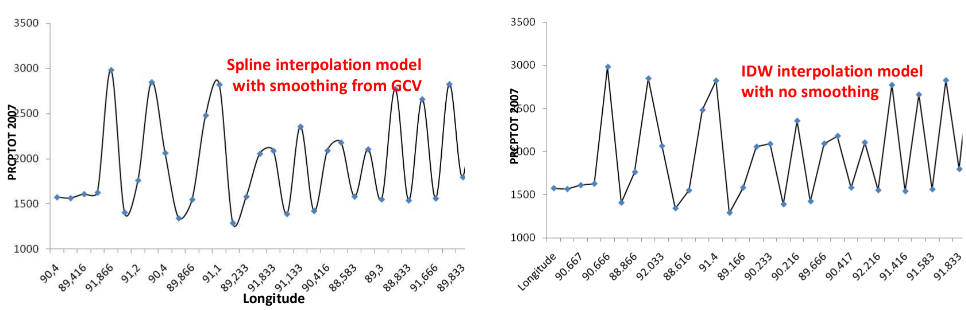
Search neighborhood basically refers to the shape of the neighborhood and the constraints of the measured points within the neighborhood that is used in the prediction of an unmeasured location. It is generally assumed in case of spatial interpolation that the farther the measured point gets from the prediction point, the less spatial autocorrelation the measured values will have to the prediction points. It is also possible that distant points may bring a detrimental effect to the predicted value if they are located in a region that has different characteristics of the phenomenon than those of the prediction location. Consequently, it is always important to define a search neighborhood formed with the nearest feasible points for interpolation. It is also important that the search neighborhood should be defined in designing variography and the same neighborhood should be used in interpolation applying this variogram (AUCHINCLOSS, et al. 2007; and WEISZ, et al., 1995). For this study, the search neighborhood has been defined using the classical method of limiting the number of neighbors utilized for the prediction by defining the maximum neighbor points (nMax) and minimum neighbor points (nMin) parameters.

Due to the low sample density of the study region, it is difficult to model the variogram in short range, rather a long ranged variation is possible to be modeled (as described in chapter 4) – typically the phenomenon behavior all over the study area is known. In addition, it is difficult to find enough points to build the experimental variogram and model it, and to interpolate the attribute data if a small search neighborhood is defined. Considering these facts, the whole study area has been defined as the search neighborhood for prediction of values at unknown locations. It has also been explored from the analysis of the indices behavior that the indices vary smoothly all over the study area and there is a spatial trend in the index behavior. In addition, maintaining the degree of smoothing is also the basic purpose of defining a search neighborhood and this can be maintained for the whole study region. Therefore, the nMax and nMin for the interpolation of the index PRCPTOT have been set to 32 and for the interpolation of TXx, they have been set to 34. Thus all the measured points have been used in the variography stage and for predicting the indices at unmeasured locations (Figure 5.1).


**Figure 5.1:** Search Neighborhood for PRCPTOT estimation (nMax=32, nMin= 32) and for TXx estimation (nMax=34, nMin=34).

## 5.2 Deterministic spatial interpolation results

As described in chapter 4, two deterministic methods have been used in this study – IDW, which predicts spatially continuous long range variation by giving more weight to nearby measurements than to distant measurements and TPS, which predicts the variation by a suitably continuous smoothing spline function. There are distinguishing features of these two methods, which characterize their performances in estimating continuous surfaces. TPS considers the spatial dependence of the phenomenon as a spline smoothed from the GCV function (Figure 5.2 (a)), whereas IDW considers the spatial dependence as straight lines connecting the measured points without any smoothing (Figure 5.2(b)) (CARUSO and QUARTA, 1998). The non-smoothing property of IDW sometimes enables it to result in better cross-validation results, since it takes the actual measurement as variability factors. Yet, it is also necessary to consider that it includes the measurement errors into the estimated values, which are unknown in most cases (BABAK and DEUTSCH, 2009). Smoothing property and taking the long range and short range variability into consideration, enables TPS to perform better for the small sampled region.




**(a)** **(b)**

**Figure 5.2:** Inherent interpolation model by (a) thin plate spline method (b) inverse distance weighting method.

Based on these fundamental features of each model (Figure 5.2), the two deterministic methods have been applied to create the continuous surfaces of PRCPTOT and TXx indices.

### 5.2.1 Thin plate spline surfaces

TPS surfaces have been created for PRCPTOT and TXx indices applying the the search neighborhood described in section 5.1 and the methodology described in chapter 4. As a result of the inherent smoothing properties, the resulted surfaces are smoothed and present the long range variability of the phenomena. The TPS surfaces of PRCPTOT and TXx are presented in the Annex A.1 and Annex A.9 respectively. A range of 0-4500mm of PRCPTOT has been calculated over the time periods and presented in the legends of created surfaces to maintain conformity. From the PRCPTOT surfaces, the spatial trend of the index is apparent, in all years the highest PRCPTOT has been experienced in the southeastern part of the country. Though, the surfaces of 1965, 1974, 1987, 1985 and 2002 present higher PRCPTOT in the northwestern part of the country, which is contrary to the general spatial trend. In 1984 and 2004, a comparatively higher PRCPTOT has been experienced almost all over the country than in the other years, although it is hard to predict any temporal trend in the surfaces for PRCPTOT. From the residual plots of PRCPTOT, presented in Annex A.5, it is clear that the spline method typically over estimated the values in prediction. The method performance in terms of prediction has been improved over time with the increase in the station availability, which can be proven by the decreased size of the residual circles. On the other hand, as described in chapter 2, the spatial trend of TXx is the opposite of PRCPTOT, which has also been represented by the spline surfaces in the Annex A.9. The surfaces represent a spatial pattern of higher TXx in the northwestern part of the country, which is dominant in almost all the years. But in 1952 and 2004, a higher TXx has been calculated in the northeastern part of the country; especially in 2004 it is significantly higher than the temporal average. The relative higher TXx has been experienced all over the country in 1960, though it is difficult to derive any temporal trend from the surfaces of TXx. Over the years, a range of 20-65$^0$C of TXx has been calculated and maintained in the legend of the surfaces. Like PRCPTOT, the residual plots of TPS surfaces of TXx show that the TPS method has typically over estimated the values and the prediction has been improved with time in terms of residuals.

### 5.2.2 Inverse distance weighting surfaces

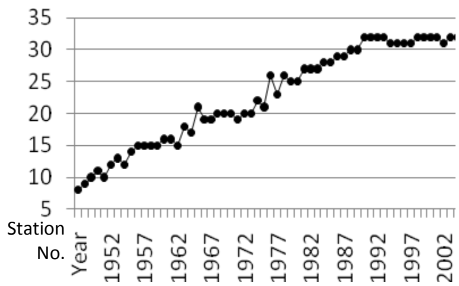
The effect of the property of the global information carried by the stationary mean in case of low density of sample (GOOVAERTS 1997), which has been described in chapter 2, has been represented by the IDW surfaces for both PRCPTOT and TXx indices. They are presented in Annex A.2 and A.10 respectively. Rather than presenting the long range variability of the indices, IDW represents more variability close to the sample location in a circular manner and shows almost no variation in distance. This is identified by the 'Bull's eyes' that appeared in the surfaces in every station location, which are expected from the modelling features of IDW described in section 5.2. Consequently, no clear spatial trend has been recognized from the surfaces; but rather some higher values have been recognized close to the areas of the representative stations. PRCPTOT surfaces of 1951,1958, 1968,1969,1975 and 1981 show very low values of PRCPTOT in some station regions located in the northern and western part of the country. On the other hand, PRCPTOT surfaces of 1959-2007 in most cases have shown a very high value of PRCPTOT in the station regions at northeastern and southeastern parts of the country. PRCPTOT surfaces of 1984 and 1987 have shown high values of PRCPTOT almost all over the country. Residual plots of PRCPTOT surfaces using IDW presented in Annex A.6 have proven that the IDW method has typically under estimated PRCPTOT values in prediction. The TXx surfaces presented in Annex A.10, have followed the similar fashion as for PRCPTOT surfaces, the spatial trend is not clearly represented, rather the short range variation of the phenomenon is predictable. Though the surfaces of 1954, 1956, 1958 and 1960 have dominantly represented higher TXx values in the northeastern region of the country, this is not visible in the surfaces from other years. Again the TXx surface of 1960 has represented the higher TXx value all over the country and surfaces of 2003 and 2004 have represented a very high TXx value in the station regions located south and southeastern parts of the country respectively. Also the residual plots of TXx surfaces using IDW presented in Annex A.14 show that the IDW method has typically under estimated the TXx values in prediction.

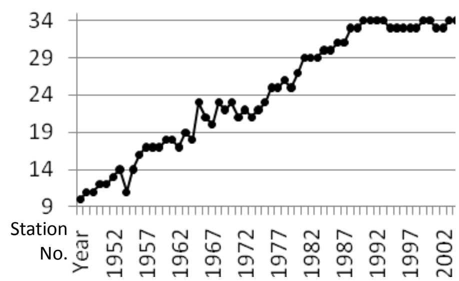
## 5.3 Stochastic spatial interpolation results

The principle of stochastic methods for which they are distinguished from the deterministic methods is that they do not design the variography exactly through the measured points. Rather they describe a smoothed variography over the area of interest and thus try to fit a smoothed line, which describes the continuous variable behavior over the region of interest (CHRISTAKOS, 2001).

### 5.3.1 Variography

As described in chapter 4, the variography for interpolation using stochastic spatial interpolation techniques has been designed based on the principle of spatially shifted temporal points. For this purpose, the temporal points have been distributed in temporal periods so that the variography has more or less same number of spatial points to describe the mean variogram. Obviously, the number of stations or spatial points has shown an increasing trend over the years for both PRCPTOT and TXx indices (Figure 5.3).

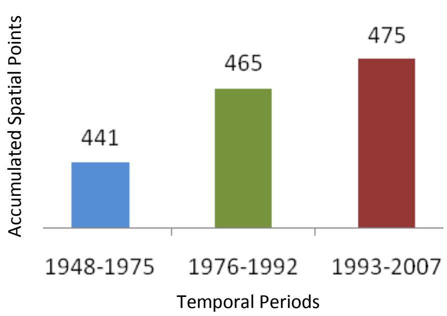


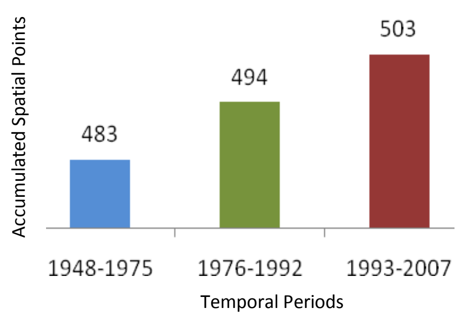




**(a)** **(b)**

**Figure 5.3:** Increasing trend of number of stations available for interpolating (a) PRCPTOT and (b) TXx.

Therefore, the total number of years has been distributed into three temporal periods and thus the number of spatial points that have been accumulated from such distribution is 441, 465 and 475 for PRCPTOT variography and 483, 494 and 503 for TXx variography (Figure 5.4).




**(a)** **(b)**

**Figure 5.4:** Number of accumulated spatial points from spatially shifted years to design variography for (a) PRCPTOT and (b) TXx.

The accumulated spatial points have been distributed to 28, 17 and 15 different spatial coordinate sets for both PRCPTOT and TXx (Figure 5.5) and thus three mean variograms have been produced for the temporal periods of 1948-1975, 1976-1992, 1993-2007 respectively for PRCPTOT and three for TXx using the same temporal periods but a different number of spatial points as described in Figure 5.5. It has been ensured, while distributing the temporal points that the distances between the coordinate sets are more than 550km, which is the maximum distance between two coordinates in a particular coordinate set. This has prevented the particular coordinate set to have a temporal effect on the other set in designing the mean variogram.

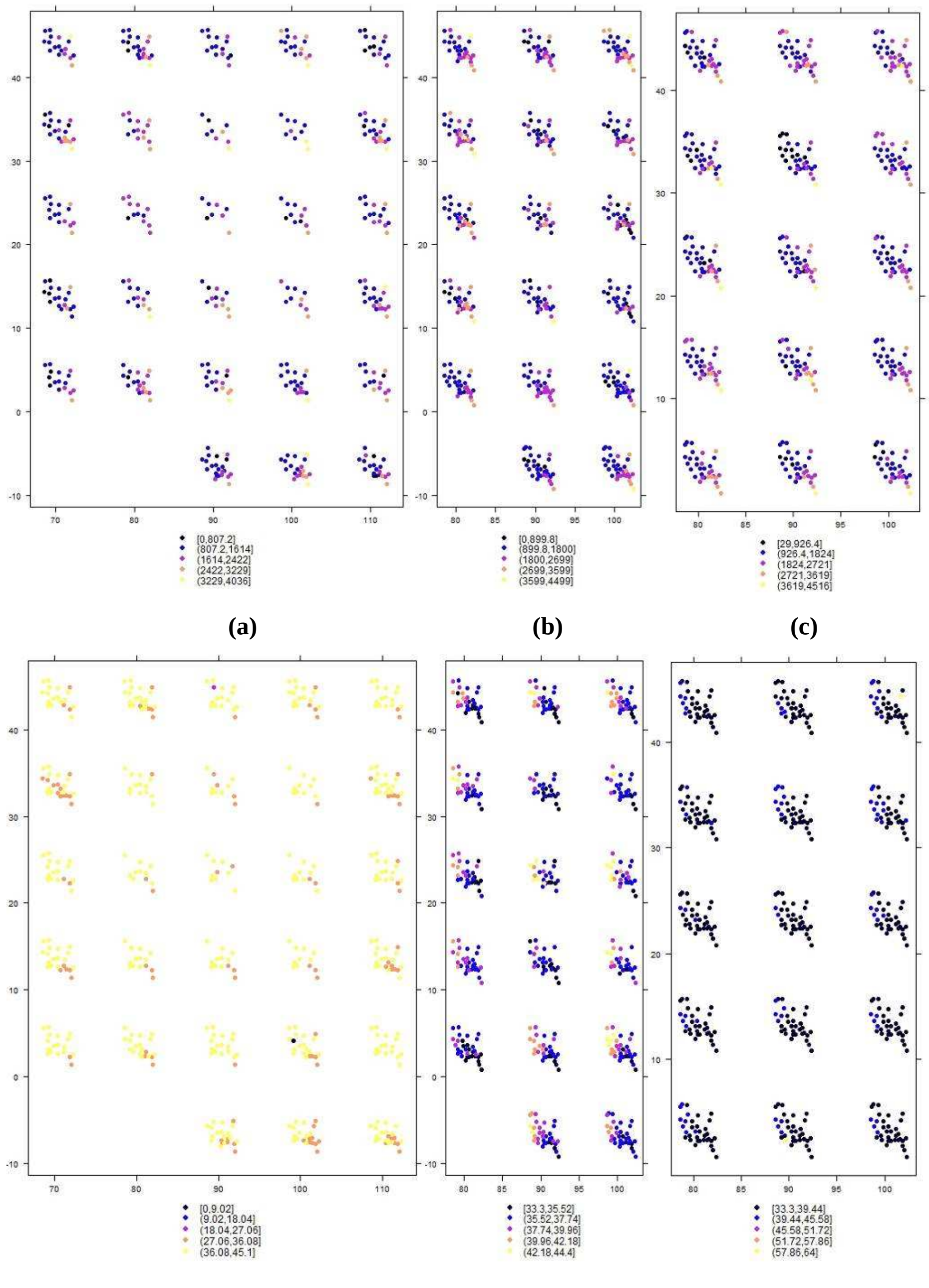


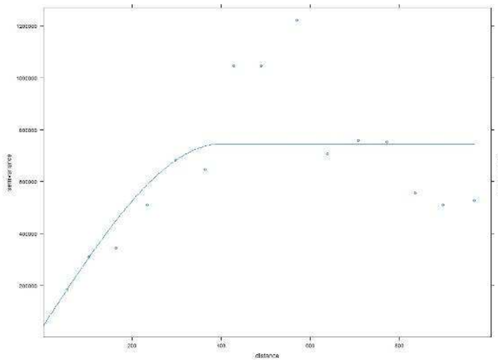
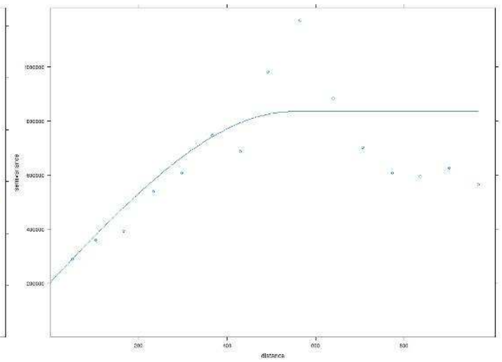

**Figure 5.5:** Spatially shifted temporal points set of PRCPTOT for the temporal periods of (a) 1948-1975 (b) 1976-1992 and (c) 1992-2007 and of TXx for the temporal periods of (d) 1948-1975 (e) 1976-1992 and (f) 1992-2007.

Thus six mean variograms have been designed for interpolating PRCPTOT and TXx using the ordinary kriging method. The variogram parameters have been presented in Table 5.1. The anisotropy has been analyzed using the 'intamap' statistical package of 'R' and has been modeled in the variogram of interest. The anisotropy for the study region is not very dominant for both indices since they have resulted in higher ratios of major and minor axes of the anisotropy ellipse (Table 5.1). The variograms for interpolating PRCPTOT have resulted in high nugget values, which represent abrupt changes in the PRCPTOT values in short distance. Variograms for TXx resulted in no nugget for the temporal periods of 1948-1975 and 1976-1992, whereas for 1993-2007 there is a nugget value of 1, which is high for this temporal period. The variogram for TXx of the temporal period of 1993-2007 also represents a lower sill value, while the variogram for PRCPTOT of the 1976-1992 period represents a higher sill value. These values show that the TXx values did not vary to a great extent during 1993-2007, while the PRCPTOT values varied to a high extent during 1976-1992 over the study region. Their ranges are similar to the extent of the whole country.

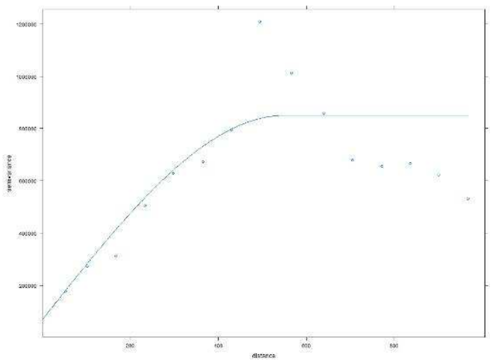
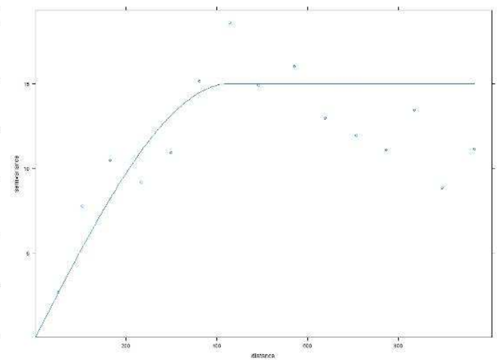
**Table 5.1:** Variogram parameters estimated from the spatially shifted temporal points set of the experimental variograms of PRCPTOT and of TXx for three temporal periods for ordinary kriging interpolation.

| Index | Temporal period | Total Spatial Points | Variography | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | Model | Sill | Range | Nugget | Anisotropy Parameters | |
| | | | | | | | Angle | Ratio of major and minor axis |
| **PRCPTOT** | 1948-1975 | 441 | Spherical | 700000 | 400 | 45000 | 2.96 | 0.83 |
| | 1976-1992 | 465 | Spherical | 630000 | 550 | 205000 | 47.28 | 0.97 |
| | 1993-2007 | 475 | Spherical | 780000 | 550 | 70000 | 83.94 | 0.87 |
| **TXx** | 1948-1975 | 483 | Spherical | 15 | 430 | 0 | 2.96 | 0.71 |
| | 1976-1992 | 494 | Spherical | 10 | 530 | 0 | 0.96 | 0.71 |
| | 1993-2007 | 503 | Spherical | 5.8 | 410 | 1 | 176.72 | 0.73 |

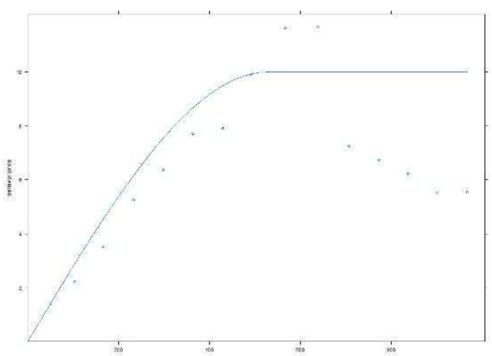
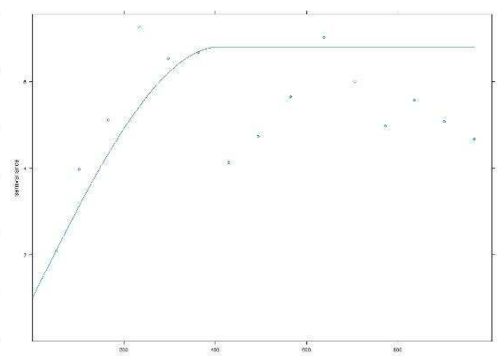
The resulting variograms based on Table 5.1 parameters are presented in Figure 5.6.

(a) (b)

(c) (d)

(e) (f)

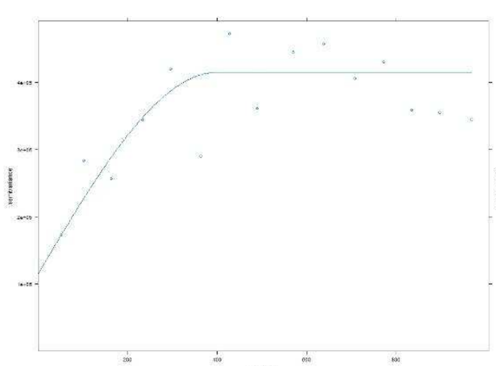
**Figure 5.6:** Mean variograms based on the parameters described in Table 5.1 using spatially shifted temporal points set of PRCPTOT for the temporal periods of (a) 1948-1975 (b) 1976-1992 and (c) 1992-2007 and of TXx for the temporal periods of (d) 1948-1975 (e) 1976-1992 and (f) 1992-2007 for ordinary kriging interpolation.

Another six mean variograms have been designed for interpolating PRCPTOT and TXx using the universal kriging method. The fundamental principle behind designing these variograms is that both PRCPTOT and TXx indices behavior over the study region are functions of longitude and latitude, which indicates that there is a spatial trend in the indices behavior. Variogram parameters considering spatial trend are presented in Table 5.2. The variograms for interpolating both PRCPTOT and TXx have resulted in high nugget values for all time periods that represent abrupt changes in the indices values in short distance considering spatial trend. The variogram for TXx of the temporal periods of 1976-1992 and 1993-2007 both represent lower sill values, which indicate that TXx values did not vary to a great extent during the mentioned temporal periods. On the other hand the sill values of PRCPTOT variograms are similar over time (Table 5.2). Their ranges are again similar to the extent of the whole study region.

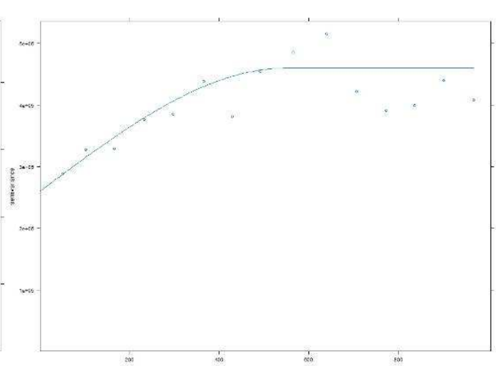
**Table 5.2:** Variogram parameters estimated from the spatially shifted temporal points set of the experimental variograms of PRCPTOT and of TXx for three temporal periods for universal kriging interpolation.

| Index | Temporal period | Total Spatial Points | Dependent Parameters | Variography | | | |
|---|---|---|---|---|---|---|---|
| | | | | Model | Sill | Range | Nugget |
| **PRCPTOT** | 1948-1975 | 441 | Longitude and Latitude | Spherical | 300000 | 400 | 115000 |
| | 1976-1992 | 465 | Longitude and Latitude | Spherical | 200000 | 550 | 260000 |
| | 1993-2007 | 475 | Longitude and Latitude | Spherical | 270000 | 500 | 136000 |
| **TXx** | 1948-1975 | 483 | Longitude and Latitude | Spherical | 10 | 420 | 1 |
| | 1976-1992 | 494 | Longitude and Latitude | Spherical | 2.1 | 480 | 1 |
| | 1993-2007 | 503 | Longitude and Latitude | Spherical | 3.9 | 450 | 1.4 |

The fitted variograms based on the parameters shown in Table 5.2 are presented in Figure 5.7.



**(a)**



**(b)**

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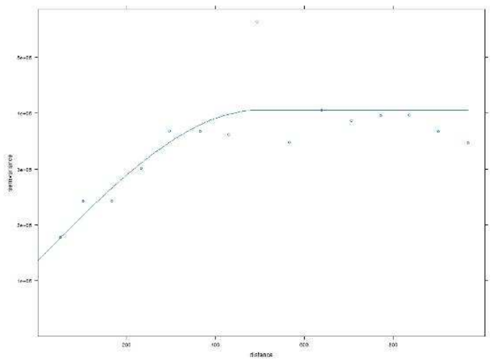
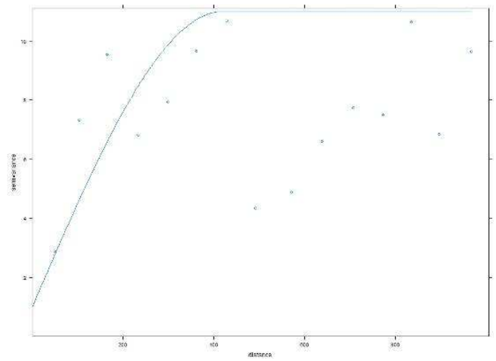
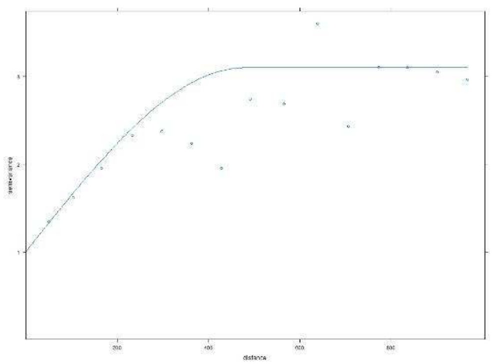
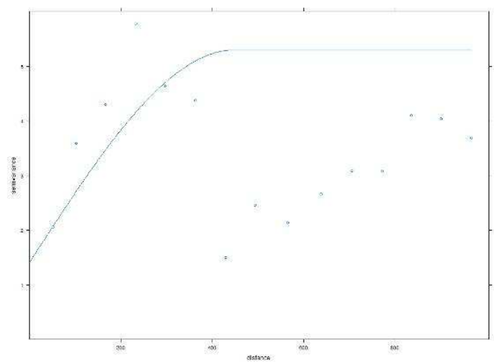




**(c)** **(d)**





**(e)** **(f)**

**Figure 5.7:** Mean variograms based on the parameters described in Table 5.2 using spatially shifted temporal points set of PRCPTOT for the temporal periods of (a) 1948-1975 (b) 1976-1992 and (c) 1993-2007 and of TXx for the temporal periods of (d) 1948-1975 (e) 1976-1992 and (f) 1993-2007 for universal kriging interpolation.

The similar ranges of the variograms equal to the extent of the whole study region indicate that both the size and density of the sample is insufficient to describe the variability of the indices in short range, especially in between the station locations. The problem is addressed in the discussion section in detail. It is obvious that designing experimental variograms provided with enough spatial pairs of points from the temporal points to model the spatial continuity in long range has reduced the uncertainty of modelling to a great extent. Spherical models seem most suitable for all variograms, which also represent a spherical change of the indices over the study region.

### 5.3.2 Ordinary kriging surfaces

The ordinary kriging method has been applied using the methodology described in chapter 4 and variography described in section 5.3.1. The produced surfaces of PRCPTOT and TXx are presented in Annex A.3 and A.11 respectively. The surfaces described well the indices behavior – both in terms of spatial trend and anisotropy. The PRCPTOT values range from 0 to 4500mm and TXx values range from $20^0$ to $65^0$C, and similar legends have been maintained for comparison. Again it is difficult to predict any temporal trend from the surfaces. PRCPTOT surfaces of 1950, 1954, 1961, 1965, 1998 and 1999 represent very high PRCPTOT values in the southeastern part of the country, and PRCPTOT surfaces of 1964, 1968, 1970, 1974, 1976, 1988 and 1989 represent very high PRCPTOT values in the northeastern corner of the country. PRCPTOT surfaces of 1956 and 1993 represent higher PRCPTOT values in the eastern part of the country while PRCPTOT surfaces of 1984, 1987, 2004 and 2007 represent higher PRCPTOT values i.e. higher monsoon rainfall all over the country. Residual plots for PRCPTOT surfaces using ordinary kriging, represented in Annex A.7, describe that the ordinary kriging method has typically under estimated the PRCPTOT values in prediction. Yet, the prediction performance is better than the deterministic methods, since the size of the circle of difference between measured and predicted values is smaller. Like PRCPTOT surfaces, TXx surfaces produced by ordinary kriging do not describe any temporal trend in particular, but the spatial trend to the northwestern part of the country is clearly represented by them, especially by the surfaces of 1954-2007. Though the TXx surfaces of 1956, 1958, 1961, 1972, 1976, 1979 and 1989 represent the higher TXx values in the whole western part of the country, the spatial trend is visible along with anisotropy. TXx surface of 2003 represents very high TXx values in the southwestern corner of the country, the surface of 2004 shows very high TXx values in the northeastern part of the country. Furthermore, the surface of 1960 represents higher TXx values all over the country. The residual plots for TXx surfaces using ordinary kriging, presented in Annex A.15, show that the ordinary kriging method has typically over estimated the TXx values in prediction but the prediction performance is also better than the performance of the deterministic methods since the size of the circle of difference between measured and predicted values is smaller. Ordinary kriging surfaces of both PRCPTOT and TXx characterize the long range variability in the index behavior.

### 5.3.3 Universal kriging surfaces

The tilted surfaces of universal kriging have been created using the methodology described in chapter 4 and the variograms designed in section 5.3.1 and are presented in Annex A.4 and A.12. The basic principle, which has been applied to create these surfaces, is the fact that there is a spatial trend in indices behavior, which is also clearly visible in the created surfaces. The spatial trend is more dominant in the east-west direction than the north-south as expected from the correlation with longitude and latitude described in chapter 3. PRCPTOT surfaces of 1948, 1949, 1950, 1953, 1954, 1961, 1965, 1968, 1974, 1982, 1988, 1991, 1998 and 1999 have predicted very high PRCPTOT values in the southeastern part of the country while PRCPTOT surfaces of 1956, 1964, 1965, 1966, 1968, 1970, 1976, 1982, 1983, 1988, 1991, 1993, 1997, 1998 and 2000 show higher PRCPTOT values in the whole eastern part of the country. PRCPTOT surfaces of 1984, 1987, 2002, 2004 and 2007 predicted higher values of PRCPTOT all over the country. Residual plots of PRCPTOT surfaces using universal kriging, presented in Annex A.8, indicate that the universal kriging method, like ordinary kriging, has in general under estimated the PRCPTOT values in prediction. The performance of the prediction is similar to the ordinary kriging method and better than the deterministic methods as presented by the size of the circle of difference between measured and predicted values. Though the spatial trend for TXx is to the north-western part of the country, this has typically been represented only by the TXx surfaces of 1954 and 1957. Rather TXx surfaces of 1956, 1958, 1961, 1970, 1972, 1975, 1976, 1979, 1980, 1989 and 1995 represent tilted surfaces of TXx to the west direction. Like all TXx surfaces from other methods, universal kriging TXx surface of 1960 represents the higher TXx values all over the country. TXx surfaces of 2003 and 2004 represent very high TXx values in the southwestern and northeastern corners of the country, respectively, similar to ordinary kriging surfaces. Unlike the residual plots for TXx surfaces using ordinary kriging, the residual plots of TXx using universal kriging (Annex A.16) indicate that the universal kriging method has mainly under estimated the TXx values in prediction and again the prediction performance is better than the deterministic methods like ordinary kriging, since the size of the circle of difference between measured and predicted value is smaller. To conclude, universal kriging surfaces of both PRCPTOT and TXx describe the long range variability in the index behavior and its spatial trend.

### 5.3.4 Differences among the surfaces created using different spatial interpolation techniques

Surfaces resulting from the differences between the surfaces of PRCPTOT and TXx generated through the different spatial interpolation techniques have been created to compare them (Annex A.23 to A.28). The difference surfaces for both PRCPTOT and TXx show significant differences in the surfaces from different spatial interpolation techniques: the maximum difference has been detected between the surfaces created by the two deterministic methods (TPS and IDW) for both PRCPTOT and TXx; and the minimum difference has been revealed between the surfaces created by the two stochastic methods (OK and UK) for both PRCPTOT and TXx. The reason of the maximum difference between the deterministic surfaces is clear from their variography concept described in 5.2 and the methodology of interpolation described in chapter 4. On the other hand, the stochastic surfaces have minimum differences because the interpolators are identical and characterize the spatial variability through the variogram in similar ways. The stochastic methods are similar in their interpolation principle but model the variables based on the variable behavior over space. Another important fact is that the differences among the surfaces have been decreasing over years with the increasing number of spatial points, which indicates that the performance of different spatial interpolation techniques becomes similar if there is an adequate density of samples.

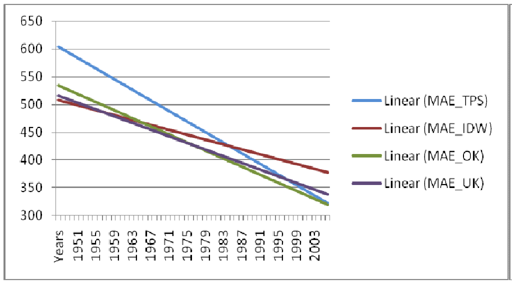
## 5.4 Performance evaluation of the spatial interpolation methods based on cross-validation

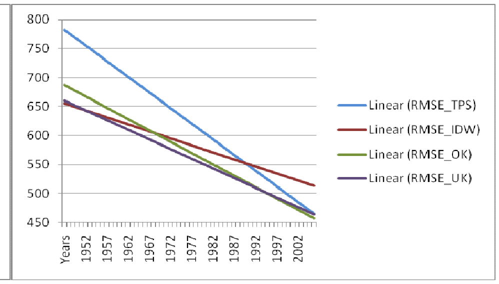
As described in chapter 4, five willmott (WILLMOTT, 1984) statistical measures ***MAE***, ***RMSE***, ***RMSEs***, ***RMSEu***, ***d*****,** and two other measures $\rho_f$, ***CP*** have been calculated from the prediction errors derived through cross-validation, which allow comparing the measured and predicted values of the indices by different methods, for each year (Annex A.29 and A.30). The linear trend of all the cross-validation measurements has been analyzed and the results are presented in Figure 5.8. It is clearly showed that all the willmott error measurements (***MAE***, ***RMSE***, ***RMSEs***, ***RMSEu***) and the coefficient of variation of errors ($\rho_f$) show a decreasing trend over years as the number of stations increases. And the index of agreement (***d***) from willmott measures and the confidence of prediction (***CP***) show an increasing trend over years. This undoubtedly results in better performance of the spatial interpolation methods over time with the increase in the number of spatial points, which means that the increased number of samples improves the performance of spatial interpolation as expected.

From the linear trends of ***MAE*** measurements for PRCPTOT (Figure 5.8(a)), it is obvious that the IDW method works better until the mid 1950s from 1948 and is then replaced by the better performance of UK until the mid 1970s. OK performs better for the rest of the years. The linear trend of ***RMSE*** for PRCPTOT (Figure 5.8(b)), which is the most important error measurement for the spatial interpolation performance evaluation, describes that IDW performed better only until the beginning of the 1950s, afterwards UK performed better until the beginning of 1990s and finally OK performed better for the rest of the years. The linear trend of ***RMSE*** for PRCPTOT (Figure 5.8(c)) describes the better performance of UK from the very beginning of the study period until the beginning of the 1980s, however OK performs better afterwards. The linear trend of ***RMSEu*** for PRCPTOT (Figure 5.8(d)) represents better performance of IDW all through the years with an increasing trend. This is due to the errors, which cannot be modeled mathematically and which are influenced by the property of fitting straight lines through measured points by IDW variography, which includes measurement errors as discussed in section 5.2. This measure supports non-smoothing methods, but the indices are smoothed in behavior which leads to the increasing trend with increased spatial points. Yet, UK and OK show good performance with their full-smoothing properties. The linear trend of $\rho_f$ for PRCPTOT (Figure 5.8(e)) indicates that IDW only performs better for two starting years followed by UK. OK performs equally good as UK at the very end of the study period. The linear trend of ***d*** for PRCPTOT (Figure 5.8(f)) shows that the UK method performs better from the beginning until the beginning of the1990s and gets then lost in hard competition with OK and TPS; OK performs slightly better then. Finally, the linear trend of ***CP*** for PRCPTOT (Figure 5.8(g)) describes that IDW performs better from 1948 to the beginning of the 1950s and then UK performs better until the mid 1990s, whereas OK performed better afterwards.

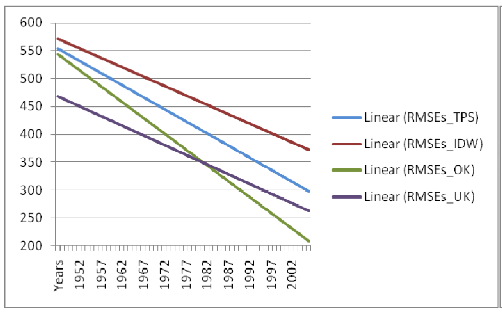
From the linear trends of the ***MAE*** measurements for TXx (Figure 5.8(h)), it is obvious that the IDW method works better from 1948 until the mid 1950s but is then being replaced by the better performance of OK until the end; UK seems performing equally good as OK during 1960s. The linear trend of ***RMSE*** for TXx (Figure 5.8(i)) indicates that IDW performed better from 1948 until the mid of 1960s, then OK performed better until the end. The linear trend of ***RMSEs*** for TXx (Figure 5.8(j)) shows very different results of spatial interpolation performance. It shows the better performance of TPS from the beginning until the mid 1980s, which is replaced by the better performance of UK until the beginning of the1990s and later by OK. The reason for this result can be the fact that when the density of sample is high enough, the smoothing of stochastic methods performs as good as the smoothing through the measured points of the deterministic methods, which is also apparent by the improved performance of UK and OK with increased points. As described in the previous paragraph, the linear trend of ***RMSEu*** for TXx (Figure 5.8(k)) shows better performance of IDW all through the years followed by TPS and then by the stochastic methods, but this measure is not a true representative of the performance. The linear trend of $\rho_f$ for TXx (Figure 5.8(l)) indicates that IDW performs better from the beginning until the mid 1960s and then OK performs better until the end. The linear trend of ***d*** for TXx (Figure 5.8(m)) shows that UK and TPS perform equally good from 1948 until the beginning of the 1980s and then OK performs better.

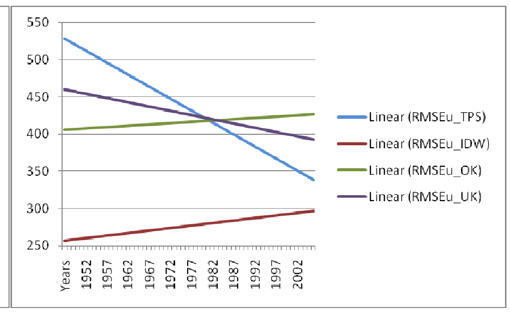
Finally, the linear trend of ***CP*** for TXx (Figure 5.8(n)) implies that IDW performs better from the 1948 to the late 1960s and finally OK performs better for the rest.



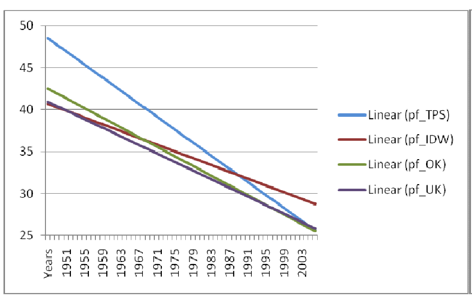



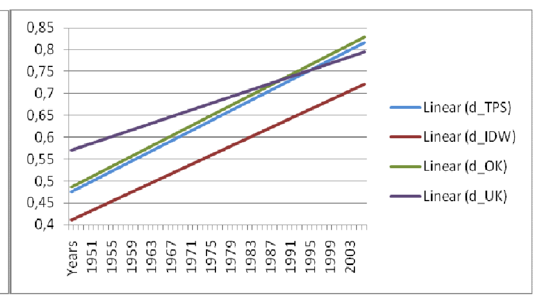

**(a)** **(b)**



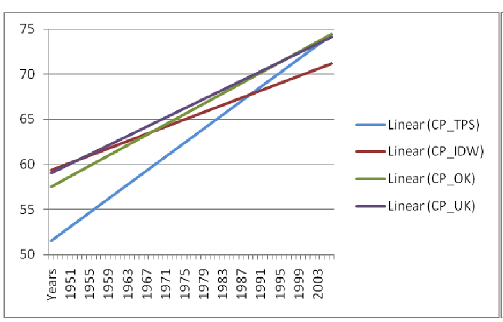



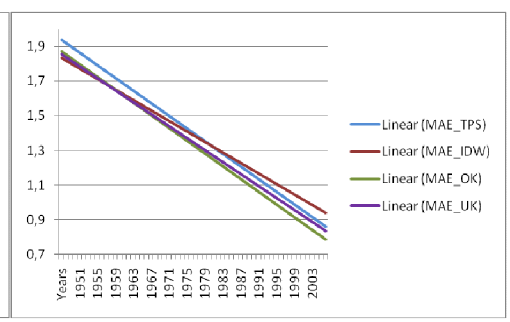

**(c)** **(d)**




**(e)** **(f)**




**(g)** **(h)**

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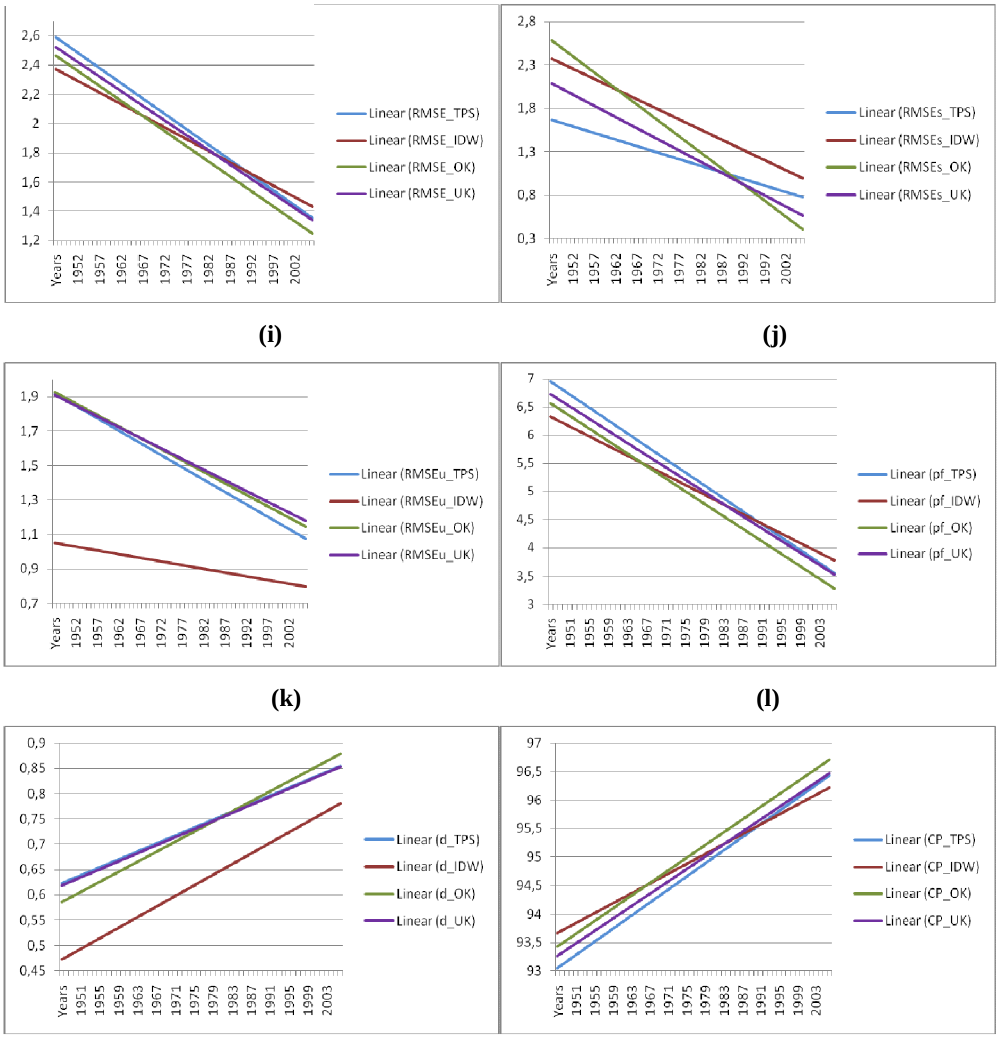


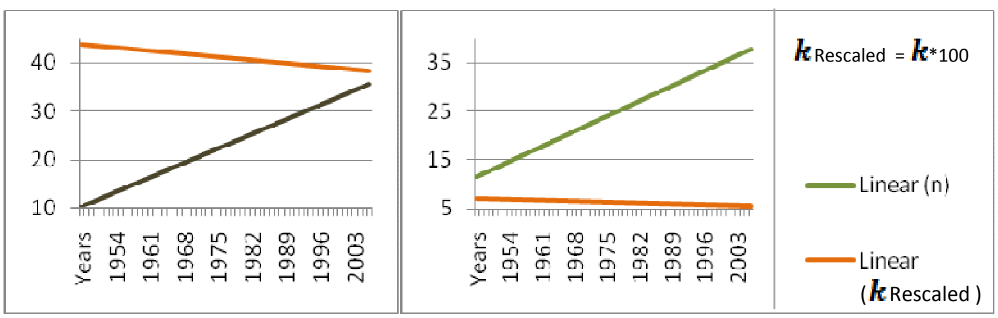

**(i)** **(j)** **(k)** **(l)** **(m)** **(n)**

**Figure 5.8:** Linear trends of the performance evaluation measurements of the spatial interpolation methods: (a) $MAE$ (b) $RMSE$ (c) $RMSE_s$ (d) $RMSE_u$ (e) $d$ (f) $\rho_f$ (g) CP of the PRCPTOT index; and (h) $MAE$ (i) $RMSE$ (j) $RMSE_s$ (k) $RMSE_u$ (l) $d$ (m) $\rho_f$ (n) CP of the TXx index from 1948-2007.

The analysis of the linear trends in the performance measurements depicts that the UK method performs better in most of the years for interpolating PRCPTOT and the OK method for interpolating TXx. It is also true that in terms of performance the spatial interpolation techniques are quite similar and differences between their performance measurement trends are minimal. But they are different in performance as described in section 5.3.4, which is obvious from the resulting surfaces despite some similarities.

## 5.5 Discussion of the results

The decreasing trend in the error measures and in the coefficient of variation of the errors, as well as the increasing trend in the index of agreement and in the confidence of prediction over time of the spatial interpolation techniques, depict clearly that the number and density of spatial points play a major role in the performance of interpolation, since the number of spatial points shows an increasing trend over time for both PRCPTOT and TXx (Figure 5.9). Considering this correlation, Figure 5.9 also shows that the coefficient of variation of the sampled indices also experiences a decreasing trend over time, though the slope of their trend lines is not as steep as the slope of the trends of spatial points. It is apparent that the behavior of the indices is spontaneous through the years and it is merely subject to the location of the meteorological stations whether the samples are well representative or not. But it is rational that the higher the number of sample points the higher the chance of the sample points to be representative, which is exactly what is represented by the decreasing trends of CV with the increasing trends of *n* over the years.




**(a)** **(b)**

**Figure 5.9:** Linear trend of number of spatial points (n) and coefficient of variation (CV) of the sampled indices for (a) PRCPTOT and (b) TXx over years. CV has been rescaled to maintain conformity with the number of spatial points to compare.

Table 5.3 concretely corroborates the fact of low sample density effect on interpolation performance. As described before, the correlation between *n* and CV does not seem very strong from Figure 5.9. Also the correlation coefficients between them are not very significant (Table 5.3), but there is an obvious negative correlation between them, which perhaps cannot be expressed in a number because of the random indices behavior over years. But it is a fact that the higher the sample number and density, the more representative are the sampled indices.

The influence of low sample density on spatial interpolation performance can be finally established by the significant strong correlation between the coefficient of variation of the samples ($\boldsymbol{k}$) and the measurements of the performance evaluation. The measurements of

TXx represent this fact better, since PRCPTOT is not a typical continuous phenomenon over time and can be measured as 0 for several technical and meteorological reasons. But for the correlation between the similar measurements like coefficient of variation of errors ($\rho_f$) and coefficient of variation of the sampled indices, the correlation is very significant for both PRCPTOT and TXx. This confirms that the more representative the samples are for the study region, the better the spatial interpolation techniques perform. The representation of the sampled indices can be ensured by the increased sample density. And thus the first proposition of the research can be proven.

**Table 5.3:** Correlation coefficient between different performance evaluation measures of the spatial interpolation techniques and coefficient of variation of samples for interpolating PRCPTOT and TXx.

| | | **Coefficient of correlation** | |
|---|---|---|---|
| **Method** | **Elements** | **PRCPTOT** | **TXx** |
| | n & $\boldsymbol{k}$ | -0.19 | -0.14 |
| | $\boldsymbol{k}$ & MAE | 0.52 | 0.90 |
| | $\boldsymbol{k}$ & RMSE | 0.59 | 0.92 |
| | $\boldsymbol{k}$ & RMSEs | 0.40 | 0.89 |
| **TPS** | $\boldsymbol{k}$ & RMSEu | 0.51 | 0.92 |
| | $\boldsymbol{k}$ & pf | -0.78 | 0.92 |
| | $\boldsymbol{k}$ & d | -0.24 | -0.49 |
| | $\boldsymbol{k}$ & CP | -0.70 | -0.92 |
| | $\boldsymbol{k}$ & MAE | 0.59 | 0.91 |
| | $\boldsymbol{k}$ & RMSE | 0.70 | 0.96 |
| | $\boldsymbol{k}$ & RMSEs | 0.51 | 0.91 |
| **IDW** | $\boldsymbol{k}$ & RMSEu | 0.44 | 0.96 |
| | $\boldsymbol{k}$ & pf | 0.81 | 0.96 |
| | $\boldsymbol{k}$ & d | -0.16 | -0.45 |
| | $\boldsymbol{k}$ & CP | -0.81 | -0.96 |
| | $\boldsymbol{k}$ & MAE | 0.54 | 0.89 |
| | $\boldsymbol{k}$ & RMSE | 0.63 | 0.94 |
| | $\boldsymbol{k}$ & RMSEs | 0.42 | 0.85 |
| **OK** | $\boldsymbol{k}$ & RMSEu | 0.35 | 0.91 |
| | $\boldsymbol{k}$ & pf | 0.73 | 0.94 |
| | $\boldsymbol{k}$ & d | -0.23 | -0.51 |
| | $\boldsymbol{k}$ & CP | -0.73 | -0.94 |
| | $\boldsymbol{k}$ & MAE | 0.54 | 0.88 |
| | $\boldsymbol{k}$ & RMSE | 0.64 | 0.89 |
| | $\boldsymbol{k}$ & RMSEs | 0.36 | 0.85 |
| **UK** | $\boldsymbol{k}$ & RMSEu | 0.36 | 0.78 |
| | $\boldsymbol{k}$ & pf | 0.75 | 0.89 |
| | $\boldsymbol{k}$ & d | -0.19 | -0.54 |
| | $\boldsymbol{k}$ & CP | -0.75 | -0.89 |

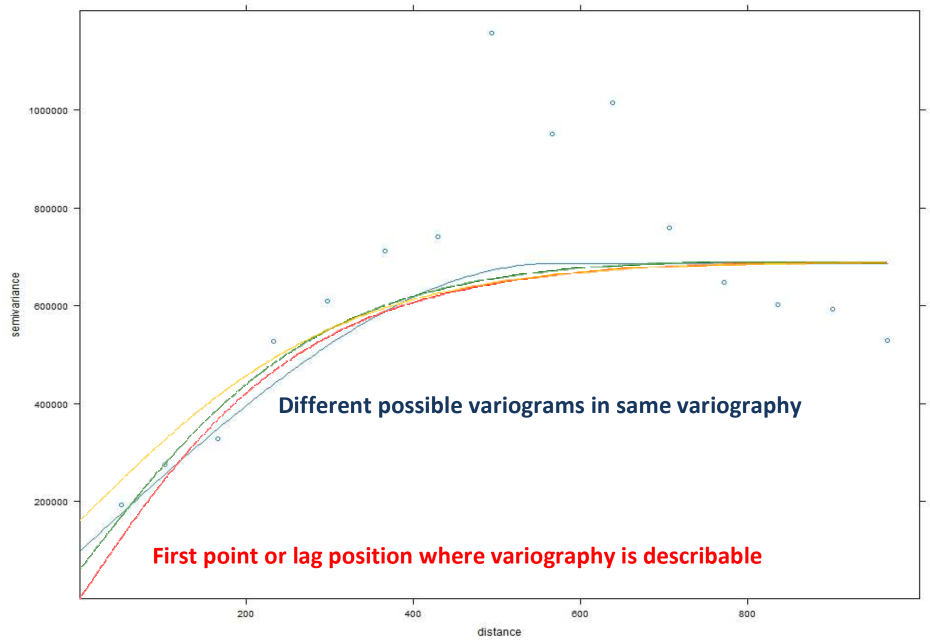



**Figure 5.10:** Problems in variography design when the lag size is too big, different possible variograms (red, green, blue and yellow curves) are available for same experimental variogram.

The reason for low sample density affecting the spatial interpolation performance is better explained by the eventual increased lag size (DUNGAN et al. 2002) in variography. In variography design, the lag size and number are the representation of the available sample number density, and the first lag distance is the average distance between the measured points (ISAACS, and SRIVASTAVA, 1989). When the sample density is too low, the first lag distance is too high with around 50km for the case of Bangladesh as described in Figure 5.10. So, the first point for which the semivariance can be measured is at 50km distance, and it cannot explain the index variability within 50km. This leads to the fact, that all possible variograms (red, green, blue and yellow), which are very different from each other, can be fitted and result in similar ranges and sills, but the values before the first lag remain uncertain. Thus it is merely impossible to describe an accurate spatial dependency for the first increased lag in low sample density and therefore the spatial interpolation results end up highly uncertain.

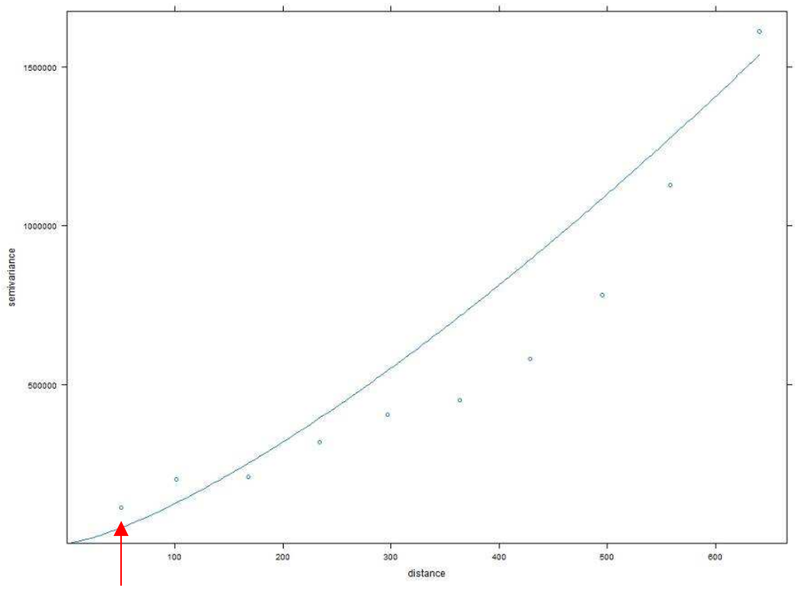



**Figure 5.11:** Variography designed with 32 available spatial points in 2007 to interpolate PRCPTOT using ordinary kriging with first lag at 50km.

The concept of designing mean variograms with spatially shifted years, which has been applied in this study, solves this low sample density problem to some extent, but not entirely. The concept provides enough measured pairs of point to describe the spatial dependence at a certain distance where measured points are missing for the earlier years at some locations. Yet, it cannot solve the problem of increased lag because the average distance cannot be decreased when only 32 and 34 spatial points are available for interpolation of PRCPTOT and TXx respectively. As shown in Figure 5.11, the designed variogram with 32 spatial points available for the year 2007 to interpolate PRCPTOT with ordinary kriging also has the first lag at 50km, so the variability of PRCPTOT within 50km is still not describable. The variogram with only spatial points from 2007 is different from the mean variogram with the accumulated spatial points of 1993-2007 (Figure 5.7(c)), it results in no anisotropy in analysis and is described by a 'Power' model with power of 1.35 (Figure 5.11). And the resulted surface created by applying it with ordinary kriging results in worse precision in all performance evaluation measurements than the surface created with the mean variogram as presented in Table 5.4. This explains why the mean variogram designed with 475 spatial points performs better than the individual variogram designed with 32 spatial points. Consequently, the mean variogram is more accurate to use. This is true for all cases and leads to an increase of precision.

**Table 5.4:** Comparison of the performance evaluation measurements between the ordinary kriging methods applied to create PRCPTOT surface of 2007 applying the individual variogram designed by available 32 spatial points and mean variogram designed by shifted 475 spatial points of the temporal period of 1993-2007.

| Variogram used in spatial interpolation | n | MAE | RMSE | RMSEs | RMSEu | d | pf | CP |
|---|---|---|---|---|---|---|---|---|
| Variogram designed only by available spatial points in 2007 | 32 | 342.8 | 456.7 | 312.7 | 400.1 | 0.80 | 22.9 | 77.0 |
| Mean variogram of 1993-2007 | 475 | 318.6 | 413.9 | 265.0 | 331.1 | 0.82 | 20.7 | 79.2 |

Finally, a relative performance overview of the applied methods is summarized in Table 5.5. The dissimilar performance attitudes of the methods result in different interpolated surfaces which prove the second research proposition.

**Table 5.5:** Summary of the comparative performance analysis of the applied spatial interpolation techniques.

| Performance Parameters | Thin Plate Spline | Inverse Distance Weighting | Ordinary Kriging | Universal Kriging |
|---|---|---|---|---|
| **Variography Design** | Smoothed spline exactly through the measured points. | Straight line exactly through the measured points. | Smoothed line that best fits through the measured semivariance. | Smoothed line that best fits through the measured semivariance considering the spatial trend. |
| **Performance with the number of spatial points and their density** | Performs comparatively worse than the stochastic methods and shows better performance in cross validation with increased number of spatial points and | Performs comparatively better than other methods when the number of spatial points and density are too little but in general shows better performance in | Performs comparatively better than the deterministic methods and shows better performance in cross validation with increased number of spatial points and | Performs comparatively better than the deterministic methods and shows better performance in cross validation with increased number of spatial points |

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| | | | | |
|---|---|---|---|---|
| | density. | cross validation with increased number of spatial points and density. | density. | and density. |
| **Measurement Errors** | Includes measurement errors in result to some extent with minimal smoothing | Includes measurement errors in result | Excludes measurement errors in result by smoothing | Excludes measurement errors in result by smoothing |

## 5.6 Chapter conclusion

This chapter has proved the research propositions with the obtained results and their discussion in the light of existing theories. The correlation between sample density and sample coefficient of variation and the strong correlation between sample coefficient of variation and spatial interpolation performance measurements clearly depict the role of sample density in spatial interpolation performance. The decreasing trends in the error measures and in the coefficient of variation of the errors together with the increasing trend in the index of agreement and in the confidence of prediction over time with increasing sample density, justify the fact even more evidently. And this fact shows conformity with the discussion of the lag problem in spatiotemporal estimation. The final chapter, conclusion and further scopes, will outline the final outcome briefly and lead to the further research scopes of the study.

# 6. CONCLUSION AND FURTHER SCOPES

The spatial interpolation methods used in this study performed irrespective of the sample size and density in terms of their methodology; none of them shows better performance, solely in case of low sample density. Though IDW shows better performance when the sample density is very small, it cannot be claimed profoundly; since this method includes measurement errors in result. As experienced from this study, the four spatial interpolation methods have their advantages and disadvantages because of their relative features, and eventually they result in significant different surfaces. But the difference between the resulted surfaces decreases over time with the increase in sample density; which proves the second proposition of this research mention in introduction chapter.

In designing variography for the stochastic methods, the spherical model has been chosen for all variograms of all indices. This is mainly related to the semivariances of the indices over space, which shows spherical spatial dependence. Variogram models were fitted interactively to the experimental variograms as recommended by GOOVAERTS (1997), thus the variogram parameters were estimated subjectively by giving more relevance to the first lags. Furthermore, other suitable variogram models have also been applied, but the spherical model also proved to be appropriate in the cross validation. The applied method to select the parameters of the variogram proves to be good as we have considered the weighted regression method and cross-validation. This method is indeed similar to the weighted least squares method (WLS). However there are scopes to try more probabilistic procedures such as Maximum Likelihood or Restricted Maximum Likelihood.

Importantly, the nugget effect under OK is small while under UK it is very high. This clearly indicates that the spatial trend is affecting the short-range variability of the indices. The reason for this effect can be seen in the spatial trend for the indices occurring in long range (across the whole country). Additionally, there is not enough sample density to model the short-range variability and to describe the gradual change in the indices' values in the trend direction. Given that the smallest accessible lag of the empirical variogram is large, the nugget effect is likely to be overestimated. Additionally, the nugget value and the semivariogram behavior at the origin cannot be cross-validated because variogram model values for lags smaller than the shortest sampling interval do not intervene in interpolation algorithms.

Last, but not least, proving the first proposition of the study, the sample or spatial point density has a profound effect on the performance of spatial interpolation techniques. It is very important to analyze the spatial point density and resulted coefficient of variation of the samples before applying any spatial interpolation. For example, if a study area provides only five spatial points for over 200,000 square kilometers of area, the spatial interpolation results for the continuous variables will end up with less accuracy. The produced surfaces in such case might not be eligible to be used in further research. Superimposing with the high-resolution auxiliary dataset (RADAR) might be useful but in case of absence of such dataset, the accuracy obtained in the estimates should be taken into account in further analysis.

## 6.1 Limitations and further scopes

The performance improvement of the spatial interpolation techniques can be subject to further analysis in future studies. Other spatial interpolation methods might perform better in response to low sample density, such as stochastic simulation algorithms, which might help researchers in the areas suffering from low sample density due to techno-economic reasons. Further research can be carried out in modeling appropriate variography for the first increased lag size. Looking at different spatial resolution may provide interesting results in this case. Furthermore, modeling with increased temporal resolution, i.e. monthly indices rather than yearly indices, may improve the results as well. Modeling the time series with the Autoregressive (AR), the Moving Average (MA) or the Autoregressive-Moving Average (ARMA) model (BROCKWELL, and DAVIS, 1991; CHAFIELD, 1989; and DIGGLE, 1990) for the yearly indices may also improve the spatial interpolation performance. In a nutshell, these further steps would eventually increase the legitimacy of the research and the proclaimed fact that sample number and density do affect the spatial interpolation result.

The research has broader perspective in applied geostatistics. Evaluating the spatial interpolation techniques and eventually determining the locally appropriate technique, will contribute to preparing appropriate climate dataset and understanding their spatial variability. Interpolation performance improvement in respect to low sample density will provide with accurate continuous surfaces of climate indices for future climate change studies. Moreover, local continuous surfaces of climate indices can be input in several climate models to forecast climate change phenomenon and related consequences on the developing countries like Bangladesh, where climate studies are suffering from unavailability of climate data recording and monitoring. To conclude, choosing appropriate methodology of spatial interpolation will result in appropriate climate forecast and climate resilience activities.

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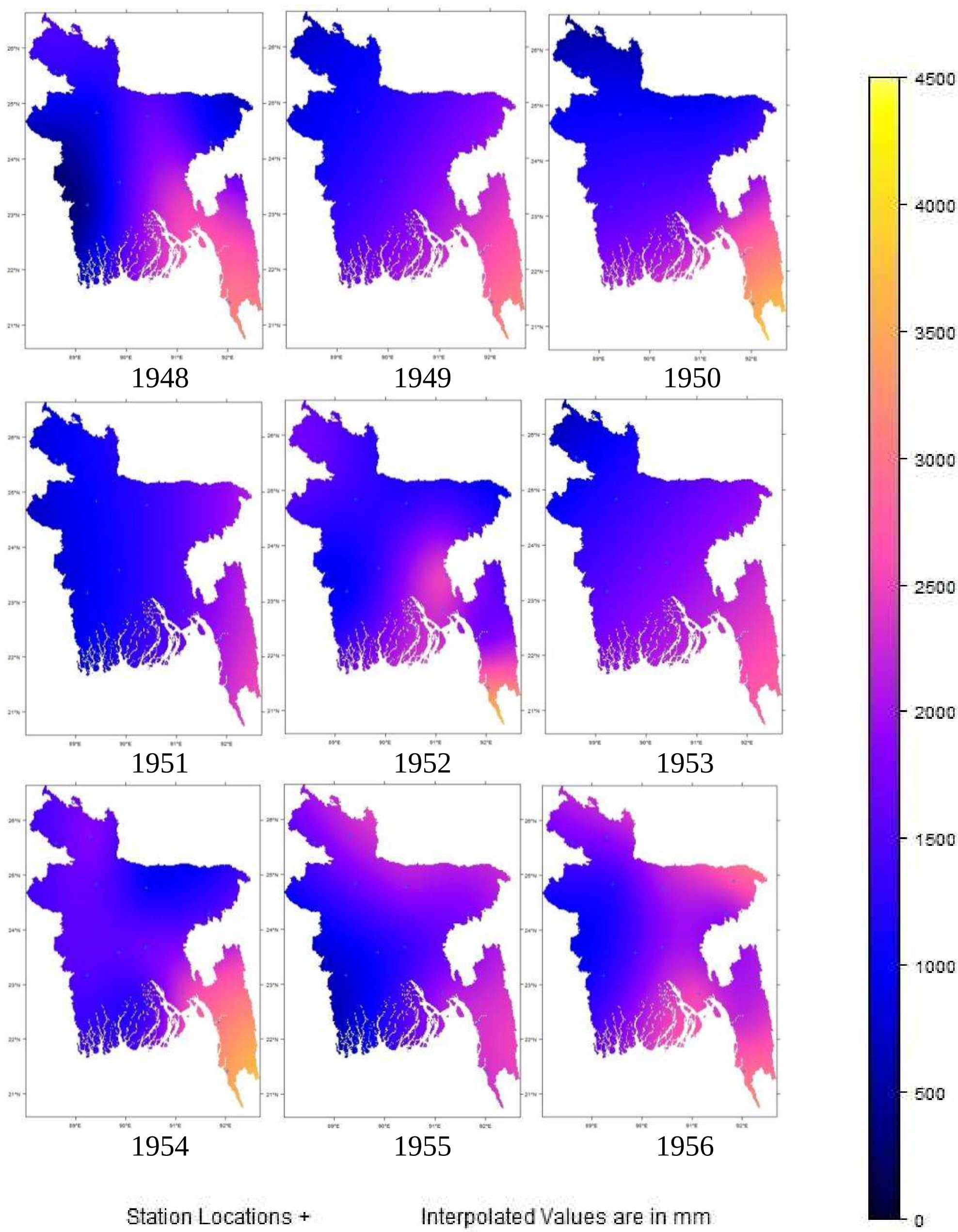
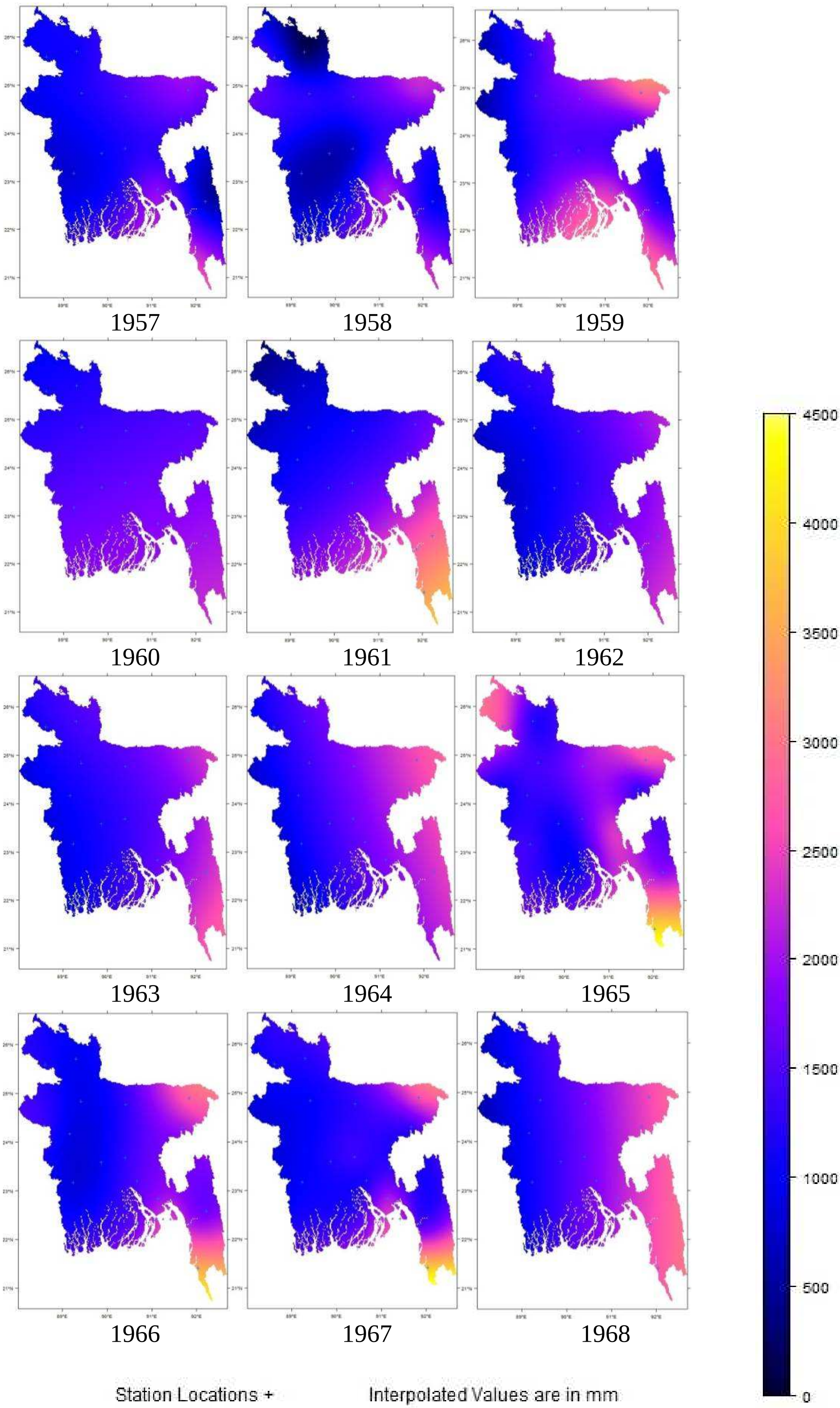
# ANNEXES

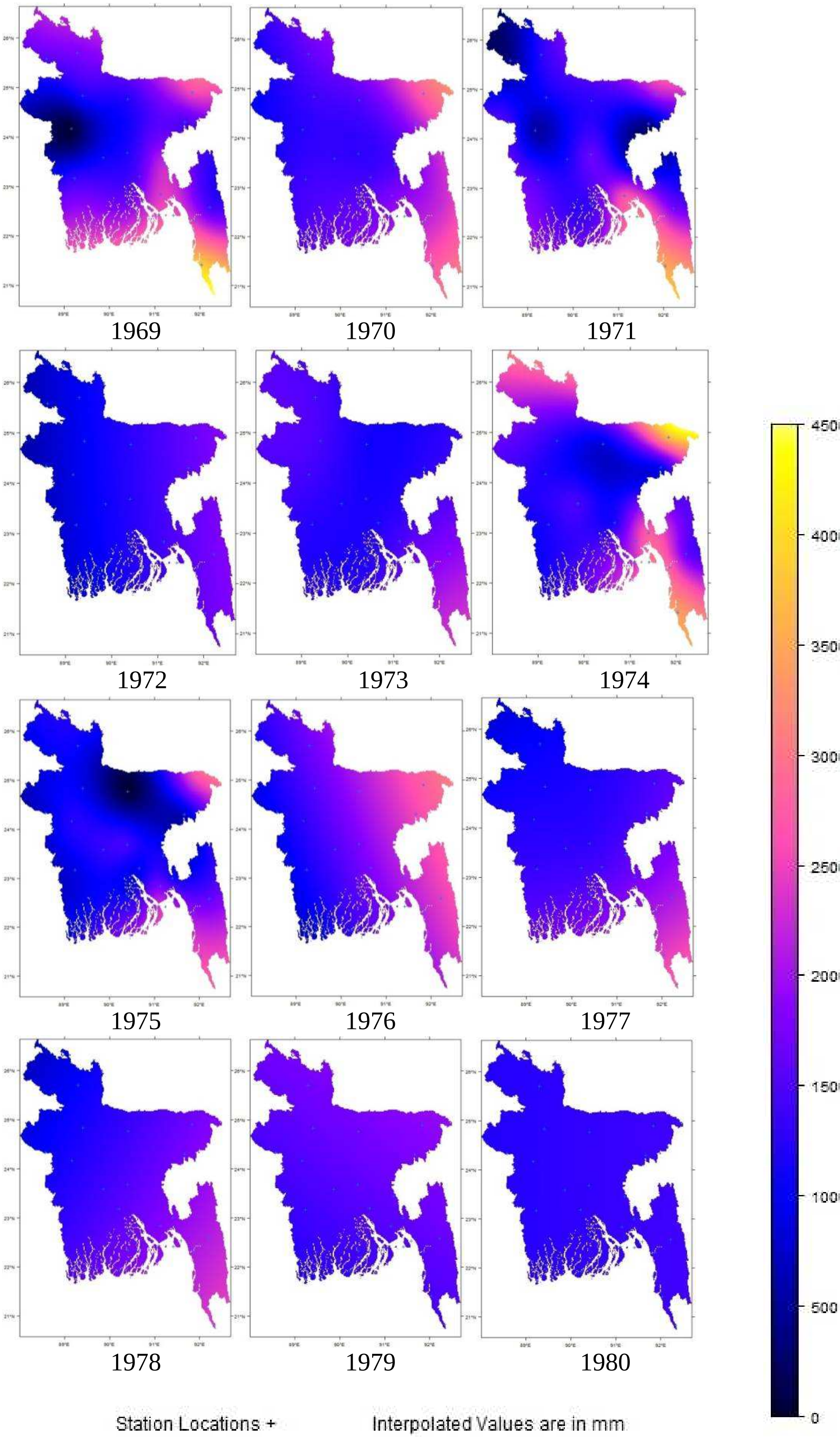
## A.1 TPS Surfaces of PRCPTOT






1957 1958 1959
1960 1961 1962
1963 1964 1965
1966 1967 1968

Station Locations + Interpolated Values are in mm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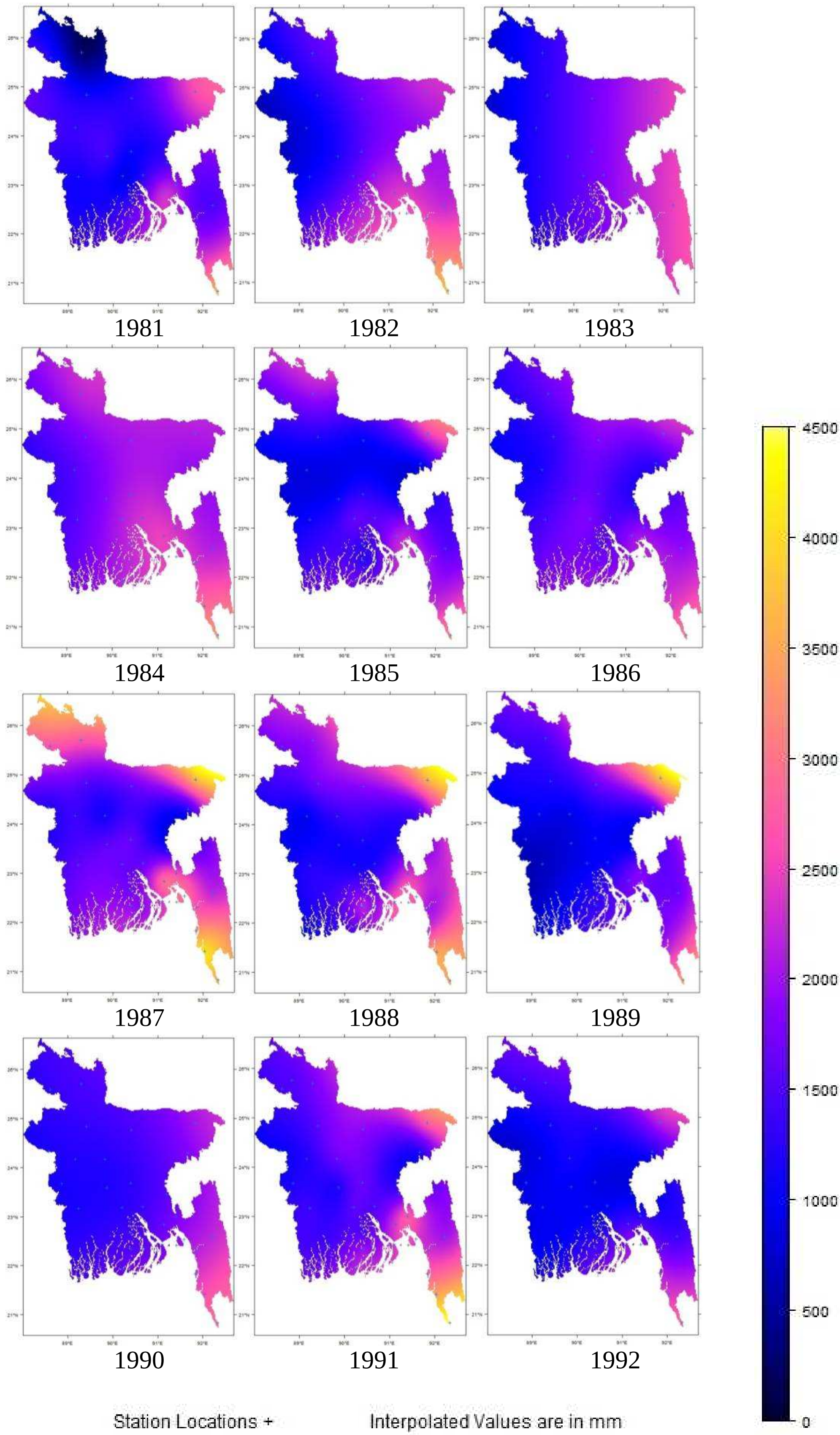
Station Locations +
Interpolated Values are in mm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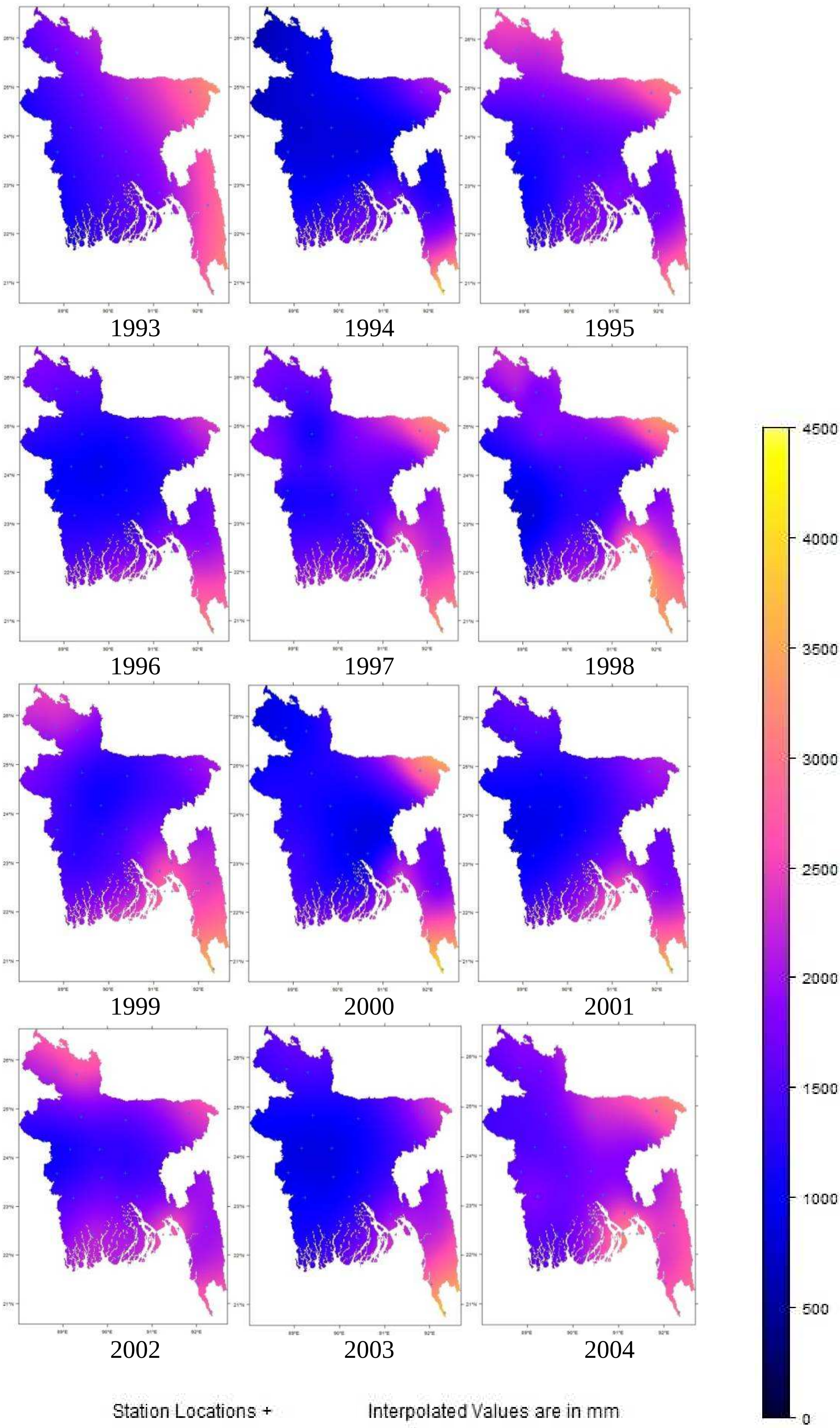
Station Locations +
Interpolated Values are in mm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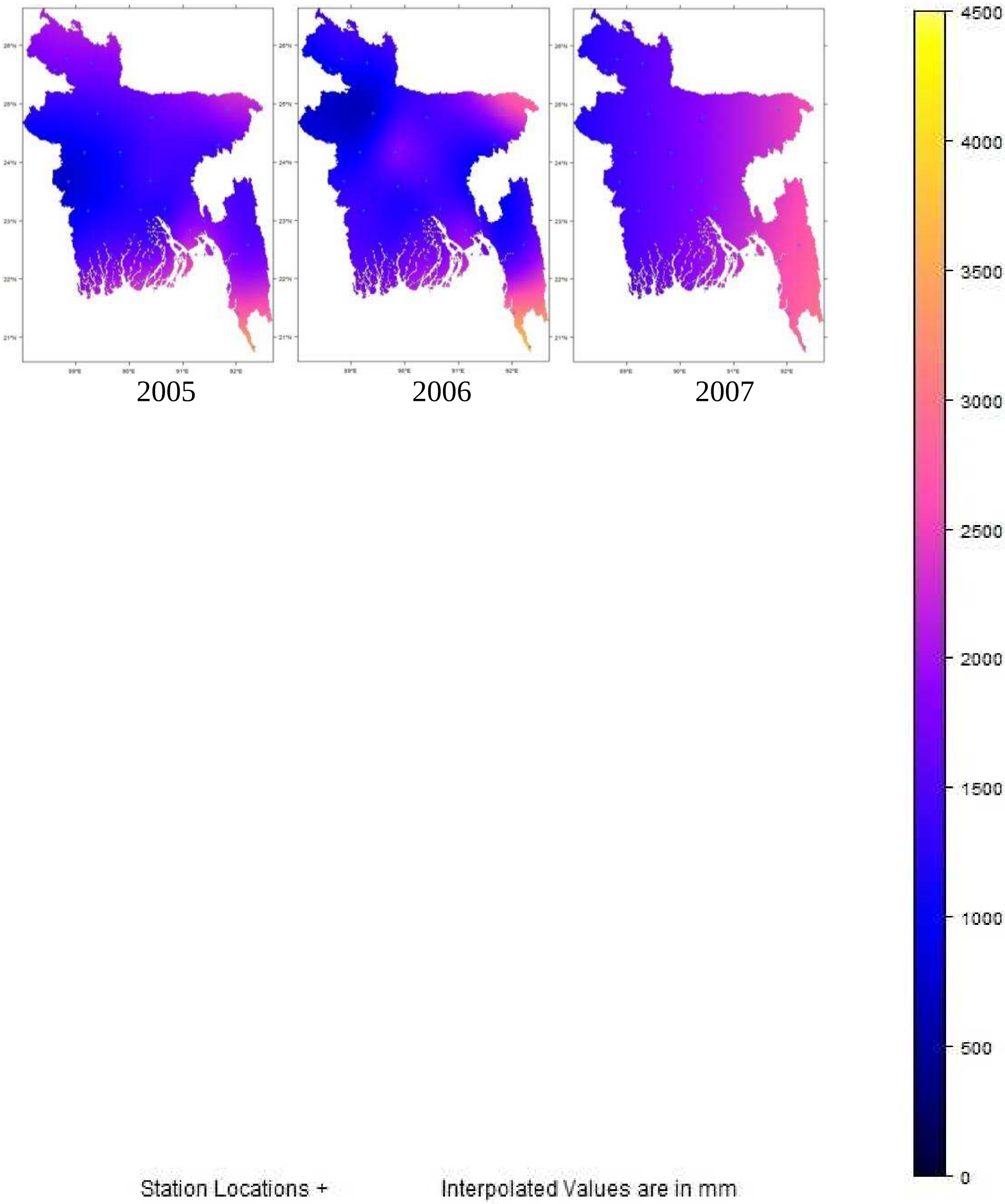
Station Locations +
Interpolated Values are in mm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Station Locations +
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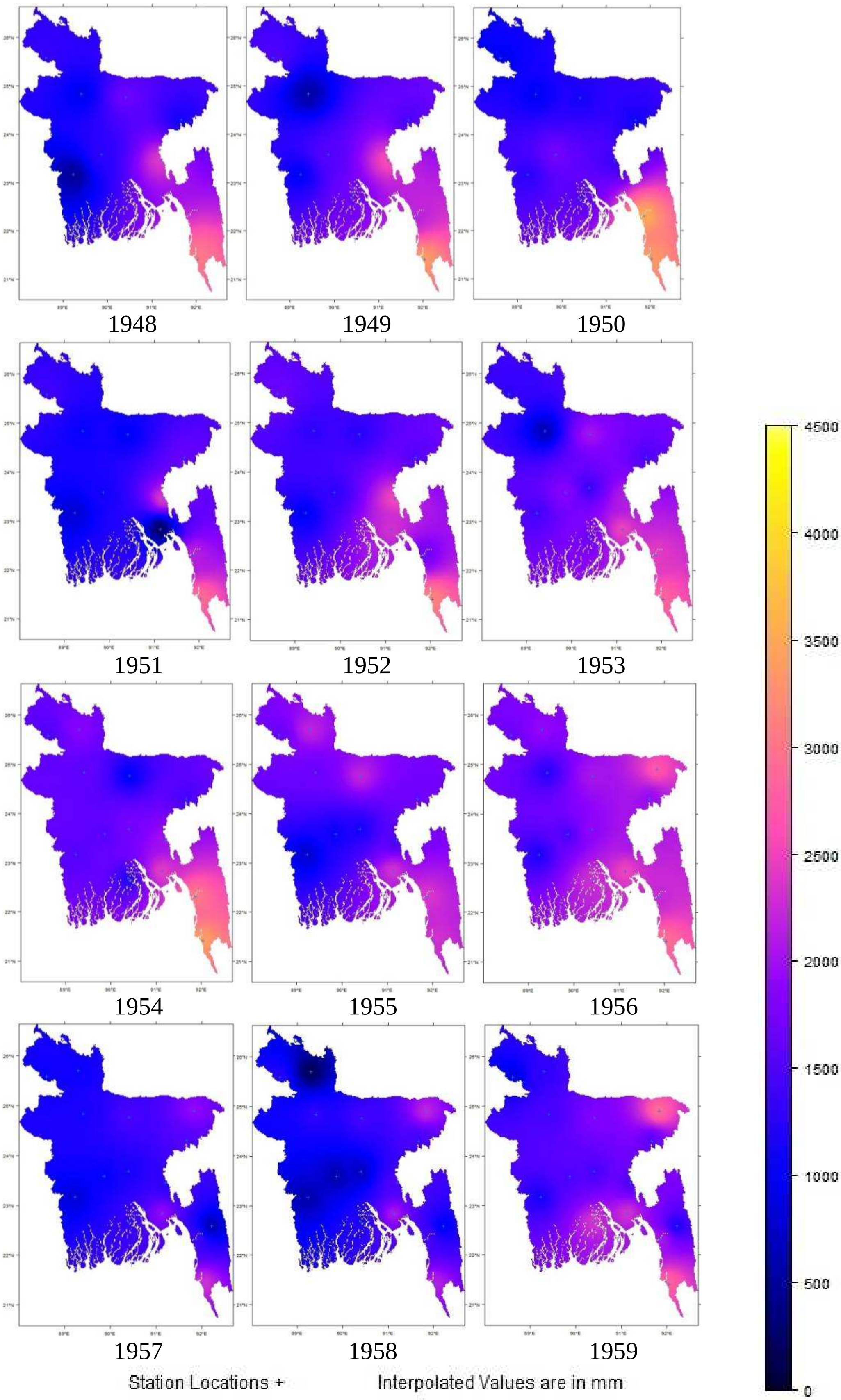
## A.2 IDW Surfaces of PRCPTOT



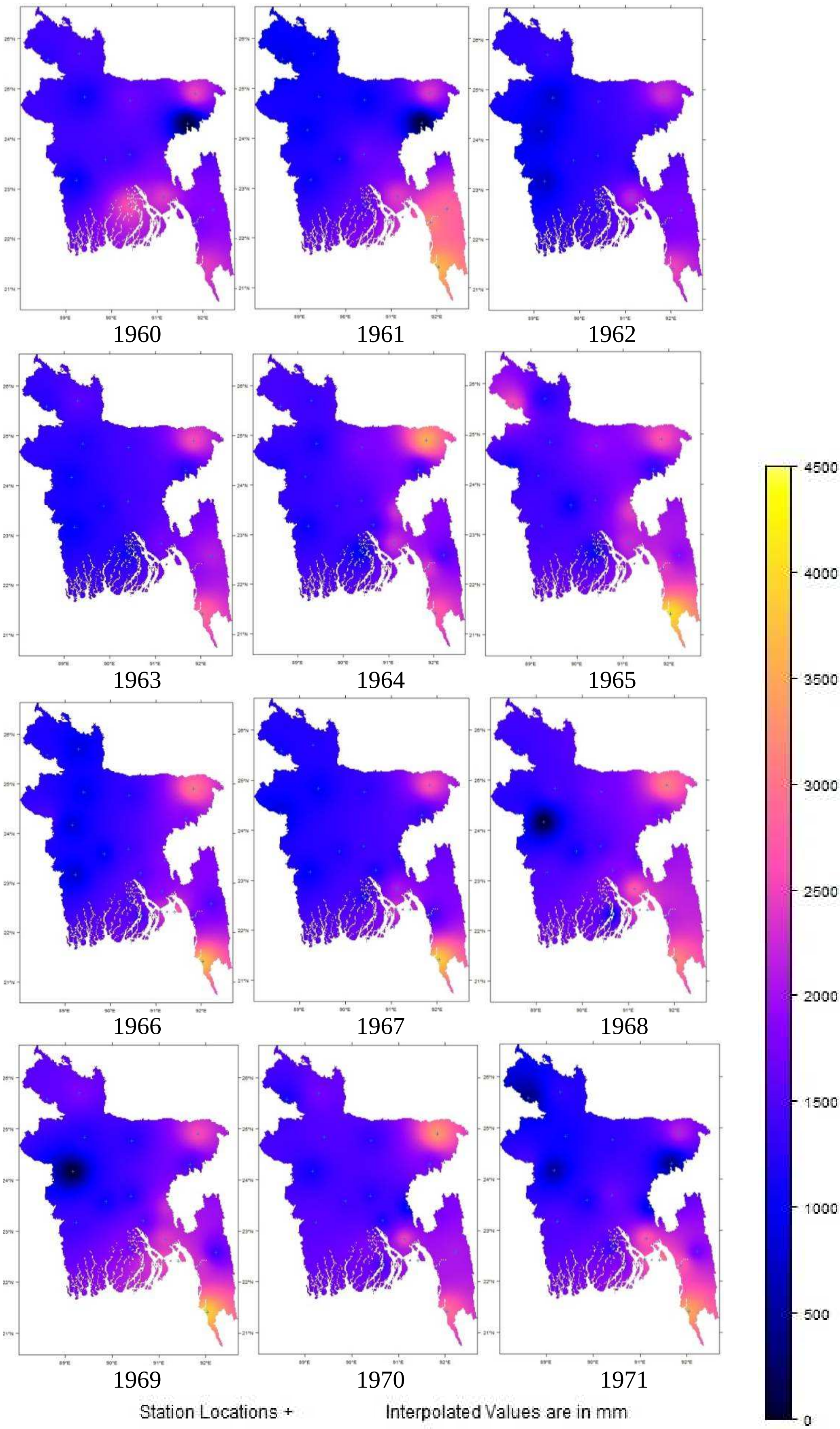

1948 1949 1950

1951 1952 1953

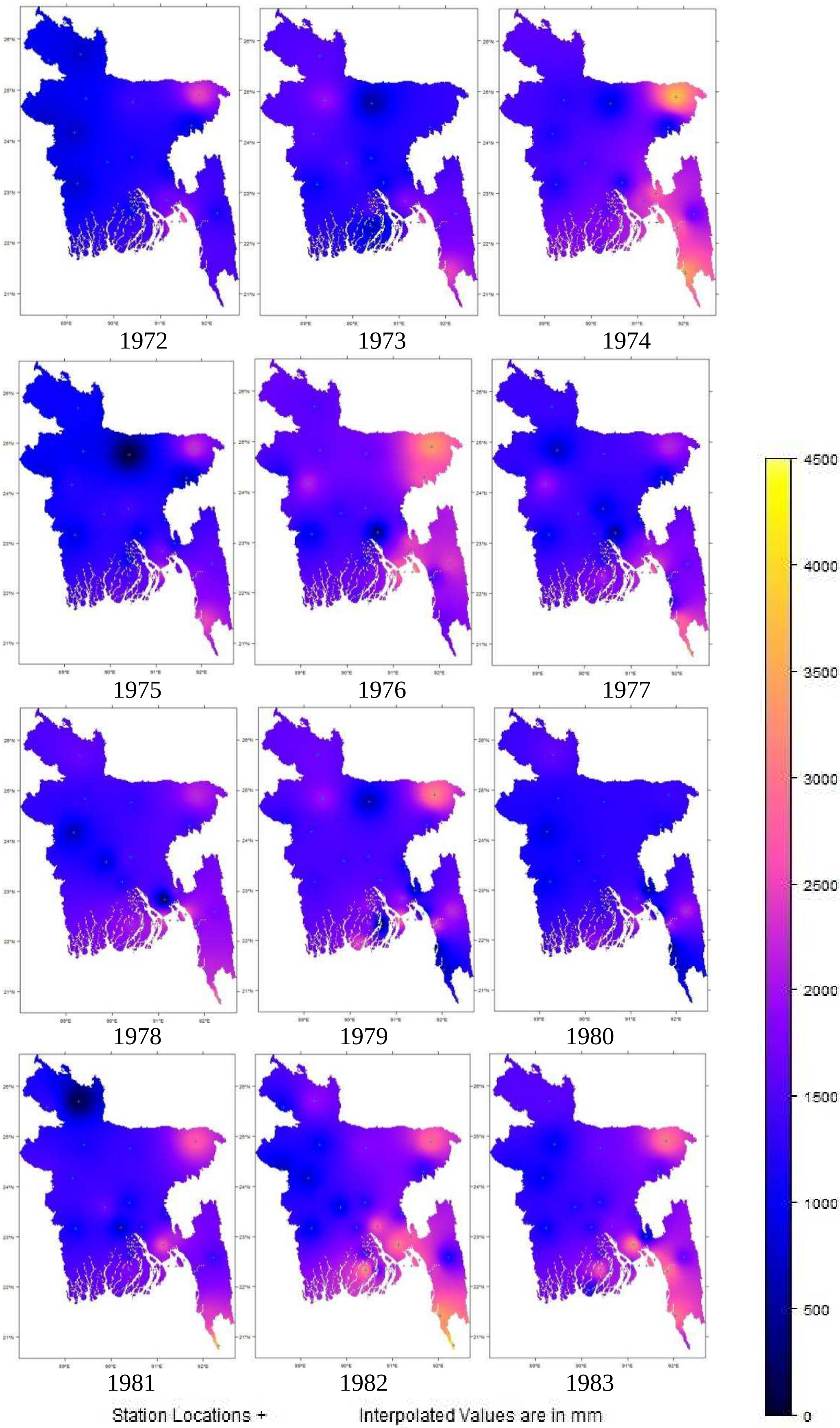
1954 1955 1956

1957 1958 1959

Station Locations + Interpolated Values are in mm



Station Locations +
Interpolated Values are in mm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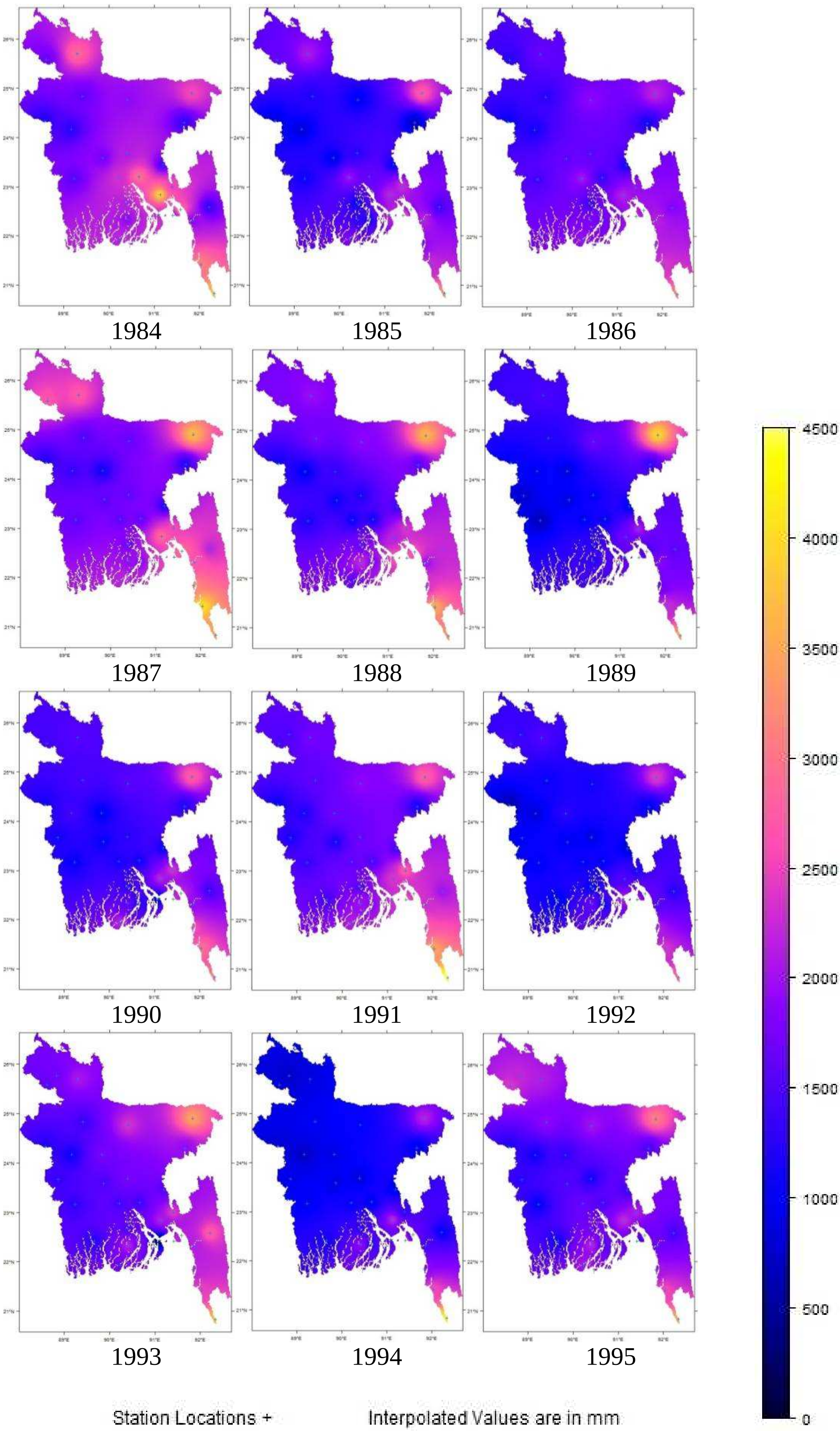
Station Locations +
Interpolated Values are in mm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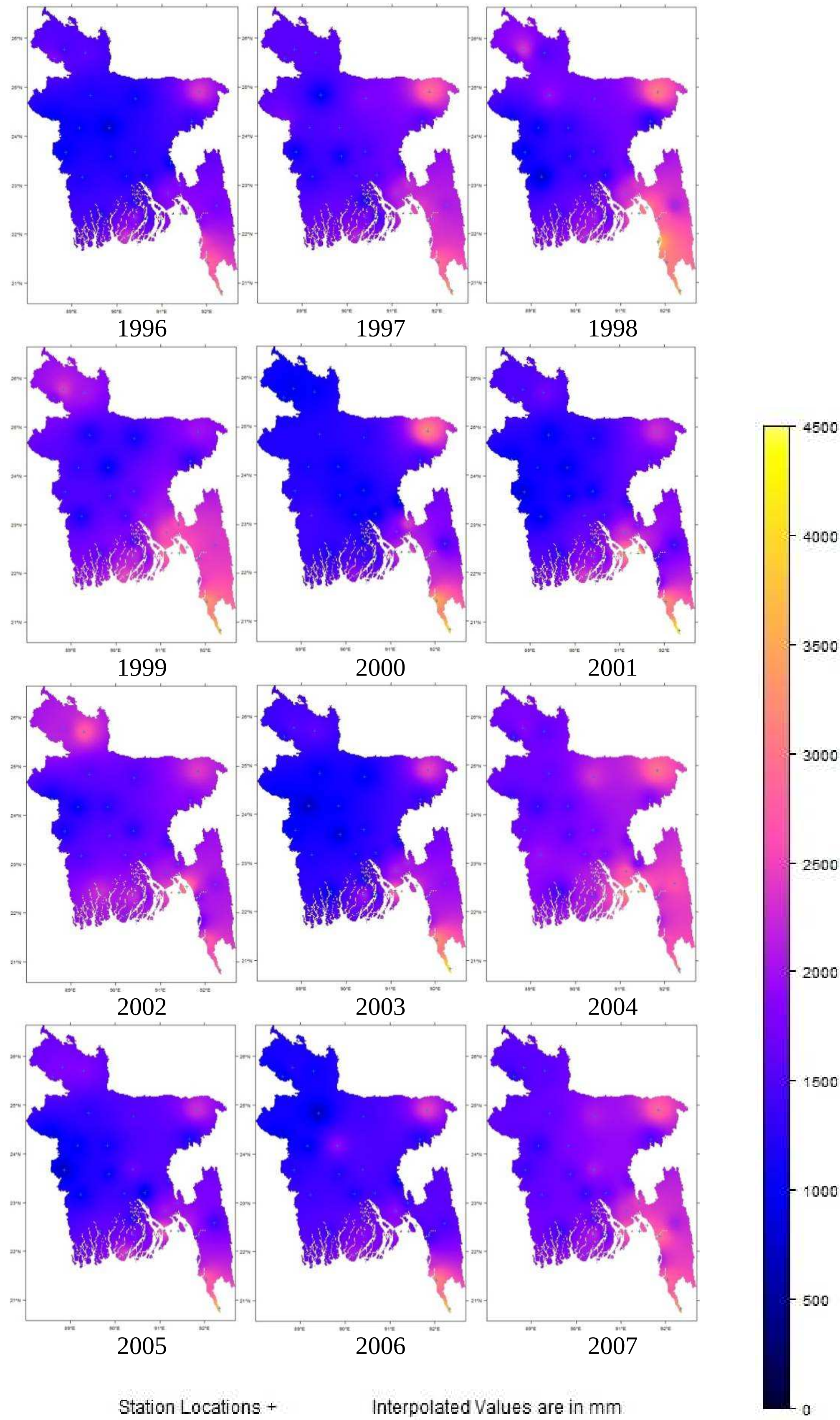
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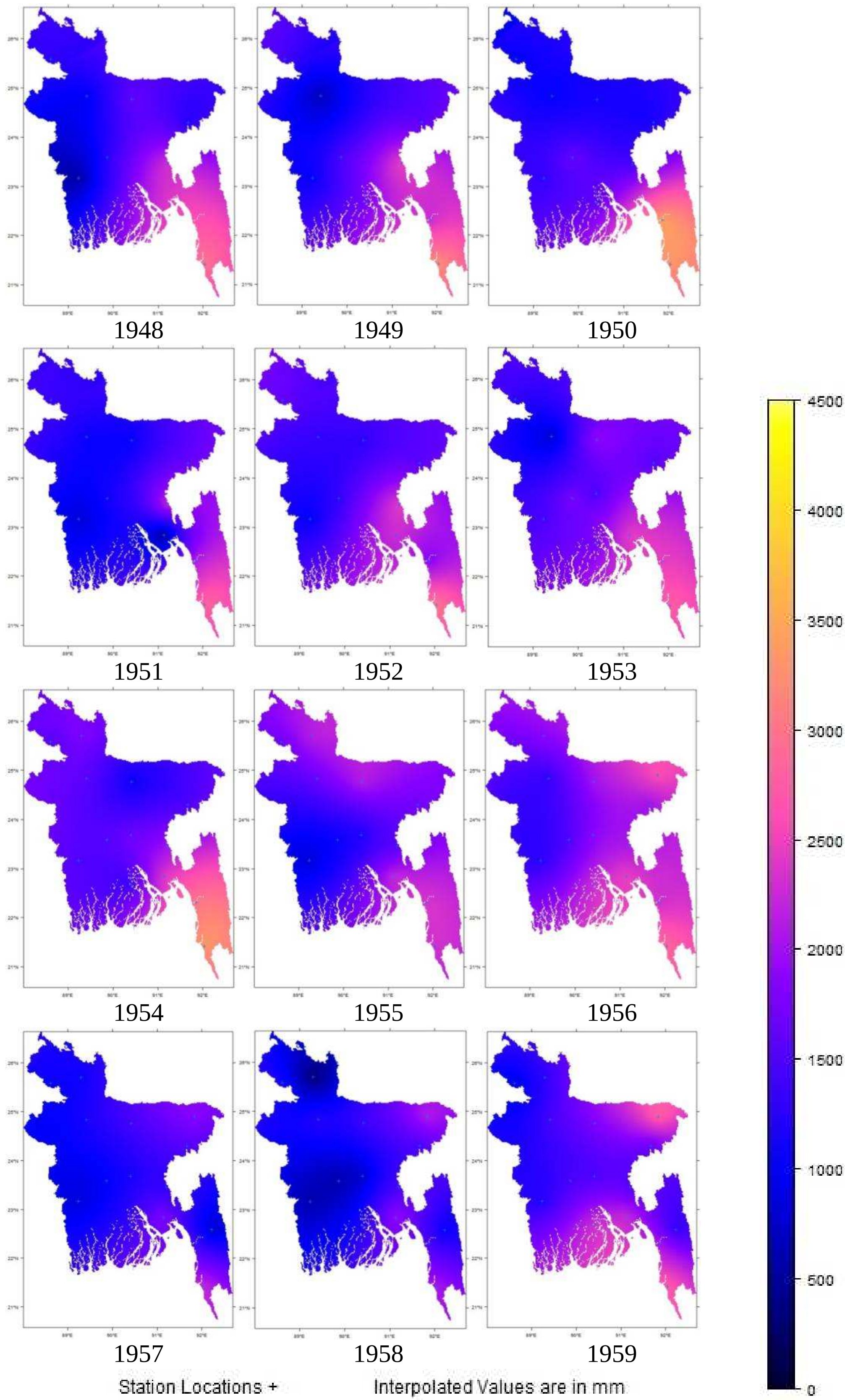
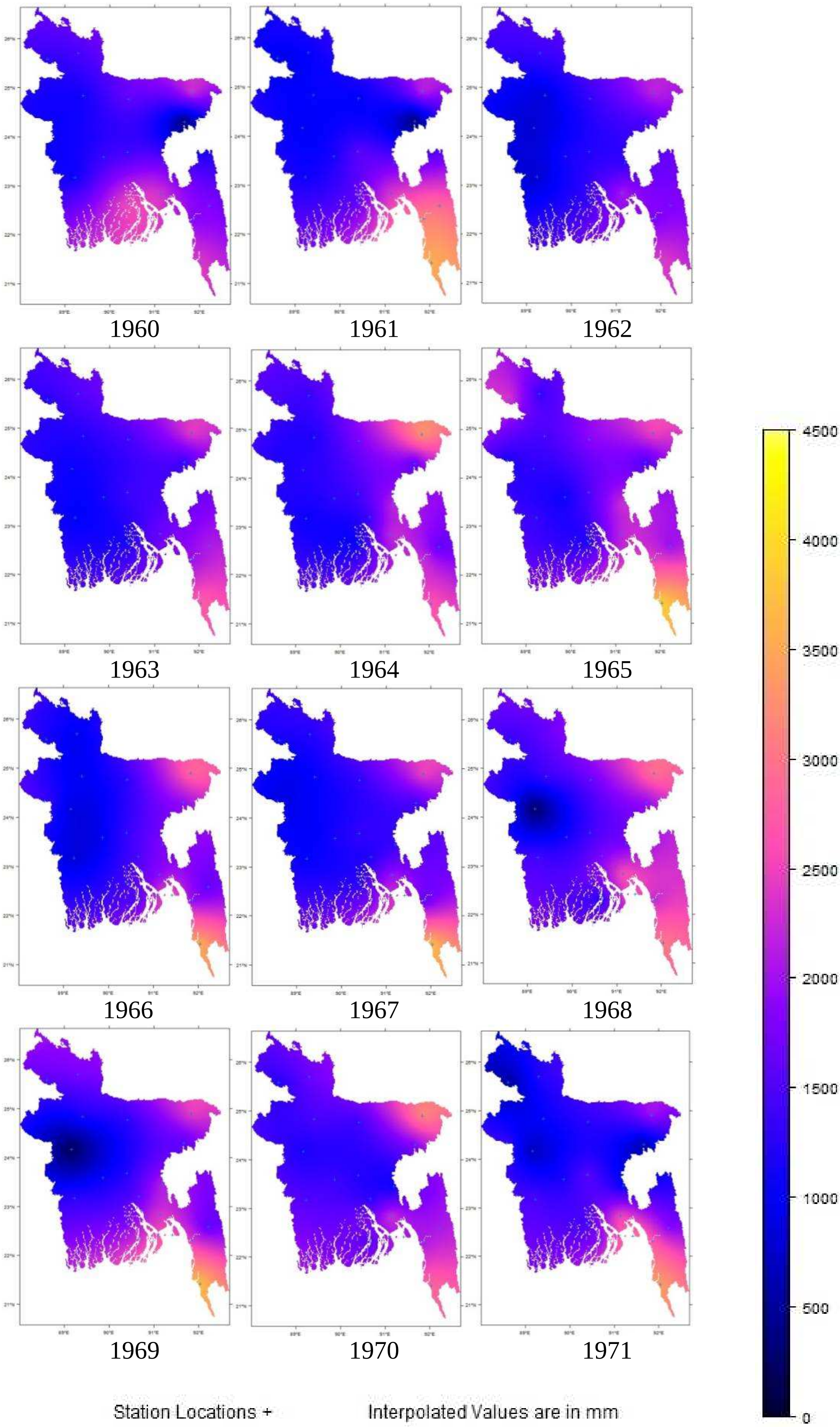
Station Locations +
Interpolated Values are in mm

## A.3 OK Surfaces of PRCPTOT

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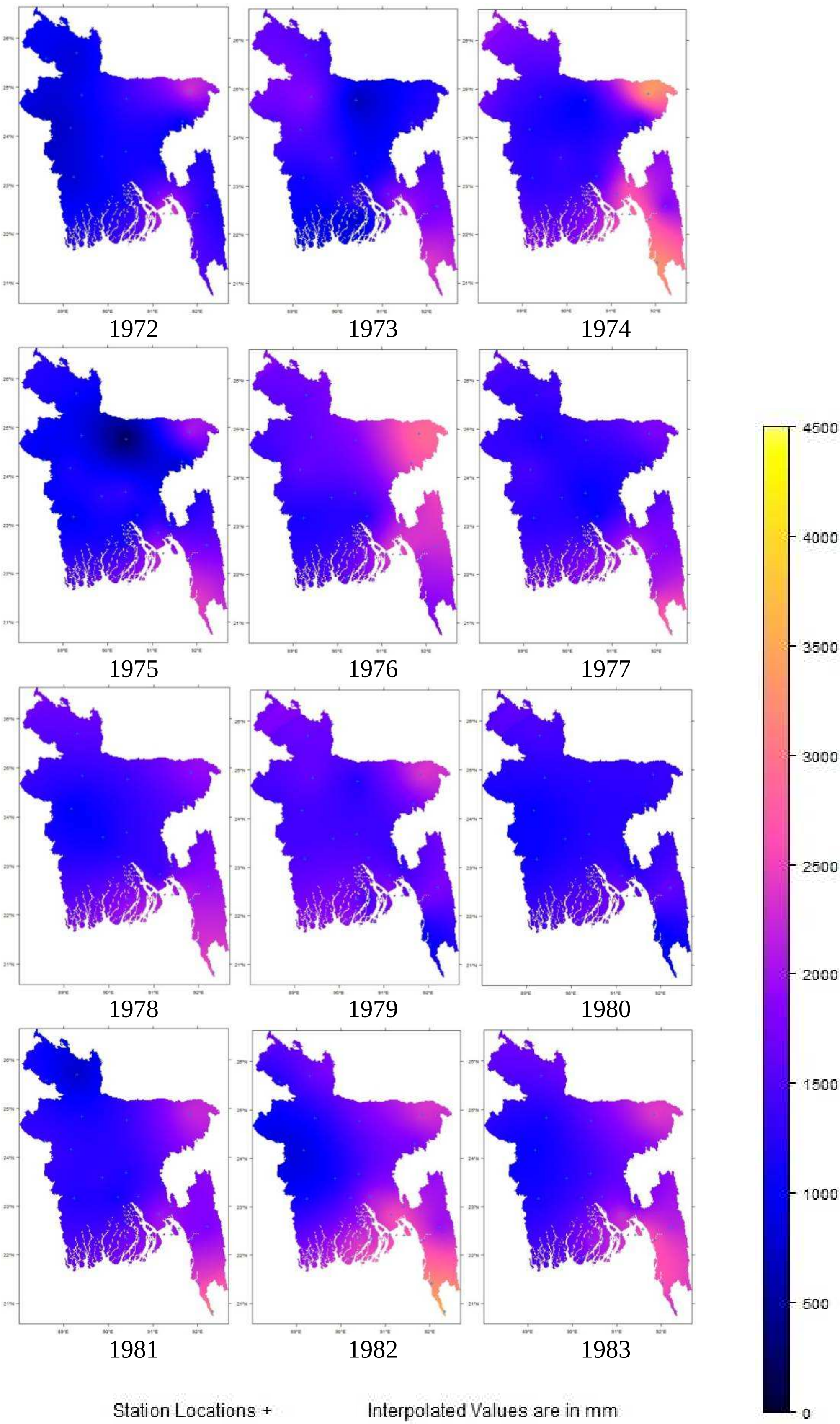
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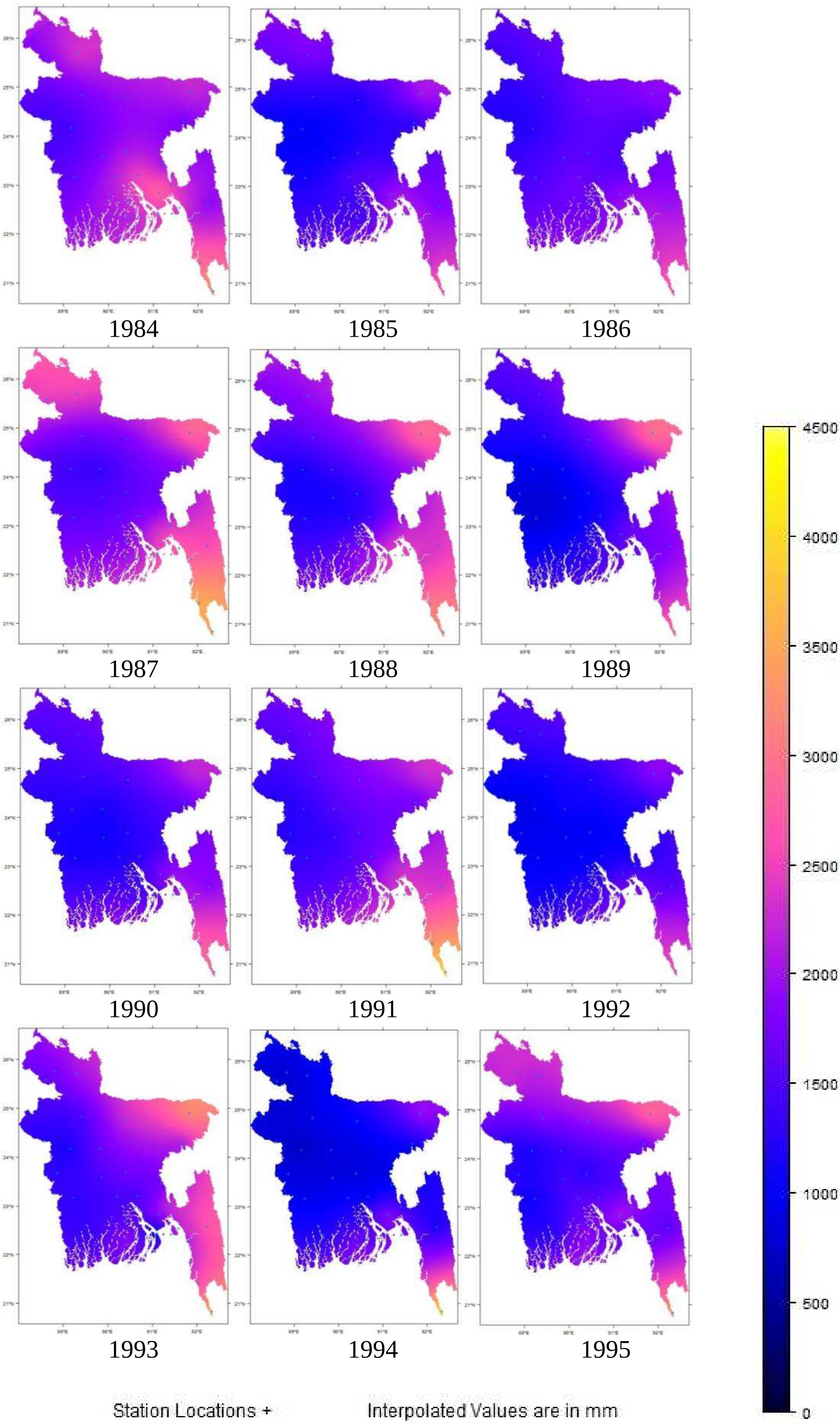
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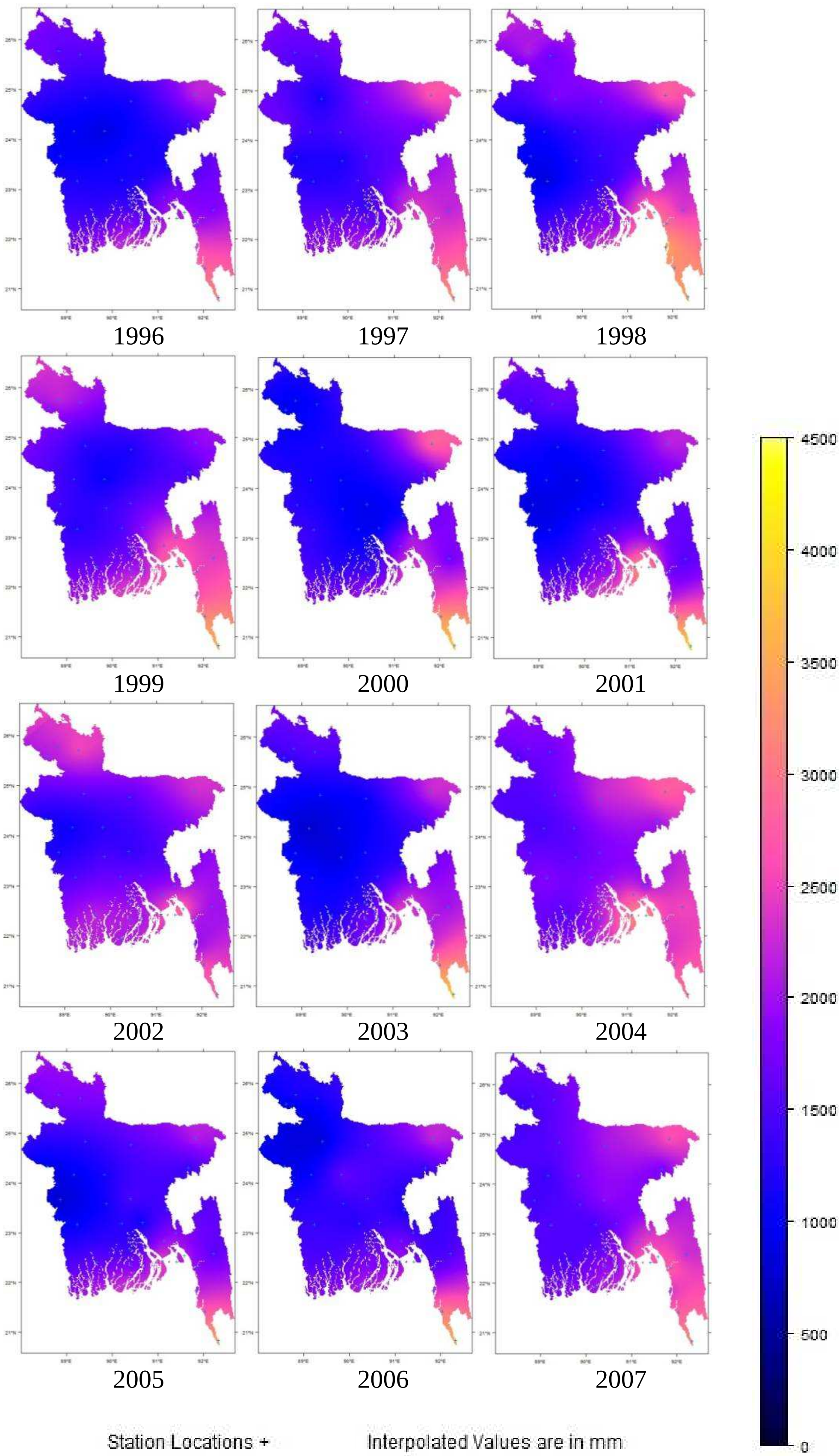
Station Locations +
Interpolated Values are in mm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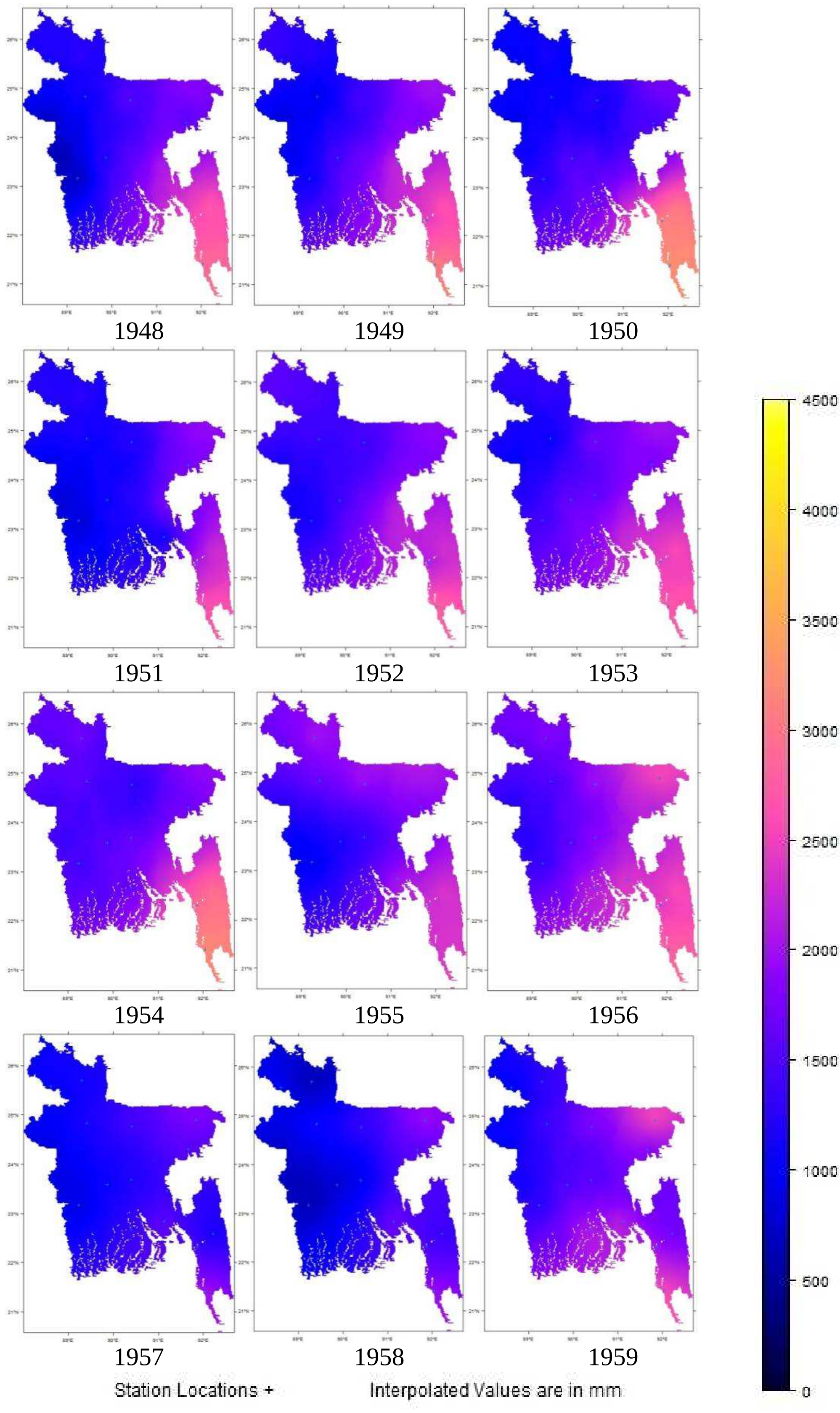
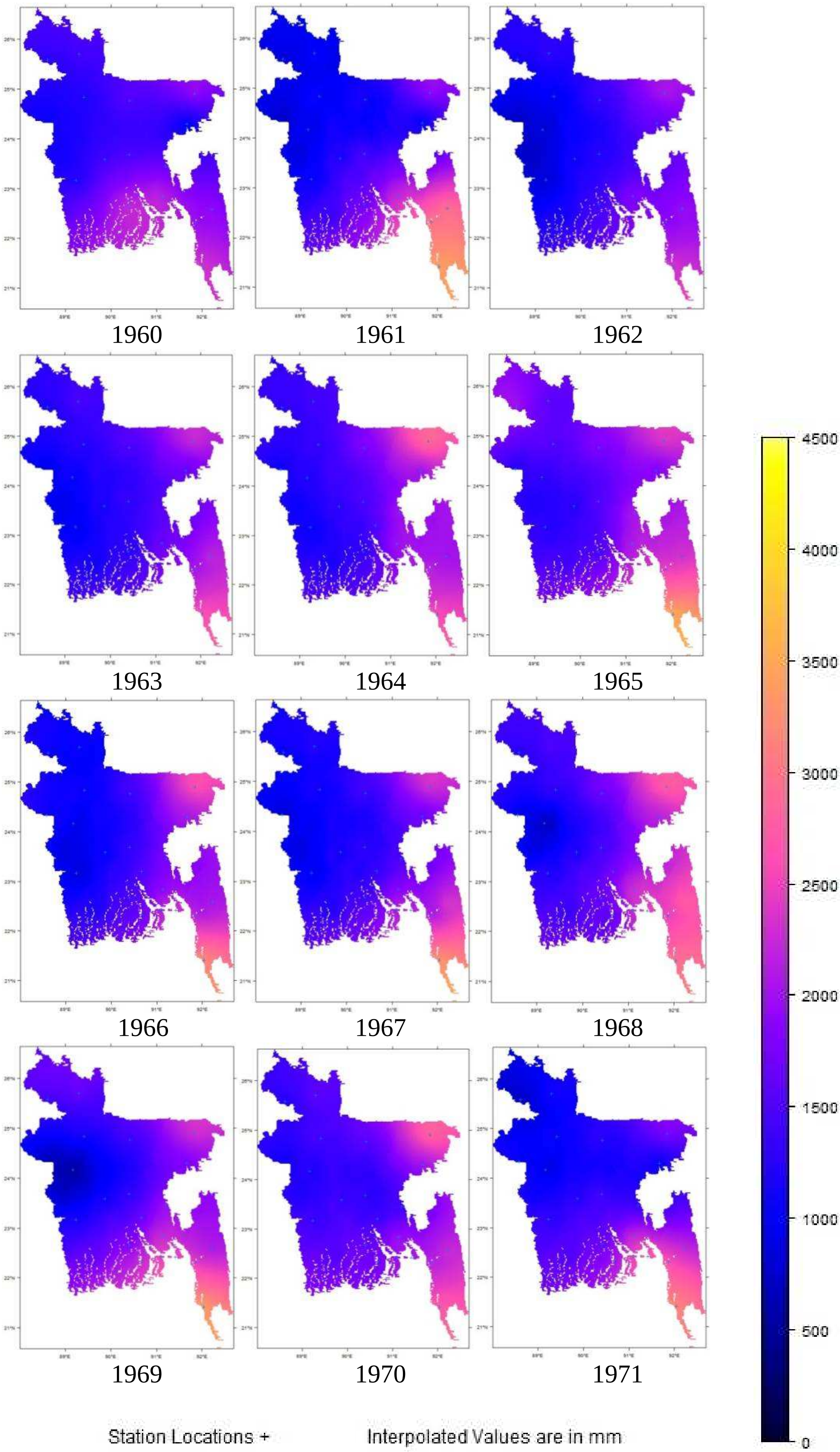
Station Locations +
Interpolated Values are in mm

## A.4 UK Surfaces of PRCPTOT

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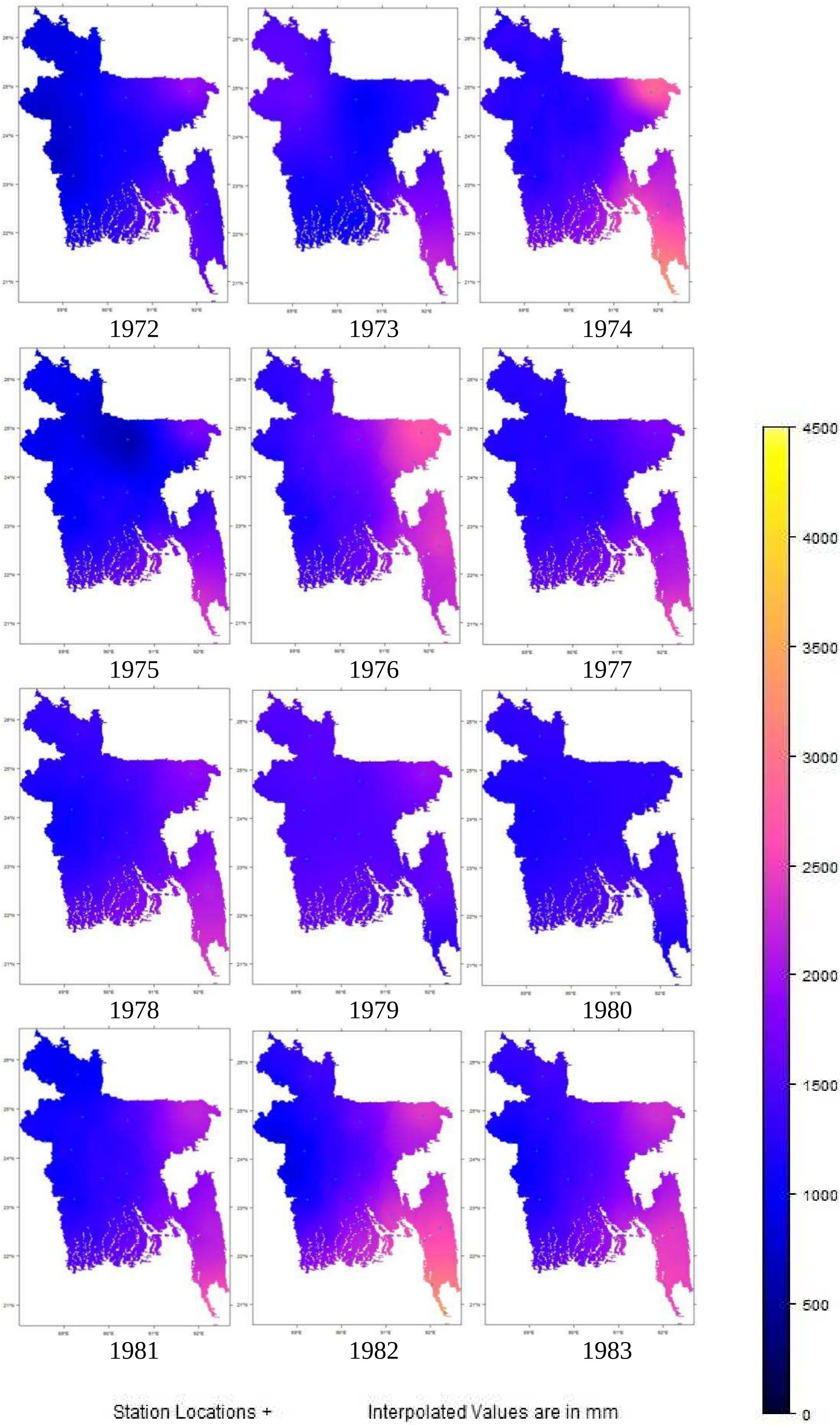
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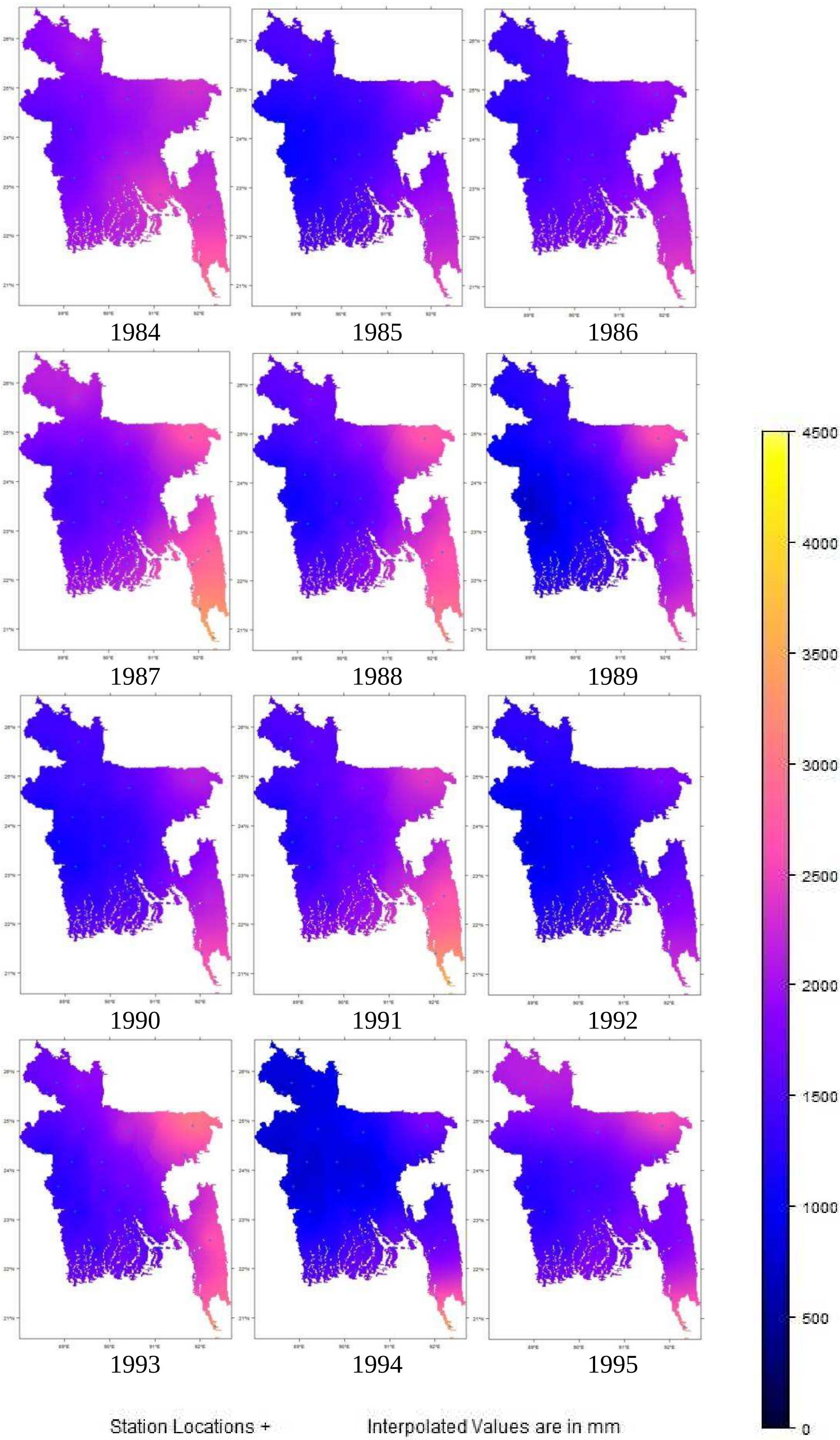
Station Locations +
Interpolated Values are in mm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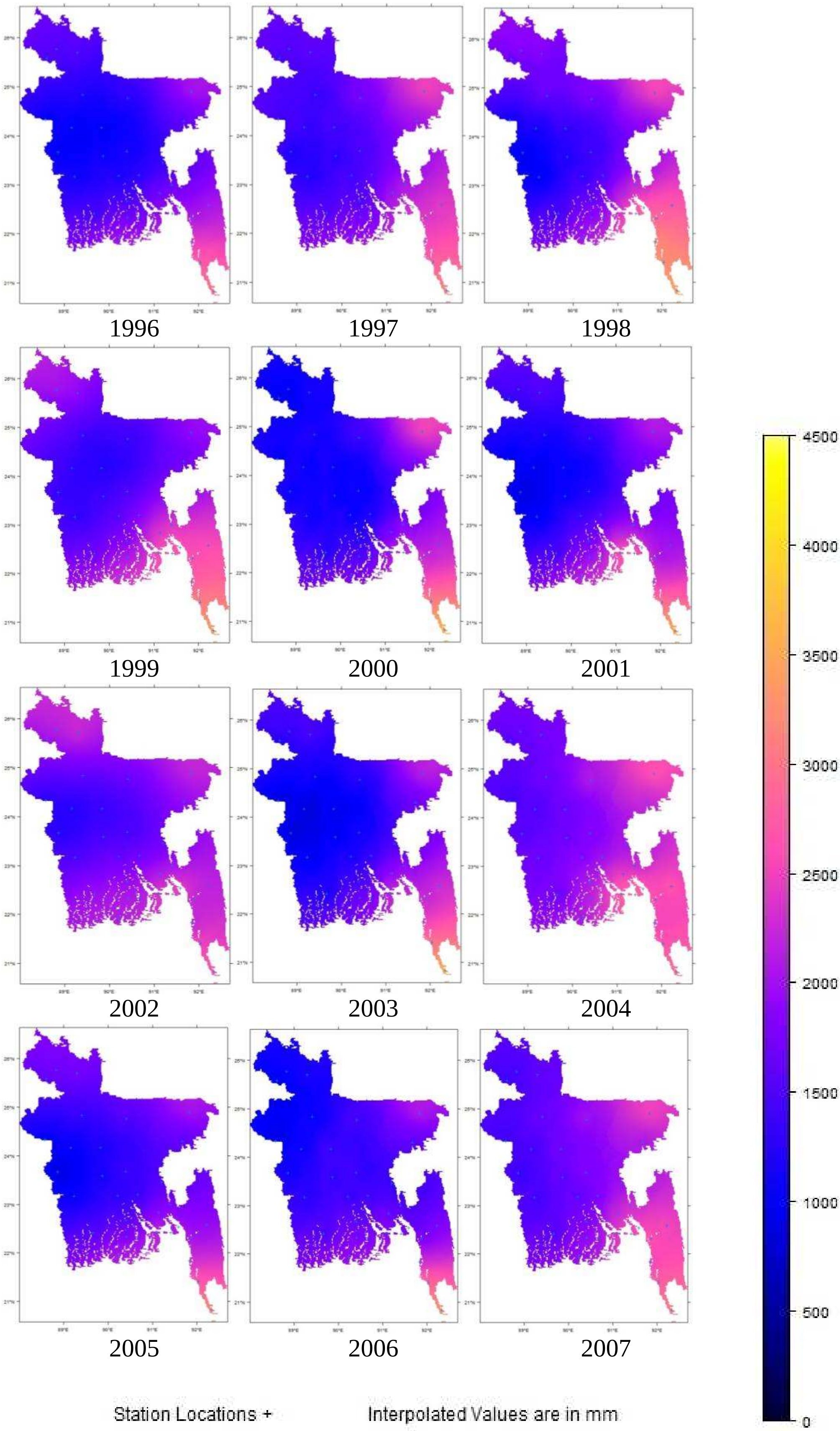
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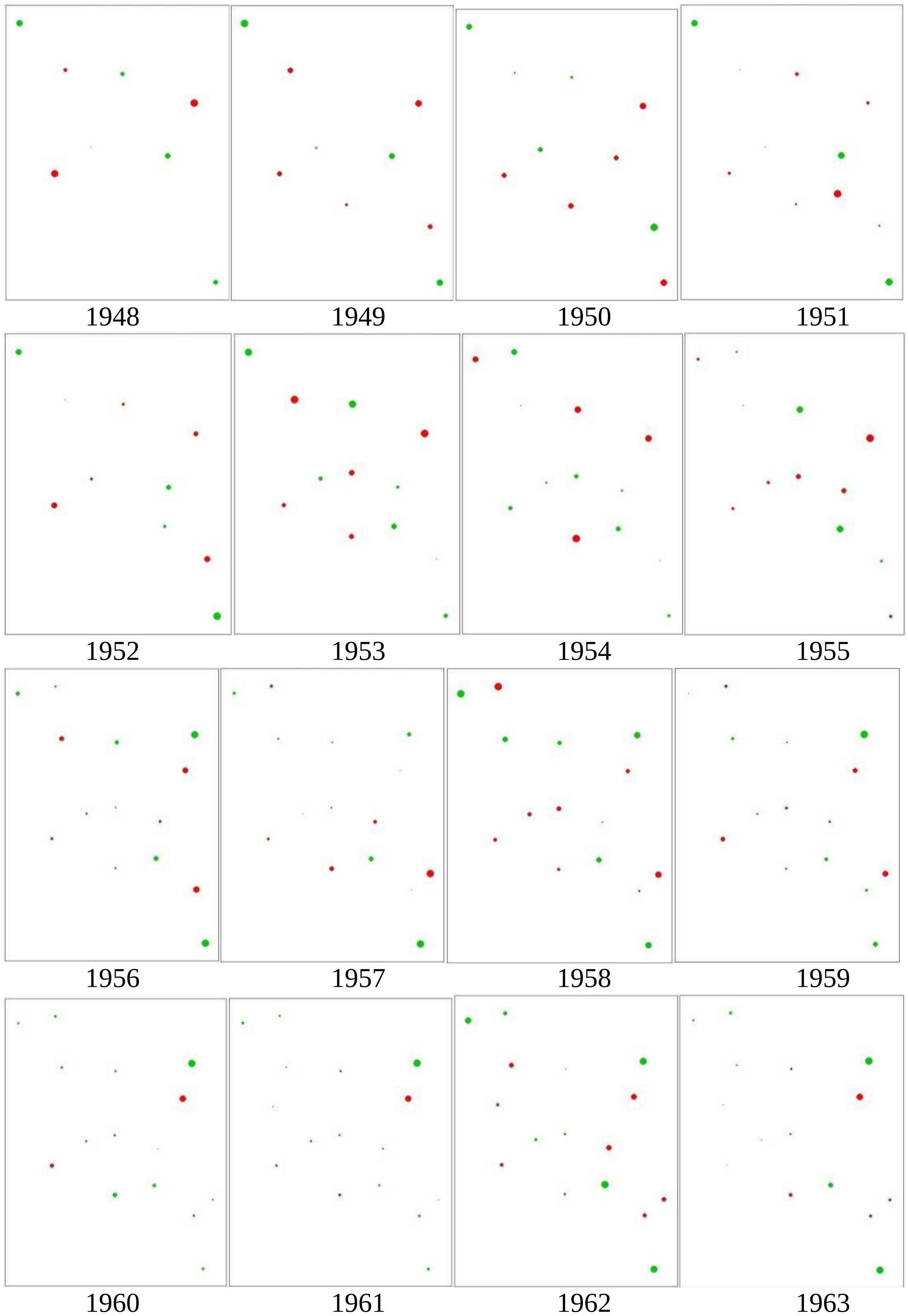
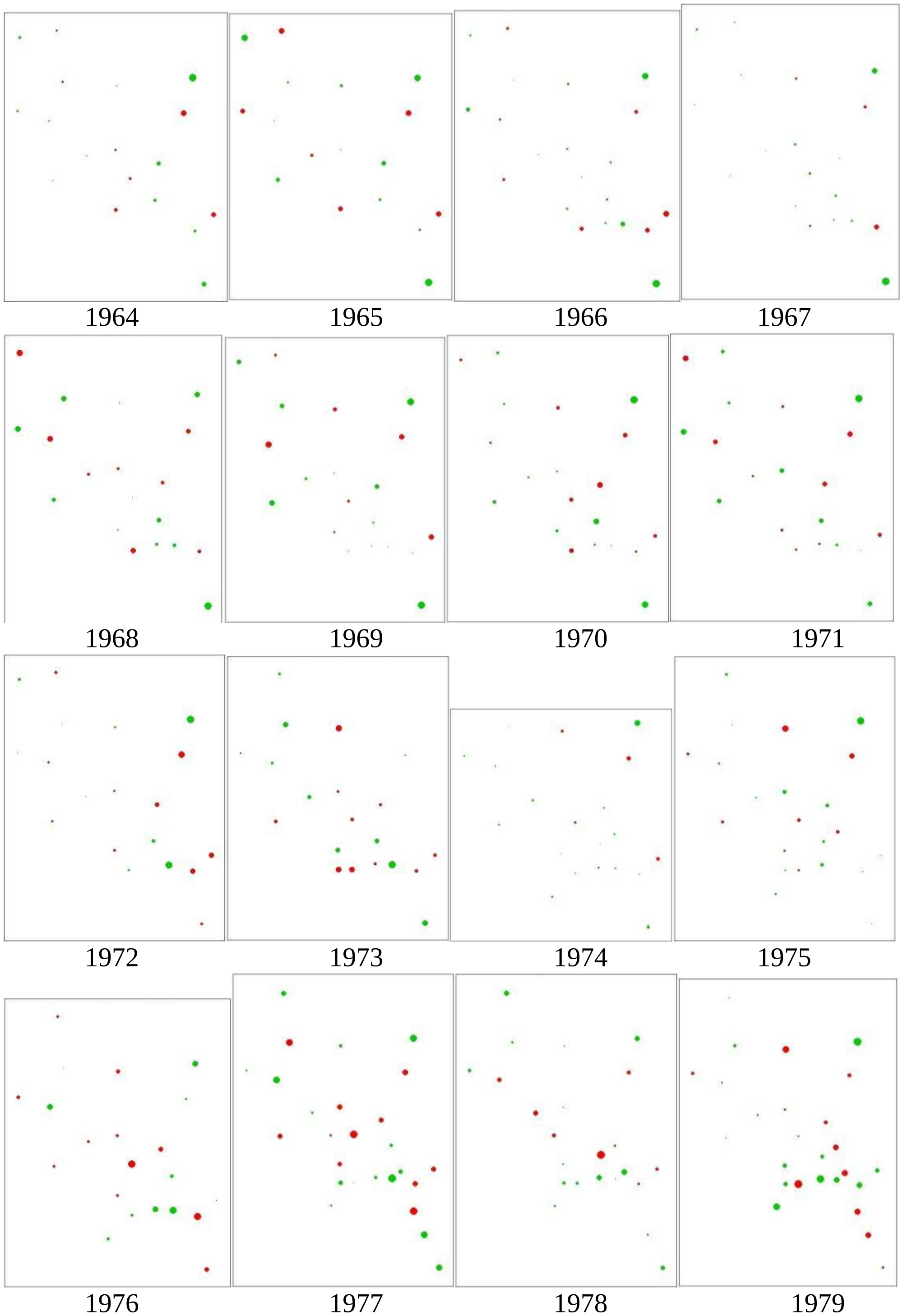
Station Locations +
Interpolated Values are in mm

## A.5 Residual Plots of TPS Surfaces of PRCPTOT



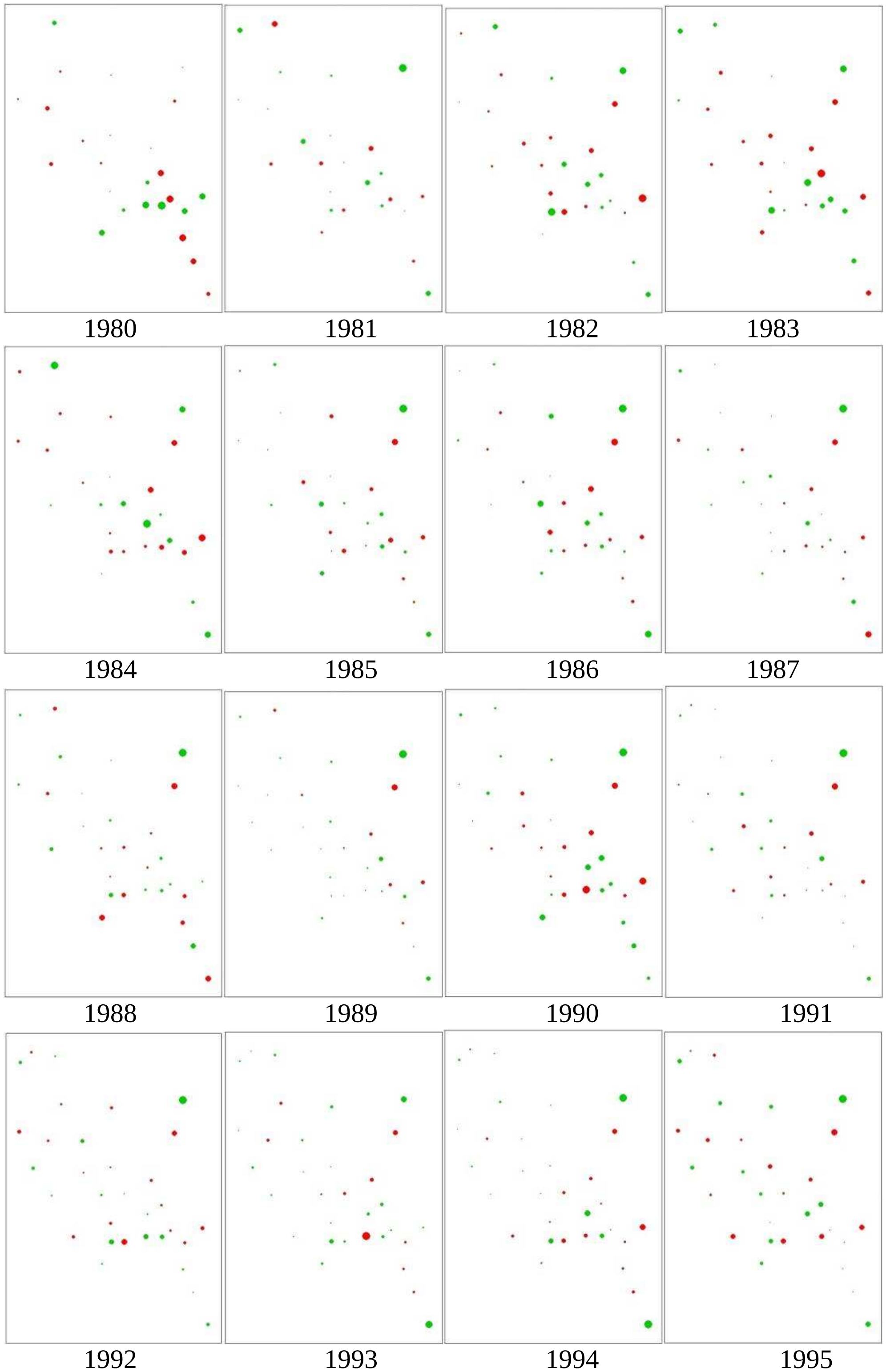

Over Estimated Values
Under Estimated Values
*Note: The relative sizes of the circles depict the degree of over or under estimation i.e the minimum the size of the circle the minimum the difference between measured and predicted values




Over Estimated Values

Under Estimated Values

*Note: The relative sizes of the circles depict the degree of over or under estimation i.e the minimum the size of the circle the minimum the difference between measured and predicted values



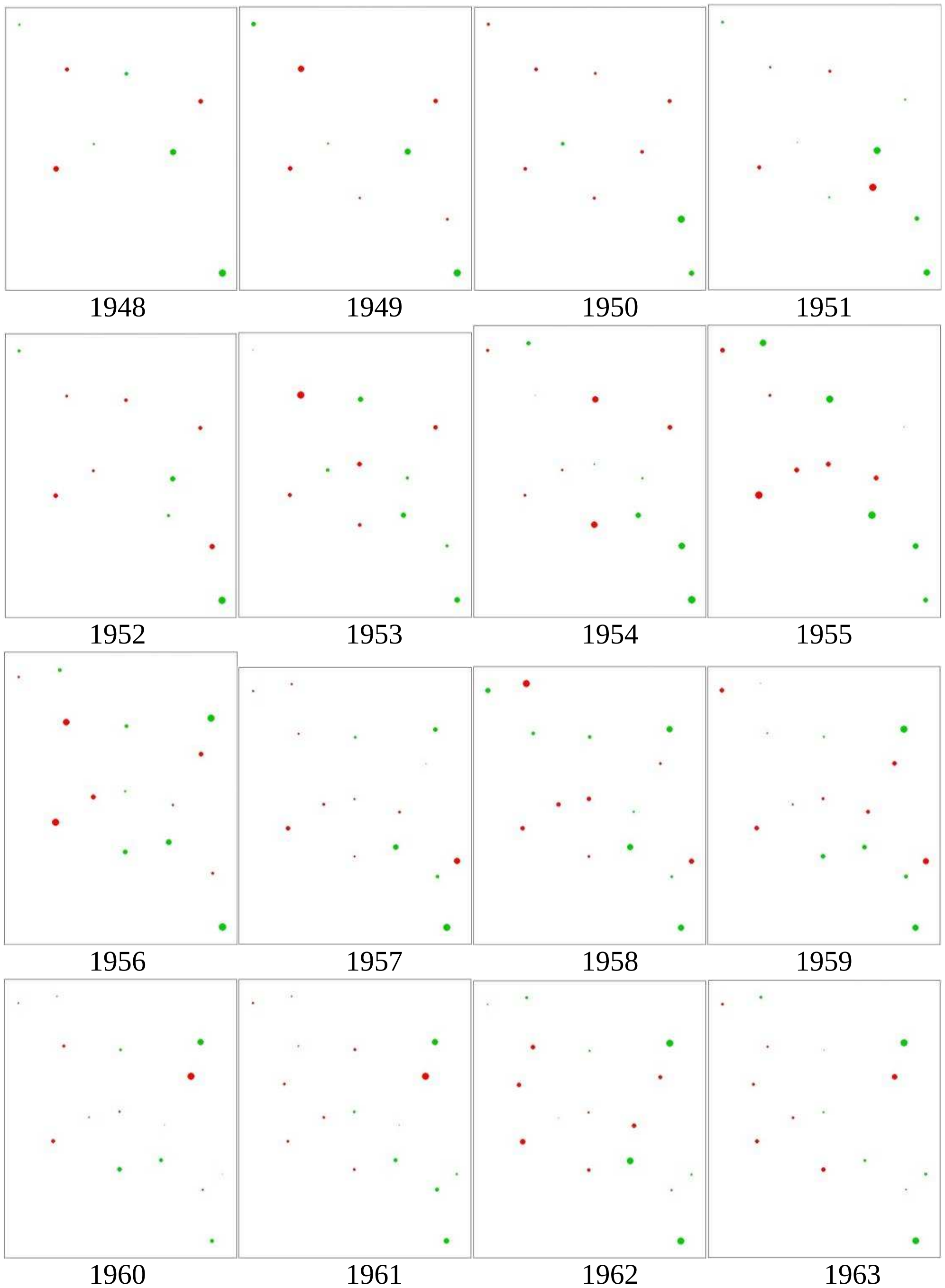

Over Estimated Values

Under Estimated Values

*Note: The relative sizes of the circles depict the degree of over or under estimation i.e the minimum the size of the circle the minimum the difference between measured and predicted values


Over Estimated Values
Under Estimated Values
*Note: The relative sizes of the circles depict the degree of over or under estimation i.e the minimum the size of the circle the minimum the difference between measured and predicted values

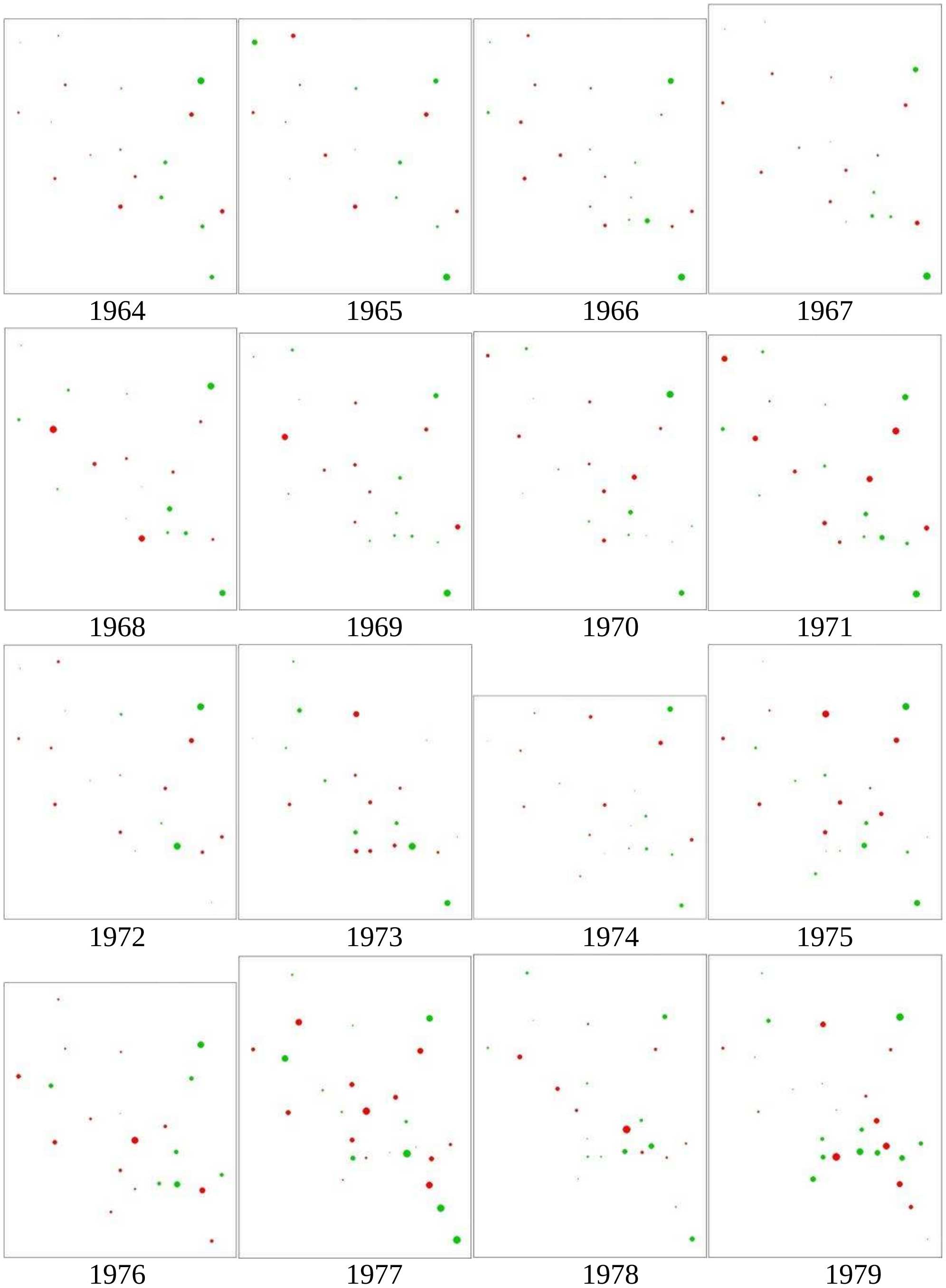
## A.6 Residual Plots of IDW Surfaces of PRCPTOT




Over Estimated Values

Under Estimated Values

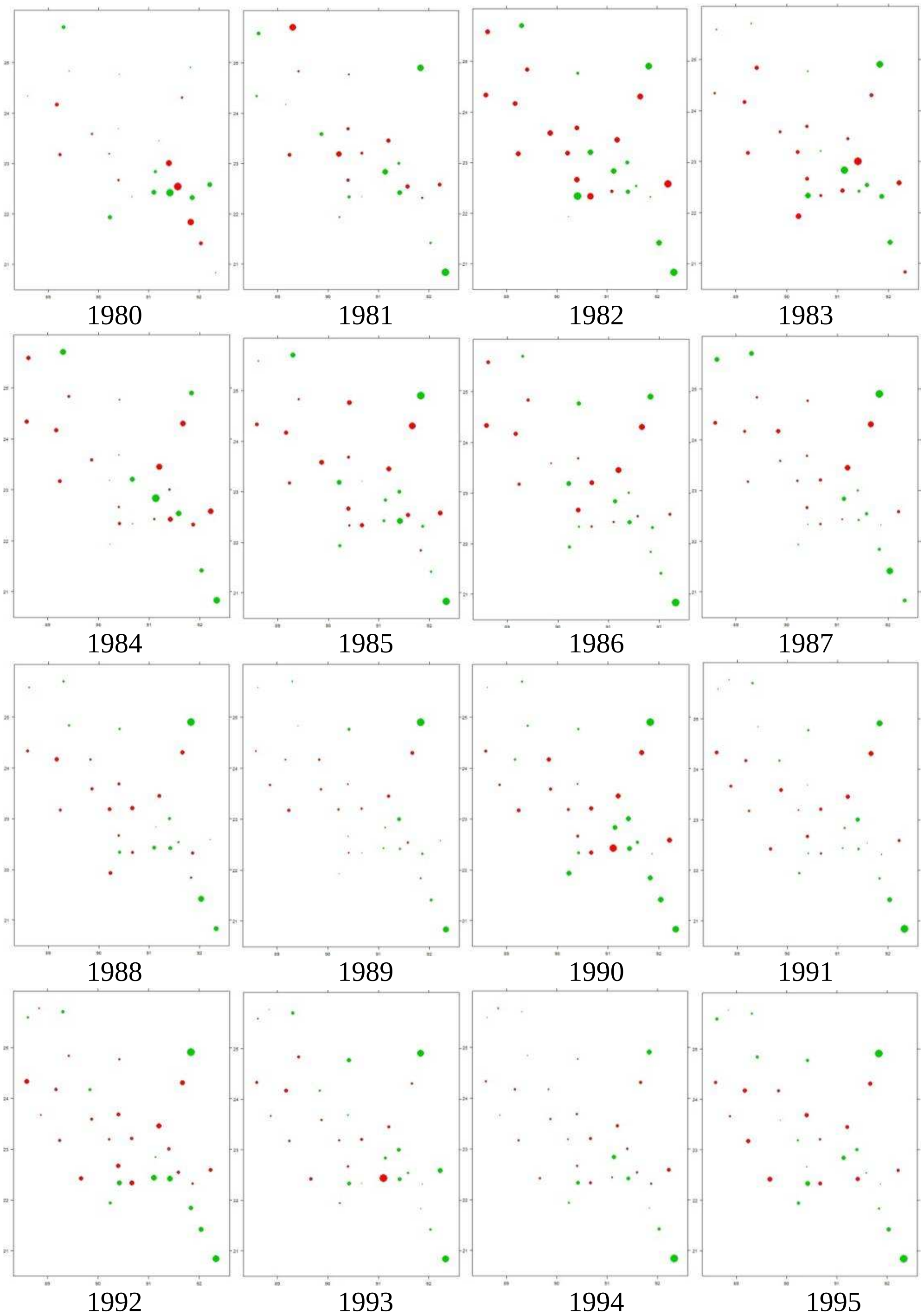
*Note: The relative sizes of the circles depict the degree of over or under estimation i.e the minimum the size of the circle the minimum the difference between measured and predicted values



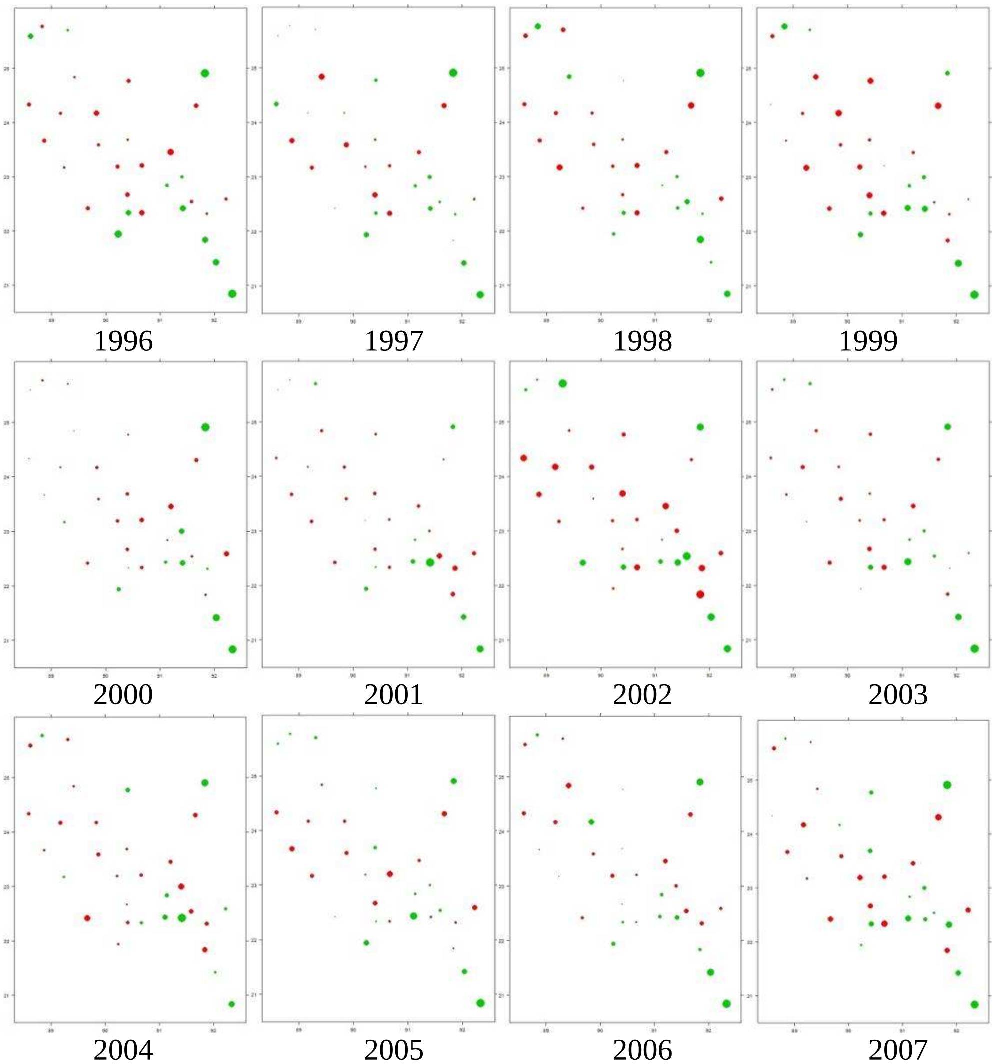

Over Estimated Values

Under Estimated Values

*Note: The relative sizes of the circles depict the degree of over or under estimation i.e the minimum the size of the circle the minimum the difference between measured and predicted values




Over Estimated Values
Under Estimated Values
*Note: The relative sizes of the circles depict the degree of over or under estimation i.e the minimum the size of the circle the minimum the difference between measured and predicted values

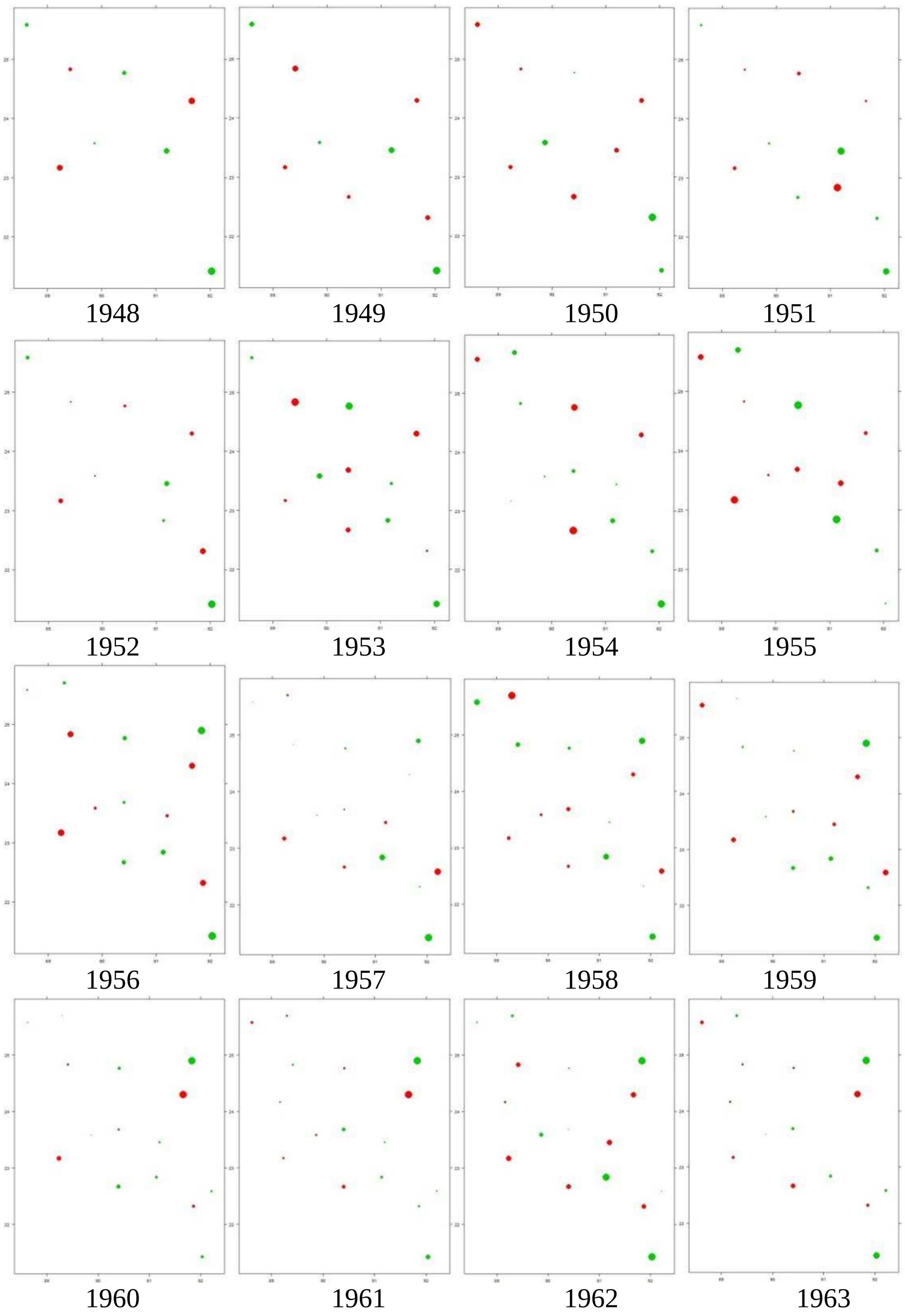



Over Estimated Values

Under Estimated Values

*Note: The relative sizes of the circles depict the degree of over or under estimation i.e the minimum the size of the circle the minimum the difference between measured and predicted values

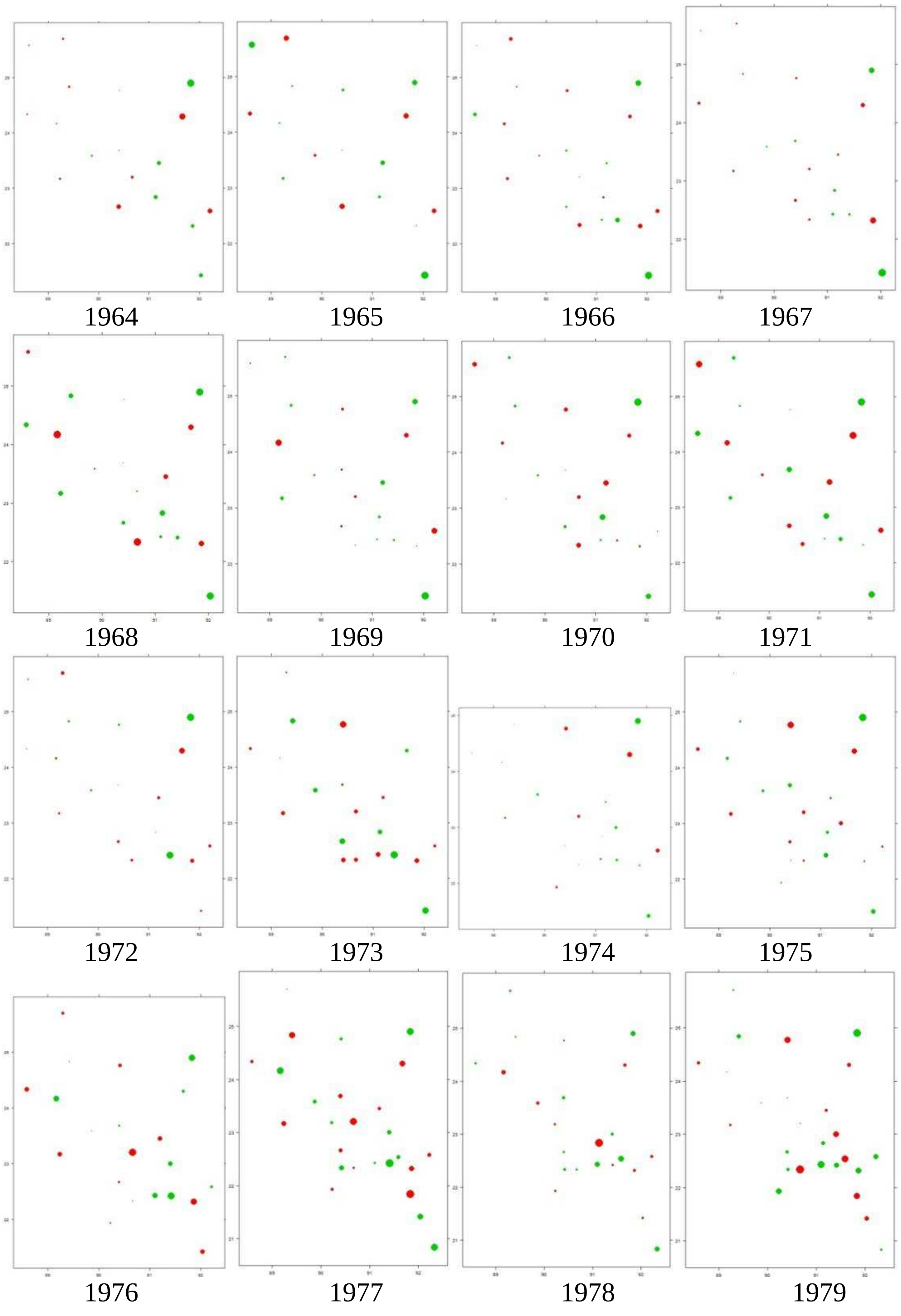
## A.7 Residual Plots of OK Surfaces of PRCPTOT




Over Estimated Values

Under Estimated Values

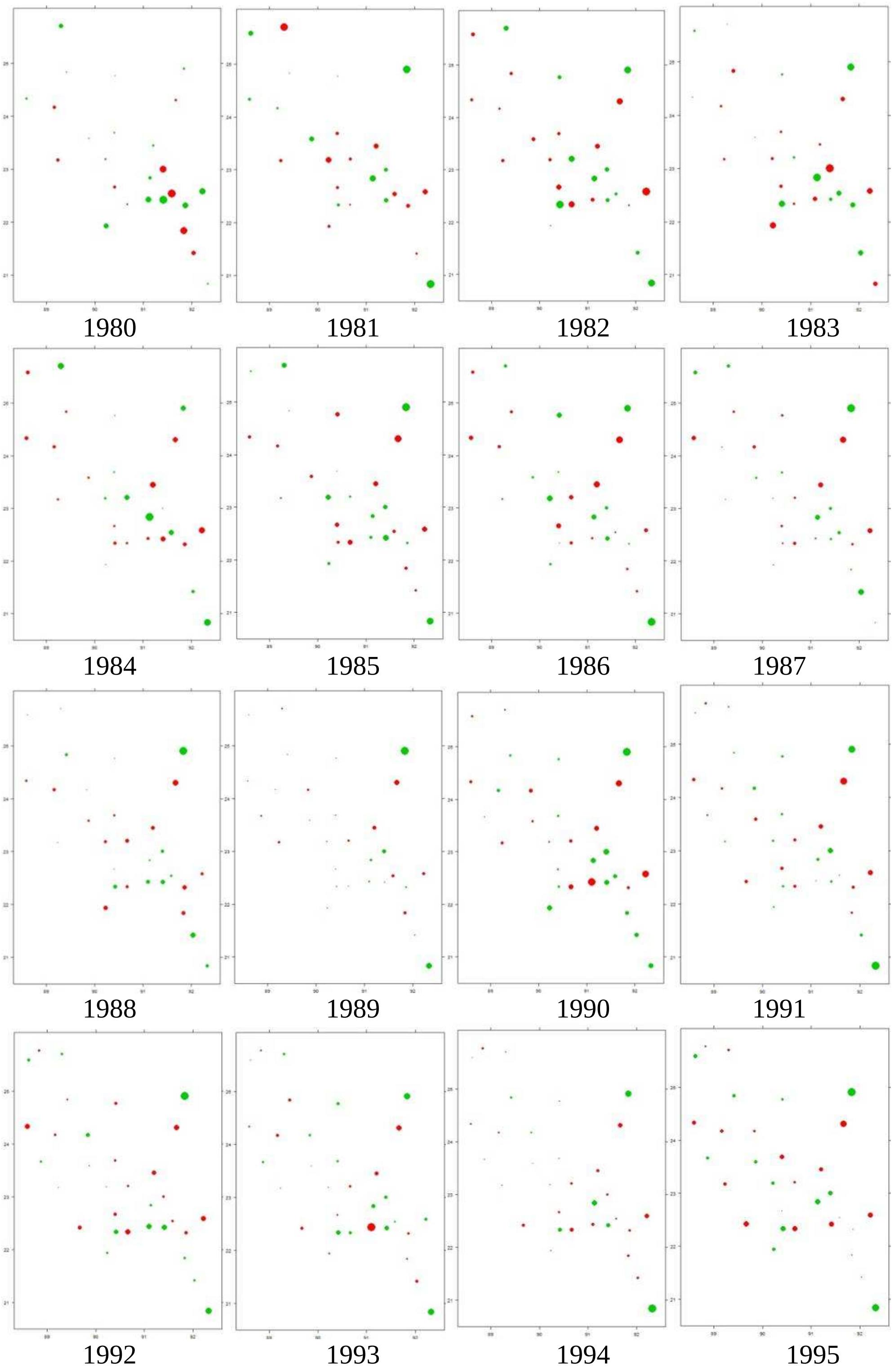
*Note: The relative sizes of the circles depict the degree of over or under estimation i.e the minimum the size of the circle the minimum the difference between measured and predicted values




Over Estimated Values

Under Estimated Values

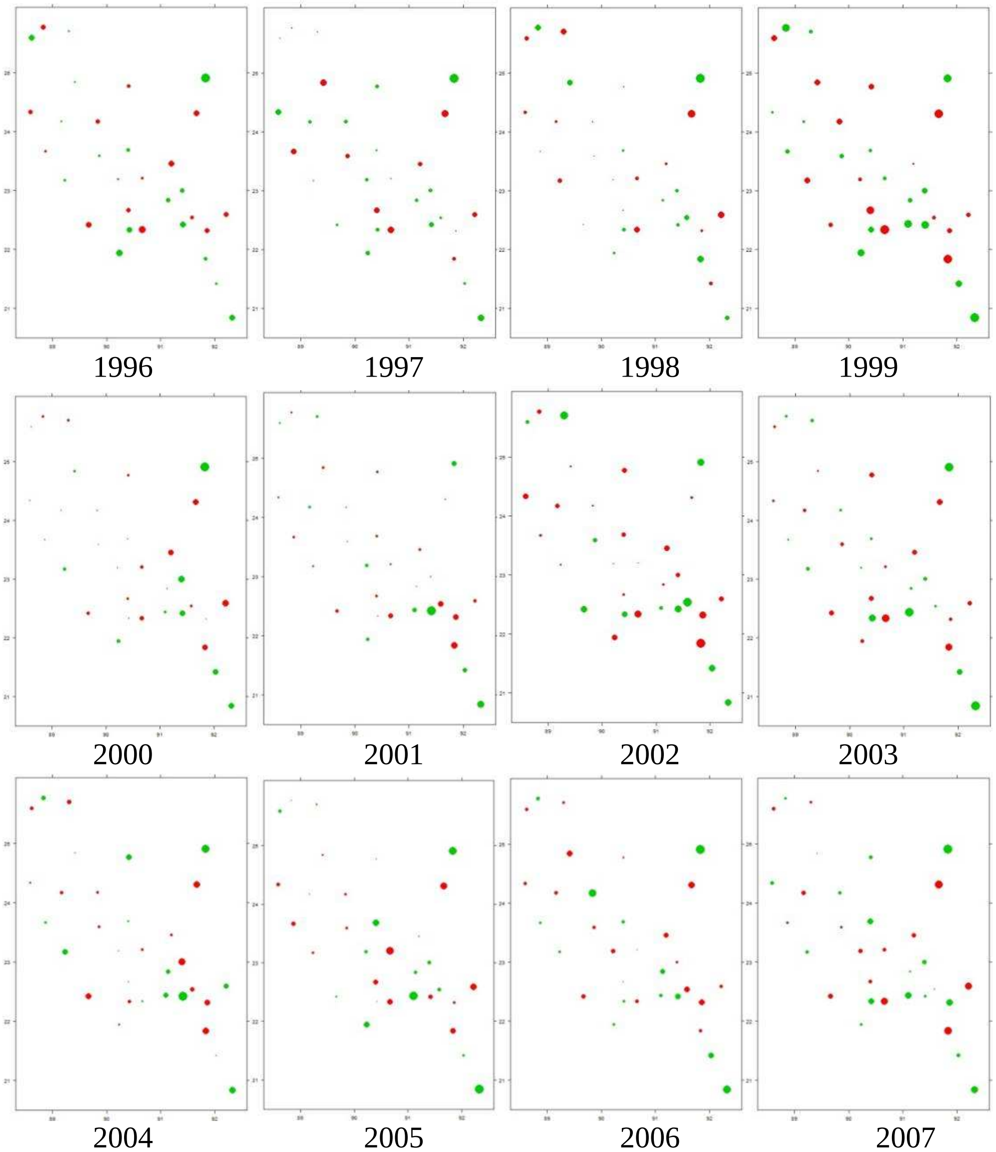
*Note: The relative sizes of the circles depict the degree of over or under estimation i.e the minimum the size of the circle the minimum the difference between measured and predicted values




Over Estimated Values

Under Estimated Values

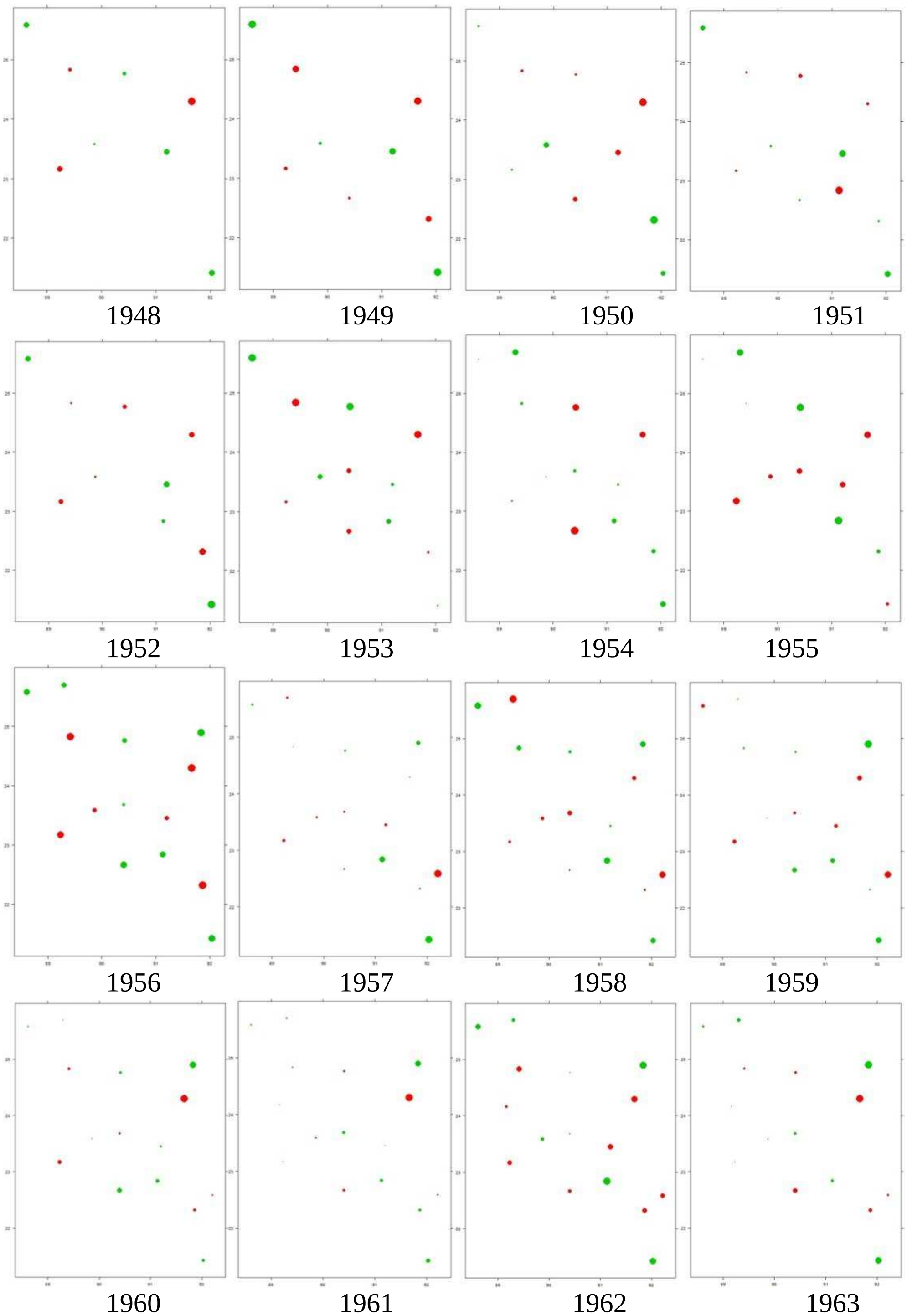
*Note: The relative sizes of the circles depict the degree of over or under estimation i.e the minimum the size of the circle the minimum the difference between measured and predicted values



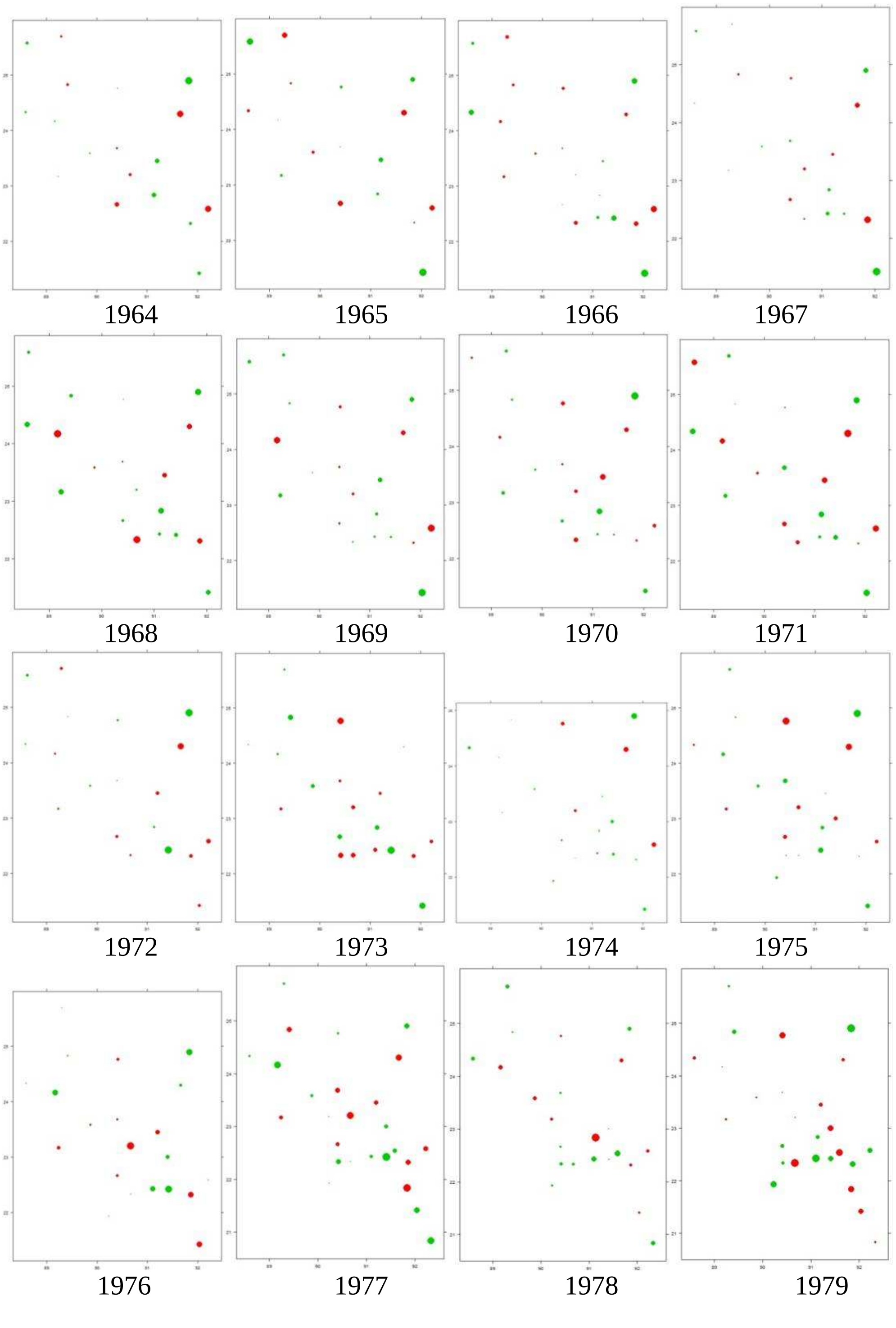

Over Estimated Values
Under Estimated Values

*Note: The relative sizes of the circles depict the degree of over or under estimation i.e the minimum the size of the circle the minimum the difference between measured and predicted values

## A.8 Residual Plots of UK Surfaces of PRCPTOT



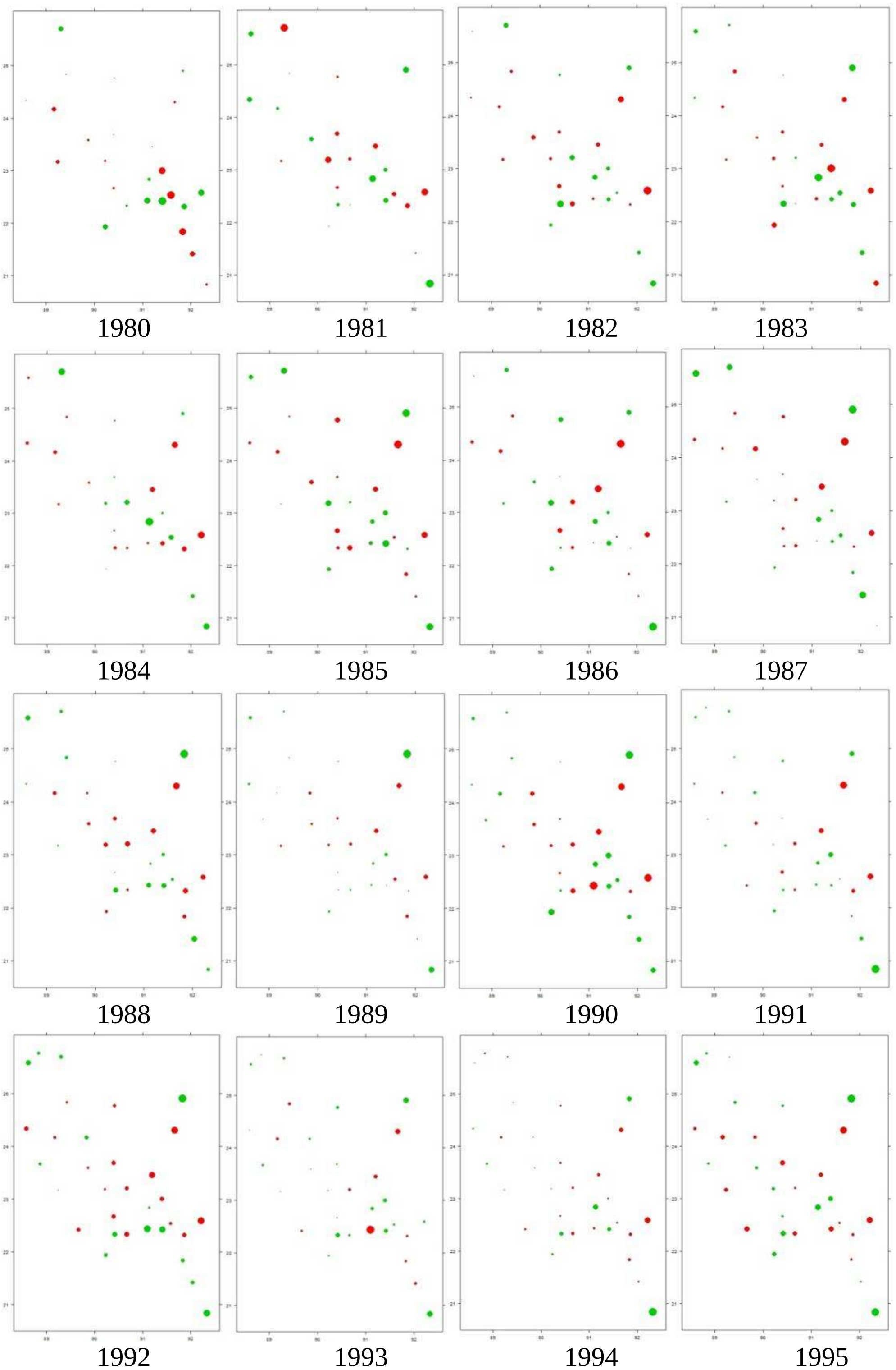

Over Estimated Values
Under Estimated Values
*Note: The relative sizes of the circles depict the degree of over or under estimation i.e the minimum the size of the circle the minimum the difference between measured and predicted values




Over Estimated Values

Under Estimated Values

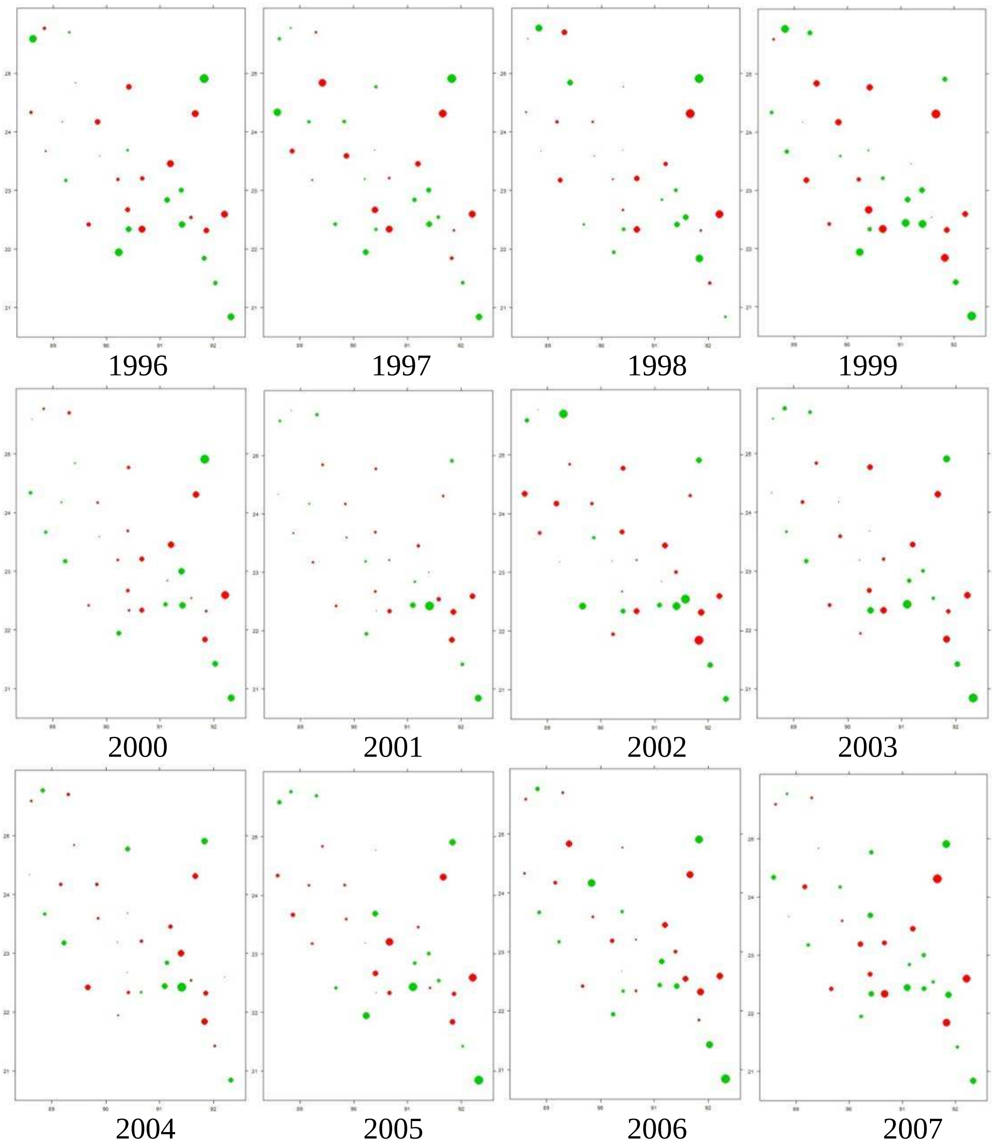
*Note: The relative sizes of the circles depict the degree of over or under estimation i.e the minimum the size of the circle the minimum the difference between measured and predicted values




Over Estimated Values

Under Estimated Values

*Note: The relative sizes of the circles depict the degree of over or under estimation i.e the minimum the size of the circle the minimum the difference between measured and predicted values

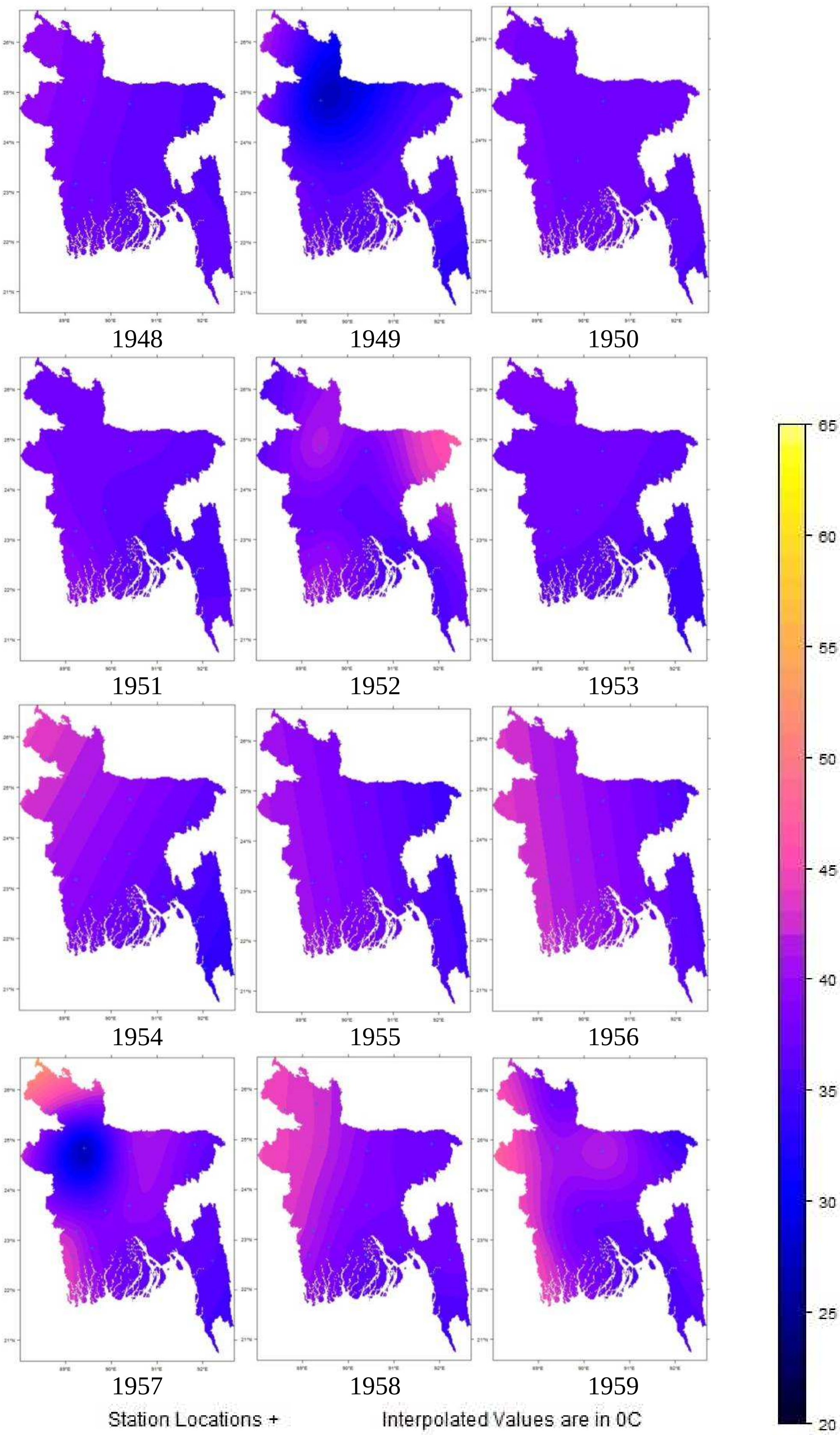


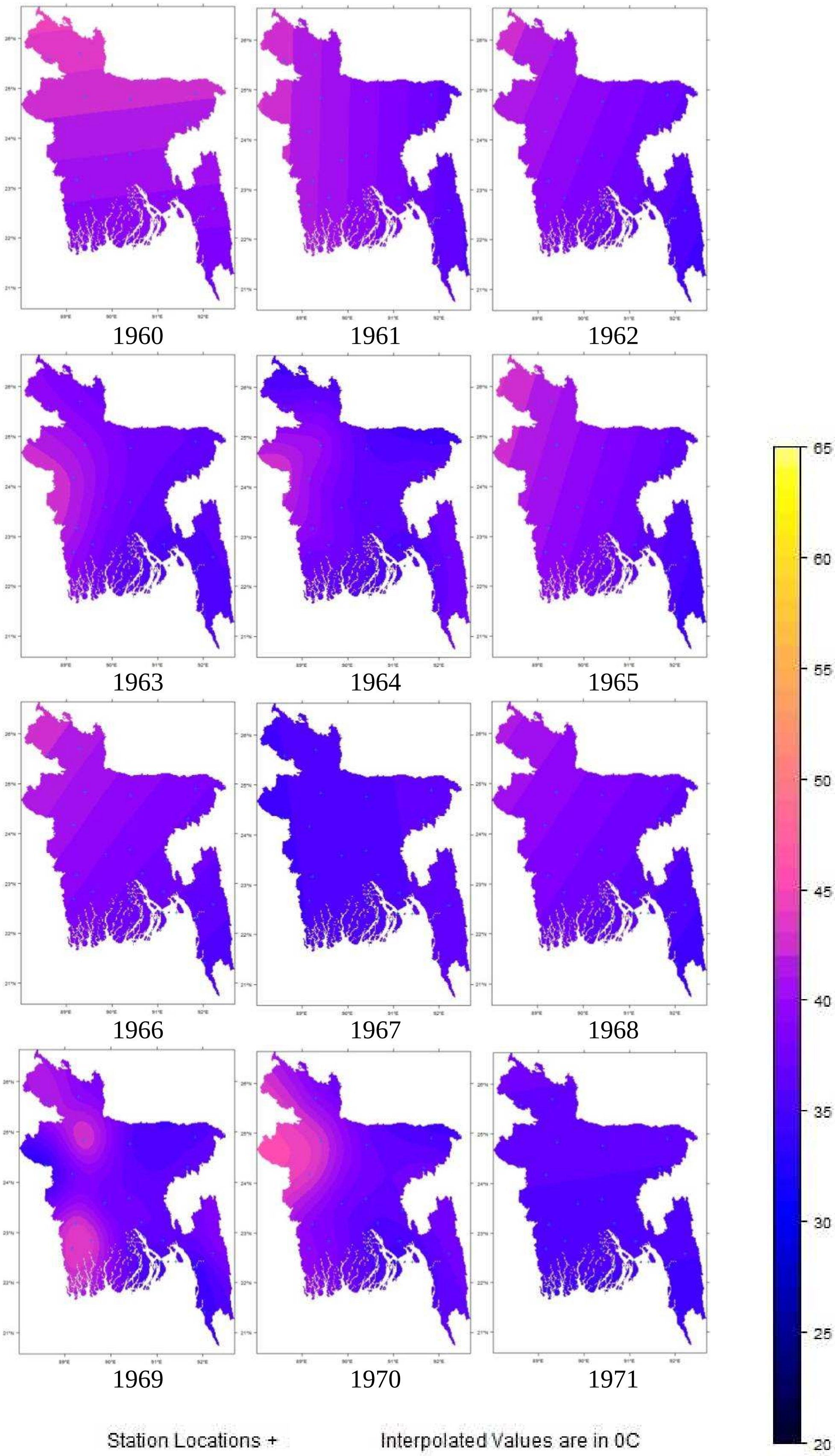

Over Estimated Values

Under Estimated Values

*Note: The relative sizes of the circles depict the degree of over or under estimation i.e the minimum the size of the circle the minimum the difference between measured and predicted values

## A.9 TPS Surfaces of TXx

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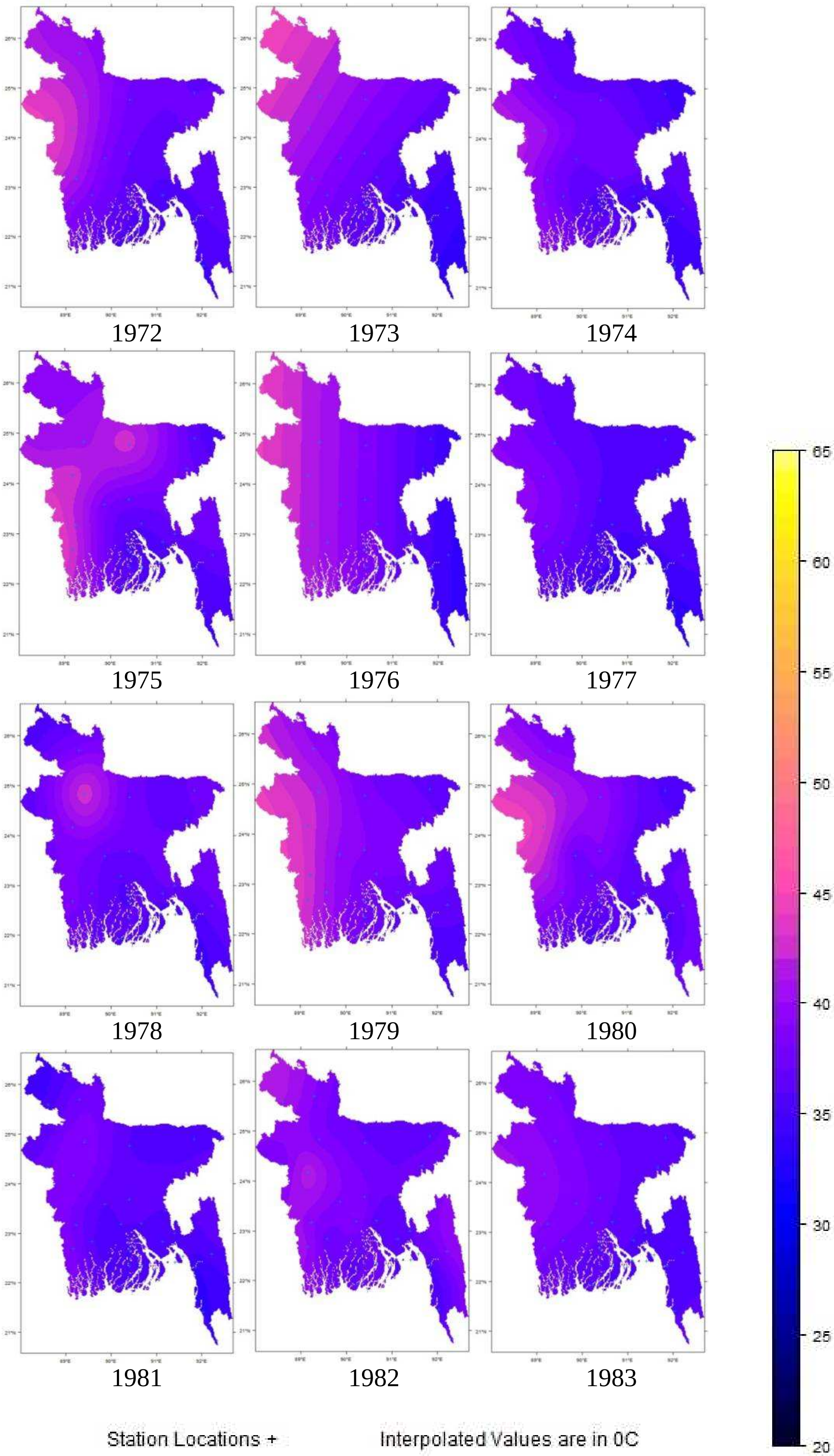
Station Locations +
Interpolated Values are in 0C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Station Locations +
Interpolated Values are in 0C
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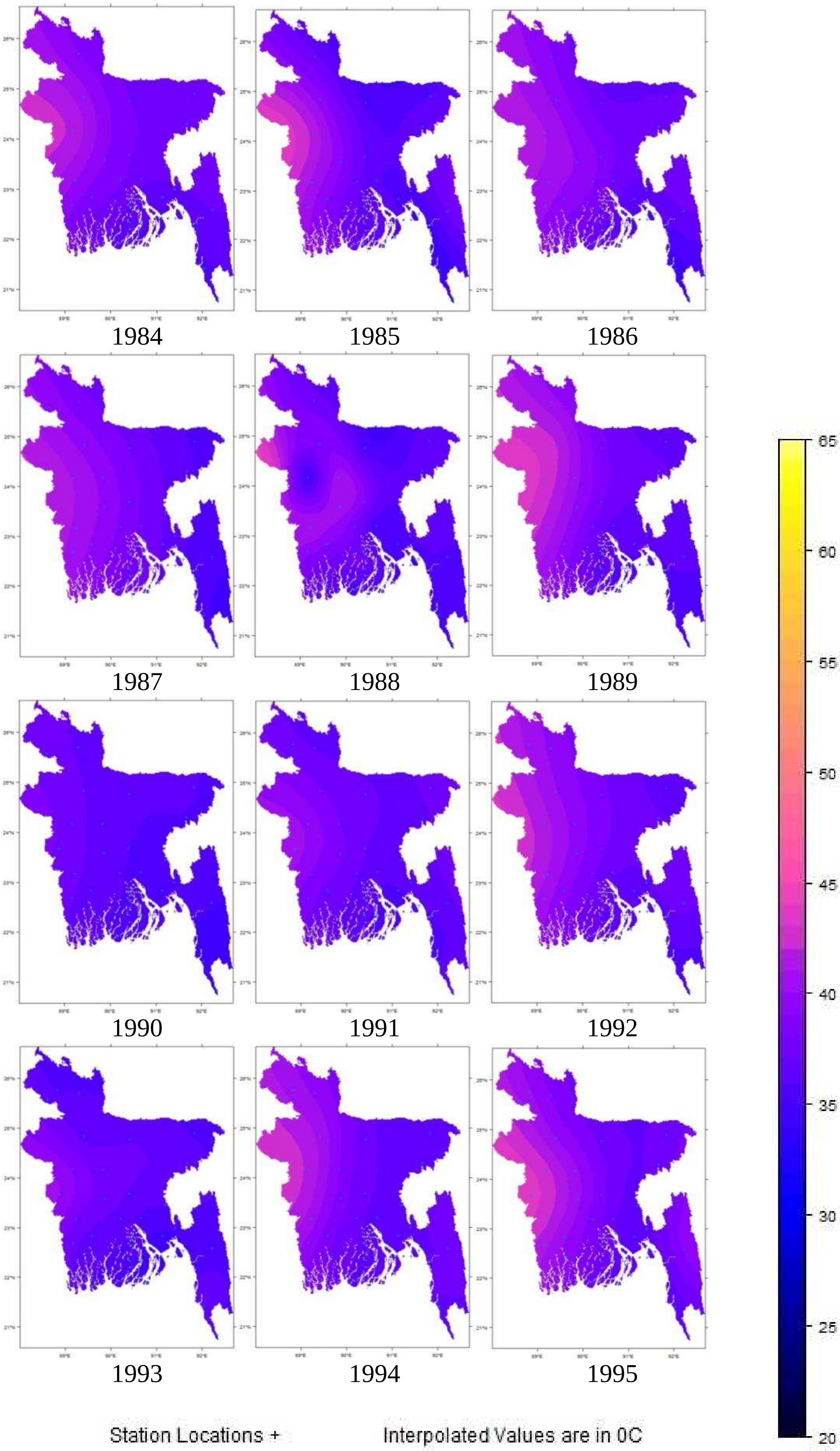
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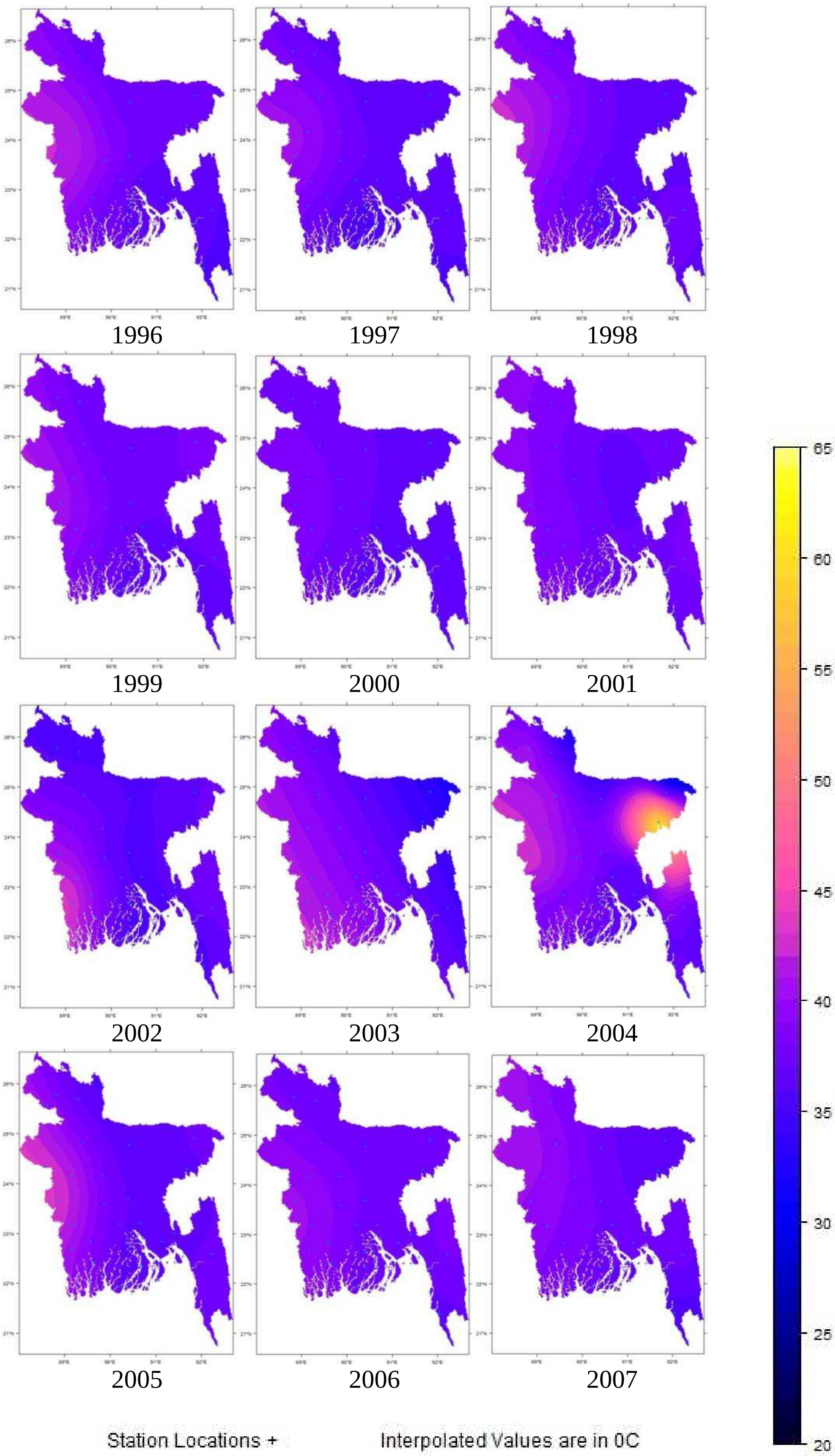
Interpolated Values are in 0C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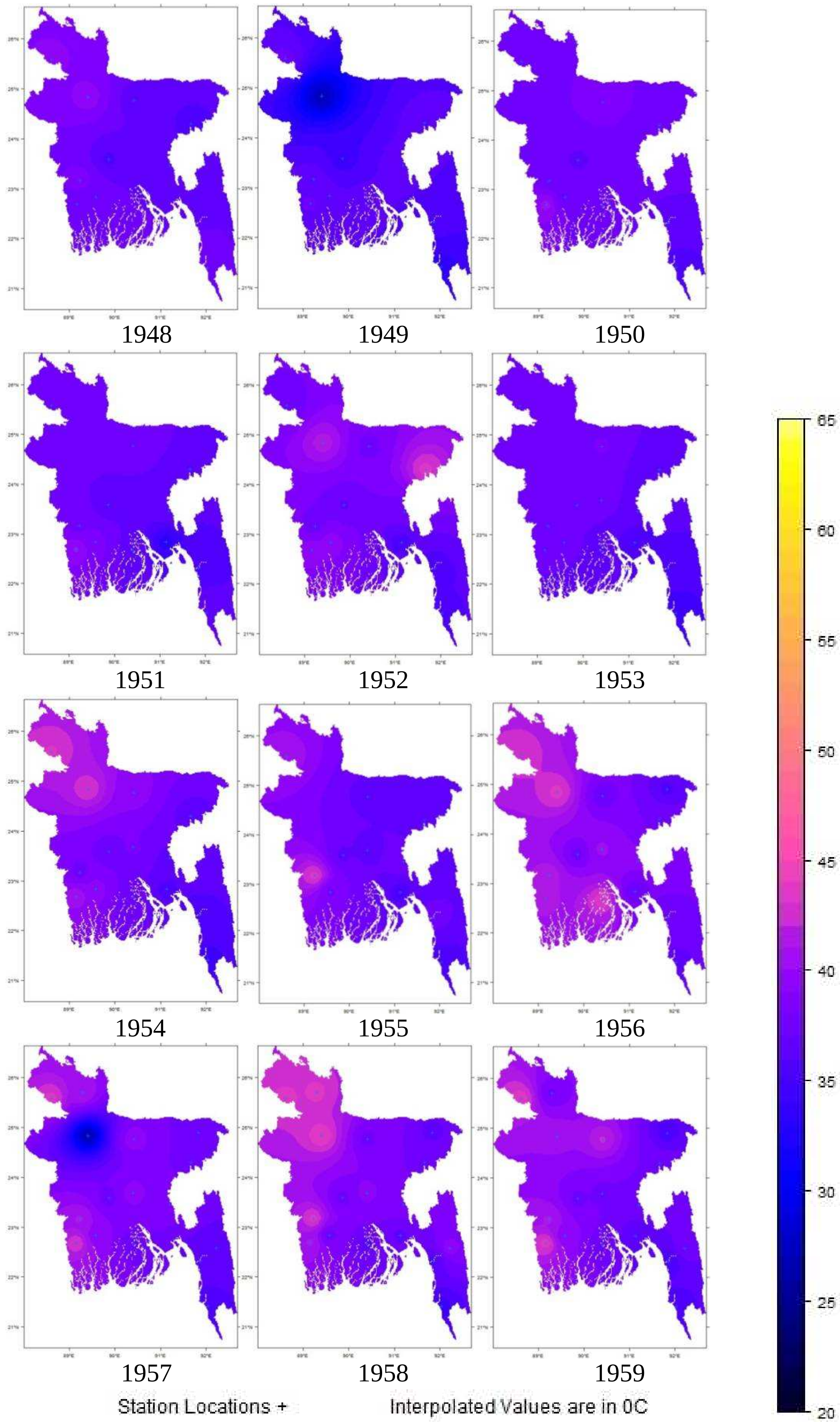
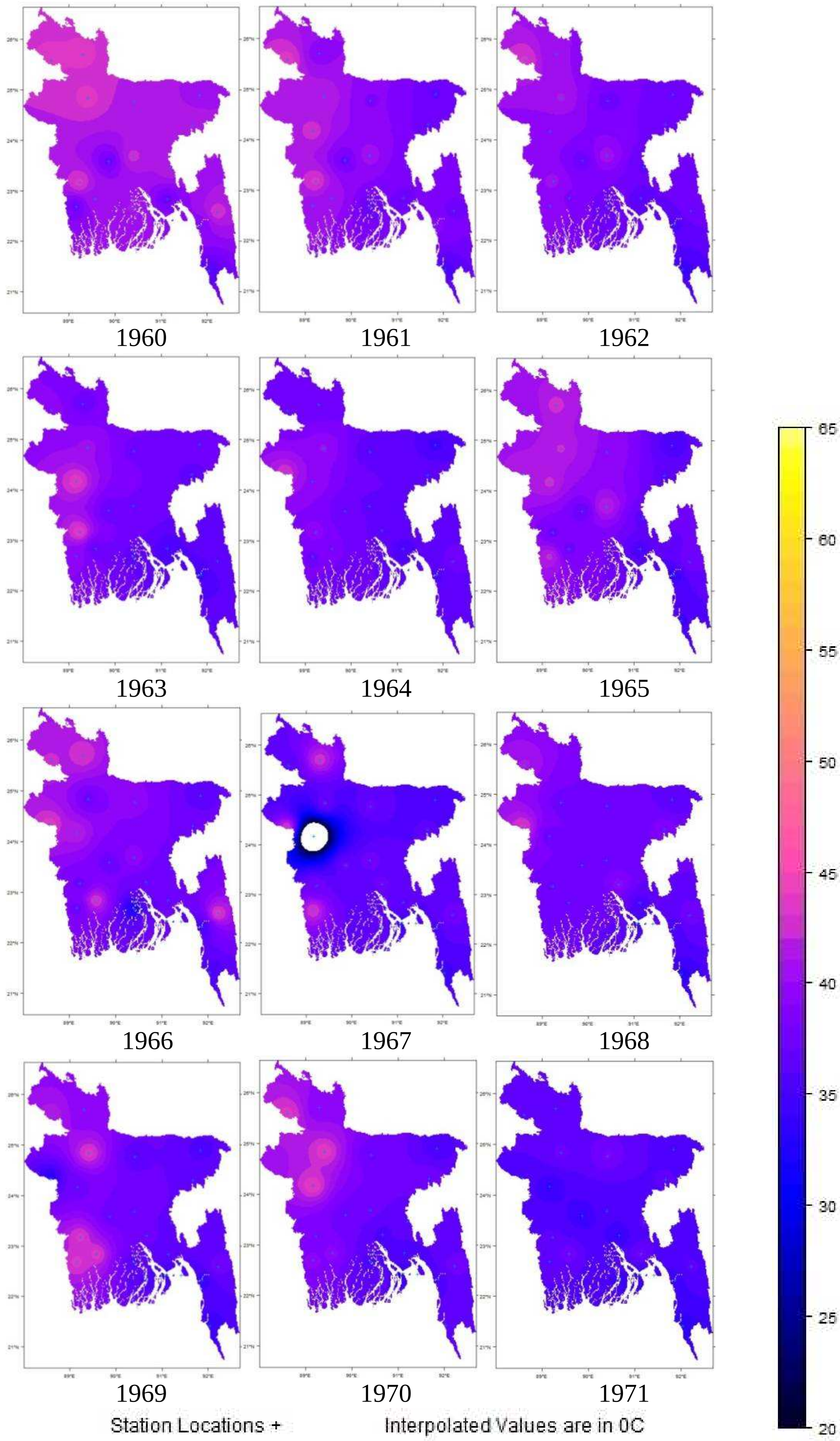
Station Locations +
Interpolated Values are in 0C

## A.10 IDW Surfaces of TXx

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Interpolated Values are in 0C
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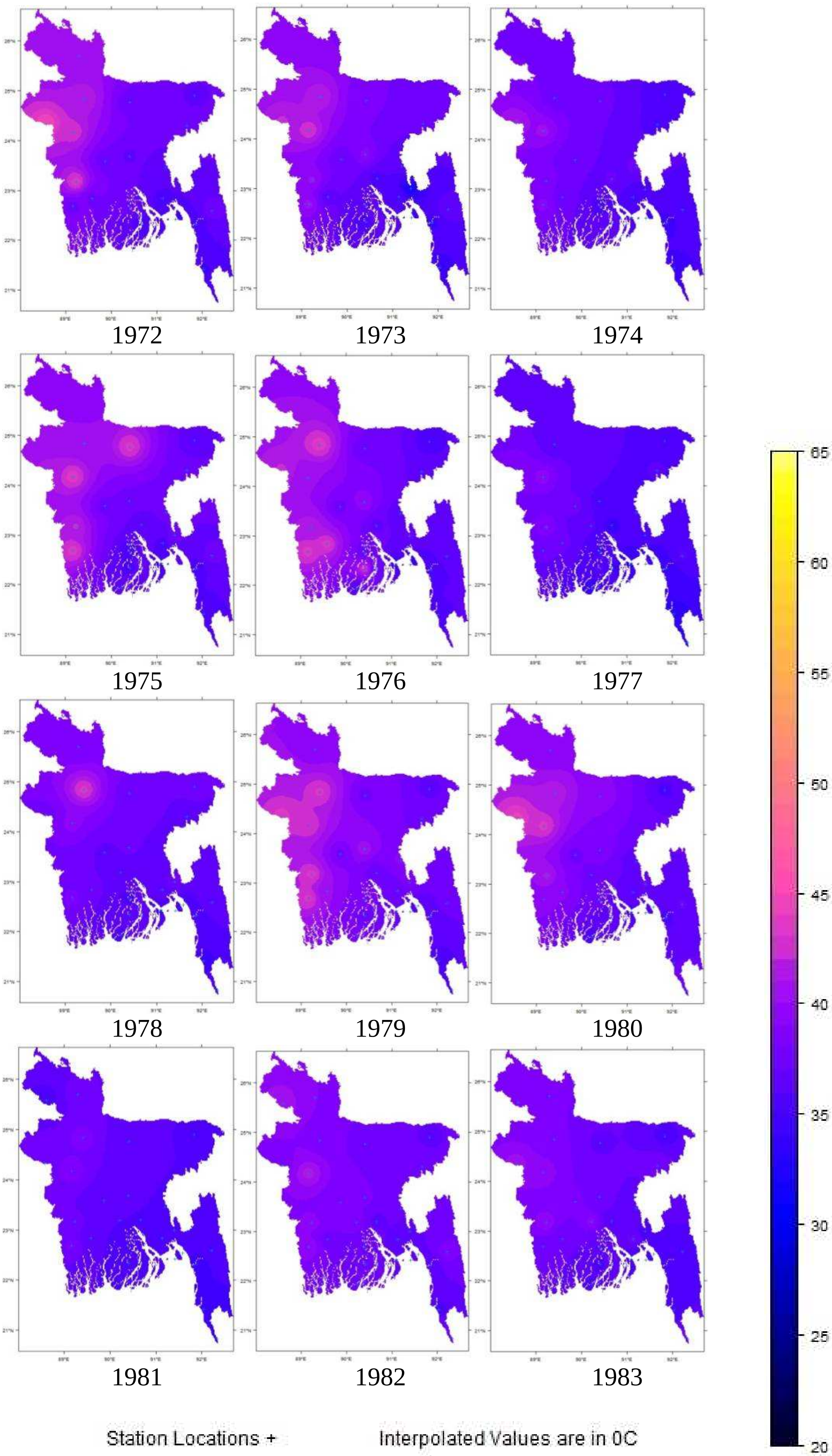
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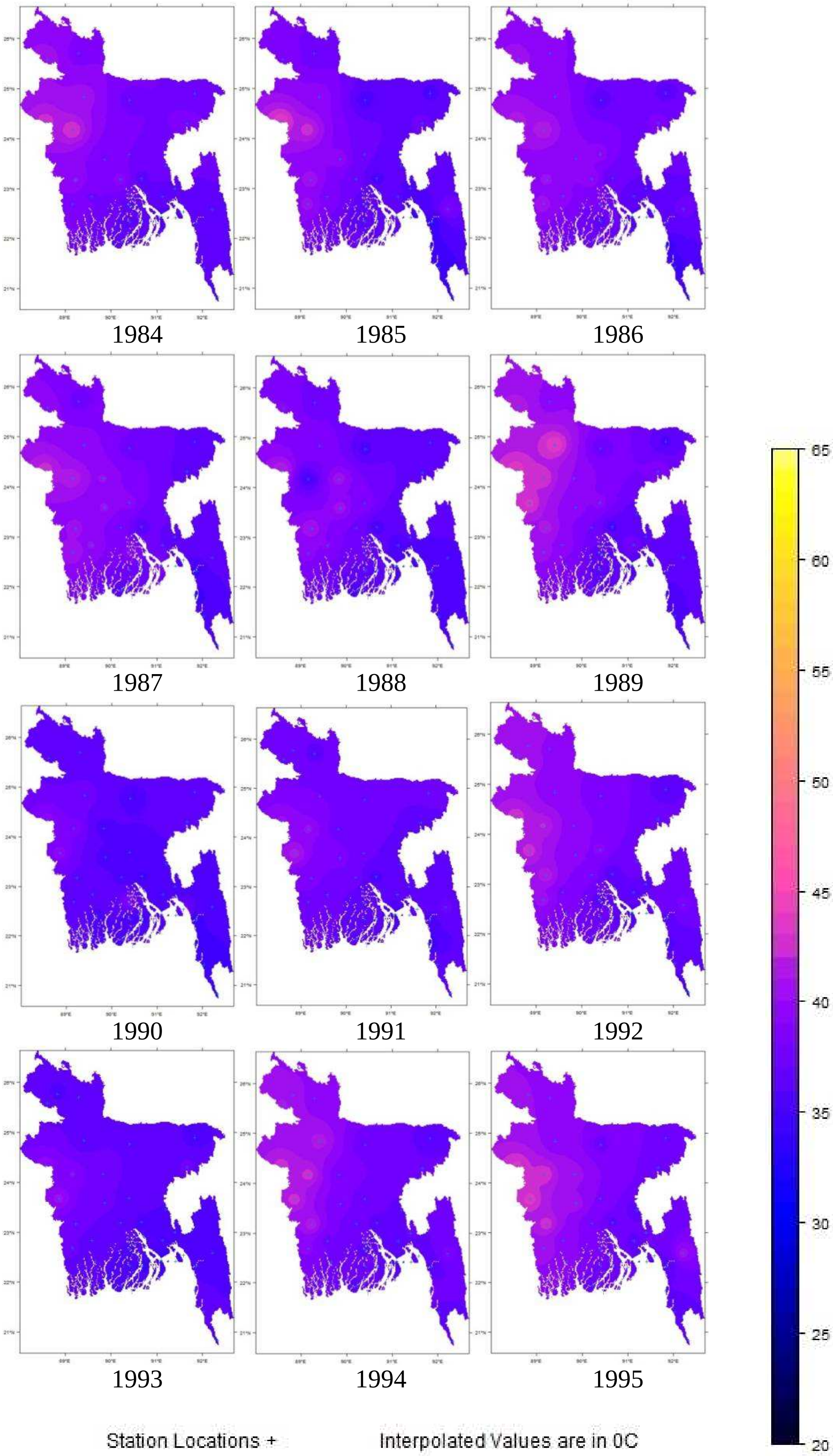
Station Locations +
Interpolated Values are in 0C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Station Locations +
Interpolated Values are in 0C
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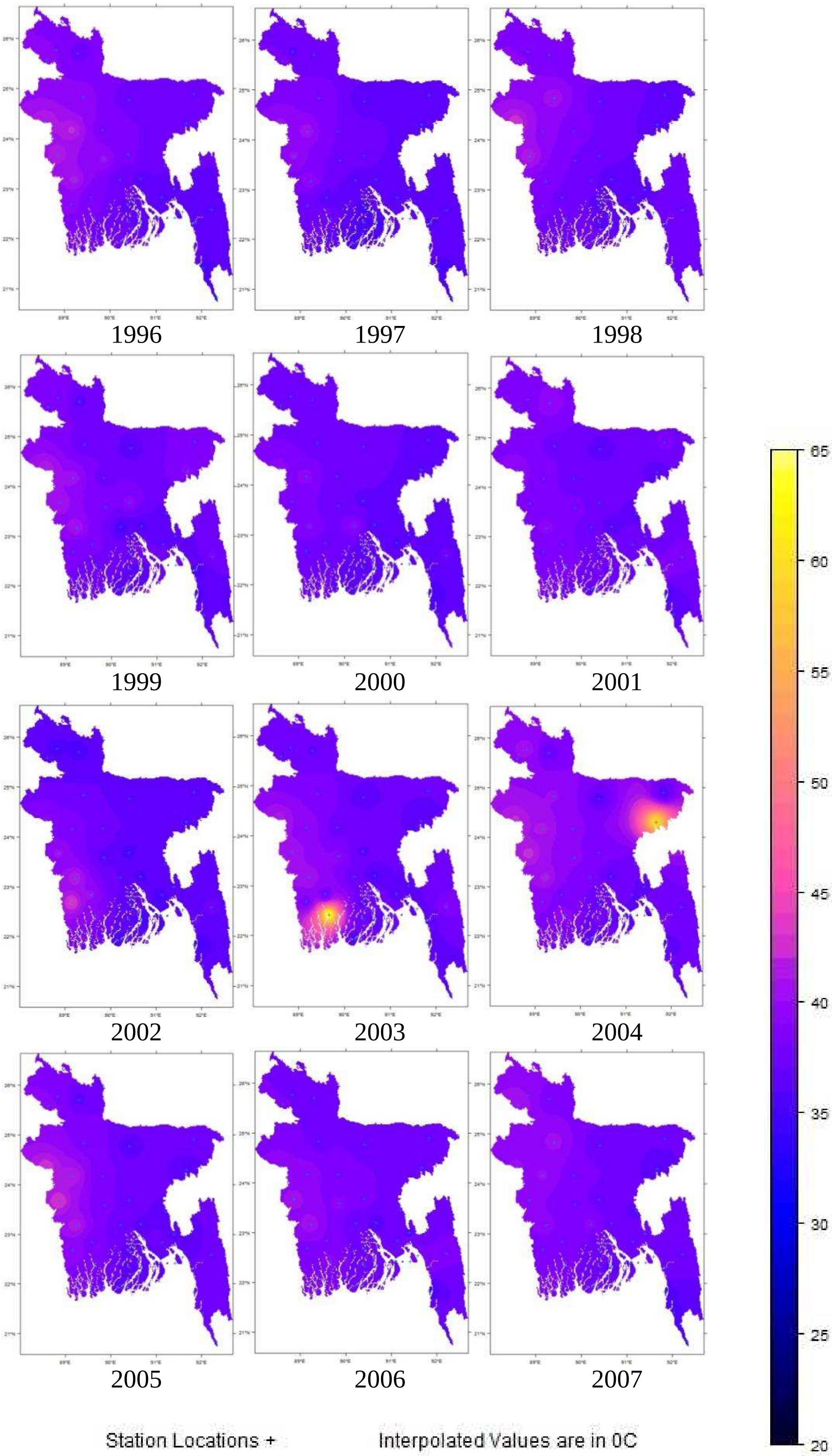
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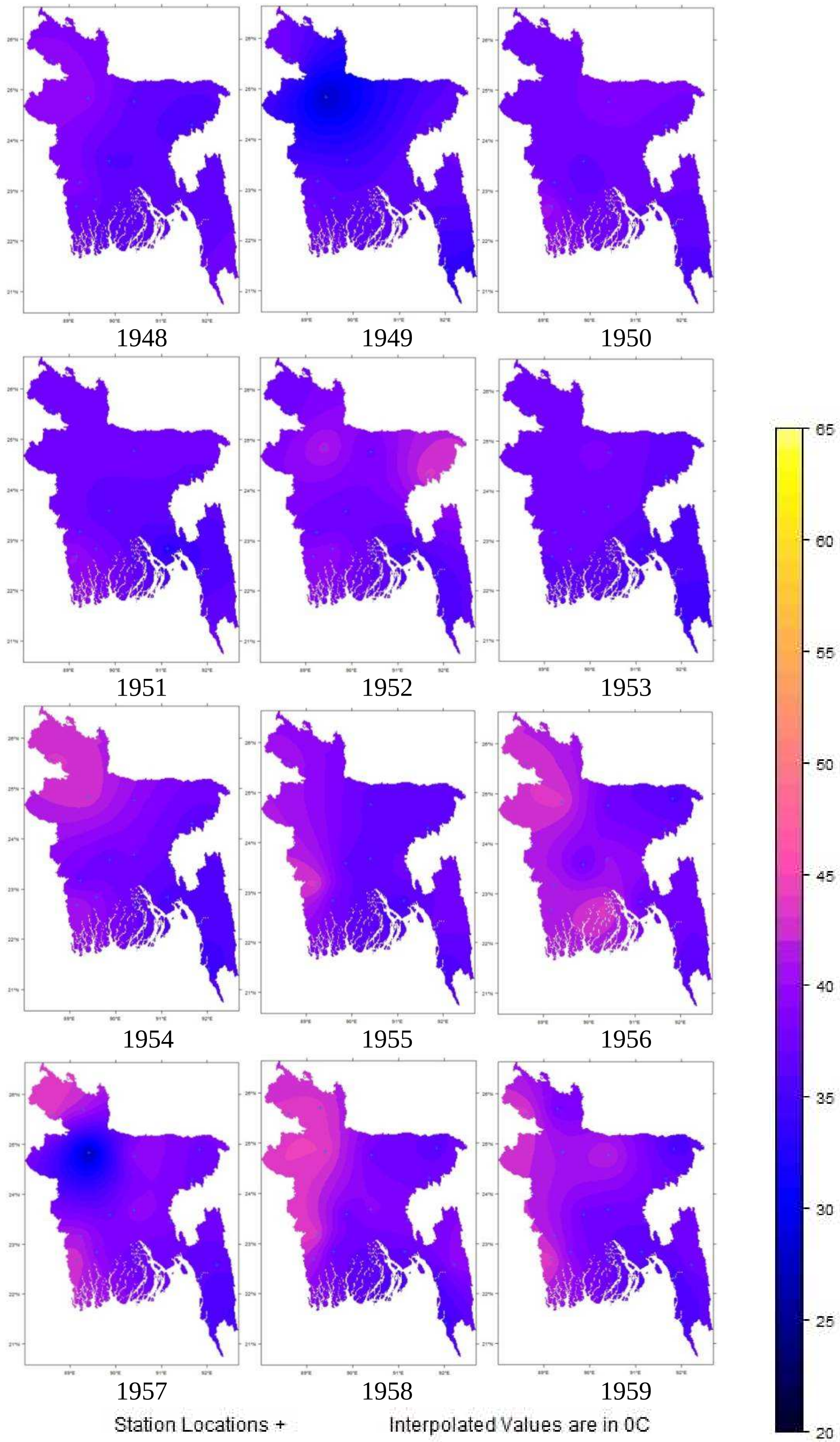
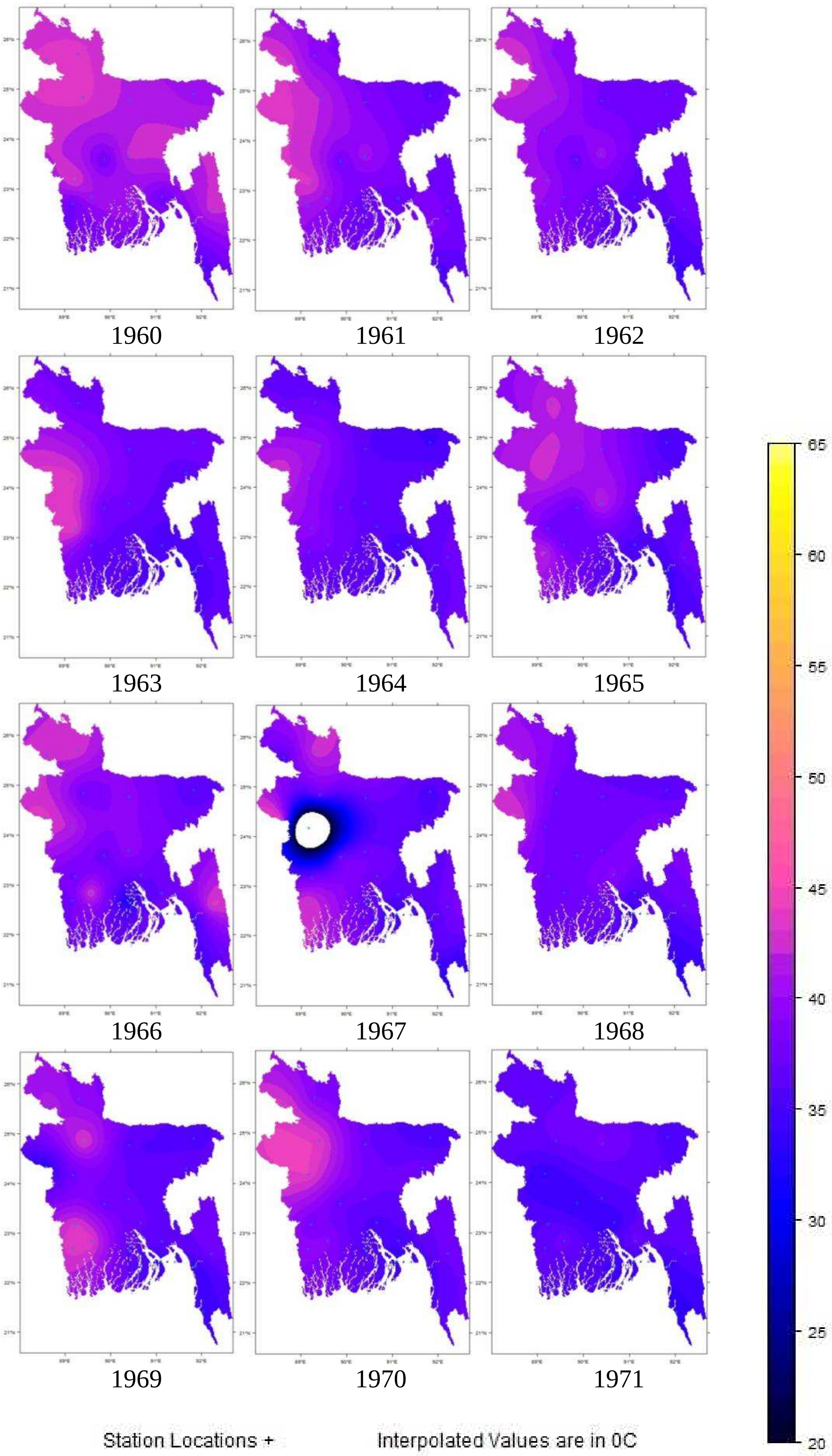
Station Locations +
Interpolated Values are in 0C

## A.11 OK Surfaces of TXx




Station Locations + Interpolated Values are in 0C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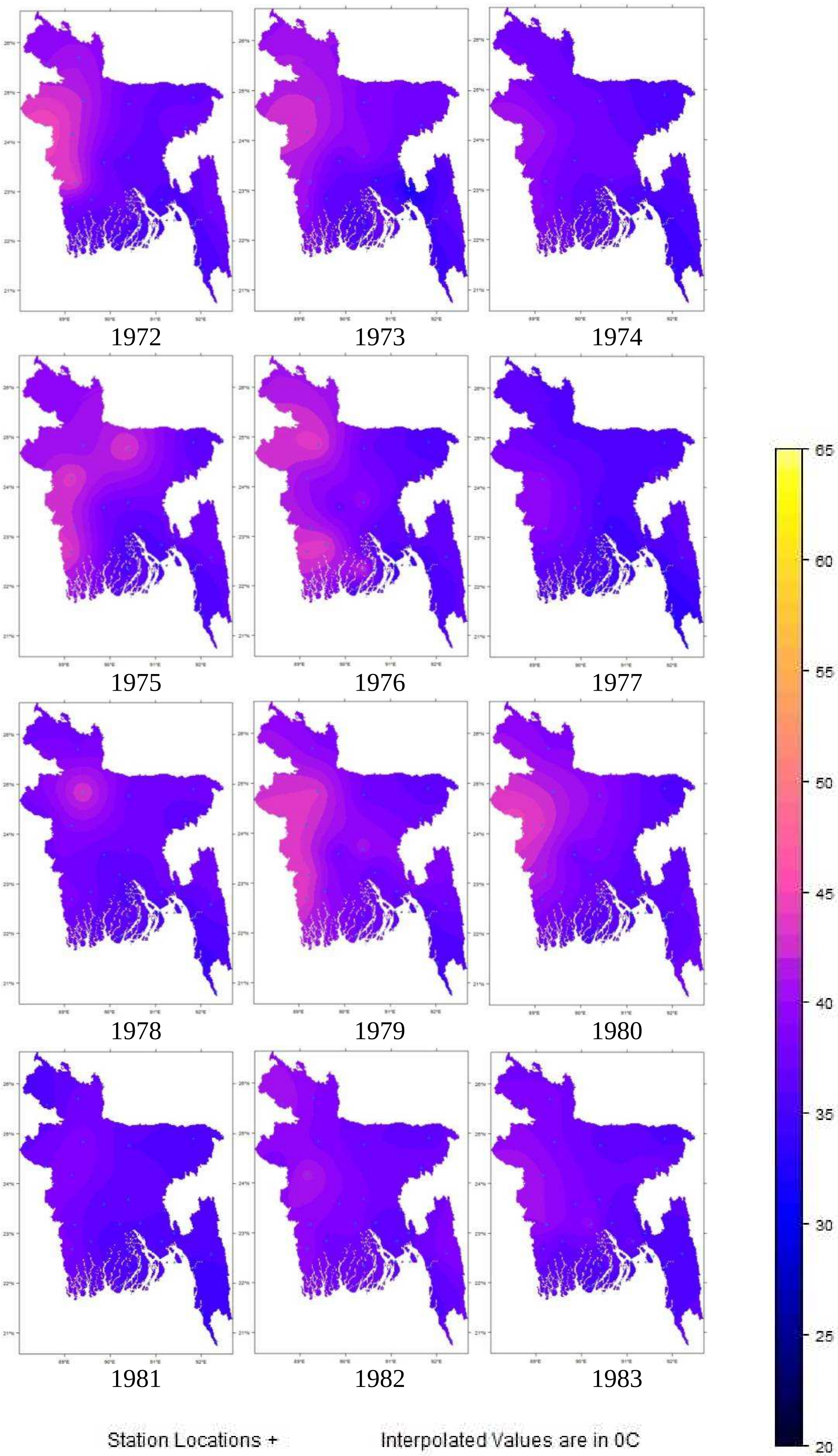
Station Locations +
Interpolated Values are in 0C


1972
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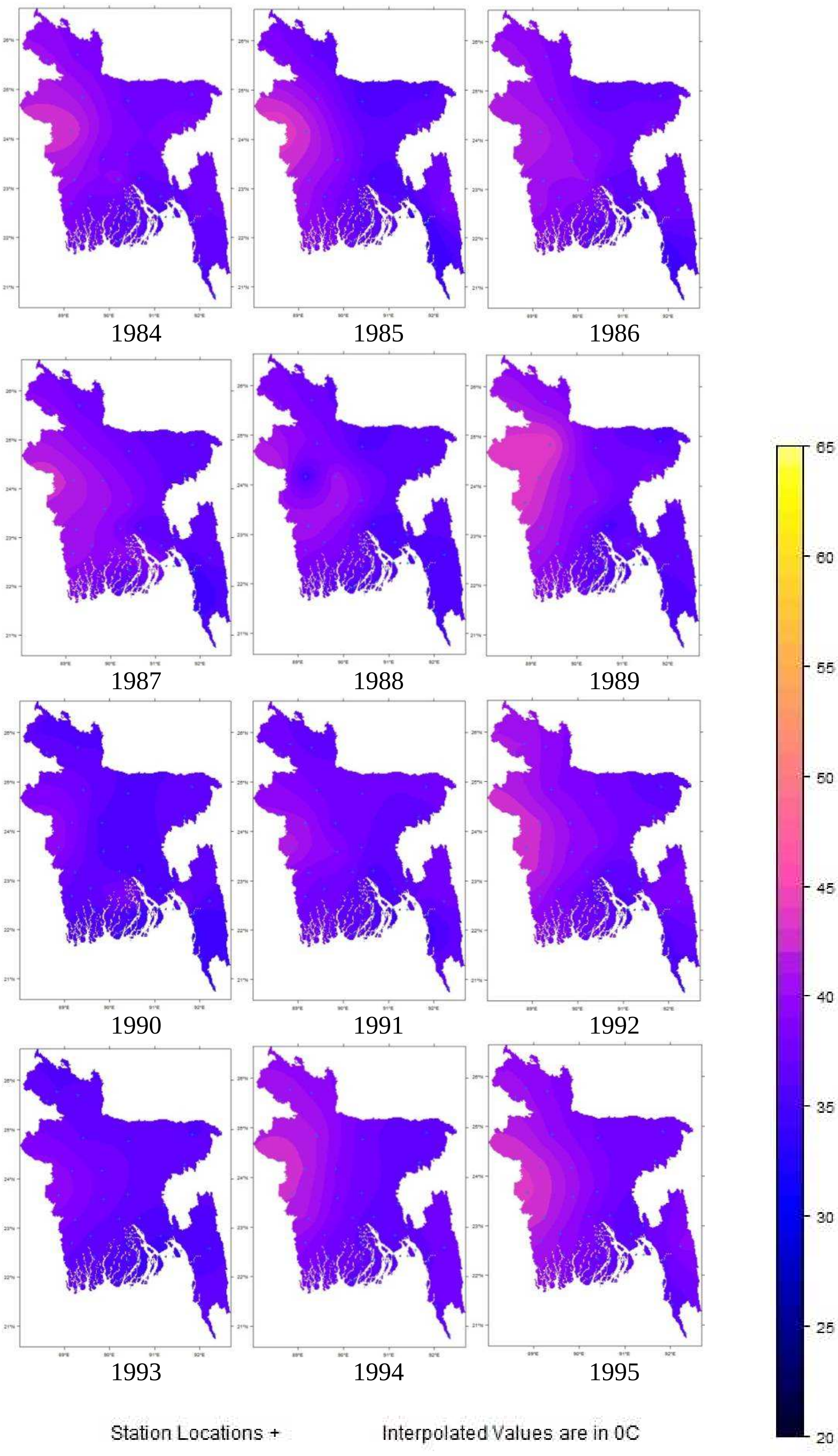
Station Locations +
Interpolated Values are in 0C


1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
Station Locations +
Interpolated Values are in 0C
65
60
55
50
45
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35
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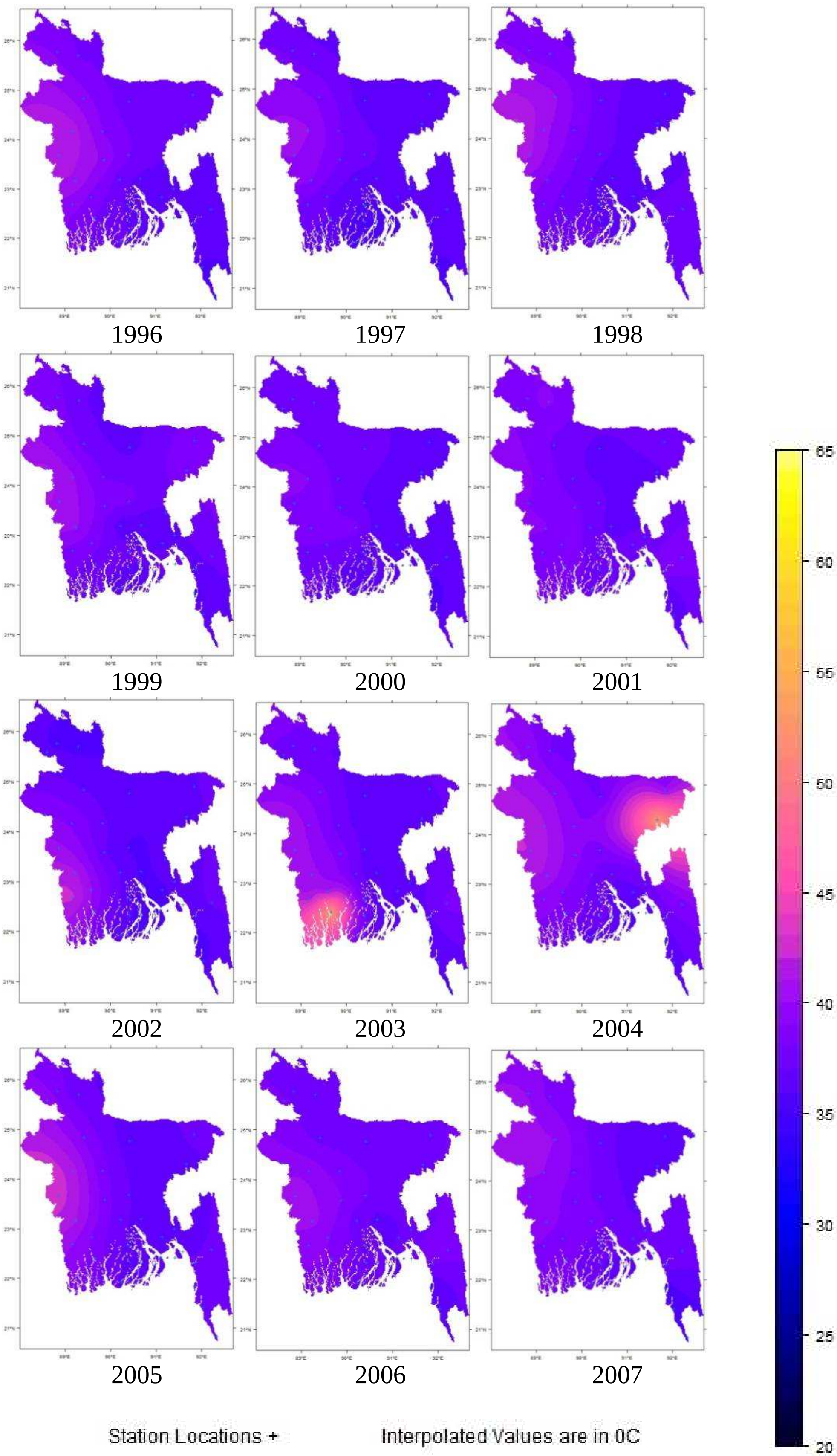
25
20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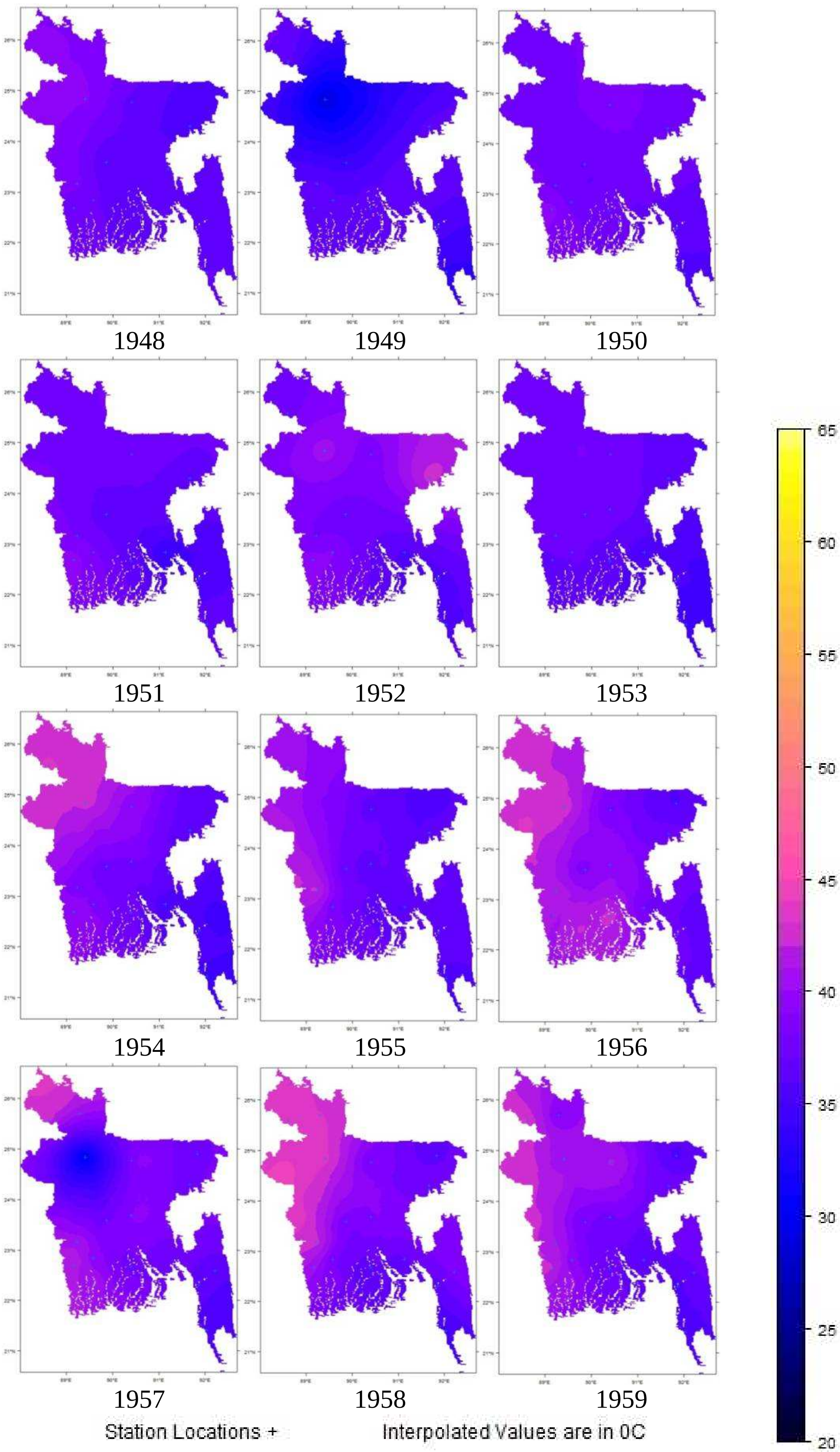
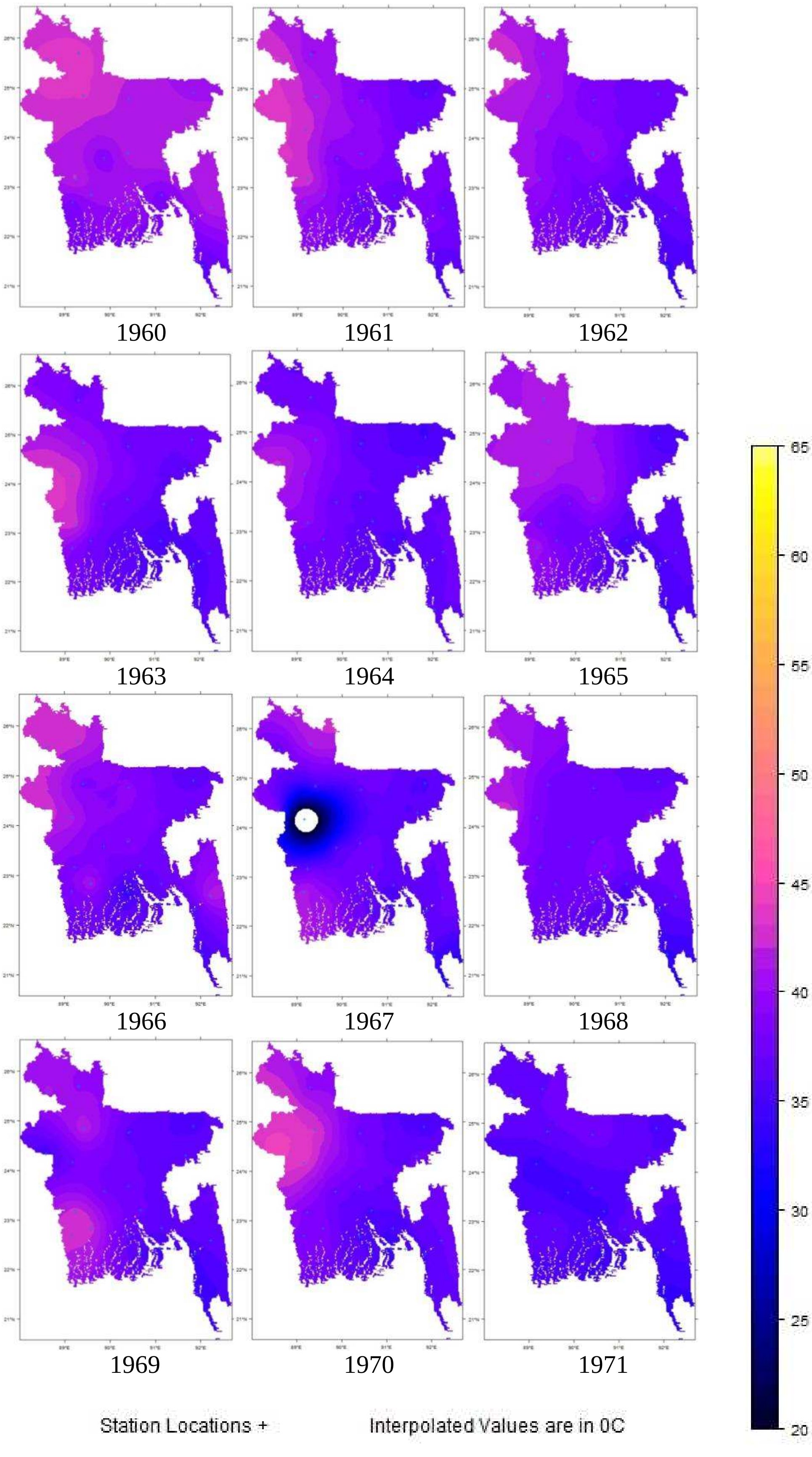
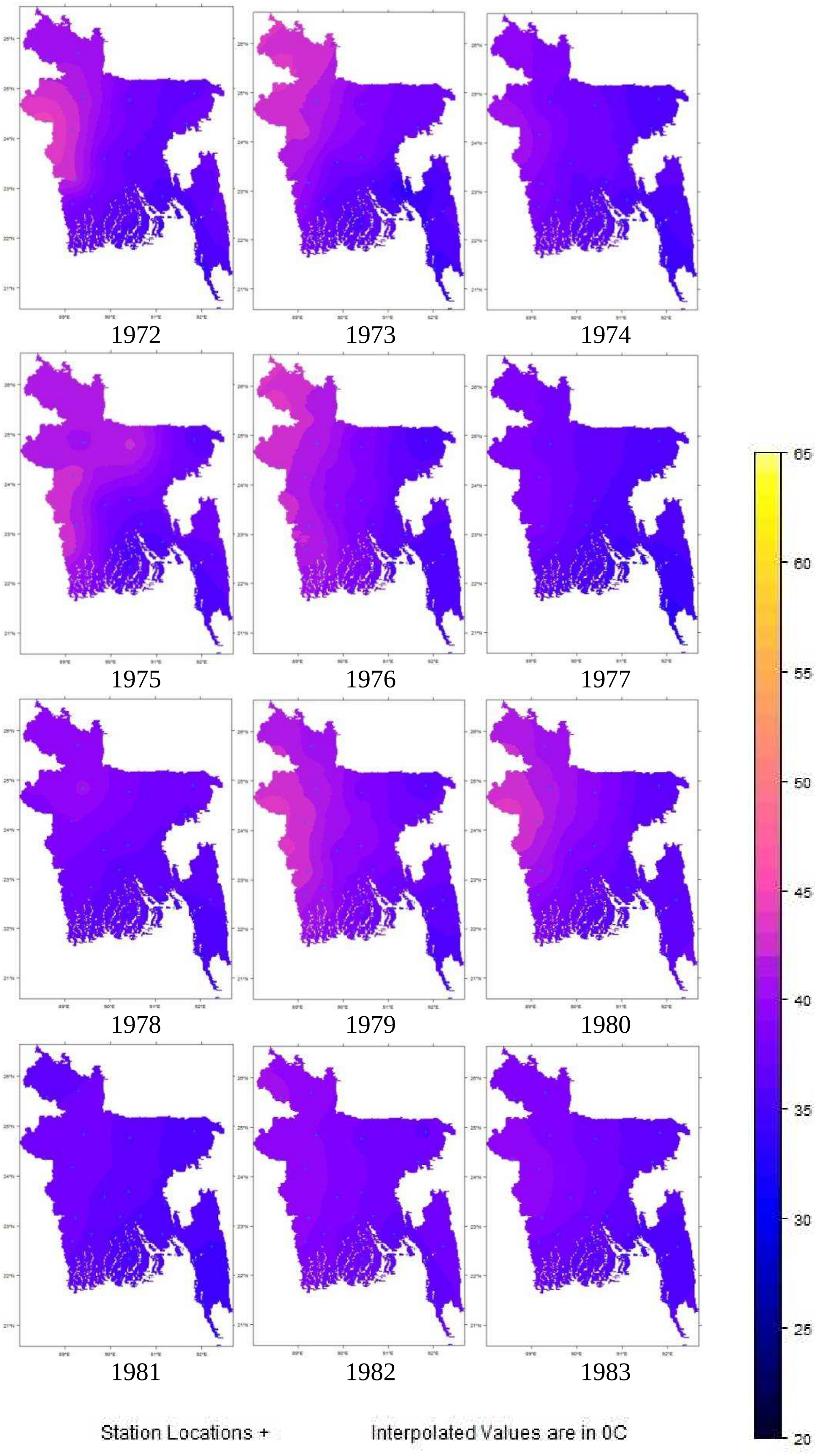
20
Station Locations +
Interpolated Values are in 0C

## A.12 UK Surfaces of TXx






Station Locations +    Interpolated Values are in 0C


1972
1973
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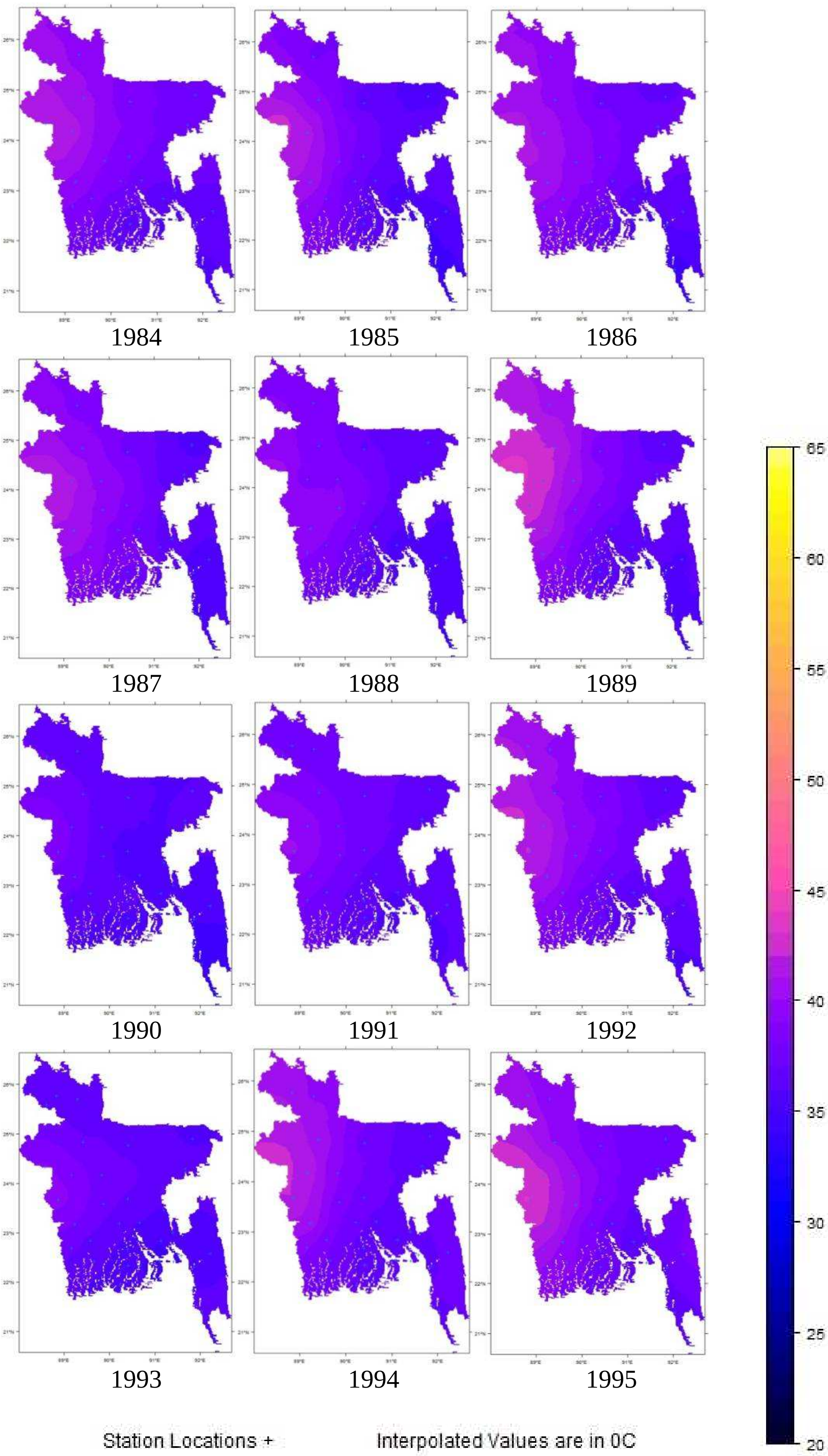
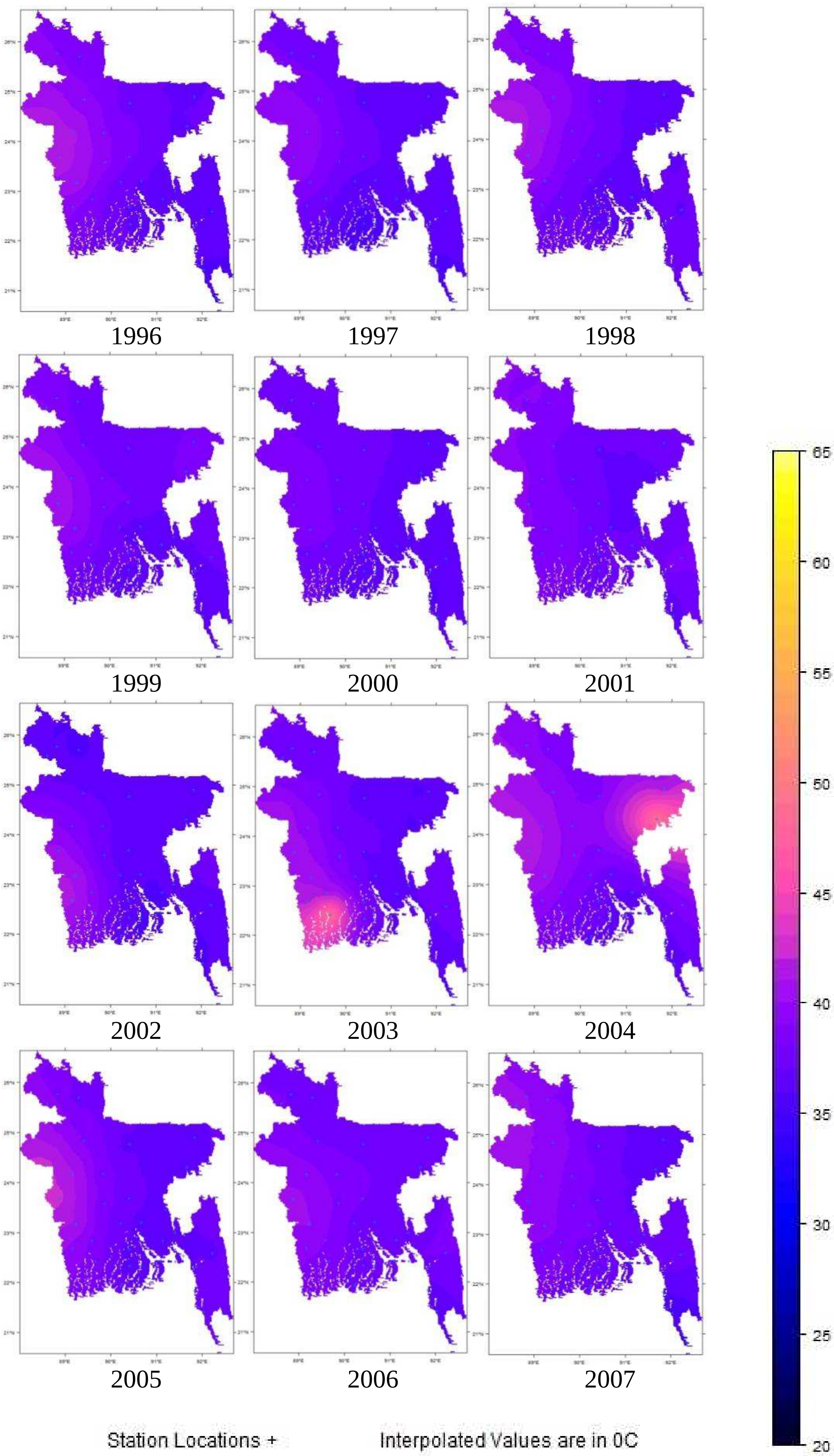
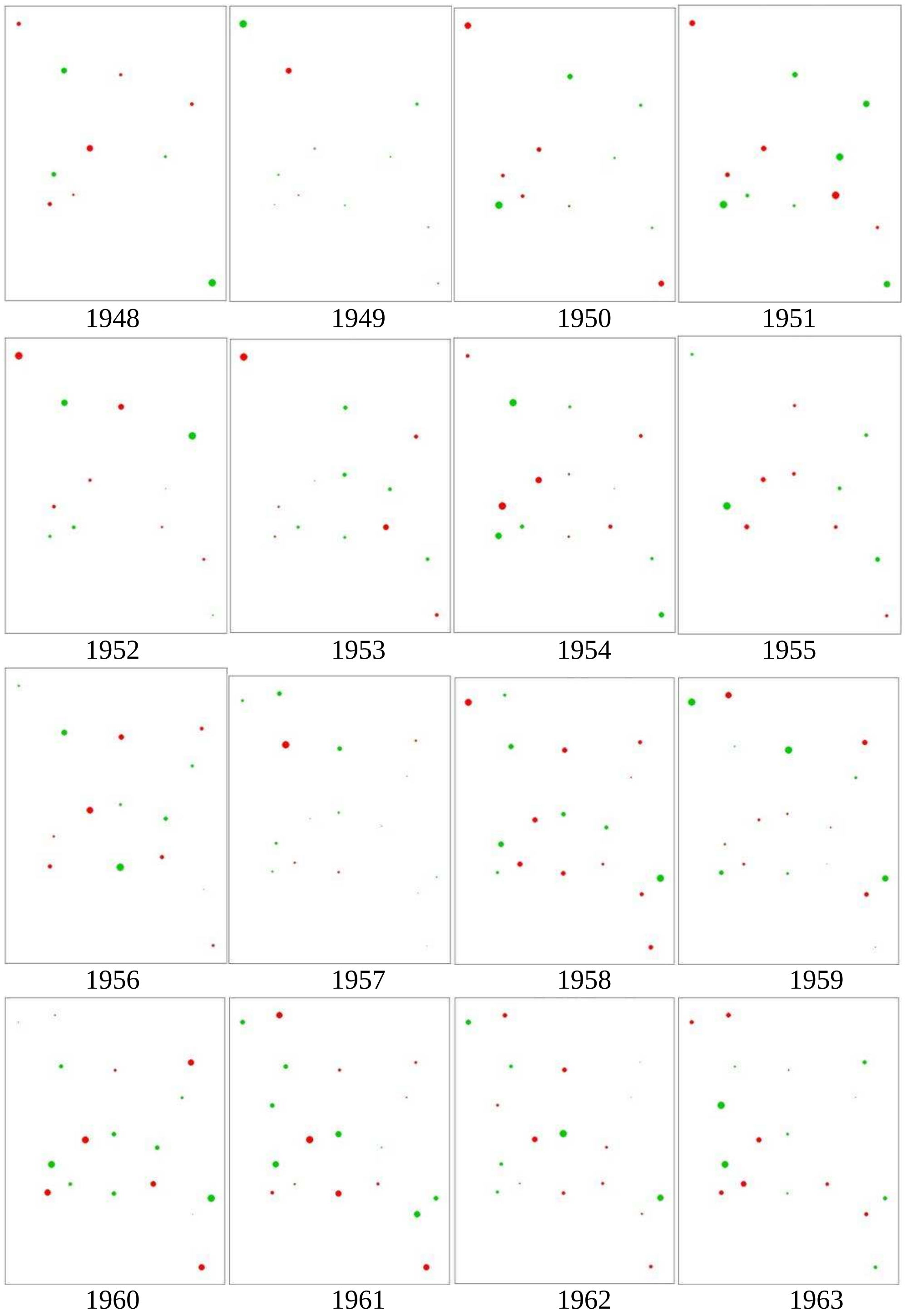
1981
1982
1983
65
60
55
50
45
40
35
30
25
20
Station Locations +
Interpolated Values are in 0C



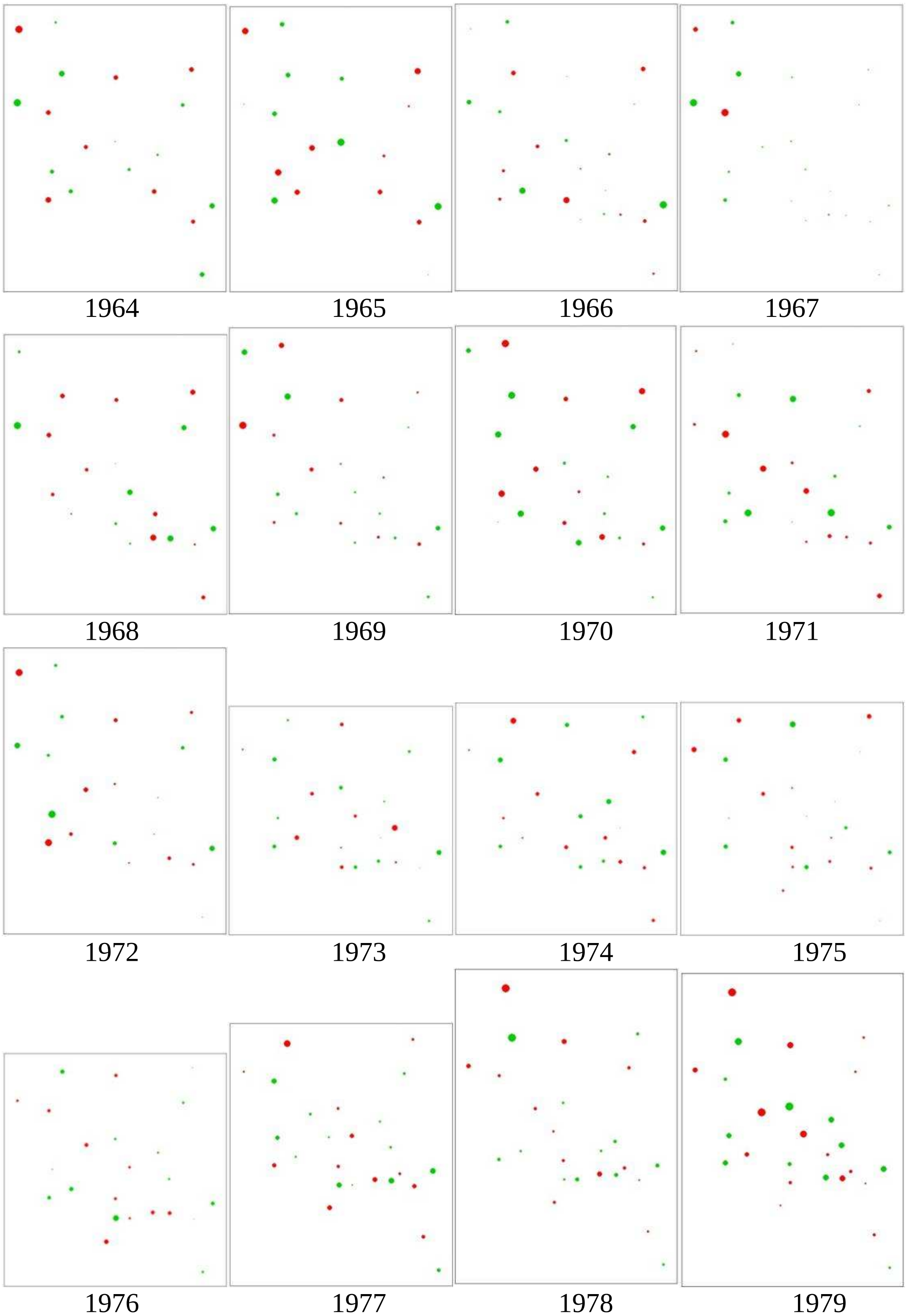

Station Locations + Interpolated Values are in 0C


1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
65
60
55
50
45
40
35
30
25
20
Station Locations +
Interpolated Values are in 0C

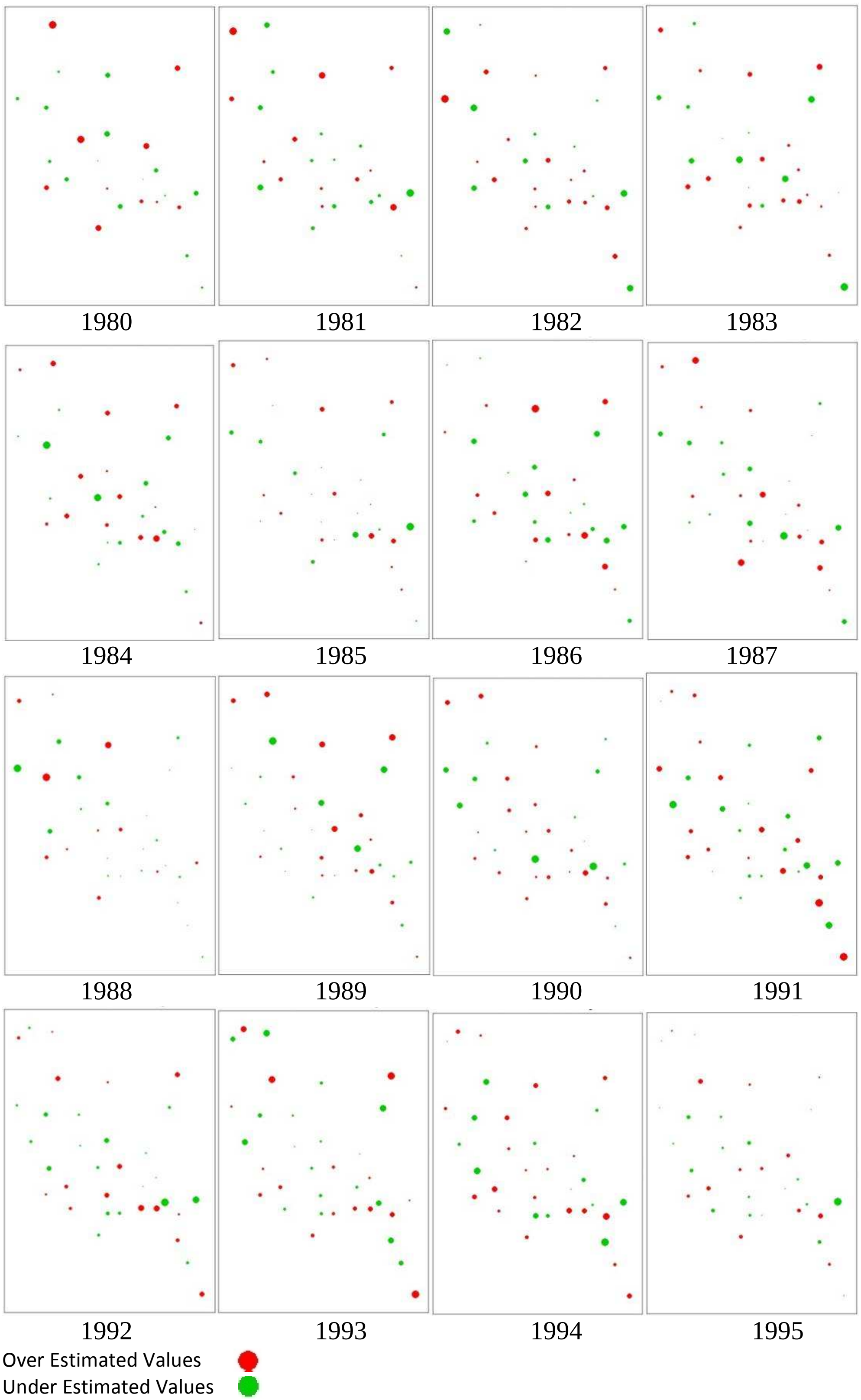
## A.13 Residual Plots of TPS Surfaces of TXx




Over Estimated Values
Under Estimated Values
*Note: The relative sizes of the circles depict the degree of over or under estimation i.e the minimum the size of the circle the minimum the difference between measured and predicted values



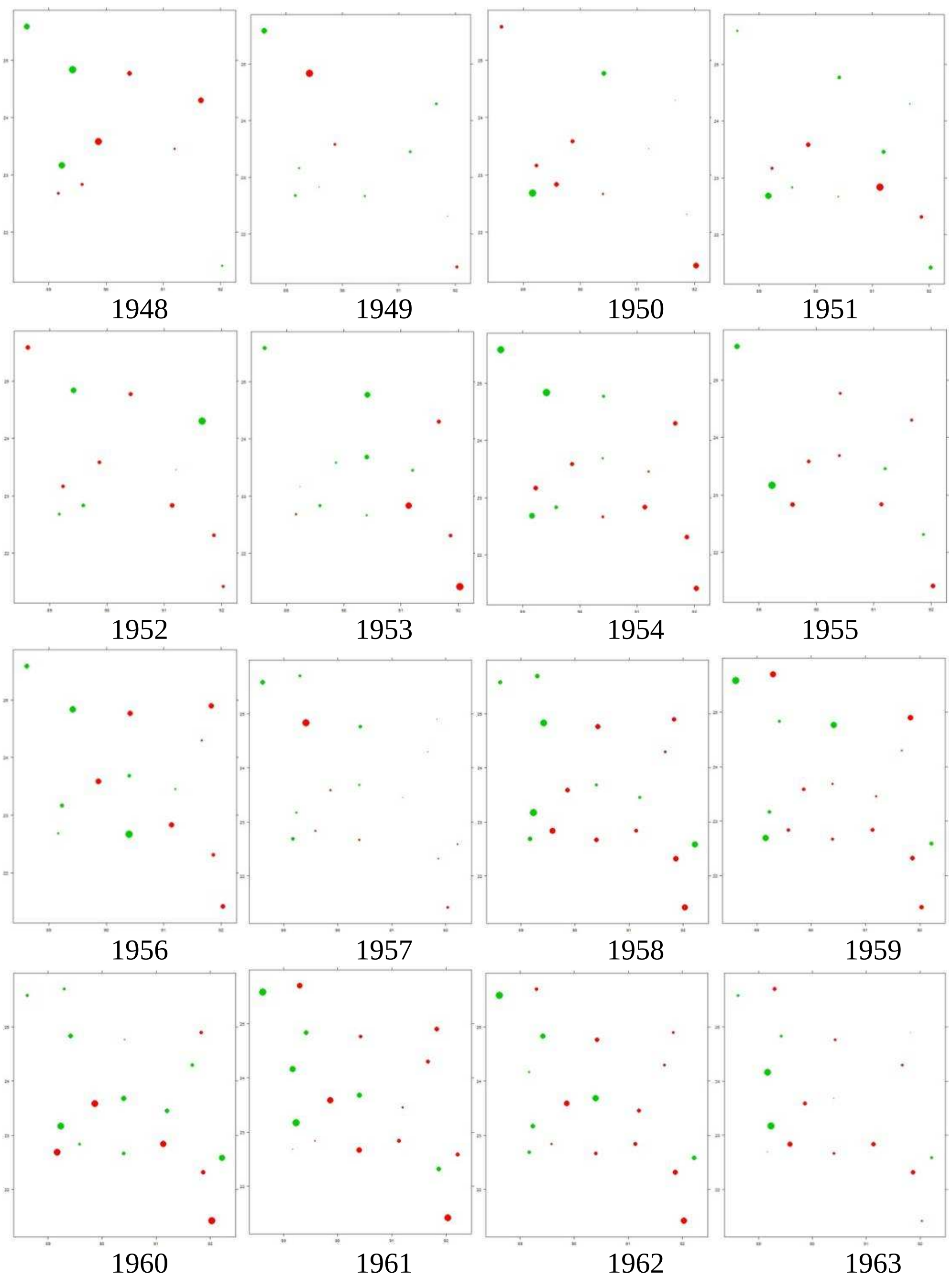

Over Estimated Values
Under Estimated Values
*Note: The relative sizes of the circles depict the degree of over or under estimation i.e the minimum the size of the circle the minimum the difference between measured and predicted values



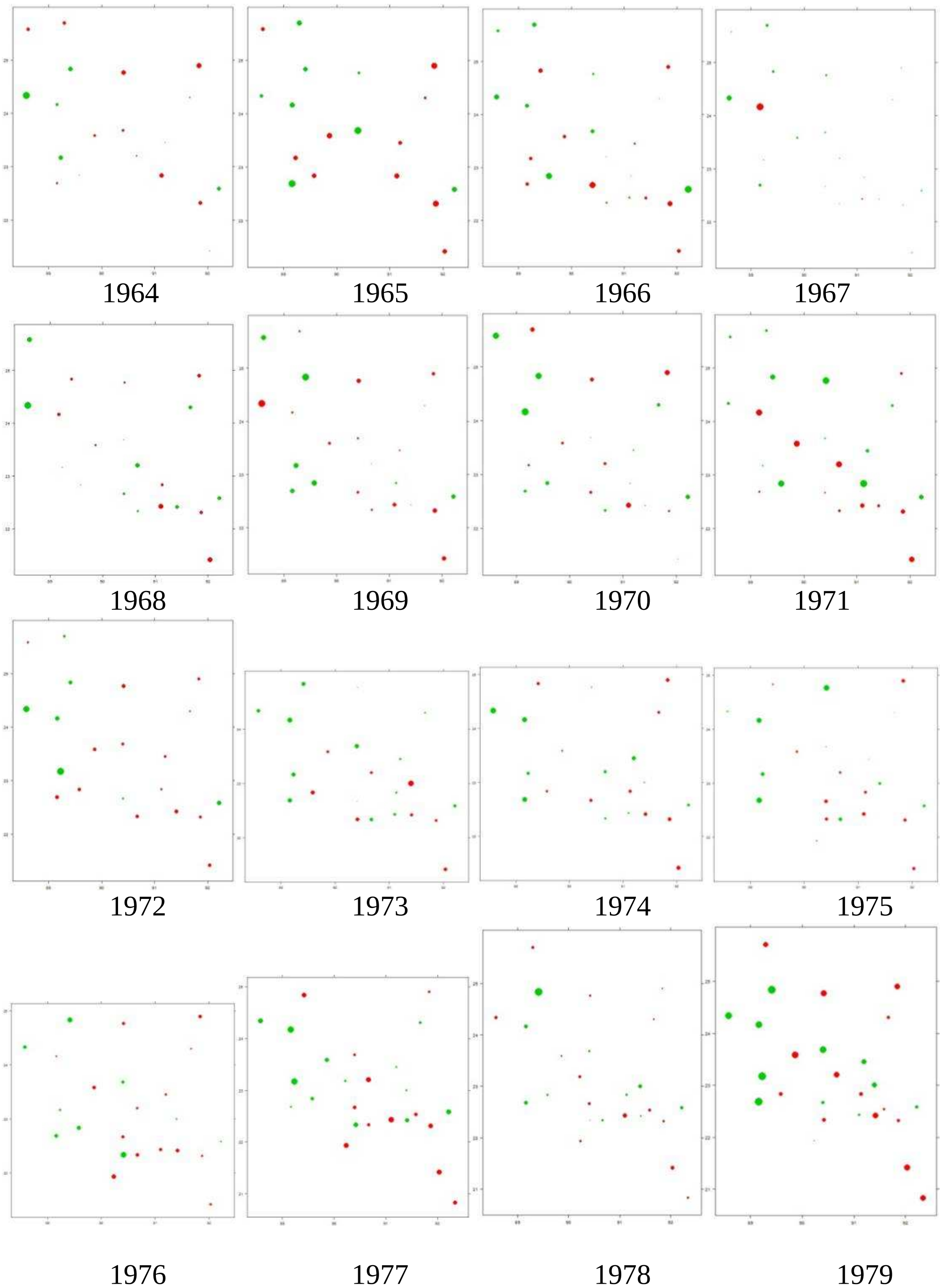

*Note: The relative sizes of the circles depict the degree of over or under estimation i.e the minimum the size of the circle the minimum the difference between measured and predicted values


Over Estimated Values
Under Estimated Values
*Note: The relative sizes of the circles depict the degree of over or under estimation i.e the minimum the size of the circle the minimum the difference between measured and predicted values

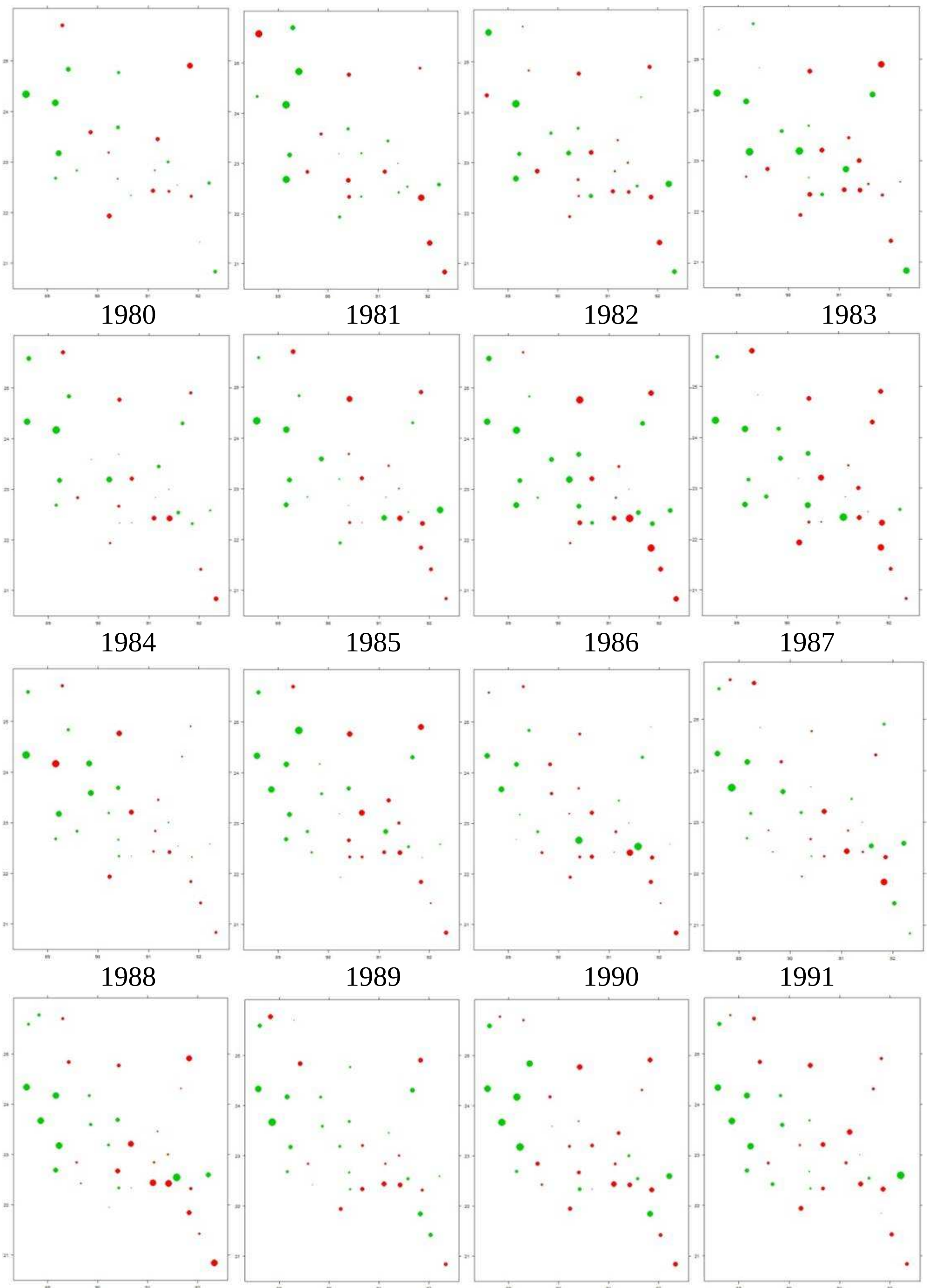
## A.14 Residual Plots of IDW Surfaces of TXx




Over Estimated Values
Under Estimated Values
*Note: The relative sizes of the circles depict the degree of over or under estimation i.e the minimum the size of the circle the minimum the difference between measured and predicted values



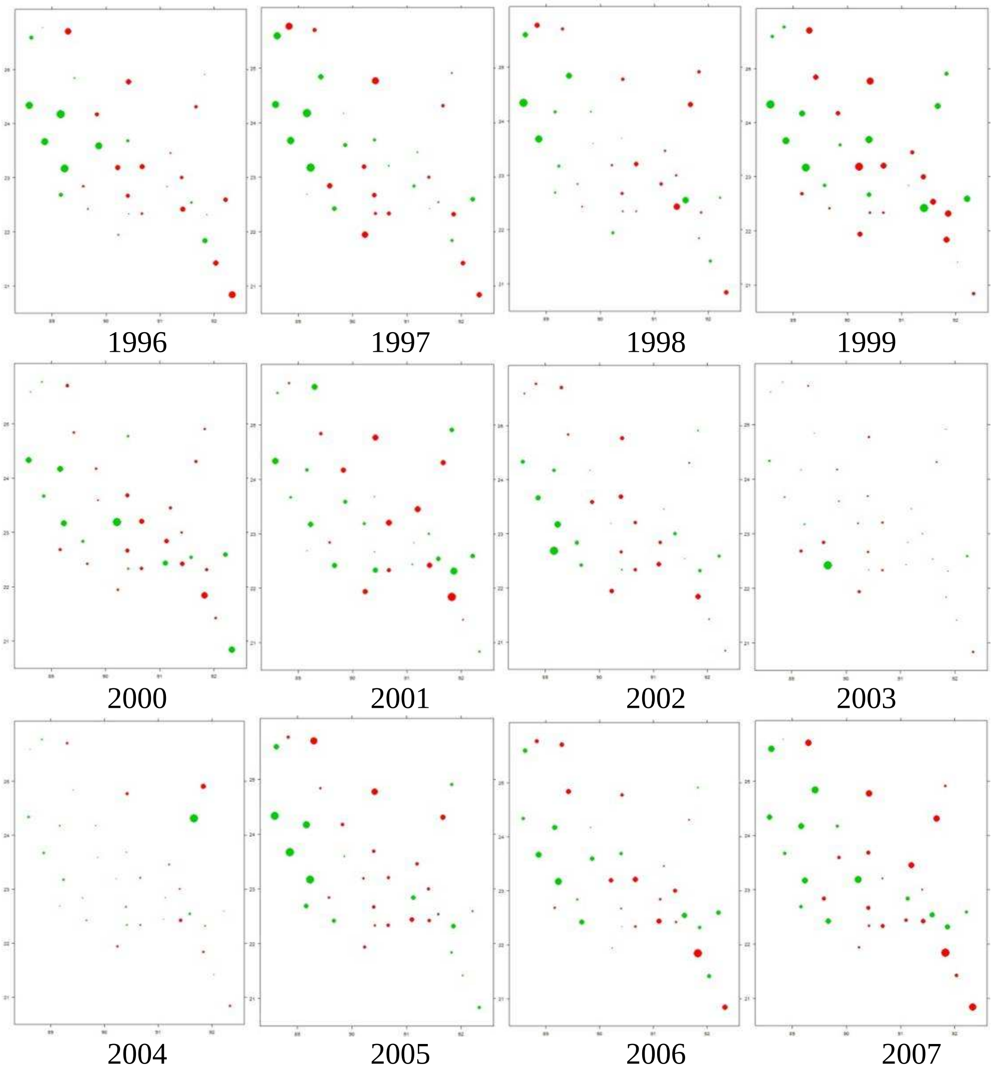

Over Estimated Values
Under Estimated Values
*Note: The relative sizes of the circles depict the degree of over or under estimation i.e the minimum the size of the circle the minimum the difference between measured and predicted values




Over Estimated Values

Under Estimated Values

*Note: The relative sizes of the circles depict the degree of over or under estimation i.e the minimum the size of the circle the minimum the difference between measured and predicted values

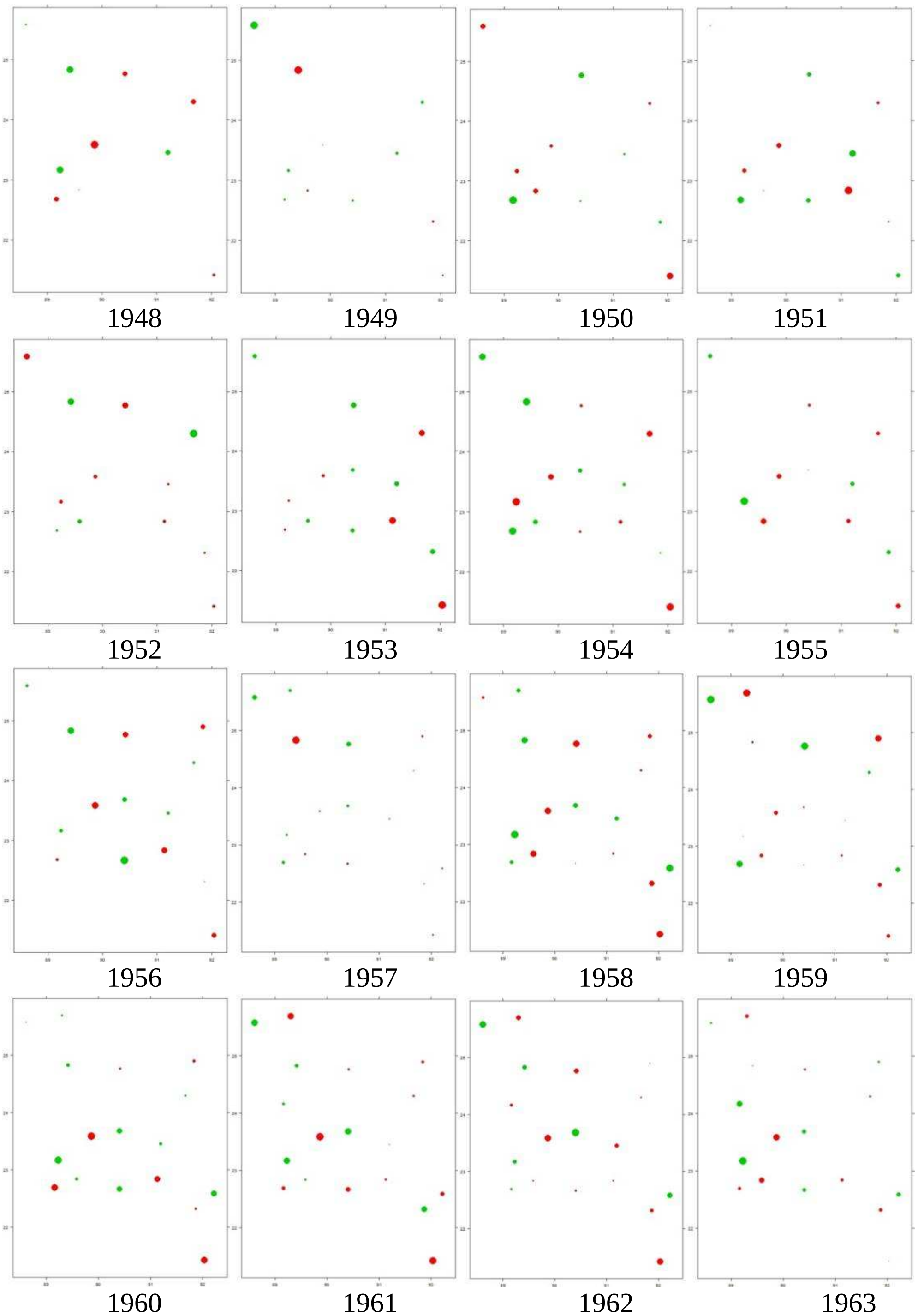



Over Estimated Values

Under Estimated Values

*Note: The relative sizes of the circles depict the degree of over or under estimation i.e the minimum the size of the circle the minimum the difference between measured and predicted values

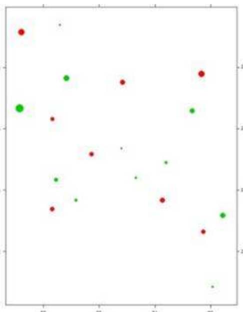
## A.15 Residual Plots of OK Surfaces of TXx

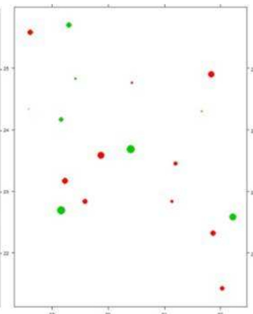


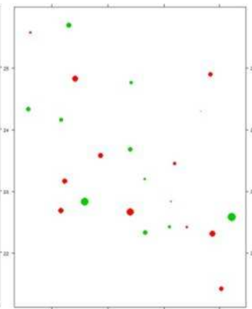

Over Estimated Values

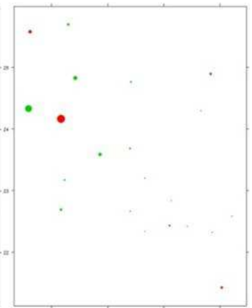
Under Estimated Values

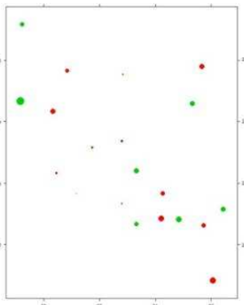
*Note: The relative sizes of the circles depict the degree of over or under estimation i.e the minimum the size of the circle the minimum the difference between measured and predicted values

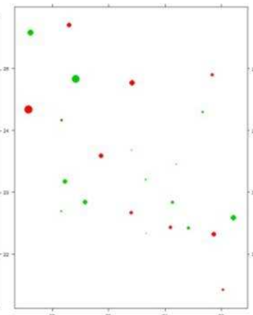

1964

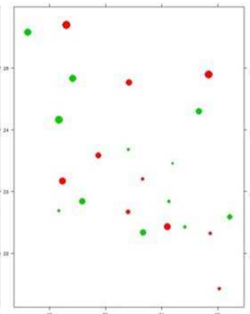

1965

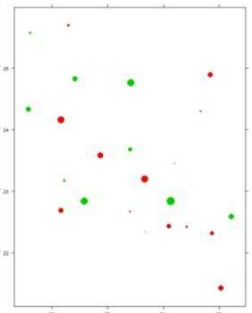

1966

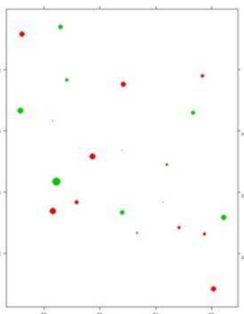

1967

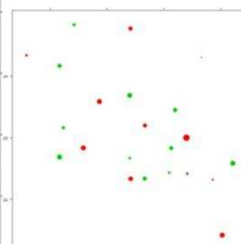

1968

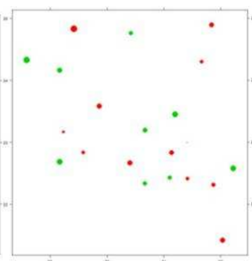

1969

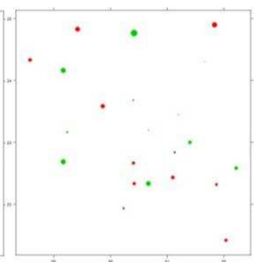

1970

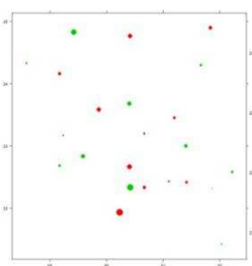

1971

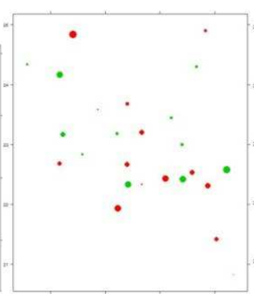

1972

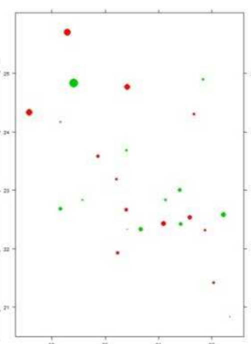

1973

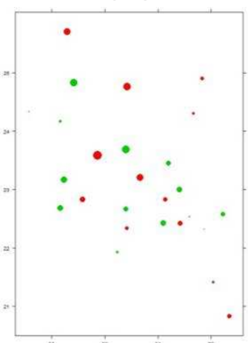

1974


1975


1976


1977

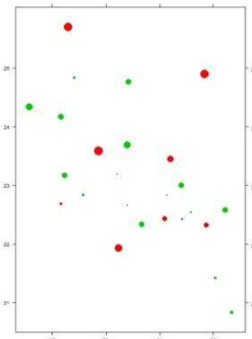
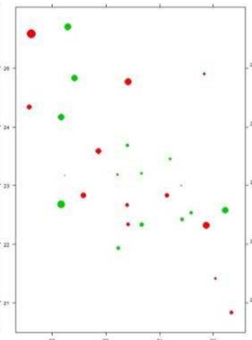
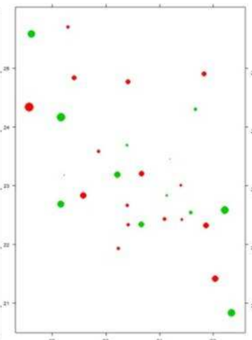
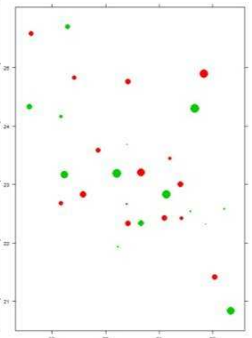
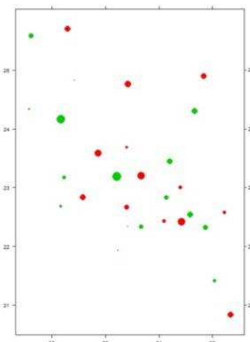
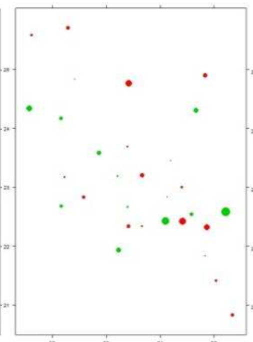

1978


1979

Over Estimated Values

Under Estimated Values

*Note: The relative sizes of the circles depict the degree of over or under estimation i.e the minimum the size of the circle the minimum the difference between measured and predicted values


1980

1981

1982

1983

1984

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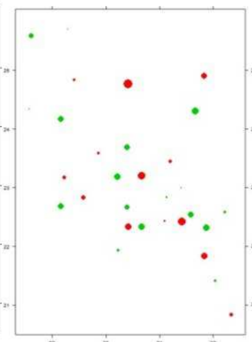
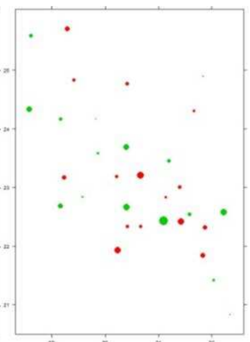
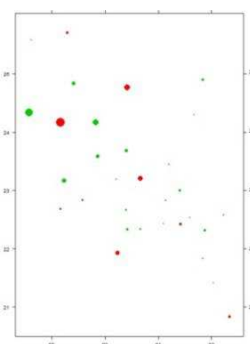
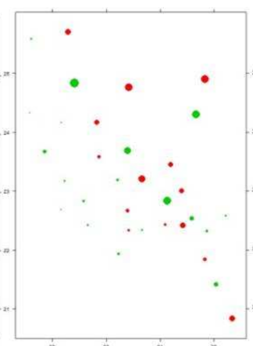
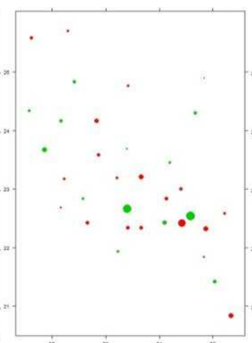
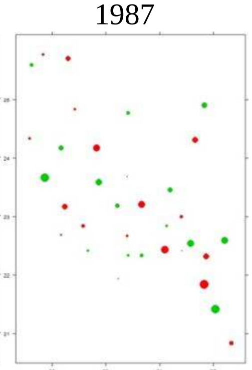
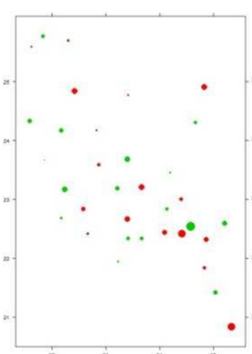
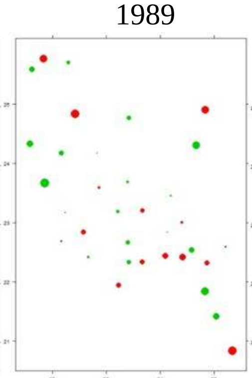
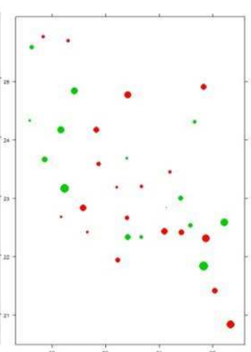
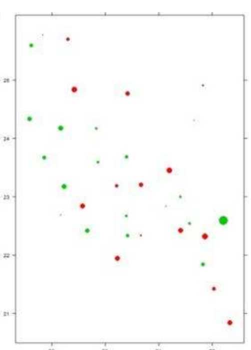
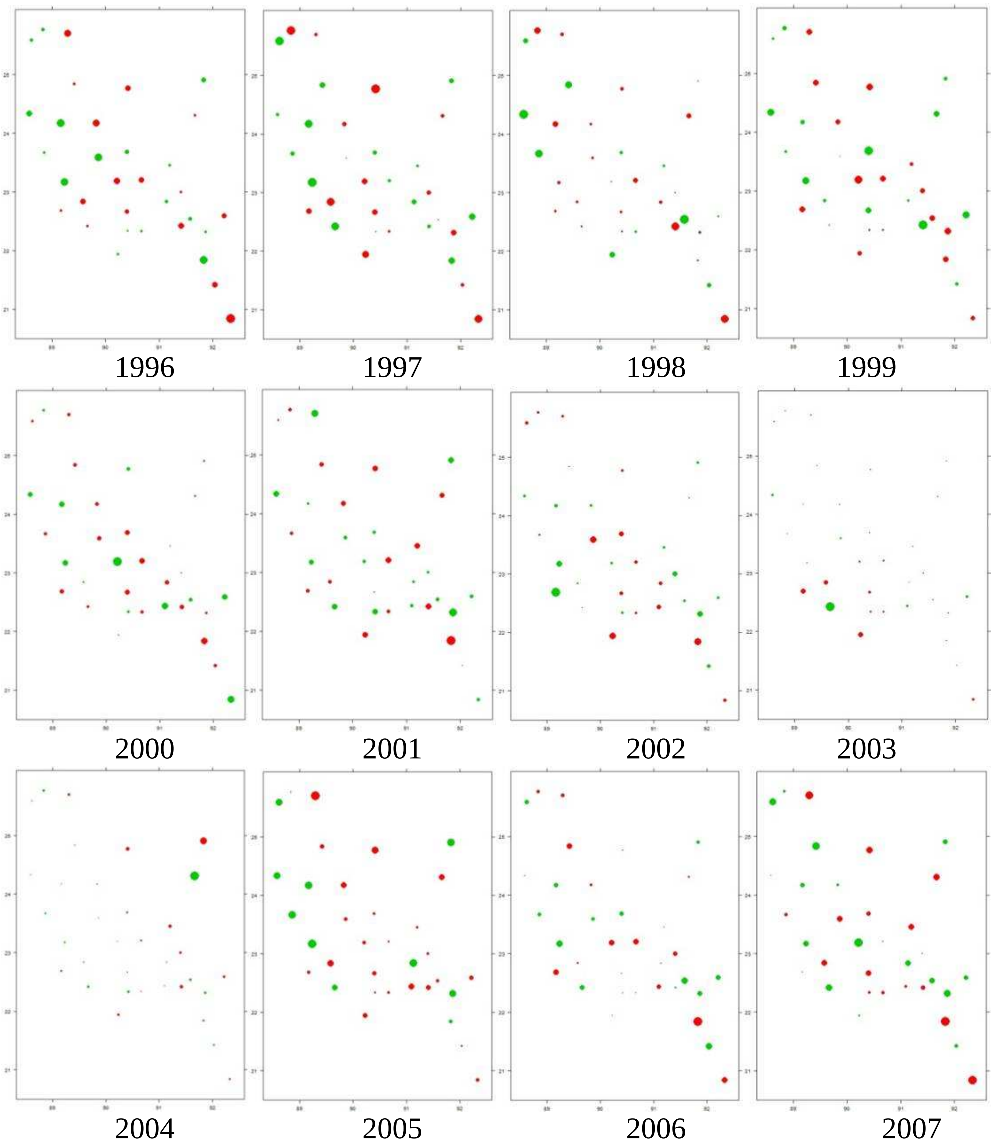

1986

1987

1988

1989

1990

1991

1992

1993

1994

1995

Over Estimated Values

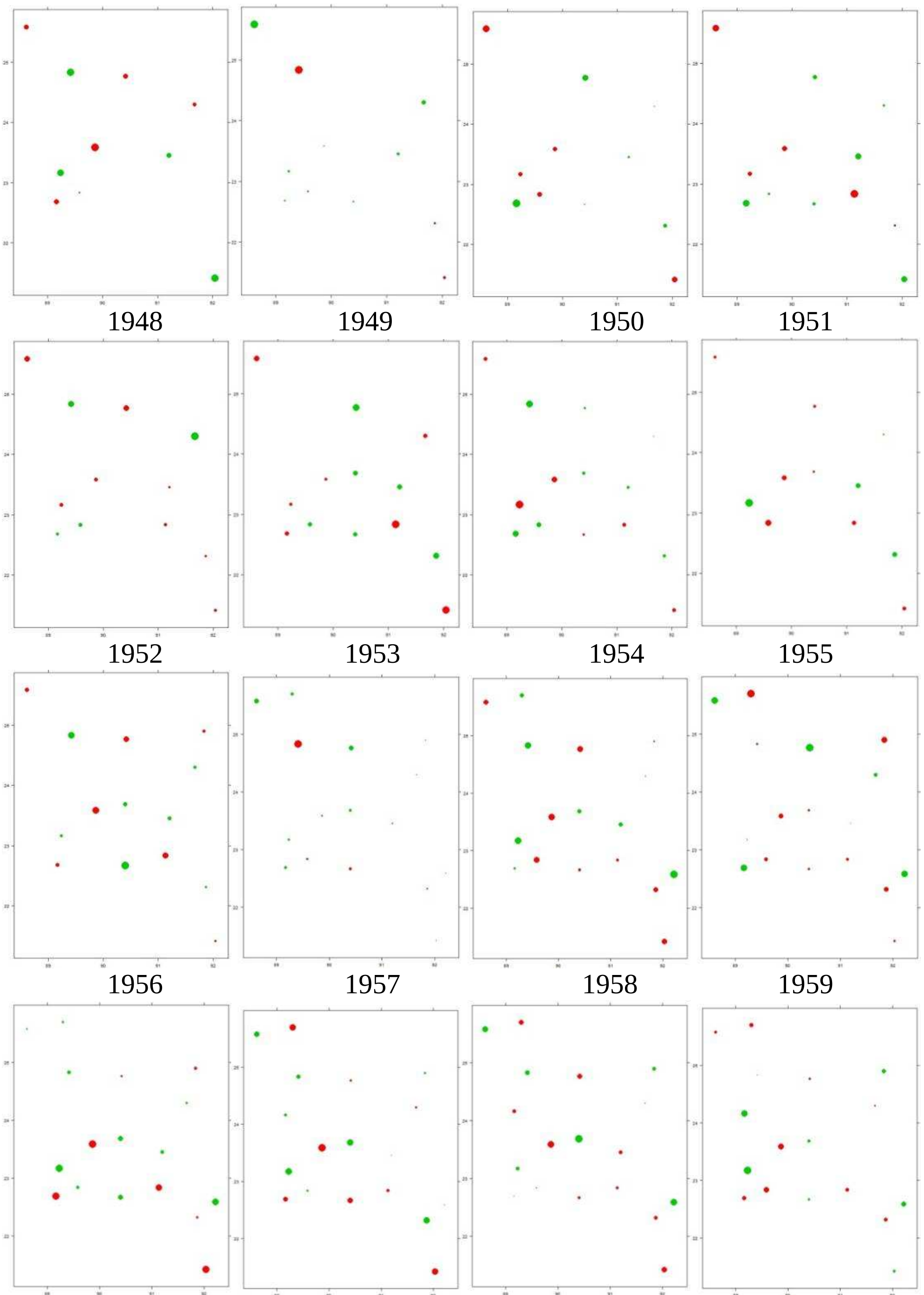
Under Estimated Values

*Note: The relative sizes of the circles depict the degree of over or under estimation i.e the minimum the size of the circle the minimum the difference between measured and predicted values




Over Estimated Values
Under Estimated Values
*Note: The relative sizes of the circles depict the degree of over or under estimation i.e the minimum the size of the circle the minimum the difference between measured and predicted values

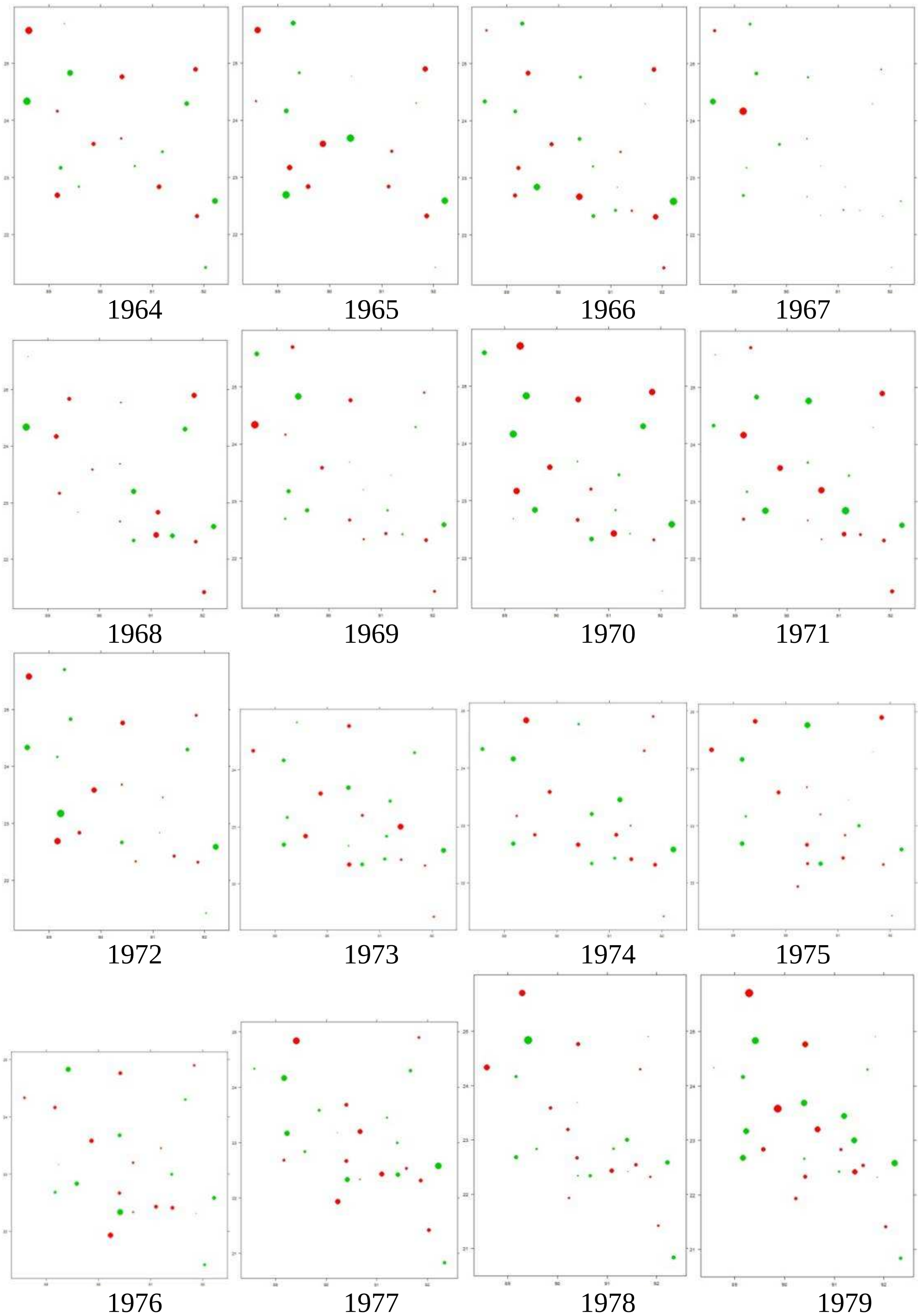
## A.16 Residual Plots of UK Surfaces of TXx




Over Estimated Values

Under Estimated Values

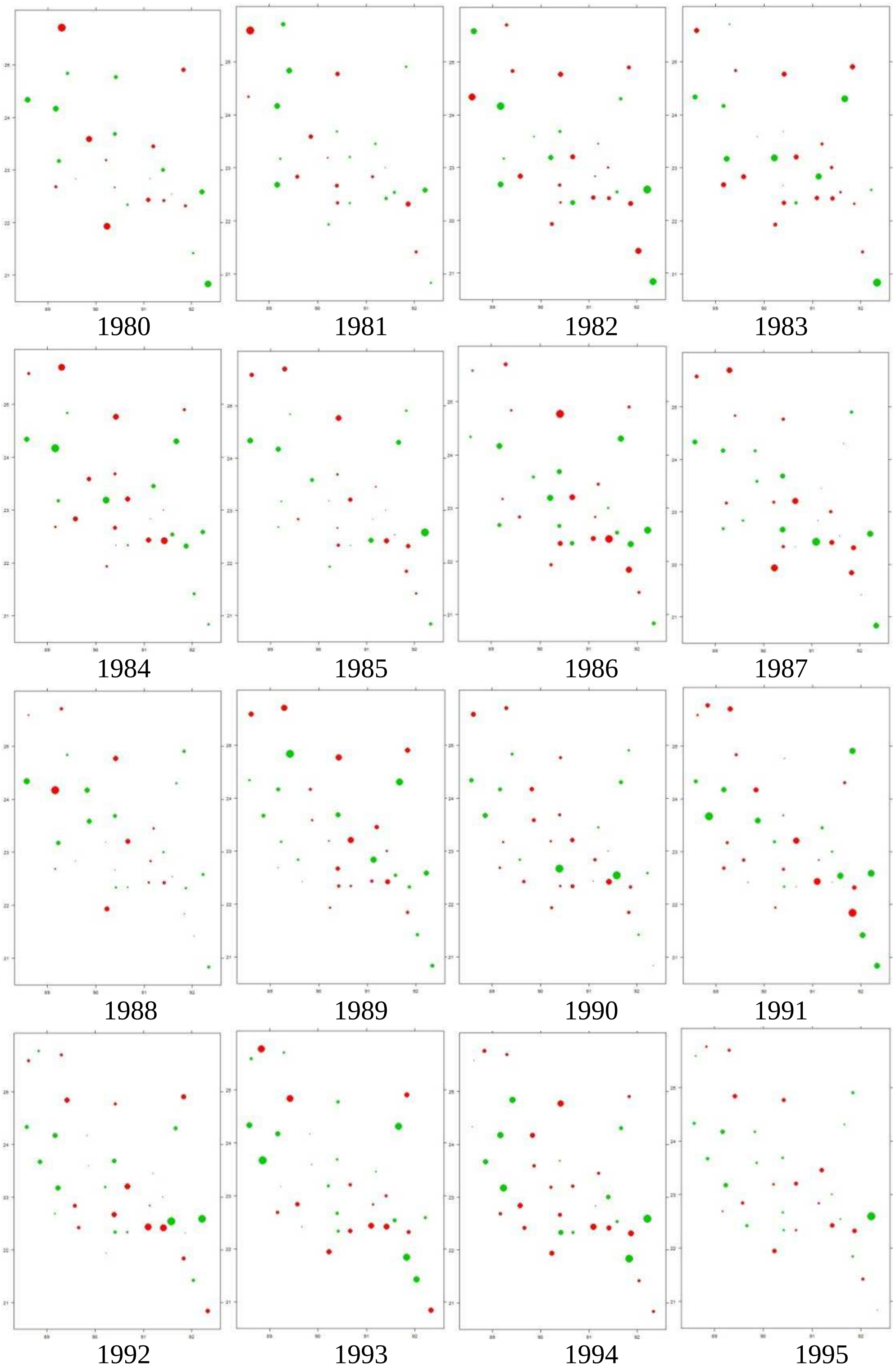
*Note: The relative sizes of the circles depict the degree of over or under estimation i.e the minimum the size of the circle the minimum the difference between measured and predicted values




Over Estimated Values

Under Estimated Values

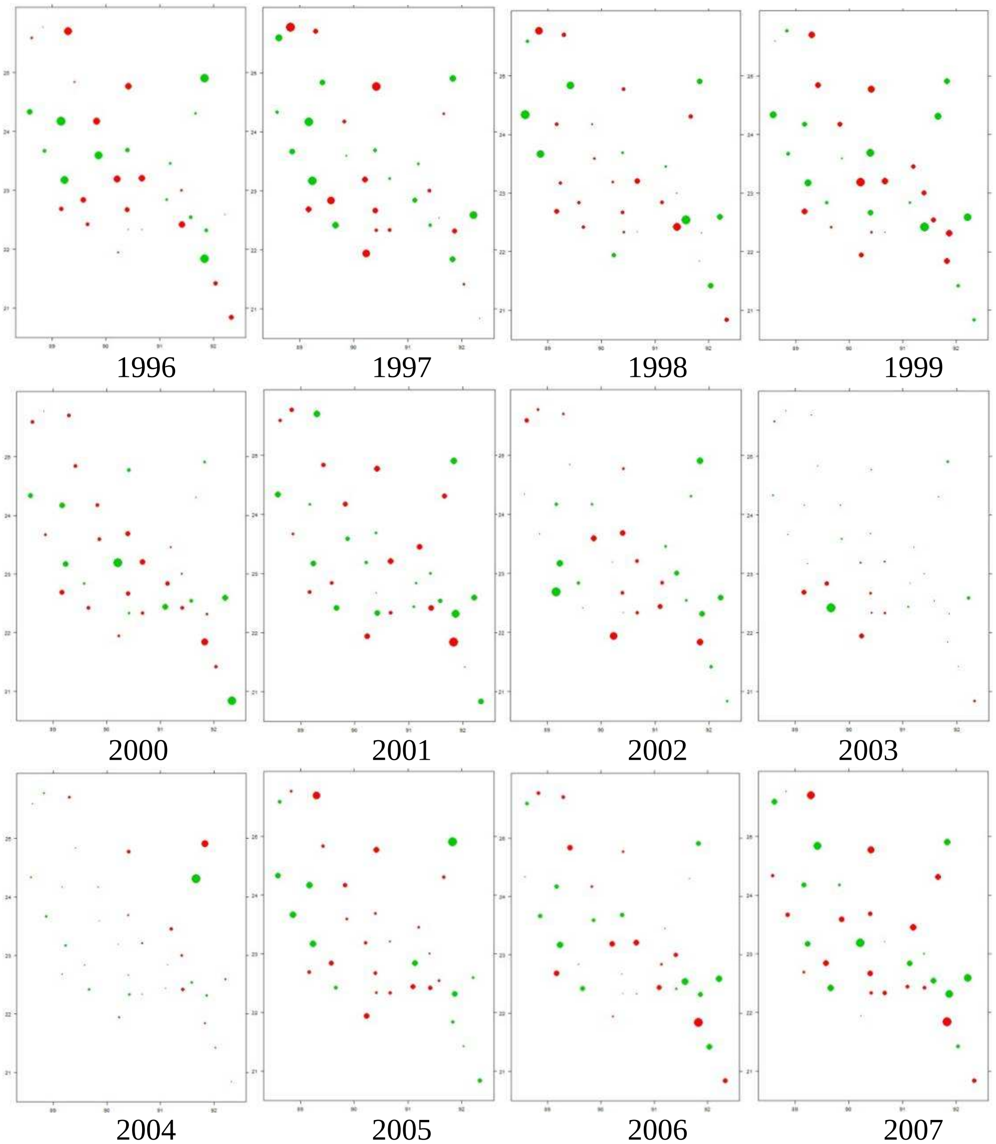
*Note: The relative sizes of the circles depict the degree of over or under estimation i.e the minimum the size of the circle the minimum the difference between measured and predicted values




Over Estimated Values

Under Estimated Values

*Note: The relative sizes of the circles depict the degree of over or under estimation i.e the minimum the size of the circle the minimum the difference between measured and predicted values

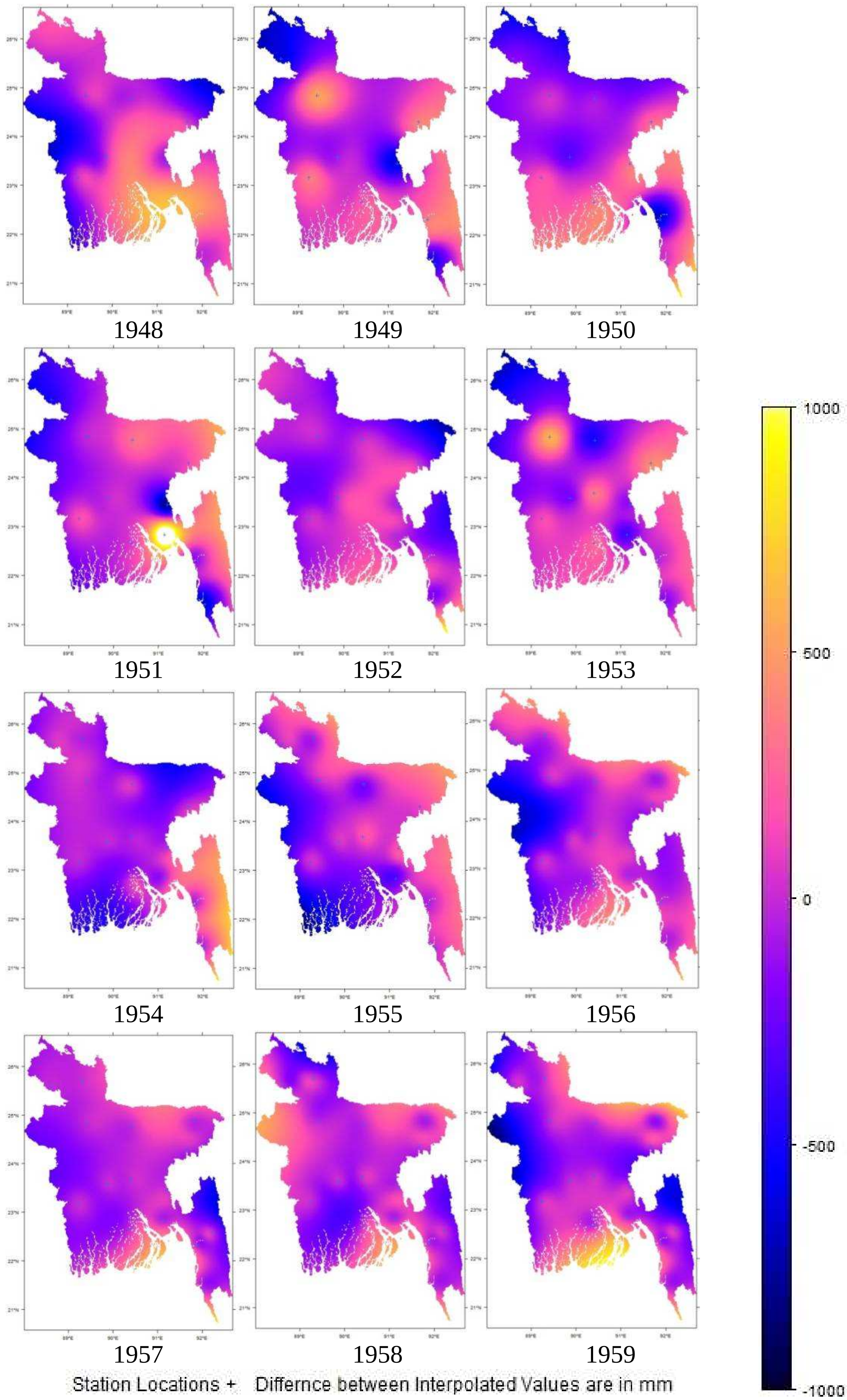


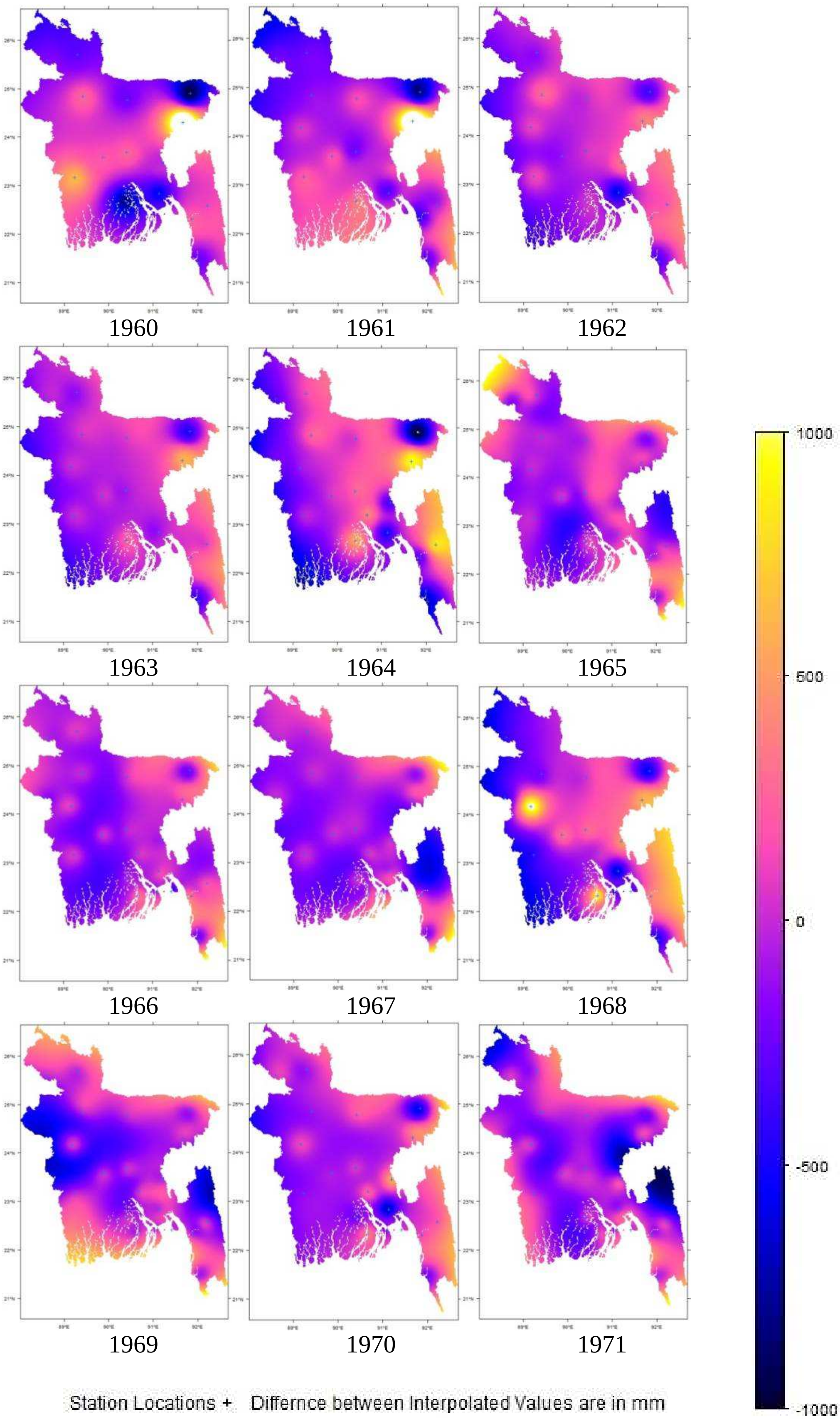

Over Estimated Values

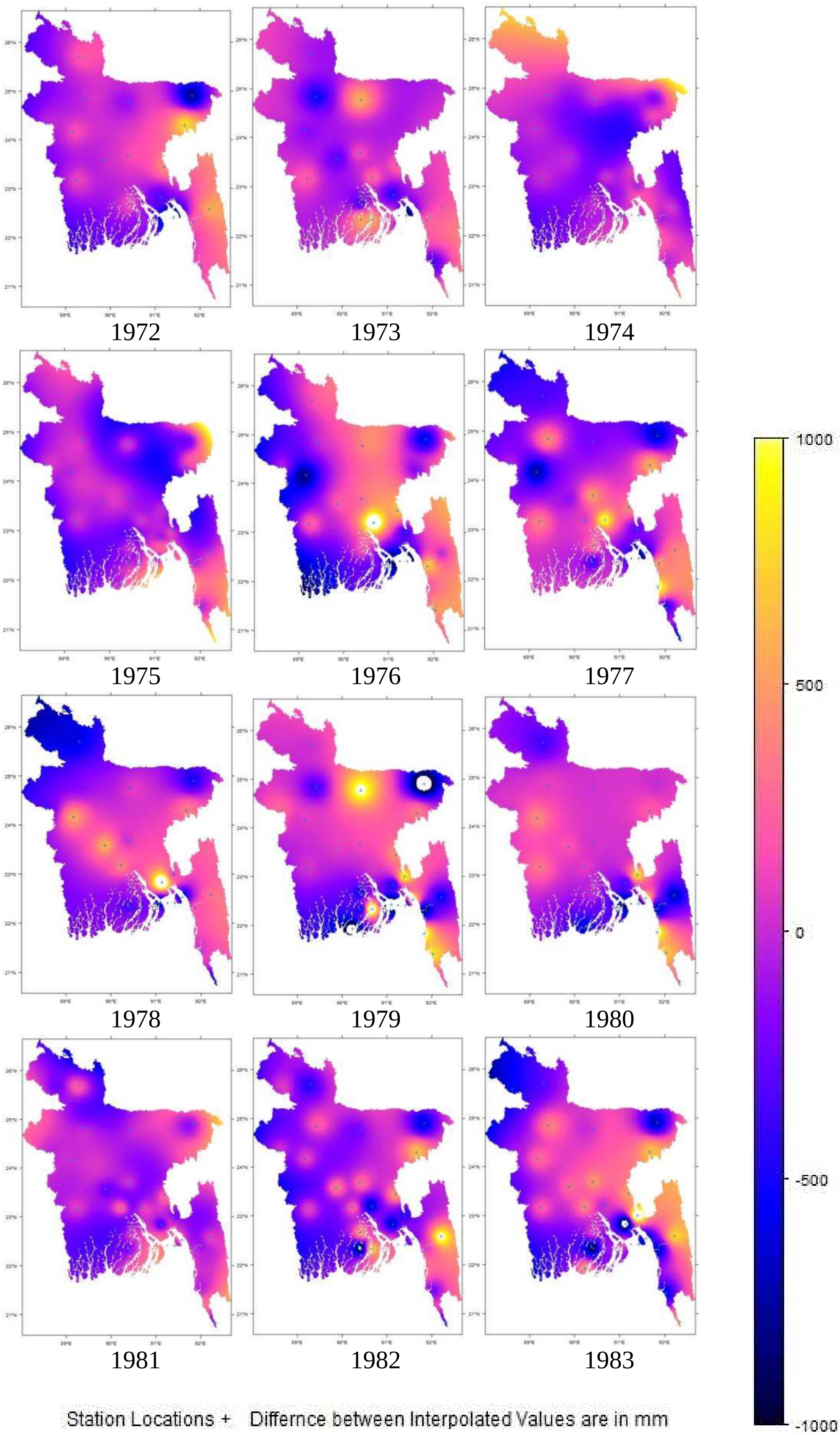
Under Estimated Values

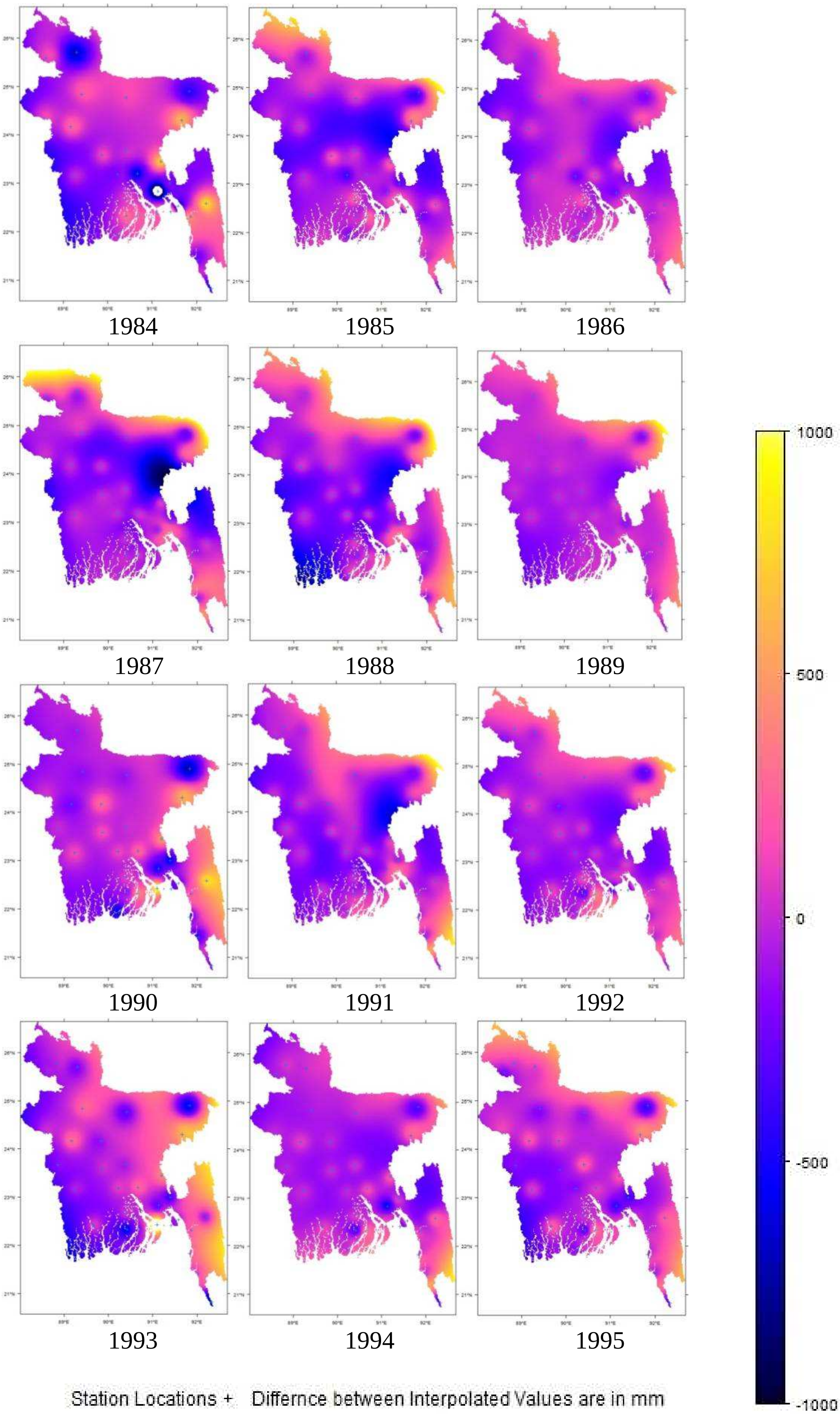
*Note: The relative sizes of the circles depict the degree of over or under estimation i.e the minimum the size of the circle the minimum the difference between measured and predicted values

## A.17 Difference Surfaces between TPS and IDW (TPS-IDW) of PRCPTOT

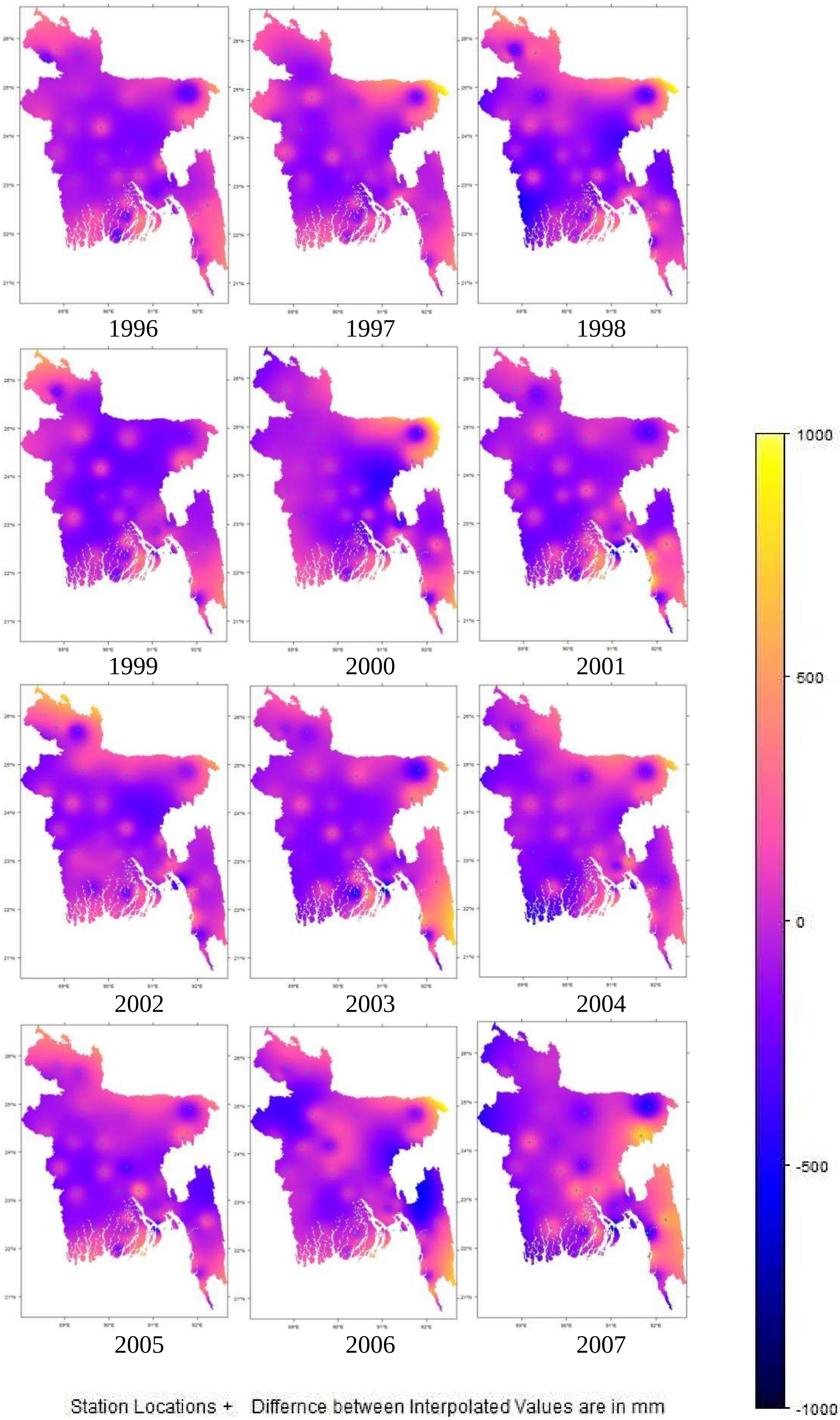



1960
1961
1962
1963
1964
1965
1966
1967
1968
1969
1970
1971
1000
500
0
-500
-1000
Station Locations +
Differnce between Interpolated Values are in mm

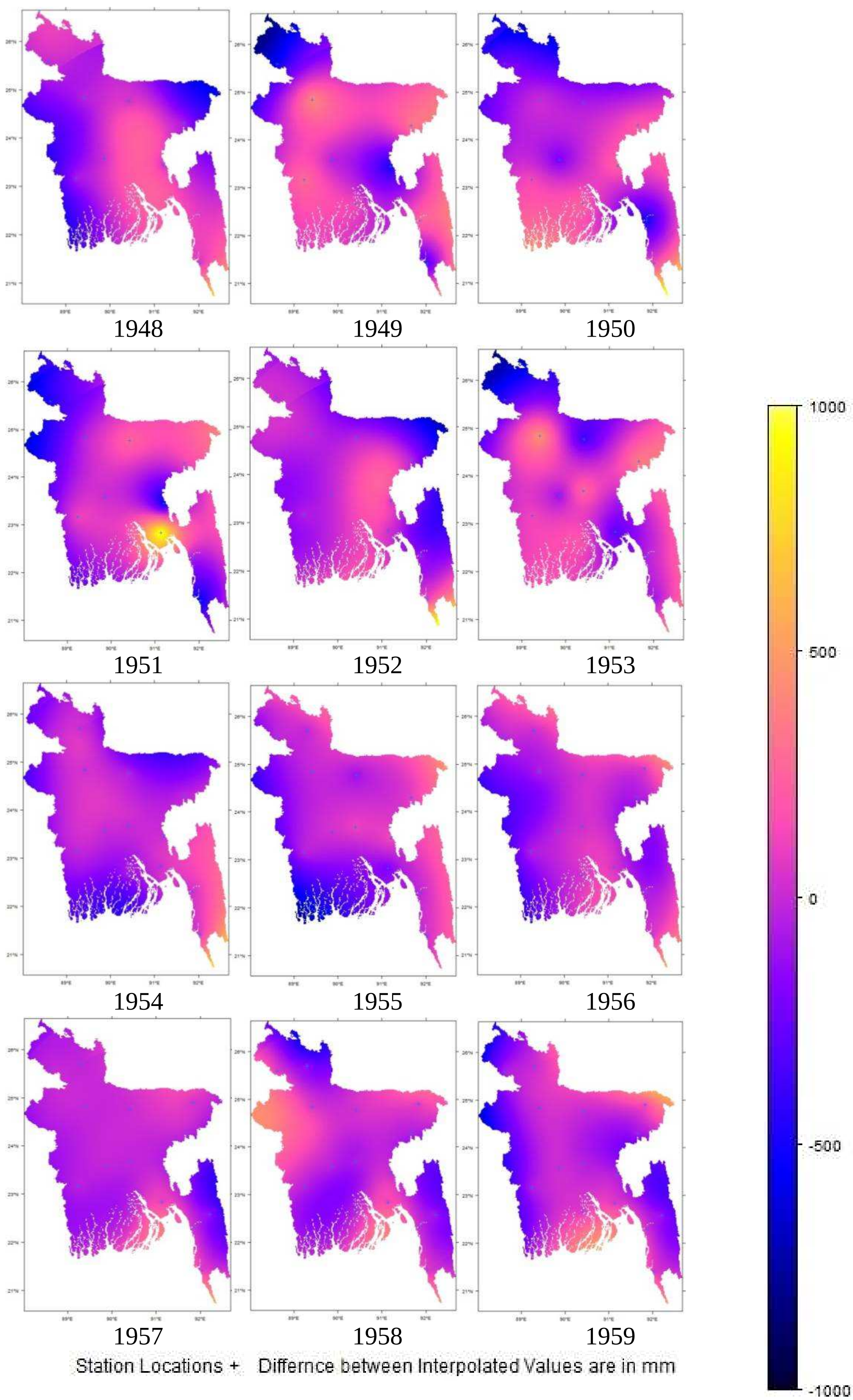

1972
1973
1974
1975
1976
1977
1978
1979
1980
1981
1982
1983
1000
500
0
-500
-1000
Station Locations +
Differnce between Interpolated Values are in mm

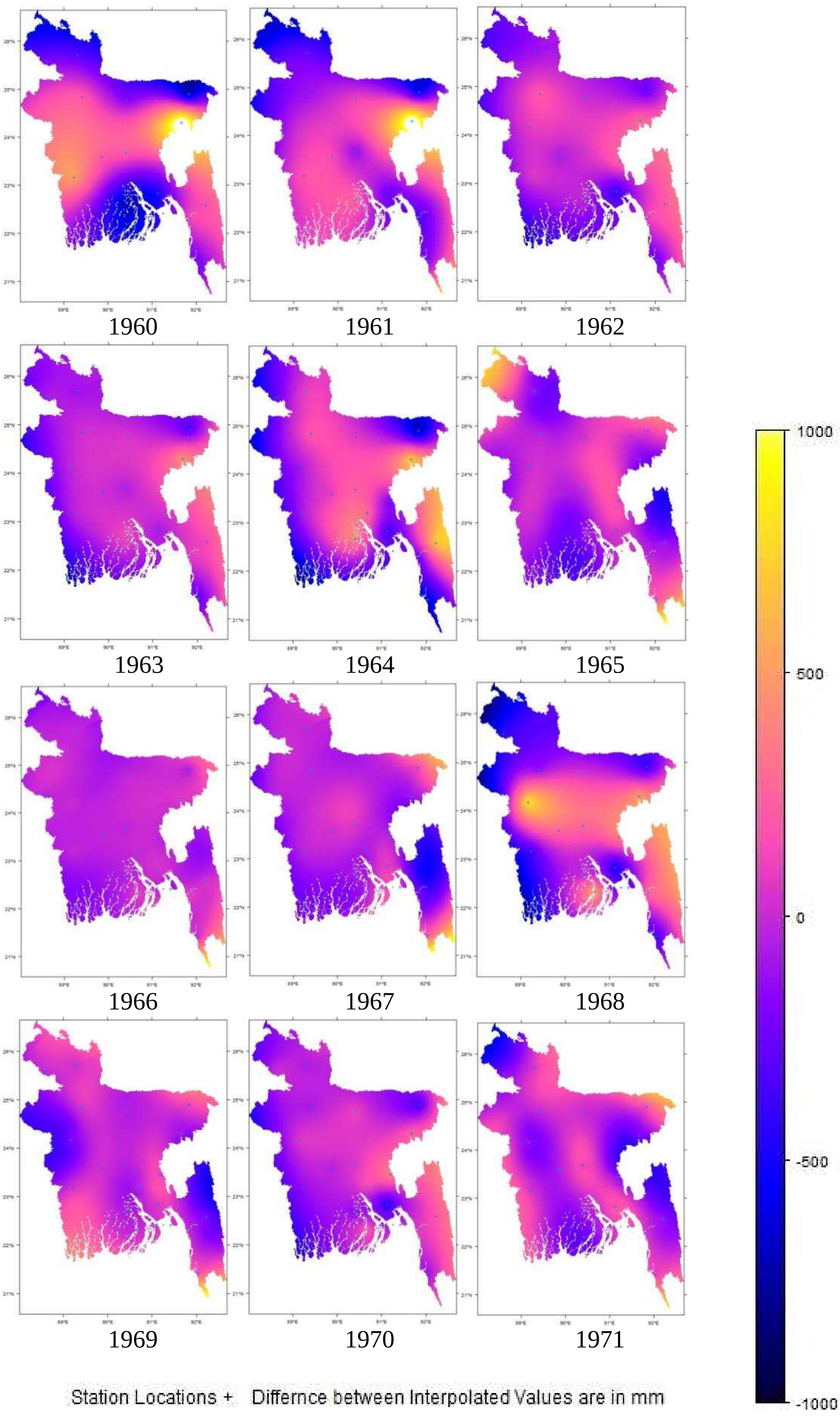


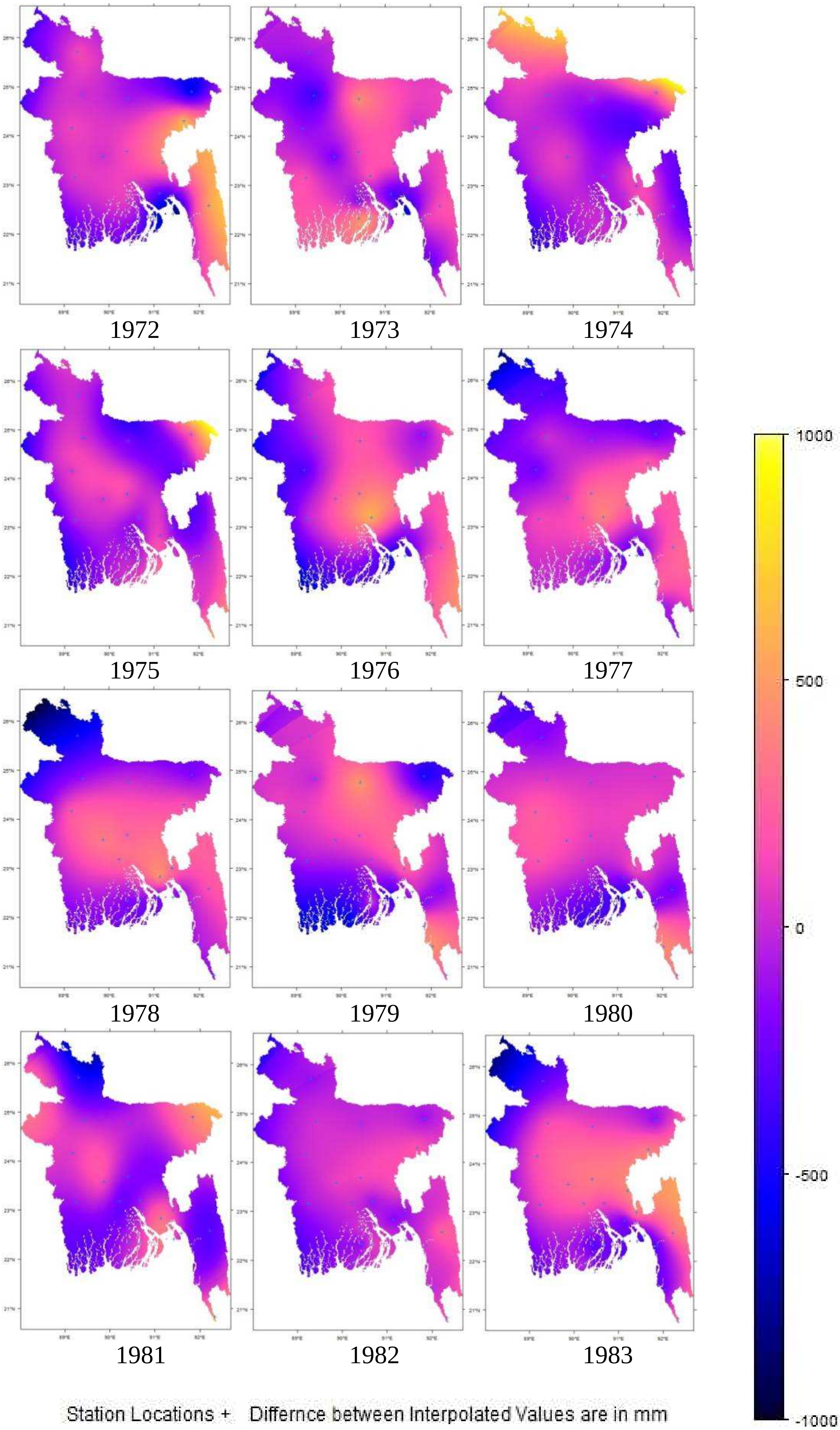

Station Locations +   Differnce between Interpolated Values are in mm


1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
1000
500
0
-500
-1000
Station Locations +
Differnce between Interpolated Values are in mm

## A.18 Difference Surfaces between TPS and OK (TPS-OK) of PRCPTOT




1960
1961
1962
1963
1964
1965
1966
1967
1968
1969
1970
1971
1000
500
0
-500
-1000
Station Locations +
Differnce between Interpolated Values are in mm


1972
1973
1974
1975
1976
1977
1978
1979
1980
1981
1982
1983
1000
500
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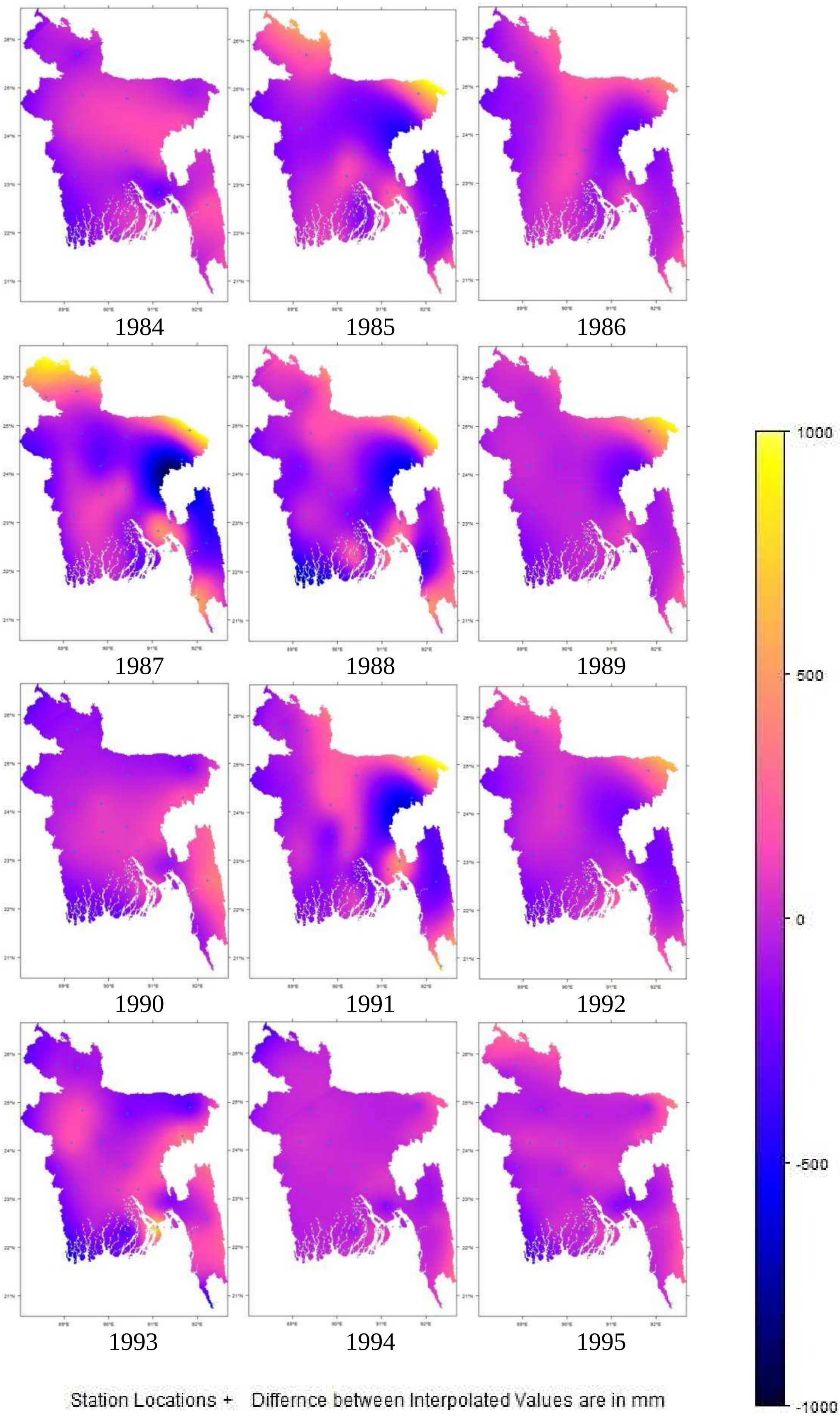
Station Locations +
Differnce between Interpolated Values are in mm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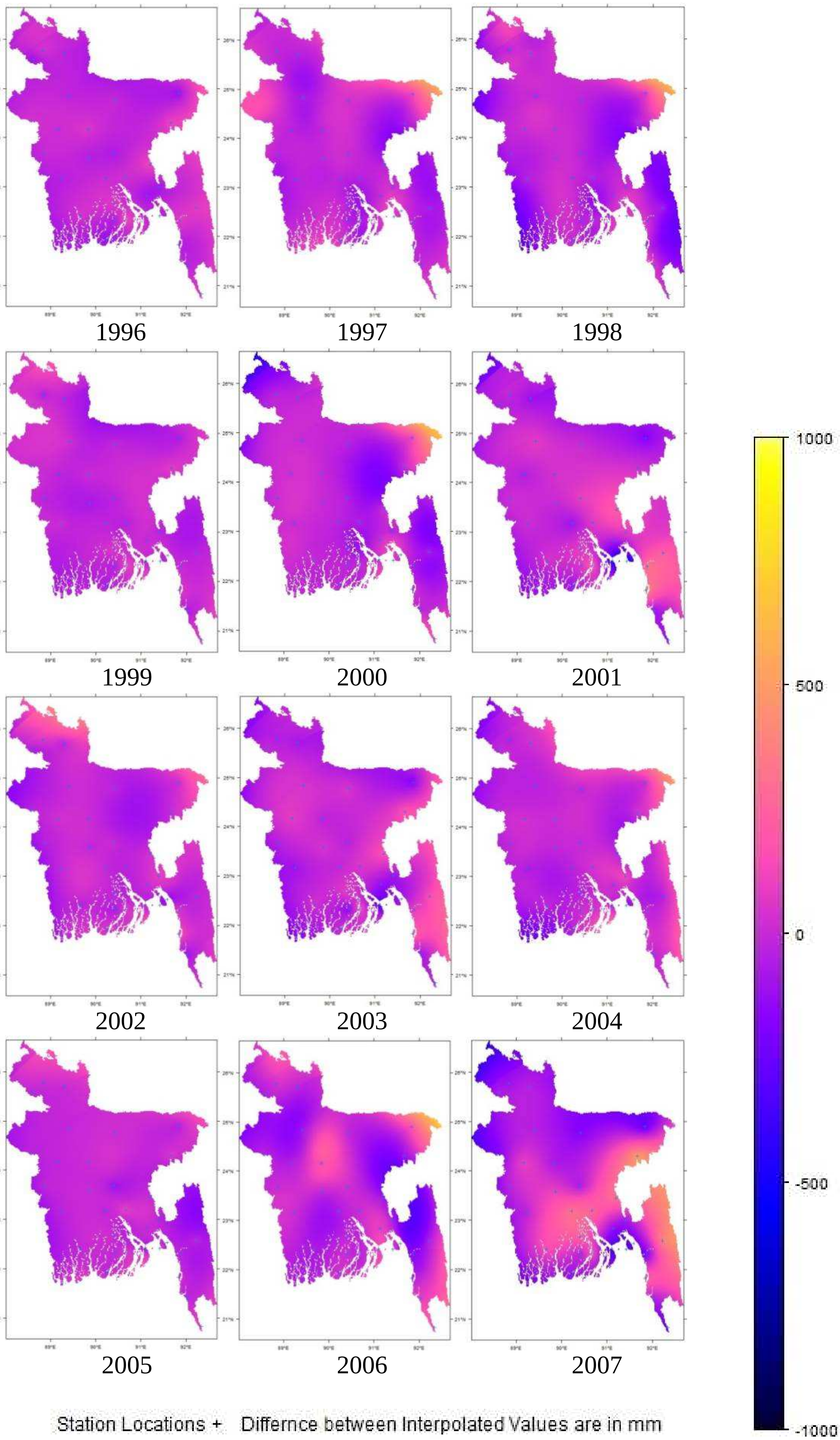
Station Locations +
Differnce between Interpolated Values are in mm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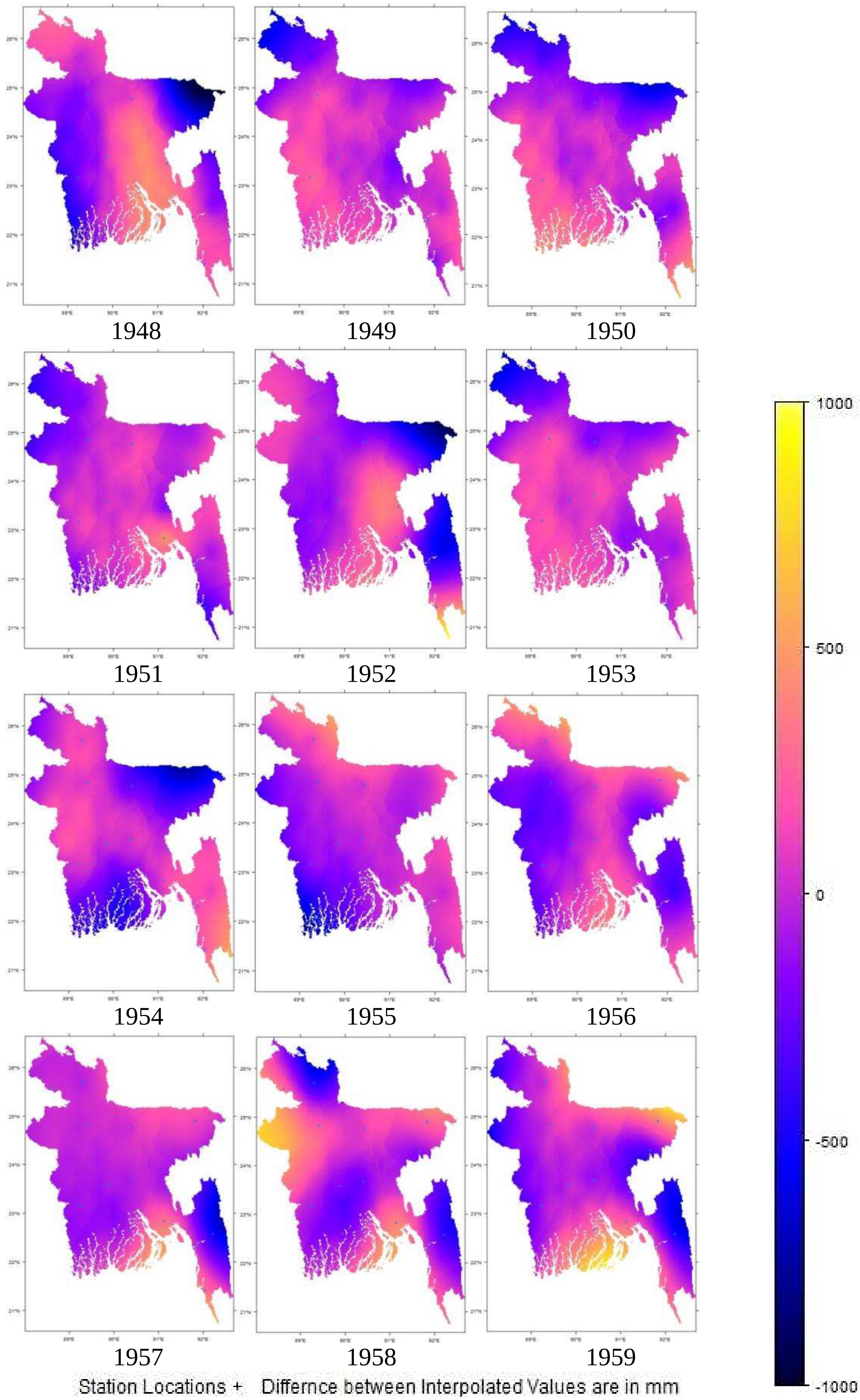
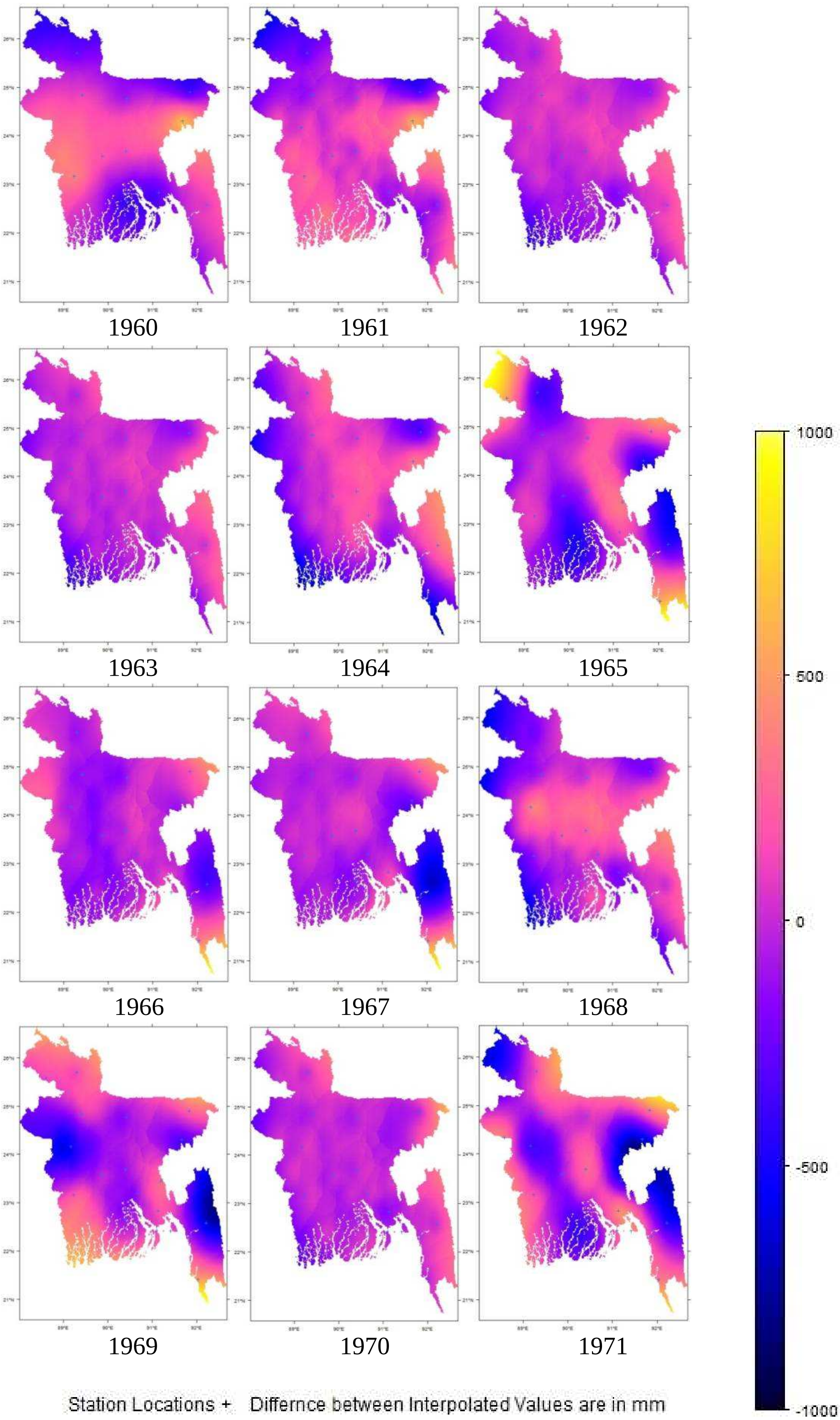
Station Locations +    Differnce between Interpolated Values are in mm

**A.19 Difference Surfaces between TPS and UK (TPS-UK) of PRCPTOT**




1960
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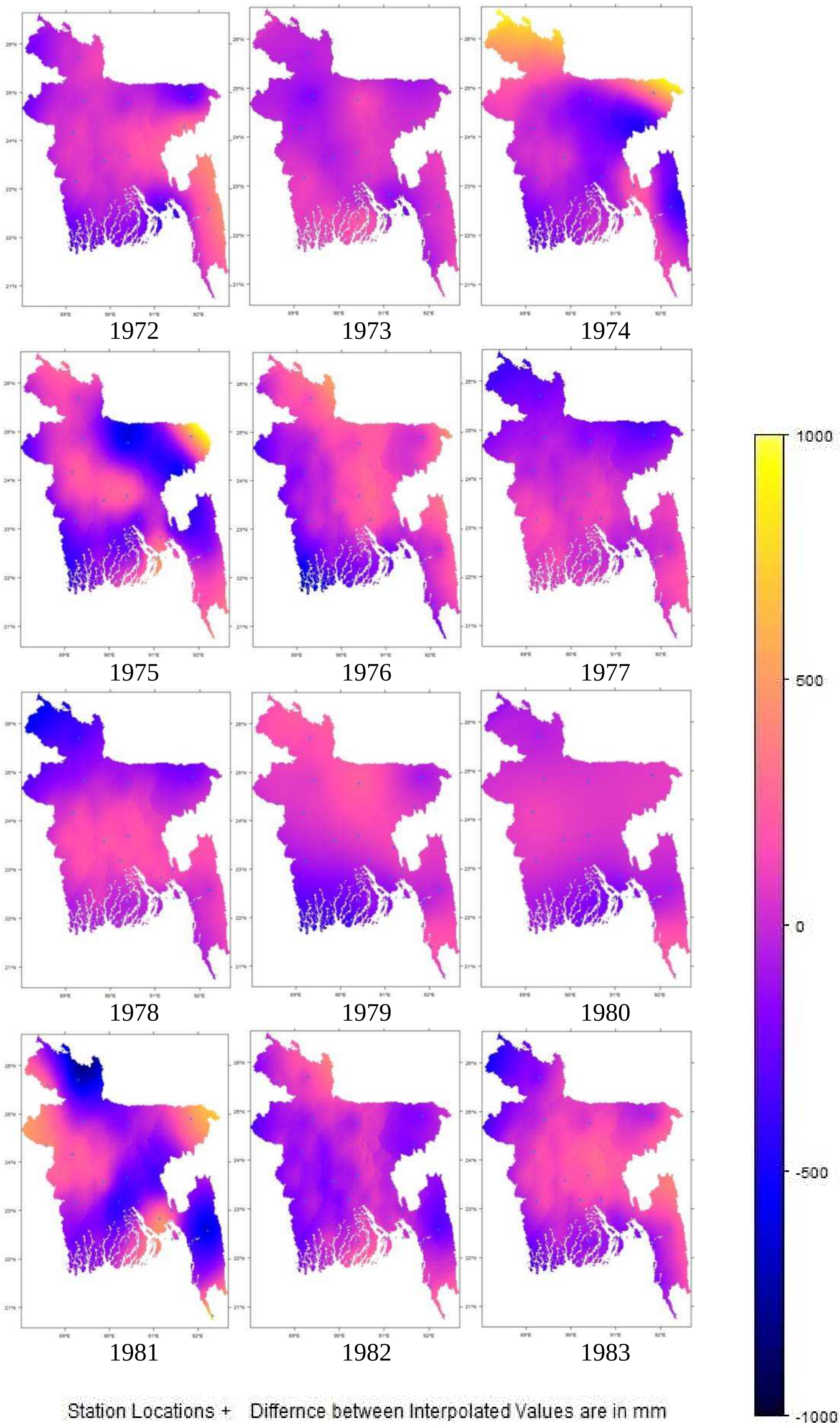
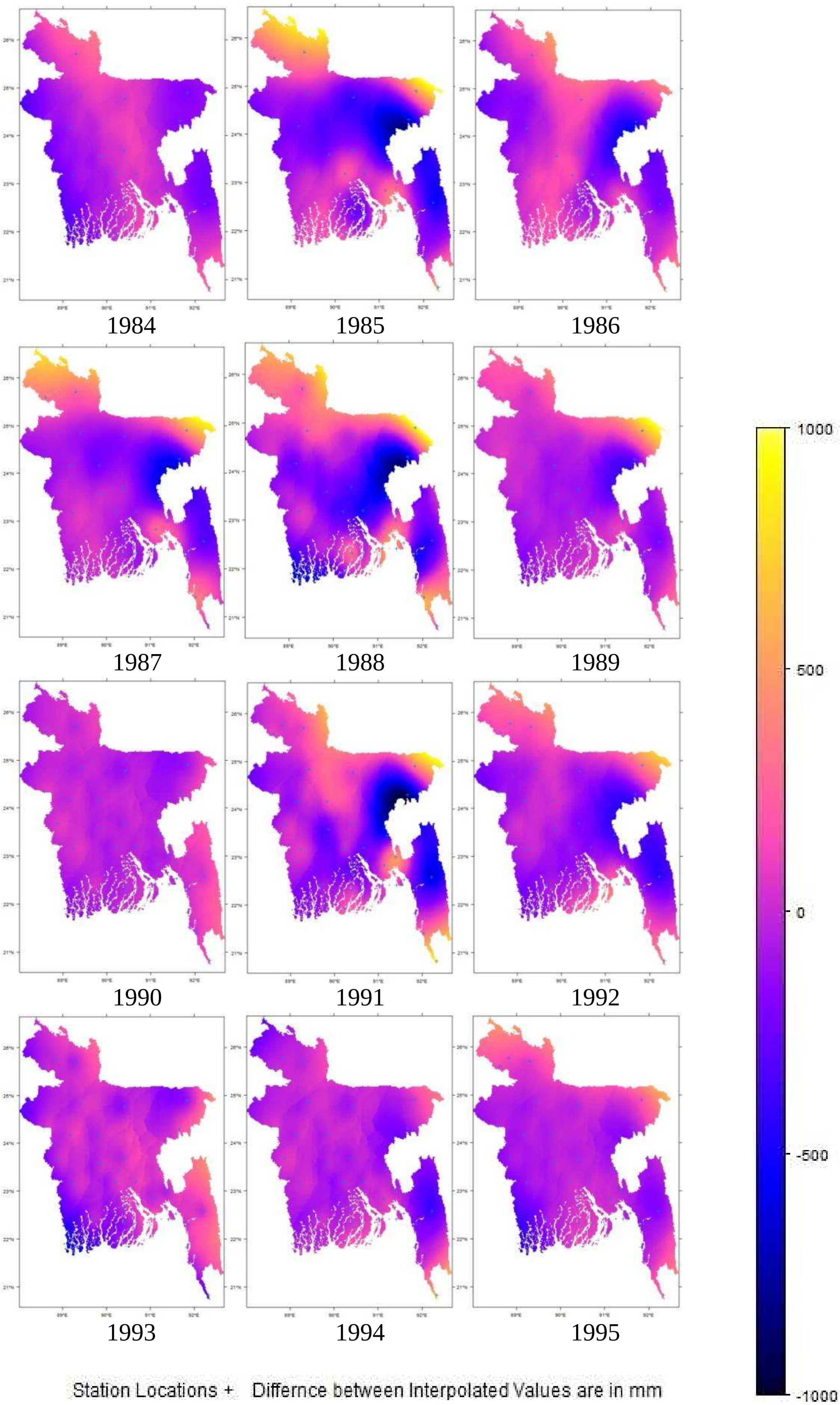
Station Locations +    Differnce between Interpolated Values are in mm




Station Locations + Differnce between Interpolated Values are in mm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Station Locations +
Differnce between Interpolated Values are in mm
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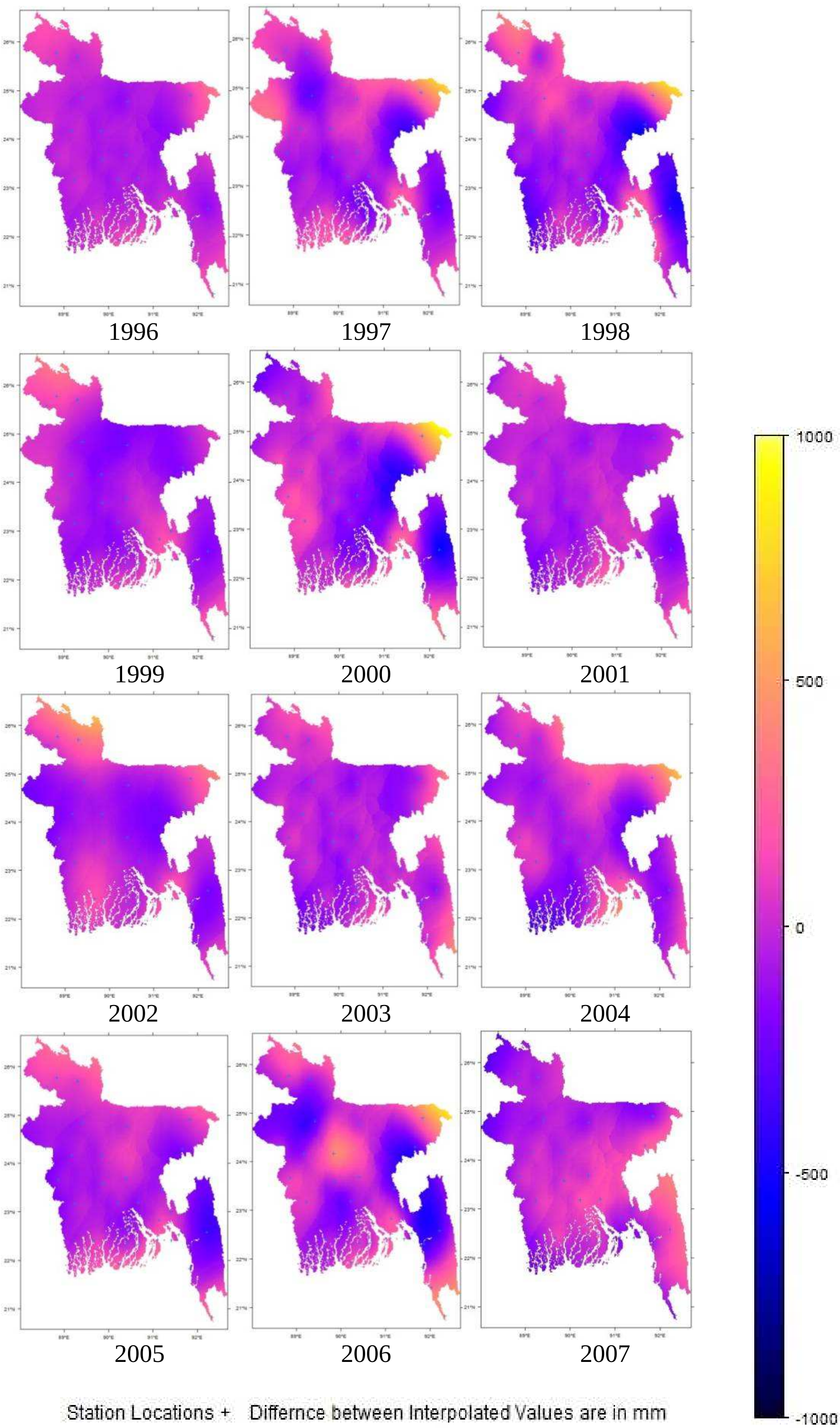
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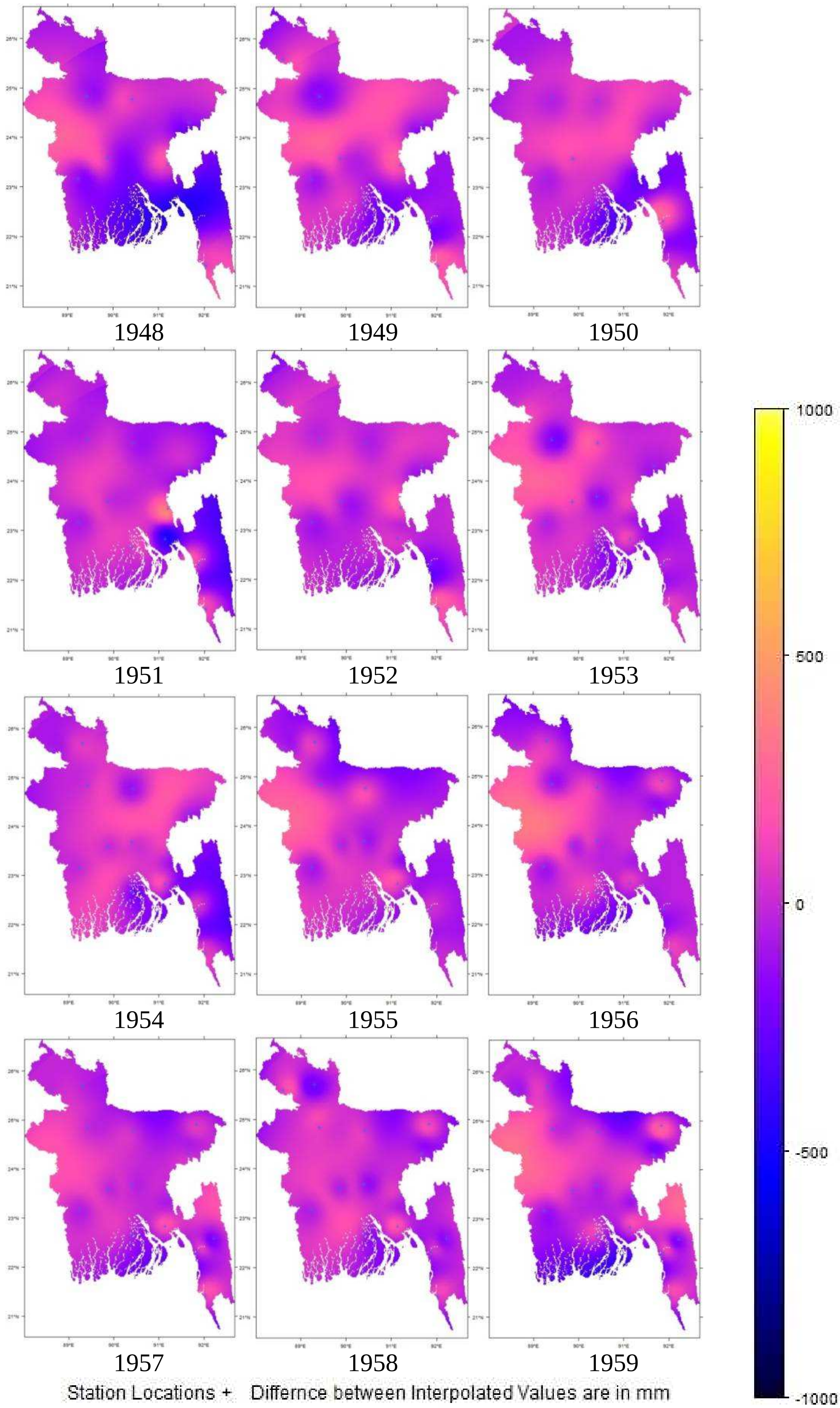
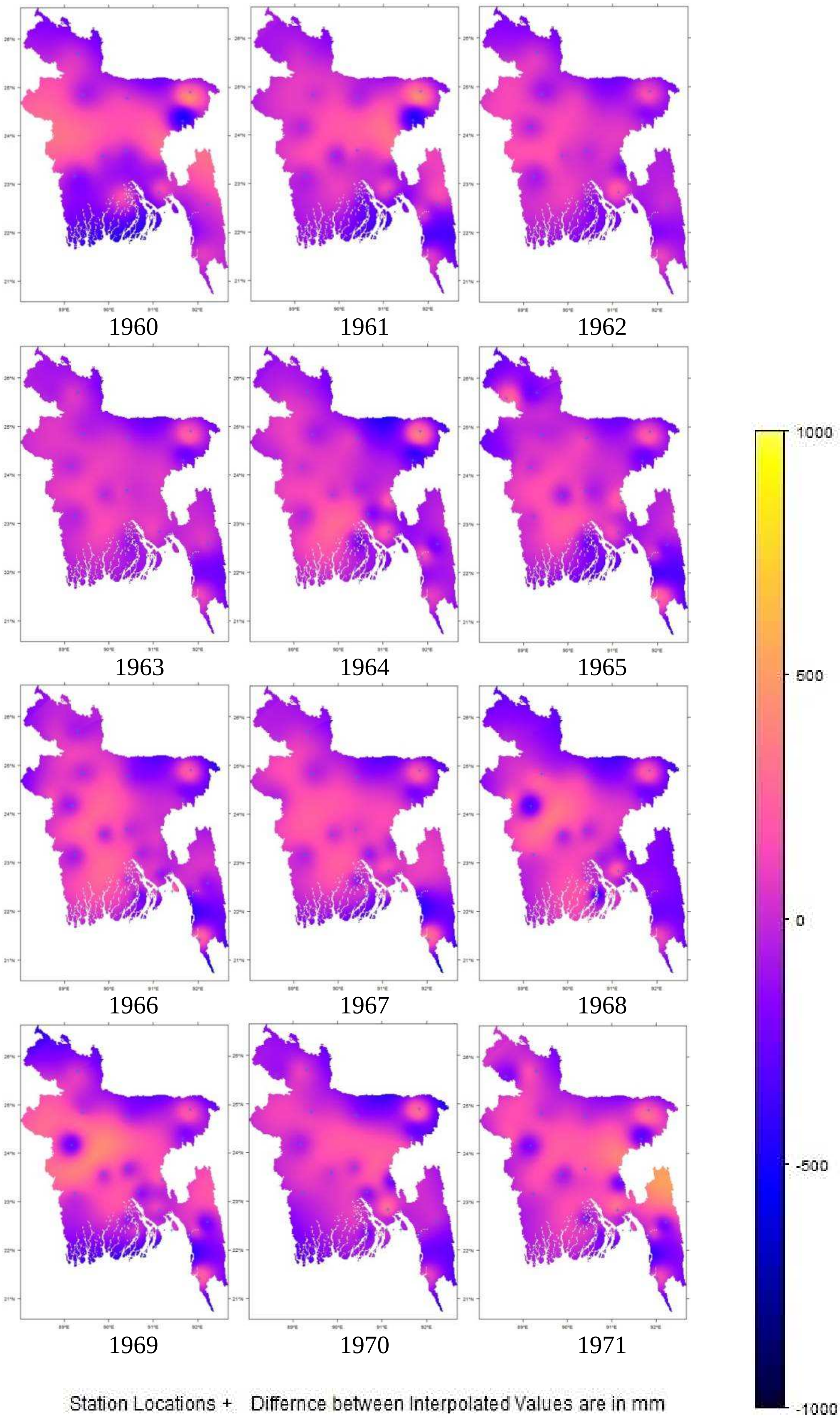
Station Locations +
Differnce between Interpolated Values are in mm

## A.20 Difference Surfaces between IDW and OK (IDW-OK) of PRCPTOT




Station Locations + Differnce between Interpolated Values are in mm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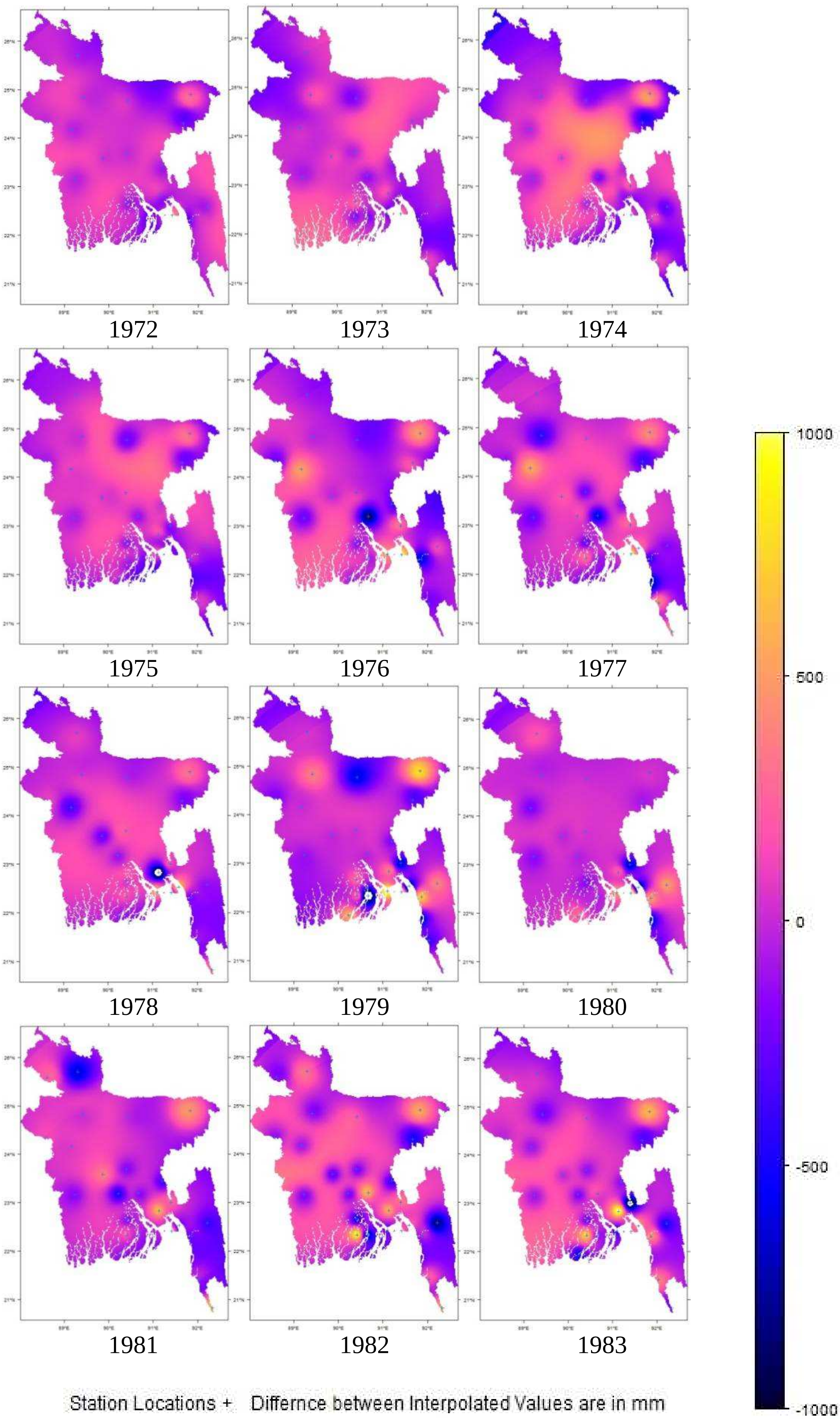
Station Locations +
Differnce between Interpolated Values are in mm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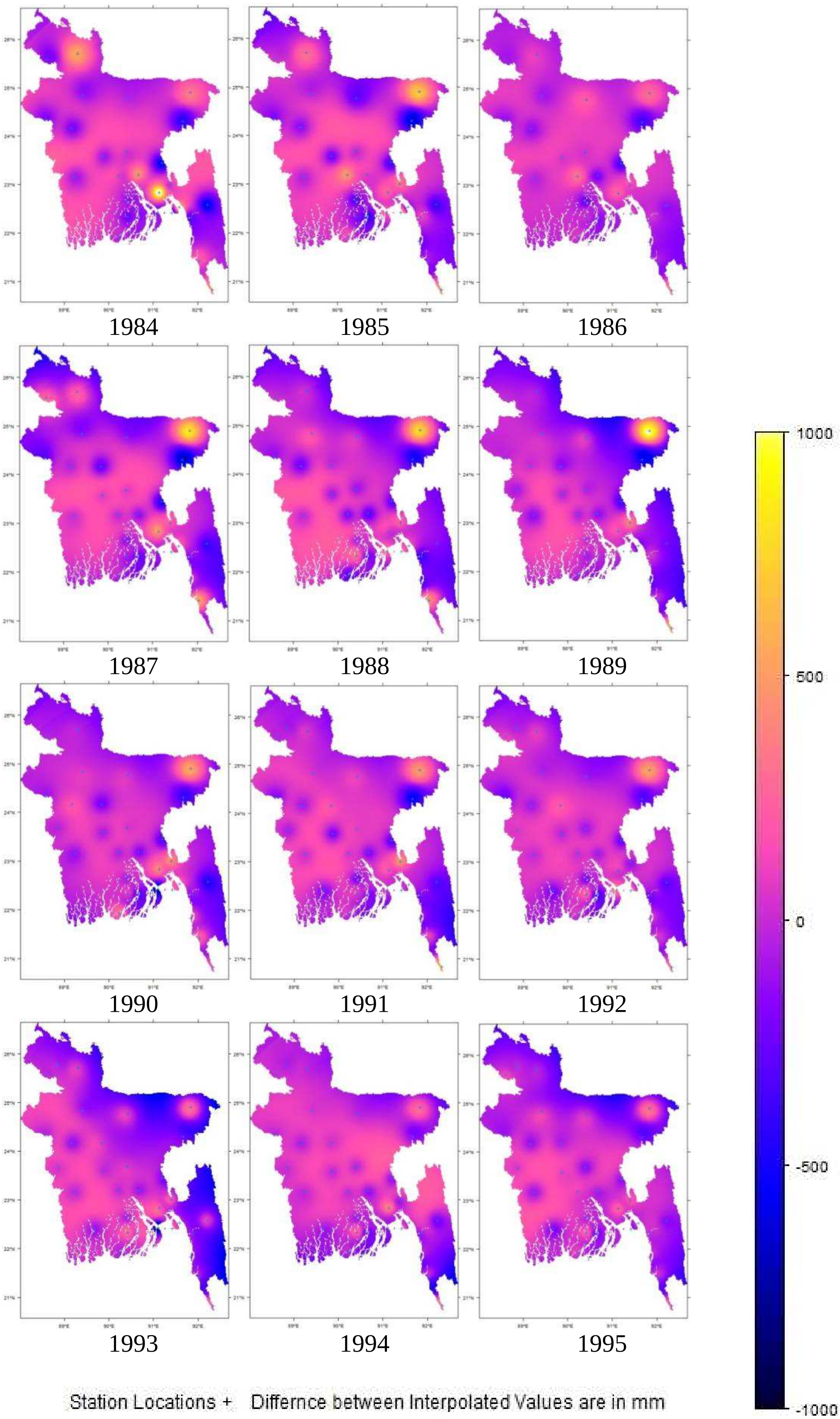
Station Locations +
Differnce between Interpolated Values are in mm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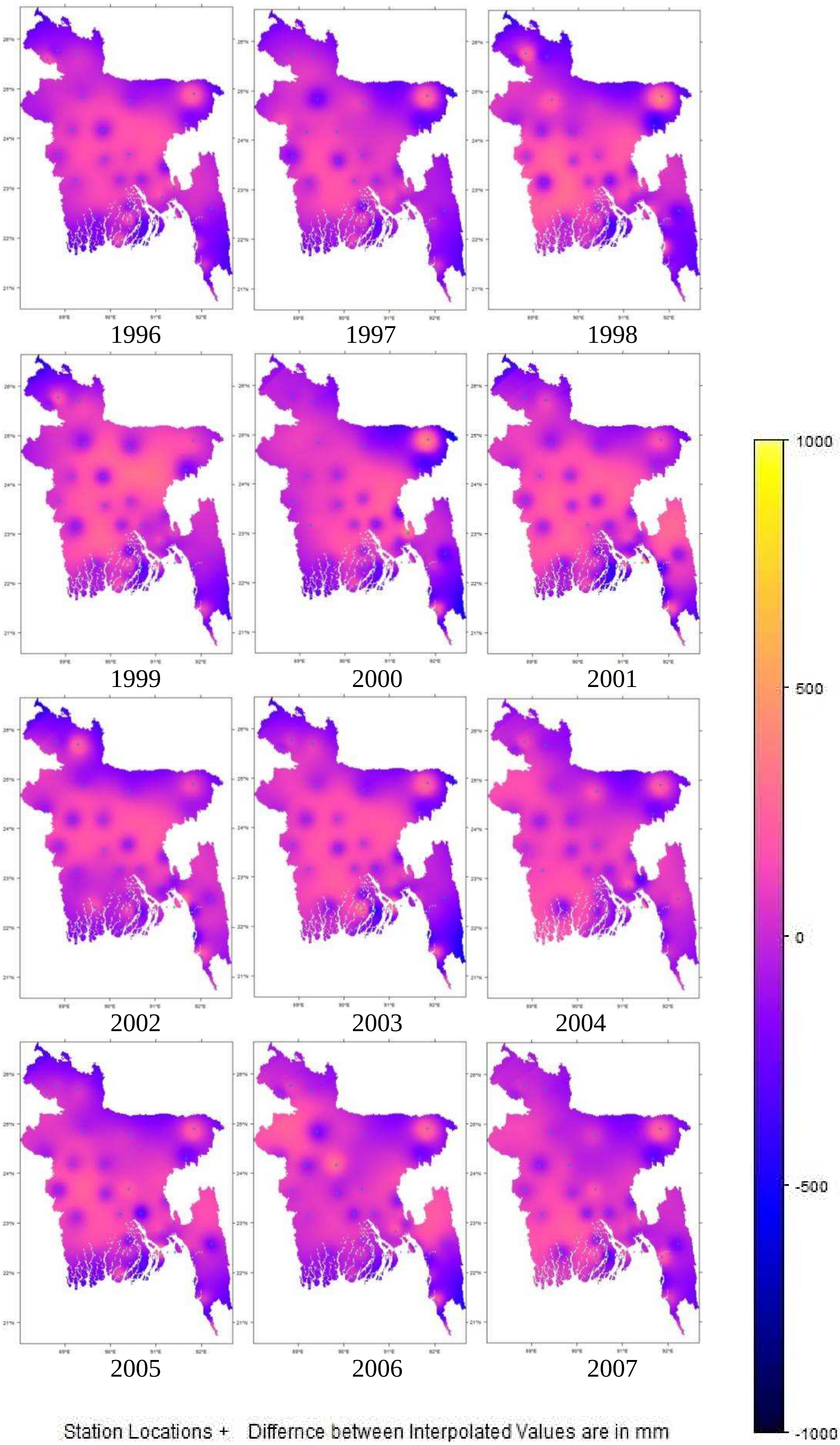
Station Locations +
Differnce between Interpolated Values are in mm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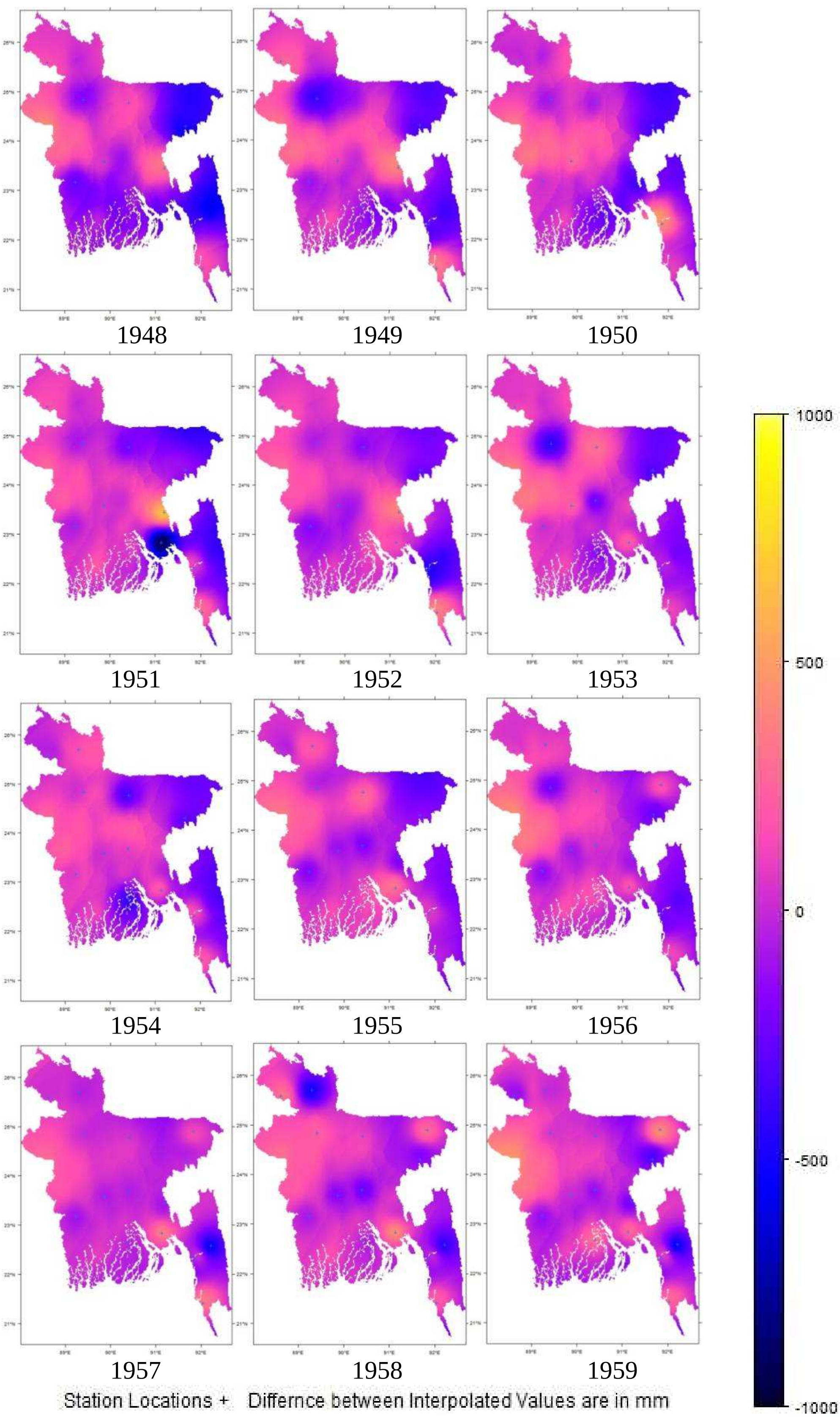
Station Locations +
Differnce between Interpolated Values are in mm

## A.21 Difference Surfaces between IDW and UK (IDW-UK) of PRCPTOT



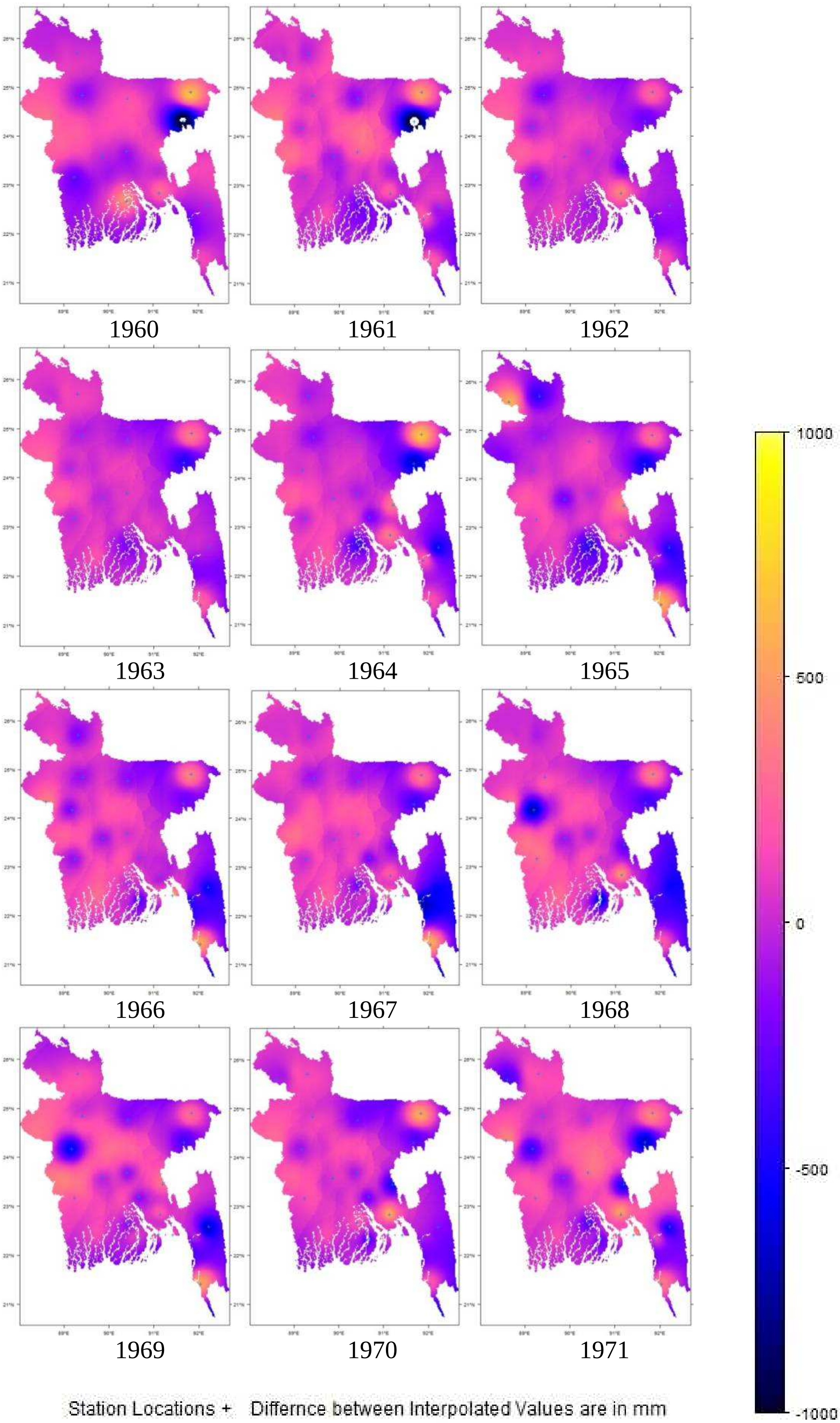

1948 1949 1950

1951 1952 1953

1954 1955 1956

1957 1958 1959

Station Locations + Differnce between Interpolated Values are in mm



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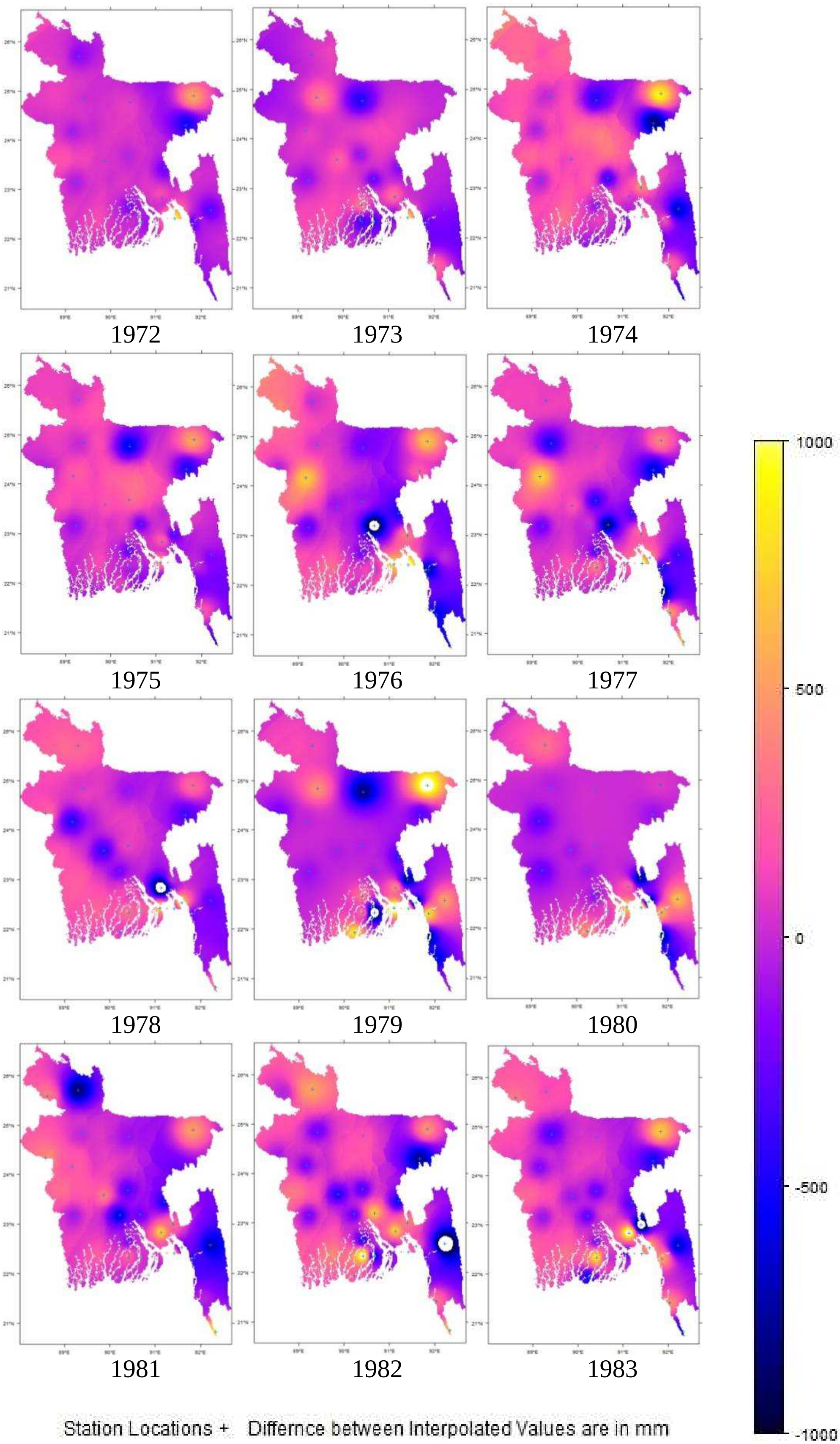
Station Locations +
Differnce between Interpolated Values are in mm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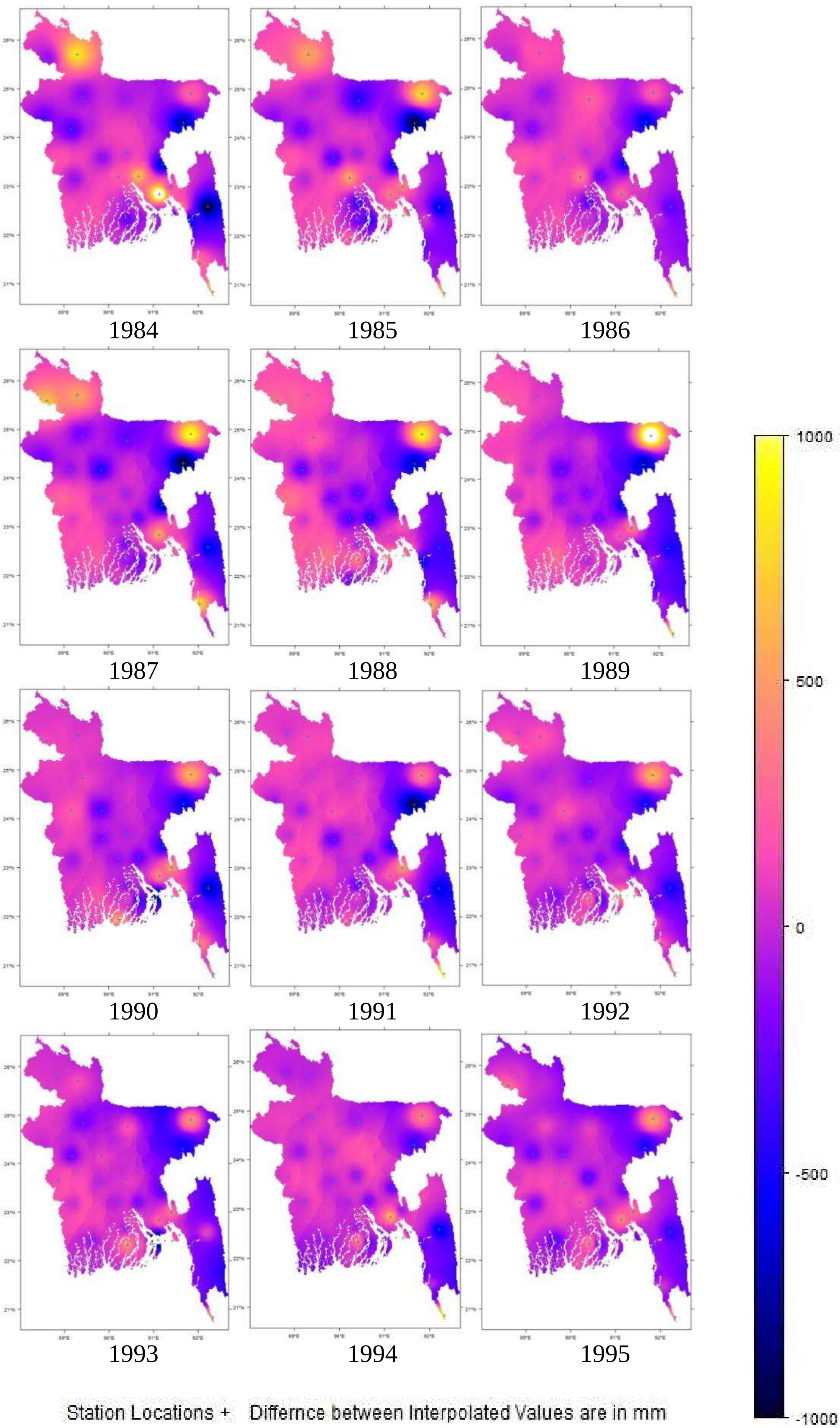
Station Locations + Differnce between Interpolated Values are in mm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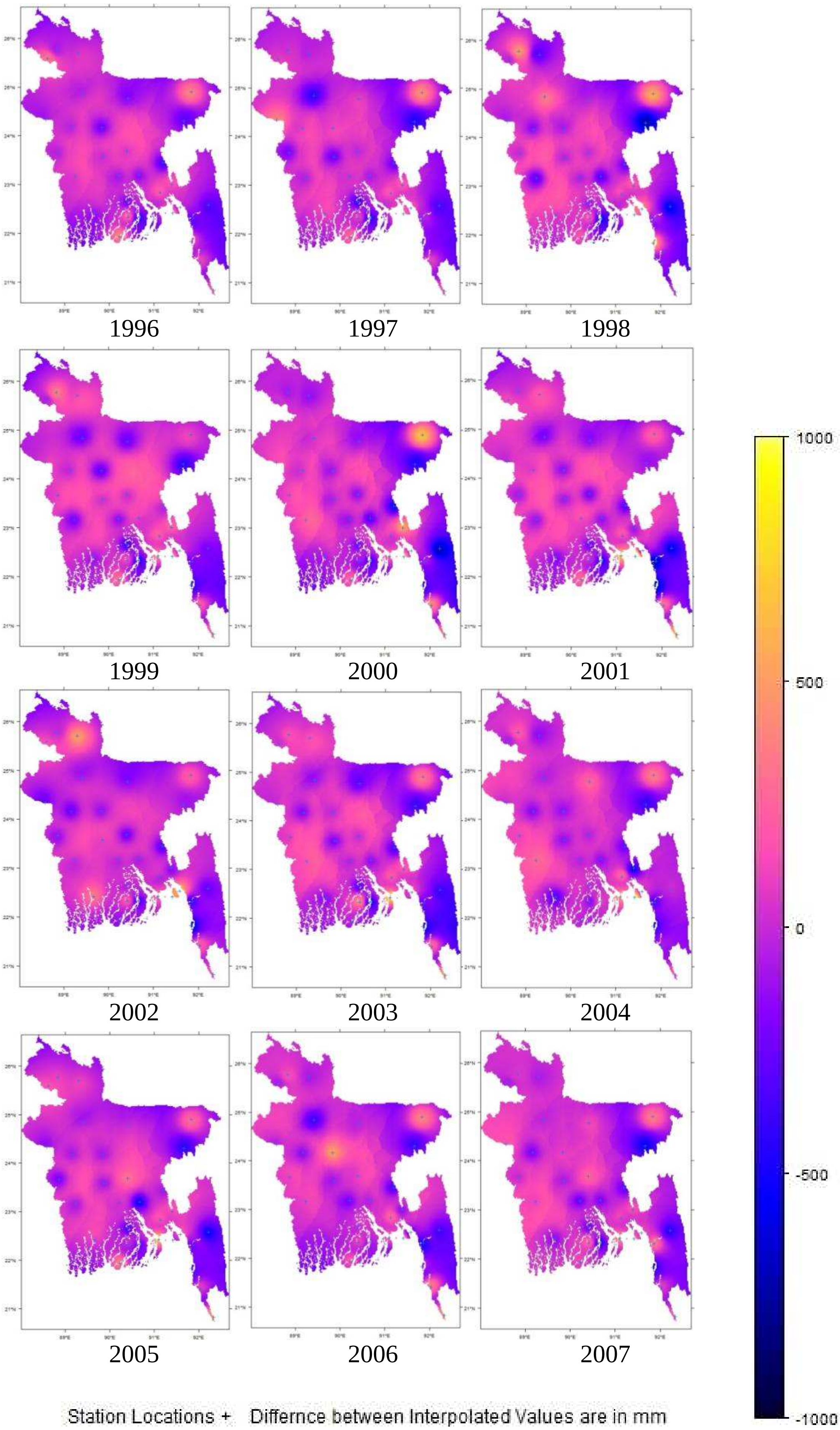
Station Locations +
Differnce between Interpolated Values are in mm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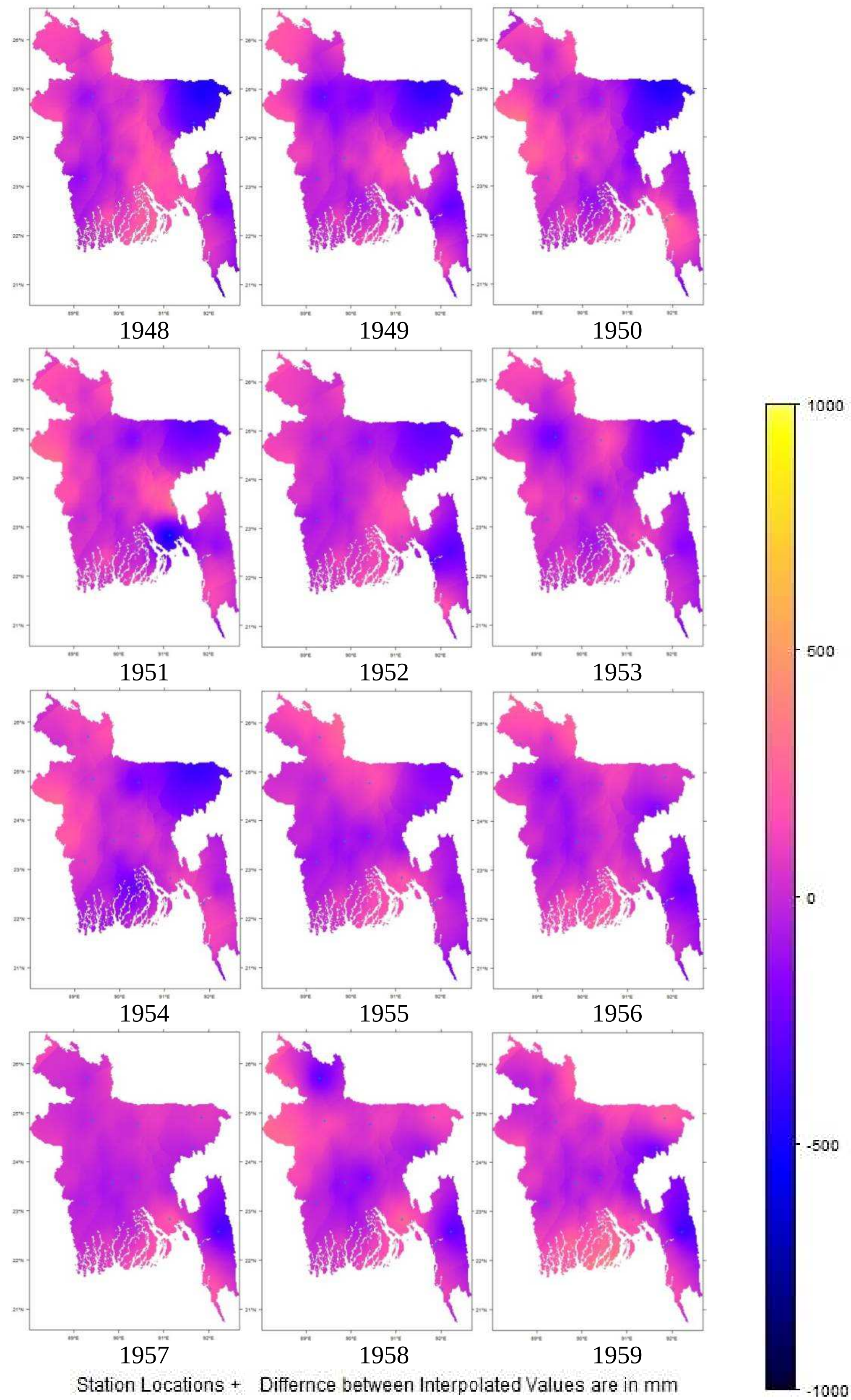
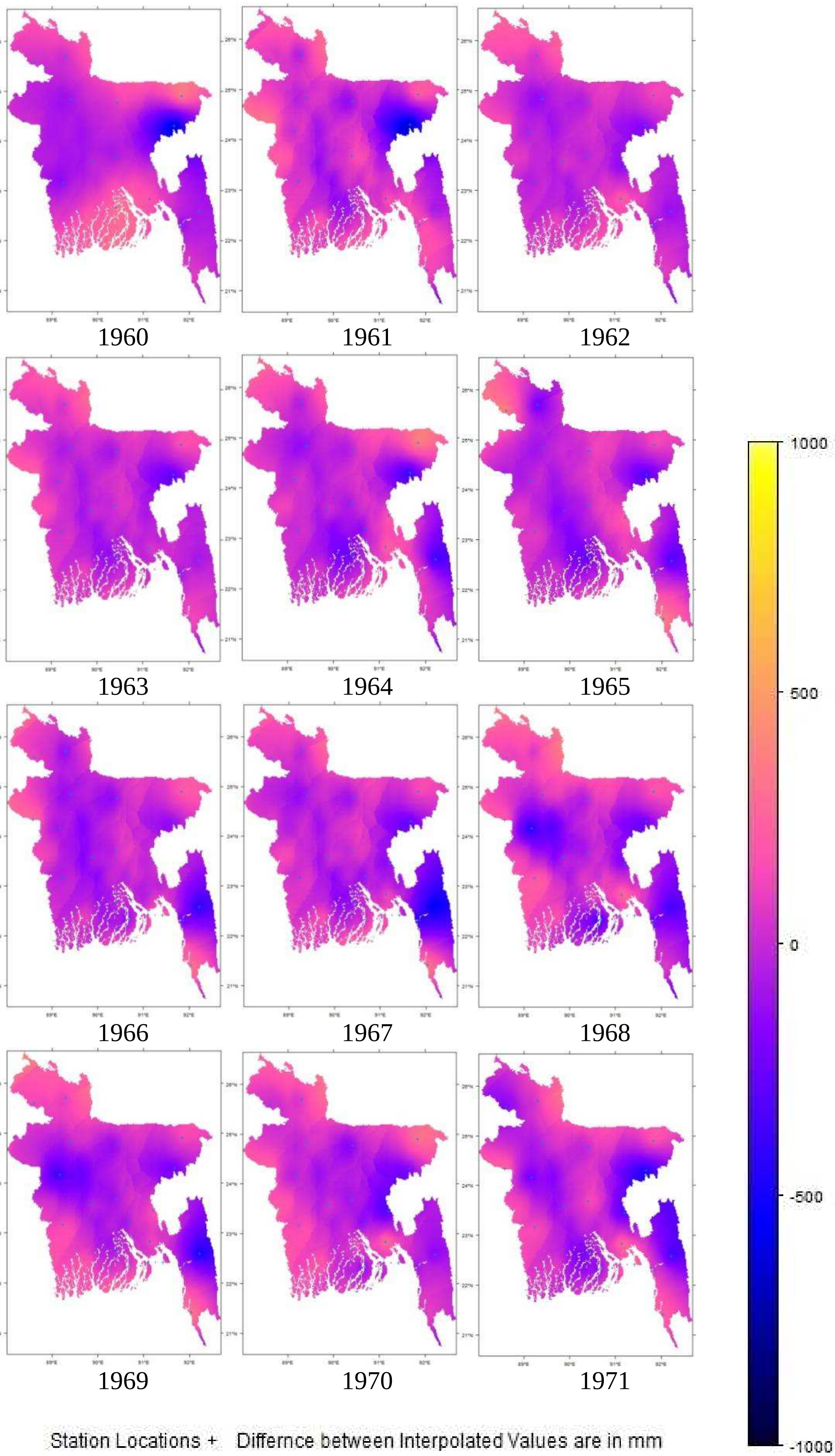
Station Locations +
Differnce between Interpolated Values are in mm

## A.22 Difference Surfaces between OK and UK (OK-UK) of PRCPTOT




Station Locations +   Differnce between Interpolated Values are in mm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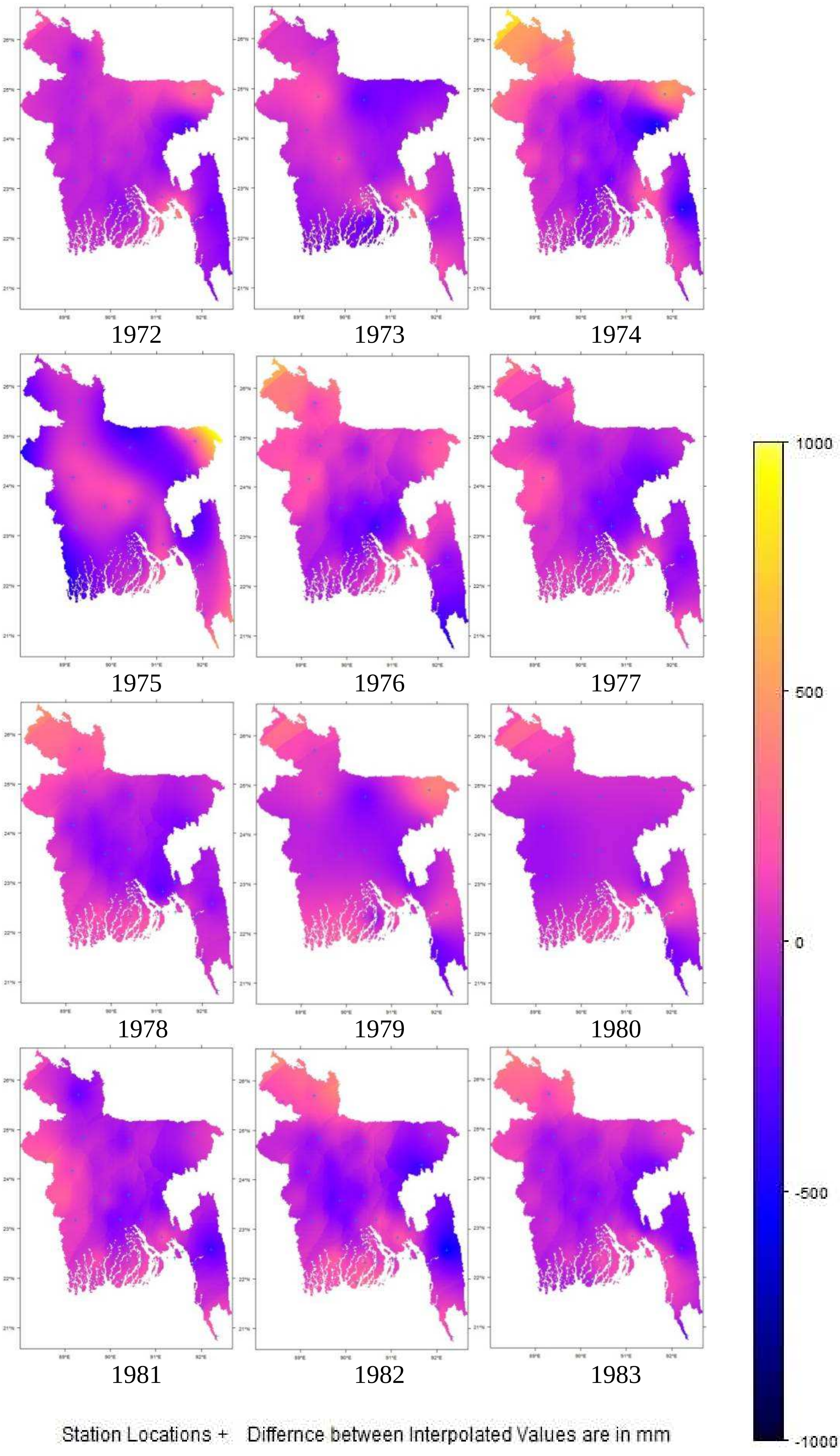
Station Locations +    Differnce between Interpolated Values are in mm



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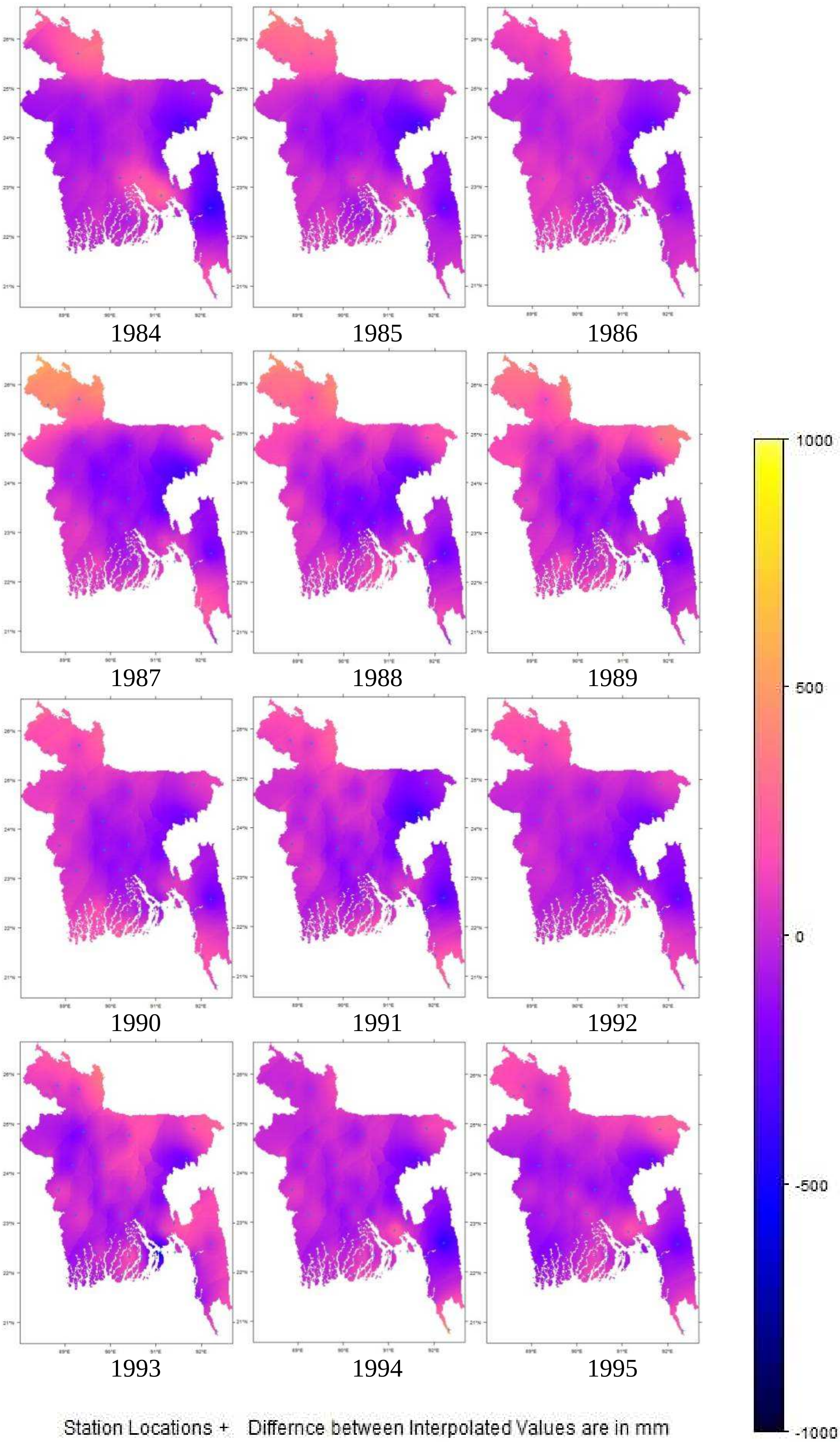
Station Locations +
Differnce between Interpolated Values are in mm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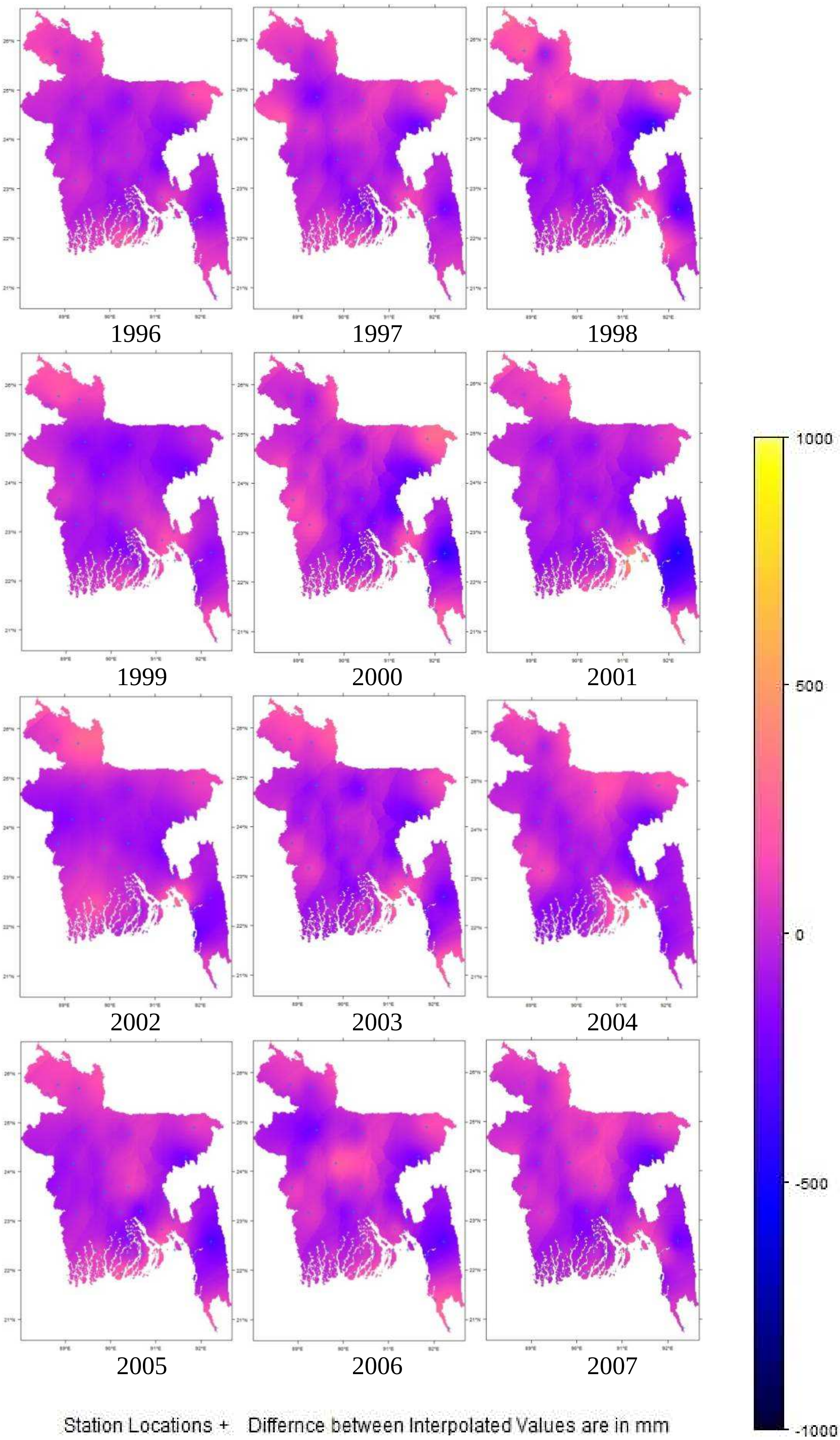
Station Locations +
Differnce between Interpolated Values are in mm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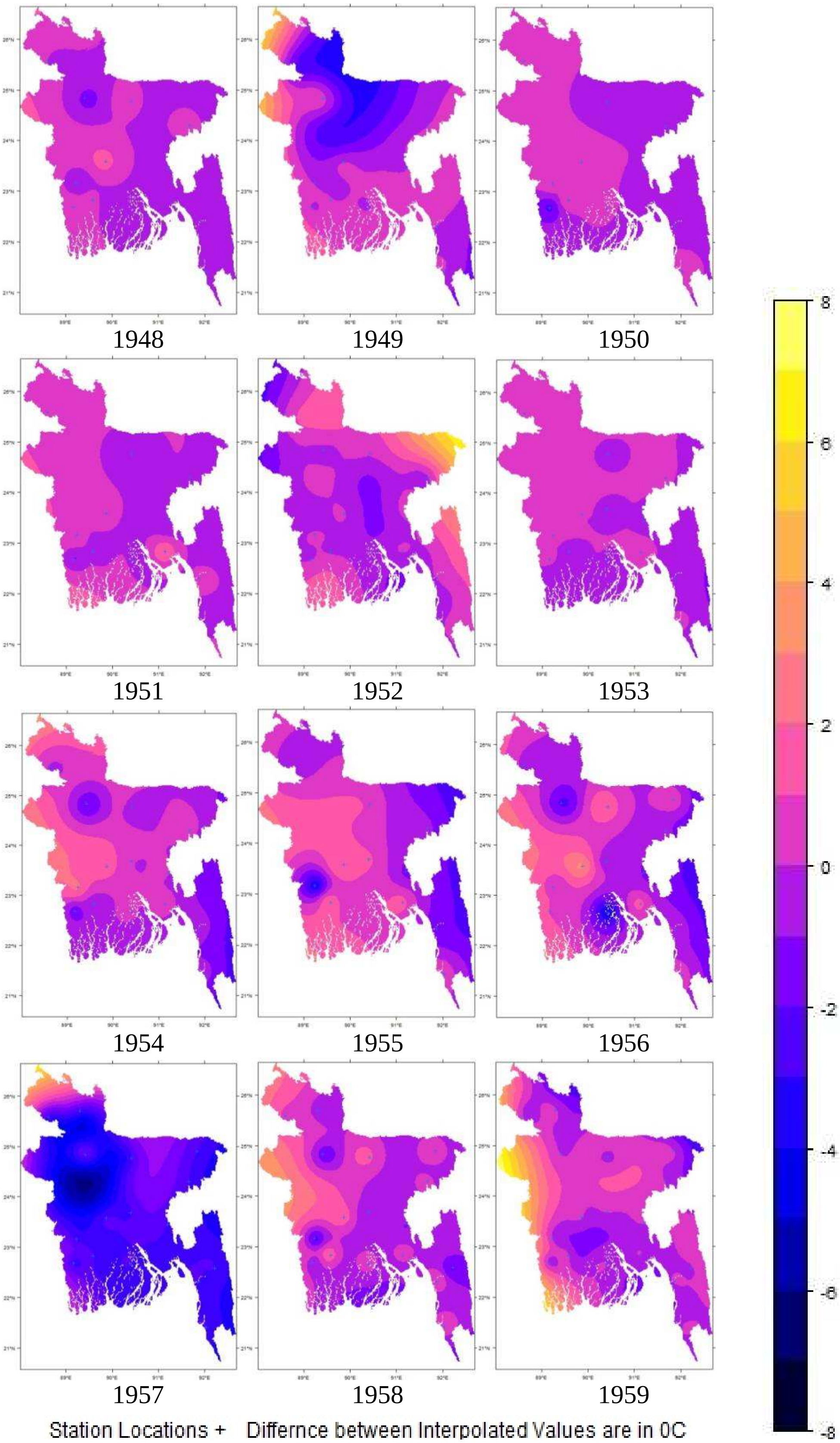
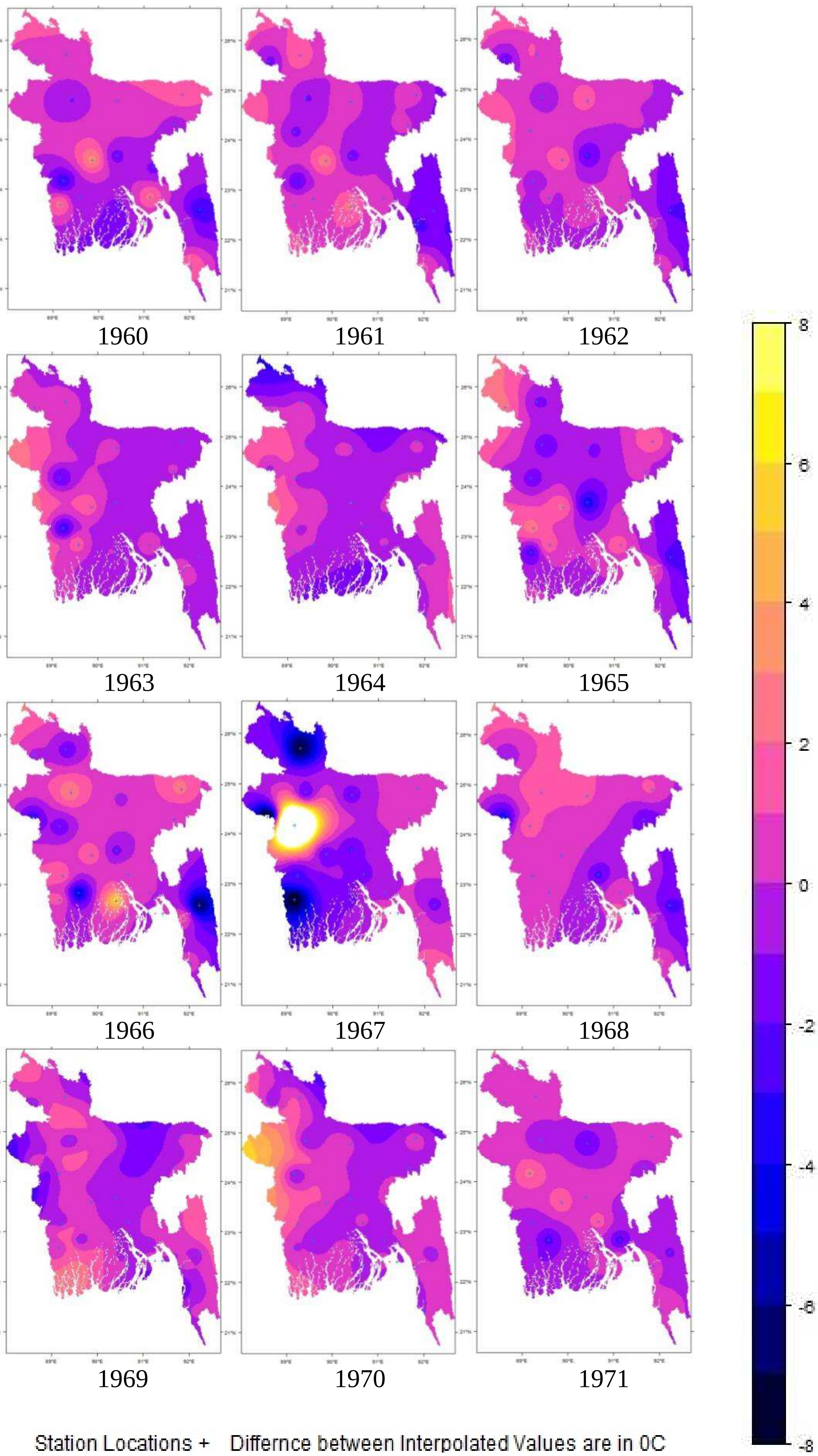
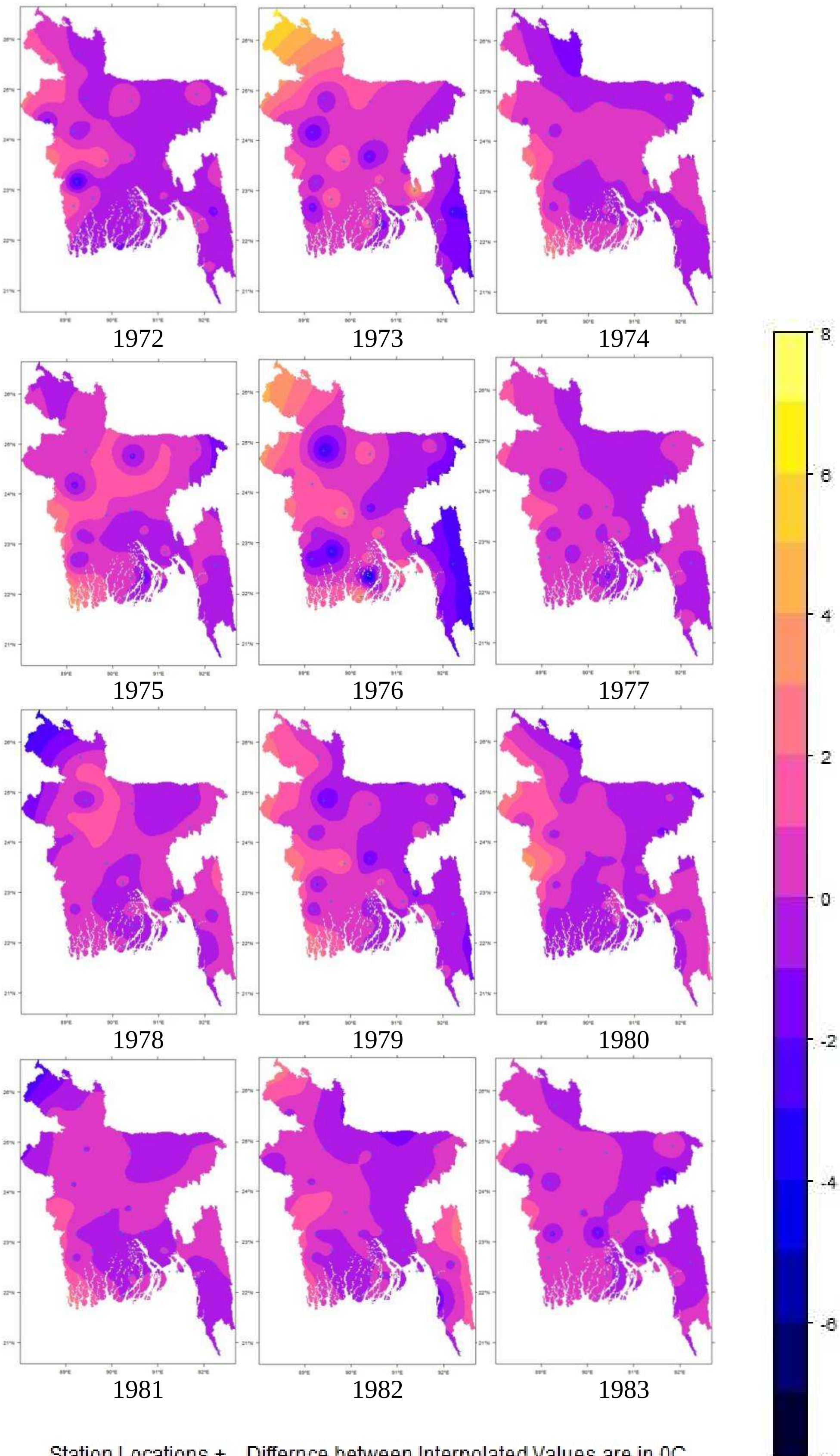
Station Locations +
Differnce between Interpolated Values are in mm

## A.23 Difference Surfaces between TPS and IDW (TPS-IDW) of TXx

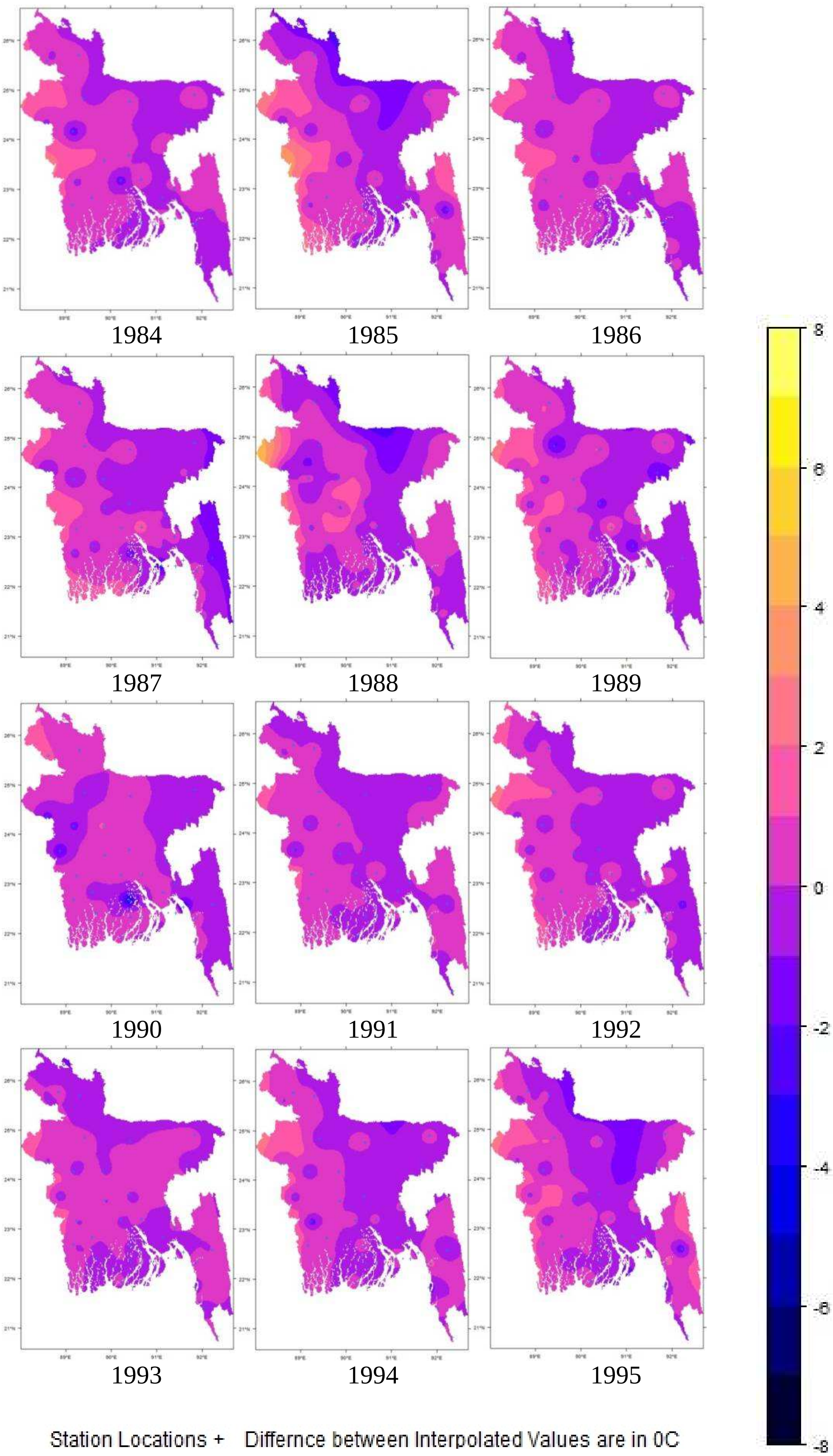




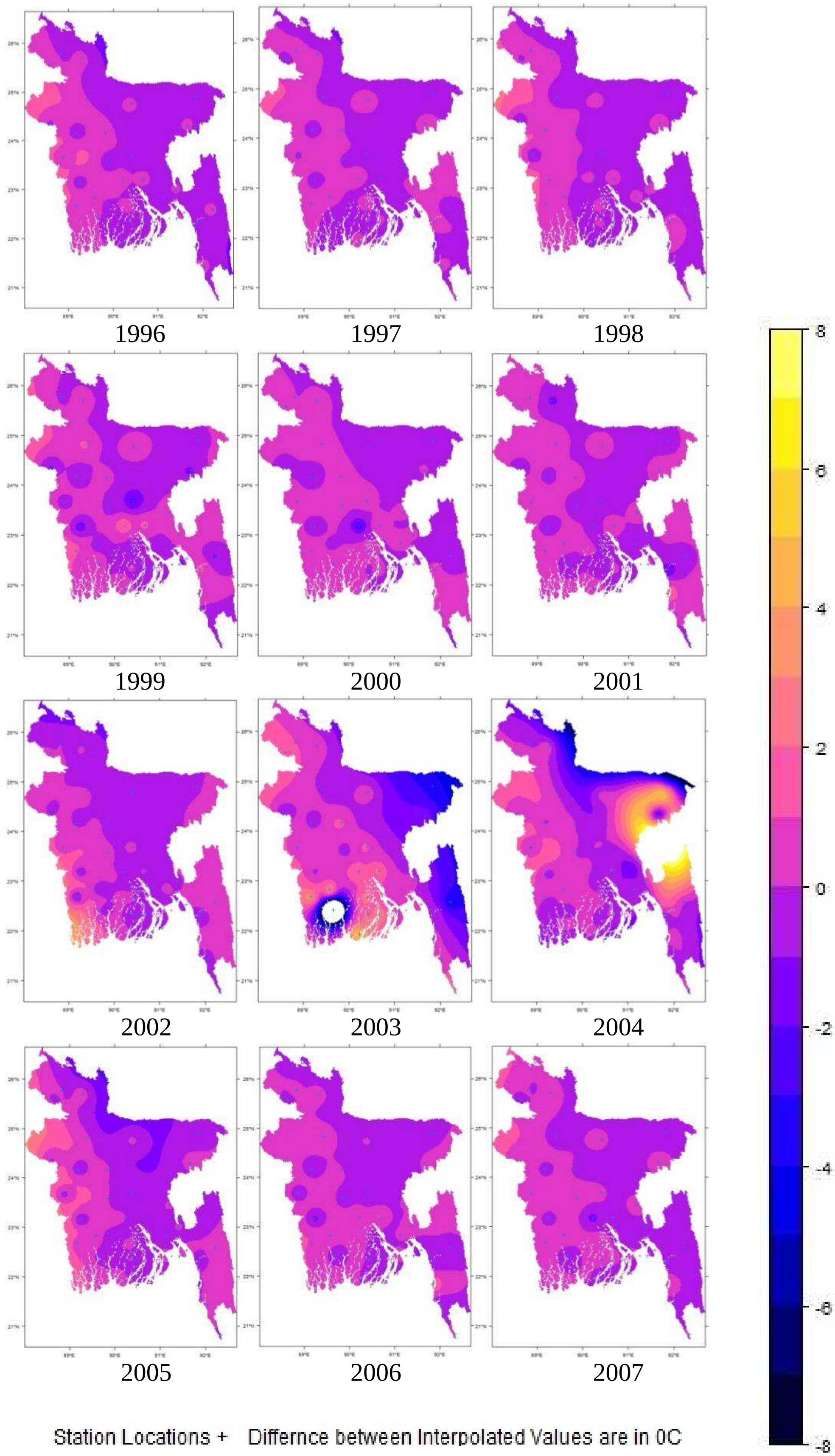
Station Locations + Differnce between Interpolated Values are in 0C




Station Locations + Differnce between Interpolated Values are in 0C




Station Locations + Differnce between Interpolated Values are in 0C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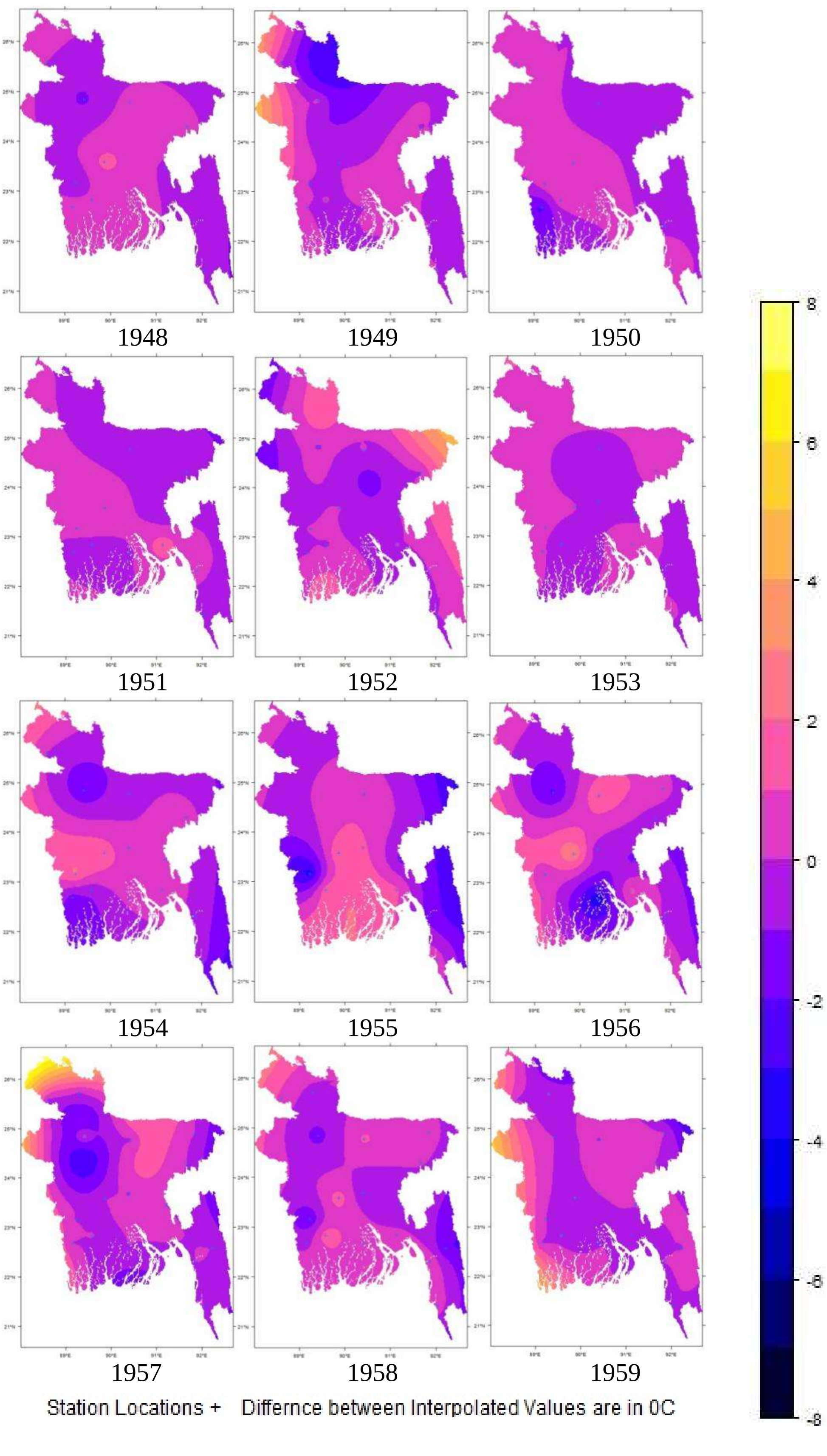
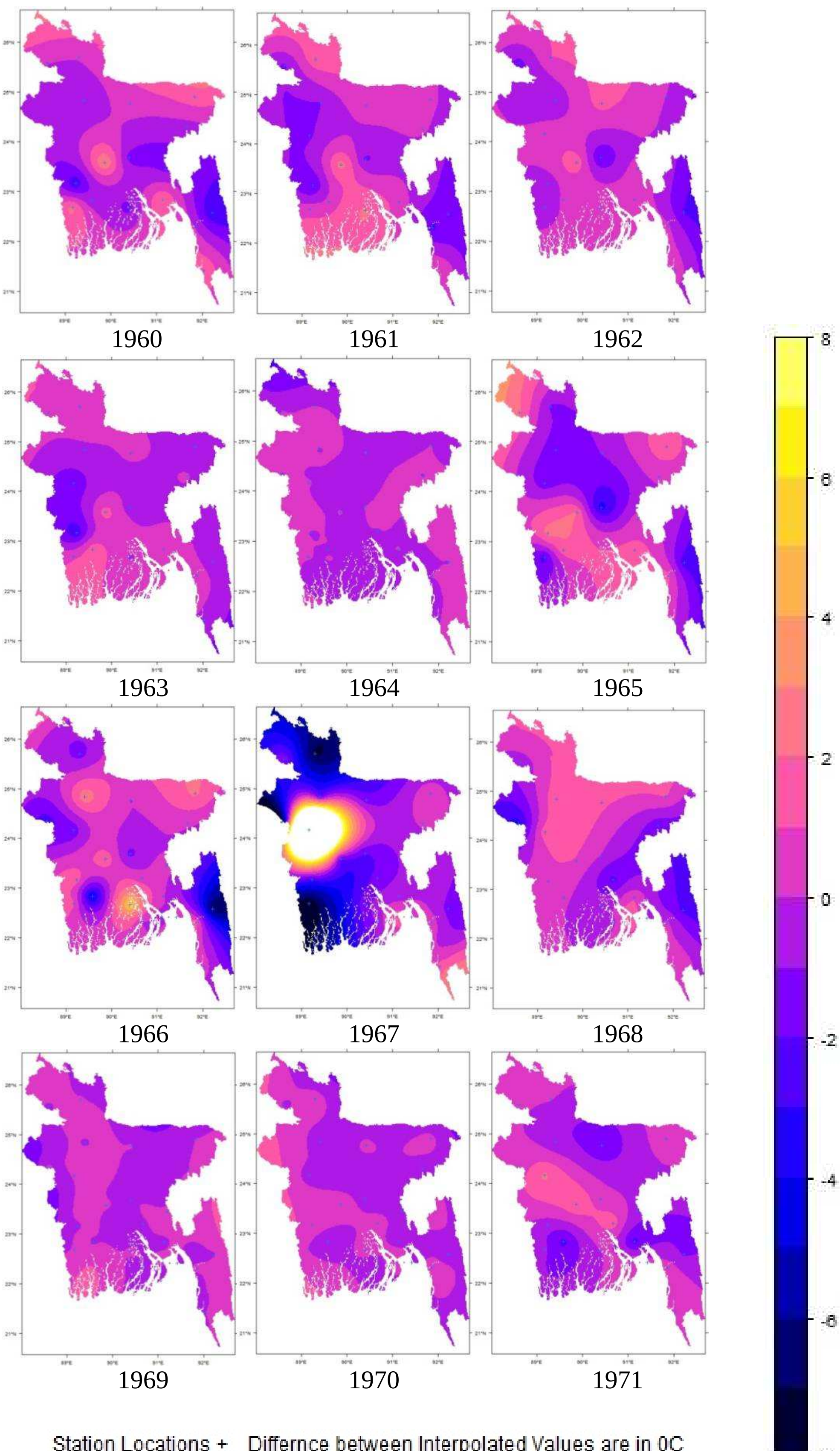
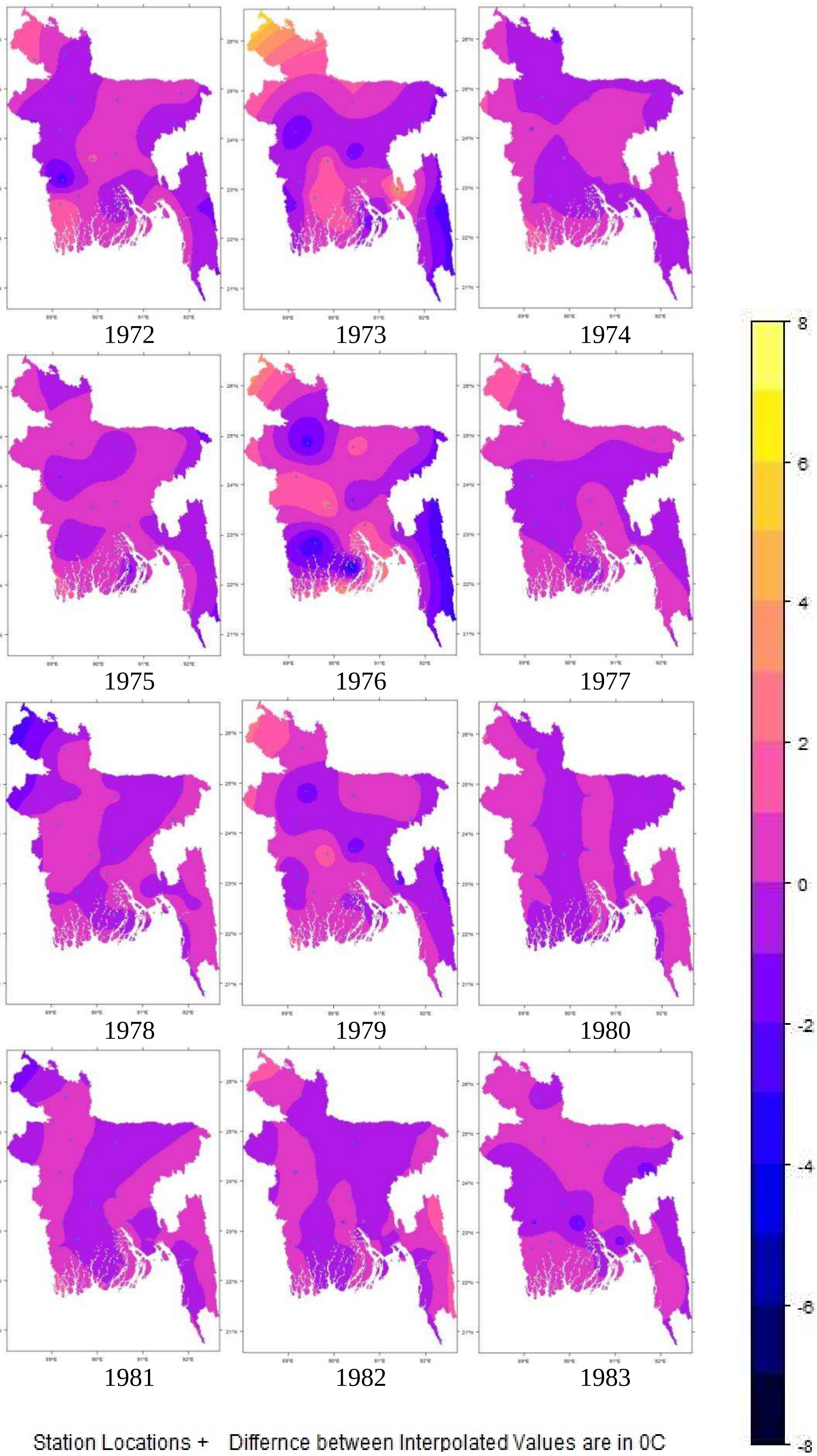
Station Locations +
Differnce between Interpolated Values are in 0C

## A.24 Difference Surfaces between TPS and OK (TPS-OK) of TXx

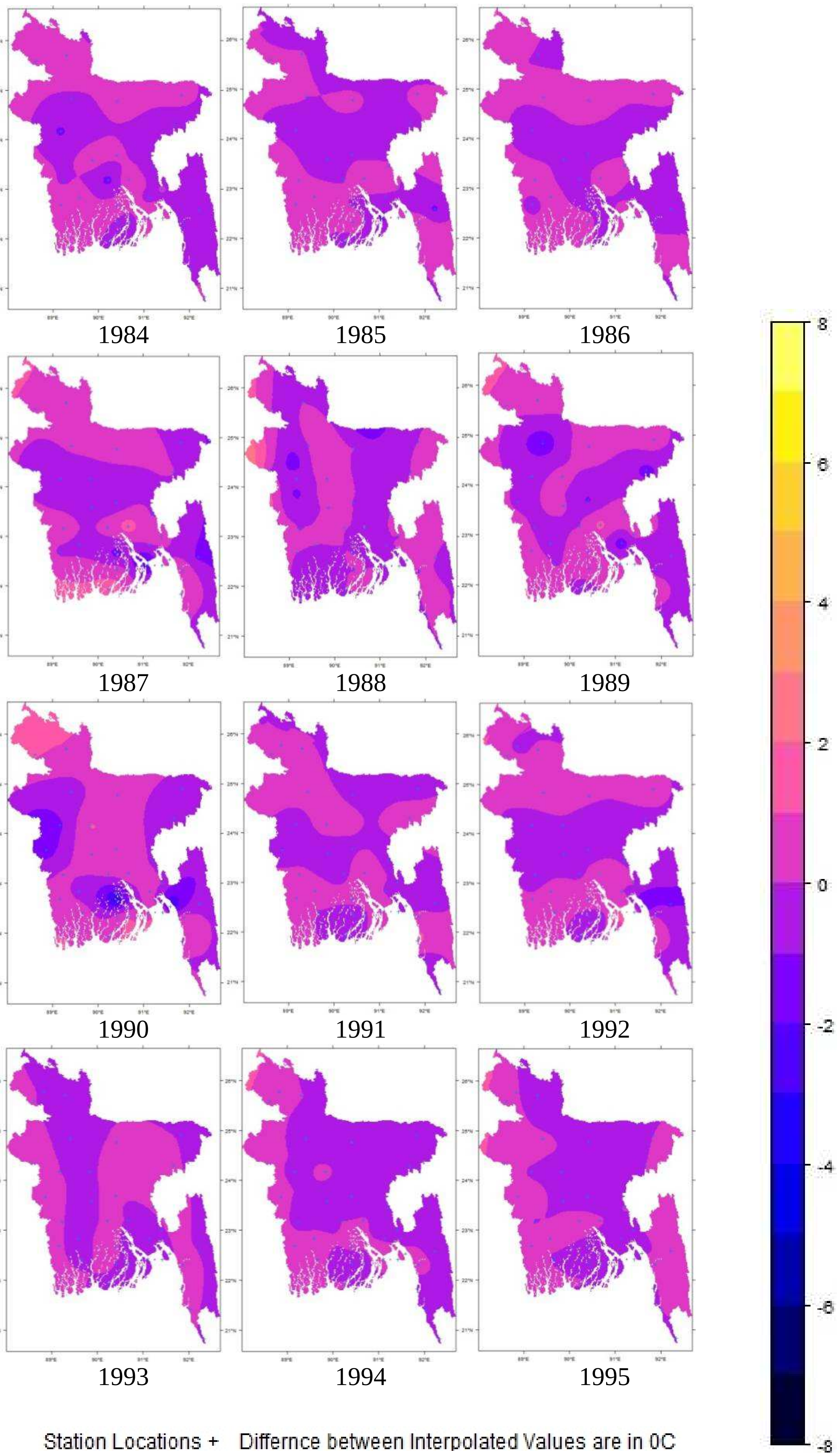




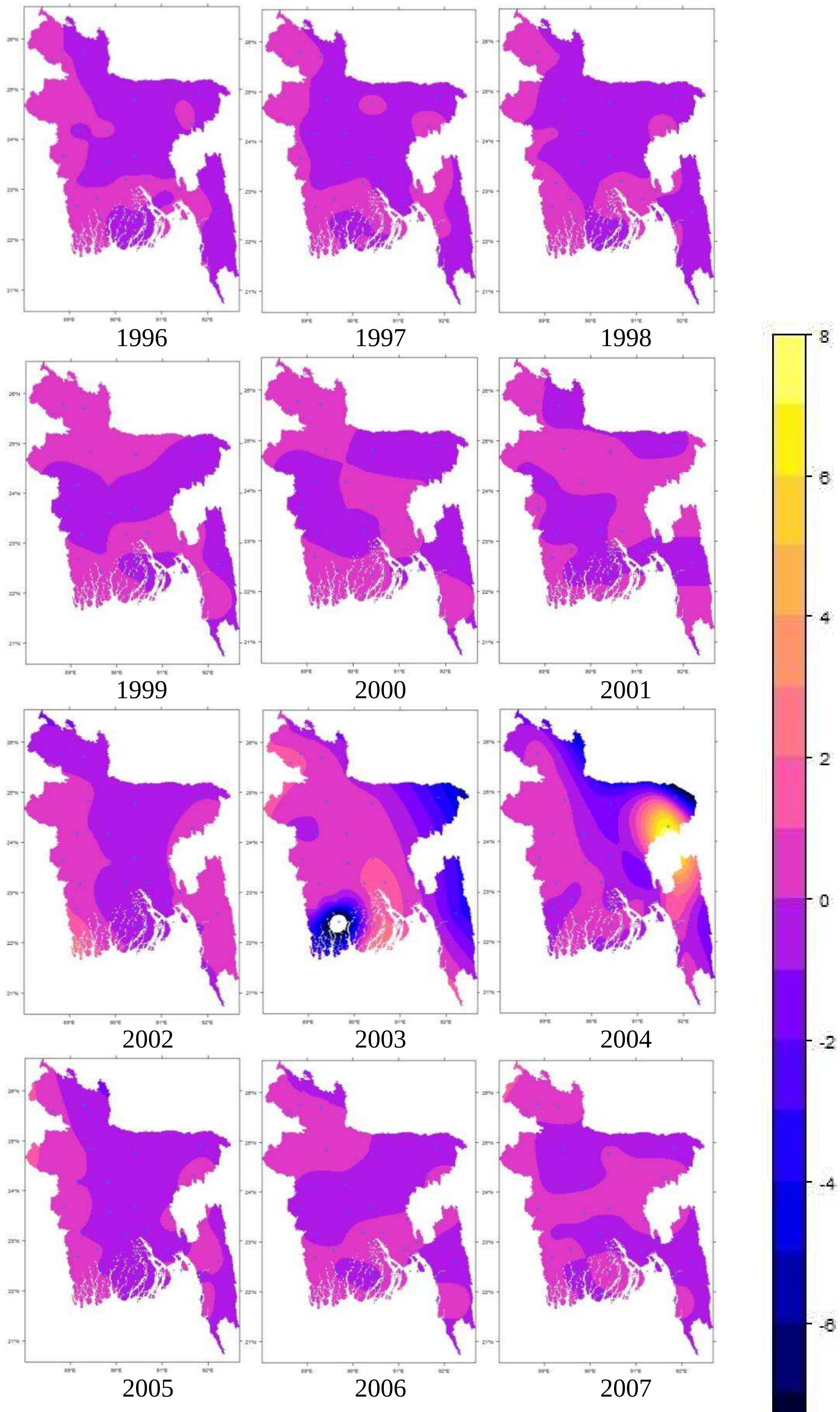

Station Locations + Differnce between Interpolated Values are in 0C




Station Locations + Differnce between Interpolated Values are in 0C

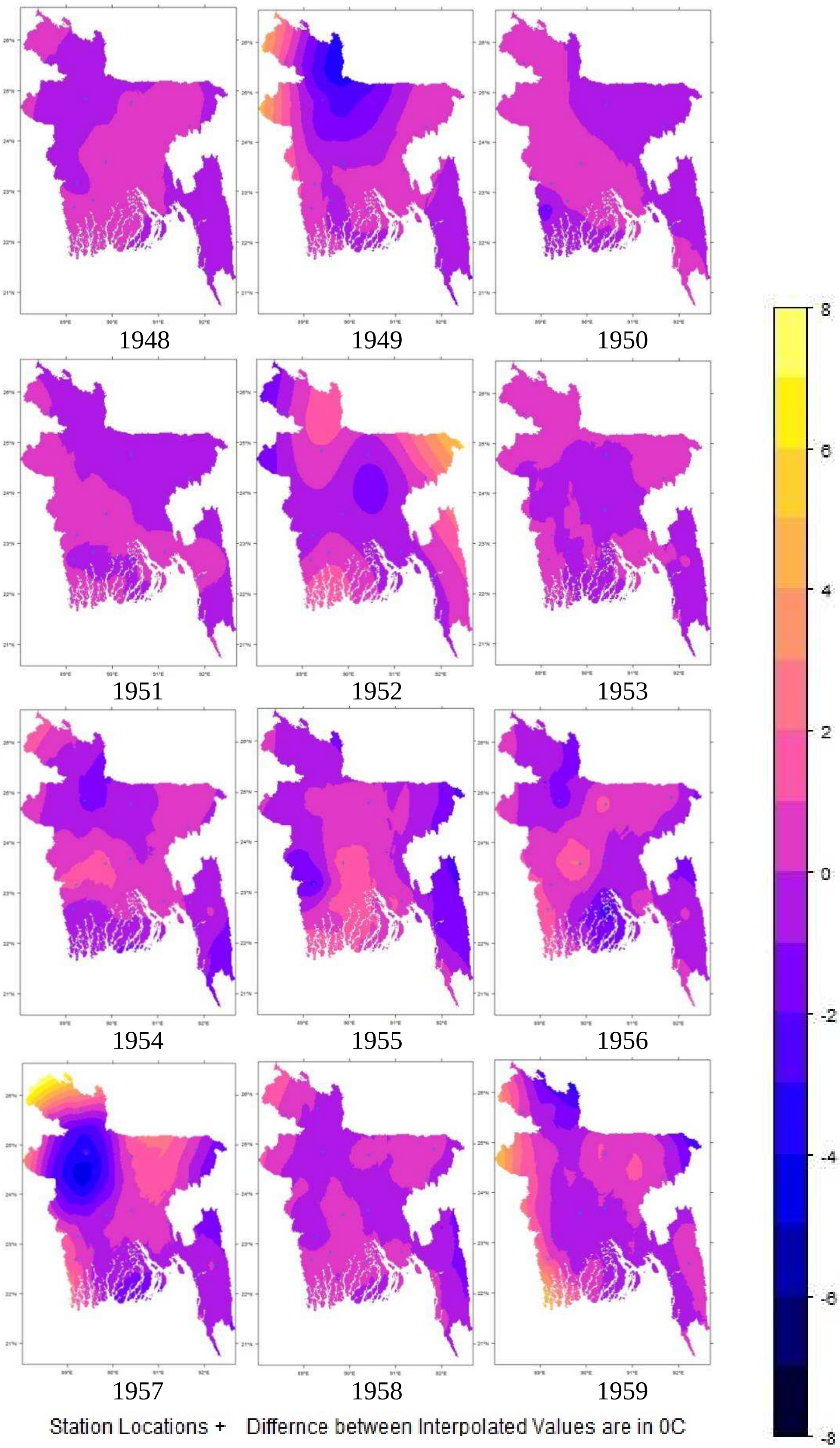


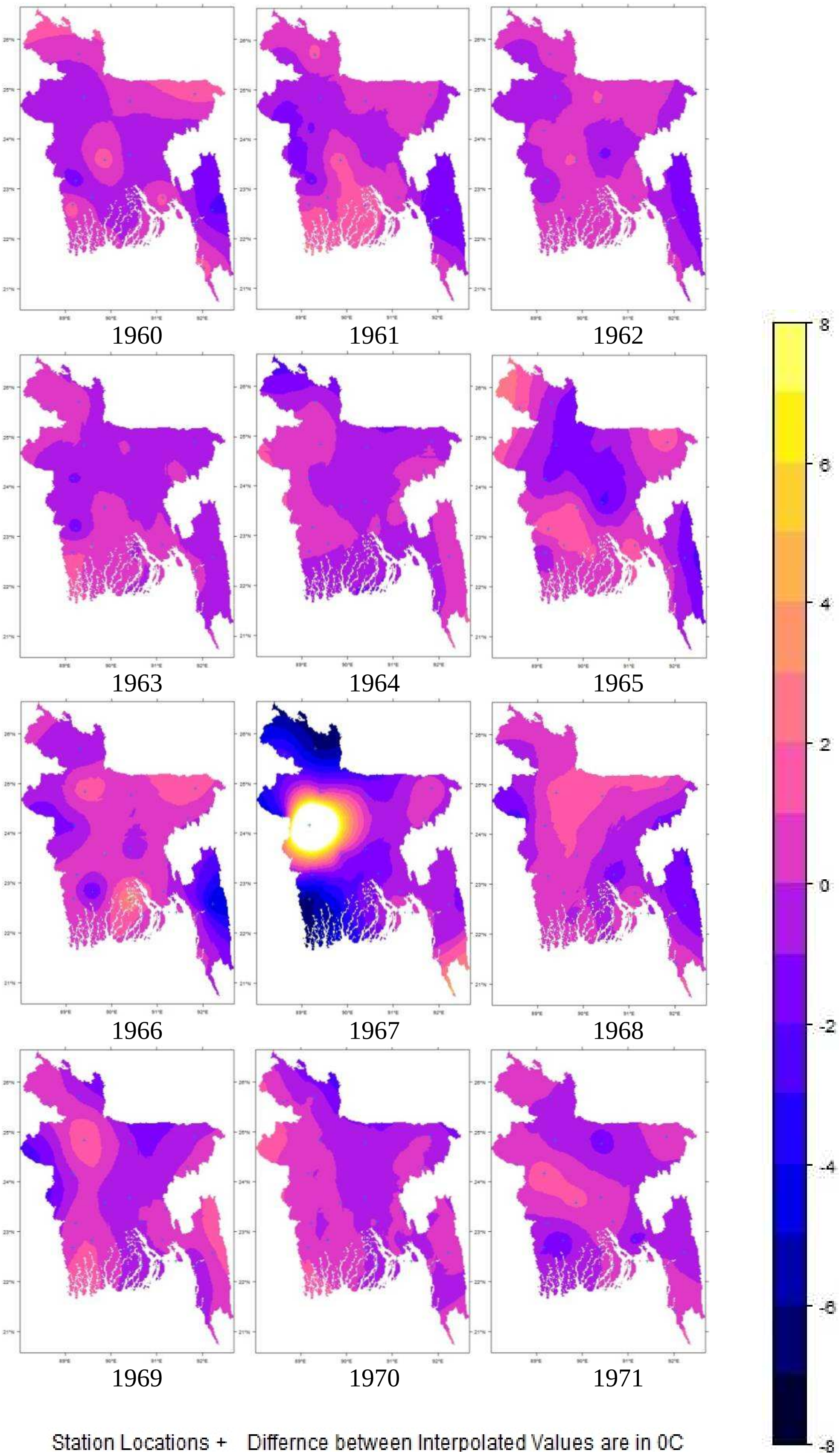

Station Locations + Differnce between Interpolated Values are in 0C



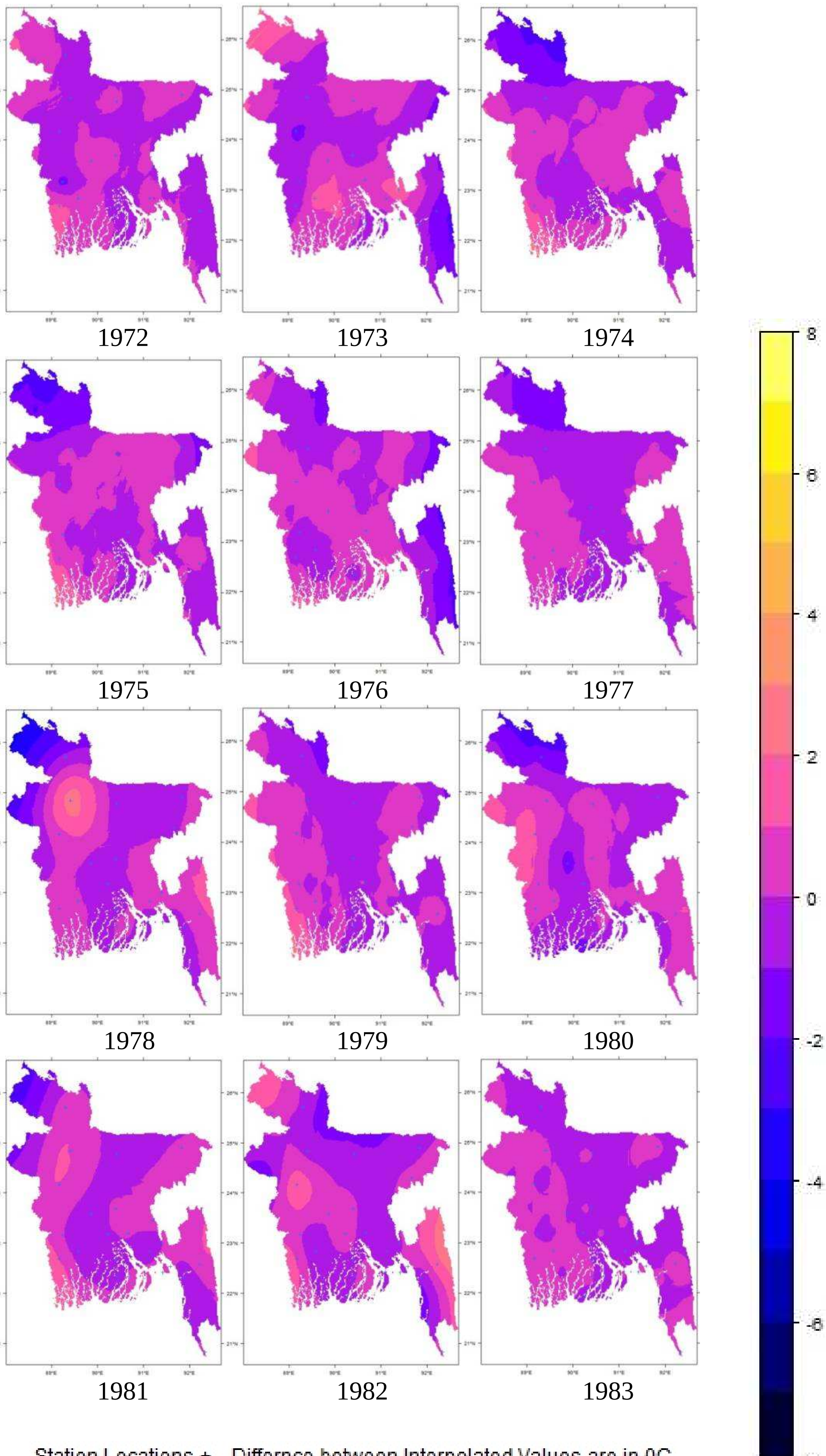

Station Locations + Differnce between Interpolated Values are in 0C

## A.25 Difference Surfaces between TPS and UK (TPS-UK) of TXx

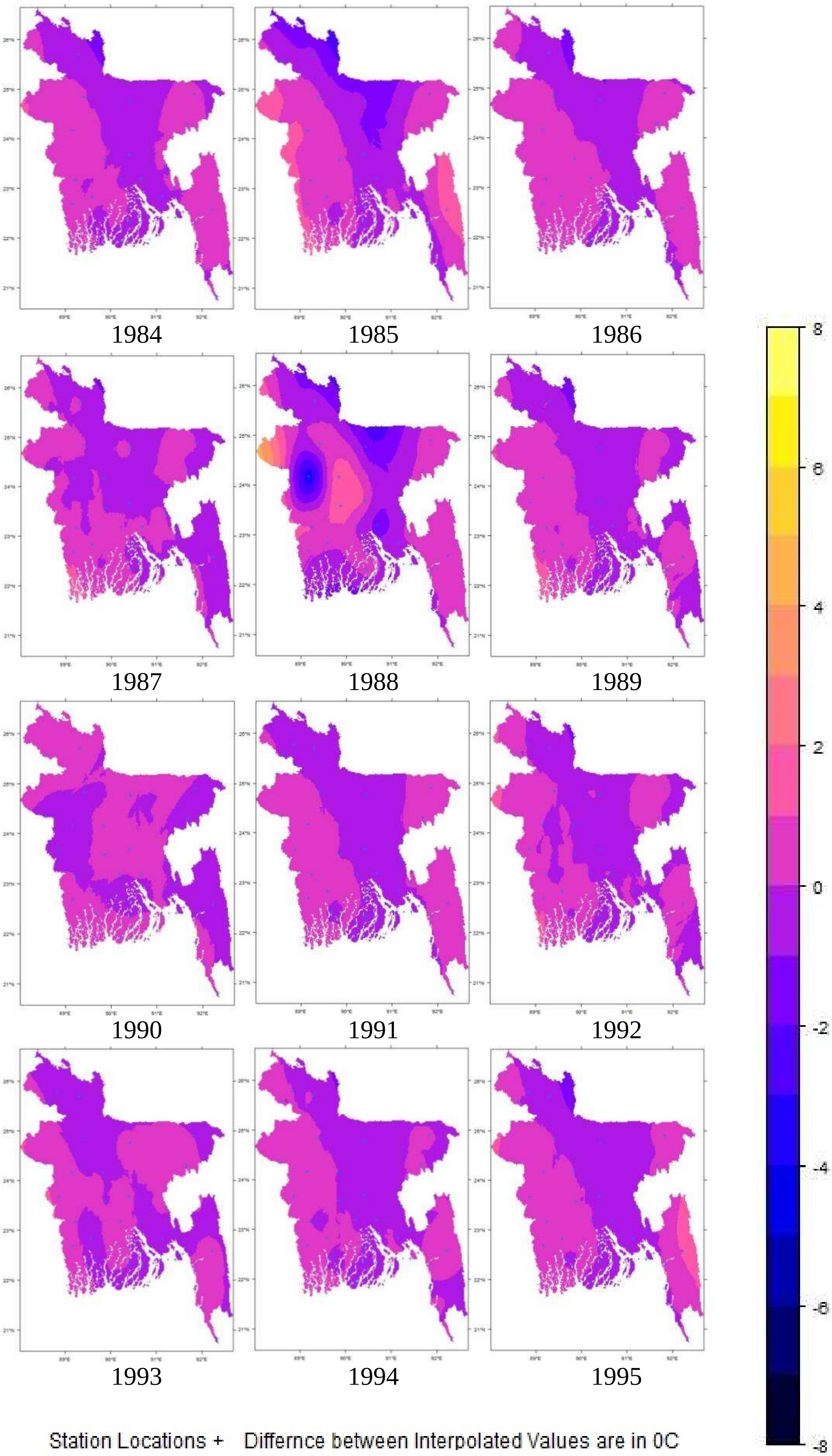




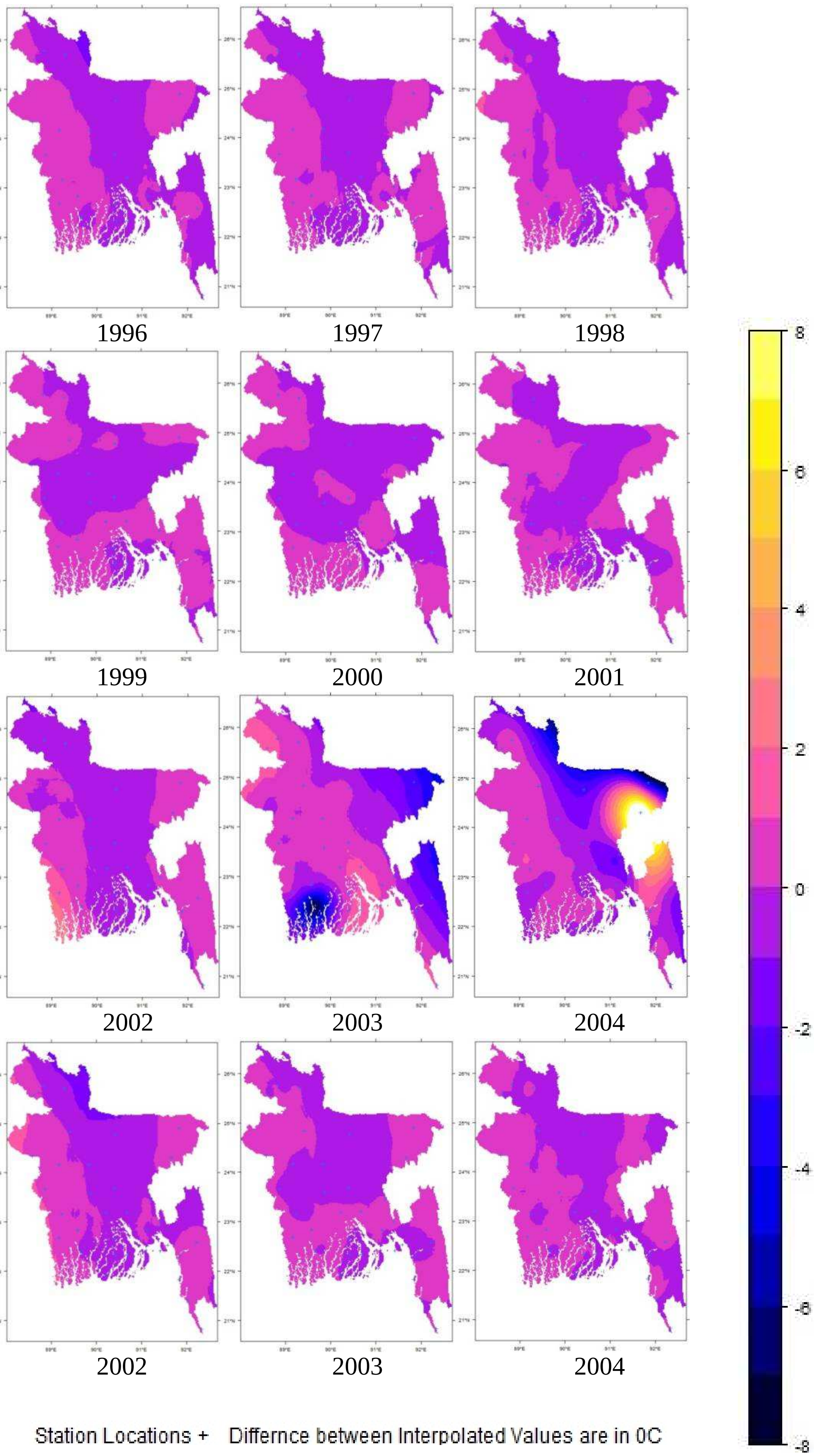

Station Locations + Differnce between Interpolated Values are in 0C




Station Locations + Differnce between Interpolated Values are in 0C




Station Locations + Differnce between Interpolated Values are in 0C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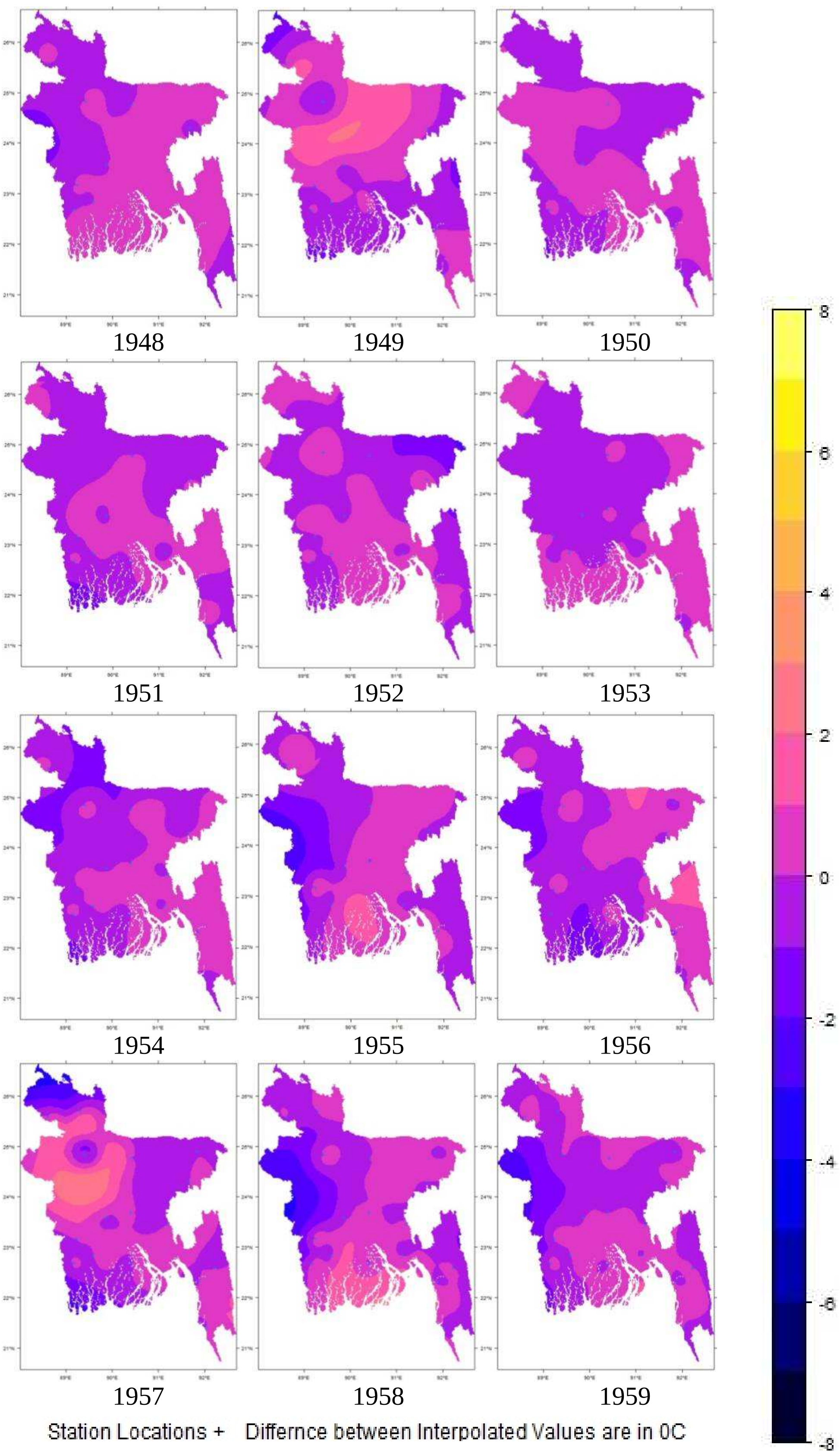
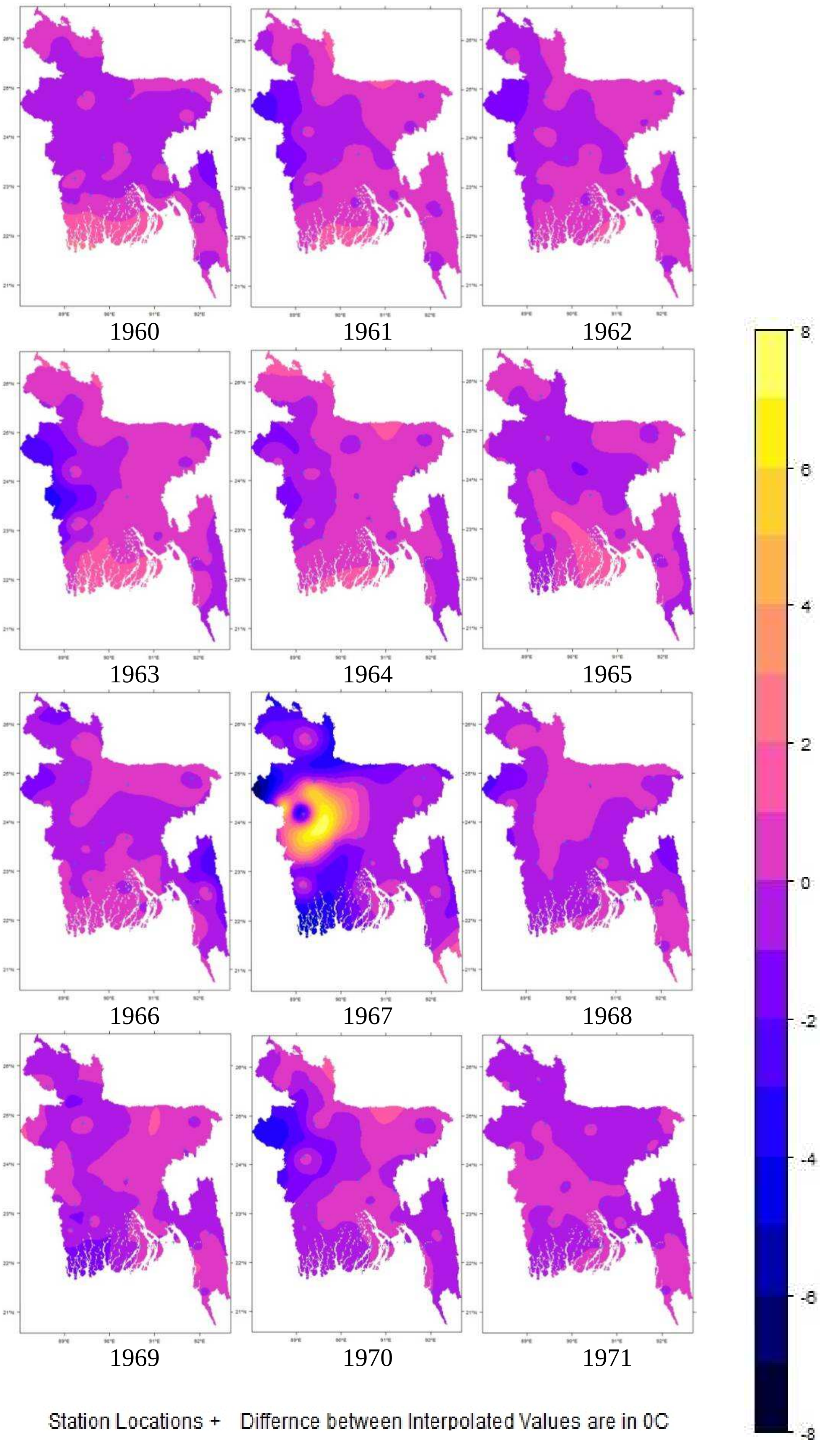
Station Locations +
Differnce between Interpolated Values are in 0C

## A.26 Difference Surfaces between IDW and OK (IDW-OK) of TXx

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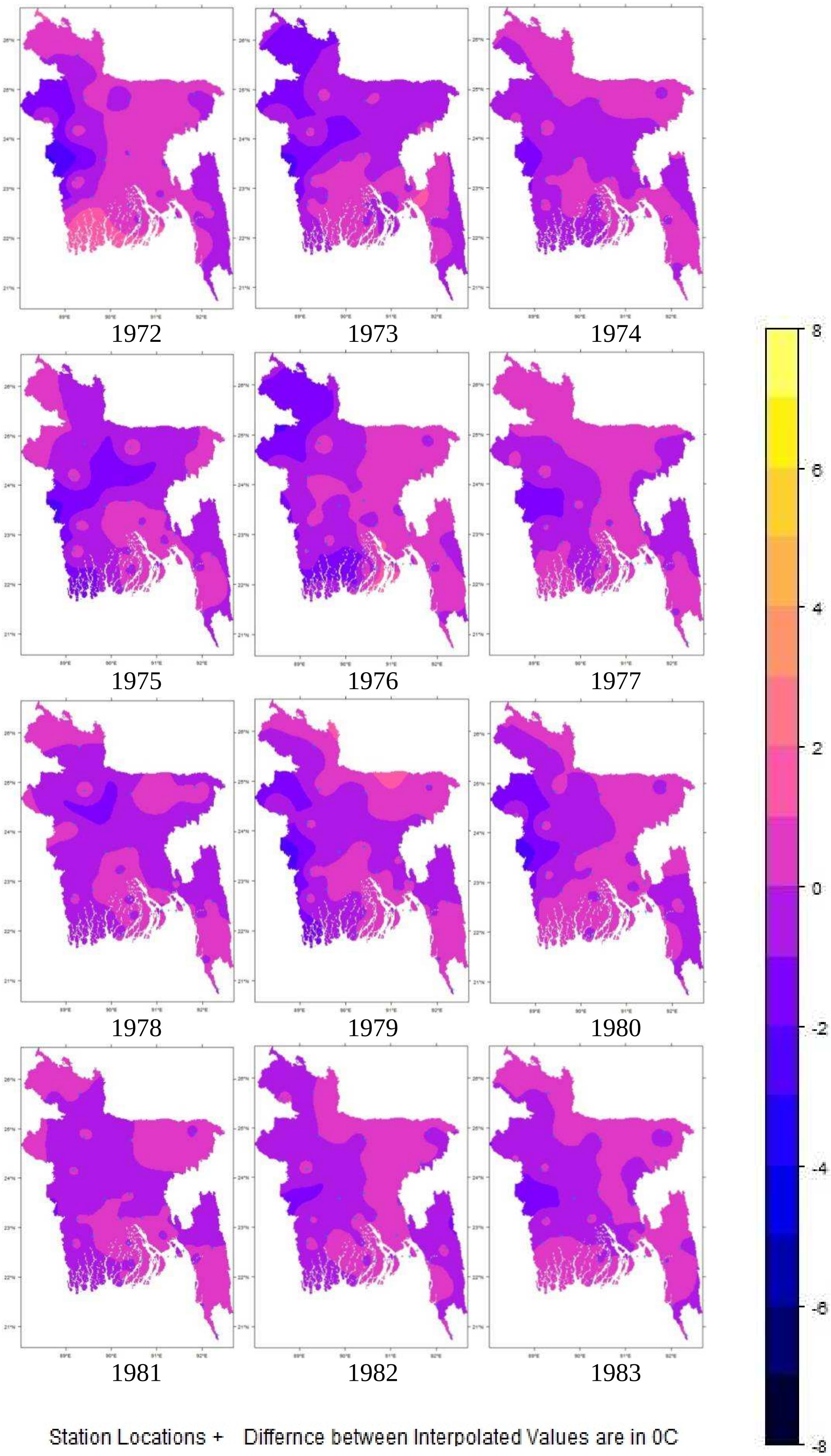
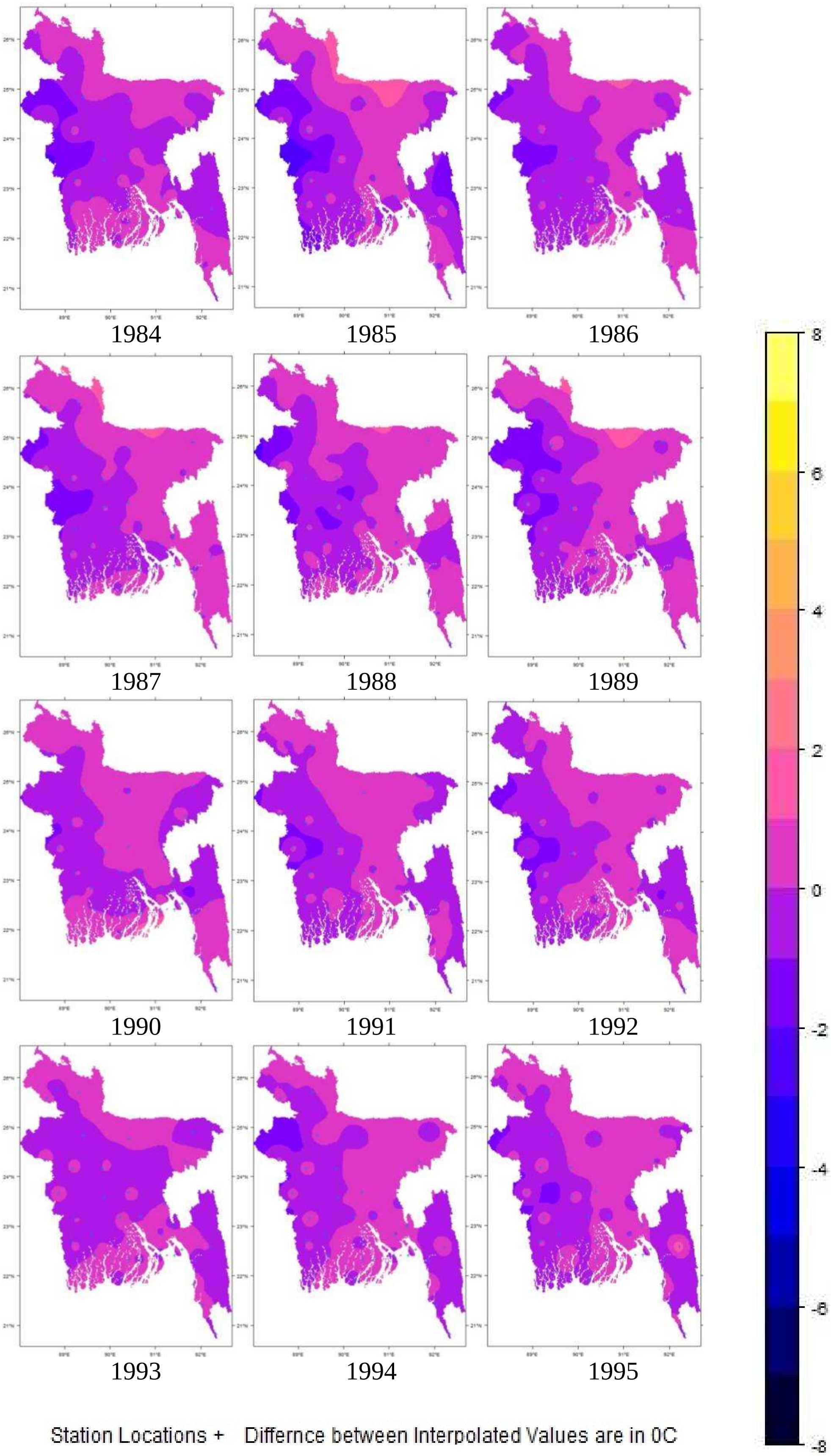
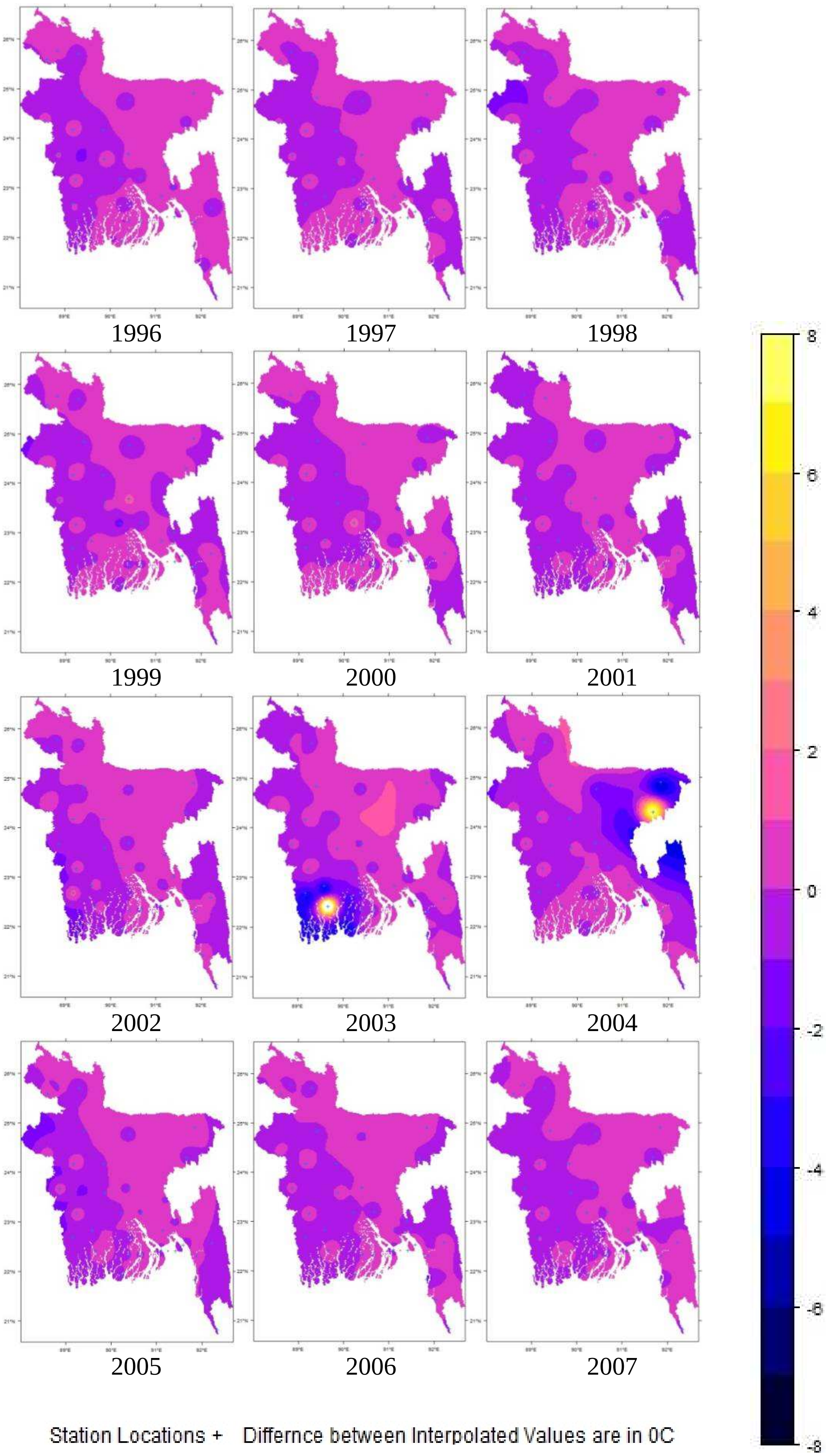
Station Locations +   Differnce between Interpolated Values are in 0C




Station Locations + Differnce between Interpolated Values are in 0C




Station Locations + Differnce between Interpolated Values are in 0C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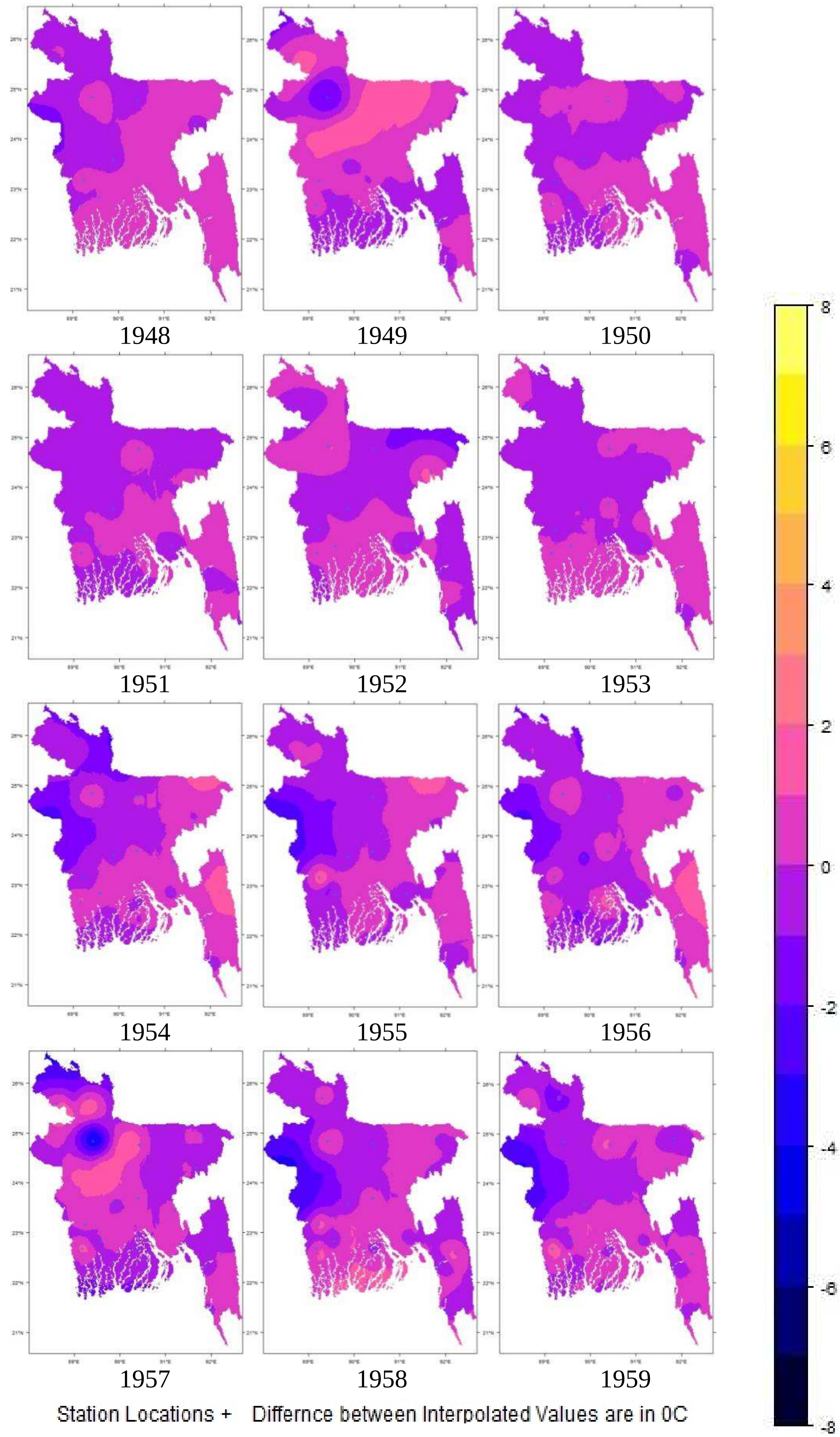
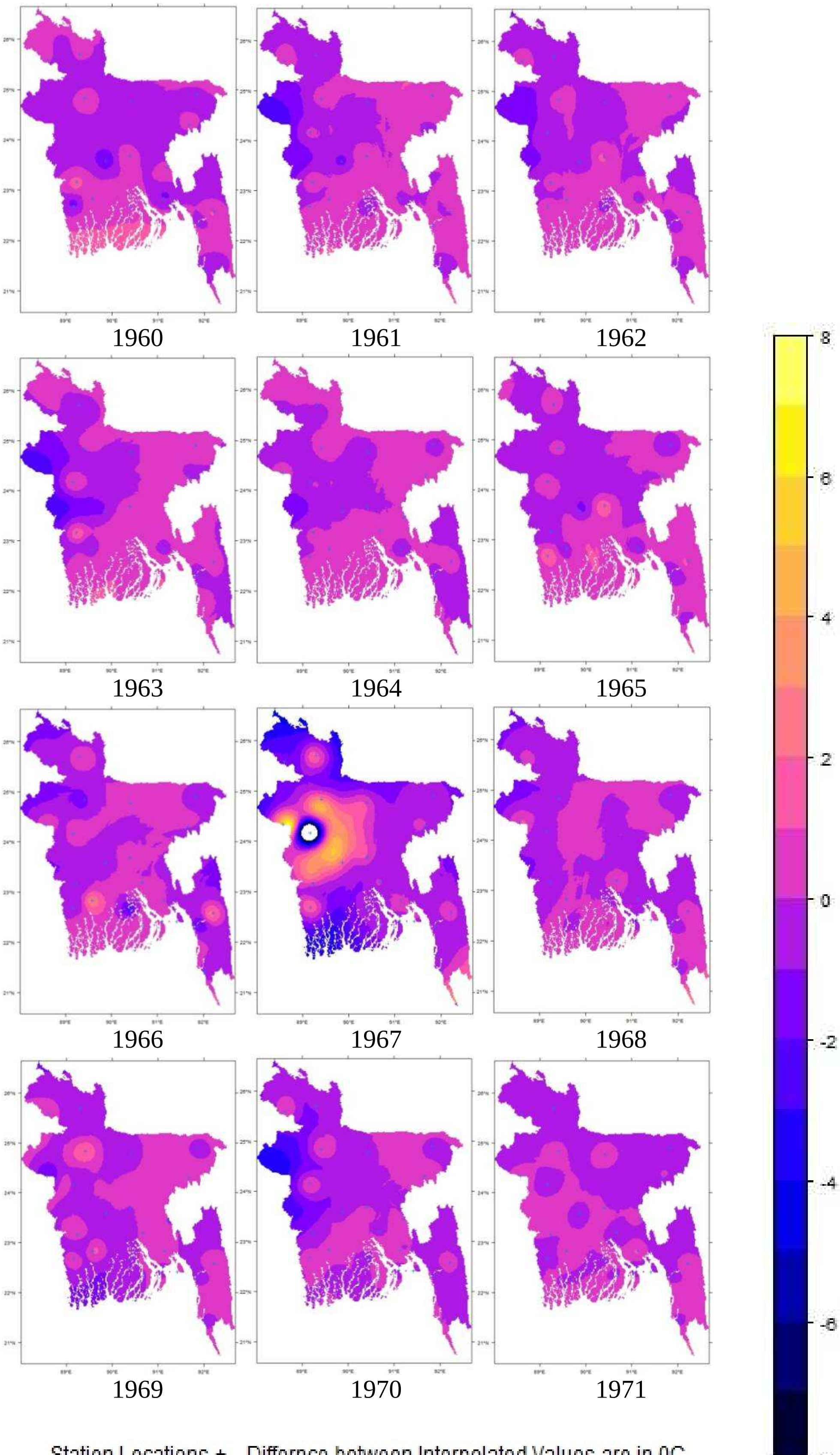
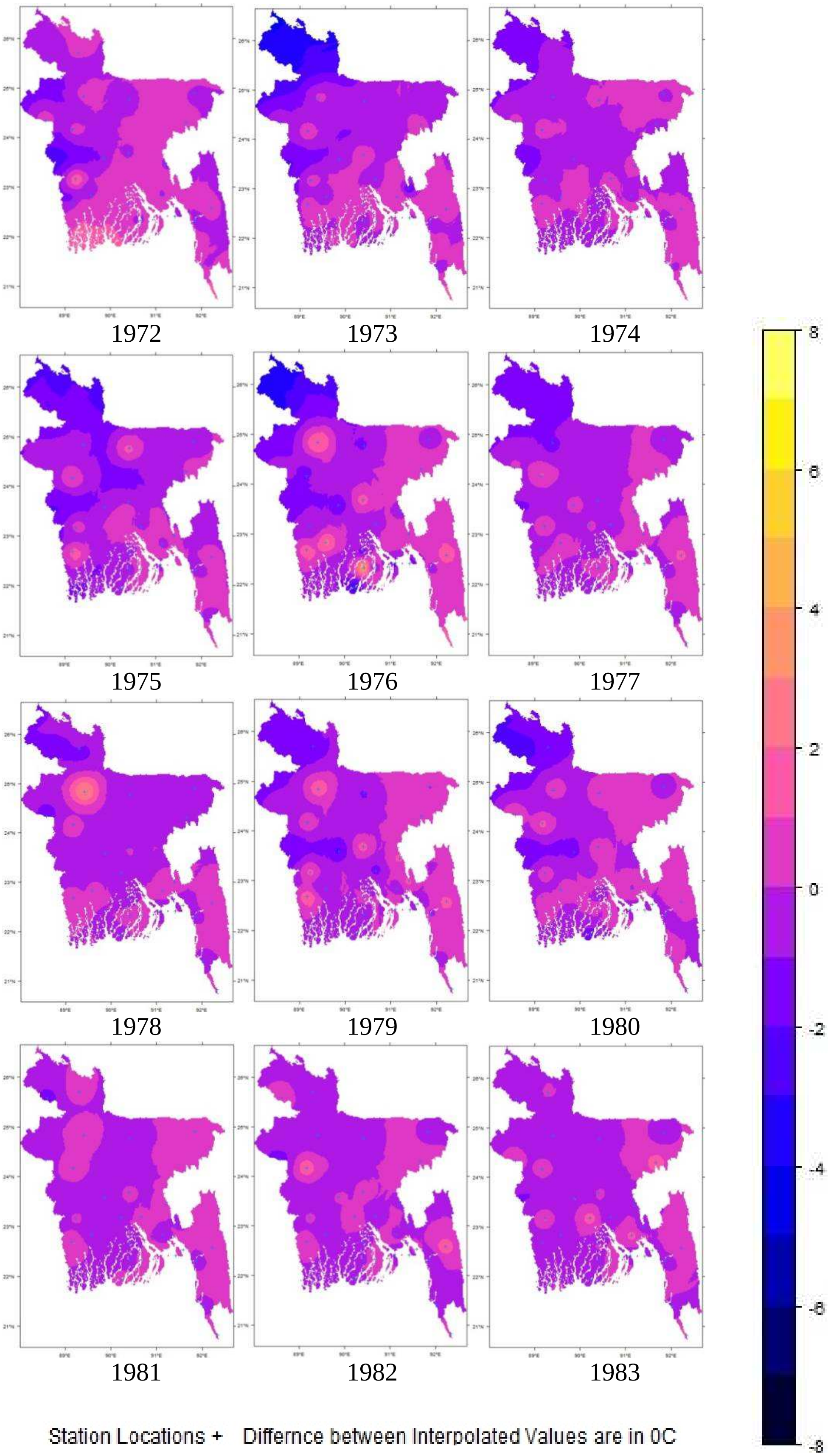
Station Locations +
Differnce between Interpolated Values are in 0C

## A.27 Difference Surfaces between IDW and UK (IDW-UK) of TXx

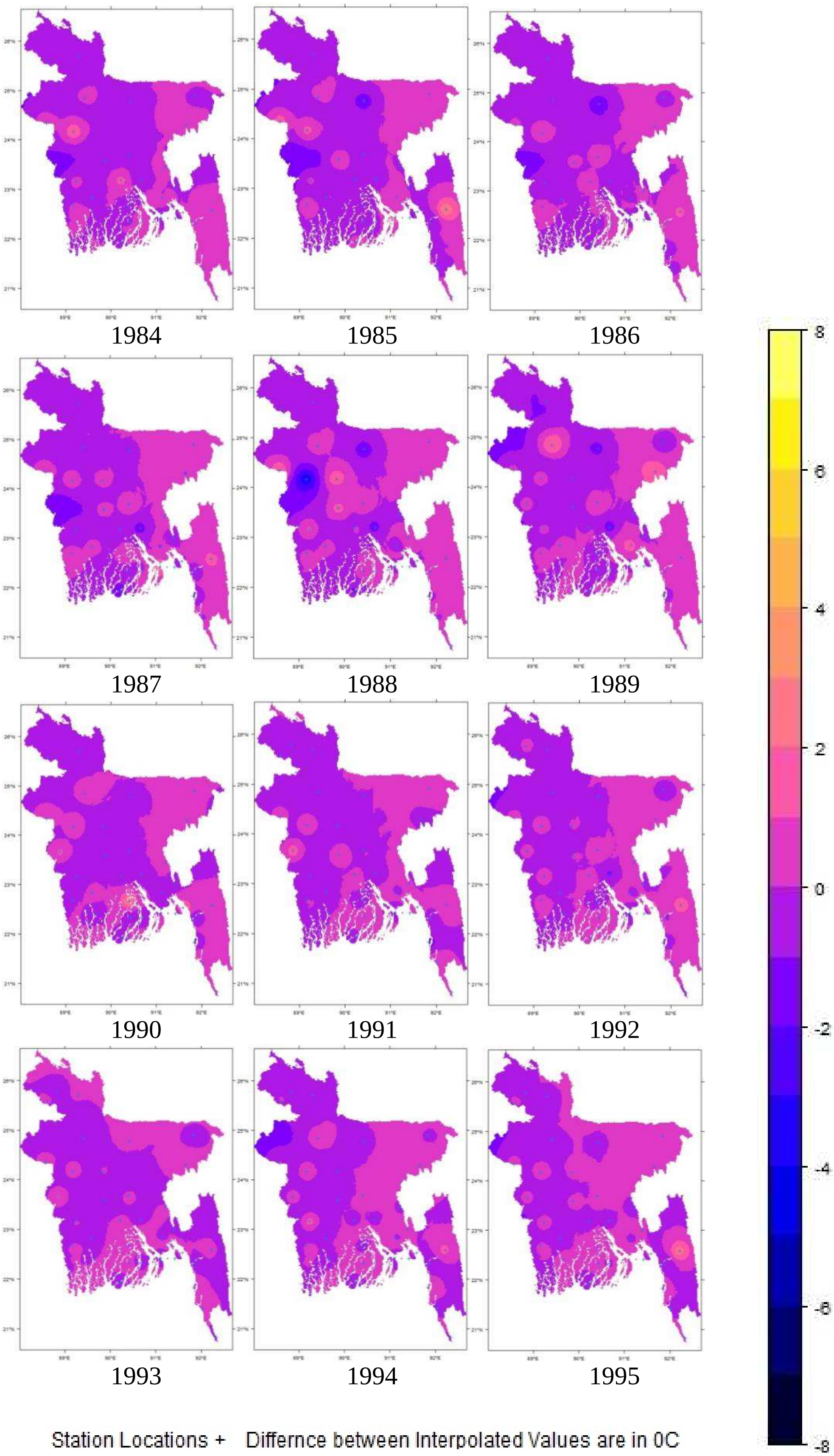





Station Locations +    Differnce between Interpolated Values are in 0C




Station Locations +    Differnce between Interpolated Values are in 0C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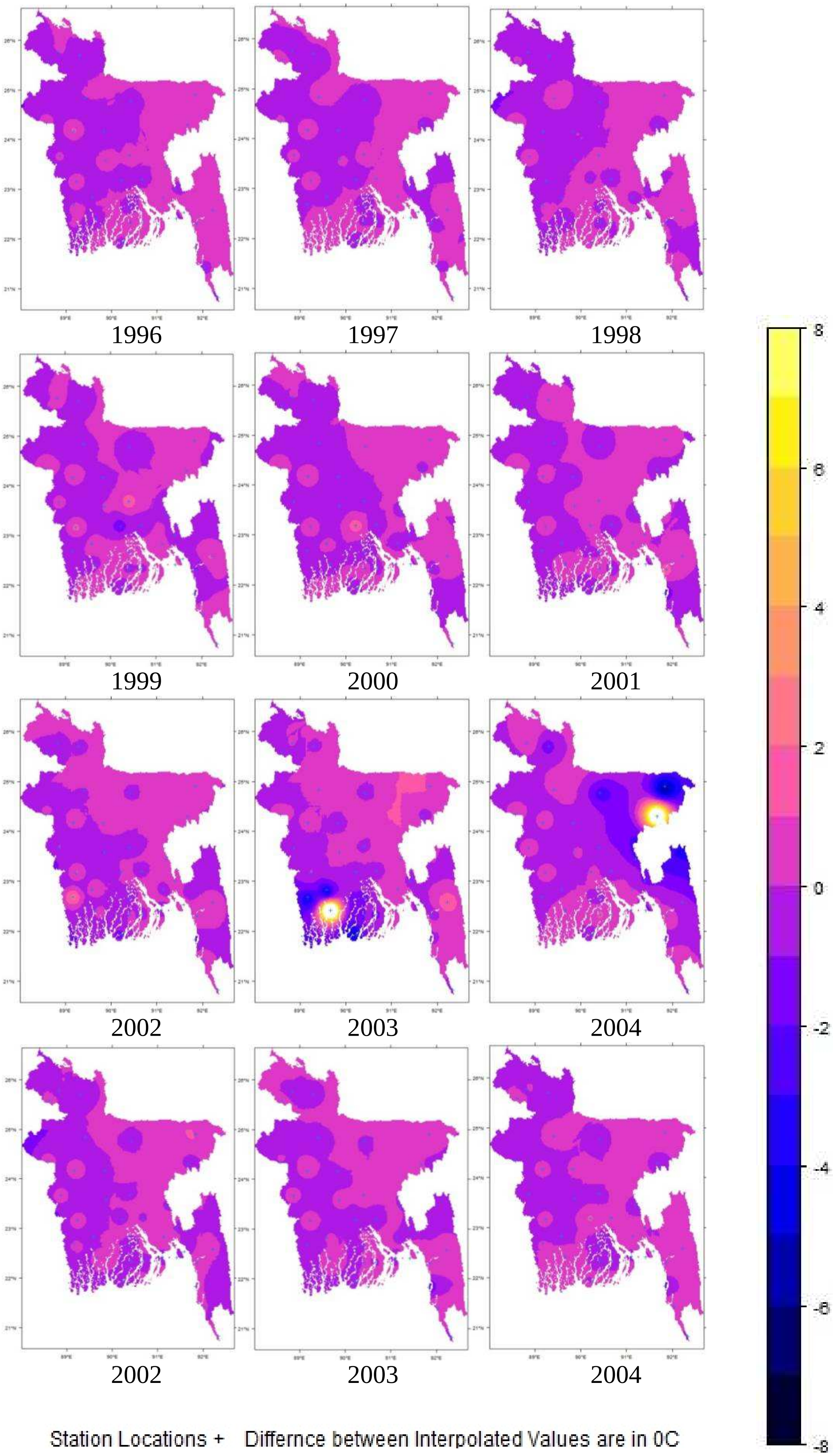
Station Locations +    Differnce between Interpolated Values are in 0C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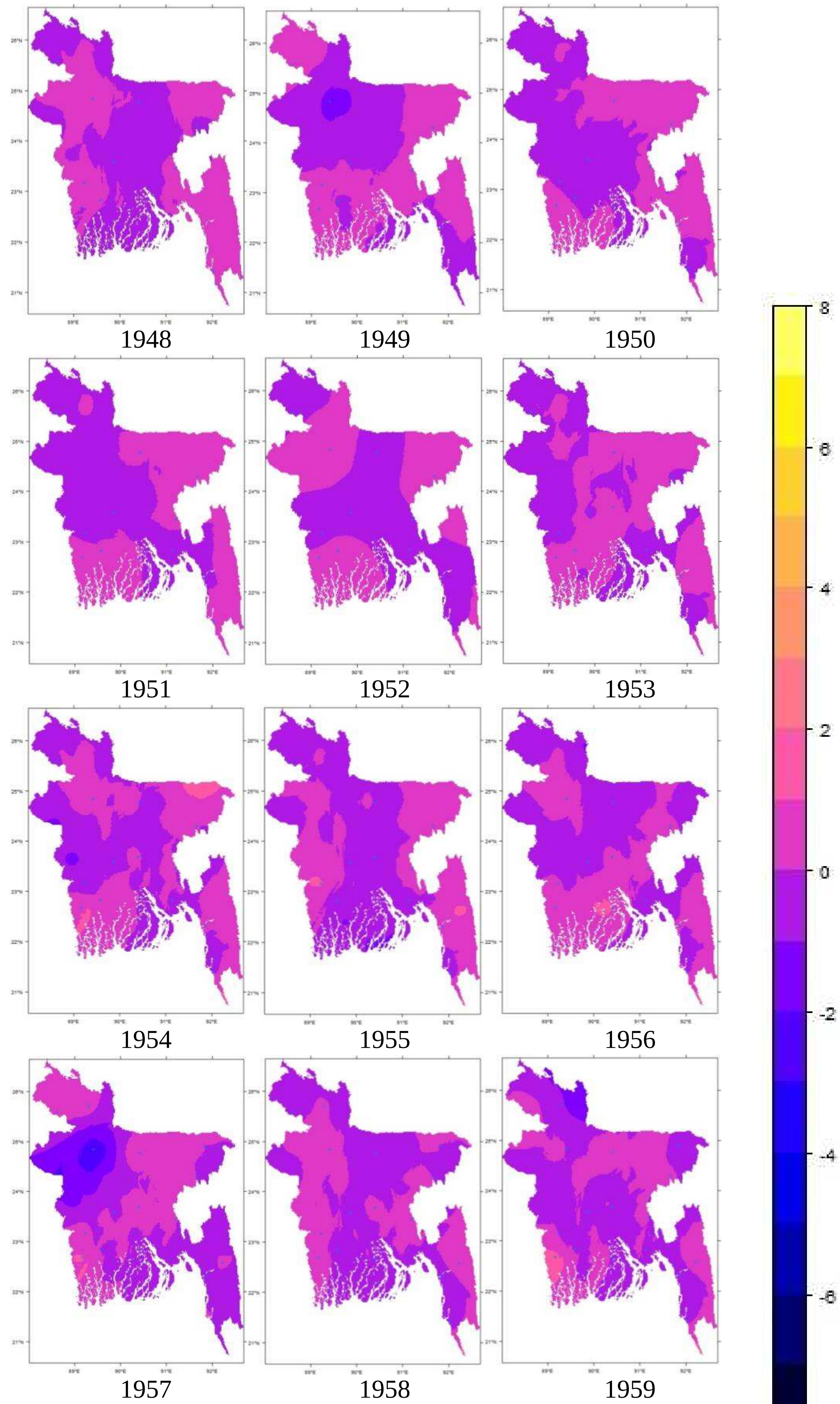
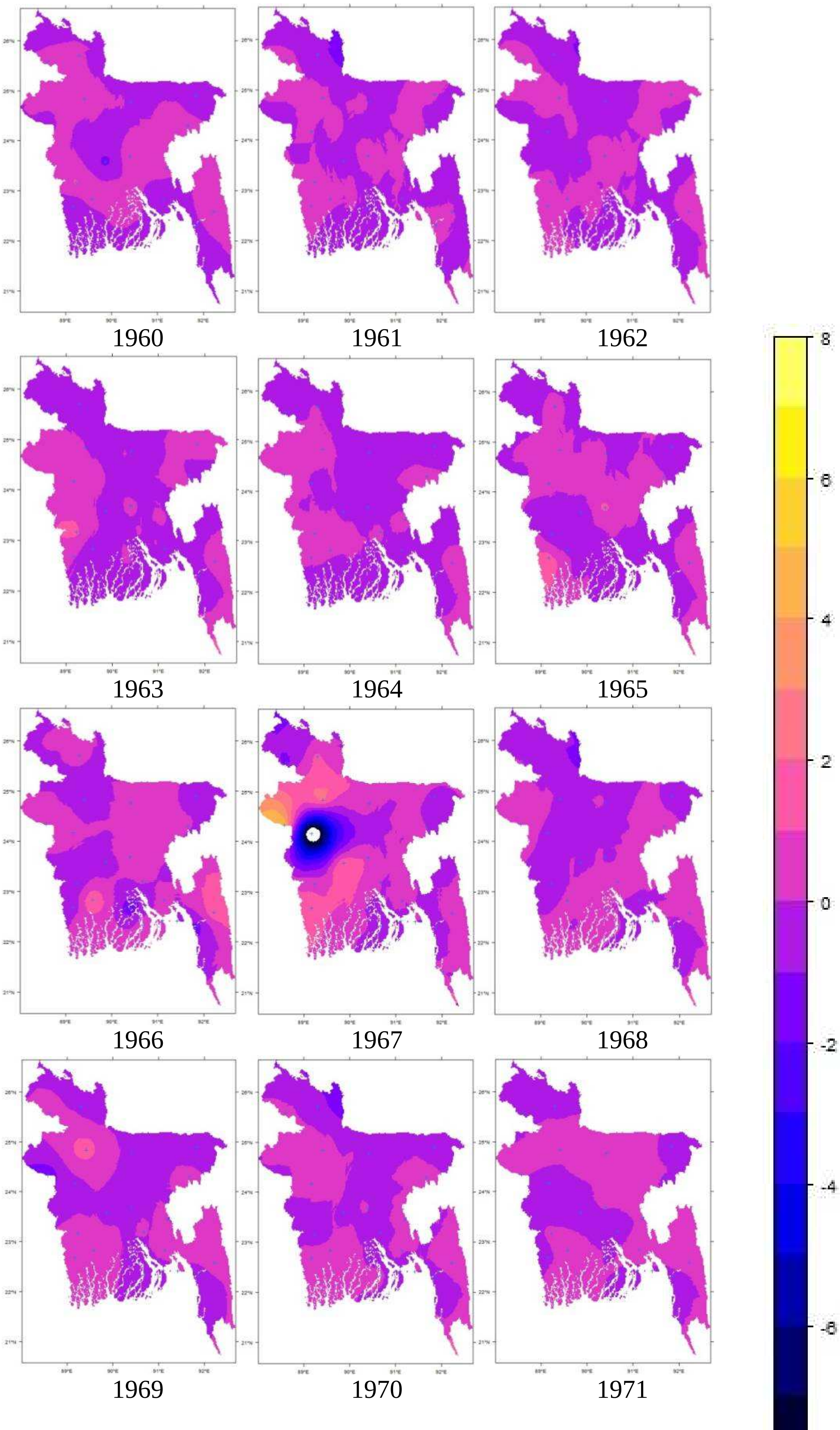
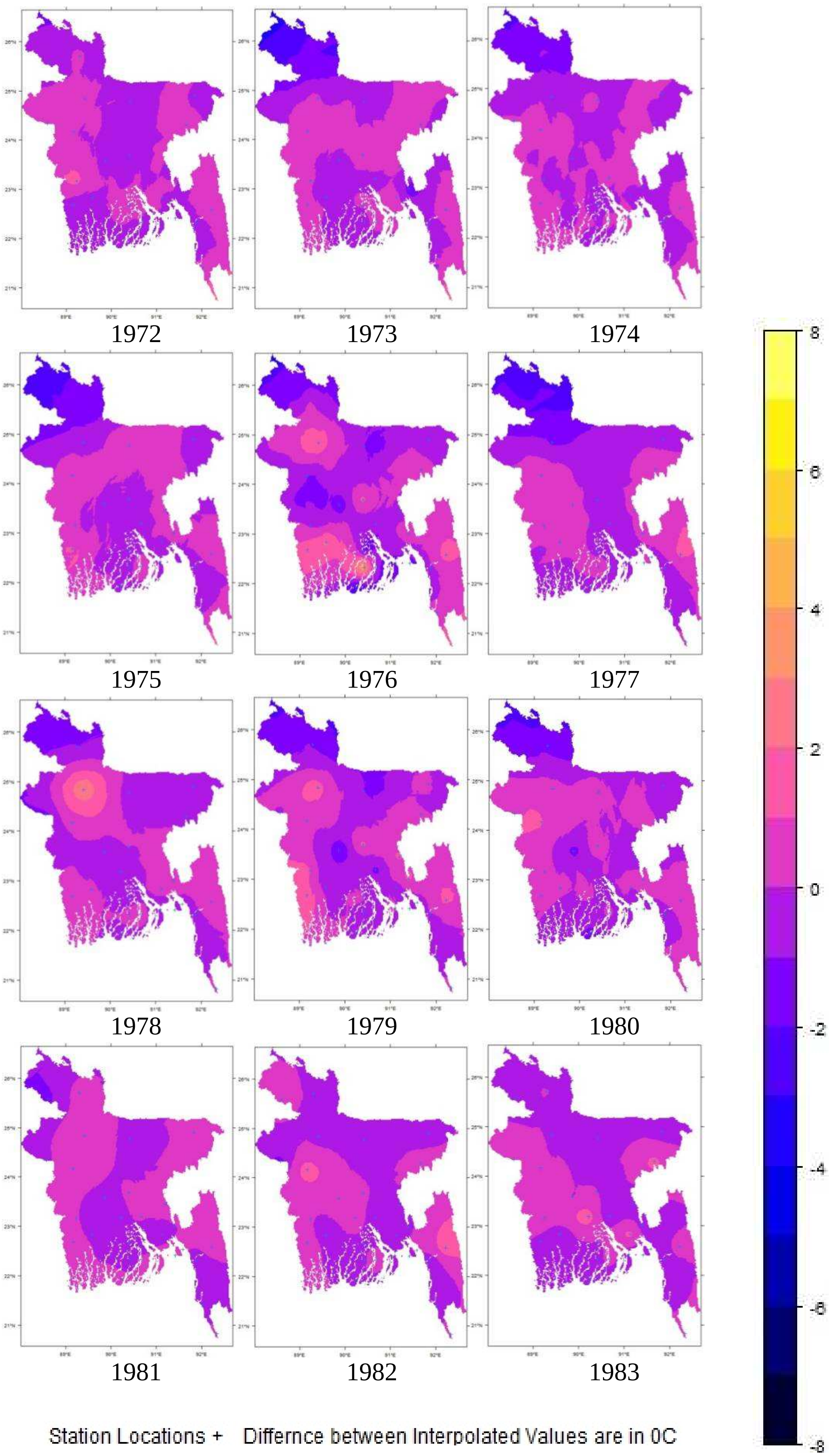
Station Locations +
Differnce between Interpolated Values are in 0C

## A.28 Difference Surfaces between OK and UK (OK-UK) of TXx






Station Locations + Differnce between Interpolated Values are in 0C


1972
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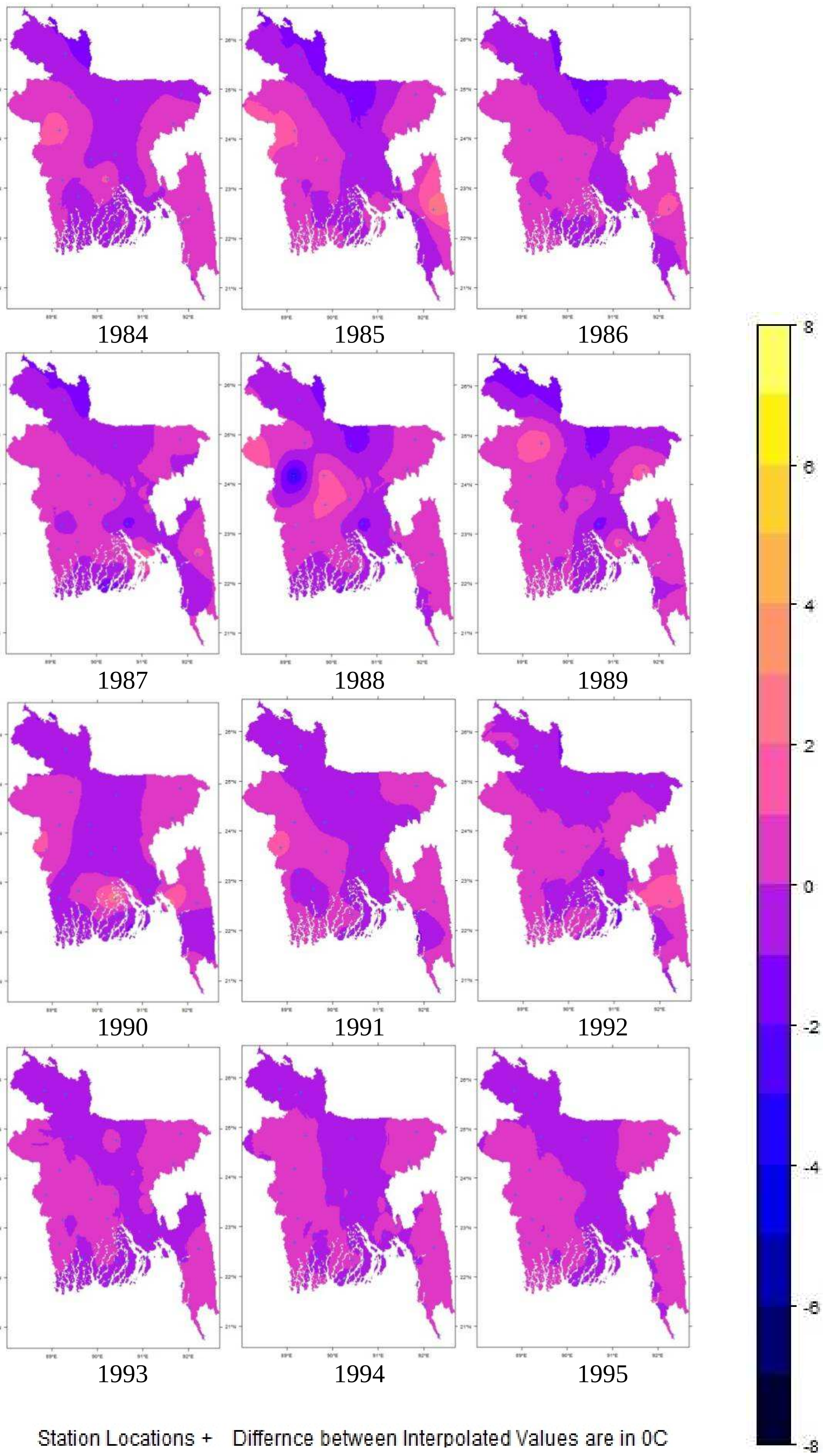
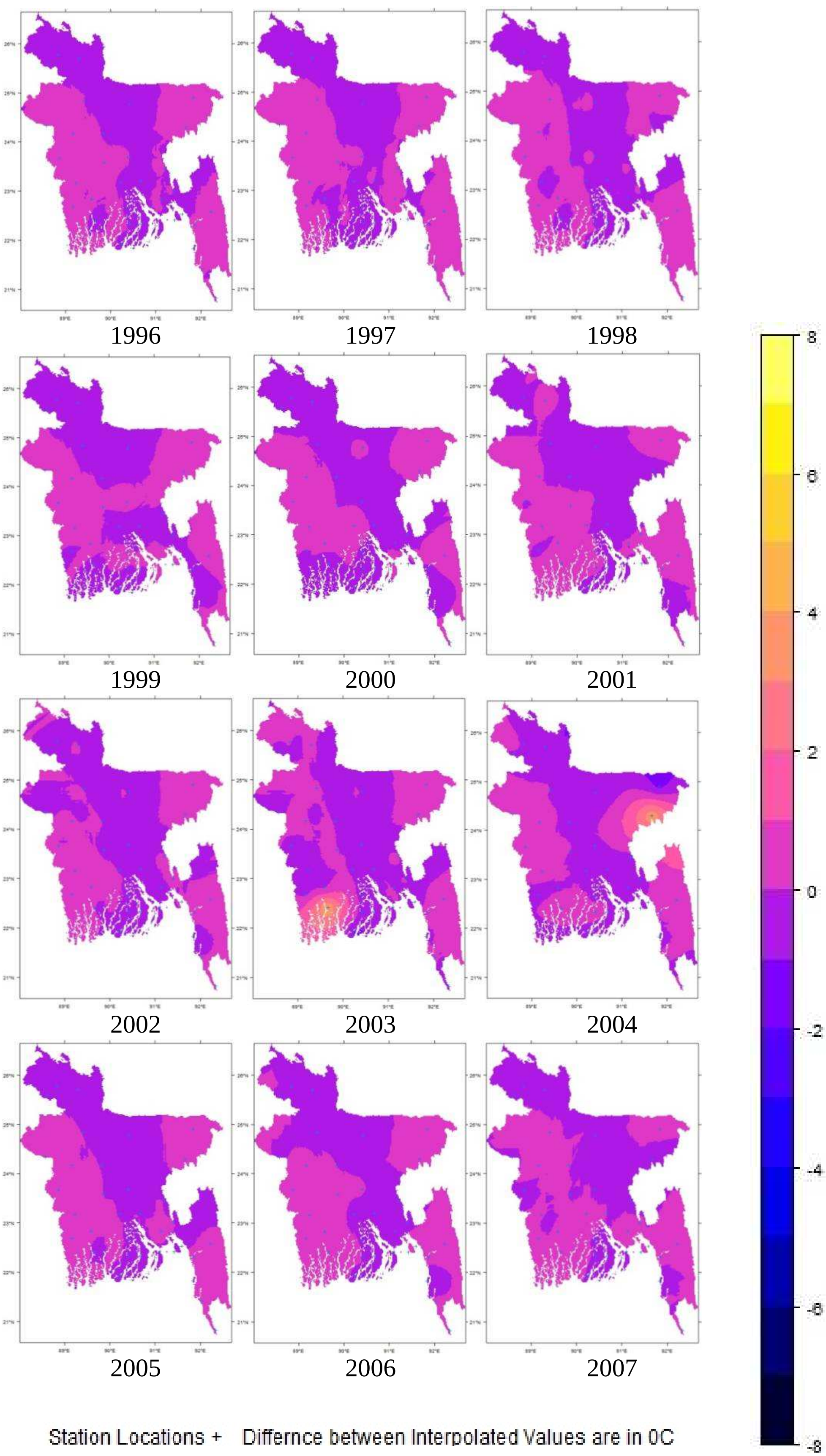
Station Locations +
Differnce between Interpolated Values are in 0C




Station Locations + Differnce between Interpolated Values are in 0C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Station Locations +    Differnce between Interpolated Values are in 0C

## A.29 Performance measurements of the four methods from cross-validation for interpolating PRCPTOT (pp. 203-205)

| | | | | MAE | | | | RMSE | | | | RMSEs | | | | RMSEu | | | | pf | | | | d | | | | CP | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Years | n | Sample Mean | *k* | TPS | IDW | OK | UK | TPS | IDW | OK | UK | TPS | IDW | OK | UK | TPS | IDW | OK | UK | TPS | IDW | OK | UK | TPS | IDW | OK | UK | TPS | IDW | OK | UK |
| 1948 | 8 | 1532.25 | 0.54 | 851.89 | 685.47 | 732.19 | 693.70 | 1006.16 | 817.73 | 851.75 | 798.92 | 650.06 | 789.37 | 778.84 | 575.74 | 767.95 | 213.70 | 344.78 | 553.85 | 65.67 | 53.37 | 55.59 | 52.14 | 0.47 | 0.21 | 0.23 | 0.56 | 34.33 | 46.63 | 44.41 | 47.86 |
| 1949 | 9 | 1772.33 | 0.47 | 802.09 | 605.95 | 749.61 | 729.75 | 907.61 | 731.38 | 835.79 | 820.77 | 506.17 | 632.84 | 623.33 | 479.91 | 753.38 | 350.20 | 545.18 | 664.72 | 51.21 | 41.27 | 47.16 | 46.31 | 0.56 | 0.50 | 0.46 | 0.61 | 48.79 | 58.73 | 52.84 | 53.69 |
| 1950 | 10 | 1708.40 | 0.55 | 541.17 | 463.50 | 461.90 | 437.70 | 610.46 | 577.11 | 532.98 | 553.68 | 133.19 | 542.99 | 455.95 | 336.09 | 595.75 | 187.37 | 314.95 | 414.13 | 35.73 | 33.78 | 31.20 | 32.41 | 0.89 | 0.79 | 0.86 | 0.86 | 64.27 | 66.22 | 68.80 | 67.59 |
| 1951 | 11 | 1446.91 | 0.54 | 625.77 | 587.73 | 616.42 | 577.84 | 862.52 | 801.70 | 828.39 | 773.73 | 710.42 | 674.97 | 617.29 | 501.23 | 489.06 | 285.95 | 460.34 | 568.60 | 59.61 | 55.41 | 57.25 | 53.47 | 0.36 | 0.30 | 0.40 | 0.54 | 40.39 | 44.59 | 42.75 | 46.53 |
| 1952 | 10 | 1778.40 | 0.38 | 761.22 | 491.39 | 538.53 | 577.33 | 916.33 | 590.04 | 695.81 | 681.62 | 701.97 | 526.05 | 561.35 | 484.21 | 589.01 | 261.64 | 410.43 | 464.27 | 51.53 | 33.18 | 39.13 | 38.33 | 0.25 | 0.47 | 0.43 | 0.47 | 48.47 | 66.82 | 60.87 | 61.67 |
| 1953 | 12 | 1818.33 | 0.33 | 412.71 | 418.51 | 472.98 | 445.71 | 481.98 | 493.46 | 543.51 | 517.19 | 256.51 | 416.73 | 353.72 | 243.45 | 408.05 | 219.62 | 384.75 | 428.34 | 26.51 | 27.14 | 29.89 | 28.44 | 0.78 | 0.59 | 0.64 | 0.73 | 73.49 | 72.86 | 70.11 | 71.56 |
| 1954 | 13 | 1876.15 | 0.37 | 361.40 | 394.11 | 346.86 | 365.49 | 453.24 | 509.75 | 442.18 | 459.82 | 233.28 | 463.50 | 337.22 | 286.27 | 388.61 | 162.05 | 278.20 | 312.69 | 24.16 | 27.17 | 23.57 | 24.51 | 0.86 | 0.66 | 0.82 | 0.82 | 75.84 | 72.83 | 76.43 | 75.49 |
| 1955 | 12 | 1744.25 | 0.31 | 323.11 | 368.50 | 293.60 | 310.42 | 424.15 | 421.46 | 355.56 | 371.62 | 220.04 | 309.95 | 203.96 | 195.26 | 362.61 | 200.46 | 278.84 | 290.25 | 24.32 | 24.16 | 20.38 | 21.31 | 0.80 | 0.65 | 0.82 | 0.80 | 75.68 | 75.84 | 79.62 | 78.69 |
| 1956 | 14 | 2016.00 | 0.25 | 262.32 | 342.02 | 315.79 | 318.83 | 334.54 | 417.86 | 376.66 | 346.14 | 215.57 | 355.34 | 250.90 | 170.80 | 255.83 | 163.37 | 237.69 | 276.05 | 16.59 | 20.73 | 18.68 | 17.17 | 0.82 | 0.52 | 0.71 | 0.81 | 83.41 | 79.27 | 81.32 | 82.83 |
| 1957 | 15 | 1334.47 | 0.33 | 344.18 | 316.46 | 273.89 | 283.98 | 513.46 | 441.46 | 427.93 | 455.26 | 417.76 | 426.29 | 391.52 | 370.70 | 298.56 | 123.70 | 177.69 | 228.41 | 38.48 | 33.08 | 32.07 | 34.12 | 0.42 | 0.30 | 0.44 | 0.44 | 61.52 | 66.92 | 67.93 | 65.88 |
| 1958 | 15 | 1263.07 | 0.47 | 664.80 | 471.98 | 476.11 | 485.83 | 784.19 | 555.09 | 572.89 | 587.77 | 599.09 | 467.47 | 411.33 | 379.15 | 506.02 | 233.58 | 351.25 | 406.80 | 62.09 | 43.95 | 45.36 | 46.53 | 0.21 | 0.34 | 0.46 | 0.49 | 37.91 | 56.05 | 54.64 | 53.47 |
| 1959 | 15 | 1753.53 | 0.34 | 351.16 | 475.30 | 420.68 | 422.00 | 474.43 | 584.62 | 535.48 | 551.92 | 367.70 | 496.96 | 376.61 | 367.31 | 299.82 | 211.94 | 273.03 | 330.42 | 27.06 | 33.34 | 30.54 | 31.47 | 0.73 | 0.36 | 0.54 | 0.57 | 72.94 | 66.66 | 69.46 | 68.53 |
| 1960 | 15 | 1645.53 | 0.42 | 651.67 | 525.21 | 549.35 | 523.09 | 975.90 | 776.17 | 833.74 | 791.23 | 812.45 | 527.75 | 365.38 | 400.08 | 540.66 | 335.53 | 548.43 | 473.05 | 59.31 | 47.17 | 50.67 | 48.08 | 0.23 | 0.23 | 0.34 | 0.34 | 40.69 | 52.83 | 49.33 | 51.92 |
| 1961 | 16 | 1654.25 | 0.60 | 578.42 | 531.30 | 503.00 | 417.07 | 965.93 | 754.02 | 789.16 | 701.54 | 517.69 | 488.09 | 289.29 | 243.76 | 815.53 | 439.84 | 634.85 | 590.70 | 58.39 | 45.58 | 47.70 | 42.41 | 0.72 | 0.73 | 0.77 | 0.82 | 41.61 | 54.42 | 52.30 | 57.59 |
| 1962 | 16 | 1446.88 | 0.37 | 369.48 | 337.14 | 329.65 | 322.43 | 449.03 | 445.13 | 426.59 | 383.74 | 294.87 | 364.55 | 289.96 | 213.46 | 338.62 | 193.39 | 278.11 | 294.39 | 31.03 | 30.77 | 29.48 | 26.52 | 0.72 | 0.58 | 0.70 | 0.80 | 68.97 | 69.23 | 70.52 | 73.48 |
| 1963 | 15 | 1570.80 | 0.36 | 343.08 | 358.49 | 350.20 | 280.31 | 519.04 | 496.61 | 488.11 | 409.07 | 361.55 | 386.18 | 299.68 | 224.78 | 372.37 | 227.81 | 316.14 | 316.49 | 33.04 | 31.62 | 31.07 | 26.04 | 0.67 | 0.56 | 0.66 | 0.80 | 66.96 | 68.38 | 68.93 | 73.96 |
| 1964 | 18 | 1690.39 | 0.41 | 586.12 | 508.17 | 504.05 | 484.23 | 844.32 | 683.47 | 719.88 | 679.29 | 657.48 | 579.45 | 493.55 | 415.74 | 529.72 | 262.52 | 428.09 | 461.18 | 49.95 | 40.43 | 42.59 | 40.19 | 0.39 | 0.34 | 0.44 | 0.54 | 50.05 | 59.57 | 57.41 | 59.81 |
| 1965 | 17 | 1859.53 | 0.43 | 824.22 | 594.09 | 642.19 | 608.98 | 1038.02 | 769.13 | 826.53 | 789.17 | 822.37 | 711.30 | 669.40 | 640.92 | 633.38 | 275.78 | 454.08 | 476.36 | 55.82 | 41.36 | 44.45 | 42.44 | 0.30 | 0.39 | 0.46 | 0.52 | 44.18 | 58.64 | 55.55 | 57.56 |
| 1966 | 21 | 1648.38 | 0.46 | 486.43 | 442.17 | 428.99 | 499.71 | 701.57 | 617.99 | 608.10 | 644.76 | 489.84 | 638.41 | 591.87 | 580.56 | 502.22 | 279.60 | 394.47 | 454.36 | 42.56 | 37.49 | 36.89 | 40.28 | 0.63 | 0.60 | 0.70 | 0.71 | 57.44 | 62.51 | 63.11 | 59.72 |
| 1967 | 19 | 1601.16 | 0.46 | 423.20 | 439.89 | 437.78 | 405.52 | 767.77 | 652.12 | 676.02 | 614.70 | 603.41 | 674.68 | 652.26 | 578.07 | 474.70 | 265.41 | 401.88 | 452.36 | 47.95 | 40.73 | 42.22 | 38.39 | 0.54 | 0.49 | 0.55 | 0.69 | 52.05 | 59.27 | 57.78 | 61.61 |
| 1968 | 19 | 1792.26 | 0.43 | 724.06 | 465.83 | 532.00 | 517.69 | 882.92 | 645.79 | 665.45 | 628.33 | 569.29 | 620.91 | 555.25 | 422.64 | 674.88 | 331.96 | 489.44 | 520.10 | 49.26 | 36.03 | 37.13 | 35.06 | 0.54 | 0.61 | 0.68 | 0.75 | 50.74 | 63.97 | 62.87 | 64.94 |
| 1969 | 20 | 1833.05 | 0.44 | 500.35 | 477.67 | 435.95 | 469.16 | 688.63 | 656.86 | 630.65 | 640.08 | 422.99 | 612.66 | 509.32 | 477.40 | 543.43 | 280.36 | 422.95 | 442.61 | 37.57 | 35.83 | 34.40 | 34.92 | 0.75 | 0.63 | 0.74 | 0.74 | 62.43 | 64.17 | 65.60 | 65.08 |
| 1970 | 20 | 1761.35 | 0.35 | 489.55 | 394.77 | 418.14 | 404.38 | 652.71 | 564.82 | 559.36 | 520.92 | 539.03 | 510.31 | 403.15 | 355.49 | 368.04 | 197.20 | 310.35 | 342.50 | 37.06 | 32.07 | 31.76 | 29.57 | 0.44 | 0.41 | 0.54 | 0.66 | 62.94 | 67.93 | 68.24 | 70.43 |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1971 | 20 | 1661.80 | 0.53 | 617.76 | 586.09 | 584.24 | 603.77 | 749.92 | 698.16 | 697.21 | 704.69 | 405.14 | 612.83 | 497.13 | 487.26 | 631.05 | 411.35 | 528.67 | 519.94 | 45.13 | 42.01 | 41.96 | 42.41 | 0.76 | 0.70 | 0.75 | 0.74 | 54.87 | 57.99 | 58.04 | 57.59 |
| 1972 | 19 | 1258.95 | 0.46 | 379.36 | 362.13 | 362.51 | 390.20 | 545.49 | 553.91 | 569.14 | 574.44 | 412.30 | 161.94 | 1072.63 | 1244.63 | 357.13 | 566.02 | 1314.61 | 1469.97 | 43.33 | 44.00 | 45.21 | 45.63 | 0.58 | 0.45 | 0.53 | 0.52 | 56.67 | 56.00 | 54.79 | 54.37 |
| 1973 | 20 | 1435.25 | 0.40 | 452.75 | 418.24 | 464.20 | 439.00 | 546.63 | 534.36 | 556.15 | 538.30 | 493.64 | 469.77 | 435.51 | 439.13 | 234.75 | 195.95 | 323.31 | 252.54 | 38.09 | 37.23 | 38.75 | 37.51 | 0.40 | 0.38 | 0.48 | 0.45 | 61.91 | 62.77 | 61.25 | 62.49 |
| 1974 | 20 | 1931.95 | 0.43 | 652.18 | 574.51 | 572.58 | 556.84 | 1005.63 | 803.85 | 868.49 | 828.74 | 724.17 | 603.89 | 388.51 | 331.45 | 697.77 | 366.90 | 652.86 | 628.93 | 52.05 | 41.61 | 44.95 | 42.90 | 0.52 | 0.49 | 0.57 | 0.61 | 47.95 | 58.39 | 55.05 | 57.10 |
| 1975 | 22 | 1353.32 | 0.44 | 425.23 | 415.37 | 421.23 | 401.10 | 637.13 | 558.37 | 581.38 | 542.81 | 435.95 | 448.36 | 228.03 | 234.13 | 464.62 | 215.54 | 422.66 | 387.64 | 47.08 | 41.26 | 42.96 | 40.11 | 0.60 | 0.44 | 0.56 | 0.62 | 52.92 | 58.74 | 57.04 | 59.89 |
| 1976 | 21 | 1839.71 | 0.42 | 564.28 | 535.91 | 501.36 | 495.35 | 729.10 | 665.86 | 652.12 | 679.74 | 593.44 | 683.54 | 654.02 | 687.90 | 423.56 | 247.59 | 317.82 | 413.81 | 39.63 | 36.19 | 35.45 | 36.95 | 0.56 | 0.52 | 0.60 | 0.60 | 60.37 | 63.81 | 64.55 | 63.05 |
| 1977 | 26 | 1653.85 | 0.44 | 585.86 | 492.80 | 517.21 | 509.70 | 702.34 | 620.67 | 626.13 | 629.52 | 589.26 | 560.47 | 434.18 | 471.46 | 382.17 | 296.44 | 393.70 | 382.89 | 42.47 | 37.53 | 37.86 | 38.06 | 0.51 | 0.60 | 0.64 | 0.62 | 57.53 | 62.47 | 62.14 | 61.94 |
| 1978 | 23 | 1696.57 | 0.39 | 460.78 | 425.47 | 414.92 | 417.66 | 621.85 | 608.00 | 578.36 | 572.77 | 502.07 | 257.09 | 65.35 | 161.58 | 366.95 | 402.39 | 526.42 | 451.34 | 36.65 | 35.84 | 34.09 | 33.76 | 0.57 | 0.54 | 0.63 | 0.63 | 63.35 | 64.16 | 65.91 | 66.24 |
| 1979 | 26 | 1552.69 | 0.51 | 717.14 | 703.29 | 716.53 | 704.76 | 899.74 | 895.80 | 918.74 | 887.86 | 891.92 | 906.52 | 946.15 | 899.78 | 118.18 | 187.65 | 198.95 | 95.88 | 57.95 | 57.69 | 59.17 | 57.18 | 0.02 | 0.08 | 0.07 | 0.02 | 42.05 | 42.31 | 40.83 | 42.82 |
| 1980 | 25 | 1326.68 | 0.45 | 491.51 | 442.28 | 446.14 | 450.76 | 648.28 | 655.73 | 631.87 | 624.90 | 644.52 | 450.92 | 459.46 | 547.00 | 69.73 | 274.96 | 241.18 | 133.53 | 48.86 | 49.43 | 47.63 | 47.10 | 0.01 | 0.25 | 0.24 | 0.12 | 51.14 | 50.57 | 52.37 | 52.90 |
| 1981 | 25 | 1628.92 | 0.45 | 513.34 | 456.12 | 480.13 | 488.89 | 708.65 | 613.20 | 630.97 | 615.99 | 477.81 | 324.32 | 212.60 | 266.86 | 523.33 | 391.44 | 628.25 | 562.62 | 43.50 | 37.64 | 38.74 | 37.82 | 0.65 | 0.56 | 0.61 | 0.65 | 56.50 | 62.36 | 61.26 | 62.18 |
| 1982 | 27 | 1981.07 | 0.47 | 595.08 | 657.56 | 580.91 | 572.70 | 768.52 | 755.16 | 708.59 | 710.09 | 522.42 | 698.90 | 510.95 | 493.70 | 563.61 | 395.28 | 495.82 | 510.79 | 38.79 | 38.12 | 35.77 | 35.84 | 0.76 | 0.68 | 0.76 | 0.77 | 61.21 | 61.88 | 64.23 | 64.16 |
| 1983 | 27 | 1821.74 | 0.42 | 577.02 | 567.57 | 525.95 | 540.38 | 702.40 | 716.94 | 703.60 | 681.02 | 556.08 | 506.87 | 413.63 | 427.13 | 429.13 | 377.95 | 449.72 | 443.09 | 38.56 | 39.35 | 38.62 | 37.38 | 0.61 | 0.56 | 0.63 | 0.65 | 61.44 | 60.65 | 61.38 | 62.62 |
| 1984 | 27 | 2181.81 | 0.33 | 564.34 | 537.77 | 505.17 | 510.97 | 740.09 | 678.02 | 661.06 | 683.28 | 637.01 | 624.23 | 554.35 | 610.26 | 376.71 | 241.72 | 305.17 | 334.91 | 33.92 | 31.08 | 30.30 | 31.32 | 0.46 | 0.43 | 0.51 | 0.48 | 66.08 | 68.92 | 69.70 | 68.68 |
| 1985 | 28 | 1667.29 | 0.40 | 469.71 | 472.59 | 463.83 | 474.07 | 646.16 | 591.06 | 601.77 | 585.83 | 460.72 | 408.63 | 283.51 | 304.64 | 453.06 | 301.08 | 441.02 | 427.72 | 38.75 | 35.45 | 36.09 | 35.14 | 0.64 | 0.56 | 0.61 | 0.63 | 61.25 | 64.55 | 63.91 | 64.86 |
| 1986 | 28 | 1786.54 | 0.26 | 256.13 | 287.53 | 269.72 | 249.61 | 337.96 | 366.33 | 351.36 | 338.77 | 206.25 | 293.77 | 209.65 | 229.58 | 267.70 | 204.31 | 304.86 | 288.02 | 18.92 | 20.50 | 19.67 | 18.96 | 0.82 | 0.67 | 0.76 | 0.79 | 81.08 | 79.50 | 80.33 | 81.04 |
| 1987 | 29 | 2204.45 | 0.36 | 380.74 | 439.05 | 392.73 | 435.99 | 595.23 | 602.01 | 579.09 | 601.55 | 241.36 | 508.67 | 371.21 | 433.27 | 544.10 | 293.36 | 404.04 | 408.23 | 27.00 | 27.31 | 26.27 | 27.29 | 0.84 | 0.72 | 0.79 | 0.76 | 73.00 | 72.69 | 73.73 | 72.71 |
| 1988 | 29 | 1958.52 | 0.38 | 417.15 | 413.95 | 362.64 | 401.15 | 574.86 | 548.03 | 508.16 | 502.46 | 217.54 | 489.59 | 343.44 | 392.50 | 532.11 | 266.80 | 353.95 | 381.73 | 29.35 | 27.98 | 25.95 | 25.66 | 0.83 | 0.73 | 0.81 | 0.82 | 70.65 | 72.02 | 74.05 | 74.34 |
| 1989 | 30 | 1507.70 | 0.47 | 331.91 | 351.94 | 305.03 | 329.23 | 590.58 | 616.42 | 582.26 | 546.47 | 405.99 | 386.86 | 163.55 | 144.82 | 428.91 | 291.34 | 479.00 | 485.26 | 39.17 | 40.88 | 38.62 | 36.25 | 0.72 | 0.53 | 0.66 | 0.74 | 60.83 | 59.12 | 61.38 | 63.75 |
| 1990 | 30 | 1655.67 | 0.39 | 404.13 | 403.62 | 375.54 | 388.21 | 533.82 | 511.06 | 493.46 | 489.52 | 386.04 | 366.25 | 239.99 | 279.33 | 368.71 | 257.54 | 364.99 | 345.61 | 32.24 | 30.87 | 29.80 | 29.57 | 0.74 | 0.68 | 0.76 | 0.76 | 67.76 | 69.13 | 70.20 | 70.43 |
| 1991 | 32 | 1959.94 | 0.38 | 258.00 | 342.24 | 305.56 | 298.89 | 423.66 | 513.47 | 465.03 | 456.68 | 189.73 | 377.90 | 189.44 | 210.23 | 378.81 | 272.09 | 416.80 | 394.51 | 21.62 | 26.20 | 23.73 | 23.30 | 0.90 | 0.76 | 0.85 | 0.86 | 78.38 | 73.80 | 76.27 | 76.70 |
| 1992 | 32 | 1395.31 | 0.40 | 277.64 | 336.56 | 286.39 | 319.01 | 389.38 | 423.65 | 386.80 | 390.38 | 234.62 | 290.39 | 176.87 | 212.57 | 310.77 | 240.55 | 355.55 | 320.85 | 27.91 | 30.36 | 27.72 | 27.98 | 0.84 | 0.69 | 0.80 | 0.79 | 72.09 | 69.64 | 72.28 | 72.02 |
| 1993 | 32 | 1872.59 | 0.37 | 362.88 | 407.57 | 381.02 | 350.08 | 575.76 | 619.84 | 575.59 | 526.55 | 446.63 | 276.00 | 187.15 | 190.68 | 363.36 | 391.91 | 670.07 | 617.04 | 30.75 | 33.10 | 30.74 | 28.12 | 0.68 | 0.46 | 0.67 | 0.75 | 69.25 | 66.90 | 69.26 | 71.88 |
| 1994 | 32 | 1366.25 | 0.54 | 302.49 | 356.41 | 304.30 | 309.60 | 484.22 | 560.99 | 494.61 | 505.15 | 321.78 | 337.33 | 182.46 | 164.87 | 361.86 | 335.33 | 579.69 | 542.48 | 35.44 | 41.06 | 36.20 | 36.97 | 0.84 | 0.68 | 0.82 | 0.81 | 64.56 | 58.94 | 63.80 | 63.03 |
| 1995 | 31 | 1800.45 | 0.32 | 368.80 | 365.29 | 339.34 | 351.84 | 468.03 | 494.50 | 450.83 | 447.64 | 333.17 | 361.67 | 90.60 | 120.11 | 328.70 | 196.44 | 398.65 | 368.53 | 25.99 | 27.47 | 25.04 | 24.86 | 0.75 | 0.55 | 0.73 | 0.72 | 74.01 | 72.53 | 74.96 | 75.14 |
| 1996 | 31 | 1613.19 | 0.38 | 292.02 | 344.14 | 281.13 | 280.58 | 375.09 | 417.87 | 346.09 | 345.44 | 182.85 | 316.47 | 205.44 | 216.54 | 327.52 | 251.72 | 369.33 | 343.77 | 23.25 | 25.90 | 21.45 | 21.41 | 0.89 | 0.78 | 0.90 | 0.89 | 76.75 | 74.10 | 78.55 | 78.59 |
| 1997 | 31 | 1840.39 | 0.33 | 313.88 | 307.37 | 296.91 | 305.87 | 423.71 | 419.07 | 392.99 | 387.62 | 209.64 | 359.40 | 297.91 | 297.21 | 368.22 | 266.32 | 384.17 | 383.44 | 23.02 | 22.77 | 21.35 | 21.06 | 0.84 | 0.77 | 0.85 | 0.85 | 76.98 | 77.23 | 78.65 | 78.94 |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1998 | 31 | 1948.00 | 0.42 | 427.87 | 438.24 | 380.20 | 359.40 | 603.36 | 538.27 | 525.37 | 497.91 | 271.03 | 298.85 | 157.29 | 168.82 | 539.07 | 371.19 | 556.28 | 492.75 | 30.97 | 27.63 | 26.97 | 25.56 | 0.84 | 0.81 | 0.87 | 0.88 | 69.03 | 72.37 | 73.03 | 74.44 |
| 1999 | 32 | 2014.69 | 0.34 | 298.17 | 355.45 | 293.42 | 312.59 | 360.51 | 430.67 | 345.20 | 375.11 | 167.24 | 378.51 | 206.84 | 220.12 | 319.38 | 221.54 | 299.68 | 312.47 | 17.89 | 21.38 | 17.13 | 18.62 | 0.92 | 0.82 | 0.92 | 0.89 | 82.11 | 78.62 | 82.87 | 81.38 |
| 2000 | 32 | 1636.59 | 0.49 | 338.35 | 387.72 | 330.56 | 380.30 | 525.58 | 571.81 | 502.73 | 513.93 | 233.27 | 513.72 | 290.42 | 320.35 | 470.99 | 283.69 | 390.81 | 400.24 | 32.11 | 34.94 | 30.72 | 31.40 | 0.87 | 0.75 | 0.86 | 0.85 | 67.89 | 65.06 | 69.28 | 68.60 |
| 2001 | 32 | 1764.91 | 0.51 | 450.87 | 518.85 | 406.37 | 447.26 | 666.21 | 715.12 | 615.85 | 664.11 | 484.89 | 578.97 | 335.34 | 360.47 | 456.84 | 375.08 | 493.47 | 536.32 | 37.75 | 40.52 | 34.89 | 37.63 | 0.78 | 0.66 | 0.81 | 0.76 | 62.25 | 59.48 | 65.11 | 62.37 |
| 2002 | 32 | 1939.81 | 0.30 | 317.90 | 344.08 | 303.79 | 331.58 | 411.20 | 412.11 | 380.02 | 420.09 | 285.75 | 365.74 | 223.73 | 234.03 | 295.72 | 205.80 | 277.52 | 312.35 | 21.20 | 21.25 | 19.59 | 21.66 | 0.80 | 0.75 | 0.83 | 0.77 | 78.80 | 78.75 | 80.41 | 78.34 |
| 2003 | 31 | 1621.42 | 0.48 | 376.83 | 408.62 | 355.49 | 361.43 | 491.37 | 551.65 | 475.85 | 465.19 | 295.55 | 458.41 | 260.16 | 254.53 | 392.57 | 283.62 | 417.13 | 412.30 | 30.31 | 34.02 | 29.35 | 28.69 | 0.86 | 0.75 | 0.87 | 0.87 | 69.69 | 65.98 | 70.65 | 71.31 |
| 2004 | 32 | 2117.47 | 0.28 | 354.98 | 368.07 | 319.90 | 323.13 | 467.27 | 456.12 | 431.03 | 437.18 | 329.11 | 213.24 | 76.80 | 105.28 | 331.73 | 320.87 | 459.16 | 461.68 | 22.07 | 21.54 | 20.36 | 20.65 | 0.76 | 0.73 | 0.80 | 0.79 | 77.93 | 78.46 | 79.64 | 79.35 |
| 2005 | 32 | 1701.94 | 0.38 | 289.73 | 351.66 | 292.61 | 320.36 | 410.04 | 476.99 | 411.68 | 428.12 | 245.18 | 386.49 | 232.91 | 264.92 | 328.69 | 239.53 | 377.17 | 386.58 | 24.09 | 28.03 | 24.19 | 25.15 | 0.86 | 0.70 | 0.84 | 0.82 | 75.91 | 71.97 | 75.81 | 74.85 |
| 2006 | 32 | 1603.88 | 0.40 | 263.33 | 362.37 | 293.49 | 321.48 | 351.83 | 484.19 | 387.63 | 409.60 | 145.29 | 429.42 | 237.36 | 297.62 | 324.70 | 213.16 | 300.61 | 296.03 | 21.94 | 30.19 | 24.17 | 25.54 | 0.91 | 0.70 | 0.87 | 0.84 | 78.06 | 69.81 | 75.83 | 74.46 |
| 2007 | 32 | 1992.88 | 0.29 | 351.89 | 335.69 | 318.62 | 316.49 | 446.05 | 408.91 | 413.94 | 388.64 | 278.38 | 360.70 | 265.03 | 240.65 | 348.52 | 244.45 | 331.07 | 322.12 | 22.38 | 20.52 | 20.77 | 19.50 | 0.78 | 0.77 | 0.82 | 0.84 | 77.62 | 79.48 | 79.23 | 80.50 |

## A.30 Performance measurements of the four methods from cross-validation for interpolating TXx (pp. 204-206)

| | | | | MAE | | | | RMSE | | | | RMSEs | | | | RMSEu | | | | pf | | | | d | | | | CP | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Years | n | Sample Mean | *k* | TPS | IDW | OK | UK | TPS | IDW | OK | UK | TPS | IDW | OK | UK | TPS | IDW | OK | UK | TPS | IDW | OK | UK | TPS | IDW | OK | UK | TPS | IDW | OK | UK |
| 1948 | 10 | 37.36 | 0.04 | 1.24 | 1.05 | 0.92 | 1.07 | 1.50 | 1.26 | 1.12 | 1.22 | 0.59 | 1.24 | 0.85 | 0.61 | 1.37 | 0.39 | 0.76 | 1.08 | 4.00 | 3.39 | 3.01 | 3.27 | 0.65 | 0.40 | 0.71 | 0.74 | 96.00 | 96.61 | 96.99 | 96.73 |
| 1949 | 11 | 35.30 | 0.08 | 2.59 | 2.10 | 2.21 | 2.41 | 4.62 | 3.29 | 3.69 | 3.89 | 3.24 | 3.23 | 3.43 | 3.54 | 3.29 | 1.18 | 2.08 | 2.17 | 13.09 | 9.31 | 10.44 | 11.02 | 0.17 | 0.14 | 0.18 | 0.14 | 86.91 | 90.69 | 89.56 | 88.98 |
| 1950 | 11 | 37.45 | 0.03 | 1.27 | 0.94 | 1.10 | 1.15 | 1.61 | 1.29 | 1.38 | 1.47 | 1.34 | 0.83 | 0.81 | 0.69 | 0.90 | 0.72 | 0.85 | 1.34 | 4.31 | 3.44 | 3.69 | 3.91 | 0.15 | 0.20 | 0.22 | 0.31 | 95.69 | 96.56 | 96.31 | 96.09 |
| 1951 | 12 | 36.96 | 0.04 | 1.54 | 1.02 | 0.95 | 1.17 | 1.71 | 1.36 | 1.22 | 1.40 | 1.05 | 0.55 | 0.19 | 0.17 | 1.35 | 0.93 | 1.23 | 1.51 | 4.63 | 3.67 | 3.30 | 3.79 | 0.57 | 0.59 | 0.76 | 0.71 | 95.37 | 96.33 | 96.70 | 96.21 |
| 1952 | 12 | 38.10 | 0.06 | 2.79 | 2.00 | 2.23 | 2.29 | 3.74 | 2.45 | 2.81 | 2.85 | 2.66 | 2.06 | 2.22 | 2.18 | 2.63 | 0.90 | 1.59 | 1.51 | 9.81 | 6.43 | 7.36 | 7.49 | 0.29 | 0.33 | 0.37 | 0.34 | 90.19 | 93.57 | 92.64 | 92.51 |
| 1953 | 13 | 36.65 | 0.03 | 0.65 | 0.66 | 0.64 | 0.61 | 0.86 | 0.87 | 0.76 | 0.69 | 0.24 | 0.72 | 0.39 | 0.15 | 0.82 | 0.30 | 0.60 | 0.63 | 2.34 | 2.36 | 2.07 | 1.89 | 0.86 | 0.68 | 0.84 | 0.88 | 97.66 | 97.64 | 97.93 | 98.11 |
| 1954 | 14 | 38.17 | 0.07 | 1.09 | 1.52 | 1.18 | 0.92 | 1.37 | 1.80 | 1.41 | 1.18 | 0.42 | 1.69 | 0.96 | 0.33 | 1.30 | 0.56 | 0.98 | 1.09 | 3.59 | 4.73 | 3.70 | 3.09 | 0.92 | 0.73 | 0.89 | 0.94 | 96.41 | 95.27 | 96.30 | 96.91 |
| 1955 | 11 | 37.62 | 0.06 | 1.82 | 1.93 | 1.99 | 1.73 | 2.17 | 2.39 | 2.38 | 2.18 | 1.56 | 2.23 | 1.98 | 1.45 | 1.51 | 0.96 | 1.37 | 1.68 | 5.77 | 6.37 | 6.34 | 5.80 | 0.60 | 0.17 | 0.36 | 0.63 | 94.23 | 93.63 | 93.66 | 94.20 |
| 1956 | 14 | 39.39 | 0.07 | 1.49 | 2.05 | 2.05 | 1.85 | 1.89 | 2.40 | 2.47 | 2.27 | 1.20 | 2.39 | 2.16 | 1.51 | 1.47 | 0.83 | 1.58 | 1.89 | 4.81 | 6.09 | 6.26 | 5.77 | 0.81 | 0.48 | 0.63 | 0.75 | 95.19 | 93.91 | 93.74 | 94.23 |
| 1957 | 16 | 38.13 | 0.10 | 2.28 | 2.43 | 2.68 | 2.43 | 3.97 | 3.93 | 4.31 | 4.17 | 3.31 | 4.11 | 4.46 | 4.32 | 2.19 | 1.23 | 1.88 | 1.87 | 10.40 | 10.30 | 11.29 | 10.94 | 0.49 | 0.27 | 0.29 | 0.34 | 89.60 | 89.70 | 88.71 | 89.06 |
| 1958 | 17 | 39.02 | 0.08 | 2.09 | 2.11 | 1.74 | 1.74 | 2.39 | 2.35 | 2.06 | 2.07 | 1.08 | 1.95 | 1.22 | 0.97 | 2.13 | 1.12 | 1.52 | 1.74 | 6.12 | 6.01 | 5.29 | 5.29 | 0.80 | 0.65 | 0.81 | 0.83 | 93.88 | 93.99 | 94.71 | 94.71 |
| 1959 | 17 | 38.53 | 0.07 | 1.68 | 1.94 | 1.81 | 1.69 | 2.36 | 2.34 | 2.48 | 2.24 | 1.47 | 2.06 | 1.73 | 1.25 | 1.84 | 1.18 | 1.76 | 1.73 | 6.11 | 6.08 | 6.42 | 5.81 | 0.72 | 0.56 | 0.64 | 0.74 | 93.89 | 93.92 | 93.58 | 94.19 |
| 1960 | 17 | 40.85 | 0.06 | 1.71 | 2.04 | 2.14 | 2.11 | 2.11 | 2.41 | 2.70 | 2.63 | 1.62 | 2.09 | 2.04 | 2.01 | 1.35 | 0.90 | 1.47 | 1.36 | 5.16 | 5.91 | 6.60 | 6.44 | 0.63 | 0.33 | 0.40 | 0.39 | 94.84 | 94.09 | 93.40 | 93.56 |
| 1961 | 18 | 39.38 | 0.06 | 1.48 | 1.69 | 1.56 | 1.34 | 1.77 | 1.97 | 1.97 | 1.72 | 1.08 | 1.99 | 1.94 | 1.54 | 1.40 | 0.92 | 1.49 | 1.42 | 4.49 | 5.00 | 5.00 | 4.36 | 0.79 | 0.57 | 0.70 | 0.80 | 95.51 | 95.00 | 95.00 | 95.64 |
| 1962 | 18 | 38.44 | 0.06 | 1.03 | 1.38 | 1.26 | 1.21 | 1.35 | 1.62 | 1.62 | 1.51 | 0.69 | 1.59 | 1.41 | 1.19 | 1.16 | 0.83 | 1.27 | 1.29 | 3.50 | 4.22 | 4.22 | 3.94 | 0.87 | 0.70 | 0.78 | 0.83 | 96.50 | 95.78 | 95.78 | 96.06 |
| 1963 | 17 | 38.07 | 0.07 | 1.55 | 1.56 | 1.32 | 1.44 | 1.95 | 2.18 | 1.75 | 1.82 | 1.21 | 1.76 | 0.85 | 0.88 | 1.53 | 1.03 | 1.36 | 1.46 | 5.12 | 5.72 | 4.60 | 4.78 | 0.78 | 0.56 | 0.82 | 0.81 | 94.88 | 94.28 | 95.40 | 95.22 |
| 1964 | 19 | 37.54 | 0.05 | 1.16 | 1.08 | 0.99 | 1.04 | 1.38 | 1.44 | 1.21 | 1.29 | 0.72 | 1.20 | 0.90 | 0.95 | 1.17 | 0.62 | 0.90 | 1.04 | 3.67 | 3.84 | 3.22 | 3.43 | 0.82 | 0.67 | 0.84 | 0.83 | 96.33 | 96.16 | 96.78 | 96.57 |
| 1965 | 18 | 38.67 | 0.07 | 1.80 | 2.08 | 1.92 | 1.95 | 2.09 | 2.30 | 2.35 | 2.41 | 1.30 | 1.92 | 1.57 | 1.33 | 1.65 | 1.25 | 1.82 | 2.00 | 5.41 | 5.95 | 6.07 | 6.23 | 0.81 | 0.69 | 0.76 | 0.76 | 94.59 | 94.05 | 93.93 | 93.77 |
| 1966 | 23 | 38.48 | 0.08 | 2.04 | 2.24 | 2.69 | 2.55 | 2.94 | 2.90 | 3.36 | 3.25 | 2.47 | 3.00 | 3.32 | 3.11 | 1.59 | 1.31 | 2.18 | 2.14 | 7.64 | 7.54 | 8.72 | 8.46 | 0.56 | 0.47 | 0.47 | 0.51 | 92.36 | 92.46 | 91.28 | 91.54 |
| 1967 | 21 | 35.65 | 0.24 | 7.42 | 4.71 | 5.91 | 5.49 | 13.73 | 9.96 | 11.46 | 10.86 | 10.73 | 13.74 | 24.42 | 15.00 | 8.57 | 4.94 | 15.67 | 7.05 | 38.52 | 27.94 | 32.14 | 30.47 | 0.01 | 0.01 | 0.07 | 0.05 | 61.48 | 72.06 | 67.86 | 69.53 |
| 1968 | 20 | 37.42 | 0.06 | 1.62 | 1.57 | 1.64 | 1.49 | 1.95 | 2.02 | 2.00 | 1.84 | 1.31 | 1.67 | 1.33 | 0.75 | 1.44 | 0.86 | 1.23 | 1.40 | 5.21 | 5.40 | 5.35 | 4.93 | 0.67 | 0.44 | 0.56 | 0.69 | 94.79 | 94.60 | 94.65 | 95.07 |
| 1969 | 23 | 37.70 | 0.08 | 2.42 | 1.79 | 1.86 | 1.95 | 3.23 | 2.40 | 2.57 | 2.73 | 1.67 | 2.75 | 2.80 | 2.84 | 2.77 | 1.41 | 2.33 | 2.27 | 8.57 | 6.36 | 6.82 | 7.25 | 0.70 | 0.71 | 0.76 | 0.72 | 91.43 | 93.64 | 93.18 | 92.75 |
| 1970 | 22 | 37.67 | 0.08 | 1.40 | 1.47 | 1.48 | 1.39 | 1.65 | 2.01 | 1.71 | 1.69 | 0.82 | 1.80 | 1.06 | 1.18 | 1.43 | 1.11 | 1.45 | 1.51 | 4.38 | 5.34 | 4.55 | 4.49 | 0.89 | 0.75 | 0.87 | 0.88 | 95.62 | 94.66 | 95.45 | 95.51 |

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1971 | 23 | 35.66 | 0.04 | 1.04 | 1.03 | 1.01 | 1.01 | 1.34 | 1.31 | 1.29 | 1.30 | 1.17 | 1.24 | 0.98 | 0.94 | 0.65 | 0.44 | 0.73 | 0.76 | 3.75 | 3.68 | 3.60 | 3.64 | 0.49 | 0.34 | 0.54 | 0.53 | 96.25 | 96.32 | 96.40 | 96.36 |
| 1972 | 21 | 38.18 | 0.08 | 1.51 | 1.62 | 1.35 | 1.42 | 1.99 | 2.03 | 1.74 | 1.84 | 0.96 | 1.09 | 0.37 | 0.38 | 1.74 | 1.27 | 1.61 | 1.95 | 5.20 | 5.32 | 4.56 | 4.82 | 0.86 | 0.78 | 0.89 | 0.88 | 94.80 | 94.68 | 95.44 | 95.18 |
| 1973 | 22 | 37.33 | 0.07 | 1.36 | 1.64 | 1.64 | 2.94 | 1.68 | 1.92 | 1.86 | 6.73 | 0.87 | 1.72 | 1.34 | 4.22 | 1.43 | 1.03 | 1.68 | 7.19 | 4.50 | 5.14 | 4.99 | 18.02 | 0.88 | 0.77 | 0.85 | 0.37 | 95.50 | 94.86 | 95.01 | 81.98 |
| 1974 | 21 | 36.72 | 0.05 | 0.89 | 0.97 | 0.90 | 0.81 | 1.04 | 1.16 | 0.98 | 0.95 | 0.29 | 1.00 | 0.55 | 0.33 | 1.00 | 0.59 | 0.84 | 0.90 | 2.83 | 3.16 | 2.68 | 2.58 | 0.92 | 0.83 | 0.91 | 0.92 | 97.17 | 96.84 | 97.32 | 97.42 |
| 1975 | 22 | 37.97 | 0.08 | 1.71 | 1.59 | 1.61 | 1.72 | 2.21 | 2.07 | 2.04 | 2.12 | 1.03 | 1.21 | 0.59 | 0.73 | 1.96 | 1.17 | 2.18 | 2.39 | 5.83 | 5.46 | 5.38 | 5.60 | 0.82 | 0.74 | 0.82 | 0.82 | 94.17 | 94.54 | 94.62 | 94.40 |
| 1976 | 23 | 38.21 | 0.08 | 1.87 | 1.99 | 1.97 | 1.91 | 2.29 | 2.38 | 2.57 | 2.31 | 1.39 | 2.30 | 2.12 | 1.72 | 1.82 | 1.23 | 2.27 | 2.00 | 5.99 | 6.22 | 6.73 | 6.05 | 0.82 | 0.73 | 0.79 | 0.82 | 94.01 | 93.78 | 93.27 | 93.95 |
| 1977 | 25 | 35.84 | 0.05 | 0.90 | 1.03 | 0.85 | 0.89 | 1.08 | 1.19 | 1.04 | 1.09 | 0.45 | 0.78 | 0.85 | 1.11 | 0.98 | 0.96 | 1.35 | 1.67 | 3.02 | 3.32 | 2.90 | 3.04 | 0.88 | 0.76 | 0.88 | 0.87 | 96.98 | 96.68 | 97.10 | 96.96 |
| 1978 | 25 | 36.88 | 0.05 | 1.29 | 0.95 | 1.10 | 1.12 | 1.79 | 1.41 | 1.52 | 1.56 | 0.77 | 1.49 | 1.26 | 1.32 | 1.61 | 0.80 | 1.40 | 1.40 | 4.84 | 3.83 | 4.13 | 4.24 | 0.69 | 0.66 | 0.73 | 0.72 | 95.16 | 96.17 | 95.87 | 95.76 |
| 1979 | 26 | 38.62 | 0.07 | 1.39 | 1.48 | 1.23 | 1.25 | 1.69 | 1.72 | 1.54 | 1.56 | 0.63 | 1.81 | 1.50 | 1.38 | 1.57 | 0.96 | 1.55 | 1.56 | 4.38 | 4.45 | 3.99 | 4.04 | 0.89 | 0.82 | 0.90 | 0.90 | 95.62 | 95.55 | 96.01 | 95.96 |
| 1980 | 25 | 38.06 | 0.07 | 0.96 | 1.08 | 0.91 | 1.09 | 1.20 | 1.43 | 1.16 | 1.42 | 0.43 | 1.05 | 0.53 | 0.56 | 1.12 | 0.74 | 1.02 | 1.26 | 3.15 | 3.76 | 3.06 | 3.73 | 0.93 | 0.85 | 0.93 | 0.90 | 96.85 | 96.24 | 96.94 | 96.27 |
| 1981 | 27 | 36.05 | 0.04 | 0.70 | 0.73 | 0.73 | 0.73 | 0.86 | 0.94 | 0.91 | 0.97 | 0.31 | 0.89 | 0.61 | 0.69 | 0.80 | 0.51 | 0.80 | 0.84 | 2.38 | 2.62 | 2.53 | 2.69 | 0.88 | 0.75 | 0.84 | 0.82 | 97.62 | 97.38 | 97.47 | 97.31 |
| 1982 | 29 | 37.81 | 0.04 | 1.25 | 0.99 | 1.11 | 1.01 | 1.56 | 1.23 | 1.39 | 1.27 | 0.94 | 1.02 | 0.92 | 0.88 | 1.24 | 0.66 | 1.10 | 0.96 | 4.13 | 3.27 | 3.69 | 3.35 | 0.59 | 0.64 | 0.67 | 0.72 | 95.87 | 96.73 | 96.31 | 96.65 |
| 1983 | 29 | 37.02 | 0.05 | 1.01 | 0.96 | 0.96 | 0.98 | 1.25 | 1.20 | 1.20 | 1.22 | 0.71 | 0.59 | 0.32 | 0.30 | 1.03 | 0.81 | 1.14 | 1.14 | 3.38 | 3.25 | 3.25 | 3.31 | 0.82 | 0.77 | 0.84 | 0.83 | 96.62 | 96.75 | 96.75 | 96.69 |
| 1984 | 29 | 37.84 | 0.05 | 0.73 | 0.94 | 0.80 | 0.82 | 0.94 | 1.24 | 0.98 | 1.04 | 0.42 | 0.79 | 0.16 | 0.27 | 0.84 | 0.70 | 0.92 | 0.95 | 2.49 | 3.28 | 2.59 | 2.74 | 0.94 | 0.84 | 0.93 | 0.92 | 97.51 | 96.72 | 97.41 | 97.26 |
| 1985 | 30 | 37.20 | 0.07 | 0.88 | 1.29 | 0.90 | 1.06 | 1.30 | 1.68 | 1.27 | 1.48 | 0.58 | 1.03 | 0.59 | 0.66 | 1.17 | 1.00 | 1.11 | 1.30 | 3.49 | 4.51 | 3.42 | 3.97 | 0.92 | 0.80 | 0.92 | 0.89 | 96.51 | 95.49 | 96.58 | 96.03 |
| 1986 | 30 | 37.78 | 0.06 | 0.81 | 1.10 | 0.81 | 0.87 | 1.00 | 1.28 | 1.01 | 1.07 | 0.40 | 0.89 | 0.38 | 0.44 | 0.92 | 0.73 | 0.97 | 0.99 | 2.66 | 3.39 | 2.66 | 2.85 | 0.94 | 0.85 | 0.94 | 0.93 | 97.34 | 96.61 | 97.34 | 97.15 |
| 1987 | 31 | 37.86 | 0.06 | 1.00 | 1.15 | 0.87 | 0.94 | 1.33 | 1.40 | 1.11 | 1.23 | 0.60 | 1.23 | 0.78 | 0.85 | 1.19 | 0.70 | 1.02 | 1.15 | 3.53 | 3.69 | 2.94 | 3.25 | 0.89 | 0.82 | 0.92 | 0.91 | 96.47 | 96.31 | 97.06 | 96.75 |
| 1988 | 31 | 37.10 | 0.06 | 1.26 | 1.15 | 1.04 | 1.06 | 2.06 | 1.64 | 1.79 | 1.58 | 1.12 | 1.17 | 0.92 | 0.88 | 1.73 | 0.89 | 1.44 | 1.25 | 5.55 | 4.42 | 4.82 | 4.25 | 0.71 | 0.70 | 0.74 | 0.80 | 94.45 | 95.58 | 95.18 | 95.75 |
| 1989 | 33 | 38.22 | 0.07 | 0.89 | 1.22 | 0.94 | 0.97 | 1.23 | 1.52 | 1.28 | 1.24 | 0.45 | 1.58 | 0.92 | 0.99 | 1.15 | 0.84 | 1.26 | 1.25 | 3.23 | 3.97 | 3.35 | 3.23 | 0.94 | 0.87 | 0.94 | 0.94 | 96.77 | 96.03 | 96.65 | 96.77 |
| 1990 | 33 | 36.06 | 0.05 | 1.08 | 1.05 | 1.00 | 0.98 | 1.45 | 1.49 | 1.43 | 1.35 | 1.23 | 1.36 | 1.09 | 1.15 | 0.88 | 0.64 | 1.06 | 0.92 | 4.03 | 4.14 | 3.96 | 3.74 | 0.64 | 0.54 | 0.72 | 0.72 | 95.97 | 95.86 | 96.04 | 96.26 |
| 1991 | 34 | 37.14 | 0.04 | 0.82 | 0.76 | 0.64 | 0.71 | 1.02 | 1.05 | 0.82 | 0.94 | 0.42 | 1.01 | 0.61 | 0.72 | 0.93 | 0.53 | 0.75 | 0.75 | 2.74 | 2.83 | 2.19 | 2.54 | 0.86 | 0.76 | 0.91 | 0.86 | 97.26 | 97.17 | 97.81 | 97.46 |
| 1992 | 34 | 38.26 | 0.06 | 0.82 | 1.08 | 0.81 | 0.79 | 1.07 | 1.39 | 1.04 | 1.06 | 0.47 | 1.04 | 0.36 | 0.38 | 0.95 | 0.78 | 1.06 | 0.96 | 2.79 | 3.62 | 2.72 | 2.77 | 0.93 | 0.84 | 0.94 | 0.93 | 97.21 | 96.38 | 97.28 | 97.23 |
| 1993 | 34 | 36.39 | 0.04 | 0.65 | 0.75 | 0.62 | 0.62 | 0.81 | 0.95 | 0.79 | 0.79 | 0.32 | 0.96 | 0.61 | 0.66 | 0.74 | 0.49 | 0.63 | 0.61 | 2.24 | 2.60 | 2.17 | 2.17 | 0.91 | 0.80 | 0.90 | 0.89 | 97.76 | 97.40 | 97.83 | 97.83 |
| 1994 | 34 | 38.09 | 0.06 | 0.79 | 1.04 | 0.82 | 0.81 | 0.97 | 1.30 | 0.99 | 1.01 | 0.41 | 1.20 | 0.64 | 0.65 | 0.87 | 0.70 | 0.81 | 0.87 | 2.55 | 3.41 | 2.59 | 2.64 | 0.94 | 0.84 | 0.93 | 0.93 | 97.45 | 96.59 | 97.41 | 97.36 |
| 1995 | 33 | 38.72 | 0.06 | 0.76 | 1.15 | 0.81 | 0.84 | 1.08 | 1.43 | 1.06 | 1.16 | 0.43 | 1.22 | 0.67 | 0.67 | 0.99 | 0.78 | 0.97 | 1.08 | 2.78 | 3.69 | 2.75 | 2.99 | 0.94 | 0.83 | 0.93 | 0.92 | 97.22 | 96.31 | 97.25 | 97.01 |
| 1996 | 33 | 37.90 | 0.05 | 0.63 | 0.83 | 0.65 | 0.61 | 0.77 | 1.12 | 0.82 | 0.81 | 0.29 | 0.85 | 0.22 | 0.22 | 0.71 | 0.56 | 0.73 | 0.74 | 2.03 | 2.95 | 2.16 | 2.13 | 0.95 | 0.85 | 0.94 | 0.95 | 97.97 | 97.05 | 97.84 | 97.87 |
| 1997 | 33 | 37.09 | 0.04 | 0.62 | 0.79 | 0.64 | 0.64 | 0.76 | 1.01 | 0.78 | 0.80 | 0.39 | 0.76 | 0.32 | 0.37 | 0.66 | 0.52 | 0.64 | 0.66 | 2.06 | 2.73 | 2.11 | 2.15 | 0.92 | 0.79 | 0.91 | 0.91 | 97.94 | 97.27 | 97.89 | 97.85 |

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1998 | 33 | 37.79 | 0.04 | 0.65 | 0.83 | 0.64 | 0.63 | 0.90 | 1.19 | 0.90 | 0.86 | 0.45 | 1.07 | 0.65 | 0.68 | 0.76 | 0.59 | 0.74 | 0.75 | 2.37 | 3.14 | 2.38 | 2.28 | 0.90 | 0.73 | 0.89 | 0.91 | 97.63 | 96.86 | 97.62 | 97.72 |
| 1999 | 33 | 37.75 | 0.05 | 1.13 | 1.25 | 1.12 | 1.15 | 1.38 | 1.48 | 1.36 | 1.39 | 1.07 | 1.12 | 0.62 | 0.71 | 0.91 | 0.67 | 0.99 | 0.98 | 3.65 | 3.93 | 3.60 | 3.68 | 0.75 | 0.61 | 0.75 | 0.74 | 96.35 | 96.07 | 96.40 | 96.32 |
| 2000 | 34 | 37.15 | 0.03 | 0.73 | 0.71 | 0.68 | 0.72 | 0.96 | 0.92 | 0.88 | 0.95 | 0.63 | 0.48 | 0.49 | 0.46 | 0.69 | 0.65 | 1.02 | 1.04 | 2.58 | 2.49 | 2.37 | 2.56 | 0.73 | 0.68 | 0.78 | 0.75 | 97.42 | 97.51 | 97.63 | 97.44 |
| 2001 | 34 | 37.81 | 0.03 | 0.97 | 0.85 | 0.85 | 0.89 | 1.14 | 1.09 | 1.04 | 1.08 | 0.90 | 1.02 | 0.78 | 0.86 | 0.72 | 0.42 | 0.61 | 0.66 | 3.02 | 2.87 | 2.76 | 2.86 | 0.57 | 0.47 | 0.62 | 0.60 | 96.98 | 97.13 | 97.24 | 97.14 |
| 2002 | 33 | 37.11 | 0.05 | 0.62 | 0.93 | 0.63 | 0.76 | 0.81 | 1.28 | 0.88 | 1.05 | 0.38 | 1.05 | 0.13 | 0.24 | 0.72 | 0.48 | 0.83 | 0.90 | 2.20 | 3.45 | 2.38 | 2.83 | 0.95 | 0.79 | 0.93 | 0.90 | 97.80 | 96.55 | 97.62 | 97.17 |
| 2003 | 33 | 38.22 | 0.13 | 2.70 | 2.14 | 2.37 | 2.35 | 5.71 | 4.95 | 5.34 | 5.27 | 4.76 | 2.94 | 1.85 | 1.76 | 3.17 | 2.26 | 3.79 | 3.80 | 14.93 | 12.95 | 13.97 | 13.78 | 0.19 | 0.15 | 0.20 | 0.22 | 85.07 | 87.05 | 86.03 | 86.22 |
| 2004 | 34 | 38.79 | 0.11 | 3.10 | 2.11 | 2.40 | 2.35 | 6.46 | 4.42 | 5.00 | 5.06 | 4.47 | 2.52 | 1.74 | 1.90 | 4.71 | 2.28 | 3.66 | 3.61 | 16.66 | 11.40 | 12.88 | 13.03 | 0.20 | 0.25 | 0.27 | 0.26 | 83.34 | 88.60 | 87.12 | 86.97 |
| 2005 | 34 | 37.96 | 0.05 | 0.58 | 0.97 | 0.65 | 0.75 | 0.81 | 1.29 | 0.80 | 0.95 | 0.28 | 1.14 | 0.41 | 0.50 | 0.76 | 0.58 | 0.63 | 0.79 | 2.12 | 3.39 | 2.10 | 2.50 | 0.95 | 0.79 | 0.95 | 0.92 | 97.88 | 96.61 | 97.90 | 97.50 |
| 2006 | 34 | 37.97 | 0.04 | 0.80 | 0.83 | 0.71 | 0.75 | 1.10 | 1.07 | 0.95 | 0.98 | 0.71 | 0.89 | 0.47 | 0.58 | 0.85 | 0.45 | 0.72 | 0.68 | 2.90 | 2.82 | 2.51 | 2.59 | 0.75 | 0.63 | 0.79 | 0.77 | 97.10 | 97.18 | 97.49 | 97.41 |
| 2007 | 34 | 37.92 | 0.04 | 0.80 | 0.86 | 0.72 | 0.73 | 0.96 | 1.04 | 0.90 | 0.88 | 0.45 | 0.82 | 0.46 | 0.47 | 0.86 | 0.58 | 0.72 | 0.77 | 2.54 | 2.74 | 2.37 | 2.32 | 0.88 | 0.80 | 0.88 | 0.90 | 97.46 | 97.26 | 97.63 | 97.68 |

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