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Integrative Complexity: An Alternative Measure for System Modularity

Complexity and modularity are important inherent properties of the system. Complexity is the property of the system that has to do with individual system elements and their connective relationship, while modularity is the degree to which a system is made up of relatively independent but interacting elements, with each module typically carrying an isolated set of functionality. Modularization is not necessarily a means of reducing intrinsic complexity of the system but is a mechanism for complexity redistribution that can be better managed by enabling design encapsulation. In this paper, the notion of integrative complexity (IC) is proposed, and the corresponding metric is proposed as an alternative metric for modularity from a complexity management viewpoint. It is also demonstrated using several engineered systems from different application domains that there is a strong negative correlation between the IC and system modularity. This leads to the conclusion that the IC can be used as an alternative metric for modularity assessment of system architectures. [DOI: 10.1115/1.4039119]

Introduction

One of the fundamental tenets of system design is to keep the system architecture as simple as possible. However, contrary to basic design rules, architectures of latest engineering systems are becoming more complex due to ever-increasing complexity of new technologies and infrastructures to accommodate them. Engineering systems across domains have adopted significant technological and architectural changes to meet the forever increasing demand for higher performance. This has led to increased system complexity and higher development effort in realizing such systems [1].

In a complex engineered system, multiple components and interfaces are designed together to perform one or more overarching set of functions. The pattern of connections and their physical behavior cannot be thought of as truly regular or fully predictable, and understanding the system behavior requires understanding of system elements and their pattern of connections [2]. This overall trend necessitates an important need for proper system architecture complexity management process. Without the smart complexity management, the system's overall architecture may become unmanageable, leading to undesirable results, such as longer development period, higher R&D and lifecycle costs, and possible increase in system's postlaunch maintenance cost.

There are two high-level categories of system complexity, which are internal and external complexity. The internal complexity is closely related to overall system design and is further divided into structural complexity, dynamic complexity, and organizational complexity [3,4], as shown in Fig. 1. The external complexity is related to factors, which are not subject to control by system architects, such as market dynamics, political complexities, and institutional complexities. In this study, the primary focus of the research is on the aspect of structural complexity and its distribution across the system.

Structural complexity of a system is closely related to the complexity of individual system elements and degree of connectivity of the underlying system architecture. As a result, the structural complexity has strong impact on the effort (and cost) of system design, development, and operation [6–8]. Even though the

primary goal of the system design organization is to optimize the overall cost related to the system being developed and operated by designing the system architecture as simple as possible, there is a need to incorporate essential complexity in the system to deliver the required level of system performance [9]. To this extent, the system architect must balance the tradeoffs between system design, development, and operation efforts, system performance, and the amount of complexity incorporated.

To better manage architectural arrangements of complex systems, one of the widely used design strategy is modular design strategy [3,10]. Modularity refers to the property of a system where the system can be divided into different number of chunks called modules, which have strong intraconnections within individual module and weak interconnections between modules [3,10]. In many instances, an individual module is responsible for a specific function required by the system, such as data storage function performed by computer hard drives and user input function performed by computer keyboard. Modular design strategy refers to variety of methodologies that attempt to decompose complex systems into manageable modules. In the context of complex system design, modular design strategies can be viewed as ways to manage the inherent complexity by effectively allocating them to individual modules. System decomposition is essentially a system organization principle and can be viewed as a means to achieve a desirable distribution of total system complexity [11].

In this paper, the notion of integrative complexity (IC) is introduced, and the corresponding metric is proposed as an alternative metric for system modularity from complexity management viewpoint. Integrative complexity is defined as the difference between the total system complexity and the sum of complexities of individual modules, as defined by the system decomposition. Therefore, one can view integrative complexity as the complexity associated with assembly or integration of a system from its constituent modules or subsystems. It is also argued that the desirability of system decomposition and the resulting complexity distribution can be represented by reduction in integrative complexity of the system. This notion of integrative complexity can be easily extended to system-of-systems.

The formal mathematical description for integrative complexity metric is introduced later in this paper. Using the proposed integrative complexity metric and a modularity metric introduced in previous literature, the relationship between integrative complexity and system modularity is quantitatively explored by performing analyses of several real-life complex engineered systems.

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Contributed by the Design Theory and Methodology Committee of ASME for publication in the JOURNAL OF MECHANICAL DESIGN. Manuscript received September 23, 2017; final manuscript received January 4, 2018; published online March 1, 2018. Assoc. Editor: Katja Holta-Otto.

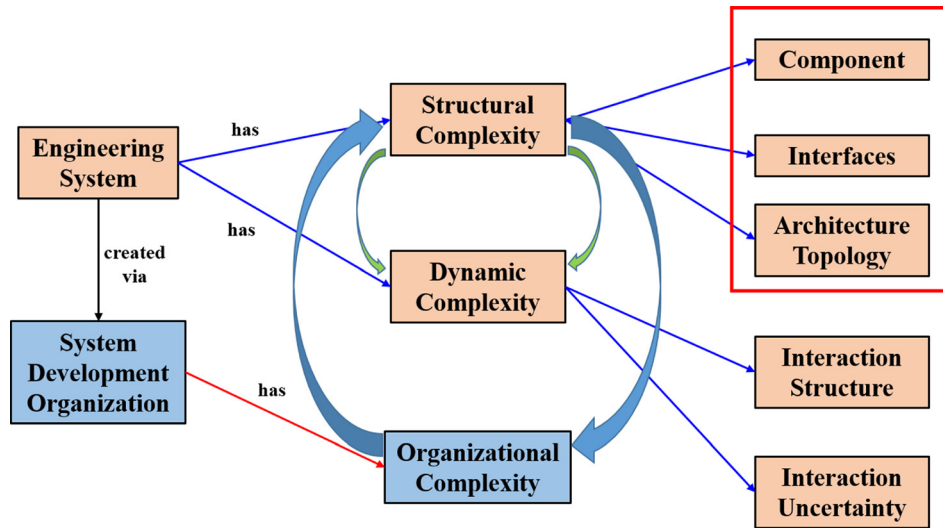


Fig. 1 Complexity typology for engineering systems (Reprinted with permission from Sinha and Suh [5]. Copyright 2018 by Springer.)

Result indicates a strong negative correlation between integrative complexity and system modularity, which states that a smaller value of integrative complexity implies higher modularity, both within and across classes of real-world engineered systems.

This paper is structured as follows: The Literature Review section presents the current state on quantification of system complexity and modularity from a metric oriented viewpoint. Subsequently, complexity quantification and complexity attribution methodology, given system decomposition, is presented. This leads to formulation of integrative complexity as an alternative metric for system modularity from complexity management viewpoint. The Complexity Management and System Decomposition section explores the relationship between integrative complexity and the Q modularity metric [2]. In order to establish the relationship between integrative complexity and modularity, empirical validation across a swath of engineered complex systems is performed in the Empirical Study section, with results demonstrating a strong negative correlation between normalized integrative complexity and Q metric of system modularity. Using the research results, it is proposed that the integrative complexity metric can be used as a surrogate for measuring the degree of system modularity. This paper concludes with related discussion, research summary, and future work.

Literature Review

Complexity and modularity are important inherent properties of complex engineering systems. As such, there has been a lot of works published in the subject of system complexity and modularity and its implication to the overall complex system design. In the context of engineered system, complexity is defined as “the property of having many interrelated, interconnected, or interwoven elements and interfaces” [4,9]. There are many complexity metrics proposed, with earlier works originating from the software engineering [12,13]. Over time, several other metrics, based on different characteristics of the system, were introduced. These include complexity metrics based on system element count-based measures [14–16], information and information transfer efficiency [17,18], entity relationship graphs decomposition [19], hierarchy extension [3], network structure heterogeneity [20], empirical measure based on similar systems [21], solvability of design [22], and graph energy of the system [23].

Complexity metrics have been used in wide variety of research applications to assess and manage system complexity. Tamaskar et al. [24] have proposed a framework for quantifying aerospace system complexity. Min et al. [25], using the complexity metric

developed by Sinha [26], analyzed complexity values and sensitivities of basic architecture structures, such as integral, modular, and bus-modular structures. Sinha et al. applied their complexity metric to quantify complexity of aircraft engines [26,27]. Kim et al. [28] analyzed complexity of product family and platforms. Kim et al. [29] established the relationship between module complexity value and actual module design effort. Additionally, there has been a recent research effort to optimize complex systems in terms of its modularity and allocation of system complexity among modules through Pareto-optimization [5].

As for modularity metrics, there are several metrics introduced in academia. According to Holtta-Otto et al. [30], modularity metrics are divided into two different types. The first type of metric measures the degree of coupling between modules, which is an indication of module independence. To this extent, several metrics were developed to measure the coupling density. Allen and Carlson-Skalak [31], Martin and Ishii [32], Newman [2], Sosa et al. [33], Guo and Gershenson [34], Holtta-Otto and de Weck [35], Whitfield et al. [36], and Jung and Simpson [37] proposed modularity metrics to measure the coupling density and demonstrated usefulness of their metrics on vehicle console, video cassette, jet engine, water cooler, camera, and computers. The second type of metric identifies and measures similar features of modules, from the perspective of materials used, manufacturing process used, suppliers involved, and overall lifecycle issues. Proposed metrics by Newcomb et al. [38] and Gershenson et al. [39] are based on life cycle similarities. Siddique et al. [40] and Mikkola and Gassman [41] proposed modularity metrics that measured similarities in components, while Mattson and Magleby [42] proposed metrics measuring similarities in functions.

Parallel to defining various modularity measuring metrics, there have been works published that propose various modular design algorithms. Yu et al. [43] and Helmer et al. [44] proposed module clustering algorithm based on minimum description length theory [45]. Van Beek et al. [46] proposed k -mean clustering algorithm based on modularity metric proposed by Whitfield et al. [36]. Li [47] proposed system decomposition method using matrix-based two-phase approach. Borjesson and Holtta-Otto [48] proposed clustering algorithms based on module function deployment. Recently, Li et al. [49] proposed module partition methods based on directed and weighted networks. Others include Idicula-Gutierrez-Thebeau algorithm [50], Cambridge advanced modeler [51], and community detection algorithm proposed by Blondel et al. [52]. In addition, there is another strand of modularity detection algorithms based on random matrix theory and spectral decomposition strategies [53]. It has been claimed that spectral

methods provide the best quality community (or module) detection technique [54]. It has been shown that there exists a fundamental resolution and detectability limit for all module detection methods [54–56] and poses a significant theoretical and practical challenge for community (or module) detection algorithms.

Until now, the literature efforts focused on solely measuring and managing system complexity or system modularity in isolation. Existing literature tends to equate increasing modularity with reduction in complexity. What this means is that higher modularity results in lower structural complexity [10]. It has been shown that there is no causal relationship between the two [26,57], and complexity reduction can be used to validate any proposed system decomposition [57]. Since modularization is essentially a desirable system organization principle that tries to encapsulate design details by way of system decomposition, it cannot fundamentally change the underlying complexity of the system. It is due to the fact that modularization only organizes a system without modifying the same. System decomposition does impact the distribution of the total system. In this paper, it is claimed that a desirable distribution of complexity, achieved by way of system decomposition, is one that reduces integrative complexity of the system and demonstrates a strong relationship between integrative complexity and system modularity as measured by the Q metric.

Complexity Management and System Decomposition

Quantifying Complexity. There is a close relationship between the engineering system's structural complexity and the "form" [9] of the system architecture, which depends on number of elements in the system, their characteristics, and connectivity between system elements. The metric adopted to measure structural complexity [23,26] in this paper captures complexity arising from (i) individual system elements, (ii) individual connections between system elements, and (iii) topology of connections for the overall system. Following is the mathematical expression of structural complexity metric for engineering systems, proposed by Sinha et al. [23,26,27]:

$$\text{structural complexity, } C = C_1 + C_2 C_3 \quad (1)$$

The overall structural complexity metric (C) shown in Eq. (1) is composed of three major terms. The first term (C_1) is the total summation of individual element's complexity. The second term (C_2) is the summation of complexities arising from individual interaction between system elements. The last term (C_3) represents the topological complexity of the system, resulting from the interface arrangements between elements in the system.

The first term (C_1) captures individual element's complexity and does not contain any system architecture information. It can be rewritten as sum of individual element's complexity (α_i) as shown below:

$$C_1 = \sum_{i=1}^n \alpha_i$$

The second term (C_2) is the sum of complexities of each pairwise interaction, which is labeled as β_{ij} in the detailed representation of C_2 below:

$$C_2 = \sum_{i=1}^n \sum_{j=1}^n \beta_{ij} A_{ij}$$

In the equation above, $A \in M_{n \times n}$ is the binary adjacency matrix which represents the connectivity structure of the system with following conditions:

$$A_{ij} = \begin{cases} 1 & \forall [(i,j)|(i \neq j) \text{ and } (i,j) \in \Lambda] \\ 0 & \text{otherwise} \end{cases}$$

where Λ represents the set of connected elements, and n is the number of elements for the entire system.

Finally, the last term (C_3) is the term that is directly related to the topological arrangements of system interfaces. In more detailed mathematical term, it is expressed as

$$C_3 = \frac{E(A)}{n}, \quad \text{where } E(A) = \sum_{i=1}^n \sigma_i(A)$$

where $\sigma_i(\cdot)$ represents the i th singular value of binary adjacency matrix (A) [26,58,59]. The C_3 term is useful for quantifying topological complexity arising from different connectivity structure within the system. This term is also related to the effort required for system integration. One should also note that in order to calculate C_3 , it is required to have the overall knowledge of connectivity structure of the system, since it must be mapped to the adjacency matrix A . Using detailed terms introduced for C_1 , C_2 , and C_3 , the complexity metric in Eq. (1) can be rewritten as

$$C = \sum_{i=1}^n \alpha_i + \left(\sum_{i=1}^n \sum_{j=1}^n \beta_{ij} A_{ij} \right) \frac{E(A)}{n} \quad (2)$$

Figure 2 shows terms shown in Eq. (2), with brief explanation of what each term represents in terms of system's overall structural complexity. For more detailed mathematical proof of the complexity equation presented, readers can refer to the work by Sinha [26]. Additionally, for estimating individual element's complexity (α_i), Sinha [26] and Dobson [60] have proposed various estimation guidelines.

Complexity Attribution and System Decomposition. Complexity attribution is a method for consistent accounting of complexity assigned to different modules or subsystems and contribution of complexity from system integration. In essence, the complexity attribution method describes how overall structural complexity is distributed within the system, given a system decomposition strategy. System decomposition strategy refers to the decomposition of any system into smaller modules or subsystems that are easier to manage [52,61]. There are other related definitions or point of view on system decomposition [10] that often uses functional view of the system. Once system decomposition is made available, the complexity attribution process performs accounting of complexity of different modules and complexity assigned to integration of modules.

The system decomposition can be defined by a map $Gr(\cdot)$, which is an element to module map. Here each element is assigned to a module, and this map is unique, meaning that an element can be a member of a unique module. It is probably best to use a motivating illustration shown in Fig. 3. The hypothetical system shown in the figure has ten elements, and the system is divided into two modules, with each module composed of five elements each. The binary symmetric adjacency matrix A for this synthetic system representation can be written in terms of submatrices A_1 , A_2 , and K . Notice that submatrix K represents the inter-module connectivity structure and is different from the number of modules, k with $k=2$ in this case. Here A_1 and A_2 represent the binary adjacency matrices of module 1 and 2, respectively.

Expanding to the general case with k modules and given system decomposition map $Gr(\cdot)$, the individual module complexity for i th module can be expressed as

$$C^{(i)} = C_1^{(i)} + C_2^{(i)} C_3^{(i)} \quad (3)$$

where each term of the complexity metric is defined as

$$C_1^{(i)} = \sum_{p=1}^{n_i} \alpha_p^{(i)}; \quad C_2^{(i)} = \sum_{p=1}^{n_i} \sum_{q=1}^{n_i} \beta_{pq}^{(i)} A_{pq}^{(i)}; \quad C_3^{(i)} = \frac{E(A^{(i)})}{n_i}$$

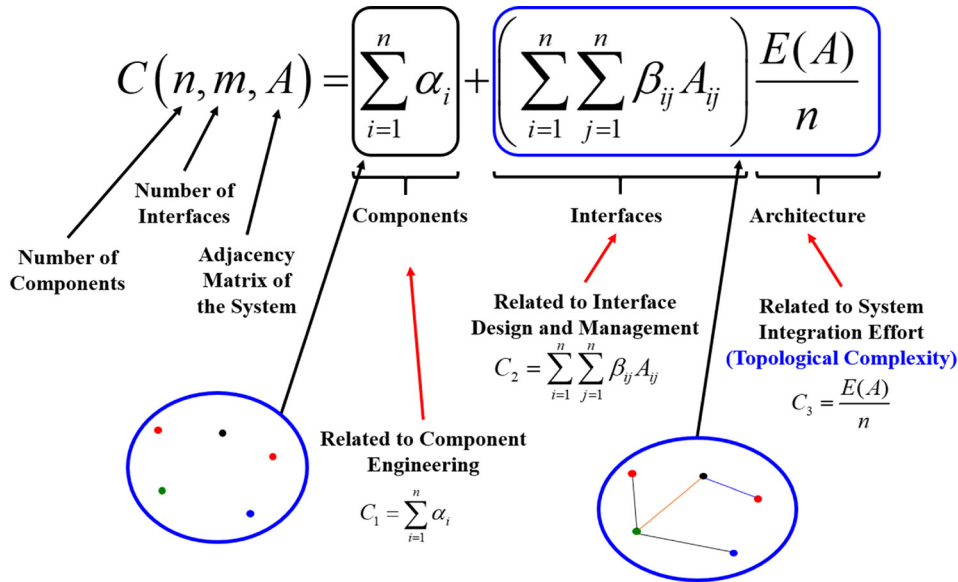


Fig. 2 Explanation of individual terms of the structural complexity metric (Reprinted with permission from Sinha and Suh [5]. Copyright 2018 by Springer.)

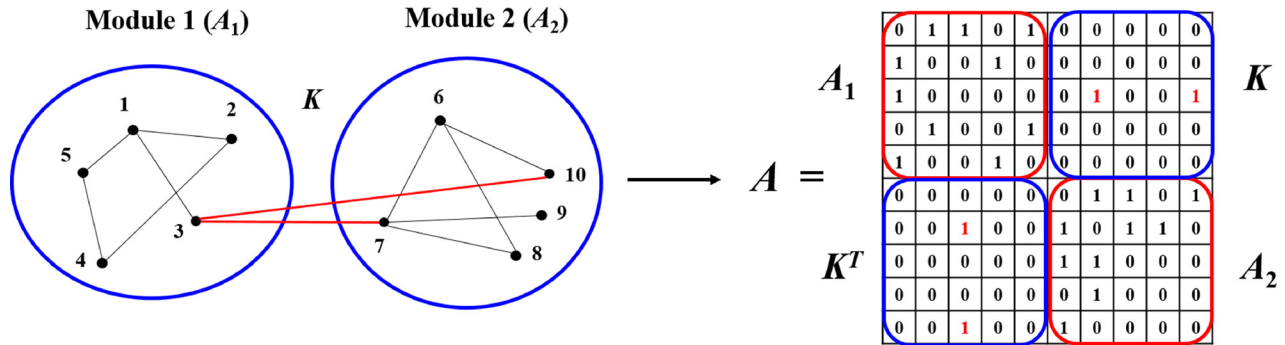


Fig. 3 A hypothetical system composed of two modules, ten elements, and ten bidirectional interfaces in network and binary adjacency matrix form

The method described above is same as that of computing structural complexity metric for a module in isolation. Given the system decomposition, the integrative complexity is defined as

$$\text{integrative complexity, } IC = C - \sum_{i=1}^k C^{(i)} \quad (4)$$

Hence, integrative complexity is the difference between total system complexity (C) and the sum of individual modules complexities. The modules are defined by the specific system decomposition method used. Since the elements are divided into modules, it can be said that $C_1 = \sum_{i=1}^k C_1^{(i)}$, and IC can be expressed as

$$IC = C_2 C_3 - \sum_{i=1}^k C_2^{(i)} C_3^{(i)} \quad (5)$$

Note that IC is a function of system decomposition, in addition to details of the system itself. It expresses the amount of complexity required to integrate the system from its module or subsystem constituents. This interpretation lends this metric directly applicable in any system-of-systems context, where integrative complexity can be interpreted as the amount of complexity required to compose the system-of-systems from its constituent systems. Conversely, one can interpret *integrative complexity* as the “hidden”

complexity that remains invisible by looking at the complexity at the aggregated level of modules alone.

As it can be seen from Eq. (5), integrative complexity is independent of components, and what matters are the interfaces and how they are topologically arranged. In order to compare different systems from multiple domains, it is helpful to use the normalized version of integrative complexity (IC_n), defined as

$$IC_n = 1 - \frac{\sum_{i=1}^k C_2^{(i)} C_3^{(i)}}{C_2 C_3} \quad (6)$$

Note that the normalized integrative complexity is a ratio with $IC_n \in [0, 1]$ and therefore a dimensionless number.

As an illustrative example, the hypothetical system shown in Fig. 3 is used. For simplicity, the assumption is that all elements have unit complexity, $\alpha_i = 1, \forall i$, and all interfaces have complexity $\beta_{ij} = 0.1, \forall i \neq j$. Each module has five elements and five within-module interfaces. Notice that while module 1 has only one module-bridging element that interfaces across module boundary, module 2 has two module-bridging elements, namely, elements 7 and 10. Applying the complexity quantification and attribution process to this hypothetical system, following result is obtained, as shown in Table 1.

Modularity and System Decomposition. For complex systems, modularity estimation is based on the given system

Table 1 Complexity quantification and attribution example based on hypothetical system shown in Fig. 3

Quantity	Value	Description
C	11.49	Using Eqs. (1) and (2)
$C_2 * C_3$	1.49	Using Eq. (2)
$C^{(b)}$	{5.56, 5.58}	Using Eq. (3)
IC	0.35	Using Eq. (4)
IC_n	0.23	Using Eq. (6)

decomposition adopted and distribution of intra- and intermodule interfaces. Let y_{ii} represents fraction of intramodule interfaces, while y_{ij} represents fraction of intermodule interfaces. For computation of modularity index Q [2], a module matrix e , also known as *community matrix*, is constructed as

$$e = \begin{bmatrix} y_{11} & y_{12}/2 & \dots & \dots & y_{1k}/2 \\ y_{21}/2 & y_{22} & \dots & \dots & y_{2k}/2 \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ y_{k1}/2 & \dots & \dots & \dots & y_{kk} \end{bmatrix}$$

For the module matrix e , the row sum is written as $a_i = \sum_{j=1}^k e_{ij} = y_{ii} + (\sum_{j=1}^k y_{ij})/2$, and the modularity metric Q is defined as

$$Q = \sum_{i=1}^K (e_{ii} - a_i^2) = Tr(e) - \|ee^T\| \quad (7)$$

Here e_{ii} represents the fraction of edges with both end vertices in the same module i , and a_i represents fraction of edges with at least one end vertex inside module i . To illustrate the process, consider the hypothetical system example shown in Fig. 3. In the example, module has five elements and five within-module interfaces. Module 1 has only one module-bridging element (element 3), while module 2 has two module-bridging elements (elements 7 and 10). Applying the method described earlier, following module matrix is obtained:

$$e = \begin{bmatrix} \frac{5}{12} & \frac{1}{12} \\ \frac{1}{12} & \frac{5}{12} \end{bmatrix}$$

Based on the module matrix above, the value of modularity metric Q for the system is

$$\begin{aligned} a_1 &= e_{11} + e_{12} = \frac{1}{2} \\ a_2 &= e_{21} + e_{22} = \frac{1}{2} \\ Q &= e_{11} + e_{22} - [a_1^2 + a_2^2] \\ &= \frac{5}{6} - 2 * \left(\frac{1}{2}\right)^2 = \frac{1}{3} \end{aligned}$$

One important issue here is to clarify the difference between integrative complexity metric and the Q modularity metric. While Q modularity metric is based only on the number of intermodule interfaces, integrative complexity considers the internal connectivity structure of modules in addition to characterization of individual intermodule interfaces. The integrative complexity can be viewed as an *apparent* loss of complexity due to aggregation of

system view through its modules or subsystems and represents the amount of complexity involved in assembling the system from its modules. This has direct relationship with complexity management aspects of engineered systems.

Relationship Between Integrative Complexity and Modularity.

From the earlier discussion, integrative complexity can be interpreted as the complexity resulting from system integration, that is, integration of modules as defined by the system decomposition strategy. For a given level of total complexity, a lower value of integrative complexity implies higher proportion of in-module complexity. Hence, a lower value of integrative complexity implies that a larger fraction of total system complexity has been attributed to modules themselves and a smaller fraction of total complexity is required to integrate those modules in order to compose the system. It is argued that smaller value of normalized integrative complexity (IC_n) is desirable from a complexity management viewpoint since it points to a reduced integration efforts and aids divide-and-conquer paradigm. In this sense, lower integrative complexity enables higher modularity, and it is hypothesized that there exists a stronger relationship between modularity and integrative complexity. To demonstrate the validity of this hypothesis, an empirical study over several real-life complex engineering systems was performed, and the analysis results are presented in the Empirical Study section.

Empirical Study

Overview. The focus of this section is to explore and establish the relationship between integrative complexity and modularity through empirical analyses of several real-life complex systems. The empirical analysis was performed at two levels: (i) study the relationship between normalized integrative complexity and modularity by using different system decomposition strategies for each individual system and (ii) study the same relationship across the set of systems, assuming the modularity index maximizing decomposition for each system. The set of complex engineered systems used in this study is listed in Table 2, shown with the size of its binary adjacency matrix system model, also known as the design structure matrix (DSM) [64]. As shown in the table, analyzed complex systems are primarily electromechanical systems, which include a train undercarriage system, two printing systems representing the industrial and office printing sectors, two aircraft engines depicting a conventional two-spool turbofan and an advanced geared turbofan, and an advanced hydrogen-enhanced combustion engine (HECE). DSMs for complex systems shown in the table are hardware-based DSMs, constructed using engineering drawings, actual system examination, system operation/maintenance manuals, and design expert interviews. For these DSMs, connections between components are physical, meaning that they are in direct contact or are connected using electrical wires or other means.

Integrative Complexity: Modularity Relationship for Individual Systems. As the first step, the relationship between the normalized integrative complexity and modularity index is quantitatively explored for each system using different system decompositions. To this end, system decomposition strategies were varied, and

Table 2 Complex systems analyzed in this study

No.	Analyzed systems	DSM size
1	Train undercarriage [28]	149 × 149
2	Industrial printing system [62]	84 × 84
3	Office printing system	50 × 50
4	Geared turbofan aircraft engine [26,27]	86 × 86
5	Two-spool turbofan aircraft engine [26,27]	69 × 69
6	HECE [63]	30 × 30

Table 3 Value of IC_n , Q , and the total number of modules defined for train undercarriage under different system decomposition configurations

Train undercarriage decomposition	Integrative complexity (IC_n)	Modularity (Q)	Number of modules
1	0.32	0.39	19
2	0.24	0.57	17
3	0.16	0.64	14
4	0.15	0.68	12
5	0.13	0.71	10
6	0.12	0.73	10
7	0.11	0.74	11

resulting behavior of normalized integrative complexity and modularity index was observed when system decomposition changes. For each system listed in Table 2, a set of component-module maps (i.e., a table that maps each component to a unique module) for various system decomposition strategies is generated. The suite of system decomposition strategies considered includes: (a) system decomposition adopted by system design team for each system, (b) multiple modularity maximization based decompositions techniques [2,50,52], and (c) decompositions that result in optimal tradeoff between modularity and diversity of in-module complexity distribution. Note that decomposition technique described in Ref. [50] is stochastic in nature and produces different decompositions based on input parameter ranges. The community detection algorithm [52] also has stochastic characteristic associated with it, while Newman algorithm [2] is deterministic. Using these three types of system decomposition techniques, a dataset of seven system decompositions, all of which aims to maximize modularity with some differences in their decomposition paradigm, is generated. As an example, Table 3 shows IC_n and the modularity index (Q) values for a subset of seven decompositions for train undercarriage system. Additionally, Fig. 4 shows the original train undercarriage decomposition DSM and the modularity (Q) maximized decomposition DSM.

In order to investigate statistical significance of the correlation between normalized integrative complexity and modularity with a larger set of data, the initial set of decompositions, suggested by approaches (a)–(c) mentioned earlier, were augmented with random permutation of system decompositions to arrive at a dataset of approximately 100 different system decompositions for each engineered system listed in Table 2. Figures 5–10 show IC_n versus Q regression plots for six systems in Table 2 with different system

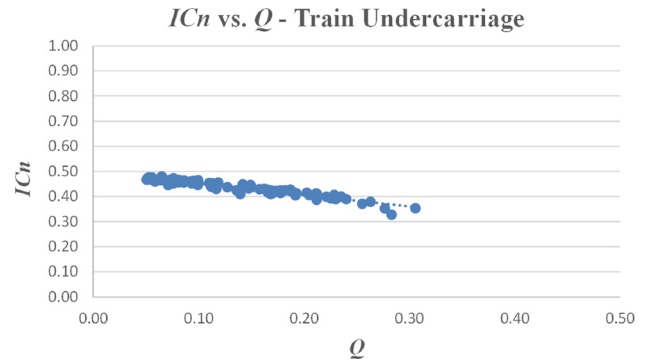


Fig. 5 Regression plots for IC_n and Q of train undercarriage system with decompositions

decompositions. As it can be observed from these plots, IC_n and Q have strong correlation.

As it can be observed from Figs. 5–10, a vast majority of these randomly perturbed system decompositions happens to generate low system modularity with $Q < 0.25$ and are densely clustered around low modularity/high integrative complexity regime of the distribution. Once these data points are obtained, rigorous statistical analysis was performed. The results are shown in Tables 4 and 5.

The reported estimates of parameters of the linear model were obtained using the pseudovalue jackknife technique [65] to mitigate the problem of biased estimates and produce robust parameter estimates. In this technique, the desired calculation for all the data is made where the data are divided into subsamples. Then, the computation is performed for each group of data obtained by leaving out one subsample [66]. For all jackknife subsamples, the estimated model parameters were found to lie within the 95% confidence interval of the all-inclusive model, the model that includes all sample points.

Results of this statistical analysis in terms of model quality indicators (R^2 , t -statistics) and parameter estimation indicate a highly significant and stable linear relationship between normalized integrative complexity and modularity index Q .

From the results shown, it is observed that within each individual system, normalized integrative complexity and modularity index have strong negative correlation that is statistically significant and lends credence to the use of integrative complexity as a surrogate for modularity. This result is not surprising since the notion of high modularity tends to emphasize higher in-module

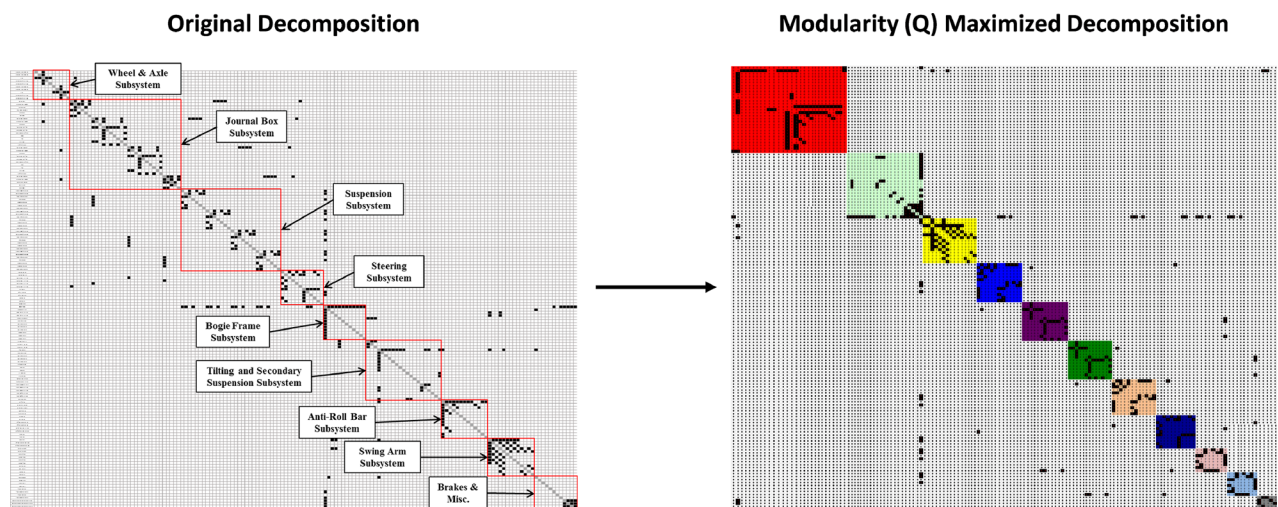


Fig. 4 Original train undercarriage decomposition and the modularity (Q) maximized decomposition in DSM format (Q -maximized decomposition from Ref. [5])

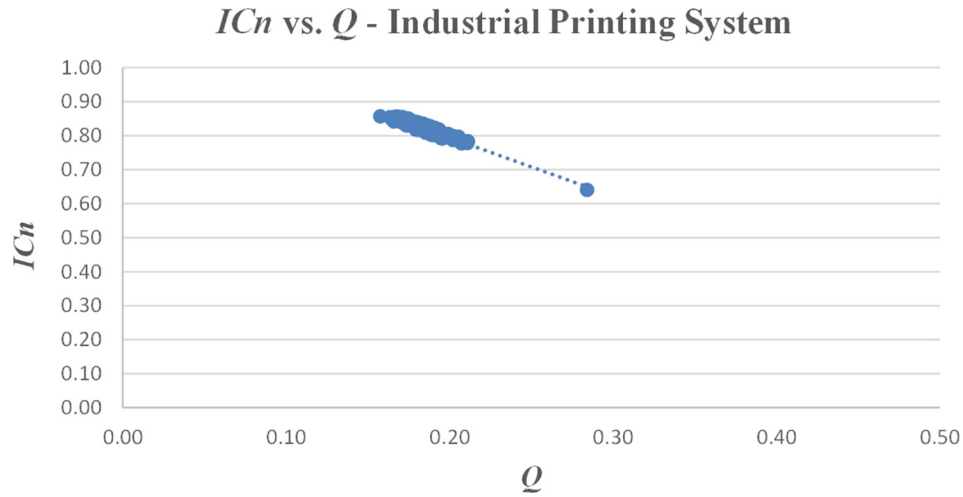


Fig. 6 Regression plots for IC_n and Q of industrial printing system with decompositions

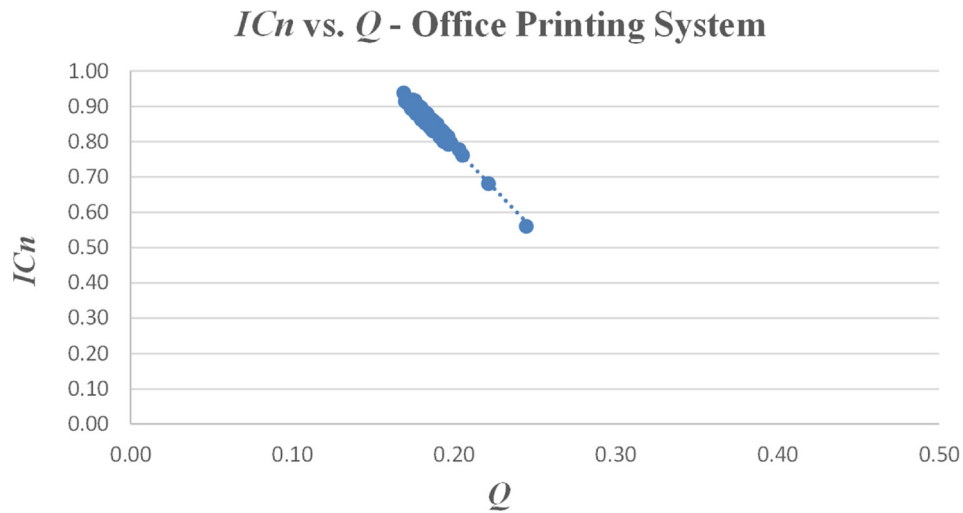


Fig. 7 Regression plots for IC_n and Q of office printing system with decompositions

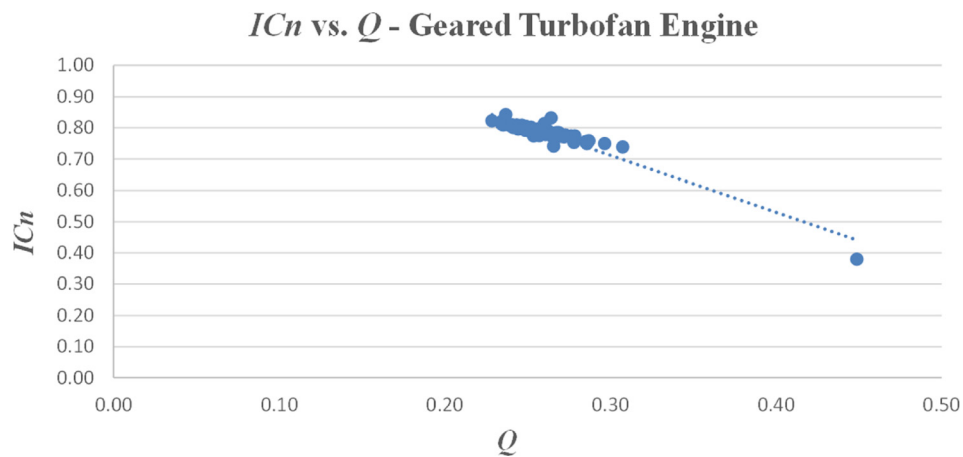


Fig. 8 Regression plots for IC_n and Q of geared turbofan aircraft engine with decompositions

complexity, and this leads to lower integrative complexity with a higher proportion of total complexity being embedded within modules. Therefore, integrative complexity and modularity are likely to be negatively correlated, and this claim is substantiated in this case study.

Integrative Complexity: Modularity Relationship for Across a Set of Systems. Once the analysis on individual complex engineered systems was completed, the relationship between IC_n and Q across the set of six systems was explored, since they are very different in terms of their functionality and spans across different

IC_n vs. Q - Two Spool Turbofan Engine

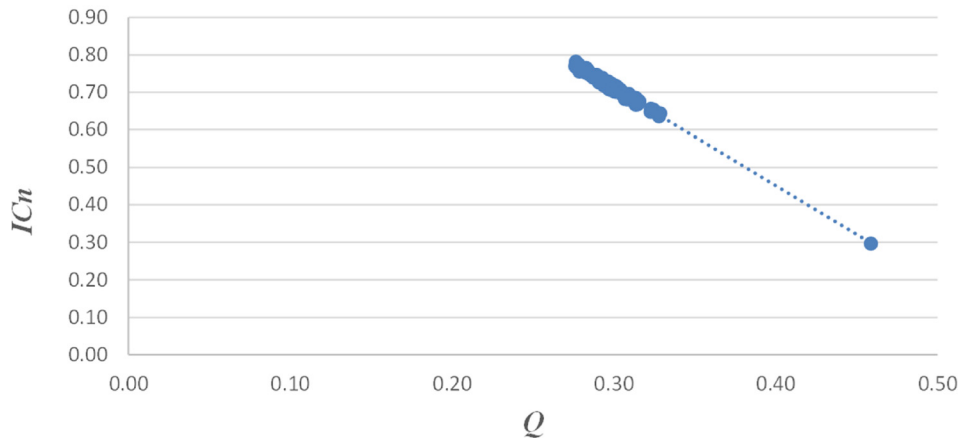


Fig. 9 Regression plots for IC_n and Q of two-spool turbofan aircraft engine with decomposition

IC_n vs. Q - HECE

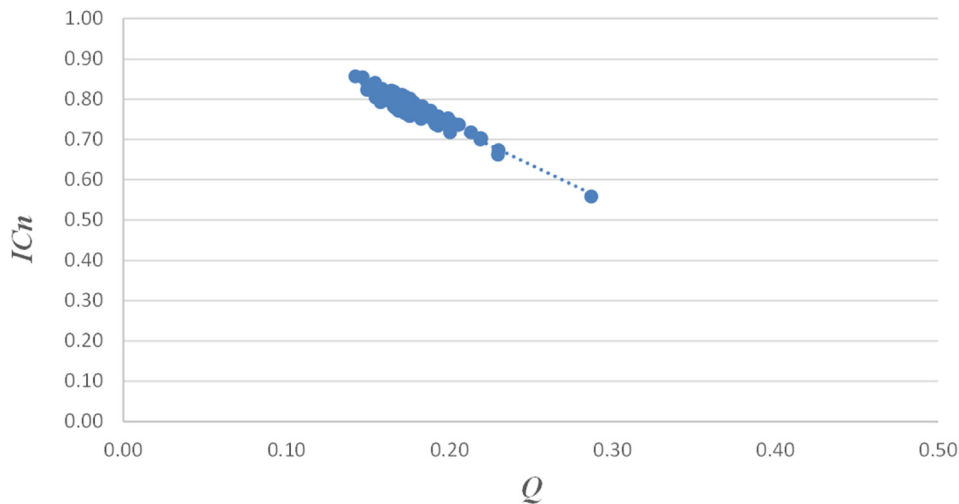


Fig. 10 Regression plots for IC_n and Q of HECE with decomposition

Table 4 Model quality statistics for Q versus IC_n relationship for systems analyzed

System	R^2	R^2_{adj}
Train undercarriage	0.92	0.92
Industrial printing system	0.97	0.97
Office printing system	0.95	0.95
Geared turbofan aircraft engine	0.89	0.89
Two-spool turbofan aircraft engine	0.95	0.95
HECE	0.92	0.91

engineering domains. The values of IC_n and Q corresponding to their individual Q-maximizing decomposition [2] for all six systems are shown in Table 6, and these values are plotted in Fig. 11.

A detailed statistical analysis accounting for very small sample size reveals a R^2 value of 0.89 with small p value of 0.015, as shown in Table 7.

Table 8 shows the linear coefficients for the regression plot shown in Fig. 11 and related statistical analysis results.

It should be noted that the model parameters are dependent on the data set used to build the parametric model and are themselves

random variables. Their estimates can vary depending on the data, especially for small datasets. Again, the pseudovalue jackknife technique [65] was applied, where the calculation is made for each subgroup of data obtained by leaving out one sample from the dataset [66]. Postapplication of the pseudovalue jackknife process, the aggregated model parameter estimates and the model quality estimates are shown in Table 9. For all jackknife subsamples, the estimated model parameters were found to lie within the 95% confidence interval of all-inclusive model, the model with all six sample points.

Results from the statistical analysis indicate a significant and stable linear relationship between normalized integrative complexity and modularity index across multiple engineered systems from different application domains. This indicates that there is a strong negative relationship between integrative complexity and modularity index even across a set of different engineering systems in very different application domains, which can potentially be generalizable.

The analysis shows that as the integrative complexity of the system decreases, modularity of the system increases. This is caused by allocation of larger fraction of total system complexity to individual modules, thereby decreasing interactions between

Table 5 Model parameter values and associated statistic for Q versus IC_n relationship for systems analyzed (linear model of the form: $IC_n = \alpha + \beta * Q$)

System	(α, β)	95% confidence interval	Standard error	t -stat	R^2
Train undercarriage	(0.50, -0.48)	({0.495, 0.504}; {-0.50, -0.45})	(0.002, 0.01)	(235.54, -34.32)	0.92
Industrial printing system	(1.14, -1.73)	({1.13, 1.15}; {-1.79, -1.68})	(0.005, 0.028)	(212.2, -61.2)	0.97
Office printing system	(1.73, -4.71)	({1.69, 1.77}; {-4.91, -4.51})	(0.01, 0.1)	(90.5, -45.9)	0.95
Geared turbofan aircraft engine	(1.26, -1.81)	({1.22, 1.29}; {-1.93, -1.69})	(0.02, 0.06)	(78.4, -33.5)	0.89
Two-spool turbofan aircraft engine	(1.48, -2.56)	({1.45, 1.51}; {-2.67, -2.45})	(0.02, 0.05)	(87.53, -45.6)	0.95
HECE	(1.15, -2.10)	({1.13, 1.17}; {-2.22, -1.98})	(0.01, 0.06)	(99.6, -36.2)	0.92

Table 6 Value of IC_n and Q for systems in Table 1 in its original decomposition configuration

System	IC_n	Q
Train undercarriage	0.17	0.74
Industrial printing system	0.50	0.28
Office printing system	0.58	0.24
Geared turbofan aircraft engine	0.21	0.44
Two-spool turbofan aircraft engine	0.36	0.46
HECE	0.56	0.29

modules, resulting in increased modularity. It was also demonstrated that this holds true for several different types of complex systems as well, thus making the proposed analytical and empirical formulation more generalizable, which enables the method to be easily extended to system-of-systems. Findings from this research can be used to integrate system analysis for complexity and modularity. System architects can assess the overall system complexity while using the integrative complexity as a surrogate measure for system modularity.

Conclusion and Future Work

In this paper, a complexity attribution approach that enables consistent complexity accounting process for effective complexity management across system representation levels, with explicit accounting for system integration, is introduced. Proposed process is based on quantification methodology described in Ref. [26] and

Table 7 Model quality statistics for Q versus IC_n relationship across systems analyzed

R^2	R^2_{adj}	p value
0.89	0.87	0.015

Table 8 Model parameter values and associated statistic for Q versus IC_n relationship across six systems analyzed (linear model of the form: $IC_n = \alpha + \beta * Q$)

Coefficients	Value	Standard error	t -stat	95% confidence interval	p value
α	0.94	0.10	9.18	{0.61, 1.27}	0.003
β	-1.46	0.29	-5.04	{-2.38, -0.54}	0.015

Table 9 The aggregated model parameter and the model quality estimates from the pseudo-value jackknife process for dataset shown in Table 6

α	β	R^2	R^2_{adj}
0.92	-1.39	0.90	0.87

the newly introduced notion of integrative complexity. Systems are deemed more modular if they have lower integrative complexity. It should be noted that realized modularity is a function of system decomposition strategy adopted, while system complexity is a

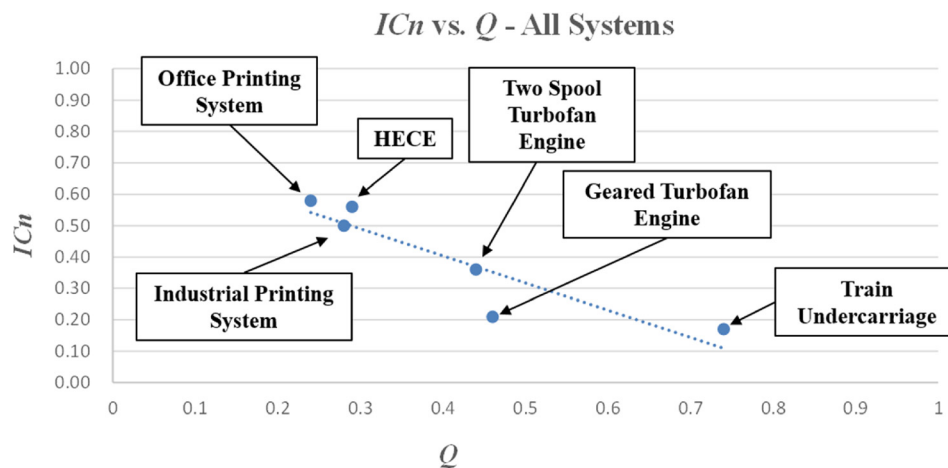


Fig. 11 Plot of integrative complexity and modularity for systems shown in Table 2

system property and is independent of system decomposition strategy. In this work, Newman's modularity index Q was used as a measure of degree of modularity to investigate the relationship between modularity and integrative complexity. Results indicate that the integrative complexity and modularity index, as defined by the Q metric, show strong negative correlation across a set of different complex systems, one might use integrative complexity as an alternative and representative measure of the degree of modularity, with a larger value indicating smaller degree of modularity.

In the future, by analyzing relationships between integrative complexity and other modularity metrics available in the literature, further insights into the relationship between complexity and modularity can be gained. In order to accomplish this, more exploratory studies linking integrative complexity and various alternative modularity metrics published in academia must be undertaken. In this work, the proposed complexity quantification and complexity attribution method was applied to electromechanical systems that can be modeled as a network. However, it is yet to be tested for other types of systems, such as complex software systems, which require additional research. Once these metric-based complexity-modularity relationships are more deeply understood, it can be used to further the knowledge in another areas, such as generation of alternative system decomposition strategies. Another future research topic is a study to create a computational/virtual system architecting "sandbox" that will enable future studies on finding effective architectural patterns for specified contexts.

Funding Data

- Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (Grant No. NRF-2016R1D1A1A09916273).

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