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The Farrell and Shapiro condition revisited

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Abstract

The purpose of this paper is to study the consequences of using the Farrell and Shapiro (1990) sufficient condition for merger approval to sectors in which a downstream horizontal merger may also affect upstream firms. As will be shown below, in some circumstances the sign of the relevant external effect can no longer be established by considering the merger as a sequence of infinitesimal mergers, each corresponding to a marginal change in output.

1 Introduction

Analyzing the effects of a horizontal merger is controversial, mainly due to the fact that the efficiencies or synergies involved are not observable by the authorities. To overcome this difficulty, Farrell & Shapiro (1990) (hereon F&S) established a sufficient condition for a merger to be welfare enhancing that does not depend on the magnitude of such cost reductions. Assuming that the merger is profitable for the participating firms (otherwise it would not take place), the sufficient condition states that the external effect of the merger (the effect on consumer surplus, CS, plus profits by firms not participating in the merger, Π_O) has to be positive. It is implicitly assumed that no other agents are affected by the merger. However, upstream firms (such as suppliers of inputs for the market in question) may be affected by a downstream horizontal merger in at least two different ways. Firstly, by changing the output level in the downstream market, the merger is likely to affect the prices and output levels in any upstream input industry. Secondly, even if output is held constant, some of the insider's cost reductions are likely to be obtained at the expense of the upstream firms. For instance, the merger may be a way of increasing buyer power and the resulting gains for insiders correspond to losses for a third party that should be considered. Such is the

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case of mergers in the retailing sector that have had some relevance in the recent past both in Europe and the US.

Figure 1 illustrates a possible decision error when upstream producers' profits, Π_P, are not considered in the assessment of the external effect of the merger. In this figure it is implicit that these domestic producers will see their profits decline as a result of the merger. This will be explained below.
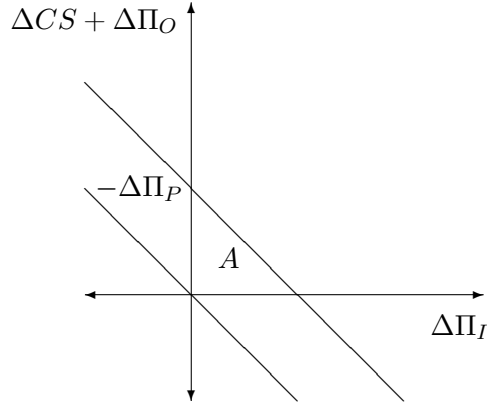

Figure 1: Error when neglecting upstream firms

The negatively sloped straight line (that does not cross the origin) represents the set of mergers that do not affect welfare, that is, mergers such that $\Delta CS + \Delta \Pi_O + \Delta \Pi_I + \Delta \Pi_P = 0$. Mergers in area A are welfare decreasing but would be approved if the F&S condition was directly applied, for instance, to the retailing sector or to any other sector with a small degree of vertical integration, where upstream firms play a relevant role. This happens because the condition only requires that $\Delta CS + \Delta \Pi_O > 0$. In order to have a sufficient condition for aggregate welfare to increase after the merger, it is necessary to include the effect on the producers' profits.

The purpose of this note is to study the consequences of using the F&S condition to sectors in which a downstream horizontal merger may also affect upstream firms. As will be shown below, in some circumstances the sign of the relevant external effect can no longer be established by considering the merger as a sequence of infinitesimal mergers, each corresponding to a marginal change in output.

Other work extending the Farrell & Shapiro (1990) results is due to Barros & Cabral (1994) and Barros (1997), while the infinitesimal approach was also followed, for instance, by Verboven (1995).

The following section presents the relevant condition when the price of inputs depends on the aggregate output level in the downstream industry. The F&S condition appears as a particular case and the differences are highlighted. The third section discusses alternative conditions when input

producers are affected not only by the change in output but also lose on the bargaining table. Finally, section 4 concludes.

2   Price taking suppliers

The technique used by Farrell & Shapiro (1990) is to measure how an infinitesimal change in the quantity produced by the insider firms affects non participating agents after the new post-merger Cournot-Nash equilibrium is reached. The merger is assumed to change aggregate quantity and its distribution amongst firms as well as the cost function of the insider firms.

Under some conditions concerning the demand and cost functions, the total change in external welfare is of the same sign of the variation resulting from a marginal change in quantity.[1] Therefore it suffices to analyze the latter and the magnitude of the eventual and unobservable change in the insiders' cost function is irrelevant as long as the sign of the change in their output is well defined. F&S establish conditions under which insiders' aggregate output declines and study the external effect in the case those conditions are verified.[2] Throughout the paper only output reducing mergers will be considered.

We assume that final demand for a given product, $P(Q)$, is negatively sloped and is served by a set of n firms that simultaneously choose the quantity they place in this market. Firms producing output q_i have total costs given by $c_i(q_i) + C(Q)q_i$ where the first term represents the costs internal to the firm and the second term the costs related to the purchase of inputs. If one considers these firms as retailers, $c_i(q_i)$ can be thought of as the marketing and selling costs while $C(Q)q_i$ represents the costs of acquiring the product that retailers re-sell. The producers of inputs are assumed to be price takers and to have an aggregate supply curve given by $C(Q)$, with $\partial C/\partial Q \geq 0$. This curve is the horizontal sum of the individual supply curves of a number of symmetric producers of which a percentage α is assumed to be domestic. Downstream firms have some degree of monopsony in the sense that they anticipate the impact that their output choices will have on the input's price.

After the announcement of a merger involving some of the downstream competitors, these can be divided in three subsets: insiders, I, domestic outsiders, O^N and foreign outsiders, O^*.

Both before and after the merger, each of the downstream firms will

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[1] The relevant conditions are that P''', P'' and c_i'' are all nonnegative and c_i''' is nonpositive for all nonparticipant firms, where x' is the first derivative of x – see Proposition 5, p. 116 in Farrell & Shapiro (1990).

[2] The new cost function must be such that the merged firm's markup at the pre-merger level of output is less than the sum of the pre-merger markups of all insiders.

maximize its profits given by

$$\pi_i(Q, q_i) = P(Q)q_i - c_i(q_i) - C(Q)q_i \qquad (1)$$

for any $i \in O^N \cup O^*$. The corresponding first-order and second-order conditions are, respectively,

$$\frac{\partial P}{\partial Q}q_i + P(Q) - \frac{\partial c_i}{\partial q_i} - C(Q) - \frac{\partial C}{\partial Q}q_i = 0 \qquad (2)$$

$$2\left(\frac{\partial P}{\partial Q} - \frac{\partial C}{\partial Q}\right) + \left(\frac{\partial^2 P}{\partial Q^2} - \frac{\partial^2 C}{\partial Q^2}\right)q_i - \frac{\partial^2 c_i}{\partial q_{i^2}} < 0 \qquad (3)$$

Let $V(Q) \equiv P(Q) - C(Q)$. We make the following assumptions regarding this function:[3]

$$\frac{\partial V}{\partial Q} + Q\frac{\partial^2 V}{\partial Q^2} < 0, \forall Q : V(Q) > 0 \qquad (4)$$

$$\frac{\partial V}{\partial Q} - \frac{\partial^2 c_i}{\partial q_i^2} < 0 \qquad (5)$$

From the first-order conditions, each firm will react to a rival's change in quantity in accordance with

$$\frac{\partial q_i}{\partial q_{-i}} = -\frac{\frac{\partial^2 V}{\partial Q^2}q_i + \frac{\partial V}{\partial Q}}{\frac{\partial^2 V}{\partial Q^2}q_i + 2\frac{\partial V}{\partial Q} - \frac{\partial^2 c_i}{\partial q_i^2}} < 0 \qquad (6)$$

where $q_{-i} \equiv Q - q_i$. This is the slope of firm i's best response function and it is negative, given the assumptions above. Assumptions 4 and 5 also guarantee that $-1 < \frac{\partial q_i}{\partial q_{-i}} < 0$. The slope of the reaction function is inferior to one, meaning that the equilibrium is stable.

We can therefore establish that

$$dq_i + \frac{\partial q_i}{\partial q_{-i}}dq_i = \frac{\partial q_i}{\partial q_{-i}}dq_{-i} + \frac{\partial q_i}{\partial q_{-i}}dq_i \Leftrightarrow dq_i = \frac{\frac{\partial q_i}{\partial q_{-i}}}{1 + \frac{\partial q_i}{\partial q_{-i}}}dQ = -\lambda_i dQ \qquad (7)$$

with

$$\lambda_i \equiv -\frac{\frac{\partial^2 V}{\partial Q^2}q_i + \frac{\partial V}{\partial Q}}{\frac{\partial V}{\partial Q} - \frac{\partial^2 c_i}{\partial q_i^2}} > 0 \qquad (8)$$

In the linear case, we have $\lambda_i = 1$.

After a merger involving some of the downstream firms, the insiders' internal costs will change. The new function, $c_I(q_I)$, is unknown to the

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[3]These conditions assure the stability of the Cournot oligopoly as shown in Hahn (1962) and Al-Nowaihi & Levine (1985). Levis (1982) shows that these conditions are sufficient for the existence of a unique Cournot-Nash equilibrium.

authorities. Assuming that these eventual cost reductions are sufficiently large so that the merger is profitable but small enough so that insiders' combined output declines, we can evaluate the impact on external welfare of a marginal change in output.

Given that the merger is profitable we only have to consider its domestic external effect, that is, the impact on consumers, domestic outsiders and domestic producers.

$$XW = \left(\int_0^Q P(u)du - P(Q)Q \right) + \sum_{i \in O^N} \left(P(Q)q_i - c_i(q_i) - C(Q)q_i \right)$$
$$+ \alpha \left(C(Q)Q - \int_0^Q C(u)du \right) \quad (9)$$

The main differences from the Barros & Cabral (1994) setting is that (i) costs are also a function of aggregate output rather than individual output and (ii) the impact of the merger on any domestic producers is accounted for. Figure 2 illustrates these differences. Total output falls from Q_0 to Q_1 while outsiders aggregate production expands from Q_0^o to Q_1^o.

In the original case, a decrease in total production will lower consumer surplus while increasing the outsiders' earnings. Consumers lose area **c** to insiders while outsiders gain area **o** to the insider firms. This trade-off is still present here, but there are other benefits for the outsiders, namely a lower input unit price. If all firms are domestic this is a mere transfer from upstream to downstream firms, given by the areas marked **t**, but when we allow for foreign producers or outsiders it may be relevant. Additionally, insiders also benefit from this lower cost and gain area **p** from the producers. Part of this area is considered a loss if there are domestic producers.

We will now follow the infinitesimal merger approach to establish sufficient conditions for the merger to be welfare increasing.

Differentiating XW and using the first-order conditions for outsiders' profit maximization, 2, we have that

$$\frac{dXW}{dQ} \frac{1}{P} = \left(\frac{1}{\varepsilon^D} + \frac{C}{P} \frac{1}{\varepsilon^S} \right) \left(s_I + s_{O^*} - \sum_{i \in O^N} s_i \lambda_i \right) - (1-\alpha) \frac{C}{P} \frac{1}{\varepsilon^S} \quad (10)$$

where $\varepsilon^D = -\partial Q/\partial P \times P/Q$ is the elasticity of demand and $\varepsilon^S = \partial Q/\partial C \times C/Q$ denotes the elasticity of the supply of inputs. The infinitesimal merger has a positive impact on external welfare if and only if

$$s_I + s_{O^*} - \sum_{i \in O^N} s_i \lambda_i < \frac{1-\alpha}{1 + \frac{\varepsilon^S}{\varepsilon^D} \frac{P}{C}} \quad (11)$$

If all producers are domestic (that is, if $\alpha = 1$) we have a condition similar to the one proposed by Barros & Cabral (1994). The impact on the

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P

c

o

t   t   p
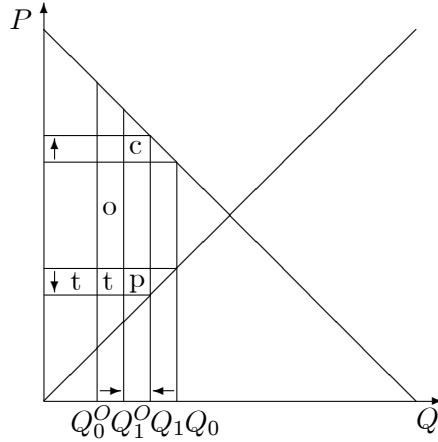
$Q_0^O Q_1^O Q_1 Q_0$ Q

Figure 2: Impact of the infinitesimal merger

external welfare of a marginal change in total output is larger but the sign of the external effect is defined by the same condition. The additional gain for domestic outsiders that results from the reduction in total output (which decreases the outsiders' marginal cost) is nothing but a loss for the domestic input producers represented by area **t** and, consequently, the net domestic effect is null. When there are foreign input producers ($\alpha < 1$) this effect is positive for the domestic economy and the condition becomes weaker. Note also the this reduction in the outsiders' marginal cost depends crucially on the elasticity of the supply of inputs. If $\varepsilon^S \to \infty$ the reduction in total output does not change the marginal cost, that is, $C(Q)$ is constant. When this happens the condition is the same as in Barros & Cabral (1994). The fact that the marginal units are sold with no profits is relevant here. If these units were sold with some profit, producers would even be worse off. This is considered in the appendix. In general, the standard condition is too strong and a number of welfare increasing mergers would not be automatically cleared by the authorities, given that the safe harbor condition was not satisfied. This may be especially relevant in sectors where a large percentage of the inputs are imported or when the supply of inputs is particularly inelastic.

3   Buyer power

In the case analyzed above, it is still possible to use the external effect approach to establish the safe harbor rule for merger approval. This happens because the merger affects all other agents via a change in the insiders' output and the external welfare is a continuous function of Q. Internal cost reductions obtained by the participating firms were only relevant to the extent that these firms had an incentive to change their output, which in its

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turn lead to a new Cournot-Nash equilibrium. There was no direct impact of $c_I(q_I)$ on consumers, rivals or producers of inputs.

However, the same is not true when the merger has an impact on the price insiders pay the producers of inputs, even if output is kept constant. The previous case had downstream firms choosing their output and the input price was such that the market cleared. An alternative to this type of transaction is one in which both downstream and upstream firms bargain for the intermediate price. We do not model the bargaining game explicitly but rather assume that larger downstream firms can have lower input unit prices. It is implicit the idea that larger clients are harder to replace (that is, are replaced at a higher cost) and therefore are able to get lower prices. It has been documented that retailers with larger market shares do tend to have lower unit prices because they are able to impose certain conditions on their suppliers. Throughout this section we will refer to the merging firms and their competitors as retailers. Naturally, the argument applies for other types of upstream/downstream interaction, such as input supplier/producer.

A retailer with a market share of $s_i = q_i/Q$ in the retailing market will pay the producers a unit price of $r_i(s_i)$, with $\partial r_i/\partial s_i < 0$. This formulation allows for retailers with the same market share to pay different prices, for instance, due to different bargaining skills. We assume that $\eta_i = (\partial r_i/\partial s_i)s_i/r_i$ is bigger than $-1/(1 - s_i)$ so that an increase in any given retailer's purchases also increases the producers' revenue, if all other firms keep their output constant, that is, marginal acquisition costs are positive for all firms. In addition to the costs of acquiring the products, each retailer faces a cost of selling or marketing the goods, given by $c_i(q_i)$. It is assumed that producers are symmetric and receive each an equal percentage of the difference between aggregate revenue and costs. The outsiders' profit function is given by

$$\pi_i(Q, q_i) = P(Q)q_i - r_i(s_i)q_i - c_i(q_i)$$

Given that the bargaining game is not modelled, it is not relevant to know each producer's cost function. The aggregate production costs are given by $C_P(Q)$. As before, we assume that a percentage α of the symmetric producers are domestic. Therefore, the share of the profits accruing to domestic producers is also α.

After the merger, insiders gain from possible marketing cost reductions (that is, a new cost function, $c_I(q_I)$, will represent the insiders' costs) as well as from a better bargaining position *vis-à-vis* the producers. It is assumed that the new unit price is $r_I(s_I)$ such that $r_I(s_I)q_I \leq \min_{i \in I} r_i(s_I)q_I$, for any s_I and q_I: for any given output level, the merged firm does not get worse conditions than the best of the merging parties could have had before the merger, when purchasing the post-merger output. The merged firm can pool its best bargaining assets and obtain better conditions for the same aggregate market share. As a consequence of the merger there will be a new

equilibrium, with insiders facing a new cost function.

Following the merger, insiders are again assumed to decrease their output. In the new equilibrium, outsiders will have increased theirs but to a lesser extent, meaning that total output falls. This means that each outsider will see its market share increase, getting lower prices when bargaining with the producers. Despite the fact that each insider will have a lower market share after the merger, insiders will bargain as a single entity and may also get lower prices.

The impact of the merger on aggregate welfare is given by

$$\Delta W = \Delta CS + \Delta R_{O^N} - \Delta T_{O^N} - \Delta C_{O^N} + \Delta R_I - \Delta T_I - \Delta C_I \\ + \alpha(\Delta T_I + \Delta T_{O^N} + \Delta T_{O^*} - \Delta C_P) \quad (12)$$

This can be written as

$$\Delta W = \Delta CS + \Delta R_{O^N} - (1-\alpha)\Delta T_{O^N} - \Delta C_{O^N} + \Delta R_I - (1-\alpha)\Delta T_I - \Delta C_I \\ + \alpha(\Delta T_{O^*} - \Delta C_P) \quad (13)$$

where ΔT_i denotes the change in acquisition costs paid to the producers and ΔR_i the change in retailers' revenue. The ΔC_i's denote the insiders' and outsiders' marketing and selling costs (respectively when $i = I, O^N, O^*$) or the production costs faced by the upstream firms (when $i = P$).

The sign of ΔW is difficult to establish because the magnitudes of ΔC_I and ΔT_I are unknown. As already mentioned, F&S circumvented this problem by evaluating the external effect of the merger, XW. Nevertheless, the domestic external effect is, in this context, given by

$$\Delta XW = \Delta CS + \Delta R_{O^N} - \Delta T_{O^N} - \Delta C_{O^N} + \alpha\left(\Delta T_I + \Delta T_{O^*} + \Delta T_{O^N} - \Delta C_P\right) \quad (14)$$

This aggregate includes the profits of domestic outsiders and producers as well as consumer surplus and leaves insider's profits aside. As mergers are expected to be profitable (that is, $\Delta R_I - \Delta C_I - \Delta T_I > 0$) a positive external effect is sufficient for a positive overall effect of the merger. But, unfortunately, the term ΔT_I is included in XW and, like ΔC_I, it is unknown to the authorities. Therefore, ΔT_I cannot be written as a succession of infinitesimal mergers. Neglecting this term and analyzing under which conditions the remainder is positive is a possible alternative but, if $\Delta T_I < 0$, we will be left with a necessary condition for the external effect to be positive, which in its turn is a sufficient condition for the merger to be welfare increasing. The relevance of such condition depends on the weight authorities give to the non-merging parties (outsiders, producers and consumers). Its non verification could lead to merger rejection if this weight is high enough. The condition will be taken up below.

Before analyzing the impact of a marginal merger on welfare, it is relevant to calculate the change in outsiders' output in response to the change in the merged firms' output.

We will start by defining

$$f_i(s_i, Q) \equiv \frac{\partial^2 (r(s_i)q_i)}{\partial q_i^2} = \frac{(1 - s_i)^2}{Q}(\frac{\partial r_i(s_i)}{\partial s_i}2 + \frac{\partial^2 r_i(s_i)}{\partial s_i^2}s_i) < 0 \qquad (15)$$

and noting that

$$\frac{\partial^2 (r(s_i)q_i)}{\partial q_i \partial q_{-i}} = \frac{s_i}{s_i - 1}f_i(s_i, Q) > 0 \qquad (16)$$

It is assumed that the inverse demand curve verifies the following condition

$$\frac{\partial P}{\partial Q} + Q\frac{\partial^2 P}{\partial Q^2} < 0, \forall Q : P(Q) > 0 \qquad (17)$$

This guarantees that the firm i's marginal revenue shifts downwards if rival firms increase production, meaning that, as the marginal cost function shifts upwards, firm i's best response function is negatively sloped. It is further assumed that

$$\frac{1}{1 - s_i}f_i(s_i, Q) + \frac{\partial^2 c_i}{\partial q_i^2} > \frac{\partial P}{\partial Q} \qquad (18)$$

This condition is sufficient for the retailer's marginal cost (the selling and marketing costs plus the acquisition cost) to intersect demand from below.

Each outsider retailer i will maximize its profit given by:

$$\Pi_i(q_i, Q) = P(Q)q_i - r_i(s_i)q_i - c_i(q_i), i \in O \qquad (19)$$

First-order conditions for profit maximization are

$$\frac{\partial P}{\partial Q}q_i + P(Q) - r_i(s_i) - \frac{\partial r_i(s_i)}{\partial s_i}(1 - s_i)s_i - \frac{\partial c_i(q_i)}{\partial q_i} = 0 \qquad (20)$$

The corresponding second-order conditions,

$$\frac{\partial^2 P}{\partial Q^2}q_i + 2\frac{\partial P}{\partial Q} - f_i(s_i, Q) - \frac{\partial^2 c_i(q_i)}{\partial q_i^2} < 0 \qquad (21)$$

are verified given the above assumptions. With the purpose of analyzing the infinitesimal merger it is necessary to express each firm's change in quantity as a function of the change in total output.

From the first-order conditions, each firm will react to a rival's change in quantity in accordance with

$$\frac{\partial q_i}{\partial q_{-i}} = \frac{\frac{\partial^2 P}{\partial Q^2}q_i + \frac{\partial P}{\partial Q} - \frac{s_i}{s_i - 1}f_i(s_i, Q)}{-\left(\frac{\partial^2 P}{\partial Q^2}q_i + 2\frac{\partial P}{\partial Q} - f_i(s_i, Q) - \frac{\partial^2 c_i(q_i)}{\partial q_i^2}\right)} \qquad (22)$$

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The corresponding λ_i is

$$\lambda_i \equiv -\frac{\frac{\partial^2 P}{\partial Q^2}q_i + \frac{\partial P}{\partial Q} + \frac{s_i}{(1-s_i)}f_i(s_i, Q)}{-\frac{\partial P}{\partial Q} + \frac{1}{(1-s_i)}f_i(s_i, Q) + \frac{\partial^2 c_i(q_i)}{\partial q_i^2}} > 0 \qquad (23)$$

In the linear case, we have

$$\lambda_i = \frac{\frac{\partial P}{\partial Q} + 2\frac{s_i(1-s_i)}{Q}\frac{\partial r_i(s_i)}{\partial s_i}}{\frac{\partial P}{\partial Q} - 2\frac{(1-s_i)}{Q}\frac{\partial r_i(s_i)}{\partial s_i}} > 1 \qquad (24)$$

It is useful to know the impact that the change in output will have on the outsiders market shares:

$$q_i ds_i = q_i\frac{dq_i Q - q_i dQ}{Q^2} = q_i\frac{-\lambda_i Q - q_i}{Q^2}dQ = -(\lambda_i + s_i)s_i dQ > 0 \qquad (25)$$

In order to establish a sufficient condition for the merger to be welfare increasing we will look at the full effect rather than the external effect. Part of this full effect depends only on the change in total output. Let us define the function X^N as the aggregation of the welfare elements that depend only on aggregate output: consumer surplus, domestic retailers profit, domestic producers' revenue when selling to all outsiders and insiders' revenue.

$$X^N \equiv \int_0^Q P(u)du - P(Q)Q + \sum_{i \in O^N}(P(Q)q_i - r_i(s_i)q_i - c_i(q_i)) +$$
$$+ \sum_{i \in O}\alpha r_i(s_i)q_i + P(Q)(Q - \sum_{i \in O}q_i) \quad (26)$$

A marginal change in the total quantity produced has the following impact on X^N, after simplification (see appendix)

$$\frac{dX^N}{PdQ} = \frac{1}{\varepsilon}\left(s_I + s_{O^*} - \sum_{i \in O^N}s_i\lambda_i\right) + \sum_{i \in O^N}\frac{r_i(s_i)}{P}\eta_i s_i(1 + \lambda_i)$$
$$- \sum_{i \in O}\alpha\frac{r_i(s_i)}{P}(\lambda_i + \eta_i(\lambda_i + s_i)) + (1 - \frac{1}{\varepsilon}s_I + \sum\lambda_i) \quad (27)$$

The change in total welfare (the full effect) is given by

$$\Delta W = \int_{Q_0}^{Q_1}\frac{dX^N}{dQ}dQ - \Delta C_I - (1 - \alpha)\Delta T_I - \Delta C_{PN} \qquad (28)$$

where the superscript 1 denotes the post-merger equilibrium.

Under the assumptions that insiders reduce production and that these can get better buying conditions, the term $(1 - \alpha)\Delta T_I$ is negative.[4] The same is assumed to hold for ΔC_I and ΔC_{PN}.Therefore, a sufficient condition for the merger to be welfare increasing is $dX^N/PdQ < 0$, that is,

$$
\frac{1}{\varepsilon}\left(s_I + s_{O^*} - \sum_{i \in O^N} s_i \lambda_i\right) + \sum_{i \in O^N} \frac{r_i}{P}\eta_i s_i (1 + \lambda_i)
$$
$$
- \sum_{i \in O} \alpha \frac{r_i}{P}(\lambda_i + \eta_i(\lambda_i + s_i)) + (1 - \frac{1}{\varepsilon}s_I + \sum_{i \in O}\lambda_i) < 0 \quad (29)
$$

The first term is the extension of the F&S condition for open economies derived by Barros & Cabral (1994). It reflects the impact on consumer surplus and on domestic retailers profits.

As the costs faced by domestic retailers depend on their rival's aggregate production (because the firm's relative size matters) it is necessary to account for this effect. This change in costs, not contemplated in the standard condition, is here represented by the second term. Given that the decrease in q_{-i} increases each retailers' market share and, consequently, reduces the acquisition costs $(-d(r_i q_i)/dq_{-i} = r_i(\eta_i s_i(1 + \lambda_i))\, dQ)$, this negative term leads to a condition that is weaker than the original one.

However, part of the gain of the domestic retailers is obtained at the expense of domestic producers. Furthermore, domestic producers may also receive a lower payment from foreign retailers. The new equilibrium will have insiders selling less and outsiders selling more. This means that each outsider firm will have a higher market share and will pay less for each of the units it re-sells but each outsider retailer will re-sell a larger quantity. If the elasticity $|\eta_i|$ is high enough, that is, $|\eta_i| > \frac{\lambda_i}{\lambda_i + s_i}$ this term will have a positive sign, meaning that the merger is less likely to be beneficial because part of the outsider's gains are, simultaneously, domestic producer's losses. The higher the percentage of domestic firms, α, the more difficult it is to satisfy the condition.

Finally, the last term reflects the change in insiders' revenue. It should be included if insiders are domestic. This term could be excluded if it was guaranteed that $\Delta R_I - \Delta C_I > 0$. However, one can no longer establish this because the profitability of the merger is not necessarily motivated by the reduction in the marketing and selling costs. The reduction in acquisition costs $(\Delta T_I < 0)$ allows for the possibility of having a profitable merger with $\Delta R_I - \Delta C_I < 0$.

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[4]Note that the insiders' combined market share declines. However, we assume that it is still larger than the $\max_{i \in I}\{s_i\}$ which guarantees that $r_I(s_I^1) < \min_{i \in I}\{r_i(s_i)\}$ and, consequently, $\Delta T_I \equiv r_I(s_I^1)q_I^1 - \sum_{i \in I} r_i(s_i)q_i < 0$.

3.1 Particular cases

3.1.1 Closed economy

Note that with $\alpha = 1$ and $O^* = \emptyset$ we have the closed economy case. Condition 29 can be simplified to

$$\sum_{i \in O} \left(\frac{\lambda_i}{P} \frac{\partial c_i}{\partial q_i} \right) < -1 \qquad (30)$$

which is never verified. When all firms are domestic and if cost reductions are not considered, no output reducing merger is desirable. The marginal valuation for the units that are no longer produced exceeds the marginal production cost. Given that all agents are domestic, total welfare will necessarily decrease.

3.1.2 Foreign producers or foreign insiders

The case that most resembles the original setting is the one in which all the producers are foreign, that is, the limit case when $\alpha = 0$. The fact that firms may have cost reductions which are, at the same time, lower revenues for the foreign producers is not relevant for the national authority. When this is the case, the external effect can be measured accurately because ΔT_I is not relevant as long as the merger is profitable, which is a reasonable assumption. The sufficient condition for the desirability of the merger is

$$\frac{dX^N}{PdQ} = \frac{1}{\varepsilon} \left(s_I + s_{O^*} - \sum_{i \in O^N} s_i \lambda_i \right) + \sum_{i \in O^N} \frac{r_i}{P} \eta_i s_i \left(1 + \lambda_i \right) < 0 \qquad (31)$$

which, as explained above, is a weaker condition than the one by Barros & Cabral (1994). The negative second term represents the extra impact on domestic outsiders expenditure which is smaller because these firms' market share has increased. As the producers are foreign this has a positive net effect for the domestic economy and the merger is more likely to be welfare enhancing.

Even when producers are domestic, this is also a necessary condition for the merger to be desirable for the set of all non-participating agents. If the authorities focus their attention on consumers, domestic outsiders and producers, the non verification of 31 leads to merger rejection. This is particularly useful when the insiders are foreign firms.

4 Conclusions

Most theoretical work on horizontal mergers considers three types of economic agents: insiders, outsiders and consumers. Other firms that sell their

products to insiders or outsiders are generally not considered, although an exception is the work by Dobson & Waterson (1997). When such firms play a relevant role we argue that the F&S condition may be inadequate to assess the desirability of a merger. This happens because upstream firms will see their profits change after the merger takes place. This note discussed the limitations of the F&S condition as well as the difficulties in the extension necessary to cope with these mergers.

Two of the features proposed by F&S that clearly simplify the analysis are the use of the external effect and the infinitesimal merger approach. When the upstream firms are only affected by the merger induced change in output in the downstream industry, the same technique can be applied. This happens when the upstream firms are price takers (or price makers with an exogenous markup). In this case, we show that the safe harbor rule for merger approval is, in general, weaker but depends on factors such as the percentage of domestic producers, the elasticity of the supply of inputs or the markup in this industry.

However, the possibility that the merger may also change the insiders' acquisition cost function, which affects directly not only the insider firms but also the upstream producers, has some implications on the use of the external welfare and infinitesimal merger approach. When both parties bargain over the intermediate price, it is likely that the merger may change the insiders' cost function (that is, for the same output for all firms, insiders may get a lower price). As the external welfare includes profits to upstream producers that depend on the payment received from the insiders, it would be necessary to estimate how the merger changes such payment. We therefore focus on total welfare and provide a sufficient condition for the merger to be welfare increasing which can be simplified under some particular conditions. Such condition requires that the elasticity of the input price in relation to the buyer's market share is estimated.

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