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**Hybrid Systems Biology:  
Application to  
*Escherichia coli***

Dissertation presented to obtain a Master degree in  
Biotechnology

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Hybrid Systems Biology: Application to *Escherichia coli*

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## Abstract

In complex biological systems, it is unlikely that all relevant cellular functions can be fully described either by a mechanistic (parametric) or by a statistic (nonparametric) modelling approach. Quite often, hybrid semiparametric models are the most appropriate to handle such problems. Hybrid semiparametric systems make simultaneous use of the parametric and nonparametric systems analysis paradigms to solve complex problems. The main advantage of the semiparametric over the parametric or nonparametric frameworks lies in that it broadens the knowledge base that can be used to solve a particular problem, thus avoiding reductionism.

In this *M.Sc.* thesis, a hybrid modelling method was adopted to describe *in silico* *Escherichia coli* cells. The method consists in a modified projection to latent structures model that explores elementary flux modes (EFMs) as metabolic network principal components. It maximizes the covariance between measured fluxome and any input “omic” dataset. Additionally this method provides the ranking of EFMs in increasing order of explained flux variance and the identification of correlations between EFMs weighting factors and input variables.

When applied to a subset of *E. coli* transcriptome, metabolome, proteome and envirome (and combinations thereof) datasets from different *E. coli* strains (both wild-type and single gene knockout strains) the model is able, in general, to make accurate flux predictions. More particularly, the results show that envirome and the combination of envirome and transcriptome are the most appropriate datasets to make an accurate flux prediction (with 88.5% and 85.2% of explained flux variance in the validation partition, respectively). Furthermore, the correlations between EFMs weighting factors and input variables are consistent with previously described regulatory patterns, supporting the idea that the regulation of metabolic functions is conserved among distinct envirome and genotype variants, denoting a high level of modularity of cellular functions.

## Keywords

System Biology; Projection to latent structures; Hybrid methods; *Escherichia coli*; Elementary flux modes; Multiple omic analysis





## Resumo

Em sistemas biológicos complexos é não é plausível que todas as funções celulares relevantes possam ser completamente descritas quer por modelos mecanísticos (paramétricos) quer por modelos estatísticos (não paramétricos). Normalmente, os modelos híbridos semiparamétricos são a forma mais apropriada para lidar com estes problemas. Estes sistemas utilizam simultaneamente sistemas paramétricos e não paramétricos para resolver problemas complexos. A principal vantagem dos sistemas semiparamétricos perante os sistemas paramétricos ou não paramétricos reside na integração complementar de conhecimentos que podem ser utilizados para resolver dado problema, o que evita o reducionismo.

Nesta tese de mestrado foi utilizada modelação híbrida para descrever células de *Escherichia coli in silico*. O método utilizado é uma versão deste modelo projecção de variáveis latentes que utiliza modos dos fluxos elementares (EFMs) como componentes principais da rede metabólica. O principal objectivo do método é a maximização da covariância entre o fluxo medido e qualquer um dos conjuntos de dados ómicos. Adicionalmente o método ordena os EFMs por ordem crescente de variância dos fluxos explicada e identifica as correlações entre factores de ponderação dos EFMs e as variáveis de entrada.

Quando aplicado a uma parte do transcriptoma, metaboloma, proteoma e ambientoma (e algumas combinações destes) de várias estirpes de *E. coli* (selvagens e com um gene deletado) o modelo, de modo geral, permitiu prever os fluxos de forma precisa. Adicionalmente, os resultados mostram que o enviroma e a sua combinação com o transcriptoma são os melhores conjuntos de dados para prever os fluxos (88.5% e 85.2% de variância explicada dos fluxos na partição de validação, respectivamente). Simultaneamente, as correlações entre os factores de ponderação dos EFMs e as variáveis de entrada são consistentes com os padrões regulatórios descritos anteriormente, suportando a ideia que a regulação das funções metabólicas é conservada entre estirpes diferentes e entre diferentes condições de crescimento, demonstrando um elevado nível de modularidade das funções celulares.

## Palavras-chave

Biologia de Sistemas; Projecção de variáveis latentes; Métodos híbridos; *Escherichia coli*; Modos dos fluxos elementares; Analisa ómica múltipla;



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## List of abbreviations

2-KG	2-Keto-glutarate
3PG	3-Phosphoglycerate
$\Lambda$	Matrix of weights in PLP; equivalent to $\mathbf{U}$ in PLS
$\lambda$	Vectors of weights for condition $d$
$\lambda$	Scalar weight for a $k$ EFM and a particular omic component
$a$	Lower metabolic rate limit for reaction $i$ in Figure 1.1
AcCoA	Acetyl-CoA
$b$	Upper metabolic rate limit for reaction $i$ in Figure 1.1
$\mathbf{B}$	Matrix of regression coefficients in PLS; equivalent to $\mathbf{RC}$ in PLP
$\mathbf{b}$	Vector of regression coefficient for the inner linear model in PLS, equivalent to $\mathbf{rc}$ in PLP
CI	Confidence interval
CO <sub>2</sub>	Carbon dioxide
$d$	Index to identify different conditions that can vary from 1 to $n_p$
E4P	Erythrose 4-phosphate
$\mathbf{E}$	Matrix of residuals for $\mathbf{X}$ , $\mathbf{U}$ or $\mathbf{Y}$ , depending on the index indication
EFM	Elementary flux mode
$\mathbf{EM}$	Matrix of EFMs and matrix of loading for the output matrix $\mathbf{R}$ in PLP, equivalent to $\mathbf{Q}$ in PLP
$\mathbf{em}$	Vector of a EFM $k$ and vector of loading for the output matrix $\mathbf{R}$ in PLP, equivalent to $\mathbf{q}$ in PLP
$eps$	Value to be used as convergence criterion
F6P	Fructose 6-phosphate
$fac$	Total number of principal components
FBA	Flux balance analysis
$\mathbf{FM}$	Functional omics matrix
FUM	Fumarate
G3P	Glyceraldehyde 3-phosphate
G6P	Glucose 6-Phosphate
$h$	Index to identify omic components that can vary from 1 to $n_x$
$i$	Index for a metabolic reaction in Figure 1.1
$j$	Index to identify metabolic reactions that can vary from 1 to $n_r$
$k$	Index to identify EFMs that can vary from 1 to $n_{em}$
KO	Single gene knockout

$k_{opt}$	Index of best performing EFM
$l$	Index of irreversible reactions
$lv$	Index number for principal components
$m$	Number of metabolites
MLR	Multivariate linear regression
mRNA	Messenger ribonucleic acid
$n_{em}$	Number of EFMs
NIPALS	Non-iterative partial least squares
$n_p$	Number of conditions
$n_r$	Number of reactions
$n_x$	Number of omic observations
OAA	Oxaloacetate
<b>p</b>	Vector of loading for the output matrix <b>X</b> in PLS
PEP	Phosphoenolpyruvate
PLP	Projection to latent pathways
PLS	Projection to latent structures
$p_{new}$	Scaled loading vector for the output matrix <b>X</b> in PLS
$p_{value}$	Statistical significance test $p_{value}$
PYR	Pyruvate
<b>Q</b>	Matrix of loading for the output matrix <b>Y</b> in PLS, equivalent to <b>EM</b> in PLP
<b>q</b>	Vector of loading for the output matrix <b>Y</b> in PLS, equivalent to <b>em</b> in PLP
qtPCR	Quantitative polymerase chain reaction
<b><math>\hat{\mathbf{R}}</math></b>	Matrix of predicted reaction rates in PLP, equivalent to <b><math>\hat{\mathbf{Y}}</math></b> in PLS
<b><math>\hat{\mathbf{r}}</math></b>	Vector of predicted reaction rates
$r^2$	Regression coefficient
R5P	Ribose 5-phosphate
<b>R</b>	Matrix of observed reaction rates, output of PLP, equivalent to <b>Y</b> in PLS
<b>r</b>	Vector of observed reaction rates, output of PLP
<b>RC</b>	Matrix of regression coefficients in PLS; equivalent to <b>B</b> in PLS
<b>rc</b>	Vector of regression coefficients in PLS; equivalent to <b>b</b> in PLS
<b>S</b>	Stoichiometric matrix
$s$	Index for a nonzero row of <b>Y</b> in PLS
<b>T</b>	Matrix of scores for the input matrix <b>X</b> for the current iteration in PLS and PLP
<b>t</b>	Vector of scores for the input matrix <b>X</b> for the current iteration in PLS and PLP
<b>t<sub>old</sub></b>	Vector of scores for the input matrix <b>X</b> for the previous iteration in PLS and PLP

<b>U</b>	Matrix of scores for the output matrix <b>Y</b> in PLS; equivalent to $\Lambda$ in PLP
<b>u</b>	Score vector for the output matrix <b>Y</b> in PLS
$var(\lambda_{\text{EFM}})$	Variance for the metabolic fluxes for the selected EFM
$var(\mathbf{r}_{\text{obs}})$	Variance for the observed metabolic fluxes
<i>var</i>	Variance
<i>v</i>	Metabolic rate for reaction <i>i</i>
<b>W</b>	Matrix of weight for the input matrix <b>X</b> in PLS and PLP
<b>w</b>	Vector of weight for the input matrix <b>X</b> in PLS and PLP
WT	Wild-type
<b>X</b>	Input (predictor) matrix in PLS and PLP constituted by independent omic observations
<b>x</b>	Input (predictor) vector in PLS and PLP for a condition <i>t</i>
$\hat{\mathbf{Y}}$	Output response matrix in PLS, constituted by independent reaction rates predictions in PLP, equivalent to $\hat{\mathbf{R}}$
<b>Y</b>	Output response matrix in PLS, constituted by independent reaction rates observations in PLP, equivalent to <b>R</b>
<b>Z</b>	Objective function in Figure 1.1



# **1 Introduction**



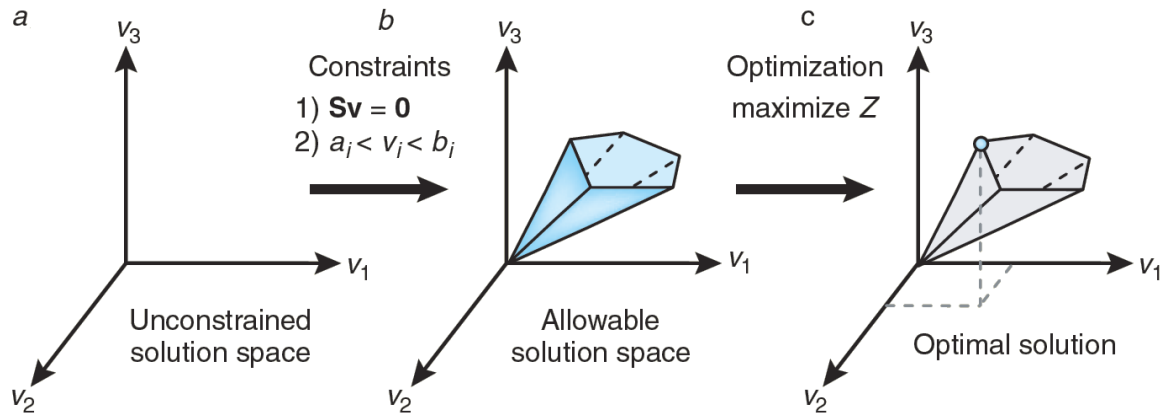


## 1. Introduction

The foundations of Systems Biology can be traced back to the writings of Aristotle around 350 B.C. [1]. At that time, the word “system” had a similar meaning as nowadays. It was connoted with an entity that represents wholeness, which can be divided or fractioned into components but whose core properties cannot be fully explained from the knowledge of such parts alone [2]. Since then scientific paradigms changed dramatically. However the fundamental concepts of Systems Biology are apparently resistant to these changes and remain pretty much the same today as they were at that time [2]. Recently, the fast-growing applications of genomics and high-throughput technologies brought to light the limitations of the reductionist view of the world. So the Systems Biology paradigm became essential to further understand the biological functions since this information cannot be obtained by studying the individual constituents on a part-by-part basis. Moreover, the contemporary development of this area also took great advantages of its similarity with the more developed area of system level engineering [3].

Particularly, *Escherichia coli* has become a model system in biology and therefore many studies have addressed the standardization of the available information [4, 5] as well as the development of *in silico* models based on this data [6]. At the heart of Systems Biology is precisely the development of large-scale mathematical models that merge together the different layers of information embodied in large datasets of different aspects of the cell. To model such large, redundant and complex systems, constraints-based modelling approaches have been extensively applied [6-9]. Such methodologies lay on the premise that the cell cannot achieve every possible combination of metabolic fluxes. Consequently, several techniques have been developed in order to get a feasible combination of fluxes that meet the imposed restrictions (Figure 1.1a and Figure 1.1b). Some commonly used constraints are stoichiometric, thermodynamic and regulatory. The more important methods are metabolic flux analysis, flux balance analysis (FBA) and network-based pathway analysis through elementary flux modes (EFMs) or extreme pathways.

In metabolic engineering the achievement of higher yields as fast as possible is critical. Thus, an *in silico* screening of possible genetic modifications and a medium design before the experimental phase is usually made. It is imperative to understand the biological system response to such genetic adjustments (across the different omic layers of biological information: transcriptome, metabolome, proteome and fluxome) and to identify a set of promising targets to improve strain performance [10]. It is also important to identify the influence of each environmental factor in the biological reaction. These two important challenges, *in silico* prediction of genetically perturbed systems and environmental and omic metabolic influence interpretation are the main objectives of the present thesis. So, in the lines below it is reviewed the most important modelling methods in Systems Biology with particular emphasis in mechanistic (metabolic reaction- and function-oriented), statistical and hybrid models.



**Figure 1.1:** The theoretical basis of constraint-based modelling and FBA. **a** – Without constraints, the hypothetical flux distribution of a biological network can be set anywhere in the solution space (any combination of metabolic fluxes). **b** – When mass balance constraints are imposed by the stoichiometric matrix  $S$  (in 1) and capacity constraints (flux  $v_i$  lower and upper bounds  $a_i$  and  $b_i$ , respectively – 2) are applied, it is defined a plausible solution space. **c** – Through optimization of an objective function  $Z$ , FBA identifies a single flux distribution that lies on the edge of the allowable solution space [11].

## 1.1 Mechanistic models of bionetwork systems

### 1.1.1 Metabolic reaction-oriented network models

Systems Biology basis lies on all types of biological networks. The network concept is *per se* very similar to the systems sciences wholeness idea. Such networks represent all the different ways in which the different systems components can interact between them in order to generate a global physical trait. In biology such networks can represent the relations concerning proteins (for instance in the signal transduction pathways) or genes (like the gene regulatory networks). These already complex networks also cooperate between them to produce the cellular phenotype. So, the main goal of Systems Biology is to understand all these relationships and integrate them in a single model. However, this task has been challenging and most of the models developed so far focused on the metabolism because it sums up all the contributions of the other layers of omic information. For instance, the signal transduction pathways are activated by the extracellular state and have an effect in the transcriptional activity as well as in the fluxome itself. On the other hand, the interaction between proteins and metabolites also controls the fluxome as some metabolites regulate the enzymatic capacities through allosteric regulation. For this reason, in the following lines will be reviewed the main metabolic modelling methods.

As described above, most of the computational methods applied in metabolic modelling use a constraint based models, from which FBA is the most used. FBA seeks for the particular flux distribution that maximizes or minimizes a pre-set objective function using linear programming methods [11] – Figure 1.1c. In order to predict the phenotype or metabolic flux

distribution of knockout (KO) strains, modifications to FBA have been proposed that, for example, include regulatory constraints [12, 13] or gene expression data [14]. More complex FBA strategies were employed with multiple objective functions including the minimization of some feature between the wild-type (WT) and the mutant strain, like minimization of metabolic adjustment [15] or regulatory on-off minimization [16]. The main difference between these two methods lies on the type of feature that these algorithms minimize, which in the former case is the Euclidean norm of the flux differences between the metabolic states [15], while the latter method minimizes the number of significant reaction flux changes, regardless of its value [16].

Benyamini *et al.* presented a FBA based method that aims to predict a feasible flux distribution under a given environmental and genetic condition. The culture medium conditions were introduced in the method in the form of growth-associated dilution of all produced intermediate metabolites. To solve the additional constraints the authors used a mixed-integer linear programming approach [17]. Additional studies were made in order to apply FBA to more objective problems. They address the engineering of an economically interesting metabolite overproducing strain [18-20].

However, when the objective function is not well posed the solution obtained by FBA is also inconsistent and inaccurate [21]. Therefore, it is very important to invest a lot of time and effort in the construction of a highly reliable metabolic network as well as in the definition of a consistent set of constraints for the optimization procedure and for the additional restrictions imposed.

### 1.1.2 Function oriented models

In alternative to FBA, pathway-oriented genetic engineering methods based on the calculation of EFMs have been developed. An EFM is a minimal set of enzymes that can operate in steady state [22]. The large number of possible combinations of metabolic reactions that can operate under these conditions makes the number of EFMs very high. Consequently, such methodology is robust but when the metabolic network grows it becomes difficult to calculate them. Recently, it has been described some methods that simplify this calculations, namely the metatool algorithm implemented as toolbox for MATLAB [23].

Stelling *et al.* [24] proposed an algorithm for the prediction of gene expression patterns on different substrates (control effective fluxes). Such model assumes that the structure of the network plays an essential role in the gene expression rate and therefore does not need any experimental data to do these calculations. Later on, the EFM-based enzyme control flux method was proposed. It calculates the correlation between the relative enzyme activity profile and its associated flux distribution. This method, together with a modified control effective fluxes model, enabled the metabolic flux distribution prediction of genetically modified microorganisms [25].

Wilhelm *et al.* [26] developed three measures of network robustness based on EFMs,

## Hybrid Systems Biology: Application to *Escherichia coli*

namely: the arithmetic mean of all ratios between the number of remaining EFMs after a KO and before it, the minimal robustness concerning an essential product (minimal number of EFMs for the production of a particular product after a KO) and the arithmetic mean of the particular product robustness values. The authors applied them to the central carbon metabolism of *E. coli* and human erythrocyte. They concluded that the bacterium was much more robust, denoting the environmental variations that can occur in that case and the much more stable growth conditions of the human erythrocytes.

Afterwards this group used a similar approach to analyse the *E. coli* and human hepatocytes metabolic networks and the effect of multiple gene KO in these systems [27]. From this study the authors also concluded that *E. coli* metabolic network is more robust than human hepatocytes.

Additional EFMs based models were applied to the improvement of some economically interesting products like heterologous protein production [28] and L-methionine [29].

### 1.2 Statistical models

As described above, statistical models represent a completely different approach to the modelling of biological networks. In opposition to the parametric models, statistical models do not use mechanistic knowledge either it takes the biochemical, thermodynamic or regulatory form. Here, the most used statistical models are briefly reviewed.

Multivariate linear regression (MLR) is one of the most used methods in top-down systems biology. One example of application is the theoretical product yield maximization proposed by Van Dien *et al.* [30]. The authors used MLR for the identification of metabolic reactions whose fluxes could be redirected in order to maximize the production of amino acids (arginine and tryptophan). They used the feasible solution space of the *E. coli* stoichiometric network to support *in silico* engineering of strains that overexpress target heterologous genes. The results showed that the increase of the glyoxylate cycle and PEP carboxylase activity as well as the elimination of malic enzyme promote the production of these amino acids [30].

A partial least squares regression, a particular form of MLR, was also used in the prediction of NADPH intracellular concentration using metabolites and protein concentrations as predictor of different *Aspergillus niger* strains (which KO genes were selected based on previous published *E. coli* and *Ralstonia eutropha* data). The motivation of this study was to overcome the rate limitation of some metabolic reactions due to the low NADPH concentrations. As conclusions, the authors identified target genes to be overexpressed in order to increase both the respective coded protein and the NADPH concentrations [31].

Another study addressed the capacity of four regression models (multiple linear regression, principal component regression, partial least-squares regression and regression using

artificial neural networks) in the estimation of physiological parameters (nine fluxes from mammalian gluconeogenesis pathway). After a calibration procedure with isotope labelling patterns from key metabolites, the model created 29 variables. In the end, the artificial neural networks showed better results (95% of captured information) than the remaining linear regression procedures (less than 75% of captured information) [32].

### 1.3 Hybrid semiparametric models

Parametric mathematical systems are expressed in the form of a functional relationship with a fixed number of parameters. Mechanistic models and phenomenological models belong to the class of parametric models and are inspired on the *a priori* knowledge of the system. On the contrary, non-parametric models do not have a fixed structure nor a fixed number of parameters. The final model structure and parameters number and values are set exclusively by the experimental data and are part of the model fitting procedure without any incorporation of a priori knowledge. Examples of nonparametric modelling methods are artificial neural networks, projection to latent structures, splines and many others. A class of hybrid models (or hybrid semiparametric systems) make simultaneous use of the parametric and nonparametric systems analysis paradigms to solve complex problems [33]. The need to use cooperatively parametric and nonparametric methods to describe a given system arises when *a priori* knowledge is not sufficient to describe the system in a mechanistic way nor the experimental data is sufficient to develop a predictive model using statistical modelling methods alone. Indeed, it is unlikely that all cellular functions could be fully described either by a mechanistic (parametric) or by a statistic (nonparametric) approach, since, in biology, as initially described, the whole complex system is greater than the “sum” of its parts [34]. Opting for the one or the other framework will invariably promote reductionism. Consequently the main advantage in the use of semiparametric models over the others frameworks lies in that it broadens the knowledge base that can be used to solve a particular problem, in other words, semiparametric models are an inclusive approach that tries to merge all available knowledge in the model [35].

This hybrid semiparametric modelling approach is especially applicable to complex systems, namely the ones addressed in Systems Biology. Generally, biological databases, usually used as source of information in Systems Biology, are constituted by large and redundant datasets of biological parts, such as the genome, transcriptome, proteome and metabolome. Some of them do not have a direct mechanistic interpretation or have such information but with some uncertainty [36]. In such cases the hybrid approach considers that *a priori* mechanistic knowledge is not the only relevant source of information but also other sources, like heuristics or information hidden in databases, are considered valuable complementary resources for model development [37].

## Hybrid Systems Biology: Application to *Escherichia coli*

The initial application of hybrid semiparametric models in bioprocess engineering occurred in the early 90s [38, 39]. In those, and subsequent, studies the authors tried to merge mechanistic or first principles models with nonparametric approaches like artificial neural networks [40, 41], mixture of experts [42] and linear and nonlinear projection of latent structures [43] in the modelling optimization and control of bioprocess.

Such hybrid models were also used in a Michaelis–Menten kinetics model and in a lin-log kinetics method. These procedures were applied to a complex large-scale network where the exact rate laws were unknown as well as some model parameters, which were calculated [44]. Others alternative studies used a hybrid mass-action rate laws that incorporated proteomic data and an aggregated rate law model for the extraction of elementary rate constants from experiment-based aggregated rate law. These techniques were used in the estimation of rate constants of a model of *E. coli* glycolytic pathways [45].

Kappal *et al.* studied the effects of extracellular stresses on the metabolic responses, in which the former was modelled with ordinary differential equations and the latter with algebraic equations composing a hybrid system [46].

Another hybrid modelling approach was proposed by Bulik *et al.* in which only the central regulatory enzymes were described in detail with mechanistic rate equations, and the majority of enzymes reactions were approximated by simplified rate equations. This model was proposed to speed up the development of reliable kinetic models for complex metabolic networks, like erythrocytes [47].

### 1.4 Objectives

Only a very limited number of studies have applied hybrid modelling methods in Systems Biology. Thus, this thesis main objective is to illustrate the applicability of hybrid modelling methodologies for *in silico* cellular modelling. In particular, the aim is to develop hybrid *in silico* *E. coli* models. The choice of this bacterium is motivated by the wealth of knowledge, data and models currently available, which facilitates benchmarking with other modelling methods. Briefly, the implemented modelling method is a constraint version of projection to latent structures (PLS). It has EFMs as additional constraints and some additional outputs (like the list of active EFMs and a regression coefficient that link the metabolic functions to inputs variables). After the enunciation of these general objectives, more specific ones can be enumerated:

- Development of several hybrid models for different *E. coli* strains (single gene KO and WT) based on envirome, metabolome or transcriptome datasets (individually and combined) to predict the flux-phenotype of this bacterium.
- Evaluate which is the best dataset to predict metabolic fluxes (envirome, metabolome or transcriptome datasets (individually and combined)).

## 1. Introduction

- Interpret the regression coefficients between the selected EFMs and the different inputs datasets.
- Assessment of regression coefficients consistency by comparing them with previously described regulatory patterns and evaluation of metabolic function regulatory mechanism conservation within the same species (across different strains and different growth conditions).

In the end it is intended to illustrate the importance of hybrid modelling in Systems Biology. As mentioned earlier, understanding biological functions cannot be obtained by studying the individual constituents on a part-by-part basis. Hybrid modelling methods provide a cost-effective method to bridge the parts in order to formulate the whole system, without the need of knowing all the mechanistic details of the model.





## **2 Methods**



## 2.1 *E. coli* data

With the model development purpose it was used the multiple high throughputs *E. coli* data published by Ishii *et al.* [48]. This data comprises envirome, proteome, metabolome, transcriptome and fluxome information, collected for different environmental conditions (namely, WT strain grown at different dilutions rates:  $0.1\text{h}^{-1}$ ,  $0.4\text{h}^{-1}$ ,  $0.5\text{h}^{-1}$  and  $0.7\text{h}^{-1}$ ) and for 24 single gene deletion mutants. Specifically, the authors removed in each strain one of the following genes: *galM*, *glk*, *pgm*, *pgi*, *pfkA*, *pfkB*, *fbp*, *fbaB*, *gapC*, *gpmA*, *gpmB*, *pykA*, *pykF*, *ppsA*, *zwf*, *pgl*, *gnd*, *rpe*, *rpiA*, *rpiB*, *tktA*, *tktB*, *talA*, and *talB*, creating the single gene KO strains analysed in this thesis. Additional data was also acquired for the reference strain (a WT strain grown at  $0.2\text{h}^{-1}$ , the same dilution rate at which every KO strain was grown).

With this data it can be analysed the effect of the external (growth conditions) and internal (genetic modifications) perturbations on the dynamics of different omic layers, namely envirome, proteome, metabolome, transcriptome and fluxome. The external perturbation data analysis motivation is to study the effect of the growth rate in the different omic layers. On the other hand, the internal perturbation data was generated for almost all the viable single gene mutants that directly affect the *E. coli* central carbon metabolism considered in the metabolic network (described in *E. coli* metabolic network section on page 15). So, this analysis also has the same objective, *i.e.*, to understand the effect of such genetic modifications in the different biological system's information sets, as well as to understand which perturbation (internal or external) have the highest effect in the system. It should also be underlined that this data was obtained through various chemostat cultures. Samples for omic analysis were taken at the same time after five complete medium volumes changes; such experimental design is consistent with the pseudo-steady state hypothesis.

In what refers to the envirome data, it was obtained for some organic compounds present in the culture medium. Specifically, envirome (E) is composed by the glucose, ethanol, acetate, D- and L-lactate, succinate, pyruvate and formate concentration values. To these concentrations values it was added the dilution rate values, thus comprising a total of nine environmental factors.

$$E = \{\text{glucose, ethanol, acetate, D-lactate, L-lactate, succinate, pyruvate, formate, dilution rate}\} \quad (\text{Eq. 2.1})$$

Proteome data was obtained for the following proteins:

$$P = \{\text{GalM, Glk, Pgm, Pgi, PfkA, PfkB, Fbp, FbaA, TpiA, GapA, Pkg, GpmA, GpmG, Eno, PykA, PykF, PpsA, Zwf, Pgl, Gnd, Rpe, RpiA, RpiB, TktA, TktB, TalA, TalB, Edd, Eda, AceE, AceF, LpdA, PckA, Ppc, SfcA, GltA, AcnA, AcnB, IcdA, SucA, SucB, SucC, SucD, SdhA, SdhB, FrdA, FumA, FumB, FumC, Mdh, AceA, AceB, GlcB, Acs, PrpC, LdhA, LldD, PoxB, PflA, PflB, PflC, AdhE, Pta, AckA, Adk, PtsH, PtsI}\} \quad (\text{Eq. 2.2})$$

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These 67 proteins are proteins involved in the central carbon metabolism of *E. coli* (Figure 2.1). Moreover, the proteins functions can be consulted in several databases, like EcoCyc [4], as well as in the original paper [48].

Metabolome data was obtained for 579 metabolites. Since the list of metabolites is very large it is presented in Appendix A. Additional metabolite information may be accessed in the previously referred database [4].

The transcriptome data was obtained by two methods. With the first one, quantitative polymerase chain reaction (qtPCR), it was obtained the messengers ribonucleic acid (mRNAs) concentration values for the genes that were directly involved in the metabolic network and it was obtained for all the previously described WT and KO strains. Namely it was obtained for 85 genes:

$$T=\{galM, glk, pgm, pgi, pfkA, pfkB, fbp, fbaA, fbaB, tpiA, gapA, gapC\ 1\ and\ 2, pgk, gpmA, gpmB, eno, pykA, pykF, ppsA, zwf, pgl, gnd, rpe, rpiA, rpiB, tktA, tktB, talA, talB, eda, edd, talC, aceE, aceF, lpdA, pckA, ppc, maeB, sfcA, gltA, acnA, acnB, icdA, sucA, sucB, sucC, sucD, fdrA, sdhA, sdhB, sdhC, sdhD, frdA, frdB, frdC, frdD, fumA, fumB, fumC, mdh, aceA, aceB, glcB, acs, prpC, ldhA, dld, lldD, poxB, pflA, pflB, pflC, pflD, adhE, pta, ackA, adk, udhA, pntA, pntB, ptsH, ptsI, crr, ptsG\}$$

(Eq. 2.3)

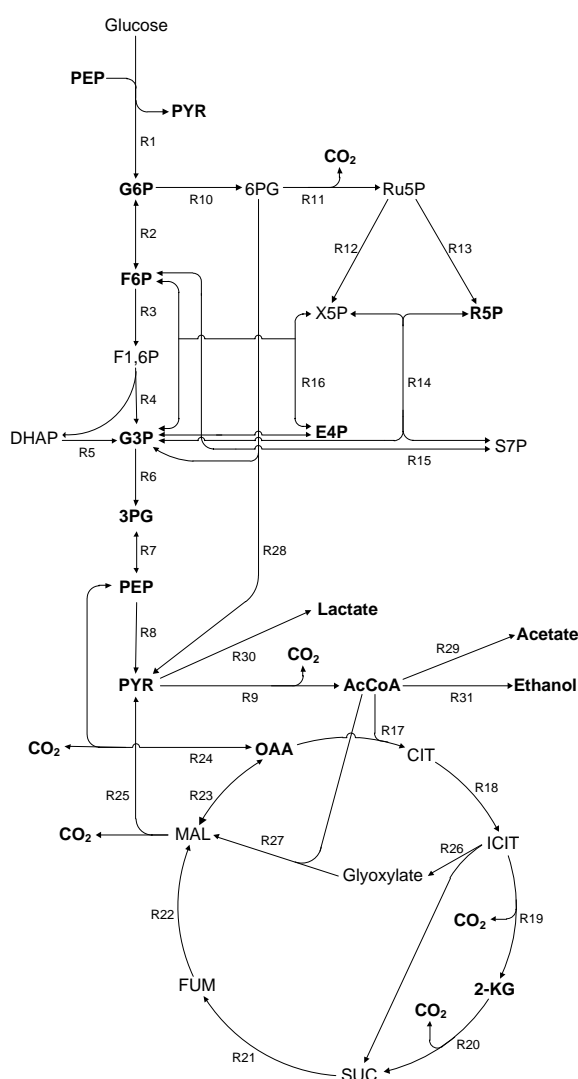
The microarrays technique was only used in the following cases: *pgm*, *pgi*, *gapC*, *zwf* and *rpe* single gene deletion mutants and WT strain grown at  $0.2h^{-1}$  (two reference strains),  $0.5h^{-1}$  and  $0.7h^{-1}$  dilution rate (full transcriptional data). From these full transcriptional datasets it was selected only the data from the regulatory genes (such selection were based in the information contained in the different biological databases [4]). So, for modelling purposes it was used the up- or down-expression level of 164 genes:

$$RT=\{accA, accB, accD, acrR, ada, adiY, agaR, aidB, alaS, alpA, appY, araC, arcA, argR, arsR, ascG, asnC, atoC, baeR, betI, bglG, bglJ, birA, bolA, cadC, caiF, cbl, cpxR, creB, crp, csgD, cspA, cynR, cysB, cytR, deoR, dicA, dnaA, dsdC, dsrA, ebgR, engA, envR, envY, exuR, fabR, fadR, feaR, fecI, fhlA, fis, flhC, flhD, fliA, fnr, fruR, fucR, fur, gadX, gadY, galR, galS, gatR, gcvA, gcvH, glcC, glnG, glpR, gntR, gutM, hcaR, hdfR, hipA, hipB, hns, hupA, hupB, hyfR, iclR, lacI, leuO, lexA, lldR, lrhA, lrp, lysR, mall, malT, marA, marR, mcbR, melR, metJ, metR, mhpR, micC, mode, mprA, mqsA, mqsR, mtlR, nac, nadR, nagC, narL, narP, nemR, nhaR, oxyR, pdhR, pepA, phoB, phoP, prpR, pspF, purR, putA, qseB, rbsR, rcsA, rcsB, relB, relE, rhaR, rhaS, rho, rne, rob, rpiR, rplA, rpoD, rpoR, rpoH, rpoN, rpoS, rpsB, rpsG, rstA, rtcR, selB, sgrR, slyA, soxR, soxS, srlR, stpA, tdcA, tdcR, torR, treR, trpR, ttdR, tyrR, uhpA, uidR, ulaR, uxuR, xapR, xylR, ycgE, yefM, yeiL, yiaJ, yqhC\}$$

(Eq. 2.4)

## 2.2 *E. coli* metabolic network

It was adopted the metabolic network specified by Ishii *et al.* [48]. The metabolic network has 43 reactions comprising the three main catabolic pathways (glycolysis, pentose phosphate and tricarboxylic acid cycle), 24 intracellular metabolites (12 of them can exit the system and participate in biosynthetic pathways or can exit the cell, the latter is the case of carbon dioxide; these 12 metabolites are marked with bold in Figure 2.1, and the detailed metabolic reaction are listed in Appendix B). Besides the biosynthetic metabolites indicated by the authors, it was also considered that lactate, ethanol and acetate can also exit the metabolic network. The criteria to choose these metabolites were the determined concentrations in the culture medium and the simultaneous metabolic production in only one reaction, with no consumption.



**Figure 2.1:** *E. coli* metabolic network composed by 43 reactions and 24 metabolites. Bold metabolites represent the ones that can exit the system; these reactions are not explicit shown in the scheme. Reactions from 32 to 43 represent the exit of G6P, F6P, R5P, E4P, G3P, 3PG, PEP, PYR, AcCoA, OAA, 2-KG and CO<sub>2</sub>, respectively.

### 2.3 *E. coli* elementary flux modes

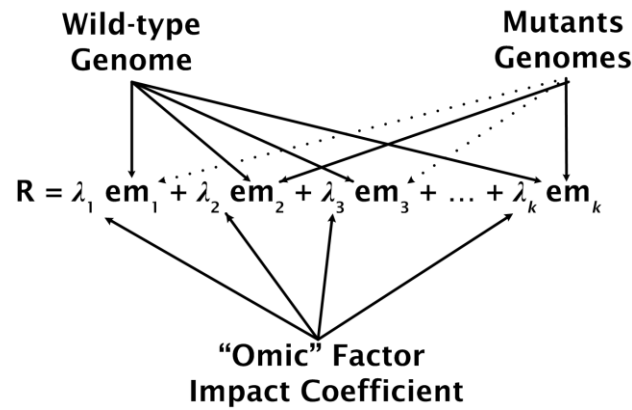
As said before, the metabolic network is one of the central pieces of Systems Biology. In this thesis it will be represented by EFMs, which becomes a central concept in the modelling method employed in the present thesis. Briefly, EFMs can be seen as a minimal set of enzymes that could operate at steady state [22], which are weighted by the relative flux they carry. EFMs represent also non-decomposable ways of linking extracellular metabolites (substrates to products). EFMs have been widely applied in Systems Biology, not only in the metabolic fluxes prediction but also as metabolic network redundancy and flexibility measures. Due to its properties, any biologically viable flux distribution can be set as a combination of EFMs and, additionally, EFMs can be used in the calculation and comparison of parallel routes of products production and substrates consumption [26].

With the described network, the EMFs were calculated using metatool 5.0 [23], resulting in a total of 275 EFMs for the complete network. They are given as supplementary data (Appendix C). Upon gene deletion the number of EFMs can be reduced (Table 7.2 in Appendix D). For instance, in *pgi* single gene deletion mutant, in which the metabolic reaction number two is unfeasible, the first plausible EFM is EFM 6, since the first five EFMs all involve the reaction two. The same EFM pre-selection procedure was applied for all EFMs and organisms. The implemented pre-selection rule will be described later on in the method detailed description (page 24).

Additionally the EFMs can be classified as energy producing EFMs, the EFMs that does not end in the production of metabolites that could exit the system and have biosynthetic functions, and biosynthetic EFMs, the EFMs that also involve the production of such metabolites. In this classification it was obtained 16 and 259, respectively. Furthermore, the classification according to the biosynthetic produced metabolite can also be set through the analysis of the possible metabolic pathways in which each metabolite can take part.

### 2.4 Statement of the modelling problem

In the models developed in this thesis, it is assumed that the genome of a given strain sets the structure of EFMs, while the relative weights of EFMs are a function of the cellular state translated by the information coded in all omic layers (Figure 2.2). Since in this thesis several KO mutants are studied, the analysed genome is variable. As was described by Klamt *et al.* [49], when a metabolic reaction is removed from a metabolic network all the EFMs in which this reaction was taking part became unfeasible, and all the others remain unaltered. In this way it was implemented a pre-selection rule in which the viable EFMs were selected for each case (as already said it will be presented later on – page 24).



**Figure 2.2:** Representation of the influence of the genome and the omic in the fluxome ( $\mathbf{R}$ ). The genome sets the metabolic capabilities of the cell (elementary flux mode –  $\mathbf{em}_k$ ). Each metabolic function is up- or down- regulated by all omic factors ( $\lambda_k$ ).

As such, in the model presented in this work, the overall EFMs structure is set by the genome of the mutant or WT strains. The relative weight of EFMs will be explored as functions of different types of information, namely: envirome, proteome, metabolome and transcriptome. One of the presented model assumptions is the close relationship between all of these biological layers and the metabolic fluxes, so it is appropriate to predict metabolic fluxes from the information contained in these datasets. Some of them have a very close association with fluxome. Such is the case of envirome. In this case the metabolite concentrations either are a result of the bacterium metabolism or they are used as substrates in the metabolic network. Moreover, the remaining abiotic conditions, like temperature, also affect the cellular metabolism.

On the other hand, the transcriptional dataset have a time-lapse between the actual mRNA production and an effect in the metabolic fluxes. These molecules can have a more direct effect in the fluxome, when the mRNA molecules code for metabolic enzymes, or can have an indirect effect when they have regulatory functions. In the latter case, the time-lapse might be even larger since the regulatory functions will have an influence over other genes or proteins and only after this event has taken place will the regulatory transcriptome have an effect in the fluxome.

Moreover, perhaps the most intuitive fluxome-omic dataset relationship occurs between proteome and the metabolic fluxes. Since in this case the proteome data is composed by catalytic proteins, the influence is clear: without such proteins the metabolic reaction would not occur and when some protein concentration increases, the correspondent flux also increases. However, this latter rule is not always true since it is dependent of others factors like, for instance, the metabolite concentration, which can influence the protein activity, thus also affects the fluxome.

As shown below, the resulting model can be classified as a hybrid model since the EFMs are hard mechanistic constraints that can be assumed with high levels of confidence, and the nature of the relationship between omic factors and the EFMs weighting factors is ‘statistic’ since it merges together many different mechanisms that are difficult to validate experimentally.

## 2.5 Hybrid modelling method

The modelling strategy used in the present thesis is called projection to latent pathways (PLP) and was developed by Teixeira *et al.* [50] and described in detail by Ferreira *et al.* [51]. It is a hybrid modelling method because it combines the EFMs modelling strategy (parametric) and the PLS model (nonparametric) previously mentioned in the introduction. In the lines below it will be described in detail PLP algorithm and its application to *E. coli* cells.

### 2.5.1 Statement of the mathematical problem

Applying the typical steady-state material balance equations enounced in the Introduction section (Figure 1.1) to a metabolic network with  $m$  metabolites and  $n_r$  metabolic reactions, the following system of linear algebraic equations is obtained:

$$\mathbf{S} \cdot \mathbf{r} = 0 \quad (\text{Eq. 2.5})$$

$$\mathbf{r}_l > 0 \quad (\text{Eq. 2.6})$$

In this case  $\mathbf{r}$  is a vector of  $n_r$  metabolic fluxes and  $\mathbf{r}_l$  is the subset of fluxes associated to irreversible reactions  $l$ .  $\mathbf{S}$  is a  $m \times n_r$  stoichiometric matrix. The null space solution of this system (Eq. 2.5 and Eq. 2.6) takes the form of a polyhedral cone [52]. Furthermore, the convex basis of system (Eq. 2.5 and Eq. 2.6) is formed by a large number of base vectors (Figure 1.1), which are the EFMs.

$$\mathbf{r} = \sum_{k=1}^{n_{em}} \lambda_k \cdot \mathbf{em}_k \quad (\text{Eq. 2.7})$$

EFMs may represent the overall flux phenotype,  $\mathbf{r}$ , of a cell. In this representation (Eq. 2.7), each  $k$  EFM is represented by  $\mathbf{em}_k$ , a  $n_r \times 1$  vector of reaction weight factors, in which  $k$  can vary from 1 to  $n_{em}$ , with  $n_{em}$  the maximum number of EFMs. Each EFM have a correspondent weighting factor,  $\lambda_k$ , a scalar variable, that defines the partial contribution of each  $\mathbf{em}_k$  to  $\mathbf{r}$ . However, not all the EFMs are active in each particular condition, *i.e.*, many times the weighting factors of a large number of EFMs are close to zero. One of the usual features of the EFMs based methods is the determination of the active EFMs, the ones that have higher weighting factors.

The PLP method also makes this selection on the basis of different omic datasets. The basic premise is that measured fluxome,  $\mathbf{r}$ , can be systematically deconvoluted into genetic dependent factors (the structure of EFMs,  $\mathbf{em}_k$ ) and omic dependent factors (the partial contribution of each EFM to flux phenotype,  $\lambda_k$ ). To implement this method, Teixeira *et al.* [50] developed a discrimination algorithm that works according to the following criteria:

1. Maximisation of explained variance of flux datasets,  $\mathbf{R} = \{\mathbf{r}(d)\}$
2. Maximization of correlation of  $\lambda_k$  against omic data,  $\mathbf{X} = \{\mathbf{x}(d)\}$
3. Minimization of the number of active EFMs

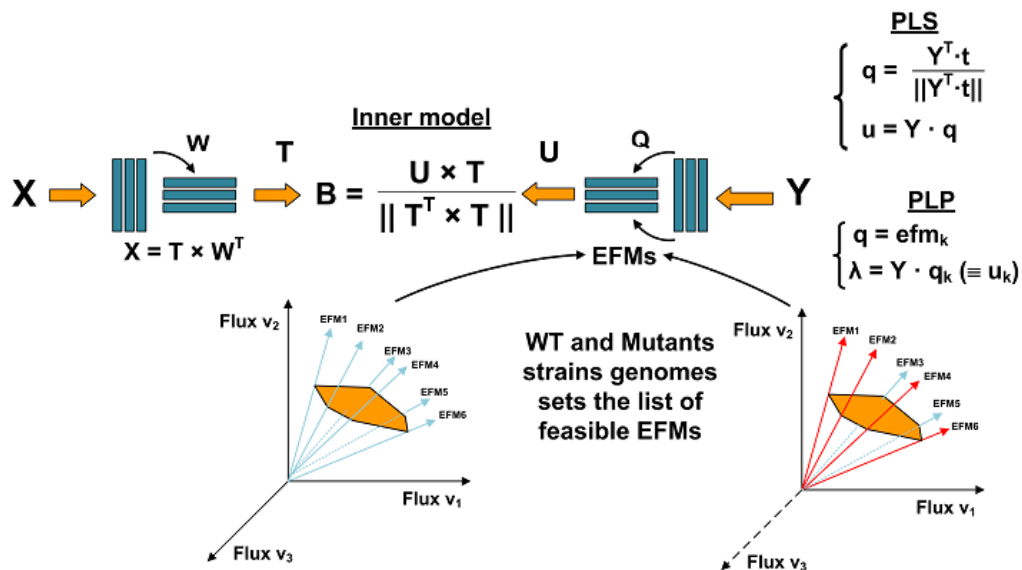


In this statement,  $\mathbf{R} = \{\mathbf{r}(d)\}$  is a  $n_p \times n_r$  matrix of  $n_p$  independent observations of reaction rates,  $\mathbf{r}(d)$  for each  $d$  condition and with  $n_r$  metabolic reaction fluxes. On the other hand,  $\mathbf{X} = \{\mathbf{x}(d)\}$  is a  $n_p \times n_x$  matrix of  $n_p$  independent observations of omic vectors  $\mathbf{x}(d)$  with information about  $n_x$  omic factors. The enumerated steps are equivalent to a covariance maximisation problem, which involves the maximisation of correlation and minimisation of redundancy between omic data,  $\mathbf{X}$ , and observed flux data,  $\mathbf{R}$ . The additional characteristic of this method lies in the additional constraints set by the universe EFMs set by the available genes.

$$\begin{aligned} & \text{Maximize} && \text{cov}(\mathbf{X}, \mathbf{R}) \\ & \text{s.t.} && \begin{cases} \mathbf{R} = \mathbf{\Lambda} \times \mathbf{E}\mathbf{M}^T \\ \mathbf{\Lambda} = \mathbf{X} \times \mathbf{R}\mathbf{C}^T \end{cases} \end{aligned} \quad (\text{Eq. 2.8})$$

In the Eq. 2.8,  $\mathbf{E}\mathbf{M}$  represent the matrix of EFMs composed by  $n_{em}$   $\mathbf{em}_k(d)$  vectors, of  $1 \times n_r$  in size, comprising a  $n_{em} \times n_r$  matrix. In the case of this metabolic network, the matrix of EFMs,  $\mathbf{E}\mathbf{M}$  is a  $24 \times 275$  matrix given in the Appendix C. Additionally,  $\mathbf{\Lambda}$  is a  $n_p \times n_{em}$  matrix of  $n_{em}$   $\lambda(d)$  weight vectors and  $\mathbf{R}\mathbf{C}$  a  $n_{em} \times n_x$  matrix of regression coefficients.

As said earlier, what distinguish this method from all the others is the EFMs additional constraint in the metabolic flux prediction and correlation with omic factors. However, unconstrained maximisation of covariance can be performed by the PLS method (also known as partial least squares). Figure 2.3 shows the structural differences between PLS and PLP. Due to the similarities between this two methods (whereas PLP is based on PLS), in the next section PLS decomposition will be described and latter it will be shown how it can be modified to PLP.



**Figure 2.3:** Schematic representation of PLS and PLP decomposition operations. Decomposition of  $\mathbf{X}$  and  $\mathbf{Y}$  are similar in PLS and PLP. These methods decompose them into loadings ( $\mathbf{W}$  and  $\mathbf{Q}$ , respectively) and scores ( $\mathbf{T}$  and  $\mathbf{U}$ , respectively). The main difference between PLS and PLP lies on the calculus of the  $\mathbf{Y}$ -loadings,  $\mathbf{Q}$ , which in PLP is a subset of EFMs obtained from the metabolic network of the cell. Also, in PLP, the scores,  $\lambda$ , have physical meaning [51].

### 2.5.2 Projection to latent structures

PLS is a multivariate linear regression technique between an input (predictor) matrix,  $\mathbf{X}$ , and an output response matrix,  $\mathbf{Y}$  (or  $\mathbf{R}$  in PLP). In the PLS method  $\mathbf{X}$  and  $\mathbf{Y}$  are decomposed into reduced sets of uncorrelated latent variables, which are then linearly regressed against each other.

Specifically, NIPALS (non-iterative partial least squares) algorithm [53] steps will be described in detail, since is one of the most used PLS derived method. This will provide the basis for PLP specification. NIPALS proceeds according to the following steps:

1. The first  $\mathbf{Y}$ -loading vector,  $\mathbf{q}$ , is set as an arbitrarily chosen nonzero row of  $\mathbf{Y}$ ,  $\mathbf{y}_s$ . When these calculi are made for univariate PLS,  $\mathbf{Y}$  is an  $n_p \times 1$  vector and  $\mathbf{q}$  is one.

$$\mathbf{q} = \frac{\mathbf{y}_s^T}{\|\mathbf{y}_s\|} \quad (\text{Eq. 2.9})$$

2. The next step is the computation of the  $n_y \times 1$   $\mathbf{Y}$ -score vector,  $\mathbf{u}$ .

$$\mathbf{u} = \mathbf{Y} \times \mathbf{q} \quad (\text{Eq. 2.10})$$

3. Followed by the calculation of the  $n_x \times 1$  weight vector,  $\mathbf{w}$ .

$$\mathbf{w} = \frac{\mathbf{X}^T \cdot \mathbf{u}}{\|\mathbf{X}^T \cdot \mathbf{u}\|} \quad (\text{Eq. 2.11})$$

4. And the  $n_p \times 1$   $\mathbf{X}$ -score vector,  $\mathbf{t}$ .

$$\mathbf{t} = \mathbf{X} \cdot \mathbf{w} \quad (\text{Eq. 2.12})$$

5. The final step is the  $\mathbf{Y}$ -loading vector,  $\mathbf{q}$ , recalculation.

$$\mathbf{q} = \frac{\mathbf{Y}^T \cdot \mathbf{t}}{\|\mathbf{Y}^T \cdot \mathbf{t}\|} \quad (\text{Eq. 2.13})$$

6. The steps 2-5 are repeated until the convergence criterion (for example, the absolute difference between  $\mathbf{t}$  and  $\mathbf{t}_{\text{old}}$ ,  $\mathbf{X}$ -score vector from the previous iteration, is lower than  $\text{eps}$  with, for instance,  $\text{eps} = 1 \times 10^{-8}$ ). In the exception enunciated in 1, univariate PLS, Eq. 2.13 yields  $\mathbf{q} = 1$  hence no iterations are performed.

7. After the criterion is fulfilled, the  $\mathbf{X}$  data block loadings,  $\mathbf{p}$ , are calculated and rescaled accordingly:

$$\mathbf{p} = \frac{\mathbf{X}^T \cdot \mathbf{t}}{\|\mathbf{t}^T \cdot \mathbf{t}\|} \quad (\text{Eq. 2.14})$$

$$\mathbf{p}_{\text{new}} = \frac{\mathbf{p}}{\|\mathbf{p}\|} \quad (\text{Eq. 2.15})$$

$$\mathbf{t} = \mathbf{t} \cdot \|\mathbf{p}\| \quad (\text{Eq. 2.16})$$

$$\mathbf{w} = \mathbf{w} \cdot \|\mathbf{p}\| \quad (\text{Eq. 2.17})$$

8. Then it is computed the regression coefficient of the inner linear model and the  $\mathbf{X}$  and  $\mathbf{Y}$  residuals,  $\mathbf{E}$ .

$$\mathbf{b} = \frac{\mathbf{u}^T \cdot \mathbf{t}}{\mathbf{t}^T \cdot \mathbf{t}} \quad (\text{Eq. 2.18})$$

$$\mathbf{E}_X = \mathbf{X} - \mathbf{t} \cdot \mathbf{p}^T \quad (\text{Eq. 2.19})$$

$$\mathbf{E}_Y = \mathbf{Y} - \mathbf{b} \cdot \mathbf{t} \cdot \mathbf{p}^T \quad (\text{Eq. 2.20})$$

9. Finally the residuals are set as  $\mathbf{X}$  and  $\mathbf{Y}$ . Now it is possible to go to the first step and repeat the procedure for the next latent variable.

$$\mathbf{X} = \mathbf{E}_X \quad (\text{Eq. 2.21})$$

$$\mathbf{Y} = \mathbf{E}_Y \quad (\text{Eq. 2.22})$$

10. Steps 1-9 are repeated for  $lv = 1, \dots, fac$  latent variables resulting into the following overall decomposition:

$$\mathbf{X} = \mathbf{T} \cdot \mathbf{W}^T + \mathbf{E}_X \quad (\text{Eq. 2.23})$$

$$\mathbf{Y} = \mathbf{U} \cdot \mathbf{Q}^T + \mathbf{E}_Y \quad (\text{Eq. 2.24})$$

$$\mathbf{U} = \mathbf{T} \cdot \mathbf{B}^T + \mathbf{E}_U \quad (\text{Eq. 2.25})$$

11. Finally, the prediction of  $\mathbf{Y}$  from  $\mathbf{X}$  is given by.

$$\hat{\mathbf{Y}} = \mathbf{X} \cdot \mathbf{RC}^T \quad (\text{Eq. 2.26})$$

12. And in PLS  $\mathbf{RC}$  is a  $n_y \times n_x$  regression coefficients matrix given by.

$$\mathbf{RC} = \mathbf{Q} \cdot \mathbf{B} \cdot \mathbf{W}^T \quad (\text{Eq. 2.27})$$

For more details about this method Geladi and Kowalski review [54] might be consulted.

### 2.5.3 Projection to latent pathways

PLP, as already said, can be viewed as a constrained version of PLS that maximises the covariance between  $\mathbf{X}$  and  $\mathbf{R}$ , an output matrix similar to  $\mathbf{Y}$ , under the constraint of known EFMs. PLP performs essentially the same decomposition described by Eq. 2.23 until Eq.2.27. The main difference resides in the computation of the output loadings,  $\mathbf{Q}$ , similar to  $\mathbf{EM}$  matrix in PLS. Since EFMs are unique and non-decomposable fluxome solutions, any observed flux distribution can be expressed as a non-negative weighted sum of EFMs (Eq. 2.7 and Figure 2.2). Additional features of PLP and some implementation details will be further enunciated. According to this analogy, PLP was modified as follows:

1. For each  $k$  EFM, loadings,  $\mathbf{q}_k$ , are set to be equal to  $\mathbf{em}_k$  and the respective score vector,  $\lambda_k$ , is computed.

$$\mathbf{q}_k = \mathbf{em}_k \quad (\text{Eq. 2.28})$$

$$\lambda_k = \mathbf{R} \cdot \mathbf{q}_k \cdot (\equiv \mathbf{u}_k) \quad (\text{Eq. 2.29})$$

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2. Perform a univariate PLS (with  $\mathbf{q} = 1$ ) with input  $\mathbf{X}$  and target  $\mathbf{R} = \lambda_k$  for *fac*  $\mathbf{X}$  latent variables as described in the previous section and compute the predicted  $\lambda_k$ .

$$\hat{\lambda}_k \text{ predicted } \lambda_k \text{ from univariate PLS} \quad (\text{Eq. 2.30})$$

3. Compute the predicted  $\mathbf{R}$  (Eq. 2.31) by the  $k$  EFM and the respective global explained variance (Eq. 2.32), with  $d$  and  $j$  denoting observation (in this case, different *E. coli* strains) and flux indexes, respectively. It was also calculated the explained variance of EFM weighting factors according to the formula (Eq. 2.33).

$$\hat{\mathbf{R}}_k = \hat{\lambda}_k \cdot \mathbf{q}_k^T \quad (\text{Eq. 2.31})$$

$$\text{var}(\text{robs})_k (\%) = 100 \cdot \left( 1 - \frac{\sum_d \sum_j (\mathbf{r}_{d,j} - \hat{\mathbf{r}}_{d,k,j})^2}{\sum_d \sum_j \mathbf{r}_{d,j}^2} \right) \quad (\text{Eq. 2.32})$$

$$\text{var}(\lambda)_k (\%) = 100 \cdot \left( 1 - \frac{\sum_d (\lambda_{d,k} - \hat{\lambda}_{d,k})^2}{\sum_d \lambda_{d,k}^2} \right) \quad (\text{Eq. 2.33})$$

4. Repeat steps 1, 2 and 3 for every  $k$  EFM, with  $k = 1, \dots, n_{em}$ , and choose the best,  $k_{opt}$ , as the one that exhibits the highest variance value given by Eq. 2.32. Furthermore other statistical significance tests were implemented, namely it was set that the  $k_{opt}$  EFMs was only selected if it satisfies the following statistical criteria: more than 0.75 in the  $r^2$  test and less than 0.05 in the  $p_{value}$  criteria.

$$k_{opt}: \text{EFM with highest } \text{var}(\text{robs})_k \text{ value} \quad (\text{Eq. 2.34})$$

5. Remove  $k_{opt}$  EFM from the full list of EFMs and also remove the predicted fluxes by the  $k_{opt}$  EFM from the  $\mathbf{R}$  matrix.  $\mathbf{X}$  matrix remain the same.

$$\mathbf{R} = \mathbf{R} - \hat{\mathbf{R}}_{k_{opt}} \quad (\text{Eq. 2.35})$$

6. Go back to step 1 and repeat the procedure until the maximum number of EFMs is reached, until the explained variance of  $\mathbf{R}$  does not increase any further or until the  $r^2$  and  $p_{value}$  minimum and maximum respective values criteria is not fulfilled.

In this model the PLS output loadings,  $\mathbf{Q}$ , hold a subset of EFMs from matrix  $\mathbf{EM}$  while the PLS output scores,  $\mathbf{U}$ , are equivalent to the EFMs weights matrix,  $\mathbf{\Lambda}$ , representing the relative weights of latent pathways. For this reason, the algorithm is called projection to latent pathways.

$$\mathbf{R} = \mathbf{\Lambda} \cdot \mathbf{EM}^T + \mathbf{E}_R \quad (\text{Eq. 2.36})$$

Although PLS and PLP are structurally equivalent, PLP has the advantage that the loadings and scores from the target matrix have a physical interpretation:

1. The number of  $\mathbf{Y}$  latent variables in PLS is analogous to the number of active EFMs in PLP (number of  $\mathbf{R}$  latent variables). Thus the subsets of EFMs that explain most of the variance of  $\mathbf{R}$  are interpreted as the set of metabolic pathways activated by omic factors.

2. The regression coefficients vector,  $\mathbf{rc}_{k_{opt}}$ , of the inner univariate PLS, being directly associated with the  $k_{opt}$  EFM, shows the contribution of each omic factor to the up- or down-regulation of EFMs.

Moreover, the regression coefficients  $\mathbf{B}$  can be used to deduce the functional omics matrix,  $\mathbf{FM}$ , as follows:

$$\mathbf{FM} = \mathbf{W} \times \mathbf{B} \quad (\text{Eq. 2.37})$$

$\mathbf{FM}$  is a  $n_x \times n_{em}$  matrix comprising the regression coefficients of EFMs against omic components, thus providing information of how EFMs are up- or down-regulated by omic components.

PLP was implemented as a Matlab<sup>TM</sup> (Mathworks, Inc) toolbox. In what follows, the PLP model execution details will be described. Such modifications are motivated by the *E. coli* omic datasets features.

## 2.6 Implementation details

### 2.6.1 Software

The previous algorithm, PLP, is available as a MATLAB toolbox. It was developed and implemented in the group and is prior to the present thesis.

### 2.6.2 Data organization

The organization in which the *E. coli* data was presented above namely the sequence of variables, will be maintained in the following sections for modelling purposes. It was established that the name of the PLP models reflect their input variables. For instance, a PLP model with envirome (E) inputs is called model (E) and has 9 input variables obeying to the sequence of vector E as previously defined by Eq. 2.1. In case of multiple omic inputs, for instance PLP model (E+P) (see Table 3.6 and Table 3.7), it has inputs of the envirome (E) followed by the proteome (P) according to the sequences of Eqs. 2.1 and 2.3 respectively. All other models also obey these rules.

### 2.6.3 Validation and calibration partitions

The different 31 *E. coli* strain's data (WT at four different dilutions rates, 24 single gene deletion mutants and three reference points, enunciated earlier) were divided in two groups:

- Calibration partition (20 points) formed by *galM*, *glk*, *pgm*, *fbaB*, *gapC*, *gpmA*, *gpmB*, *pykA*, *pykF*, *ppsA*, *pgl*, *rpiA*, *rpiB*, *tktA*, *tktB*, *talA*, WT grown at  $0.1\text{h}^{-1}$ ,  $0.5\text{h}^{-1}$  and two reference points.

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- Validation partition (11 points) formed by *pgi*, *pfkA*, *pfkB*, *fbp*, *zwf*, *gnd*, *rpe*, *talB*, WT grown at two high dilution rates ( $0.4\text{h}^{-1}$  and  $0.7\text{h}^{-1}$ ) and a reference data point.

Model parameters were calibrated with the calibration partition only while predictive power was assessed with the validation partition.

Such division was maintained for all developed models with the exception of regulatory transcriptome (microarrays technique) since the data was only available for the *pgm*, *pgi*, *gapC*, *zwf* and *rpe* single gene deletion mutants and WT strain grown at  $0.5\text{h}^{-1}$ ,  $0.7\text{h}^{-1}$  and two strains grown at  $0.2\text{h}^{-1}$  (reference). In this case the validation partition was formed by *pgi*, *rpe* single gene KO and a strain grown at  $0.7\text{h}^{-1}$ . The calibration partition was composed by *pgm*, *gapC*, *zwf* and WT strains grown at  $0.5\text{h}^{-1}$  and two reference strains ( $0.2\text{h}^{-1}$ ).

This division was not randomly made. Instead, it was intended to ensure a minimal number of *E. coli* strains in which each EFMs was feasible (higher than the input data principal components number, *fac*).

Moreover, the validation dataset also comprises the *E. coli* strains with more distinct phenotypes (namely the examples given by Ishii *et al.* in the main text: *pfkB*, *zwf*, *rpe* and WT grown at  $0.7\text{h}^{-1}$ ) in order to impose tough conditions for the assessment of model predictive power.

### 2.6.4 Principal components optimization

Before each actual PLP run, it was made an omics input's principal components optimization (since the fluxes principal components are the EFMs, it does not need previous optimization). This procedure was done by defining the number of best performing principal components, *i.e.*, the number of principal components that comprise the needed information to do a good metabolic flux prediction. The criterion used to evaluate such performance was the fluxes predictions mean squared error minimization for the strains defined in the validation subset. The MATLAB statistical toolbox was used to perform the mean squared error calculus. The optimal principal component number will be defined for each model (*fac* value).

### 2.6.5 EFMs feasibility examination

As said before, in this particular case, the *E. coli* strain genome was variable. An obvious consequence is the inability to catalyse a metabolic reaction. Such information was added in the method through the exclusion of all the EFMs that were in part constituted by such removed reaction [49]. Additionally, all the others EFMs remain unaltered.

The implemented pre-selection rule in which the viable EFMs were selected for each case was applied at the algorithm's third step (Eq. 2.27). Before the showed calculus, it was calculated in which organisms were valid the *k* EFM. It was set in the form of a 1 to  $n_p$  vector of ones and zeros. One means that the removed reaction in the strain does not takes part in the *k* EFM, thus

such  $k$  EFM is feasible; zero means otherwise. A consequence of such pre-selection rule was that only the fluxes from the *E. coli* strains in which the  $k$  and  $k_{opt}$  EFM was valid will be predicted. So, only the feasible fluxes will be considered for the  $k_{opt}$  EFM selection and the remaining will be set to zero in the predicted fluxes matrix.

### 2.6.6 Consistency analysis

The bootstrapping method was employed to check the model consistency. It was used a resampling method that generates new datasets from the available ones through the introduction of artificial perturbations. This consistency analysis is done by the spread in the results obtained for these new datasets [55].

More specifically, the general PLP method described in the previous section was repeated  $n_p$  times, being  $n_p$ , only in this case the number of *E. coli* strains in the calibration partition. In each time it was removed all the information regarding a  $d$  *E. coli* strain, with  $d$  an index value that changed from 1 to  $n_p$ . This was the artificial perturbation introduced in the model. For instance, in the model (E) first PLP-bootstrapping iteration it was removed all the information regarding the *galM* *E. coli* strain from the calibration partition (first strain in this partition). Thus it was developed a PLP model with the remaining 19 strains in the calibration partition. A similar procedure was followed for all the 19 remaining strains present in this partition, as well as for all the other PLP models (with single and multiple omic input datasets).

With the resulting information it was calculated the frequency of selection of EFMs and respective regression coefficients. Additionally it was also calculated the regression coefficients confidence interval (CI) of the inner model.

Moreover, it was calculated the mean squared error for each PLP run, for all data points and for the validation and calibration partition, separately. In order to do that, it was used the Matlab<sup>TM</sup> (Mathworks, Inc) function for this calculus.





## **3 Results**



### 3.1 Model structure discrimination

The general goal in this section is the synthesis of a model for *E. coli* cells with the ability to predict flux-phenotype. The major concern is to identify the optimal model structure and to estimate model parameters with high statistical confidence. In the hybrid modelling strategy employed in this work the general model structure is set by the EFMs of the metabolic network of *E. coli* cells as previously shown in Figure 2.2. This part of the model is fixed and represents the *a priori* mechanistic cellular knowledge. The number of active EFMs and the respective weighting factors values are however not known *a priori*. The identification of these unknown structural features is accomplished with the PLP algorithm as previously described. With PLP the EFMs weighting factors are linearly regressed against the input data in order to fulfil two criteria:

- Maximise explained variance of target fluxome datasets
- Minimise the number of active EFMs

In this section, the models are explored with EFMs weighting factors linearly regressed against different input omic dataset. More precisely, it is investigated different scenarios where individual omic datasets are set as inputs to the PLP algorithm.

#### 3.1.1 Models with single omic information layers

The Tables 3.2, 3.3, 3.4 and 3.5 compile the PLP modelling results when the input datasets are the envirome, proteome, metabolome, transcriptome and regulatory transcriptome, respectively. The first column represents the index of selected EFMs,  $r^2$  is the correlation coefficient of EFM weighting factor and input data,  $p_{value}$  is an alternative measure of correlation and it should be as low as possible,  $var(\lambda_{EFM})$  is the explained variance of the EFM weighting and  $var(\mathbf{r}_{obs})$  is the explained variance of measured fluxome data. In all the enunciated PLP runs, the optimal number of input data principal components (*fac*) was two. As said before, the flux principal components were the EFMs. The number of selected EFMs was limited by the not fulfillment of the statistical rules (minimum and maximum values for  $r^2$  and  $p_{value}$ , respectively), the cease of improvement in the flux explained variance or by a maximum number of 20 EFMs.

Here, as well as in all the remaining PLP runs, the *E. coli* data was separated in two partitions, as referred before. Usually, from the 31 *E. coli* strain's dataset it was made a calibration partition (*galM*, *glk*, *pgm*, *fbaB*, *gapC*, *gpmA*, *gpmB*, *pykA*, *pykF*, *ppsA*, *pgl*, *rpiA*, *rpiB*, *tktA*, *tktB*, *talA*, WT grown at  $0.1\text{h}^{-1}$ ,  $0.5\text{h}^{-1}$  and two reference points) and a validation partition (*pgi*, *pfkA*, *pfkB*, *fbp*, *zwf*, *gnd*, *rpe*, *talB*, WT grown at two high dilution rates ( $0.5\text{h}^{-1}$  and  $0.7\text{h}^{-1}$ ) and a reference data point). The exception was the PLP runs which included the regulatory transcriptional dataset. In such cases the 9 different *E. coli* strains were separated in a calibration partition composed by *pgm*, *gapC*, *zwf* and WT strains grown at  $0.5\text{h}^{-1}$  and two reference strains ( $0.2\text{h}^{-1}$ ), while the validation partition were constituted by *pgi*, *rpe* and  $0.7\text{h}^{-1}$  *E. coli* strains. Still,

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the EFMs selection and remaining calculations were done for the separated calibration and validation partitions while the data presented in the following Tables were calculated for the global data, unless it was said otherwise.

With this information, it can be highlighted that the number of model parameters was always much lower than the number of data points. The former is highest in the model (RT). In this model the number of selected EFMs is maximum (20), thus the number of model parameters was 40 (selected EFMs  $\times$  *fac* – 20  $\times$  2). On the other hand, in this same model the number of independent measurements is also the lowest (9, as said before). So, the number of data points in the calibration partition was 258 (6 strains in the calibration partition  $\times$  43 metabolic reactions). All the others presented models have more data points and less model parameters.

**Table 3.1:** Ranking of statistically significant EFMs with high correlation with the envirome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
92	0.76	$8.998 \times 10^{-5}$	60.6	73.4
205	0.87	$6.754 \times 10^{-7}$	91.6	84.4
232	1.00	$1.584 \times 10^{-2}$	55.9	84.8
34	1.00	$3.237 \times 10^{-2}$	99.5	84.8
209	0.84	$3.623 \times 10^{-6}$	77.2	86.6
202	0.76	$1.119 \times 10^{-4}$	87.4	88.7

**Table 3.2:** Ranking of statistically significant EFMs with high correlation with the proteome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
260	0.84	$8.00 \times 10^{-6}$	53.1	30.8
275	0.82	$8.71 \times 10^{-6}$	56.6	38.6
263	0.89	$2.08 \times 10^{-7}$	52.5	50.9
271	0.91	$4.51 \times 10^{-8}$	60.5	51.4
217	0.81	$2.89 \times 10^{-5}$	45.2	52.7
250	0.93	$4.83 \times 10^{-9}$	55.6	53.1
242	0.74	$2.55 \times 10^{-4}$	42.2	53.1
232	1	$2.79 \times 10^{-2}$	95	53.2
100	0.76	$9.25 \times 10^{-5}$	45.2	59.1

**Table 3.2 (cont.):** Ranking of statistically significant EFMs with high correlation with the proteome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
244	0.82	$1.41 \times 10^{-5}$	30	59.3
220	0.77	$1.13 \times 10^{-4}$	37.2	60
229	0.68	$1.35 \times 10^{-3}$	39.7	60.4
92	0.78	$4.23 \times 10^{-5}$	34.2	69.3
124	0.87	$1.14 \times 10^{-6}$	32.8	69.3
220	0.77	$1.13 \times 10^{-4}$	37.2	60
229	0.68	$1.35 \times 10^{-3}$	39.7	60.4
92	0.78	$4.23 \times 10^{-5}$	34.2	69.3
124	0.87	$1.14 \times 10^{-6}$	32.8	69.3
249	0.64	$3.46 \times 10^{-3}$	22.4	69.4
34	1	$3.56 \times 10^{-2}$	99.5	69.4
236	0.87	$1.70 \times 10^{-6}$	31.3	69.5
105	1	$1.37 \times 10^{-4}$	50.9	69.9
99	1	$1.15 \times 10^{-2}$	26.4	69.9

**Table 3.3:** Ranking of statistically significant EFMs with high correlation with the metabolome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
260	0.86	$2.55 \times 10^{-6}$	66.6	31
219	0.88	$3.91 \times 10^{-7}$	52.4	54.5
271	0.89	$3.56 \times 10^{-7}$	68.6	54.7
275	0.82	$8.04 \times 10^{-6}$	48.6	59
249	0.93	$1.37 \times 10^{-8}$	66.9	61.5
274	0.83	$4.78 \times 10^{-6}$	38.8	64.3
228	0.84	$8.51 \times 10^{-6}$	56.9	64.7
253	0.98	$8.45 \times 10^{-3}$	58	65.1
242	0.81	$3.13 \times 10^{-5}$	47.2	65.1
252	0.78	$8.45 \times 10^{-5}$	45.9	65.2
250	0.97	$3.91 \times 10^{-12}$	79.3	65.4
240	0.78	$9.61 \times 10^{-5}$	42.1	65.4

**Table 3.3 (cont.):** Ranking of statistically significant EFMs with high correlation with the metabolome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
236	0.9	$1.87 \times 10^{-7}$	57.6	65.5
34	0.99	$1.03 \times 10^{-1}$	94.7	65.5
246	0.87	$1.67 \times 10^{-6}$	46.8	65.6
48	0.85	$1.73 \times 10^{-6}$	51	69.5
227	0.86	$1.89 \times 10^{-6}$	41.8	69.7
30	0.88	$8.14 \times 10^{-7}$	28.1	69.8

**Table 3.4:** Ranking of statistically significant EFMs with high correlation with the transcriptome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
92	0.81	$1.37 \times 10^{-5}$	55.9	72.6
205	0.92	$6.44 \times 10^{-9}$	64.9	82.4
34	1.00	$8.27 \times 10^{-4}$	100	82.5
30	0.80	$3.80 \times 10^{-5}$	57.7	82.5
202	0.90	$6.57 \times 10^{-8}$	51.3	84.8

**Table 3.5:** Ranking of statistically significant EFMs with high correlation with the regulatory transcriptome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
92	0.98	$6.99 \times 10^{-4}$	70.9	73
263	0.98	$4.69 \times 10^{-3}$	68.3	82.7
271	1	$2.56 \times 10^{-3}$	98.6	82.9
269	1	$2.45 \times 10^{-3}$	87.3	82.9
3	0.99	$8.37 \times 10^{-5}$	67.9	87.5
122	1	$2.14 \times 10^{-4}$	53.7	87.5
197	0.98	$2.68 \times 10^{-3}$	53.5	89.9
157	0.99	$5.66 \times 10^{-3}$	41.3	90
242	1	$5.28 \times 10^{-4}$	87.4	90
129	0.98	$4.43 \times 10^{-3}$	-10.3	90.3

**Table 3.5 (cont.):** Ranking of statistically significant EFMs with high correlation with the regulatory transcriptome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM through the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
241	1	$8.32 \times 10^{-4}$	60.2	90.3
240	0.99	$5.60 \times 10^{-3}$	54.8	90.3
250	0.99	$7.00 \times 10^{-3}$	49.1	90.4
158	1	$3.32 \times 10^{-4}$	99.6	90.4
244	0.98	$2.17 \times 10^{-2}$	49.9	90.4
162	0.97	$2.87 \times 10^{-2}$	37	90.4
164	1	$1.46 \times 10^{-3}$	55.4	90.4
124	1	$4.92 \times 10^{-3}$	58.3	90.4
249	0.99	$7.17 \times 10^{-3}$	43.5	90.4
121	1	$3.83 \times 10^{-3}$	99.2	90.4

From these results it can be observed that the best performing model in terms of fluxome prediction accuracy is the one that has as input data the regulatory transcriptome omic layer (90.4% of explained variance of fluxome data) closely followed by the one that uses the envirome layer (88.9% of explained variance of fluxome data). However, this transcriptional data is available for fewer strains (9) than the remaining input datasets (31). This means that fewer independent measurements are available. So, it precludes a fair comparison with the other structures. Furthermore, when combined with other omic information, the validation partition variance decreased (see Table 3.6 discussed below). For these two reasons (few data points and possible overfitting) this structure was not selected for complete model identification in the next sections. Later on, flux prediction from the regulatory transcriptome dataset will be discussed (namely the reasons because it was not used for detailed examination and the possible applications of a wider data collection) and also the results for the envirome dataset, second best performing dataset in terms of explained variance and best mean squared error results, will be the closely analysed.

### 3.1.2 Models with multiple omic information layers

Tables 3.6 and 3.7 show the model performance criteria for different combinations of omic datasets as inputs for the metabolic fluxes calculation (the detailed EFMs selection results for each case is shown in Appendix E). The data shown in the referred Appendix is similar to the data showed earlier: the first column represents the index of selected EFMs,  $r^2$  is the correlation coefficient of EFM weighting factor and input data,  $p_{value}$  is another alternative measure of

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correlation and it should be as low as possible,  $var(\lambda_{\text{EFM}})$  is the explained variance of the EFM weightings and  $var(\mathbf{r}_{\text{obs}})$  is the explained variance of measured fluxome data. In all the next PLP runs, the optimal number of input data principal components (*fac*) was also two. On the other hand, the flux principal components were the EFMs. The number of selected EFMs was also limited by the not fulfillment of the statistical rules (minimum and maximum values for  $r^2$  and  $p_{\text{value}}$ , respectively), by the cease of the improvement of explained variance or by a maximum number of 20 EFMs. Each line presents the fluxomes explained variance and the mean squared error for the overall, calibration and validation datasets. As in the models described earlier, the comparison between the number of model parameters and data points can also be done. The same results can be seen here: the number of model parameters is much lower than the number of data points.

Similarly, the *E. coli* data was separated in two partitions. Usually, from the 31 *E. coli* strain's dataset it was separated in a calibration partition composed by *galM*, *glk*, *pgm*, *fbaB*, *gapC*, *gpmA*, *gpmB*, *pykA*, *pykF*, *ppsA*, *pgl*, *rpiA*, *rpiB*, *tktA*, *tktB*, *talA*, WT grown at  $0.1\text{h}^{-1}$ ,  $0.5\text{h}^{-1}$  and two reference points) and a validation partition (*pgi*, *pfkA*, *pfkB*, *fbp*, *zwf*, *gnd*, *rpe*, *talB*, WT grown at two high dilution rates ( $0.5\text{h}^{-1}$  and  $0.7\text{h}^{-1}$ ) and a reference data point). The exception was the PLP runs which included the regulatory transcriptional dataset. In such cases the nine different *E. coli* strains were separated in a calibration partition composed by *pgm*, *gapC*, *zwf* and WT strains grown at  $0.5\text{h}^{-1}$  and two reference strains ( $0.2\text{h}^{-1}$ ), while the validation partition were constituted by *pgi*, *rpe* and  $0.7\text{h}^{-1}$  *E. coli* dataset.

These results show a coarse comparison between different model structures that integrate different layers of omic information. It can be observed that the envirome alone is the most informative omic layer in terms of flux prediction since this dataset obtained the lowest mean squared error results for the validation partition, Table 3.6, and the second highest explained variance for the validation partition, Table 3.7.

In the next sections, the best performing models are analysed in more detail. These include the envirome model (E) and also the envirome and transcriptome combined model (E+T), which ranked in second place in terms of mean squared error in the validation partition (20722), Table 3.7, and explained variance of flux data in the validation dataset (85.2%), Table 3.6.

It should be noted again that despite the apparent good results of the regulatory transcriptome models (model RT), due to the lack of measurements and potential overfitting problems when joint together with other omic datasets, this models will not be considered for further detailed analysis.



**Table 3.6:** Explained variance for all data points and for calibration and validation partition for each PLP run with different omic factors. E: Envirome; P: Proteome; M: Metabolome; T: Transcriptome; RT: Regulatory transcriptome; +: Different omic data was put together.

Omic factors	Overall explained variance (%)	Calibration partition explained variance (%)	Validation partition explained variance (%)
E	88.7	88.83	88.52
P	69.9	76.71	64.40
M	69.8	84.32	58.14
T	84.8	90.04	80.55
RT	90.4	89.23	91.03
E+P	87.8	94.20	82.72
E+M	74.3	85.27	65.55
E+T	87.1	89.56	85.16
E+RT	89.7	91.07	64.26
P+M	71	78.92	64.55
P+T	89.1	93.12	85.85
P+TR	89.3	90.38	69.13
M+T	87.9	90.77	85.58
M+RT	92.1	93.03	74.83
T+RT	88.8	90.11	64.53
E+P+M	71.7	79.74	65.30
E+P+T	88.9	92.54	86.07
E+P+RT	89.1	90.15	70.63
E+M+T	86.7	89.50	84.45
E+M+RT	92.1	93.00	75.17
E+T+RT	88.8	90.18	64.09
P+M+T	87.2	90.13	84.83
P+M+RT	90.5	91.41	75.14
M+T+RT	92	92.89	76.27
E+P+M+T	86.2	89.56	83.58
E+P+M+RT	90.5	91.40	75.34
E+P+T+RT	88.8	90.20	64.53
E+M+T+RT	92	92.87	76.43
P+M+T+RT	90.2	91.13	73.83
E+P+M+T+RT	90.2	91.12	74.00

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**Table 3.7:** Mean squared error for all data points, calibration and validation partition for each PLP run with different omic factors. E: Envirome; P: Proteome; M: Metabolome; T: Transcriptome; RT: Regulatory transcriptome; +: Different omic data was put together.

Omic factors	Overall mean squared error	Calibration partition mean squared error	Validation partition mean squared error
E	10819	8751	14579
P	20373	15607	29037
M	21727	9238	44434
T	14355	7285	27211
RT	18436	16778	21751
E+P	13797	6657	26777
E+M	19716	11704	34282
E+T	14956	11785	20722
E+RT	43004	31583	65848
P+M	22025	12632	39104
P+T	14071	10304	20922
P+RT	39598	29345	60103
M+T	14871	11691	20852
M+RT	24533	18861	35878
T+RT	38849	28369	59810
E+P+M	17504	10847	29608
E+P+T	14421	10932	20765
E+P+RT	32259	24000	48778
E+M+T	15708	12758	21072
E+M+RT	24532	18840	35915
E+T+RT	39216	27951	61748
P+M+T	15967	12525	22226
P+M+RT	37299	25458	60982
M+T+RT	28109	20088	44151
E+P+M+T	15515	11998	21909
E+P+M+RT	37334	25472	61060
E+P+T+RT	28150	20126	44198
E+M+T+RT	39713	28068	63003
P+M+T+RT	38241	25478	63768
E+P+M+T+RT	38260	25453	63872

## 3.2 Envirome as input to PLP

### 3.2.1 Discriminated EFMs

Table 3.1 shows the subset of EFMs with highest correlation with the envirome data determined by the PLP algorithm. It should be highlighted that a high percentage of variance (approximately 89%) can be explained by few EFMs (six; as said and as it was described by Wlaschin, *et al.*, a small set of EFMs are able to predict the full metabolic state of the cell [56]).

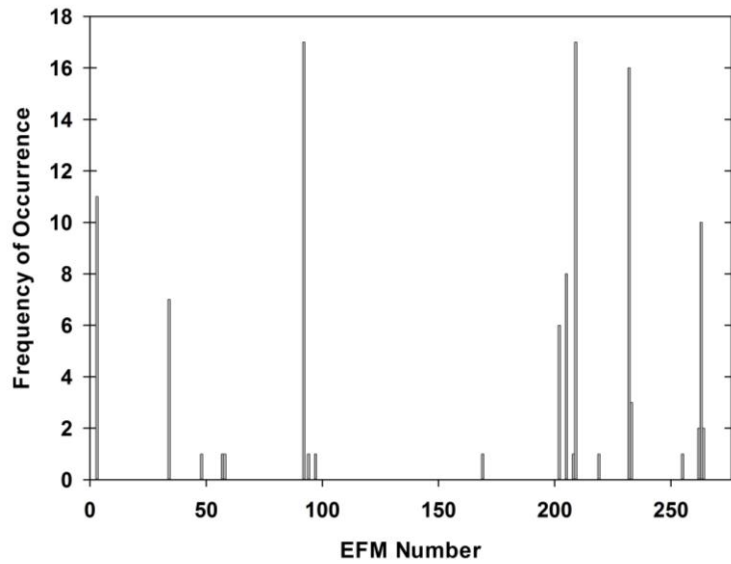
To assess the confidence of EFMs selection, a bootstrapping method was employed (see methods section). Briefly, in the bootstrapping method different PLP runs were made removing an *E. coli* strain data at each time. At each PLP run, the EFMs, regression coefficients values and remaining data was saved. From this data it was calculated an EFM frequency of selection map and it was also calculated a CI for the regression coefficients (Figure 3.1 and Table 3.8, respectively). The EFMs with highest frequency of selection define a plausible active set of EFMs activated by the envirome alone.

### 3.2.2 Predictive power

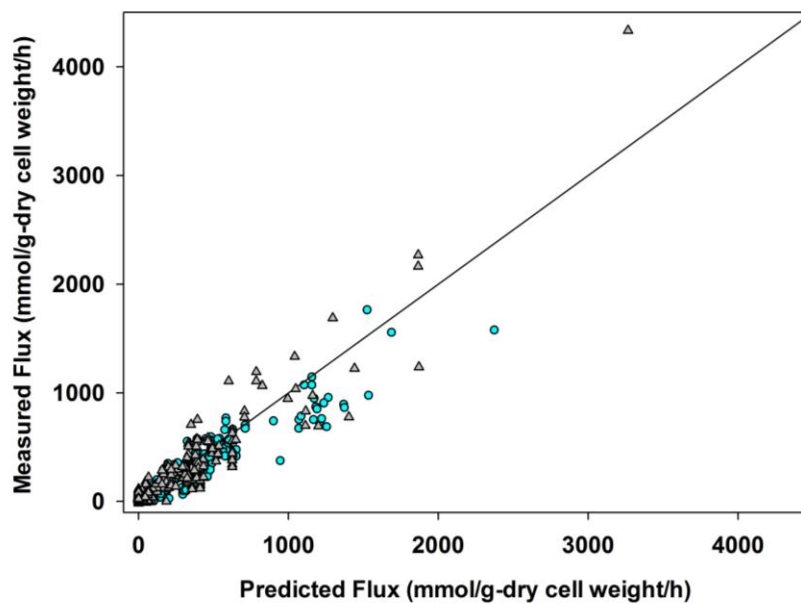
Figure 3.2 represents predicted against measured fluxes for the calibration partition (20 observations with 43 reactions) and a validation partition (11 observations with 43 reactions). The validation dataset comprised eight mutants (namely, *pgi*, *pfkA*, *pfkB*, *fbp*, *pgl*, *gnd*, *rpe* and *talB* strains), some of each show differences when compared with the others mutants or with the reference strain, as described by Ishii *et al.* [48]. Moreover the validation dataset also include a WT point grown with a dilution rate of  $0.2\text{h}^{-1}$ , one with dilution rate of  $0.4\text{h}^{-1}$  and another with a dilution rate of  $0.7\text{h}^{-1}$ . On the other hand, the calibration partition is composed by the remaining 16 mutants (specifically, *galM*, *glk*, *pgm*, *fabB*, *gapC*, *gpmA*, *gpmB*, *pykA*, *pykF*, *ppsA*, *zwf*, *rpiA*, *rpiB*, *tktA*, *tktB* and *talA* strains) and four WT strains, two grown at different dilutions rates ( $0.1\text{h}^{-1}$  and  $0.5\text{h}^{-1}$ ) and two reference points grown at  $0.2\text{h}^{-1}$  dilution rate.

It can be observed from these results that the model was able to do a good global flux prediction. It should be emphasized that these results indicate that the hybrid model succeed in the prediction of the fluxome of 8 single gene deletion mutants (included in the validation dataset). The relative errors go from -185 to 1 in the validation partition and its average is -5.3. The outer points (namely the two points with higher fluxes) will be further discussed, but in general they represent only one reaction, exit of carbon dioxide from the system (as it is shown in the last metabolic reaction presented in Figure 3.3). These outer points are the ones that presented the higher error, namely -185.

Moreover, the mean squared error for the calibration and validation partitions are comparable, as well as the lowest in relation to the other models (Table 3.7), denoting a very consistent model that captured the essential features of the system.

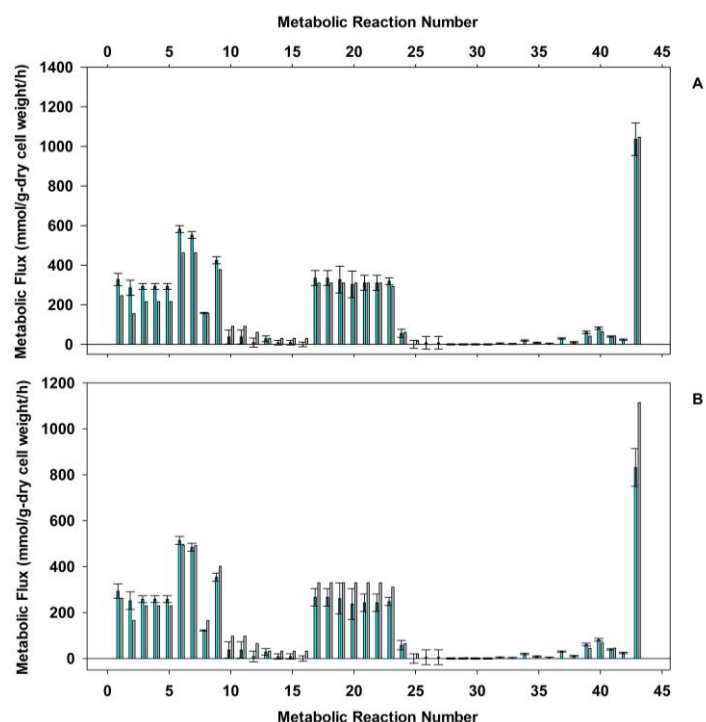


**Figure 3.1:** Frequency of selection of each EFM in the bootstrapping validation procedure when envirome is the input data.



**Figure 3.2:** Predicted against measured fluxes with envirome as input to PLP for the calibration partition (blue circles) and validation partition (grey triangles).

As illustrative examples Figure 3.3A and Figure 3.3B show the prediction of a reference WT and mutant *talB*, respectively (both included in the validation dataset). The behaviour of both strains is quite different (as described by Ishii *et al.* [48]). Such biological alterations in the mutant strain when compared with the WT are especially noticeable in the protein concentration, gene expression, metabolite concentration and metabolic reaction rate that were directly related with such modification, this will be detailed examined in the Discussion section.



**Figure 3.3:** Measured (blue bar) and predicted (grey bar) metabolic flux for the reference strain grown at  $0.2\text{h}^{-1}$  as dilution rate (A) and mutant *talB* (B) both present in validation partition of PLP algorithm with envirome as input data.

### 3.2.3 Envirome-to-function relationship

Table 3.8 shows the regression coefficients matrix (**RC**) of EFMs weighting factors against envirome components and the confidence interval. Additional information about the confidence interval determination procedure (bootstrapping method) is described in the methods section.

**Table 3.8:** Regression coefficients, and respective confidence interval, between environmental components and selected EFM.

	Glucose	Ethanol	Acetate	D-Lactate	L-Lactate	Succinate	Pyruvate	Formate	Dilution rate
<b>92</b>	0.0454	0.111	0.0664	0.048	0.0625	0.0811	0.1881	0.0762	0.1328
	$\pm 0.0142$	$\pm 0.0074$	$\pm 0.0052$	$\pm 0.0244$	$\pm 0.0446$	$\pm 0.0283$	$\pm 0.0325$	$\pm 0.0054$	$\pm 0.0097$
<b>205</b>	0.1793	0.1316	0.1204	0.2735	-0.0221	0.0721	0.0168	-0.0089	0.3417
	$\pm 0.0219$	$\pm 0.0256$	$\pm 0.0094$	$\pm 0.0172$	$\pm 0.0508$	$\pm 0.1178$	$\pm 0.0667$	$\pm 0.0416$	$\pm 0.0898$
<b>232</b>	0.0403	0.0714	0.0423	-0.2556	-0.3387	0.112	0.329	0.0022	0.0482
	$\pm 0.0696$	$\pm 0.1111$	$\pm 0.0675$	$\pm 0.0396$	$\pm 0.1075$	$\pm 0.1743$	$\pm 0.1514$	$\pm 0.0607$	$\pm 0.0750$
<b>34</b>	-0.0214	-0.0384	-0.0214	0.3295	0.1193	-0.0602	-0.5217	0.0216	-0.0259
	$\pm 0.0897$	$\pm 0.1418$	$\pm 0.0859$	$\pm 0.5136$	$\pm 0.3407$	$\pm 0.2225$	$\pm 0.8750$	$\pm 0.0996$	$\pm 0.0957$
<b>209</b>	0.1534	0.1039	0.1028	0.1417	-0.0531	0.0555	-0.0143	-0.026	0.2036
	$\pm 0.0697$	$\pm 0.0245$	$\pm 0.0241$	$\pm 0.0633$	$\pm 0.0795$	$\pm 0.0309$	$\pm 0.0772$	$\pm 0.0525$	$\pm 0.0508$
<b>202</b>	0.2234	0.103	0.1181	0.2545	-0.1561	0.0092	0.0857	-0.1248	0.47
	$\pm 0.0335$	$\pm 0.0201$	$\pm 0.0054$	$\pm 0.0321$	$\pm 0.0162$	$\pm 0.0349$	$\pm 0.0939$	$\pm 0.0718$	$\pm 0.1018$

Each column represents the contribution of envirome components to EFMs weighting factors. In some cases, the magnitude of  $\mathbf{I}_{k,h}$  may be interpreted as the strength of up- or down-regulation of a given cellular function  $k$  by the environmental factor  $h$  (see Methods).

### 3.3 Envirome and transcriptome as input to PLP

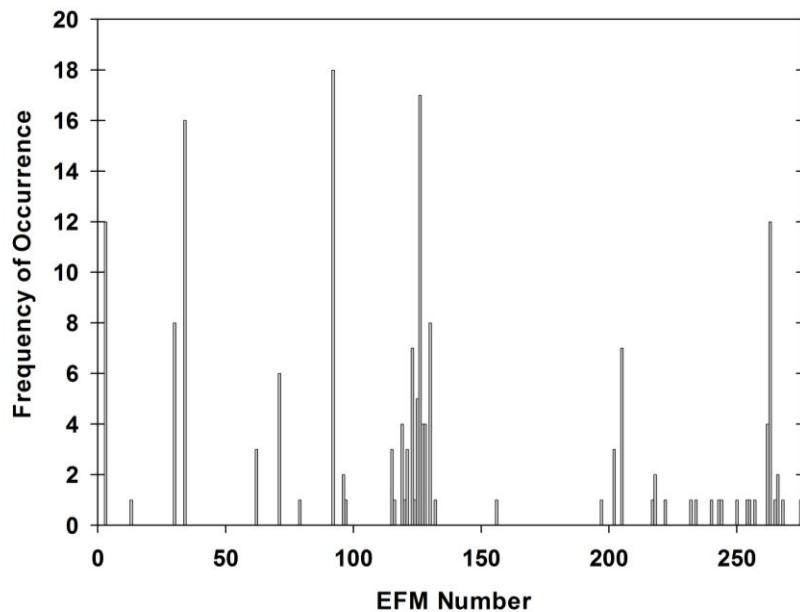
#### 3.3.1 Discriminated EFMs

As already mentioned, the envirome and transcriptome PLP results are listed in Appendix E (Table 7.5). Additional information about the EFMs selection frequency in the bootstrapping procedure can be seen in Figure 3.4.

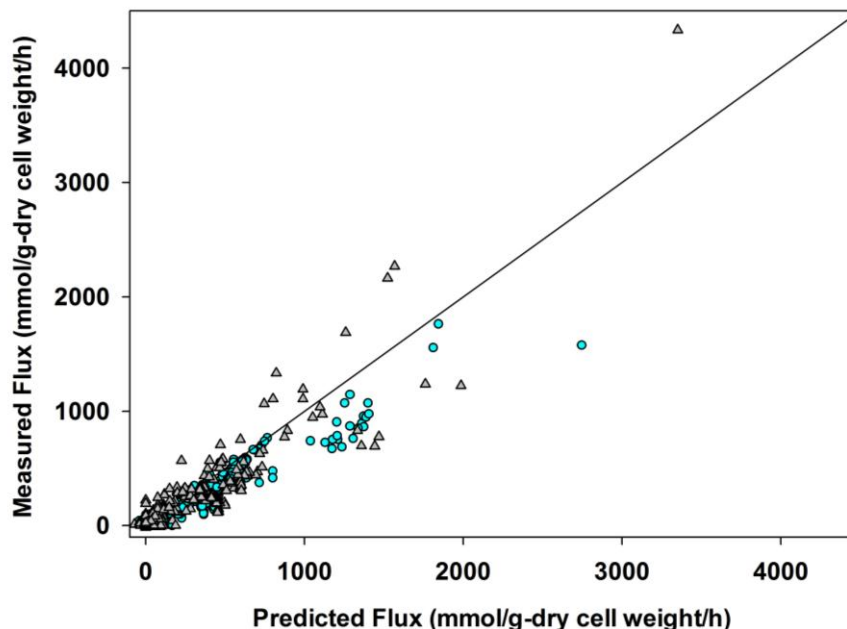
#### 3.3.2 Predictive power

Figure 3.5 represents predicted against measured fluxes for the calibration partition and a validation partition, which are the same as the ones described earlier.

It can be observed from these results that the model was able to accurately predict the fluxome from the eight mutants present in the validation partition. The higher explained variance when compared with the envirome results can be substantiated by the more accurate prediction of the lower fluxes, with the one drawback: the worse flux prediction of the highest metabolic fluxes (namely the two highest fluxes, one in the calibration and the other in the validation partition). This interpretation can also be seen in the analysis of its relative error. These values go from -806 to 116 and its average is -20.4.



**Figure 3.4:** Frequency of selection of each EFM in the bootstrapping validation procedure when envirome and transcriptome are the input data.



**Figure 3.5:** Predicted against measured fluxes with proteome and transcriptome as input to PLP for the calibration partition (blue circles) and validation partition (grey triangles).

Figure 3.6A and Figure 3.6B represent the same illustrative examples shown before (reference WT and mutant *talB*, respectively; both included in the validation dataset). In these examples, the over prediction of the last metabolic reaction on Figure 3.6B is more evident, this may be due to the number of selected EFMs as will be proposed.

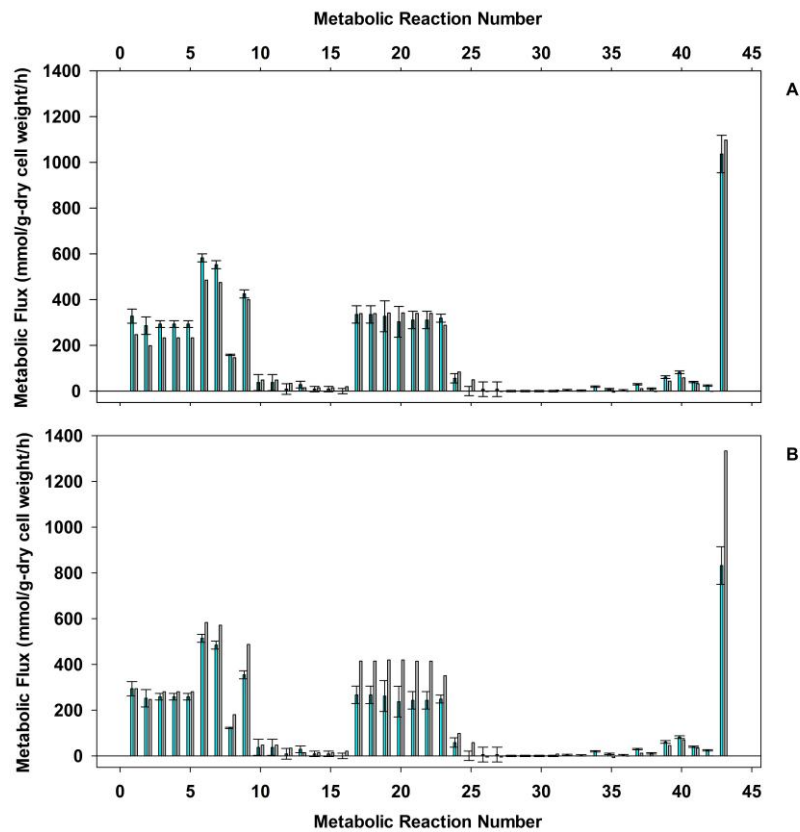
### 3.3.3 Relationship between input and cellular function

Figure 3.7 shows the regression coefficients of EFMs (cellular functions) weighting factors against envirome and transcriptome components (input data). This information is not shown in the form of a table due to its overwhelming size. Here the only CI values presented will be the ones attached to the EFMs up- or down-regulation effects discussed in the next section. The remaining CI values will not be presented due to its size.

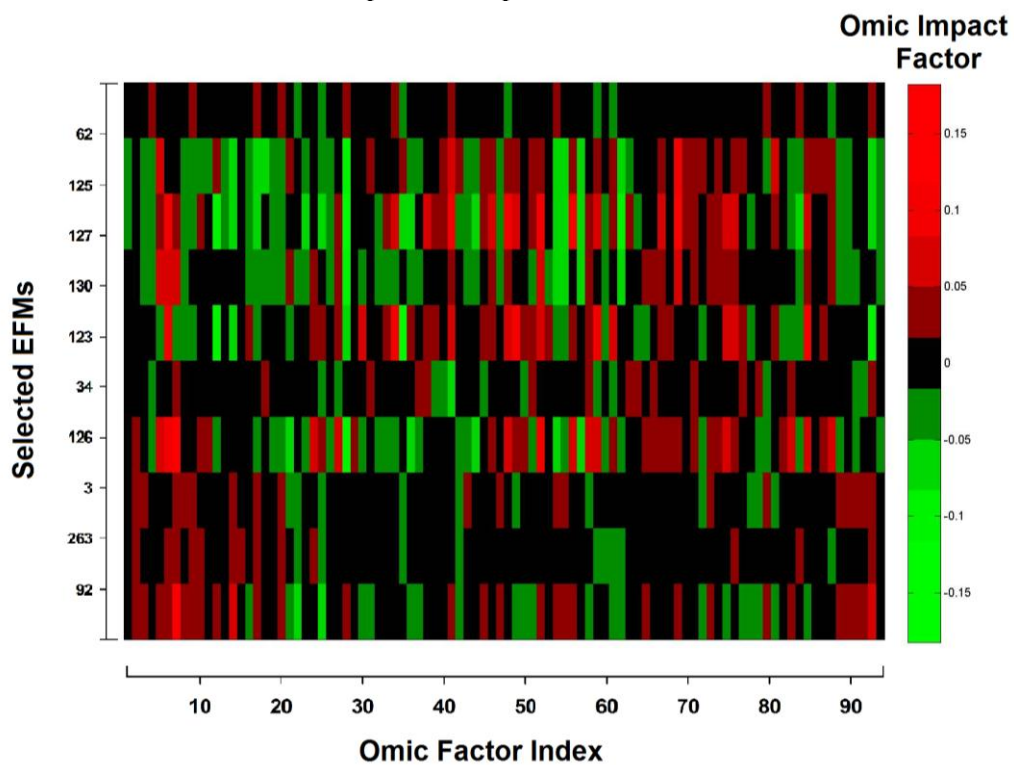
A global analysis over the Figure 3.7 shows that the main up-regulation effects on the EFMs 92, 126 and 127 undergoes by the last environmental factor (high concentrations of succinate and pyruvate in the culture medium) and by the sixty-ninth transcriptional factors, *fumC*, in the EFM 127.

On the other hand, the negative influences are more distributed over all the metabolic functions. However, they are more present in the last selected EFMs (125 and 127). Such down-regulation is clear in the tenth to twentieth omic index factor zone.

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**Figure 3.6:** Measured (blue bar) and predicted (grey bar) metabolic flux for the reference strain grown at  $0.2\text{h}^{-1}$  as dilution rate (A) and mutant *talB* (B) both present in validation partition of PLP algorithm with envirome and transcriptome as input data.



**Figure 3.7:** Regression coefficients between the input components (envirome – index 1 to 9 – and transcriptome – index 10 to 94) and selected EFMs



## **4 Discussion**



### 4.1 Relevance of omic information for flux-phenotype prediction

In the previous section, flux prediction models were developed based on single or multiple omic information layers. An important aspect in the analysis of these results is assessing the relevance of the information contained in the different omic layers in respect to the final goal of flux-phenotype prediction.

The first noteworthy observation is the apparent very high explained variance of flux data achieved when the regulatory transcriptional dataset is the only input to PLP. This model explains the highest percentage of flux variance of the calibration dataset (see Table 3.6). It is unquestionable that the regulatory transcriptome (only dataset with information for the regulatory proteins) hold very relevant information about the up- or down-regulation of the metabolic pathways. However the most likely explanation for this good performance is probably the much lower number of data points available of DNA microarrays data for the different strains (nine against 31 for the other biological layers); this implied a much lower number of points for model fitting leading to a lack of trustworthiness in these results. Still it is very important to generate enough data to evaluate this dataset performance and compare it to the other biological layers.

As for the proteome model, one would expect such dataset to be more directly related with the target fluxome than the regulatory transcriptome, since the proteome holds information linked to enzymatic activity. On the other hand, the regulatory transcriptome still needs to be translated in order to have an effect in the transcriptional regulation of the metabolic genes. Only then an effect in the metabolic network can be noticed, normally with a significant time delay associated. This was however not translated into a better model with the percentage of variance captured of the fluxome data being only 64.4% compared to 91% for the regulatory transcriptome (see Table 3.6), for the validation partition. Despite the complex dynamics with time delays, it was here hypothesized that this dataset could be useful when used together with some other inputs since this data encodes information that does not appear directly in any other biological layer. This was impossible to show due to the low number of measurements and potential overfitting when this dataset was joined together with other datasets (see Table 3.6). Moreover, this dataset is also comprised by high levels of noise, which complicates even further the analysis.

The PLP model with the metabolome as single input shows the lowest explained variance of the fluxome data set. This is not a surprising result given the more stable nature of the metabolome information layer when compared with the other omic information layers. In fact, when cells undergo a given genetic or environmental perturbation the resulting proteome and transcriptome dynamical responses have normally the objective of maintaining the concentrations of the various metabolites relatively constant in order to assure cellular viability. Indeed, despite the fact that the metabolome was the most complete biological layer quantified, fluxome

explained variance was substantially lower than others much more incomplete layers of information such as the proteome (model P) or the envirome (model E).

The PLP model that uses the envirome dataset as input (model E) is the second best performing model in terms of explained variance of fluxome data and also the one with lowest mean squared error of prediction (see Table 3.6 and Table 3.7, respectively). This result is concordant with the research of Kell *et al.* and Allen *et al.*. These studies have shown that the number of metabolites excreted or secreted to the environment is very high and that the variations of the envirome are much higher than the metabolome and also more easily and accurately measured [57]. These authors have clearly demonstrated through statistical analysis that exometabolome data enables to discriminate single gene deletion mutants of yeast cells [58]. Although the characterization of the envirome in this work was the most incomplete in relation to the other omic layers, the results obtained in the present thesis clearly support the discrimination of single gene deletion mutants using the envirome only. But even further than that, it is shown here that the envirome analysis enables not only the discrimination but also an accurate prediction of flux phenotype of the KO mutants.

It could be argued that the regulatory transcriptome responds to the external impulses and, although it may have a time lapse, it comprises more or complementary information to the partial environmental data used in this work (only organic compounds). Contrary to this hypothesis, the results obtained with the E+T model, although quite good (the second best), they do not show a significant improvement in relation to the envirome model.

## 4.2 Envirome flux prediction analysis

In this section, it will be discussed the results obtained with the envirome as input to PLP model.

### 4.2.1 Discriminated EFMs

The first EFM selected by PLP is EFM 92, with 85% of frequency of selection (calculated by the bootstrapping method – Figure 3.1). EFM 92 is partially constituted by glycolysis and tricarboxylic acid cycle, and it belongs to the small group of EFMs that exclusively produces energy. Moreover, within the analysed KO mutants group present in both calibration and validation partitions (see section 2.6.3 on page 23), EFM 92 is a plausible EFM in all KO mutants except in the strain in which *pgi* gene is deleted, which adds consistency to this result.

Others frequently selected EFMs are EFMs 209 and 232 (see Table 3.1). These EFMs together with EFM 92 comprise all the metabolic reactions involved in the three considered pathways (glycolysis, tricarboxylic acid cycle and pentose phosphate pathway) in such a combination that at least one of them is in theory plausible in every *E. coli* strain considered. As said before the EFM 92 is the only pathway that functions exclusively for energy production. All

the remaining EFMs besides energy also produce additional building blocks that could be used in biosynthesis (acetate in the case of EFM 209 and fructose-6-phosphate and erythrose-4-phosphate in the case of EFM 232). Depending on the global EFMs reaction products, the EFMs functions can be inferred through the analysis of the fate of such end-products in the cell. For instance, fructose-6-phosphate can be used in the production of others hydrocarbons or in the production of the cellular wall, namely through the production of peptidoglycan. On the other hand, the erythrose-4-phosphate utilization can be traced to the production of several amino acids like tryptophan, phenylalanine and tyrosine. All the remaining EFMs functions listed in Table 3.1 can be consulted in the supplementary material through the participating reactions and its global products (as described by Schwartz, *et al.* [59]).

The second layer of frequently selected EFMs is formed by EFMs 3, 202, 205 and 263. However they are usually selected in two sets (EFM 3+263 and EFM 202+205). The two sets confer the same functionality, namely the production of oxaloacetate, pyruvate and acetyl coenzyme-A required for biosynthesis. These are vital precursors for the cell to grow, whose fluxes carry a significant variance that needs to be explained. What seems to happen is that for some KO mutants EFM 3+263 are unfeasible while for other it is the group EFM 202+205 that is unfeasible. This justifies the mutually excluding selection observed in the PLP results.

The EFM selected with lowest frequency is EFM 34. This may be related to the fact that the metabolic function of this EFM is redundant with the function of EFM 232, *i.e.*, both EFMs produce erythrose 4-phosphate. Moreover, this EFM is only feasible in *pgl*, *ripA* and *tktA E. coli* strains, so this may also be one fact for the EFM selection since these cases may have unexplained fluxes that have been fully explained in the remaining cases.

### 4.2.2 Predictive power

As shown in Figure 3.2, the envirome model succeeds in the prediction of the fluxome of several gene deletion mutants with high accuracy. This result is to some extent surprising in light of the potential perturbations that the deletion of a gene can have in the physiology of a cell, which includes perturbations in the metabolome, proteome and transcriptome.

As said before, usually, after the internal (genetic) perturbation, not many metabolic reactions are in theory directly affected. When compared to the full size metabolic network, such perturbations are very localized to the metabolic reactions tightly related to the deleted one. This localized nature of genetic perturbations could be confirmed through the analysis of the metabolome, proteome and transcriptome.

One example is the *talB* single gene deletion mutant (Figure 3.3B). In this strain the concentration of sedoheptulose 7-phosphate, a reactant that participates in the metabolic reaction catalysed by the removed protein (reaction 16 in the metabolic network) drastically increased. Moreover, the concentration of some others metabolites not directly related to this protein also

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changed (like the concentration of glucose-6-phosphate, ribose-5-phosphate and ribulose-5-phosphate). The effects of these perturbations can also be traced in the proteome (since AcnB expression index reduced to half and SdhB expression index doubled, both proteins participate in the tricarboxylic acid cycle).

Nevertheless, despite the visible but localized perturbations in the biological system, as whole one can conclude that the deletion of a single gene does not have a dramatic effect on the metabolism. An important indication of this is that all the analysed mutants are viable. Particularly, it seems that the deletion of a single gene does not perturb significantly the relationship between the cellular metabolism and the envirome, which is likely to be conserved within the same species. This could explain the success of the envirome model in predicting several single gene deletion mutants.

Among the 43 metabolic reactions, the last reaction in the network is systematically over predicted with a higher error associated. This can be observed in the Figure 3.2 as the two highest fluxes correspond to the referred metabolic reaction in the *E. coli* strain grown at  $0.5\text{h}^{-1}$  and  $0.7\text{h}^{-1}$ . Reaction 43 is peculiar in that it codes for the exit of  $\text{CO}_2$  from the system and is involved in all the EFMs. This modelling discrepancy is mostly associated to the difficulty in closing the carbon balance in the identified reduced metabolic models with a small number of EFMs.

### 4.2.3 Envirome function mapping

In this section, the identified relation patterns between envirome components and metabolic functions are analysed on the basis of the regression coefficients matrix delivered by the PLP algorithm (see Table 3.8 and Figure 2.3).

As first note, it should be commented that the computation of confidence intervals of regression coefficients by the bootstrapping method clearly shoes that many of the values obtained are not statistically consistent, precluding taking rigorous conclusions about the effect of several envirome components on several metabolic functions. This is the case of, for instance, EFM 34. This EFM is only feasible in *pgl*, *ripA* and *tktA* *E. coli* strains. So, since there are few data points that can be used to calculate these regression coefficients, their confidence intervals are very large. Moreover, EFM 34 has a similar function to the EFM 232, which is also theoretically feasible in the referred strains. Framed by these practical constraints, the discussion that follows will be centred in those results that are statistical consistent.

So, one of the first things that stands out is the generally strong positive correlation between the weighting factors of almost all the EFMs and the glucose concentration and also the reaction dilution rate (see Table 3.8). These results are very consistent with the fact that glucose is the single carbon source and also that all the EFMs have either an energy production function or a biosynthetic function. As such, the higher the glucose concentration or the dilution rate value, the higher are the fluxes through the active pathways. Since glucose is the only carbon source in this

system, a positive regulation coefficient should be the most likely outcome.

In the same way, the acetate, ethanol and (D- and L-) lactate regression coefficients show mostly a positive correlation with the pathways presented in the Table 3.8. This may suggest that when the fluxes upstream to the tricarboxylic acid cycle are very high they might be deflected to the production of these by-products. In this case, the positive correlation does not correspond to a regulatory effect triggered by environmental factors. Rather it represents an environmental change caused by an overflow in the metabolic pathways, which resulted in the accumulation of metabolic by-products in the extracellular medium. This condition is especially obvious in the case of EFM 202. The main product of EFM 202 is acetate, thus not surprisingly, EFM 202 weighting factor shows positive correlation with acetate concentration in the medium.

In another example, formate can be produced as an incomplete oxidized by-product of the formation of acetyl coenzyme-A from pyruvate (not included in the metabolic network). As expected the accumulation of formate in the medium correlates positively with EFM 92, which includes a reaction composed by the formation of acetyl coenzyme-A from pyruvate, and negatively with EFM 202, which does not include the referred reaction. Both regression coefficients have low CI suggesting a high confidence in this result.

Until now the discussion was focused on global interaction patterns between the envirome and fluxome. However, this relationship is the result of several molecular level interaction mechanisms that goes across the multiple omic layers. The envirome components can interact with genes through sensing and regulatory proteins, which will activate or repress their expression [60]. Environmental factors can also exhibit, for instance, some allosteric interactions with enzymes (metabolites) or have other regulatory effects caused by abiotic factors such as temperature [61]. In many cases, a given environmental perturbation is followed by changes (either they are in a set of proteins activities or the transcriptional expression of some genes), which eventually will influence the fluxome.

So, looking closely at the regulation properties, for instance in the case of pyruvate, the effect can be traced through the regulatory protein IclR, in which pyruvate works as ligand. It has been reported [62] that when pyruvate is present in high concentration, a AceB-IclR-pyruvate complex is formed as a safety mechanism to quickly repress the glyoxylate shunt. In Table 3.8 can be seen that pyruvate correlates negatively with the EFM 34 that involves the glyoxylate shunt (reactions 26 and 27).

Furthermore, the reflex of this effect can be seen in the *ripA* and *tktA* mutant. In these cases the extracellular and intracellular pyruvate concentration is one of the highest among all mutants. This had the consequence of decreasing the protein expression from the operon *aceBAK* (also the concentration of these proteins are one of the lowest in the referred cases when compared with the others mutant strains). This might be also a reason for the negative regression

coefficients between pyruvate and EFM34.

Other metabolic effects can also be seen in the relationship between the PykF expression index and the extracellular pyruvate concentration, which show a positive correlation, that is, when the protein expression increases, pyruvate concentration also increases. The protein expression increase may occur due to the increase of fructose-1-phosphate intracellular concentration, which is the only metabolite with reported regulation properties of this gene expression (if the metabolite is present, the FruR transcriptional regulator becomes inactive and does not block the *pykF* transcriptional initiation [62]). A plausible explanation for the fructose-1-phosphate concentration increase is the increase of others closely related metabolites like fructose-6-phosphate and fructose-1,6-phosphate, involved in glycolysis. So, if these metabolites increase the remaining glycolysis metabolite can also increase, namely the pyruvate intracellular concentration (last metabolite in glycolysis) and so, when pyruvate intracellular concentration is high the respective extracellular concentration also increases. Again, this may be one of the regulation mechanism reflected in the regression coefficient of pyruvate in EFM 92, which involves the reaction 8 (production of pyruvate from phosphoenolpyruvate, catalysed by PykF).

Similar mechanisms can be found with regulatory protein LldR which senses L-lactate. However, it has little influence in the protein concentration because it is maintained constant in the majority of the referred cases. This may be explained by others regulatory mechanisms that influences this metabolic function, like for instance translation regulation or its turnover process.

### 4.3 Envirome and transcriptome flux prediction analysis

Here discussion is focused on the differences between the envirome (E) and the envirome and transcriptome (E+T) models rather than exhaustive regulation interpretation.

#### 4.3.1 Discriminated EFMs

When comparing Table 7.5 with Table 3.1, it is clear that the global prediction power of both methods is comparable (87.1% and 88.7%, respectively). The same occurs in the explained flux variance in the validation partition (88.5% and 85.1%, respectively). It is unquestionable that the transcriptome dataset may have important information for the flux prediction, yet it can also have some redundant information or some transcriptional evidence that is not relevant for these metabolic functions. This may complicate model development eventually resulting in a degradation of overall flux prediction results (as has been described, the mRNA concentration does not have a direct relationship between the protein concentration nor with its metabolic rate rather it depends on many other interactions besides the described one [63]).

However, when it is added this dataset to the environmental information, there are some modification in the selected metabolic function. For instance, in this PLP run there were selected



the 2-keto-glutarate, phosphoenolpyruvate, 3-phosphoglycerate, glyceraldehyde 3-phosphate and ethanol production, which was not been selected when the envirome dataset were given to PLP as only input. This means that the metabolic function of, for instance, production of some amino acids like tyrosine (through the utilization of 2-keto-glutarate), or serine (using 3-phosphoglycerate) are in some way related with the transcriptional level of the analysed genes. However, if there were to be determined exhaustively all the environmental factors concentration probably the flux prediction could be even better or it could select more metabolic function.

### 4.3.2 Input data to function relationship

Just as illustrative example, some of the omic factors that always show positive correlation with the selected EFMs are succinate and *acs*, column six and 64 from Figure 3.7. In the former case, it may be related with the utilization of succinate in almost all the selected EFMs (except in the last one, which has the lowest regression coefficient, 0.0053 and with relatively high confidence interval, 0.072). On the other hand, the other example given is the *acs* gene, which codes for the acetyl-CoA synthetase. Although the formation of acetyl-CoA is included in the metabolic network, this enzyme is not directly involved since the catalysed reaction involves the consumption of, for instance, acetate (which corresponds to the inverse reaction 29 in Figure 2.1). Nevertheless, this relationship also makes sense, since all the EFMs selected have the production of acetyl-CoA. So, although the mRNA production does not have a direct relationship with the protein level nor with its enzymatic rates, this regulation property was expected due to the close relationship between the acetyl-CoA production and the EFMs activation status.

On the other end are the other omic factors that have a consistently negative influence in all the selected EFMs like *fbp* and *lldd* mRNA expression (16 and 78 columns in Figure 2.1, respectively). These two genes code for two enzymes (fructose-1,6-bisphosphatase I and L-lactate dehydrogenase) that catalyse two reactions that are not represented in the metabolic network. Yet they represent the inverse reaction of reaction 3 and 30, respectively. In this way it is obvious that they should have a negative influence in the EFMs that involve those reactions.



## **5 Conclusions**



## 5. Conclusion

In this thesis, mathematical modelling tools were employed for the interpretation of different layers of omic information of *E. coli* cells, collected from 24 single gene deletion mutant experiments and WT strain experiments grown in 5 different conditions. In particular, a hybrid computational algorithm was employed to identify correlation patterns between input omic data sets (envirome, metabolome, proteome, transcriptome and regulatory transcriptome) with target fluxome data sets. In the adopted algorithm, the target fluxome data was deconvoluted into weighting factors of metabolic functions, which were linearly regressed against many input variables. This enabled to map in a robust way redundant input data sets into sets of active metabolic functions. Several combinations of input information were assessed in terms of achieved prediction power of the target fluxome data.

One of the main conclusions to be taken from this study is that the envirome layer of information is very important for metabolic function inference and ultimately metabolic flux prediction. It should be highlighted in particular the ability of the PLP algorithm to predict the fluxome of genetically modified strains or WT strains grown at different conditions on the basis of envirome data and genome information alone.

The analysis of the PLP regression coefficients showed that the regression coefficients could be used to infer envirome regulation mechanisms. Several of the identified regulation mechanisms could be confirmed by the analysis of the remaining datasets. Such regulatory mechanisms were previously described in other papers. However, it should be underlined that these regulation factors are computed at a pathway level, so the most straightforward interpretation scenario is that such regulatory coefficients are related to the rate-limiting step within each EFM.

As corollary of the results above, one can conclude that the relationship between envirome and metabolic function is to some extent conserved among genotype variants lacking different genes. The deletion of a particular metabolic gene has the effect of deleting one or several metabolic functions within the cells. Interestingly, this does not perturb the remaining viable functions. As future study, it should be considered the full envirome data acquisition for a genome wide fluxome prediction. It should also be studied if the environmental factors regulation patterns are conserved not only among genotype variants of the same species but also among different species, in different phylogenetic classes or even in different genus.

It is well known that the optimization of environmental factors, such as medium composition or reactor nutrients feeding, play an important role in industrial biotechnological process performance. Historically, these two factors (medium composition and genetic modifications) have contributed to the increase of bioprocess performance. In a way, the results produced in this thesis confirm the importance of the envirome information layer to infer and predict metabolic functions. Furthermore, an important lesson to be learned is that the envirome

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layer can be used for effective metabolic function engineering. It can be used to systematically discover regulatory patterns over metabolic functions, paving the way for a function oriented environment engineering approach.

Finally, besides the many developed and applied modelling methods, the PLP algorithm might have an important role in the future to unravel the regulatory patterns in redundant biological systems that have not yet been mechanistically described in the literature. This can be performed not only at an environmental level, but also at the level of metabolites and proteins, particularly for elucidation of the role of transcription factors. With this knowledge, standard biological parts can be better designed as building block for pathway level synthetic biology developments.

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## **7 Appendix**



## 7.1 Appendix A

Ordered list of metabolites present in the metabolome dataset.

- (Methylthio) acetate
- Malonyl-CoA
- 1,3-Diaminopropane
- 1,4-Butanediamine
- 1,5-Diaminopentane
- 1,5-Diphenylcarbohydrazide
- 10-Hydroxydecanoate
- 1-Adamantanamine
- 1-Amino-1-cyclopentanecarboxylate
- 1-Aminocyclopropane-1-carboxylate
- 1-Aminoethylphosphonate
- 1-Methyl-2-pyrrolidone
- 1-Methyl-4-phenyl
- 1-1,2,3,6-tetrahydropyridine
- 1-Methyladenine
- 1-Methyladenosine
- 1-Methylnicotinamide
- 1-Methylhistamine
- 2,3-Diaminopropionate
- 2,3-Diphosphoglycerate
- 2,4-Diaminobutyrate
- 2,4-Dihydroxypyrimidine-5-carboxylate
- 2,4-Dimethylaniline
- 2,5-Dihydroxybenzoate
- 2,6-Diaminoheptanedioate
- 2,6-Diethylaniline
- 2,6-Dimethylaniline
- 2-Acetamido-1-amino-1,2-dideoxyglucopyranose
- 2-Amino-2-(hydroxymethyl)-1,3-propanediol
- 2-Amino-2-methyl-1,3-propanediol
- 2-Amino-3-phosphonopropionate
- 2-Aminobenzimidazole
- 2-Aminoethylphosphonate
- 2-Aminophenol
- 2-Carboxybenzaldehyde
- 2-Cyanopyridine
- 2'-Deoxyadenosine
- 2'-Deoxyadenosine+
- 5'-Deoxyadenosine
- 2'-Deoxycytidine
- 2-Deoxyglucose 6-phosphate
- 2'-Deoxyguanosine
- 2-Deoxyribose 1-phosphate
- 2-Deoxystreptamine
- 2'-Deoxyuridine
- 2-Furoate
- 2-Guanidinobenzimidazole
- 2-Hydroxy-4-methylpentanoate
- 2-Hydroxyoctanoate
- 2-Hydroxypentanoate
- 2-Hydroxypyridine
- 2-Isopropylmalate
- 2-Mercapto-1-methylimidazole
- 2-Oxadipate
- 2-Oxoglutarate
- 2-Oxoctanoate
- 2-Oxopentanoate
- 2-Quinolinecarboxylate

## Hybrid Systems Biology: Application to *Escherichia coli*

- 2-Thiopheneacetate
- 3-(2-Hydroxyphenyl)propionate
- 3,3',5-Triiodothyronine
- 3,5-Diiodo-tyrosine
- 3,5-Dinitrosalicylate
- 3-Acetylacrylate
- 3-Amino-1,2,4-triazole
- 3-Amino-1,2-propanediol
- 3-Amino-3-(4-hydroxyphenyl) propionate
- 3-Aminopropionitrile
- 3-Chloroalanine
- 3-Hydroxy-3-methylglutarate
- 3-Hydroxyanthranilate
- 3-Hydroxykynurenine
- 3-Indoleacetonitrile
- 3-Indolebutanoate
- 3-Indoxyl sulphate
- 3-Iodotyrosine
- 3-Methyladenine
- 3-Methylbutanoate
- 3-Methylguanine
- 3-Methylhistidine
- 3-Phenyllactate
- 3-Phenylpropionate
- 3-Phosphoglycerate
- 3-Ureido propionate
- 4,4'-Methylene bis (o-chloroaniline)
- 4-Acetylbutanoate
- 4-Amino-3-hydroxybutyrate
- 4-Aminoindole
- 4-Aminoindole+5-Aminoindole
- 4-Aminophenylsulfone
- 4-Hydroxy-3-methoxybenzoate
- 4-Hydroxy-3-methoxymandelate
- 4-Hydroxyindole
- 4-Hydroxymandelate
- 4-Hydroxymethylimidazole
- 4-Methyl-2-oxopentanoate
- 4-Methyl-5-thiazoleethanol
- 4-Methylpyrazole
- 4-Methylthio-2-oxobutanoate
- 4-Nitrophenyl phosphate
- 4-Oxohexanoate
- 4-Oxopentanoate
- 4-Pyridoxate
- 4-Sulfobenzoate
- 5,6-Dimethylbenzimidazol
- 5-Aminoimidazole-4-carboxamide-1-ribofuranosyl 5'-monophosphate
- 5-Aminoindole
- 5-Aminolevulinate
- 5-Aminopentanoate
- 5'-Deoxy-5'-Methylthioadenosine
- 5'-Deoxyadenosine
- 5-Hydroxy-3-indoleacetate
- 5-Hydroxylysine
- 5-Hydroxytryptophan
- 5-Methoxy-3-indoleacetate
- 5-Methoxy-N,N-dimethyltryptamine
- 5-Methoxytryptamine
- 5-Methoxytryptamine+2,6-Diaminoheptanedioate
- 5-Methyl-2'-deoxycytidine
- 5-Methylcytosine
- 5-Methyltetrahydrofolate
- 5-Oxoproline
- 5-Phosphorylribose 1-pyrophosphate
- 6,8-Thioctate



- 6-Aminohexanoate
- 6-Aminopenicillanate
- 6-Hydroxyhexanoate
- 6-Hydroxynicotinate
- 6-Methylaminopurine
- 6-Phosphogluconate
- 7,8-Dihydrobiopterin
- 7,8-Dihydroneopterin
- 7-Methylguanin
- 8-Anilino-1-naphthalene sulfonate
- 8-Hydroxyoctanoate
- 9-Amino-1,2,3,4-tetrahydroacridine
- Acetanilide
- Acetoacetamide
- Acetohydroxamate
- Acetylcholine
- Adenine
- Adenosine
- Adenylosuccinate
- Adipate
- Adenosine diphosphate
- Adenosine diphosphate-glucose
- Adenosine diphosphate-ribose
- Agmatine
- alpha-Hydroxybutanoate
- alpha-Hydroxybutanoate+  
beta-Hydroxybutanoate
- alpha-Hydroxyisobutanoate
- Alanine
- Alanine-Alanine
- Albizziine
- Allantoate
- Allantoin
- Alliin
- alpha-Aminoadipate
- alpha-Aminoisobutyrate
- alpha-Methylbenzylamine
- alpha-Methylserine
- Adenosine monophosphate
- Aniline
- Anserine
- Anserine + Homocarnosine
- Anthranilate
- Arginine
- Arginine ethyl ester
- Argininosuccinate
- Asparagine
- Aspartic acid
- Aspartate
- Adenosine triphosphate
- Azelaate
- Azetidine-2-carboxylate
- Barbiturate
- Benzamide
- Benzamidine
- Benzimidazole
- Benzoate
- Benzoylformate
- Benzyl viologen
- Benzylsuccinate
- beta-Alanine
- beta-Alanine-Lys
- beta-Cyanoalanine
- beta-Guanidinopropionate
- beta-Hydroxyphenethylamine
- beta-Imidazolelactate
- Betaine
- Betaine aldehyde
- beta-Hydroxybutanoate
- beta-Hydroxypropionate

## Hybrid Systems Biology: Application to *Escherichia coli*

- Biotin
- Bis(3-aminopropyl)amine
- Bis(p-nitrophenyl)phosphate
- Butanoate
- Cyclic adenosine monophosphate
- Canavanine
- Carbachol
- Carbamoylaspartate
- Carnitine
- Carnosine
- Castanospermine
- Cyclic cytosine monophosphate
- Cytosine diphosphate
- Cyclic guanine monophosphate
- Chelidonate
- Cholate
- cis,cis-Muconate
- cis-Aconitate
- Citraconate
- Citramalate
- Citrate
- Citrate
- Citrulline
- Cytosine monophosphate
- Cytosine monophosphate+Cytosine monophosphate -N-acetylneuramate
- Cytosine monophosphate-N-acetylneuramate
- Creatine
- Creatinine
- Crotonate
- Cyclic thymine monophosphate
- Cytosine triphosphate
- Cumate
- Cyclohexanecarboxylate
- Cysteine
- Cysteine-Glycine
- Cystathionine
- Cysteamine
- Cysteate
- Cysteine S-sulfate
- Cysteine sulfinat
- Cytidine
- Cytosine
- Deoxyadenosine diphosphate
- Deoxyadenosine monophosphate
- Deoxyadenosine triphosphate
- Deoxycytosine diphosphate
- Deoxycytosine monophosphate
- Deoxycytosine triphosphate
- Decanoate
- Deisopropylatrazine
- Desethylatrazine
- Desthiobiotin
- Diethanolamine
- Diethyl-2-phenylacetamide
- Digalacturonate
- Dihydroorotate
- Dihydrouracil
- Dihydroxyacetone phosphate
- Dihydrozeatine
- Diphenylamine
- Divalent ion from Acetyl coenzyme A
- Divalent ion from coenzyme A
- Divalent ion from D111\*
- Divalent ion from Isobutyryl coenzyme A
- Divalent ion from Lauroyl coenzyme A

- Divalent ion from Malonyl coenzyme A
- Divalent ion from nicotinamide adenine dinucleotide phosphate (NADPH)
- Divalent ion from n-Propionyl coenzyme A
- Divalent ion from guanosine tetraphosphate
- Divalent ion from succinyl coenzyme A
- Divalent ion from uridine diphosphate -glucuronate
- Uridine diphosphate -N-acetylglucosamine
- Dodecanedioate
- Dodecanoate
- L-3,4-dihydroxyphenylalanine
- Dopamine
- Ectoine
- Eflornithine
- Epinephrine
- Erythrose 4-phosphate
- Ethanolamine phosphate
- Etidronate
- Flavin adenine dinucleotide
- Flavin mononucleotide
- Folate
- Fructose 1,6-diphosphate
- Fructose 6-phosphate
- Fumarate
- Fumarate
- gamma-Aminobutyric acid
- Galacturonate
- Galacturonate 1-phosphate
- gamma-Guanidinobutyrate
- Guanosine diphosphate
- Gibberellate
- Glutamine
- Glutamine+Albizzine
- Gluconate
- Glucosaminatate
- Glucosamine
- Glucosamine 6-phosphate
- Glucose 1-phosphate
- Glucose 6-phosphate
- Glucuronate
- Glutamic acid- Glutamic acid
- Glutamate
- Glutarate
- Glutathione
- Glycine
- Glycerate
- Glycerophosphate
- Glycerophosphorylcholine
- Glycocholate
- Glycolate
- Glycyrrhetinate
- Glycine-Glycine
- Glycine-Leucine
- Glyoxylate
- Guanosine monophosphate
- Gramine
- Guanosine triphosphate
- Guanidinosuccinate
- Guanidoacetate
- Guanine
- Guanosine
- Harman
- Heptanoate

## Hybrid Systems Biology: Application to *Escherichia coli*

- Hexamethylene tetramine
- Hexanoate
- Hexylamine
- Hippurate
- Histidine
- Histamine
- Histidinol
- Homoarginine
- Homocarnosine
- Homocysteine
- Homocystine
- Homoserine
- Homovanillate
- Hydroxyatrazine
- Hydroxyproline
- Hydroxyurea
- Hypotaurine
- Hypoxanthine
- Ibotenate
- Inosine diphosphate
- Isoleucine
- Isoleucine+ Leucine
- Imidazole-4-acetate
- Inosine monophosphate
- Indole-3-acetaldehyde
- Indole-3-acetamide
- Indole-3-acetate
- Indole-3-ethanol
- Inosine
- Isatin
- Isethionate
- Isoamylamine
- Isobutylamine
- Isocitrate
- iso-Citrate
- Isocitrate + Citrate
- Isonicotinamide
- Isonicotinate hydrazide
- Isopropanolamine
- Isoquinoline
- Itaconate
- Inosine triphosphate
- Keramine
- Kynurenine
- Lactate
- Leucine
- Leucine-Leucine-Tyrosine
- Leupeptin hemisulfate
- Lumazine
- Lysine
- Malate
- Malonate
- Malonyl coenzyme A
- Mandelate
- Mannosamine
- Melamine
- Melatonin
- Mesalamine
- Methionine
- Metformin
- Methanesulfonate
- Methionine sulfoximine
- Methyl sulfate
- Methylguanidine
- m-hydroxybenzoate
- Mucate
- Muramate
- Muscimol
- N,N-Dimethylaniline  
+Phenethylamine

- N,N-Dimethylaniline
- N,N-Dimethylglycine
- N1-Acetylspermine
- N6-Methyl-2'-deoxyadenosine
- N8-Acetylspermidine
- N-Acetyl muramate
- N-Acetylaspartate
- N-Acetyl-b-alanine
- N-Acetylgalactosamine 6-sulfate
- N-Acetylglucosamine
- N-Acetylglucosamine 1-phosphate
- N-Acetylglucosamine 6-phosphate
- N-Acetylglutamate
- N-Acetylhistidine
- N-Acetylleucine
- N-Acetylmethionine
- N-Acetylneuraminic acid
- N-Acetylornithine
- N-Acetylphenylalanine
- N-Acetylputrescine
- Nicotinamide adenine dinucleotide (NAD<sup>+</sup>)
- Nicotinamide adenine dinucleotide (NADH)
- Nicotinamide adenine dinucleotide phosphate (NADP)
- Nicotinamide adenine dinucleotide phosphate (NADPH)
- Nalpha,Nalpha-Dimethylhistidine
- Nalpha-Benzenolarginine ethylester
- N-a-t-boc-asparagine
- N-Benzoyloxycarbonylglycine
- N-Carbamylglutamate
- Neamine
- Nepsilon-Acetyllysine
- N-ethylglutamine
- N-Formylaspartate
- N-Formylmethionine
- N-Glycolylneuraminic acid
- Nicotinamide
- Nicotinamide hypoxanthine dinucleotide
- Nicotinate
- Nicotine
- N-Methylalanine
- N-Methylaniline
- N-Methylantranilate
- N-Methylglutamate
- N-Methyl-N-propagylbenzylamine
- Nomega-Acetylhistamine
- Nomega-Methyltryptamine
- Nonanoate
- Noradrenaline
- Nornicotine
- n-Propionyl CoA
- o-Acetylcarnitine
- o-Acetylserine
- Octanoate
- Octopine
- Octylamine
- o-Hydroxybenzoate
- o-Hydroxyhippurate
- o-Phenanthroline
- o-Phosphoserine
- Ornithine
- Orotidine 5'-monophosphate
- o-Succinylhomoserine
- Oxamate
- Oxidized glutathione
- p-Aminobenzoate

## Hybrid Systems Biology: Application to *Escherichia coli*

- Pantothenate
- p-Coumarate
- Pentanoate
- Phenylalanine
- Phenaceturate
- Phenethylamine
- Phenyl phosphate
- Phenylhydrazine
- Phloretate
- Phosphoarginine
- Phosphocreatine
- Phosphoenolpyruvate
- Phosphonoacetate
- Phosphoramidon
- Phosphorylcholine
- Phthalate
- p-hydroxybenzoate
- p-Hydroxybenzoate+
- m-Hydroxybenzoate
- p-Hydroxyphenylacetate
- p-Hydroxyphenylpyruvate
- Picolinamide
- Pimelate
- Pipecolate
- Piperazine
- Piperidine
- Porphobilinogen
- Guanosine tetraphosphate
- Propionate
- Prostaglandine2
- ProstaglandinF2a
- Pseudopelletierine
- Psychosine
- Pterin
- Purine
- Purine riboside
- Pyridine
- Pyridoxal
- Pyridoxal 5-phosphate
- Pyridoxamine
- Pyridoxamine 5'-phosphate
- Pyridoxine
- Pyrimidine
- Pyrrole-2-carboxylate
- Pyruvate
- Quinate
- Quinoline
- Quisqualate
- Riboflavin
- Ribose 5-phosphate
- Ribulose 1,5-diphosphate
- Ribulose 5-phosphate
- Saccharate
- S-Adenosylhomocysteine
- S-Adenosylmethionine
- Sarcosine
- S-Carboxymethylcysteine
- Scopolamine
- Sebacate
- Sedoheptulose-7-phosphate
- Sepiapterin
- Serine
- Serine O-sulfate
- Serotonin
- Shikimate
- Sinapate
- S-Lactoylglutathione
- Sorbitol 6-phosphate
- Spermidine
- Spermine

- Succinate
- Succinimide
- Sulfanilate
- Synephrine
- Syringate
- Tartarate
- Taurine
- Taurochenodeoxycholate
- Taurocholate
- Taurodeoxycholate
- Taurodeoxycholate and Taurochenodeoxycholate
- Taurolithocholate
- Timine diphosphate
- Terephthalate
- Thiamine
- Thiamine monophosphate
- Threonine
- Threo-beta-Hydroxyaspartate
- Threonate
- Thymidine
- Thymidine+1,5-Diphenylcarbohydrazide
- Tiglate
- Timine monophosphate
- Tolazoline
- T-4-Hydroxy-3-methoxycinnamate
- T-Aconitate
- T-Cinnamate
- T-Zeatin
- Trehalose 6-phosphate
- Trientine
- Triethanolamine
- Trimethylamine N-oxide
- Trimethylsulfonium
- Tryptophan
- Tryptamine
- Tryptophanamide
- Timine triphosphate
- Tryptophanamide+O-Acetylcarnitine
- Tyrosine
- Tyramine
- Tyrosine methyl ester
- Uridine diphosphate
- Uridine diphosphate-glucose
- Uridine diphosphate-glucuronate
- Uridine diphosphate-N-acetylglucosamine
- Uridine monophosphate
- Undecanoate
- Uracil
- Uridine
- Urocanate
- Uridine triphosphate
- Valine
- Xanthopterin
- Xanthosine
- Xanthurenate
- Xanthosine monophosphate
- Glycine-Proline

## 7.2 Appendix B

Input for metatool 5.1 for the EFMs calculation

-METEXT

CO2ex G6Pex F6Pex R5Pex e4Pex G3Pex 3PGex PEPex PYRex AcCoAex OAAex 2-KGex Glucose Acetate Lactate Ethanol

-CAT

r1 : Glucose + PEP => G6P + PYR

r2 : G6P = F6P

r3 : F6P => F1,6P

r4 : F1,6P => DHAP + G3P

r5 : DHAP => G3P

r6 : G3P => 3PG

r7 : 3PG = PEP

r8 : PEP => PYR

r9 : PYR => AcCoA + CO2

r10 : G6P => 6PG

r11 : 6PG => Ru5P + CO2

r12 : Ru5P => X5P

r13 : Ru5P => R5P

r14 : R5P + X5P = S7P + G3P

r15 : S7P + G3P = E4P + F6P

r16 : X5P + E4P = F6P + G3P

r17 : AcCoA + OAA => CIT

r18 : CIT => ICT

r19 : ICT => 2-KG + CO2

r20 : 2-KG => SUC + CO2

r21 : SUC => FUM

r22 : FUM => MAL

r23 : MAL = OAA

r24 : PEP + CO2 = OAA

r25 : MAL => PYR + CO2

r26 : ICT => Glyoxylate + SUC

r27 : Glyoxylate + AcCoA => MAL



r28 : 6PG => G3P + PYR

r29 : AcCoA => Acetate

r30 : PYR => Lactate

r31 : AcCoA => Ethanol

r32 : G6P => G6Pex

r33 : F6P => F6Pex

r34 : R5P => R5Pex

r35 : E4P => E4Pex

r36 : G3P => G3Pex

r37 : 3PG => 3PGex

r38 : PEP => PEPex

r39 : PYR => PYRex

r40 : AcCoA => AcCoAex

r41 : OAA => OAAex

r42 : 2-KG => 2-KGex

r43 : CO2 => CO2ex

### 7.3 Appendix C

Table 7.1: Elementary Modes Matrix – Metabolic reactions x EFMs.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	1	2	4	1	1	1	1	2	2	1	1	1	1	1	4	1	1	1	1	1	4	3	2	4
2	1	2	4	1	1	0	1	1	2	1	1	1	1	1	4	1	1	1	1	1	4	2	1	3
3	1	2	4	1	1	0	1	1	1	1	1	1	1	1	4	1	1	1	1	1	4	2	1	3
4	1	2	4	1	1	0	1	1	1	1	1	1	1	1	4	1	1	1	1	1	4	2	1	3
5	1	2	4	1	1	0	1	1	1	1	1	1	1	1	4	1	1	1	1	1	4	2	1	3
6	2	4	8	2	2	1	2	2	2	2	2	1	2	2	8	2	2	2	2	2	8	4	2	5
7	2	4	8	2	2	1	2	2	2	2	2	1	1	2	8	2	2	2	2	2	8	4	2	5
8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	2	1	0	2	0	2	0	0	1	3	0	0	0	2	0	1	3	2	1	2	0	0	0
10	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1
17	0	1	1	0	1	0	1	0	0	0	1	0	0	0	1	0	0	1	1	0	1	0	0	0
18	0	1	1	0	1	0	1	0	0	0	1	0	0	0	1	0	0	1	1	0	1	0	0	0
19	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	1	1	0	1	0	1	0	0	0	1	0	0	0	1	0	0	1	1	0	1	0	0	0
22	0	1	1	0	1	0	1	0	0	0	1	0	0	0	1	0	0	1	1	0	1	0	0	0
23	0	2	0	-1	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0
24	0	2	4	1	1	0	1	0	0	1	1	0	0	0	4	1	1	1	1	1	4	1	0	1
25	0	0	1	1	1	0	2	0	0	0	2	0	0	0	2	1	0	2	2	0	2	0	0	0
26	0	1	0	0	0	0	1	0	0	0	1	0	0	0	1	0	0	1	1	0	1	0	0	0
27	0	1	0	0	0	0	1	0	0	0	1	0	0	0	1	0	0	1	1	0	1	0	0	0
28	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0
30	2	0	0	2	0	2	1	2	2	0	0	1	1	1	4	0	0	0	0	0	0	3	2	4
31	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
32	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
36	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
39	0	0	4	0	0	0	0	0	0	0	0	0	0	0	2	0	0	1	0	4	0	0	0	0
40	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0
41	0	3	3	0	0	0	0	0	0	1	0	0	0	0	3	0	1	0	0	1	3	1	0	1
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
43	0	0	0	0	4	0	3	0	0	0	4	0	0	0	0	0	0	4	3	0	0	0	1	0

Table 7.1 (cont.): Elementary Modes Matrix – Metabolic reactions x EFMs.

	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48
1	3	4	1	1	2	4	2	1	3	3	2	2	2	1	1	2	4	2	4	1	2	2	1	1
2	2	1	-2	1	0	-2	2	1	3	-2	0	0	0	1	0	0	2	0	2	0	1	2	1	1
3	2	3	0	1	1	1	2	1	3	0	1	1	1	1	0	1	3	1	3	0	1	1	1	1
4	2	3	0	1	1	1	2	1	3	0	1	1	1	1	0	1	3	1	3	0	1	1	1	1
5	2	3	0	1	1	1	2	1	3	0	1	1	1	1	0	1	3	1	3	0	1	1	1	1
6	3	7	1	2	2	2	4	2	6	1	2	2	2	2	0	2	6	2	6	1	2	2	1	2
7	3	7	1	2	2	2	4	2	6	1	2	2	2	2	0	2	6	2	6	1	2	2	1	1
8	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
9	0	0	0	3	2	4	1	1	1	4	2	2	2	0	2	0	0	0	0	0	0	0	0	0
10	1	3	3	0	2	6	0	0	0	5	2	2	2	0	1	2	2	2	2	1	0	0	0	0
11	1	3	3	0	2	6	0	0	0	4	2	2	2	0	0	2	2	2	2	0	0	0	0	0
12	0	2	2	0	1	3	0	0	0	2	1	1	1	0	0	1	1	1	1	0	0	0	0	0
13	1	1	1	0	1	3	0	0	0	2	1	1	1	0	0	1	1	1	1	0	0	0	0	0
14	1	1	1	0	1	3	0	0	0	2	1	1	1	0	0	1	1	1	1	0	0	0	0	0
15	1	1	1	0	1	3	0	0	0	2	1	1	1	0	0	1	1	1	1	0	0	0	0	0
16	-1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	1	2	2	1	1	1	2	0	0	0	0	1	0	0	0	0	0	0	0	0	0
18	0	0	0	1	2	2	1	1	1	2	0	0	0	0	1	0	0	0	0	0	0	0	0	0
19	0	0	0	0	2	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	2	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	1	2	2	0	0	1	2	0	0	0	0	1	0	0	0	0	0	0	0	0	0
22	0	0	0	1	2	2	0	0	1	2	0	0	0	0	1	0	0	0	0	0	0	0	0	0
23	0	0	0	0	2	4	0	0	1	4	0	0	0	0	2	0	0	0	0	0	0	0	0	0
24	0	3	0	1	0	-2	2	1	3	-2	0	0	0	0	-1	0	2	0	2	0	0	0	0	0
25	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	0	0	0	1	0	2	0	0	0	2	0	0	0	0	1	0	0	0	0	0	0	0	0	0
27	0	0	0	1	0	2	0	0	0	2	0	0	0	0	1	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0
30	3	4	1	0	0	0	1	0	2	0	0	0	0	0	0	2	4	0	0	0	0	0	0	0
31	0	0	0	1	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	2	0	0	0	1	3	0	0	0	2	1	1	1	0	0	1	1	1	1	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	2	4	2	2	2	1	1
40	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0
41	0	3	0	0	0	0	1	0	3	0	0	0	0	0	0	0	2	0	2	0	0	0	0	0
42	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
43	1	0	3	4	8	12	0	1	0	10	4	4	4	0	3	2	0	2	0	0	0	0	0	0

Table 7.1(cont.): Elementary Modes Matrix – Metabolic reactions x EFMs.

	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72
1	1	1	1	1	1	1	1	1	1	1	2	2	1	1	1	2	2	2	2	1	1	1	1	1
2	1	1	1	1	1	1	1	0	0	0	1	2	1	1	1	1	1	2	2	1	1	1	1	1
3	1	1	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
6	2	2	2	2	2	2	2	1	1	1	2	2	1	2	2	2	2	2	2	1	1	2	2	2
7	2	2	2	2	2	2	2	1	1	1	2	2	1	1	2	2	2	2	2	1	1	1	1	2
8	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	2	2	2	2	2	2	2	2	2	2	2	1	1	1	2	2	2	2	1	1	1	1	1
10	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	2	0	0	2	0	0	2	0	0	0	0	0	0	2	0	2	0	1	0	1	0
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31	0	2	0	0	2	0	0	2	0	0	2	2	1	1	1	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	1	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	1	0
38	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1
39	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	0	0	2	0	0	2	0	0	2	0	0	0	0	0	0	2	0	2	0	1	0	1	0	1
41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
43	0	2	2	2	2	2	2	2	2	2	2	2	1	1	1	2	2	2	2	1	1	1	1	1

Table 7.1 (cont.): Elementary Modes Matrix – Metabolic reactions x EFMs.

	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95
1	1	1	1	1	3	2	4	3	4	1	1	1	4	4	2	2	2	2	1	1	2	3	2
2	1	-2	1	-2	2	1	3	2	1	-2	-2	1	1	4	2	2	2	2	0	1	1	2	1
3	1	0	1	0	2	1	3	2	3	0	0	1	1	1	2	2	2	2	0	1	1	2	1
4	1	0	1	0	2	1	3	2	3	0	0	1	1	1	2	2	2	2	0	1	1	2	1
5	1	0	1	0	2	1	3	2	3	0	0	1	1	1	2	2	2	2	0	1	1	2	1
6	2	1	2	1	4	2	5	3	7	1	1	2	2	2	1	4	4	4	1	2	2	3	2
7	2	1	2	1	4	2	5	3	7	1	1	2	2	2	1	1	4	4	1	2	2	3	2
8	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	1	0	0	0
9	1	0	4	1	0	0	0	0	0	1	1	4	4	4	2	2	2	1	4	2	2	3	2
10	0	3	0	3	1	1	1	1	3	3	3	0	0	0	0	0	0	0	1	0	1	1	1
11	0	3	0	3	1	1	1	1	3	3	3	0	0	0	0	0	0	0	0	0	1	1	1
12	0	2	0	2	0	0	0	0	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0
13	0	1	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1
14	0	1	0	1	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	1	0
15	0	1	0	1	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	1	0
16	0	1	0	1	0	0	-1	-1	1	1	1	0	0	0	0	0	0	0	0	0	0	-1	0
17	0	0	2	0	0	0	0	0	0	0	0	2	2	2	1	1	1	1	2	2	0	0	0
18	0	0	2	0	0	0	0	0	0	0	0	2	2	2	1	1	1	1	2	2	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	2	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0
21	0	0	2	0	0	0	0	0	0	0	0	2	2	2	1	1	1	0	2	2	0	0	0
22	0	0	2	0	0	0	0	0	0	0	0	2	2	2	1	1	1	0	2	2	0	0	0
23	0	0	4	0	0	0	0	0	0	0	0	1	4	4	2	2	2	0	4	2	0	0	0
24	0	0	-2	0	1	0	1	0	3	0	0	1	-2	-2	-1	-1	-1	2	-2	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0
26	0	0	2	0	0	0	0	0	0	0	0	2	2	2	1	1	1	0	2	0	0	0	0
27	0	0	2	0	0	0	0	0	0	0	0	2	2	2	1	1	1	0	2	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
29	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	3	0
32	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0
34	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1
35	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0
38	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0
39	0	1	0	0	3	2	4	3	4	0	0	0	0	0	0	0	0	1	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	2
41	0	0	0	0	1	0	1	0	3	0	0	0	0	0	0	0	0	1	0	0	0	0	0
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
43	1	3	6	4	0	1	0	1	0	4	4	6	6	6	3	3	3	0	6	6	3	4	3

Table 7.1 (cont.): Elementary Modes Matrix – Metabolic reactions x EFMs.

	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119
1	2	3	3	1	1	3	3	1	1	1	1	2	2	1	1	1	3	1	3	1	1	3	5	2
2	1	2	2	0	1	0	2	0	0	0	0	1	2	1	1	1	3	1	0	-2	-2	0	0	-2
3	1	2	2	0	1	0	0	0	0	0	0	1	1	1	1	1	3	1	0	0	0	0	0	0
4	1	2	2	0	1	0	0	0	0	0	0	1	1	1	1	1	3	1	0	0	0	0	0	0
5	1	2	2	0	1	0	0	0	0	0	0	1	1	1	1	1	3	1	0	0	0	0	0	0
6	2	3	3	1	2	1	1	1	1	1	1	2	2	1	2	2	6	2	3	1	1	1	1	1
7	2	3	3	1	2	1	1	0	1	1	1	2	2	1	1	2	6	2	3	1	1	1	1	1
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
9	2	3	3	4	2	4	4	2	2	2	2	2	2	1	1	1	1	1	6	2	2	4	8	2
10	1	1	1	1	0	1	1	1	1	1	1	0	0	0	0	0	0	0	3	3	3	3	5	3
11	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	2	2	3
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	0	0	2
13	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	2	2	1
14	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	2	1
15	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	2	1
16	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	-2	1
17	0	0	0	2	2	2	2	1	1	1	2	2	2	1	1	1	1	1	4	1	1	2	4	1
18	0	0	0	2	2	2	2	1	1	1	2	2	2	1	1	1	1	1	4	1	1	2	4	1
19	0	0	0	0	2	0	0	0	0	0	2	2	2	1	1	1	1	1	2	0	0	0	0	0
20	0	0	0	0	2	0	0	0	0	0	2	2	2	1	1	1	1	1	0	0	0	0	0	0
21	0	0	0	2	2	2	2	1	1	1	2	2	2	1	1	1	1	1	2	1	1	2	4	1
22	0	0	0	2	2	2	2	1	1	1	2	2	2	1	1	1	1	1	2	1	1	2	4	1
23	0	0	0	2	1	4	4	2	2	2	2	2	2	1	1	1	1	1	4	2	1	4	8	2
24	0	0	0	0	1	-2	-2	-1	-1	0	0	0	0	0	0	0	3	1	0	-1	0	-2	-4	-1
25	0	0	0	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
26	0	0	0	2	0	2	2	1	1	1	0	0	0	0	0	0	0	0	2	1	1	2	4	1
27	0	0	0	2	0	2	2	1	1	1	0	0	0	0	0	0	0	0	2	1	1	2	4	1
28	0	0	0	1	0	1	1	1	1	1	1	0	0	0	0	0	0	0	3	0	0	1	3	0
29	2	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	2	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1
33	0	0	0	0	0	0	2	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
34	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0
35	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0
40	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
41	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	3	1	0	0	0	0	0	0
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0
43	3	4	4	6	6	6	6	3	3	2	6	6	6	3	3	3	0	2	8	6	6	8	14	6

Table 7.1 (cont.): Elementary Modes Matrix – Metabolic reactions x EFMs.

	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145
1	2	2	4	2	2	2	2	2	2	1	3	2	3	1	1	4	1	1	1	1	2	2	1	1	1	2
2	-1	-2	1	-3	1	-4	-4	-4	-4	-2	-6	1	2	1	1	4	1	1	1	1	1	2	1	1	1	1
3	0	0	1	0	1	0	0	0	0	0	0	1	2	1	1	4	1	1	1	1	1	1	1	1	1	1
4	0	0	1	0	1	0	0	0	0	0	0	1	2	1	1	4	1	1	1	1	1	1	1	1	1	1
5	0	0	1	0	1	0	0	0	0	0	0	1	2	1	1	4	1	1	1	1	1	1	1	1	1	1
6	1	1	2	1	1	1	2	2	2	1	3	2	3	2	2	8	2	2	2	2	2	2	2	1	2	2
7	1	1	2	1	1	1	1	2	2	1	3	2	3	2	2	8	2	2	2	2	2	2	2	1	1	2
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0
9	2	2	4	2	2	2	2	2	2	1	3	2	3	1	2	1	2	1	4	2	4	4	2	2	2	2
10	3	4	3	5	1	6	6	6	6	3	9	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
11	3	4	3	5	1	6	6	6	6	3	9	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
12	2	2	0	3	0	4	4	4	4	2	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	1	2	3	2	1	2	2	2	2	1	3	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
14	1	1	0	2	1	2	2	2	2	1	3	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
15	1	1	0	2	1	2	2	2	2	1	3	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
16	1	1	0	1	-1	2	2	2	2	1	3	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0
17	1	1	2	1	1	1	1	1	1	1	2	2	3	1	1	1	1	1	2	1	2	2	1	1	1	1
18	1	1	2	1	1	1	1	1	1	1	2	2	3	1	1	1	1	1	2	1	2	2	1	1	1	1
19	0	0	0	0	0	0	0	0	0	1	1	2	3	1	1	1	1	1	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	1	0	2	3	1	1	1	1	1	0	0	0	0	0	0	0	0
21	1	1	2	1	1	1	1	1	1	1	1	2	3	1	1	1	1	1	2	1	2	2	1	1	1	1
22	1	1	2	1	1	1	1	1	1	1	1	2	3	1	1	1	1	1	2	1	2	2	1	1	1	1
23	2	2	4	2	2	2	2	2	2	1	2	2	3	0	0	0	0	0	2	2	2	2	1	1	1	2
24	-1	-1	-2	-1	-1	-1	-1	-1	0	0	0	0	0	1	1	4	1	1	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	2	0	2	2	1	1	1	0
26	1	1	2	1	1	1	1	1	1	0	1	0	0	0	0	0	0	0	2	1	2	2	1	1	1	1
27	1	1	2	1	1	1	1	1	1	0	1	0	0	0	0	0	0	0	2	1	2	2	1	1	1	1
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	1	0	0	0	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1
33	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
34	0	1	3	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	0	0	0	1	2	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
37	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
38	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
39	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
41	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	3	0	0	0	1	0	0	0	0	0	1
42	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
43	6	7	9	8	4	9	9	9	8	6	13	7	10	3	4	0	4	3	6	2	6	6	3	3	3	2

# Hybrid Systems Biology: Application to *Escherichia coli*

**Table 7.1 (cont.):** Elementary Modes Matrix – Metabolic reactions x EFMs.

	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170
1	2	2	2	2	3	6	6	3	3	3	2	2	6	2	3	2	6	6	3	3	3	5	5	3	3
2	2	2	2	2	3	3	6	3	3	3	1	1	3	0	2	0	4	0	2	0	0	0	0	0	0
3	1	2	2	2	3	3	3	3	3	3	1	1	3	1	2	1	4	3	2	2	2	2	2	1	1
4	1	2	2	2	3	3	3	3	3	3	1	1	3	1	2	1	4	3	2	2	2	2	2	1	1
5	1	2	2	2	3	3	3	3	3	3	1	1	3	1	2	1	4	3	2	2	2	2	2	1	1
6	2	2	4	4	6	6	6	3	6	6	2	2	6	2	3	2	6	6	3	5	5	5	5	3	3
7	2	2	2	4	6	6	6	3	3	6	2	2	6	2	3	2	6	6	3	5	5	5	5	3	3
8	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	0	0	0	0
9	2	2	2	2	6	6	6	3	3	3	4	2	6	4	6	2	6	6	3	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	1	1	3	2	1	2	2	6	1	3	3	3	3	3	3
11	0	0	0	0	0	0	0	0	0	0	1	1	3	2	1	2	2	6	1	3	3	3	3	3	3
12	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	3	0	2	2	2	2	2	2
13	0	0	0	0	0	0	0	0	0	0	1	1	3	1	1	1	2	3	1	1	1	1	1	1	1
14	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	2	3	1	1	1	1	1	1	1
15	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	2	3	1	1	1	1	1	1	1
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	-2	0	-1	1	1	1	1	1	1
17	1	1	1	1	4	4	4	2	2	2	2	1	4	2	3	1	3	4	2	0	0	0	0	0	0
18	1	1	1	1	4	4	4	2	2	2	2	1	4	2	3	1	3	4	2	0	0	0	0	0	0
19	0	0	0	0	2	2	2	1	1	1	0	0	2	0	0	0	0	2	1	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	1	1	1	1	2	2	2	1	1	1	2	1	2	2	3	1	3	2	1	0	0	0	0	0	0
22	1	1	1	1	2	2	2	1	1	1	2	1	2	2	3	1	3	2	1	0	0	0	0	0	0
23	2	2	2	2	4	4	4	2	2	2	2	2	4	2	3	2	6	4	2	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	2	0	0	2	3	0	0	0	0	0	0	0	0	0	0
26	1	1	1	1	2	2	2	1	1	1	2	1	2	2	3	1	3	2	1	0	0	0	0	0	0
27	1	1	1	1	2	2	2	1	1	1	2	1	2	2	3	1	3	2	1	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	0	5	0	3
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	0	0
33	1	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
34	0	0	0	0	0	0	0	0	0	0	1	1	3	0	0	0	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	1	4	3	2	0	0	0	0	0	0
36	0	2	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	2	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	2	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	0	5	0	3	0
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
41	1	1	1	1	0	0	0	0	0	0	0	1	0	0	0	1	3	0	0	0	0	0	0	0	0
42	0	0	0	0	2	2	2	1	1	1	0	0	2	0	0	0	0	2	1	0	0	0	0	0	0
43	2	2	2	2	8	8	8	4	4	4	7	3	11	8	10	4	8	14	5	3	3	3	3	3	3



Table 7.1 (cont.): Elementary Modes Matrix – Metabolic reactions x EFMs.

	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192	193	194
1	5	5	3	3	3	3	3	3	3	3	3	5	3	5	3	3	3	5	5	3	3	5	5	3
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	2	2	2	2	2	2	2	2	2	2	2	2	1	2	2	2	2	2	2	1	1	2	2	2
4	2	2	2	2	2	2	2	2	2	2	2	2	1	2	2	2	2	2	2	1	1	2	2	2
5	2	2	2	2	2	2	2	2	2	2	2	2	1	2	2	2	2	2	2	1	1	2	2	2
6	5	5	3	3	5	5	5	5	5	5	5	5	3	5	3	5	5	5	5	3	3	5	5	3
7	5	5	3	3	3	3	5	5	5	5	5	5	3	5	3	3	5	5	5	3	3	5	5	3
8	0	0	0	0	0	0	0	0	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	5	5	5	5	3	5	3	3	3	5	5	3	3	5	5	3
10	5	5	3	3	3	3	3	3	3	3	3	3	3	5	3	3	3	3	3	3	3	5	5	3
11	5	5	3	3	3	3	3	3	3	3	3	3	3	5	3	3	3	3	3	3	3	5	5	3
12	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
13	3	3	1	1	1	1	1	1	1	1	1	1	1	3	1	1	1	1	1	1	1	3	3	1
14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	5	0	0	0	0	0	0	0	5	0	3	0	5	0
30	0	5	0	3	0	3	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31	0	0	0	0	0	0	0	0	5	0	0	5	3	5	3	3	3	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	2	2	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	1	0	0	0
34	2	2	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	2	2	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	2	2	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	2
37	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0
39	5	0	3	0	3	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	5	0	0	0	0	0	0	0	5	0	3	0	5	0	3
41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
43	5	5	3	3	3	3	3	3	8	8	8	8	6	10	6	6	6	8	8	6	6	10	10	6

# Hybrid Systems Biology: Application to *Escherichia coli*

**Table 7.1 (cont.):** Elementary Modes Matrix – Metabolic reactions x EFMs.

	195	196	197	198	199	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	
1	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	2	6	3	
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
3	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	4	2	
4	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	4	2	
5	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	4	2	
6	3	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	2	10	5	
7	3	3	3	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	2	10	5	
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	4	7	
9	3	3	3	3	3	0	0	0	0	3	3	3	5	5	5	3	2	3	3	2	10	4	10	10	
10	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	2	6	3	
11	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	2	6	3	
12	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	4	2	
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	
14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	2	1	
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	2	2	2	5	2	5	5
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	2	2	2	5	2	5	5
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	2	2	2	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	2	5	5
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	2	5	5
23	0	0	0	0	0	0	0	-2	-2	0	0	0	-2	-2	-2	0	0	0	0	0	0	5	4	10	10
24	0	0	0	0	0	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	0	-2	0	-5
25	0	0	0	0	0	0	0	2	2	0	0	0	2	2	2	0	0	0	0	0	0	5	0	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	2	5	5
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	2	5	5
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	3	0	3	0	3	0	0	0	0	0	0	3	0	0	5	0	0	0	1	0	0	0	0	0	0
30	0	0	0	0	0	0	3	0	5	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0	3	0	0	5	0	0	1	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
36	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	3	0	5	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
40	0	3	0	3	0	0	0	0	0	0	3	0	0	5	0	0	0	1	0	0	0	0	0	0	0
41	0	0	0	0	0	2	2	0	0	2	2	2	0	0	0	0	0	0	0	0	0	0	0	5	0
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	2	2	2	0	0	0	0
43	6	6	6	6	6	1	1	3	3	4	4	4	8	8	8	6	5	6	6	5	18	8	16	18	

Table 7.1 (cont.): Elementary Modes Matrix – Metabolic reactions x EFMs.

	219	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243
1	3	3	5	3	5	3	3	3	3	3	3	3	3	3	7	4	10	10	4	6	12	4	10	10	2
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	2	2	2	1	2	2	2	2	2	2	2	2	2	0	0	1	4	2	1	2	1	1	4	2	1
4	2	2	2	1	2	2	2	2	2	2	2	2	2	0	0	1	4	2	1	2	1	1	4	2	1
5	2	2	2	1	2	2	2	2	2	2	2	2	2	0	0	1	4	2	1	2	1	1	4	2	1
6	5	5	5	3	5	3	5	5	5	5	5	5	5	1	3	2	10	5	2	6	6	2	10	5	1
7	5	5	5	3	5	3	3	5	5	5	5	5	5	1	3	2	10	5	2	6	6	2	10	5	1
8	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	5	7	10	6	10	6	6	6	4	7	7	4	10	4	8	4	10	10	4	6	12	4	10	10	2
10	3	3	3	3	5	3	3	3	3	3	3	3	3	3	7	2	6	3	4	6	12	4	10	10	2
11	3	3	3	3	5	3	3	3	3	3	3	3	3	2	6	2	6	3	4	6	12	4	10	10	2
12	2	2	2	2	2	2	2	2	2	2	2	2	2	1	4	1	4	2	2	4	8	1	4	2	1
13	1	1	1	1	3	1	1	1	1	1	1	1	1	1	2	1	2	1	2	2	4	3	6	8	1
14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	2	1	2	2	4	1	2	1	1
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	2	1	2	2	4	1	2	1	1
16	1	1	1	1	1	1	1	1	1	1	1	1	1	0	2	0	2	1	0	2	4	0	2	1	0
17	5	2	5	3	5	3	3	3	2	2	2	2	5	2	4	2	5	5	2	3	6	2	5	5	1
18	5	2	5	3	5	3	3	3	2	2	2	2	5	2	4	2	5	5	2	3	6	2	5	5	1
19	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	5	2	5	3	5	3	3	3	2	2	2	2	5	2	4	2	5	5	2	3	6	2	5	5	1
22	5	2	5	3	5	3	3	3	2	2	2	2	5	2	4	2	5	5	2	3	6	2	5	5	1
23	5	0	5	3	5	3	3	3	0	0	0	0	3	4	8	4	10	10	4	6	12	4	10	10	2
24	0	2	0	0	0	0	0	0	2	2	2	2	2	-2	-4	-2	0	-5	-2	0	-6	-2	0	-5	-1
25	0	4	5	3	5	3	3	3	4	4	4	4	7	0	0	0	0	0	0	0	0	0	0	0	0
26	0	2	5	3	5	3	3	3	2	2	2	2	5	2	4	2	5	5	2	3	6	2	5	5	1
27	0	2	5	3	5	3	3	3	2	2	2	2	5	2	4	2	5	5	2	3	6	2	5	5	1
28	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0
31	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	2	4	7	0	0	0	0	0	0	0
33	0	0	0	1	0	0	0	0	0	0	0	0	0	1	4	0	0	0	1	2	7	0	0	0	0
34	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	4	7	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	2	0	0	1	0	0	1
36	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
37	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	0	0	3	0	0	5	0	0	0
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
43	18	12	18	12	20	12	12	12	9	12	12	9	18	8	18	8	16	18	10	12	30	10	20	25	5

# Hybrid Systems Biology: Application to *Escherichia coli*

**Table 7.1 (cont.):** Elementary Modes Matrix – Metabolic reactions x EFMs.

	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268		
1	2	2	8	6	6	6	6	6	6	6	5	3	5	3	3	3	9	3	3	3	3	3	3	3	15	3	
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
3	1	1	3	4	4	4	4	4	4	4	2	1	2	2	2	2	6	2	2	2	2	2	2	2	6	1	
4	1	1	3	4	4	4	4	4	4	4	2	1	2	2	2	2	6	2	2	2	2	2	2	2	6	1	
5	1	1	3	4	4	4	4	4	4	4	2	1	2	2	2	2	6	2	2	2	2	2	2	2	6	1	
6	2	2	4	6	3	10	10	10	10	10	5	3	5	3	5	5	15	5	5	5	5	5	5	5	15	3	
7	1	2	4	6	3	6	3	10	10	10	5	3	5	3	3	5	15	5	5	5	5	5	5	5	15	3	
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	0	0	0	0	0	0	0	0	0	
9	2	2	8	6	6	6	6	6	6	6	5	3	5	3	3	3	15	5	2	5	5	2	5	5	15	3	
10	2	2	8	6	6	6	6	6	6	6	3	3	5	3	3	3	9	3	3	3	3	3	3	3	9	3	
11	2	2	8	6	6	6	6	6	6	6	3	3	5	3	3	3	9	3	3	3	3	3	3	3	9	3	
12	1	1	3	4	4	4	4	4	4	4	2	2	2	2	2	2	6	2	2	2	2	2	2	2	6	2	
13	1	1	5	2	2	2	2	2	2	2	1	1	3	1	1	1	3	1	1	1	1	1	1	1	3	1	
14	1	1	5	2	2	2	2	2	2	2	1	1	1	1	1	1	3	1	1	1	1	1	1	1	3	1	
15	1	1	5	2	2	2	2	2	2	2	1	1	1	1	1	1	3	1	1	1	1	1	1	1	3	1	
16	0	0	-2	2	2	2	2	2	2	2	1	1	1	1	1	1	3	1	1	1	1	1	1	1	3	1	
17	1	1	4	3	3	3	3	3	3	3	5	3	5	3	3	3	10	2	2	2	2	2	2	5	10	2	
18	1	1	4	3	3	3	3	3	3	3	5	3	5	3	3	3	10	2	2	2	2	2	2	5	10	2	
19	0	0	0	0	0	0	0	0	0	0	5	3	5	3	3	3	5	2	2	2	2	2	2	5	5	1	
20	0	0	0	0	0	0	0	0	0	0	5	3	5	3	3	3	0	2	2	2	2	2	2	5	0	0	
21	1	1	4	3	3	3	3	3	3	3	5	3	5	3	3	3	5	2	2	2	2	2	2	5	5	1	
22	1	1	4	3	3	3	3	3	3	3	5	3	5	3	3	3	5	2	2	2	2	2	2	5	5	1	
23	2	2	8	6	6	6	6	6	6	6	5	3	5	3	3	3	10	0	0	0	0	0	0	3	10	2	
24	-1	-1	-4	0	-3	0	-3	0	-3	4	0	0	0	0	0	0	0	2	2	2	2	2	2	2	0	0	
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	2	2	2	2	0	0	
26	1	1	4	3	3	3	3	3	3	3	0	0	0	0	0	0	5	0	0	0	0	0	0	0	5	1	
27	1	1	4	3	3	3	3	3	3	3	0	0	0	0	0	0	5	0	0	0	0	0	0	0	5	1	
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	
32	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	0
33	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	
34	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	
35	1	1	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
36	0	0	0	4	7	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	
37	1	0	0	0	0	4	7	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	
38	0	1	0	0	0	0	0	4	7	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	
41	0	0	0	3	0	3	0	3	0	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	0	0	0	0	0	0	0	5	1	
43	5	5	20	12	15	12	15	12	15	8	18	12	20	12	12	12	29	12	9	12	12	9	18	29	7	7	

**Table 7.1 (cont.):** Elementary Modes Matrix – Metabolic reactions x EFMs.

	269	270	271	272	273	274	275
1	15	3	3	3	9	3	3
2	0	0	0	0	0	0	0
3	6	2	2	2	6	2	2
4	6	2	2	2	6	2	2
5	6	2	2	2	6	2	2
6	15	3	5	5	15	5	5
7	15	3	3	5	15	5	5
8	0	0	0	0	0	0	0
9	15	3	3	3	9	3	3
10	15	3	3	3	9	3	3
11	15	3	3	3	9	3	3
12	6	2	2	2	6	2	2
13	9	1	1	1	3	1	1
14	3	1	1	1	3	1	1
15	3	1	1	1	3	1	1
16	3	1	1	1	3	1	1
17	10	2	2	2	8	3	3
18	10	2	2	2	8	3	3
19	5	1	1	1	7	3	3
20	0	0	0	0	0	3	1
21	5	1	1	1	1	3	1
22	5	1	1	1	1	3	1
23	10	2	2	2	2	3	1
24	0	0	0	0	6	2	2
25	0	0	0	0	0	0	0
26	5	1	1	1	1	0	0
27	5	1	1	1	1	0	0
28	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0
31	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0
34	6	0	0	0	0	0	0
35	0	0	0	0	0	0	0
36	0	2	0	0	0	0	0
37	0	0	2	0	0	0	0
38	0	0	0	2	0	0	0
39	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0
41	0	0	0	0	0	2	0
42	5	1	1	1	7	0	2
43	35	7	7	7	19	10	8

### 7.4 Appendix D

**Table 7.2:** Unfeasible EFMs for the analysed strains – EFMs x group of stains. Rows represent groups of strains: 1 – WT reference grown at  $0.2h^{-1}$ , *tktB*, *talA*, *talB*, *rpiB*, *pfkA*, *pfkB*, *fbp*, *fbaB*, *gapC*, *gpmA*, *gpmB*, *pykA*, *pykF*, *ppsA*, *galM*, *glk*, *pgm*; 2 – WT grown at  $0.4h^{-1}$ ,  $0.5h^{-1}$  and  $0.7h^{-1}$ ; 3 – *pgi*; 4 – *gnd*; 5 – *zwf*; 6 – *rpe*; 7 – In the remaining cases (*pgl*, *rpiA* and *tktA* gene deletions strains) all EFMs are plausible.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1						x																		
2		x				x	x			x					x			x	x		x			
3	x	x	x	x	x		x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
4																						x	x	x
5						x																x	x	x
6																								
7																								

**Table 7.2 (cont.):** Unfeasible EFMs for the analysed strains.

	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48
1									x					x					x					
2			x		x				x					x						x				
3	x	x	x		x	x	x	x	x					x			x		x		x	x	x	x
4	x	x		x	x				x	x	x	x			x	x	x	x						x
5	x	x		x	x				x	x	x	x		x	x	x	x	x	x					x
6	x	x		x	x				x	x	x	x			x	x	x	x						x
7																								

**Table 7.2 (cont.):** Unfeasible EFMs for the analysed strains.

	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71
1								x	x	x													
2								x	x	x													
3	x	x	x	x	x	x	x				x	x	x	x	x	x	x	x	x	x	x	x	x
4																							
5								x	x	x													
6																							
7																							

**Table 7.2 (cont.):** Unfeasible EFMs for the analysed strains.

	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94					
1																								x				
2				x										x	x	x	x	x	x						x			
3	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x						x	x	x	
4			x		x	x	x	x	x	x	x	x														x	x	
5			x		x	x	x	x	x	x	x	x														x	x	x
6			x		x						x	x	x															
7																												

**Table 7.2 (cont.):** Unfeasible EFMs for the analysed strains.

	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118					
1					x		x	x	x	x	x	x									x			x	x				
2					x		x	x	x	x	x	x									x	x	x	x	x				
3	x	x	x	x		x		x						x	x	x	x	x	x						x	x			
4	x	x	x	x																						x	x	x	x
5	x	x	x	x	x		x	x	x	x	x	x									x	x	x	x	x	x	x	x	
6																											x	x	
7																													

**Table 7.2 (cont.):** Unfeasible EFMs for the analysed strains.

	119	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	
1																											
2	x	x	x	x	x	x	x	x	x	x	x	x									x	x	x	x	x	x	x
3	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
4	x	x	x	x	x	x	x	x	x	x	x	x	x	x													
5	x	x	x	x	x	x	x	x	x	x	x	x	x	x													
6	x	x	x		x		x	x	x	x	x	x															
7																											

**Table 7.2 (cont.):** Unfeasible EFMs for the analysed strains.

	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	
1																										
2	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x					
3	x	x	x	x	x	x	x	x	x	x	x	x	x	x		x					x					
4													x	x	x	x	x	x	x	x	x	x	x	x	x	x
5												x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
6															x		x				x	x	x	x	x	x
7																										

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**Table 7.2 (cont.):** Unfeasible EFMs for the analysed strains.

	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192	193
1																								
2																								
3																								
4	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
5	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
6	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
7																								

**Table 7.2 (cont.):** Unfeasible EFMs for the analysed strains.

	194	195	196	197	198	199	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	
1																									
2																							x	x	x
3																									
4	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	
5	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	
6	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	
7																									

**Table 7.2 (cont.):** Unfeasible EFMs for the analysed strains.

	218	219	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242
1															x	x									
2	x		x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
3																									
4	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
5	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
6	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
7																									

**Table 7.2 (cont.):** Unfeasible EFMs for the analysed strains.

	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261	262	263			
1																								
2	x	x	x	x	x	x	x	x	x	x	x							x						
3																								
4	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
5	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
6	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
7																								



**Table 7.2 (cont.):** Unfeasible EFMs for the analysed strains.

	264	265	266	267	268	269	270	271	272	273	274	275
1												
2				x	x	x	x	x	x	x		
3												
4	x	x	x	x	x	x	x	x	x	x	x	x
5	x	x	x	x	x	x	x	x	x	x	x	x
6	x	x	x	x	x	x	x	x	x	x	x	x
7												

## 7.5 Appendix E

**Table 7.3:** Ranking of statistically significant EFMs with high correlation with the envirome and proteome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
92	0.87	$6.75 \times 10^{-7}$	56	72.6
205	0.89	$2.18 \times 10^{-7}$	53	82.1
164	0.81	$2.98 \times 10^{-5}$	56.1	82.3
158	0.76	$1.71 \times 10^{-4}$	50.9	82.5
126	0.78	$7.59 \times 10^{-5}$	57.3	82.6
34	1	$3.92 \times 10^{-2}$	99	82.6
115	0.8	$3.29 \times 10^{-5}$	58.9	84.3
30	0.93	$1.21 \times 10^{-8}$	73.4	84.3
130	0.79	$5.81 \times 10^{-5}$	49.5	84.4
175	0.94	$4.35 \times 10^{-10}$	52	87.8
102	1	$1.54 \times 10^{-2}$	58.7	87.8

**Table 7.4:** Ranking of statistically significant EFMs with high correlation with the envirome and metabolome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
260	0.87	$1.67 \times 10^{-6}$	67.5	31
219	0.93	$2.44 \times 10^{-9}$	57.5	56
275	0.9	$7.87 \times 10^{-8}$	52.5	60.6
249	0.93	$8.05 \times 10^{-9}$	70.7	62.9
272	0.87	$1.63 \times 10^{-6}$	59.6	63.1
228	0.85	$3.33 \times 10^{-6}$	54.6	63.8
227	0.81	$2.65 \times 10^{-5}$	55.1	63.9
242	0.77	$1.04 \times 10^{-4}$	46.9	63.9
250	0.97	$7.00 \times 10^{-12}$	77.6	64.2
3	0.91	$3.32 \times 10^{-8}$	55.5	70.7
236	0.92	$2.55 \times 10^{-8}$	60.2	70.8
34	0.99	$1.03 \times 10^{-1}$	95	70.9
246	0.83	$9.01 \times 10^{-6}$	55.8	70.9

**Table 7.4 (cont.):** Ranking of statistically significant EFMs with high correlation with the envirome and metabolome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
110	0.81	$1.48 \times 10^{-5}$	43	73.1
252	0.95	$4.43 \times 10^{-10}$	37	73.1
93	0.83	$4.90 \times 10^{-6}$	37.4	73.5
69	0.85	$1.79 \times 10^{-6}$	45.3	74.3

**Table 7.5:** Ranking of statistically significant EFMs with high correlation with the envirome and transcriptome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
92	0.88	$3.28 \times 10^{-7}$	64	74
263	0.89	$1.64 \times 10^{-7}$	50.7	81.2
3	0.94	$8.88 \times 10^{-10}$	63.2	86.1
126	0.8	$4.35 \times 10^{-5}$	58	86.1
34	1	$2.46 \times 10^{-2}$	99.8	86.2
123	0.79	$6.21 \times 10^{-5}$	47.1	86.3
130	0.66	$2.09 \times 10^{-3}$	26.1	86.4
127	0.8	$4.13 \times 10^{-5}$	3.5	86.4
125	0.72	$4.70 \times 10^{-4}$	-4.6	86.4
62	0.94	$1.29 \times 10^{-9}$	50.6	87.1

**Table 7.6:** Ranking of statistically significant EFMs with high correlation with the envirome and regulatory transcriptome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
92	0.98	$8.40 \times 10^{-5}$	95.6	78.3
263	0.99	$1.96 \times 10^{-3}$	90.4	85.8
264	0.98	$3.61 \times 10^{-3}$	87.4	87.7
274	0.98	$3.25 \times 10^{-3}$	76	88.4
273	0.99	$8.62 \times 10^{-2}$	50.1	88.5
164	1	$8.91 \times 10^{-4}$	37.6	88.5
272	0.99	$1.10 \times 10^{-1}$	13.8	88.5

**Table 7.6 (cont.):** Ranking of statistically significant EFMs with high correlation with the envirome and regulatory transcriptome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
262	0.96	$8.65 \times 10^{-3}$	68	88.5
162	1	$1.09 \times 10^{-3}$	50.3	88.5
275	0.99	$4.72 \times 10^{-4}$	53.5	88.6
227	1	$2.80 \times 10^{-2}$	32.4	88.6
126	1	$9.77 \times 10^{-3}$	70.5	88.6
124	1	$7.18 \times 10^{-4}$	90.8	88.6
258	0.98	$3.43 \times 10^{-3}$	44.7	88.7
211	1	$1.03 \times 10^{-4}$	92	88.8
158	1	$2.40 \times 10^{-4}$	46.6	88.8
157	1	$1.05 \times 10^{-3}$	51.8	88.8
122	1	$1.61 \times 10^{-3}$	74.3	88.9
154	0.98	$3.30 \times 10^{-3}$	68.4	89.1
3	0.95	$1.21 \times 10^{-3}$	79.9	89.7

**Table 7.7:** Ranking of statistically significant EFMs with high correlation with the proteome and metabolome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
260	0.9	$1.86 \times 10^{-7}$	66.6	31
219	0.9	$7.60 \times 10^{-8}$	57.2	55.8
263	0.88	$2.56 \times 10^{-7}$	47.3	61
249	0.86	$2.73 \times 10^{-6}$	60.5	61.6
241	0.86	$2.03 \times 10^{-6}$	62.2	61.7
228	0.96	$7.91 \times 10^{-11}$	55.8	62.5
235	0.96	$1.66 \times 10^{-10}$	45.9	62.5
242	0.92	$2.49 \times 10^{-8}$	39.9	62.5
34	0.99	$8.21 \times 10^{-2}$	97.2	62.6
244	0.9	$1.17 \times 10^{-7}$	42.2	62.6
3	0.89	$1.02 \times 10^{-7}$	53.7	68.6
79	0.88	$3.22 \times 10^{-7}$	19.4	68.9
71	0.89	$1.50 \times 10^{-7}$	49.7	71

**Table 7.8:** Ranking of statistically significant EFMs with high correlation with the proteome and Transcriptome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
92	0.89	$1.04 \times 10^{-7}$	54.1	72.2
263	0.95	$1.13 \times 10^{-10}$	51.8	80
164	0.86	$1.86 \times 10^{-6}$	37.9	80.2
30	0.82	$1.77 \times 10^{-5}$	50.4	80.3
228	0.85	$3.18 \times 10^{-6}$	39.7	80.6
3	0.96	$2.47 \times 10^{-11}$	54.3	85.4
126	0.81	$2.74 \times 10^{-5}$	51.7	85.5
124	0.96	$1.24 \times 10^{-10}$	39.1	85.5
34	1	$1.21 \times 10^{-2}$	100	85.5
121	0.79	$5.55 \times 10^{-5}$	49.1	85.5
119	0.76	$1.73 \times 10^{-4}$	46.2	85.7
130	0.85	$4.24 \times 10^{-6}$	38.2	85.7
120	0.85	$3.19 \times 10^{-6}$	43.6	85.8
115	0.87	$1.75 \times 10^{-6}$	54.1	86.9
244	0.96	$3.57 \times 10^{-11}$	50.7	87.4
127	0.96	$1.26 \times 10^{-10}$	66.5	87.5
157	0.93	$1.34 \times 10^{-8}$	37.8	87.6
71	0.97	$6.12 \times 10^{-12}$	49.7	89.1

**Table 7.9:** Ranking of statistically significant EFMs with high correlation with the proteome and regulatory transcriptome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
92	0.97	$1.88 \times 10^{-4}$	94.1	78
263	0.98	$3.86 \times 10^{-3}$	88.8	85.5
264	0.98	$4.33 \times 10^{-3}$	87.7	87.5
274	0.99	$4.67 \times 10^{-4}$	81.5	88.2
273	1	$8.14 \times 10^{-3}$	50.7	88.3
164	1	$4.49 \times 10^{-4}$	43.7	88.3
260	1	$1.03 \times 10^{-3}$	13.5	88.4
262	0.91	$3.27 \times 10^{-2}$	64.5	88.4

**Table 7.9 (cont.):** Ranking of statistically significant EFMs with high correlation with the proteome and regulatory transcriptome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
162	1	$9.20 \times 10^{-4}$	75.3	88.4
275	0.98	$3.56 \times 10^{-3}$	49	88.5
227	1	$4.35 \times 10^{-4}$	38.3	88.5
124	1	$1.08 \times 10^{-3}$	95.1	88.5
158	1	$1.34 \times 10^{-3}$	49	88.6
115	1	$6.77 \times 10^{-4}$	43.1	88.8
253	1	$3.11 \times 10^{-3}$	77.5	88.8
126	1	$1.39 \times 10^{-3}$	99	88.8
211	1	$1.25 \times 10^{-5}$	95.3	89
157	1	$8.43 \times 10^{-5}$	65.2	89
122	1	$1.46 \times 10^{-4}$	90.6	89
154	1	$1.53 \times 10^{-4}$	83.6	89.3

**Table 7.10:** Ranking of statistically significant EFMs with high correlation with the metabolome and transcriptome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
92	0.93	$3.71 \times 10^{-9}$	59.3	73.1
263	0.92	$1.13 \times 10^{-8}$	51.5	80.5
3	0.95	$2.90 \times 10^{-10}$	53.4	85
116	0.8	$3.56 \times 10^{-5}$	62.6	85.8
30	0.77	$1.07 \times 10^{-4}$	56.9	85.8
126	0.79	$6.08 \times 10^{-5}$	50.1	86.1
123	0.79	$6.65 \times 10^{-5}$	44.9	86.1
125	0.89	$3.68 \times 10^{-7}$	57.5	86.2
102	1	$6.12 \times 10^{-2}$	72.2	86.2
71	0.95	$1.87 \times 10^{-10}$	47.4	87.7
120	0.87	$1.06 \times 10^{-6}$	55.9	87.8
121	0.87	$1.30 \times 10^{-6}$	61	87.9

**Table 7.11:** Ranking of statistically significant EFMs with high correlation with the metabolome and regulatory transcriptome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
92	1	$3.35 \times 10^{-7}$	99.3	79.1
263	1	$1.51 \times 10^{-5}$	95.8	86.3
264	1	$7.50 \times 10^{-6}$	97.7	88.2
274	1	$1.82 \times 10^{-5}$	63.9	88.8
164	1	$6.66 \times 10^{-5}$	59.6	88.8
262	1	$3.47 \times 10^{-5}$	36.5	88.9
162	1	$1.08 \times 10^{-3}$	61.5	88.9
158	1	$8.72 \times 10^{-4}$	52.5	88.9
126	1	$2.29 \times 10^{-2}$	86.5	89
211	1	$1.85 \times 10^{-6}$	81.6	89.1
157	1	$1.61 \times 10^{-3}$	56.6	89.1
154	1	$1.33 \times 10^{-5}$	92.5	89.4
3	0.97	$3.55 \times 10^{-4}$	83.7	90
155	1	$6.49 \times 10^{-5}$	81.4	90
153	1	$8.31 \times 10^{-5}$	80	90.1
151	0.99	$5.78 \times 10^{-4}$	78.6	90.1
202	0.99	$6.34 \times 10^{-4}$	81.9	90.1
134	0.99	$1.02 \times 10^{-5}$	93.9	92
273	1	$1.91 \times 10^{-3}$	68	92
275	1	$1.01 \times 10^{-5}$	96.6	92.1

**Table 7.12:** Ranking of statistically significant EFMs with high correlation with the transcriptome and regulatory transcriptome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
92	0.97	$4.14 \times 10^{-4}$	91.8	77.5
263	0.98	$2.47 \times 10^{-3}$	88.7	85.3
264	0.99	$1.20 \times 10^{-3}$	86.7	87.4
271	1	$2.72 \times 10^{-3}$	59.4	87.7
274	0.99	$4.96 \times 10^{-4}$	83.3	88.2

**Table 7.12 (cont.):** Ranking of statistically significant EFMs with high correlation with the transcriptome and regulatory transcriptome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM through the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
164	1	$2.03 \times 10^{-4}$	40.2	88.2
269	0.99	$9.10 \times 10^{-2}$	34.9	88.3
260	1	$3.13 \times 10^{-2}$	20.1	88.4
275	0.98	$2.30 \times 10^{-3}$	88	88.4
273	1	$6.00 \times 10^{-2}$	36.2	88.4
272	1	$5.14 \times 10^{-2}$	30.9	88.4
253	1	$1.61 \times 10^{-2}$	75	88.5
162	1	$4.87 \times 10^{-4}$	58.8	88.5
262	0.99	$1.83 \times 10^{-3}$	75.9	88.5
227	0.99	$9.95 \times 10^{-2}$	66.2	88.5
258	0.98	$2.61 \times 10^{-3}$	79.3	88.6
247	1	$4.89 \times 10^{-2}$	21.8	88.6
124	1	$1.97 \times 10^{-3}$	98.8	88.6
158	1	$1.23 \times 10^{-3}$	73.2	88.7
211	1	$7.03 \times 10^{-5}$	98.3	88.8

**Table 7.13:** Ranking of statistically significant EFMs with high correlation with the envirome, proteome and metabolome. Each EFM is selected with increasing explained flux variance. Statistical relevance of each EFM can also be examined ( $p_{value}$  and  $r^2$  value).

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
260	0.9	$1.11 \times 10^{-7}$	67.3	31
219	0.93	$1.67 \times 10^{-9}$	59.8	56.5
262	0.92	$7.83 \times 10^{-9}$	48.3	60.9
249	0.86	$3.08 \times 10^{-6}$	58.4	61.5
241	0.84	$7.37 \times 10^{-6}$	62.7	61.5
227	0.92	$2.42 \times 10^{-8}$	53.9	62.2
236	0.94	$1.39 \times 10^{-9}$	68.6	62.3
233	0.99	$1.00 \times 10^{-1}$	69.4	62.3
231	0.86	$2.31 \times 10^{-6}$	46.7	62.7
253	0.96	$6.32 \times 10^{-11}$	54.2	63
3	0.95	$3.07 \times 10^{-10}$	50	67.3



**Table 7.13 (cont.):** Ranking of statistically significant EFMs with high correlation with the envirome, proteome and metabolome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
228	0.87	$1.70 \times 10^{-6}$	47.8	67.4
250	0.98	$4.56 \times 10^{-13}$	23.9	67.4
94	0.85	$2.28 \times 10^{-6}$	39.3	67.7
70	0.92	$9.88 \times 10^{-9}$	50.9	70.8
34	0.99	$8.37 \times 10^{-2}$	97.2	70.9
246	0.93	$1.31 \times 10^{-8}$	55.9	71.1
106	1	$5.37 \times 10^{-2}$	53.9	71.7

**Table 7.14:** Ranking of statistically significant EFMs with high correlation with the envirome, proteome and transcriptome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
92	0.9	$8.68 \times 10^{-8}$	55	72.4
263	0.96	$6.64 \times 10^{-12}$	55.8	80.4
164	0.89	$4.21 \times 10^{-7}$	40.5	80.6
30	0.85	$4.45 \times 10^{-6}$	48.2	80.7
3	0.96	$1.71 \times 10^{-11}$	56.1	85.3
126	0.84	$6.93 \times 10^{-6}$	59.5	85.4
34	1	$2.59 \times 10^{-2}$	99.8	85.4
121	0.83	$9.72 \times 10^{-6}$	52.3	85.4
130	0.84	$6.45 \times 10^{-6}$	43.4	85.6
123	0.9	$1.91 \times 10^{-7}$	46.6	85.6
127	0.88	$7.48 \times 10^{-7}$	45.2	85.6
125	0.78	$9.22 \times 10^{-5}$	16.1	85.6
115	0.87	$1.17 \times 10^{-6}$	56.8	86.9
244	0.96	$1.03 \times 10^{-10}$	53.9	87.3
119	0.99	$1.78 \times 10^{-14}$	58.4	87.4
120	0.96	$4.45 \times 10^{-11}$	29.7	87.4
128	0.9	$1.03 \times 10^{-7}$	31.5	87.4
71	0.97	$3.01 \times 10^{-12}$	50.6	88.9

**Table 7.15:** Ranking of statistically significant EFMs with high correlation with the envirome, proteome and regulatory transcriptome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
92	0.98	$1.29 \times 10^{-4}$	94.9	78.2
263	0.98	$2.46 \times 10^{-3}$	90.4	85.7
264	0.98	$2.72 \times 10^{-3}$	89.4	87.6
274	1	$8.99 \times 10^{-5}$	79.7	88.3
273	1	$8.68 \times 10^{-3}$	46.1	88.4
164	1	$4.53 \times 10^{-4}$	42.6	88.4
262	0.97	$5.73 \times 10^{-3}$	73.3	88.5
162	1	$6.01 \times 10^{-4}$	64.1	88.5
275	0.99	$1.18 \times 10^{-3}$	50.5	88.5
124	1	$4.96 \times 10^{-4}$	92.4	88.5
229	1	$5.87 \times 10^{-3}$	75.7	88.7
253	1	$3.72 \times 10^{-3}$	52.4	88.7
126	1	$7.07 \times 10^{-4}$	69.1	88.8
121	1	$3.77 \times 10^{-3}$	41.6	88.8
132	1	$1.47 \times 10^{-5}$	98.3	88.9
246	1	$4.65 \times 10^{-3}$	73.2	88.9
160	1	$4.04 \times 10^{-4}$	46.4	88.9
163	1	$3.62 \times 10^{-3}$	53.2	89
216	1	$1.99 \times 10^{-3}$	12	89
244	1	$4.63 \times 10^{-4}$	72.4	89.1

**Table 7.16:** Ranking of statistically significant EFMs with high correlation with the envirome, metabolome and transcriptome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
92	0.93	$2.38 \times 10^{-9}$	60.4	73.3
263	0.93	$3.36 \times 10^{-9}$	51.3	80.6
3	0.96	$4.31 \times 10^{-11}$	53.7	85.2
119	0.8	$4.63 \times 10^{-5}$	63.2	85.4
126	0.89	$3.04 \times 10^{-7}$	76.2	85.5

**Table 7.16 (cont.):** Ranking of statistically significant EFMs with high correlation with the envirome, metabolome and transcriptome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
30	0.91	$6.62 \times 10^{-8}$	72.3	85.6
125	0.8	$4.44 \times 10^{-5}$	49.7	85.7
121	0.81	$2.77 \times 10^{-5}$	64	85.7
123	0.81	$2.39 \times 10^{-5}$	39.3	85.7
127	0.69	$1.10 \times 10^{-3}$	42.6	85.7
102	1	$6.11 \times 10^{-2}$	73.6	85.8
130	0.78	$7.16 \times 10^{-5}$	38.3	85.8
62	0.96	$5.39 \times 10^{-11}$	49.6	86.5
13	0.81	$1.39 \times 10^{-5}$	35.2	86.7

**Table 7.17:** Ranking of statistically significant EFMs with high correlation with the envirome, metabolome and regulatory transcriptome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
92	1	$4.54 \times 10^{-7}$	99.3	79.1
263	1	$1.45 \times 10^{-5}$	96	86.3
264	1	$8.96 \times 10^{-6}$	97.9	88.2
274	1	$2.35 \times 10^{-5}$	63.8	88.8
164	1	$1.05 \times 10^{-4}$	60.1	88.8
162	1	$1.56 \times 10^{-3}$	60	88.8
262	1	$6.67 \times 10^{-5}$	34.4	88.9
158	1	$9.12 \times 10^{-4}$	52.3	88.9
126	1	$2.48 \times 10^{-2}$	86.6	88.9
211	1	$1.71 \times 10^{-6}$	81	89.1
157	1	$1.59 \times 10^{-3}$	57.6	89.1
154	1	$2.24 \times 10^{-5}$	92.4	89.4
3	0.97	$3.85 \times 10^{-4}$	83.9	90
155	1	$5.62 \times 10^{-5}$	81.4	90
153	1	$7.84 \times 10^{-5}$	79.8	90.1
151	0.99	$6.04 \times 10^{-4}$	78.3	90.1

**Table 7.17 (cont.):** Ranking of statistically significant EFMs with high correlation with the envirome, metabolome and regulatory transcriptome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
202	0.99	$5.78 \times 10^{-4}$	81.6	90.1
134	0.99	$1.03 \times 10^{-5}$	93.9	92
273	1	$4.40 \times 10^{-3}$	59.7	92
275	1	$8.42 \times 10^{-6}$	95.5	92.1

**Table 7.18:** Ranking of statistically significant EFMs with high correlation with the envirome, transcriptome and regulatory transcriptome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
92	0.97	$3.15 \times 10^{-4}$	92.8	77.7
263	0.99	$1.64 \times 10^{-3}$	89.9	85.5
264	0.99	$7.83 \times 10^{-4}$	88.1	87.5
271	1	$5.48 \times 10^{-3}$	54.7	87.8
274	0.99	$4.99 \times 10^{-4}$	82.6	88.3
164	1	$1.17 \times 10^{-4}$	38.8	88.3
269	0.99	$9.12 \times 10^{-2}$	29.6	88.3
262	0.99	$1.02 \times 10^{-3}$	75.8	88.4
253	1	$2.00 \times 10^{-2}$	74.2	88.4
162	1	$1.81 \times 10^{-4}$	55.9	88.4
275	1	$4.15 \times 10^{-4}$	79.3	88.5
270	0.99	$9.20 \times 10^{-2}$	29.9	88.5
272	0.99	$1.02 \times 10^{-1}$	32.7	88.5
227	0.99	$9.16 \times 10^{-2}$	67.4	88.5
258	0.99	$1.51 \times 10^{-3}$	74.9	88.6
124	1	$2.15 \times 10^{-3}$	98.3	88.6
121	1	$3.55 \times 10^{-2}$	92.1	88.6
273	1	$2.35 \times 10^{-2}$	90.1	88.6
158	0.99	$6.85 \times 10^{-3}$	69.9	88.7
211	1	$7.32 \times 10^{-5}$	93.7	88.8

**Table 7.19:** Ranking of statistically significant EFMs with high correlation with the proteome, metabolome and transcriptome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
92	0.93	$2.66 \times 10^{-9}$	57.8	72.9
263	0.93	$2.75 \times 10^{-9}$	52.4	80.4
3	0.95	$8.55 \times 10^{-11}$	53.3	85
125	0.79	$6.79 \times 10^{-5}$	59.8	85.2
126	0.8	$4.13 \times 10^{-5}$	55.7	85.3
120	0.85	$5.01 \times 10^{-6}$	63.5	85.5
121	0.88	$7.61 \times 10^{-7}$	69.5	85.5
127	0.83	$1.17 \times 10^{-5}$	44.3	85.6
130	0.79	$6.08 \times 10^{-5}$	39.1	85.6
30	0.8	$3.53 \times 10^{-5}$	50.7	85.6
102	1	$2.61 \times 10^{-2}$	57.8	85.6
71	0.96	$1.93 \times 10^{-11}$	46.7	87.2

**Table 7.20:** Ranking of statistically significant EFMs with high correlation with the proteome, metabolome and regulatory transcriptome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
92	1	$5.02 \times 10^{-7}$	99.3	79.1
263	1	$2.01 \times 10^{-7}$	96.2	86.3
264	1	$9.51 \times 10^{-7}$	98	88.2
274	1	$1.29 \times 10^{-5}$	62.2	88.7
164	1	$2.32 \times 10^{-4}$	60.8	88.8
162	1	$3.82 \times 10^{-4}$	61.4	88.8
158	1	$4.14 \times 10^{-3}$	54.8	88.8
262	1	$4.83 \times 10^{-5}$	33.3	88.9
126	1	$2.36 \times 10^{-2}$	79.9	88.9
211	1	$4.17 \times 10^{-6}$	80.1	89.1
157	1	$3.19 \times 10^{-4}$	61.8	89.1
154	1	$5.15 \times 10^{-5}$	94.2	89.3
3	0.98	$8.12 \times 10^{-5}$	89.4	89.9

**Table 7.20 (cont.):** Ranking of statistically significant EFMs with high correlation with the correlation with proteome, metabolome and regulatory transcriptome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
155	1	$8.58 \times 10^{-5}$	83.4	90
153	1	$9.38 \times 10^{-5}$	82	90
151	1	$3.35 \times 10^{-4}$	80.6	90.1
150	0.99	$4.87 \times 10^{-4}$	77.9	90.3
272	1	$2.66 \times 10^{-2}$	54.4	90.4
273	1	$4.26 \times 10^{-3}$	53	90.4
212	1	$6.44 \times 10^{-7}$	97.8	90.5

**Table 7.21:** Ranking of statistically significant EFMs with high correlation with the metabolome, transcriptome and regulatory transcriptome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
92	1	$3.07 \times 10^{-6}$	98.8	79
263	1	$4.24 \times 10^{-6}$	95.9	86.1
264	1	$4.59 \times 10^{-6}$	97.5	88
274	1	$3.51 \times 10^{-6}$	66.8	88.6
164	1	$5.55 \times 10^{-4}$	51.6	88.6
262	1	$2.02 \times 10^{-4}$	42.5	88.6
162	1	$1.41 \times 10^{-4}$	60.4	88.6
158	1	$3.97 \times 10^{-3}$	46.8	88.7
126	1	$1.01 \times 10^{-2}$	86.2	88.7
211	1	$6.47 \times 10^{-7}$	85.2	88.9
157	1	$1.61 \times 10^{-3}$	52.5	88.9
154	1	$1.07 \times 10^{-5}$	85.5	89.1
155	1	$3.27 \times 10^{-4}$	76.9	89.2
3	0.97	$2.47 \times 10^{-4}$	82.1	89.8
134	0.99	$1.68 \times 10^{-5}$	89.7	91.7
275	1	$1.36 \times 10^{-5}$	75.2	91.8
228	1	$1.89 \times 10^{-2}$	76.6	91.9
258	1	$1.66 \times 10^{-5}$	64.4	92

**Table 7.21:** Ranking of statistically significant EFMs with high correlation with the metabolome, transcriptome and regulatory transcriptome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
212	1	$1.26 \times 10^{-4}$	73.5	92
213	1	$7.31 \times 10^{-5}$	90.9	92

**Table 7.22:** Ranking of statistically significant EFMs with high correlation with the envirome, proteome, metabolome and transcriptome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
92	0.93	$1.58 \times 10^{-9}$	58.8	73.1
263	0.94	$1.16 \times 10^{-9}$	52	80.5
30	0.8	$3.64 \times 10^{-5}$	62.5	80.6
3	0.96	$1.64 \times 10^{-11}$	53.5	85.2
126	0.84	$5.77 \times 10^{-6}$	67.7	85.2
121	0.82	$1.99 \times 10^{-5}$	62.4	85.3
127	0.86	$2.92 \times 10^{-6}$	56.8	85.4
130	0.78	$8.63 \times 10^{-5}$	51.3	85.4
123	0.94	$3.60 \times 10^{-9}$	59.5	85.4
119	0.92	$2.75 \times 10^{-8}$	65.5	85.5
102	1	$4.69 \times 10^{-2}$	74	85.5
62	0.96	$2.96 \times 10^{-11}$	46.7	86.2

**Table 7.23:** Ranking of statistically significant EFMs with high correlation with the envirome, proteome, metabolome and regulatory transcriptome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
92	1	$5.89 \times 10^{-7}$	99.3	79.1
263	1	$2.64 \times 10^{-7}$	96.3	86.2
264	1	$5.69 \times 10^{-7}$	98.1	88.2
274	1	$7.97 \times 10^{-6}$	62.1	88.7
164	1	$1.44 \times 10^{-4}$	60.9	88.8

**Table 7.23 (cont.):** Ranking of statistically significant EFMs with high correlation with the envirome, proteome, metabolome and regulatory transcriptome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
162	1	$4.28 \times 10^{-4}$	61.6	88.8
158	1	$4.09 \times 10^{-3}$	54.3	88.8
262	1	$3.40 \times 10^{-5}$	32.2	88.9
126	1	$2.39 \times 10^{-2}$	79.8	88.9
211	1	$2.56 \times 10^{-6}$	79.8	89.1
157	1	$4.87 \times 10^{-4}$	61.6	89.1
154	1	$4.81 \times 10^{-5}$	94.3	89.3
3	0.98	$8.88 \times 10^{-5}$	89.6	89.9
155	1	$8.51 \times 10^{-5}$	83.5	90
153	1	$9.79 \times 10^{-5}$	82.1	90
151	1	$3.80 \times 10^{-4}$	80.4	90.1
150	1	$3.85 \times 10^{-4}$	78.9	90.3
272	1	$2.44 \times 10^{-2}$	52.5	90.4
273	1	$1.45 \times 10^{-3}$	63.7	90.4
212	1	$4.00 \times 10^{-7}$	98	90.5

**Table 7.24:** Ranking of statistically significant EFMs with high correlation with the envirome, proteome, transcriptome and regulatory transcriptome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
92	0.97	$2.83 \times 10^{-4}$	93	77.8
263	0.99	$1.27 \times 10^{-3}$	90.7	85.5
264	1	$3.66 \times 10^{-4}$	89.5	87.5
271	1	$1.15 \times 10^{-2}$	52.2	87.8
274	0.99	$5.84 \times 10^{-4}$	85.1	88.4
164	1	$2.27 \times 10^{-4}$	41.4	88.4
272	1	$4.42 \times 10^{-2}$	40.5	88.4
253	1	$4.11 \times 10^{-2}$	53.4	88.4
262	0.99	$1.79 \times 10^{-3}$	81.9	88.5
162	1	$7.50 \times 10^{-6}$	62.5	88.5



**Table 7.24 (cont.):** Ranking of statistically significant EFMs with high correlation with the envirome, proteome, transcriptome and regulatory transcriptome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
275	0.99	$8.25 \times 10^{-4}$	78.1	88.5
270	1	$1.69 \times 10^{-2}$	16.8	88.6
227	1	$4.97 \times 10^{-2}$	56.9	88.6
258	0.99	$1.77 \times 10^{-3}$	68.6	88.6
124	1	$8.16 \times 10^{-4}$	98.9	88.6
121	1	$3.33 \times 10^{-2}$	75	88.7
273	1	$9.18 \times 10^{-3}$	65.6	88.7
158	0.99	$5.35 \times 10^{-3}$	60.7	88.7
211	1	$4.42 \times 10^{-5}$	95.7	88.8
157	1	$5.83 \times 10^{-5}$	65.2	88.8

**Table 7.25:** Ranking of statistically significant EFMs with high correlation with the envirome, metabolome, transcriptome and regulatory transcriptome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
92	1	$3.25 \times 10^{-6}$	98.8	79
263	1	$5.52 \times 10^{-6}$	96.1	86.1
264	1	$4.93 \times 10^{-6}$	97.6	88
274	1	$1.61 \times 10^{-6}$	66.6	88.6
164	1	$5.99 \times 10^{-4}$	51.7	88.6
262	1	$1.13 \times 10^{-4}$	40.7	88.6
162	1	$1.67 \times 10^{-4}$	60.6	88.6
158	1	$3.73 \times 10^{-3}$	46.5	88.7
126	1	$9.87 \times 10^{-3}$	86.1	88.7
211	1	$2.38 \times 10^{-7}$	85.1	88.9
157	1	$1.79 \times 10^{-3}$	52.4	88.9
154	1	$1.03 \times 10^{-5}$	85.5	89.1
155	1	$3.63 \times 10^{-4}$	76.8	89.2
3	0.97	$2.74 \times 10^{-4}$	82.3	89.8

**Table 7.25 (cont.):** Ranking of statistically significant EFMs with high correlation with the envirome, metabolome, transcriptome and regulatory transcriptome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
134	0.99	$1.77 \times 10^{-5}$	89.7	91.7
275	1	$1.05 \times 10^{-5}$	74.5	91.8
228	1	$1.27 \times 10^{-2}$	75.6	91.9
258	1	$2.16 \times 10^{-5}$	63.7	92
212	1	$1.66 \times 10^{-4}$	72.3	92
213	1	$6.76 \times 10^{-5}$	91	92

**Table 7.26:** Ranking of statistically significant EFMs with high correlation with the proteome, metabolome, transcriptome and regulatory transcriptome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
92	0.99	$4.18 \times 10^{-6}$	98.6	79
263	1	$1.49 \times 10^{-6}$	96.1	86.1
264	1	$4.61 \times 10^{-6}$	97.6	88
274	1	$1.72 \times 10^{-7}$	66.6	88.6
164	1	$1.72 \times 10^{-3}$	53.2	88.6
262	0.99	$6.99 \times 10^{-4}$	42.6	88.6
162	1	$3.29 \times 10^{-5}$	61.4	88.6
158	0.99	$7.03 \times 10^{-3}$	49.9	88.7
126	1	$4.65 \times 10^{-3}$	81.9	88.7
211	1	$6.52 \times 10^{-7}$	86	88.9
157	1	$8.97 \times 10^{-4}$	55.5	88.9
154	1	$6.83 \times 10^{-5}$	89.6	89.1
3	0.98	$1.84 \times 10^{-4}$	85.5	89.8
155	1	$5.13 \times 10^{-5}$	75.8	89.8
152	1	$2.73 \times 10^{-4}$	73.7	89.9
153	1	$9.09 \times 10^{-6}$	63.9	89.9
151	1	$3.63 \times 10^{-4}$	62.2	89.9
214	1	$1.18 \times 10^{-7}$	92.5	90
213	1	$1.87 \times 10^{-5}$	87.7	90
150	1	$3.91 \times 10^{-4}$	76.7	90.2

**Table 7.27:** Ranking of statistically significant EFMs with high correlation with the envirome, proteome, metabolome, transcriptome and regulatory transcriptome. Each EFM is selected with increasing explained flux variance. It can also be examined the statistical relevance of each EFM trough the analysis of  $p_{value}$  and  $r^2$  value.

EFM	$r^2$	$p_{value}$	$var(\lambda_{EFM})$	$var(\mathbf{r}_{obs})$
92	0.99	$4.17 \times 10^{-6}$	98.7	79
263	1	$1.17 \times 10^{-6}$	96.2	86.1
264	1	$3.51 \times 10^{-6}$	97.7	88
274	1	$1.45 \times 10^{-7}$	66.3	88.6
164	1	$1.77 \times 10^{-3}$	53.1	88.6
262	0.99	$5.05 \times 10^{-4}$	40.9	88.6
162	1	$3.54 \times 10^{-5}$	61.5	88.6
158	0.99	$7.54 \times 10^{-3}$	49.5	88.7
126	1	$4.17 \times 10^{-3}$	81.8	88.7
211	1	$3.71 \times 10^{-7}$	85.8	88.9
157	1	$1.02 \times 10^{-3}$	55.4	88.9
154	1	$6.98 \times 10^{-5}$	89.7	89.1
3	0.97	$1.99 \times 10^{-4}$	85.6	89.8
155	1	$4.80 \times 10^{-5}$	76	89.8
152	1	$2.78 \times 10^{-4}$	73.8	89.9
153	1	$1.38 \times 10^{-5}$	63.7	89.9
151	1	$3.41 \times 10^{-4}$	62.1	89.9
214	1	$4.53 \times 10^{-8}$	92.4	90
213	1	$1.46 \times 10^{-5}$	82.3	90
150	0.99	$4.39 \times 10^{-4}$	77.2	90.2