A reliability-based measure of robustness for concrete structures subjected to corrosion

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ABSTRACT: This work is a contribution to the definition and assessment of structural robustness. Special emphasis is given to reliability of reinforced concrete structures under corrosion of longitudinal reinforcement. On this communication several authors' proposals in order to define and measure structural robustness are analyzed and discussed. The probabilistic based robustness index is defined, considering the reliability index decreasing for all possible damage levels. Damage is considered as the corrosion level of the longitudinal reinforcement in terms of rebar weight loss. Damage produces changes in both cross-sectional area of rebar and bond strength. The proposed methodology is illustrated by means of an application example. In order to consider the impact of reinforcement corrosion on failure probability growth, an advanced methodology based on the strong discontinuities approach and an isotropic continuum damage model for concrete is adopted. The methodology consist on a two-step analysis: on the first step an analysis of the cross section is performed in order to capture phenomena such as expansion of the reinforcement due to the corrosion products accumulation and damage and cracking in the reinforcement surrounding concrete; on the second step a 2D deteriorated structural model is built with the results obtained on the first step of the analysis. The referred methodology combined with a Monte Carlo simulation is then used to compute the failure probability and the reliability index of the structure for different corrosion levels. Finally, structural robustness is assessed using the proposed probabilistic index.

1 INTRODUCTION

The deterioration and maintenance of existing structures are issues of increasing concern since massive costs are expected in a near future. This concern is particularly important in bridges reaching their design life time since their social and economic impact is huge. According to Costs (2002), the direct costs associated with bridge deterioration in the United States reach 8.3 billions of dollars annually, and it is expected that the indirect costs associated to users can be up to ten times higher.

One of the main reasons responsible for the deterioration is member corrosion and reinforcement corrosion in steel and reinforced concrete structures, respectively. In the last case structural repairing could be a harder task since the reinforcement is not easily accessible. Also in reinforced concrete (RC) structures the corrosion effect cannot be seen as simply as a reinforcement area reduction. In fact, corrosion mechanism leads to the development of several side effects (rust expansion, concrete cracking, bond strength decreasing, among others) responsible for the bridge deterioration acceleration. For this reason it is fundamental to correctly assess the reliability of an existing corroded RC structure in order to adequate a safety service level.

On the other hand, the structure reliability or the safety level decreasing due to damage occurrence is related to the robustness concept which has seen growing interest in the last decades as a result of the occurrence of tragic consequences (Eagar and Musso 2001, Pearson et al. 2003, NTSB 2008) due to extreme events such as terrorist attacks. However the robustness concept can also be very useful when applied to deterioration scenarios allowing for instance for a reinforcement concrete structure to evaluate the safety susceptibility to corrosion.

Therefore this paper intends to be a contribution to the robustness assessment of reinforced concrete structures subjected to corrosion.

2 BACKGROUND ON ROBUSTNESS

Robustness is an emergent concept related with structural damage tolerance. Despite being a desirable property the fact is that no consensus has been reached about its definition and the framework to assess it. In fact there are also some robustness related concepts, such as vulnerability, redundancy and ductility, among others, that are frequently misunderstood.

In the last two decades several authors appeared with different robustness approaches (Frangopol and Curley 1987, Lind 1995, Goshn and Moses 1998, Biondini and Restelli 2008, Baker et al. 2008, Starossek and Haberland 2008, Cavaco et al. 2010). Among the proposals, some are deterministic based and other are probabilistic based. Another point of interest is the fact that some authors support the idea that robustness is an intrinsic structural property depending on the structural ability to maintain an adequate performance level after damage occurrence. The works of Frangopol and Curley (1987), Lind (1995), Biondini and Restelli (2008), Starossek and Haberland (2008) and Cavaco et al. (2010) follow this perspective.

On the other hand, other authors (Goshn and Moses 1998 and Baker et al. 2008) prefer to consider robustness as a property of the structure and its environment. In this case robustness is related with the magnitude of the damage trigger event and the consequences extent. At the same time this robustness perspective is much broader since to compute the consequences of structural failure it is necessary to have in consideration all the social and economic environment aspects. Therefore, in this case robustness supersedes the structural engineer domain.

In Figure 1 the different proposals for the robustness concept and the framework to assess it are presented and organized accordingly to the perspective assumed by the respective author.

Robustness

Structural Property

Deterministic
- Frangopol and Curley (1987)
- Biondini and Restelli (2008)
- Starossek and Haberland (2008)
- Cavaco et al. (2010)

Probabilistic
- Frangopol and Curley (1987)
- Lind (1995)
- Cavaco et al. (present paper)

Structural and Environment Property

Probabilistic
- Goshn and Moses (1998)
- Baker et al. (2008)

Figure 1. Different perspectives for the robustness concept.

In this paper the proposal of Cavaco et al. (2010) was adopted. Accordingly, robustness can be defined as a measure of the degree of structural performance lost after damage occurrence. The structural performance can assume many forms, and can be related to service limit states or to ultimate limit states. Damage concept should also be considered with a broader sense, i.e., damage can vary from a simple degradation state to a more serious damage as a column or a beam failure. Errors during the design or the construction stage can also be seen as types of damages.

Associated to this definition the authors also propose a framework to assess robustness obtained through equation (1), which gives the area above the curve defined by the normalized structural performance f subjected to a normalized damage d (Figure 2).

$$R_d = \int_{d=0}^{d=1} f(x)dx \qquad (1)$$

where f is given by the ratio between the structural performances on the intact and damage states, and d is given by the ratio between actual and maximum possible damage.

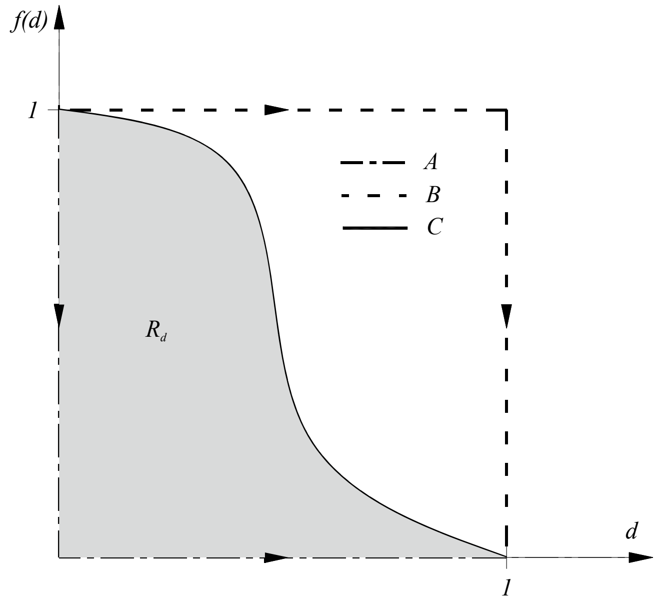


Figure 2. Robustness assessment. Normalized structural performance f as a function of the normalized damage d.

Robustness index R_d may vary from 0 to 1 respectively if a minimum damage level produces the entire loss of structural performance, curve A (Figure 2), or if the damage does not influence the structural performance, curve B. In Figure 2, curve C represent intermediate robustness approximately equal to *0.5*.

This approach can be either deterministic or probabilistic by simply considering a deterministic or a probabilistic measure of the degree of the structural performance lost. In this paper a probabilistic measure was adopted. Since the reliability index, β, is one of the most used parameters to assess the safety of existing structures, it was defined here as the structural performance indicator f used to assess robustness:

$$\beta = \Phi\left(1 - P(F)\right)^{-1} \qquad (2)$$

where Φ is the cumulative normal distribution function and P(F) is the failure probability. In this case robustness indicator R_d results on equation (3):

$$R_d = \int_{d=0}^{d=1} \frac{\beta(d=x)}{\beta(d=0)} dx \quad (3)$$

3 AN APPLICATION EXAMPLE

3.1 *Structure description*

The case study considered in this paper consists on a simply supported reinforced concrete beam subjected to both permanent, *g*, and live, *q*, uniform loads (Figure 3) and with a 5.0m x 2.0m influence area (being 2 m, the width of the loaded area). The live load considered results from people concentration in accordance to Faber and Vrouwenvelder (2000).

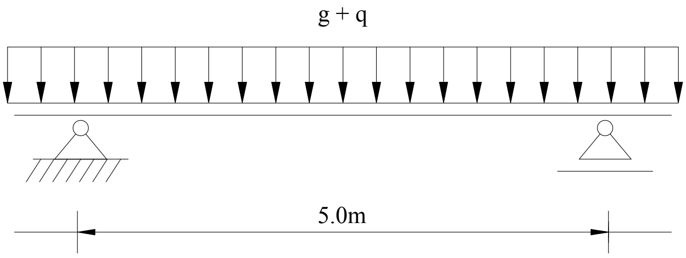


Figure 3. Simply supported RC beam under corrosion.

A simple 0.20m x 0.40m rectangular cross section (Figure 4) was designed in accordance with Eurocode prescriptions (CEN 2002) corresponding to the case of a pedestrian bridge. In order to assess robustness, defined as explained previously, the damage variable considered was the corrosion level X_p of the longitudinal reinforcement, measured in terms of weight percentage loss .

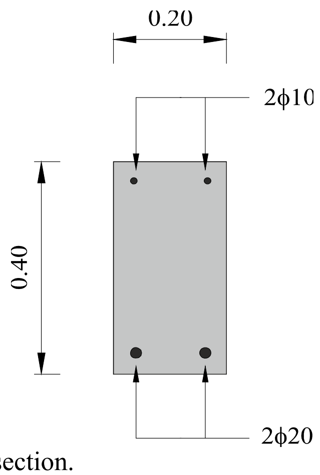


Figure 4. Cross section.

Transversal reinforcement was overdesigned and considered protected against corrosion in order to simplify the analysis.

As explained above, the reliability index, β (equation (2)), was the performance indicator, *f*, considered to assess robustness.

3.2 *Corrosion analysis methodology*

To perform a structural analysis, having in consideration the reinforcement corrosion effects on RC structures, a methodology proposed by Sánchez et al. (2008) was adopted. The main concept behind this methodology is to perform the structural analysis in two steps separately. In the first step the corrosion deteriorating effect is considered in the cross section. Then the corroded cross section properties are used to build a 2D longitudinal model of the deteriorated structure in order to assess its reliability index. The competence of the proposed method was demonstrated comparing numerical with experimental results (Sánchez et al. (2008)).

3.2.1 *Cross section Analysis*

In the first step of the referred methodology a cross section analysis is performed in order to capture the corrosion deteriorating effects. The most important effects are the expansion of corrosion products, the concrete deterioration and the concrete cracking and spalling. In order to consider these phenomena, corrosion was simulated as a steel bar expansion. For concrete an isotropic continuum damage model, ICDM, (Oliver et al. 1990) was adopted (Figure 5).

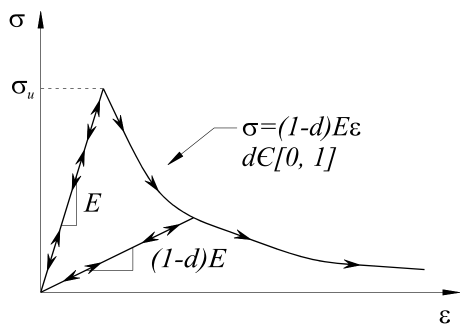
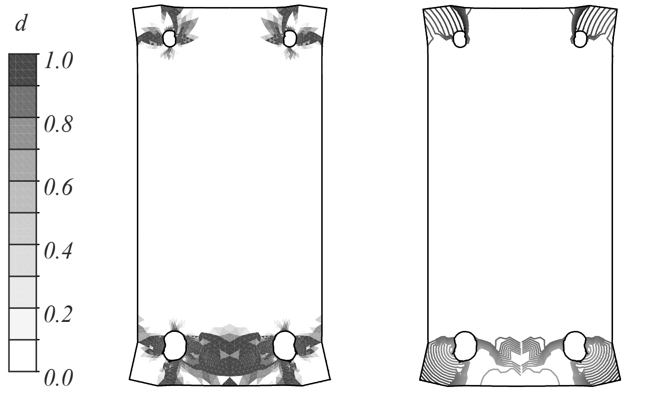


Figure 5. Isotropic Continuum damage model.

This type of constitutive relation coupled with the kinematics provided by the strong discontinuities approach, CSDA (Oliver et al. 2002), allows crack modeling on concrete. In fact, the degradation of concrete strength is the result of the initiation, growth and coalescence of micro cracks. Accordingly to the ICDM this process may be modeled by introducing an internal damage variable, *d*, which can be a scalar quantity and may vary from 0, for undamaged concrete, to 1, for full damaged concrete (Figure 5). When concrete loses all the strength, *d*=1, the kinematics provided by the CSDA characterize crack occurrence as a material discontinuity that can be understood as jump on the displacement field.

A full detailed description of ICDM and CSDA can be found, respectively, in Oliver et al. (1990) and Oliver et al. (2002).

The results obtained during the cross section analysis are presented in Figure 6. During the corrosion process a volumetric expansion of the steel bars occurs. This expansion produces concrete deterioration as can be observed in Figure 6 (a). The darker areas correspond to the most damage parts of concrete. Where damage variable *d*, reach values near the unity it means concrete lost almost all the strength and displacements will concentrate on these areas.



(a) *(b)*

Figure 6. Cross section analysis results. (a) Concrete damage; (b) isodisplacement lines

In Figure 6 (b), isodisplacement lines are presented. It is possible to observe the direct relation between Figures 6 (a) and (b). Cracks appear where isodisplacement lines tend to concentrate, i.e, in the same areas where concrete appear to be more deteriorated.

In Figure 6 (b) it is possible to observe that at the beam's top, small cracks appear producing concrete corner to detach. At the bottom, as reinforcement spacing is smaller and reinforcement radius is larger, a single crack appears connecting the two bars and leading to the delamination of the concrete cover.

The cross section analysis explained above was performed for corrosion levels X_p varying from 0% to 100%.

3.2.2 *Longitudinal analysis*

The longitudinal analysis was realized in order to assess the load carrying capacity used to calculate the reliability index. The longitudinal model of the impaired structure was built accordingly to the results obtained for the cross section during the first step of the analysis.

In order to achieve the compatibility between the cross section model and the 2D longitudinal model, it was necessary to project the concrete damage variable *d* from the first to the second model. Firstly, the cross section was divided into thin horizontal slices. After this, the average damage on concrete, for each slice, was calculated, considering the parts of disconnected concrete with damage *d* equal to 1. This means that these parts of concrete are no longer working together with the rest of the cross section. Finally a horizontal projection of concrete average damage *d* of each slice between both models was conducted and the 2D longitudinal model of the corroded structure built.

In the 2D longitudinal model, reinforced concrete was modeled as a composite material (Figure 7), as proposed by Oliver et al. (2008), constituted by a matrix, representing concrete, with embedded fibers representing reinforcement. The ICDM, together with the CSDA and upgraded with the initial damage obtained from the cross section analysis, was used in order to model de deteriorated matrix behavior.

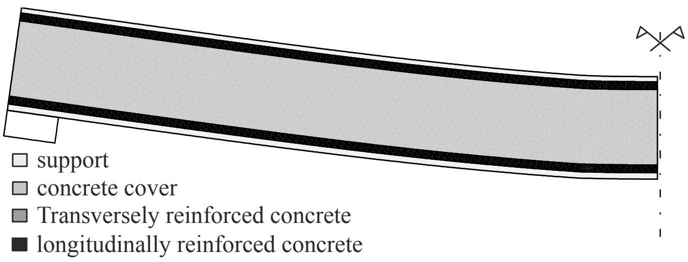


Figure 7. 2D Longitudinal model.

For the embedded fibers, the goal was to model both bond and reinforcement and its interaction since it plays a fundamental role on load carrying capacity deterioration. To consider the bond-slip effect, the slipping-fiber model proposed by Oliver et al. (2008) was adopted. This model mainly consists on the use of two components, representing reinforcement and its interface with concrete, in a serial system (Figure 8).

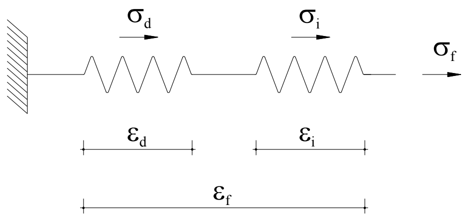


Figure 8. Slipping-fiber model.

The slipping-fiber total strain ε^f is given by the sum of the mechanical strain on the reinforcement, ε^d, and the deformation due to sliding, ε^i:

$$\varepsilon^f = \varepsilon^d + \varepsilon^i \tag{4}$$

For the serial system the stresses are identical in both components:

$$\sigma^f = \sigma^d + \sigma^i \tag{5}$$

Assuming, for each spring, an elastoplastic model, the constitutive relation of the serial system results also elastoplastic with the following characteristics:

$$\sigma_y^f = min(\sigma_y^d, \sigma_i^i)$$
$$E^f = \frac{1}{\frac{1}{E^d} + \frac{1}{E^i}} \quad (6)$$

where E^d and σ_y^d are the steel Young's modulus and yield stress, respectively, E^i is the interface elastic modulus and σ_y^i is the interface bond limit stress. Regard that when $E^i \to \infty$ and $\sigma_y^d < \sigma_y^i$, the serial system replicates perfect adhesion between concrete and reinforcement. Since there are no consistent knowledge about bond behavior, in the present work it was assumed perfect adhesion for the uncorroded states, i.e., $E^i \to \infty$ and, $\sigma_y^d = \sigma_y^i$.

For the corroded states a perfect rigid-plastic model was considered, with $E^i \to \infty$ and $\sigma_y^i < \sigma_y^d$. This means that it is not possible for reinforcement to yield because bond slips first. In order to characterize bond strength σ_y^d decreasing due to corrosion, the M-pull model proposed by Bhargava et al. (2004) was adopted. This model, summarized in equation (7) and in Figure 9, gives the normalized bond strength $\sigma_y^i(X_p)/\ \sigma_y^i(X_p=0)$ as a function of the corrosion level X_p and is based on the available experimental data.

$$\frac{\sigma_y^i(X_p)}{\sigma_y^i(X_p=0)} = \begin{cases} 1.0 & if \;\; X_p \le 1.5\% \\ 1.192 \cdot e^{-0.117 X_p} & if \;\; X_p > 1.5\% \end{cases} \quad (7)$$

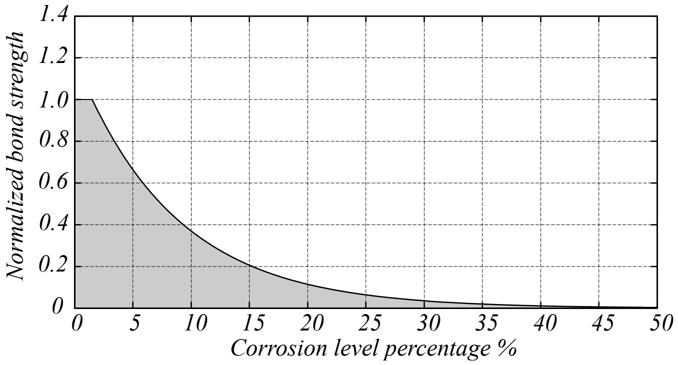


Figure 9. Bond strength deterioration as a function of the corrosion level.

Besides bond strength deterioration, reinforcement area reduction was also considered for the corroded states:

$$A_{eff} = A_0 \times (1 - X_p) \quad (8)$$

where A_{eff} and A_0 are respectively the effective and initial reinforcement areas.

In Figure 10, an example of the results obtained with the 2D longitudinal model is presented. Since the transversal reinforcement was overdesigned and considered not affected by corrosion, the failure mechanism consist on a mid-span plastic hinge formation. In Figure 10, damage variable on concrete is shown, and it is possible to observe the formation of two large cracks near mid-span.

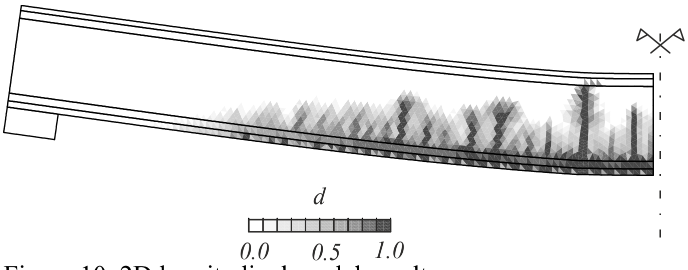


Figure 10. 2D longitudinal model results.

3.3 *Random variables*

Since a probabilistic index was chosen to be the structural performance indicator used to assess robustness, it was necessary to characterize in a probabilistic form the random variables that have influence in the reliability index assessment. Due to the high complexity involving the corrosion analysis methodology presented, it was necessary to choose only the most important parameters.

From the resistance viewpoint, concrete compressive strength and reinforcement yielding stress were characterized as random variables.

For concrete compressive strength it was assumed a lognormal distribution with a 38.5MPa mean and a 15% coefficient of variation.

For reinforcement the model proposed in the Probabilistic Model Code (Faber and Vrouwenvelder 2000) was adopted. Accordingly to this manual, for a good quality steel production, reinforcement strength can be considered normally distributed with mean yielding stress μ_1 given by:

$$\mu_1 = S_{nom} + 2\sigma_1 \quad (9)$$

where S_{nom} is the minimum specified yield stress limit, accordingly to the steel grade, and σ_1 is the overall standard deviation that can be taken equal to 30MPa. In this work the S400 steel grade was considered.

From the exposure perspective, the structure was considered subjected to self-weight, g, and live load, q. Accordingly to Faber and Vrouwenvelder (2000), reinforced concrete self-weight was considered normally distributed with a 25kN/m^3 mean and 0.75kN/m^3 standard deviation. The live load was considered as the result of people concentration. For this case, Faber and Vrouwenvelder (2000) recommend to consider null the sustained load and gamma distributed the intermittent load with 1.25kN/m^2 mean and 2.5kN/m^2 standard deviation.

In order to consider the uncertainties on both resistance and load models, Faber and Vrouwenvelder (2000) recommend accounting with two more random variables:

- a resistance model variable θ_R, lognormal distributed with 1.2 mean and 0.15 variation coefficient;
- a load effect model variable θ_E, lognormal distributed with 1.0 mean and 0.10 variation coefficient.

The distribution parameters of the six random variables considered are presented in Table 1.

Table 1. Random variables distribution and parameters

Random Variable	Dist.	mean	std. dev.
fc	logn	38.5MPa	5.8MPa
fy	norm	460MPa	30MPa
g	norm	$25kN/m^3$	$0.75kN/m^3$
q	gamma	$1.25kN/m^2$	$2.5kN/m^2$
θ_R	logn	1.2	0.15
θ_E	logn	1.0	0.10

3.4 *Reliability analysis*

The limit state function to consider is shown in equation (10)

$$G = S - R \qquad (10)$$

The failure probability can be assessed if both f_S and f_R which represent respectively the load effect S and resistance R probability density functions, are known (Figure 11).

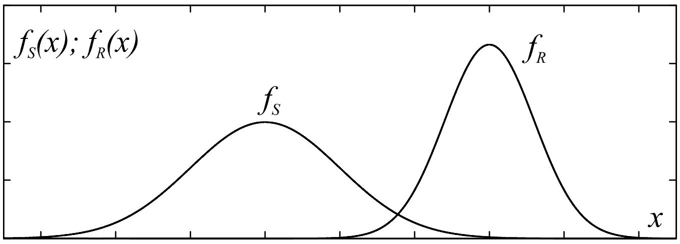


Figure 11. Load effect S and resistance R probability density functions.

In order to compute P_f, Monte Carlo simulation was used with $n=10^8$ samples. Accordingly with this simulation method P_f can be obtained through the equation (11)

$$P_f = \frac{n_f}{n} \qquad (11)$$

where n_f is the number of trials where failure occurs, i.e., $S>R$.

The generation of the load effect values was a simple task since S is a random variable with an explicit function given by the follow expression:

$$S = (g+q) \times \theta_E \times W_{\inf} \quad (kN/m) \qquad (12)$$

The influence width, W_{inf} in equation (12), was considered deterministic and equal to 2.0m.

On the other hand, it is not possible to have an explicit expression for the resistance R due to the complexity involving the presented corrosion analysis methodology. Consequently, it was necessary to achieve an adequate approximation for the resistance probability density function f_R. Latin hypercube sampling technique, accordingly to Olson et al. (2003), with 100 samples was used with the objective of reducing correlation between resistance variables f_c, f_y and θ_R.

The corrosion analysis methodology was performed for each sample element and for each corrosion level X_p varying from 0% to 100%. For each X_p value, a normal distribution was fitted to the resistance probability density function f_R. The maximum likelihood parameters estimation technique was used.

To evaluate the distribution fitting performed, a one hundred dimension sample from the fitted resistance probability density function f_R was generated. Both original and fitted samples were compared using a Kolmogorov-Smirnov hypothesis test (Massey 1951). The null hypothesis was that both samples are from the same continuous distribution. The alternative hypothesis was that they are from different continuous distributions. The result was negative if the test rejects the null hypothesis at the 5% significance level and positive otherwise. For every corrosion levels X_p, the null hypothesis was never rejected.

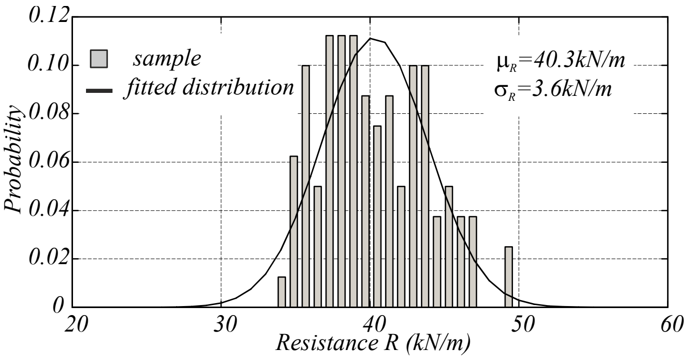


Figure 12. Fitting the resistance distribution for X_p=0%.

In Figure 12 the described resistance R fitting process performed for a 0% corrosion level is presented. The resistance R, measured in terms of beam's linear load carrying capacity, is normally fitted with mean $\mu_R=40.3kN/m$ and standard deviation $\sigma_R=3.6kN/m$.

4 RESULTS

In Figure 13 it is possible to observe the failure probability evolution from the uncorroded to the full corroded state. For a corrosion level X_p<1.55% the failure probability increases at a slow rate due specially to reinforcement area reduction. Regard that in accordance to Figure 13, there is no bond strength deterioration for this corrosion level. For X_p>1.5% the failure probability rapidly increases due to bond strength deterioration. For X_p>40% the failure probability curve slope tends to zero since reinforcement loses almost all the adhesion to concrete and the composite interaction between the two materials no longer exists. This means that from this stage forward, the beam resistance is provided by the concrete only.

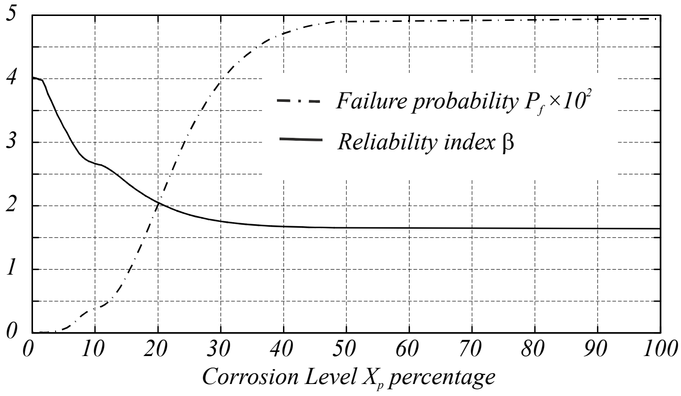


Figure 13. Failure probability P_f and reliability index β as function of the corrosion level X_p.

In Figure 13 the reliability index β is also shown. As expected for X_p<1.5% the reliability index has a very similar shape to bond strength deterioration curve, presented in Figure 9. Regard that the results shown in Figure 13 if combined with a model of prediction of corrosion depth with time, can be very useful in defining lifetime and safety of existing structures.

Robustness can be computed according to equation (1). As referred previously, the structural performance indicator chosen was the reliability index β and the damage considered is reinforcement corrosion depth X_p. Normalizing this parameters and computing the area bellow the normalized curve β^{norm} (X_p^{norm}) the robustness indicator R_d accordingly to equation (1) results in a value of 0.48 (Figure 14). This is an average percentage of the reliability index of the structure in the pristine state, considering that the structure was subjected to generalized corrosion depths varying from 0% to 100%. If for instance a plain concrete structure would be considered its reliability index wouldn't decrease with corrosion since reinforcements does not exist. In this case robustness indicator R_d would be equal to one reflecting a 100% corrosion damage tolerant structure.

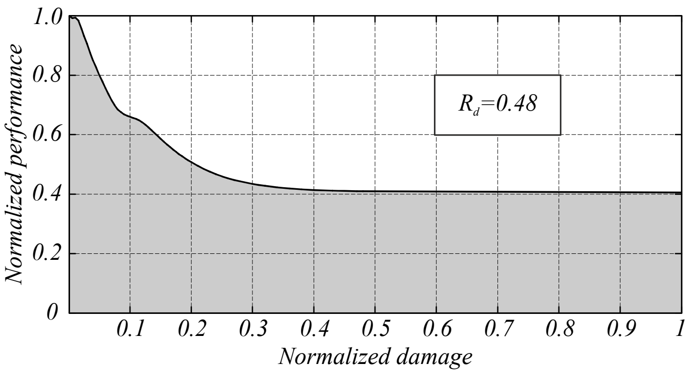


Figure 14. Robustness assessment

5 CONCLUSIONS

The robustness assessment framework presented in Cavaco et al. (2010) was used in this paper in order to characterize the behavior of a reinforced concrete beam subjected to corrosion. The structural performance indicator selected was the reliability index β since it is commonly used as a probabilistic safety measure. For damage quantification, corrosion level X_p measured in terms of weight percentage was considered.

Deteriorating corrosion effects were considered through the use of an advance methodology consisting on a two steps analysis. On the first step the beam cross section was subjected to corrosion exposure in order to consider the follow effects: expansion of corrosion products; concrete deterioration; and concrete cracking and spalling. With the results obtained in the first step of the analysis a 2D longitudinal model of the deteriorated beam was built in order to perform a reliability analysis. Effects such as reinforcement area reduction and bond strength deterioration were also considered.

Due to the complexity involving the corrosion analysis methodology the number of random variables considered was limited. It was also necessary to make an approximation to the resistance probability density function using the Kolmogorov-Smirnov hypothesis test with a population number of one hundred for each corrosion level using the latin hypercube sampling technique. The reliability analysis was performed using Monte Carlo simulation with a trial number $n=10^8$.

The results obtained give a precise evaluation of the failure probability and the reliability index with the corrosion level increase and can be very useful, if combined with a model of corrosion depth, in defining safety and lifetime of existing structures. Robustness assessment revealed, an average ratio of 0.48 between the reliability index of the corroded and uncorroded beam, when subjected to generalized corrosion values varying from 0% to 100%. This is a measure of the beam "robustness" to corrosion risk.

ACKNOWLEDGMENTS

The authors would like to acknowledge COST Action TU-0601 and Fundação para Ciência e Tecnologia, scholarship SFRH/BD/45799/2008, for help supporting this research.

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