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New parameterizations for neutral and ion-induced two-component sulfuric acid-water particle formation in nucleation and kinetic regimes

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Key Points: Improved particle formation parameterizations have been developed Both neutral and ion-induced pathways are considered and the kinetic regime is accounted for We recommend using these parameterizations from now on

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16 Abstract

We have developed new parameterizations of two-component neutral homogeneous and 17 ion-induced sulfuric acid - water particle formation for large ranges of environmental condi-18 tions, based on an improved model that has been validated against Cosmics Leaving OUtdoor 19 Droplets (CLOUD) experiments at CERN. The neutral parameterization is valid for the fol-20 lowing ranges: temperatures 165-400 K, sulfuric acid concentrations 10⁴-10¹³ cm⁻³ and rel-21 ative humidities 0.001-100%. The ion-induced parameterization is valid for temperatures 195-22 400 K, sulfuric acid concentrations 10^4 - 10^{16} cm⁻³ and relative humidities 10^{-5} -100%. The 23 new parameterizations are thus applicable for the full range of Earth's atmospheric conditions 24 where binary sulfuric acid - water particle formation is relevant, including both tropospheric 25 and stratospheric conditions. They can also be used for describing particle formation in the 26 atmosphere of Venus. The model is based on a thermodynamically consistent version of Clas-27 sical Nucleation Theory normalized using quantum chemical data. The model is also appli-28 cable to extreme dry conditions where the one-component sulfuric acid limit is approached. 29 Parameterizations are presented for critical cluster sulfuric acid mole fraction, radius, total num-30 ber of molecules, and particle formation rate. If the critical cluster contains only one sulfu-31 ric acid molecule, a simple formula for kinetic particle formation can be used: this threshold 32 has also been parameterized. 33

34 **1 Introduction**

Aerosols affect Earth's climate by scattering solar radiation and acting as cloud conden-35 sation nuclei [Boucher et al., 2013] and thus they significantly affect the radiation balance within 36 the Earth's atmosphere. Sulfur is transmitted into the Earth's atmosphere as gaseous SO_2 and 37 other gaseous sulfur species. These species are transformed into gaseous sulfuric acid through 38 atmospheric oxidation. The gaseous sulfuric acid generates new atmospheric aerosol particles 39 through secondary particle formation processes. Global aerosol models show that secondary 40 particles formed in the atmosphere dominate the total global aerosol number concentrations 41 and generate up to half of global cloud condensation nuclei (CCN)[Spracklen et al., 2008; Merikanto 42 et al., 2009]. Observed particle formation rates are often proportional to the abundance of gaseous 43 sulfuric acid [Weber et al., 1996; Sihto et al., 2006; Kuang et al., 2008; Petäjä et al., 2009], sug-44 gesting that sulfuric acid is a key compound in atmospheric new particle formation in the Earth's 45 troposphere. However, in the lower troposphere also additional compounds are required to sta-46 bilize the sulfuric acid clusters [Sipilä et al., 2010; Zhao et al., 2010; Petäjä et al., 2011] and 47

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allow their growth to atmospherically relevant sizes [*Ortega et al.*, 2012]. However, in the up per troposphere pure sulfuric acid -water particle formation is likely an efficient mechanism
 for new particle formation [*Kirkby et al.*, 2011; *Merikanto et al.*, 2015]. A significant fraction
 of boundary layer CCN originate from particles formed in the upper troposphere [*Raes*, 1995;
 Merikanto et al., 2009].

⁵³ Sulfate aerosols also cause ozone destruction through heterogeneous chemical reactions ⁵⁴ in the stratosphere [*Peter and Grooß*, 2012] and are important components of present and past ⁵⁵ atmospheres of other planets of the solar system, such as Venus or Early Mars [*McGouldrick* ⁵⁶ *et al.*, 2011].

Besides its tropospheric significance, sulfuric acid -water particle formation mechanism 57 is likely responsible for generating a persistent aerosol layer in the stratosphere (the Junge layer) 58 composed of sulfuric acid-water particles [Lee et al., 2003]. This layer has a predominantly 59 cooling effect on the global climate [Solomon et al., 2011]. Volcanic eruptions inject variable 60 amounts of gaseous sulfur and sulfate particles into the Earth's stratosphere, leading to vari-61 ations in the sulfate aerosol concentrations [Vernier et al., 2011; Sawamura et al., 2012], while 62 also changes in anthropogenic sulfur dioxide emissions can lead to substantial changes in lower 63 stratospheric sulfate loading [Pitari et al., 2002; Hofmann et al., 2009; Hommel et al., 2015]. 64 These effects need to be taken into account in climate models extending up to the stratosphere. 65

Venus' clouds, which cover the whole planet, and are responsible for Venus' high vis-66 ible wavelength albedo [Esposito et al., 1983], are mainly formed of sulfuric acid solution droplets 67 at around 50-70 km above the Venusian surface at conditions comparable to those on the Earth 68 with temperatures and pressures of the order of T=200-350 K and p=0.01-1 bar. These lay-69 ered clouds distributed over 20 km effectively block the sunlight from reaching the surface of 70 the planet, where only a few percent of solar radiation is received. The surface temperatures 71 on Venus are, however, on average around 700 K because of the greenhouse effect of the very 72 thick CO_2 atmosphere, although without the clouds the planet would be even hotter. It is likely 73 that binary sulfuric acid - water particle formation plays a significant role in the formation of 74 the clouds in Venus. Previous models have assumed [James et al., 1997; McGouldrick and Toon, 75 2007; Gao et al., 2014] that cloud droplet formation involves heterogeneous nucleation of sul-76 furic acid on pre-existing particles. No observations have allowed the identification of the pos-77 sible CCN, although several candidates have been suggested [such as sulfur allotropes and me-78 teor dust, Young, 1983; Gao et al., 2014]. However, the conditions in Venus could also allow 79

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the droplet formation to be initiated by homogeneous or ion-induced unary/binary nucleation and followed by subsequent growth to cloud droplet sizes.

Large scale atmospheric aerosol models require computationally efficient parameterized 82 representations of dynamics of the particle formation process in various ambient conditions. 83 Such representations can be produced by either calculating theoretical formation rates based 84 on physical properties of the system in different conditions and by parameterizing the data or 85 interpolating from a look-up-table of the data, or by gathering experimental data points and fitting suitable functions to the data. Since particle formation has a complex dependence on 87 the concentrations of participating species and on temperature, the latter approach, while pos-88 sibly more accurate for describing measured data, is not trustworthy for conditions outside the 89 measurement range. Therefore, it is much safer to base parameterizations on a theory that can 90 reproduce the measured data and predict the dynamics of particle formation outside the mea-91 sured range. 92

The physical understanding of atmospheric particle formation processes has not been sup-93 ported by a firm basis, even for the relatively simple binary sulfuric acid - water system. Dif-94 ferent theoretical formulations for binary particle formation for the neutral system [Wexler et al., 95 1994; Pandis et al., 1994; Kulmala et al., 1998; Vehkamäki et al., 2002; Kazil and Lovejoy, 2007; 96 Yu, 2008], and for the ion-induced system in particular [Modgil et al., 2005; Kazil and Love-97 joy, 2007; Yu, 2010], can produce binary formation rates that vary by several orders of mag-98 nitude [Zhang et al., 2010]. The lack of precision and coverage of binary particle formation 99 experiments that could be used for quantitative validation of theoretical models has also con-100 tributed to the variation in the formation rates predicted by the different parameterizations. For 101 example, the theory used by Yu [2010] probably contains a double counting of the dipole-dipole 102 interaction [Donald et al., 2008], leading to too high binding energies and predictions of the 103 formation rate [Donald et al., 2011]. New particle formation measurements in controlled con-104 ditions [Duplissy et al., 2015] finally allow constraining the theoretical models used. 105

The Classical Nucleation Theory [CNT, *Doyle*, 1961; *Binder and Stauffer*, 1976; *Trinkaus*, 1983; *Seinfeld and Pandis*, 1998] is an old and elegant description of the nucleation process, but suffers from the assumption of macroscopic bulk properties for the substances at microscopic scales. Recent Cosmics Leaving OUtdoor Droplets (CLOUD) project in CERN has, for the first time, provided accurate and demonstrably contaminant-free measurements of binary sulfuric acid - water particle formation allowing precise model-data comparisons. In the pro-

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vided parameterization we use the thermodynamically consistent Classical Nucleation Theory
 normalized with quantum chemical data on sulfuric acid hydration as described in *Merikanto et al.* [2015]. The model has been validated against the CLOUD experiments [*Duplissy et al.*,
 2015]. The framework of Classical Nucleation Theory ensures a physically based behavior for
 the developed parameterizations even outside the experimental range.

117 **2 Methods**

In this paper we present new parameterizations of neutral and ion-induced sulfuric acid-118 water particle formation. The thermodynamic data and the applied theoretical model used for 119 generating the parameterizations are described in *Merikanto et al.* [2015]. In the framework 120 of the revised theory of Merikanto et al. [2015], both neutral and ion-induced particle forma-121 tion can take place in the range of parameterized conditions either through nucleation, where 122 particle formation involves crossing of the free energy barrier for formation of stable particles, 123 or through kinetic particle formation, where such free energy barrier does not exist. Through-124 out the paper we generally employ the term "particle formation" to describe the process in gen-125 eral, but may substitute it with "nucleation" when the particle formation involves a free en-126 ergy barrier. When the process is barrierless we use the term kinetic particle formation. 127

The binary particle formation rates at conditions close to zero relative humidity tend closely 128 towards one-component nucleation rates, but the binary theory cannot mathematically handle 129 zero relative humidity due to singularities in the kinetic prefactor for the nucleation rate in this 130 case. Furthermore, the zero relative humidity limit of the binary kinetic prefactor does not ap-131 proach exactly the unary prefactor, producing an inconsistency, albeit small, between the bi-132 nary and unary models. For mathematical completeness, we discuss the difference in binary 133 and unary prefactors in the Appendix. Within the accuracy required for the parameterization, 134 the predicted particle formation approaches one-component sulfuric acid particle formation at 135 very low relative humidities (x > 0.99), and at zero relative humidity the one-component the-136 ory is used to produce the data. However, the parameterizations do not include the exact RH=0137 point, since they contain logarithms of RH. 138

The neutral sulfuric acid -water parameterization can be viewed as an update to the widely used [*Vehkamäki et al.*, 2002] particle formation parameterization. The temperature and sulfuric acid concentration ranges of the new parameterizations are larger than those of *Vehkamäki et al.* [2002] and *Vehkamäki et al.* [2003], and the calculations are also extended to very low

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relative humidities. This was motivated by the present development of sulfuric acid cloud models for the Venusian atmosphere, which is much drier than the Earth's atmosphere.

The model behind the present parameterization differs form the model used in Vehkamäki 145 et al. [2002] and Vehkamäki et al. [2003] in three respects: first we now use analytical formu-146 lae for the second derivatives of the formation free energy, since the numerical derivatives were 147 unstable at the one-component limit. Second, the cluster distribution in the nucleating vapor 148 is normalized via a known reference concentration, which in Noppel et al. [2002] and Vehkamäki 149 et al. [2002] was taken to be the sulfuric acid dihydrate. Here, here we use the full monomer 150 hydrate distribution in the normalization process instead of only the dihydrate. This modified 151 approach ensures a smooth transition into a one-component system in the pure sulfuric acid 152 limit. Third, the reference sulfuric acid monomer hydrate distribution is based on more accu-153 rate quantum chemical results [Kurtén et al., 2007; Henschel et al., 2014] than in Vehkamäki 154 et al. [2002] and Vehkamäki et al. [2003] 155

In the presence of water, the total concentration of sulphuric acid molecules can be sig-156 nificantly larger that the concentration of free sulphuric acid monomers, and a large fraction 157 of acid molecules is bound to hydrates. In the conditions of Earth's atmosphere, the concen-158 tration of water is approximately ten orders of magnitude higher than that of sulphuric acid, 159 and thus the total concentration of water molecules can be considered equal to the concentra-160 tion of free water monomers, as pure water cluster formation does not play a significant role 161 either. In the very dry conditions studied in this work, this is not necessarily true and the dis-162 tinction between total concentration of water and concentration free of water molecules must 163 be kept in mind. By definition relative humidity used as an input parameter in the parameter-164 izations is proportional to the concentration of free water molecules. 165

¹⁶⁶ **3 Size and nature of the ions**

If the free energy of formation as a function of cluster size in ion-induced particle formation exhibits a barrier, the equation yielding the critical cluster radius has two solutions: a stable (in one dimensional representation free energy minimum) pre-nucleation cluster representing the ion in surrounded by a few molecules, r_1^* , and the actual metastable (free energy maximum) critical cluster r_2^* . [see Figure 2 of *Merikanto et al.*, 2015]. The classical version of ion-induced nucleation theory used in this work assumes that the size of the bare ion in smaller than the lower radius $r_{ion} < r_1^*$, and in this case the size of the ion does not affect the en-

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ergetics of nucleation. While an extension of the theory to relax this condition is possible, it 174 is beyond the scope of this study. Thus, care should be taken to ensure that the ions are small 175 enough to satisfy this condition always when this parametrization is applied. For the whole 176 validity range of the parametrization the radius r_1^* varies between 0.6 nm - 1.4 nm. Ion size 177 has a very minor effect on the kinetics of nucleation, which has been ignored in this study as 178 this produces an error insignificant compared to other sources of uncertainty in the nucleation 179 rates. We have used ion radius 0.487 nm corresponding to the bisulphate ion in all the cal-180 culations. 181

In the Earths atmosphere, the dominant molecular anions (HSO₄⁻, NO₃⁻, Cl⁻) forming 182 the cores of negatively charged cluster ions are mostly the conjugate bases of the strongest at-183 mospheric acids, which in turn are small inorganic molecules. Similarly, the dominant atmo-184 spheric molecular cations tend to be the conjugate acids of the strongest bases (e.g. NH₄⁺, pyrid-185 ium, different aminium ions), though H₃O⁺ -based clusters are also often present due to the 186 much larger concentration of water. With the possible exception of some of the nitrogen-containing 187 organic cations, these molecular ions are all rather small - well below 1 nm in diameter [Hir-188 sikko et al., 2011; Ehn et al., 2010; Beig and Brasseur, 2000] 189

Though the precise chemical identities of the acids and bases present of course vary between different planetary atmospheres, the fundamental result that atmospheric molecular ions tend to be rather small is likely universal, especially for the anions. As the complicated large organic molecules present in our atmosphere are almost invariably biogenic, it could even be argued that the probability of encountering large molecular ions should be even smaller in other planetary atmospheres compared to ours.

It should be noted that in cases where the core molecular ions are formed by charging 196 large organic molecules (e.g. biomolecules), application of classical ion-induced nucleation 197 theory may suffer from difficulties above and beyond that related to ion size. In such cases, 198 the negative or positive charge is very likely localized to one or more specific functional group 199 of the molecule (e.g. sulfate, nitrate or carboxylate groups in the case of anions). Thus, treat-200 ing the molecular ion as a uniformly charged sphere (an excellent approximation for simple 201 inorganic ions such as bisulfate or ammonium) may lead to large errors even if the ion size 202 is appropriately accounted for. 203

In laboratory conditions, molecular ions of arbitrary size can be created for example in the form of charged metal clusters. In contrast to the charged biomolecules discussed above,

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heterogeneous nucleation onto such core ions may in principle be described fairly well by clas-

sical theory, as the charge can be delocalized over the entire (more or less spherical) cluster.

²⁰⁸ When applying the classical theory to such studies, the effect of ion size on the nucleation bar-

rier must therefore be taken into account.

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4 Particle formation parameterizations

Parameterizations are given for neutral and ion-induced particle formation rates and crit-211 ical cluster sulfuric acid mole fraction, radius and total number of molecules in the cluster. We 212 also provide parameterizations for the limit at which particle formation becomes kinetic in both 213 neutral and ion-induced cases, that is, the conditions where the free energy barrier limiting the 214 particle formation rate disappears. Analytical equations are given for calculating the particle 215 formation rate in the kinetic regime. Moreover, a parameterization for the threshold sulfuric 216 acid concentration that yield a neutral particle formation rate of 1 cm $^{-3}$ s $^{-1}$ is given for three 217 separate, but contiguous, temperature ranges (in the lowest temperature range particle forma-218 tion is kinetic). 219

In the following we present the parameterizations and compare the obtained theoretical and parameterized results with each other and also, in the neutral case, with the theory and parameterization of *Vehkamäki et al.* [2002].

The validity of the neutral nucleation parameterization is limited to relative humidities from 0.001% to 100%, sulphuric acid concentration from 10^4 to 10^{13} cm⁻³, and to temperatures from 165 to 400 K. The results of the parameterization are valid only if the resulting particle formation rate is $J > 10^{-7}$ cm⁻³s⁻¹ and the number of sulfuric acid molecules in the critical cluster is $n_a^* > 1$. The limit $n_a^* = 1$ is related to particle formation becoming kinetic. We have developed another parameterization that gives the sulfuric acid concentration required for the kinetic limit as a function of relative humidity and temperature.

The validity of the ion-induced nucleation parameterization is limited to relative humidities from 10^{-5} % to 100%, sulphuric acid concentration from 10^4 to 10^{16} cm⁻³, and the temperature range 195-400 K. The results of the parameterization are valid only if the resulting particle formation rate (assuming a negative ion concentration of 1 cm⁻³) is $J > 10^{-10}$ cm⁻³s⁻¹ and, as for the neutral case, total number of sulfuric acid molecules in the critical cluster is $n_a^* > 1$, since below this limit particle formation becomes again kinetic. This ion-induced kinetic limit is described by another parameterization. The ion-induced particle formation rate

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has to be multiplied by the actual negative ion concentration (typically some hundreds to some thousand negative ions cm⁻³) to get rates in the conditions investigated. Depending on the chemistry of the atmosphere studied, negative ion concentration can depend on the sulphuric acid concentration and vice versa. As the input values of our parameterization, these two concentrations are treated as independent variables, and the coupling between them in the studied case can be specified before calling the parametrization.

The particle formation calculations using the parameterizations are orders of magnitude more computationally efficient than with the full theory, and hence the parameterizations are suitable for large scale atmospheric models. A Fortran code for the new parameterization is included in the supplementary electronic material. When using the parameterizations in, for example, a microphysical/atmospheric model, it is advisable to use the same thermodynamical data (density, surface tension, activities, etc., see *Merikanto et al.* [2015]) as used here for consistency, and double precision for floats should be used.

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4.1 Neutral particle formation

The mole fraction of sulfuric acid in the critical cluster, valid for both neutral and ioninduced case, is given by

$$\begin{aligned} x^* &= 7.9036365428891719 \cdot 10^{-1} - 2.8414059650092153 \cdot 10^{-3} T \tag{1} \\ &+ 1.4976802556584141 \cdot 10^{-2} \ln(\rho_a) - 2.4511581740839115 \cdot 10^{-4} T \ln(\rho_a) \\ &+ 3.4319869471066424 \cdot 10^{-3} \ln(\frac{RH}{100}) - 2.8799393617748428 \cdot 10^{-5} T \ln(\frac{RH}{100}) \\ &+ 3.0174314126331765 \cdot 10^{-4} \left[\ln(\frac{RH}{100})\right]^2 - 2.2673492408841294 \cdot 10^{-6} T \left[\ln(\frac{RH}{100})\right]^2 \\ &- 4.3948464567032377 \cdot 10^{-3} \left[\ln(\frac{RH}{100})\right]^3 + 5.3305314722492146 \cdot 10^{-5} T \left[\ln(\frac{RH}{100})\right]^3, \end{aligned}$$

where ρ_a is the total gas phase concentration of sulfuric acid (1/cm³), *T* is the absolute temperature and *RH* is the relative humidity in %.

The particle formation rate is given by an exponential of a third order polynomial of $\ln(RH/100)$ and $\ln(\rho_a)$

$$\begin{split} J[1/(\mathrm{cm}^{3}\mathrm{s})] &= \exp \left\{ a(T,x^{*}) \right. \\ &+ b(T,x^{*}) \ln(RH/100) + c(T,x^{*}) [\ln(RH/100)]^{2} \\ &+ d(T,x^{*}) [\ln(RH/100)]^{3} + e(T,x^{*}) \ln(\rho_{a}) \\ &+ f(T,x^{*}) \ln(RH/100) \ln(\rho_{a}) \\ &+ g(T,x^{*}) [\ln(RH/100)]^{2} \ln(\rho_{a}) \\ &+ h(T,x^{*}) [\ln(\rho_{a})]^{2} \\ &+ i(T,x^{*}) \ln(RH/100) [\ln(\rho_{a})]^{2} \\ &+ j(T,x^{*}) [\ln(\rho_{a})]^{3} \right\}, \end{split}$$

(2)

where the coefficients $a(T,x^*)\ldots j(T,x^*)$ are functions of temperature and critical cluster

mole fraction x^* (calculated using equation (1)):

$$\begin{split} a(T,x^*) &= 2.1361182605986115\cdot 10^{-1} + 3.3827029855551838\,T - 3.2423555796175563\cdot 10^{-2}\,T^2 \\ &+ 7.0120069477221989\cdot 10^{-5}\,T^3 + \frac{8.0286874752695141}{x^*} \\ b(T,x^*) &= -2.6939840579762231\cdot 10^{-1} + 1.6079879299099518\,T - 1.9667486968141933\cdot 10^{-2}\,T^2 \\ &+ 5.524475597970844\cdot 10^{-5}\,T^3 + \frac{7.8884704837892468}{x^*} \\ c(T,x^*) &= 4.6374659198909596 - 8.2002809894792153\cdot 10^{-2}\,T + 8.5077424451172196\cdot 10^{-4}\,T^2 \\ &- 2.6518510168987462\cdot 10^{-6}\,T^3 - \frac{1.4625482500575278}{x^*} \\ d(T,x^*) &= -5.2413002989192037\cdot 10^{-1} + 5.2755117653715865\cdot 10^{-3}\,T - 2.9491061332113830\cdot 10^{-6}\,T^2 \\ &- 2.4815454194486752\cdot 10^{-8}\,T^3 - \frac{5.2663760117394626\cdot 10^{-2}}{x^*} \\ e(T,x^*) &= 1.649666458266762 - 8.0809397859218401\cdot 10^{-1}\,T + 8.9302927091946642\cdot 10^{-3}\,T^2 \\ &- 1.9583649496497497\cdot 10^{-5}\,T^3 - \frac{8.9505572767891685}{x^*} \\ f(T,x^*) &= -3.0025283601622881\cdot 10^1 + 3.0783365644763633\cdot 10^{-1}\,T - 7.4521756337984706\cdot 10^{-4}\,T^2 \\ &- 5.7651433870681853\cdot 10^{-7}\,T^3 + \frac{1.2872868529673207}{x^*} \\ g(T,x^*) &= -6.1739867501526535\cdot 10^{-1} + 7.2347385705333975\cdot 10^{-3}\,T - 3.0640494530822439\cdot 10^{-5}\,T^2 \\ &- 5.944609194346214\cdot 10^{-8}\,T^3 - \frac{2.8681650332461055\cdot 10^{-2}}{x^*} \\ h(T,x^*) &= 6.5213802375160306 - 4.7907162004793016\cdot 10^{-2}\,T - 1.0727890114215117\cdot 10^{-4}\,T^2 \\ &+ 5.6401818280534507\cdot 10^{-7}\,T^3 + \frac{5.4113070888923099\cdot 10^{-1}}{x^*} \\ i(T,x^*) &= 5.2062808476476330\cdot 10^{-1} - 6.0696882500824584\cdot 10^{-3}\,T + 2.3851383302608477\cdot 10^{-5}\,T^2 \\ &- 1.5243837103067096\cdot 10^{-8}\,T^3 - \frac{5.6543192378015687\cdot 10^{-2}}{x^*} \\ i(T,x^*) &= -1.1630806410696815\cdot 10^{-1} + 1.3806404273119610\cdot 10^{-3}\,T - 2.0199865087650833\cdot 10^{-6}\,T^2 \\ &- 3.0200284885763192\cdot 10^{-9}\,T^3 - \frac{6.9425267104126316\cdot 10^{-3}}{x^*}. \end{split}$$

The total number of molecules in the critical cluster $n_{tot}^* = n_a^* + n_w^*$ (where n_w^* is the number of water molecules in the critical cluster) is given by

$$n_{tot}^{*} = \exp\{A(T, x^{*})$$

$$+B(T, x^{*}) \ln(RH/100) + C(T, x^{*})[\ln(RH/100)]^{2}$$

$$+D(T, x^{*})[\ln(RH/100)]^{3} + E(T, x^{*}) \ln(\rho_{a})$$

$$+F(T, x^{*}) \ln(RH/100) \ln(\rho_{a})$$

$$+G(T, x^{*})[\ln(RH/100)]^{2} \ln(\rho_{a})$$

$$+H(T, x^{*})[\ln(\rho_{a})]^{2}$$

$$+J(T, x^{*}) \ln(RH/100)[\ln(\rho_{a})]^{2}$$

$$+J(T, x^{*})[\ln(\rho_{a})]^{3}\},$$
(3)

where the coefficients $A(T,x^*)\ldots J(T,x^*)$ again depend on temperature and critical cluster

mole fraction x^* (from equation (1)):

$$\begin{split} &A(T,x^*) &= -3.5863435141979573 \cdot 10^{-3} - 1.0098670235841110 \cdot 10^{-1}T + 8.9741268319259721 \cdot 10^{-4}T^2 \\ &\quad -1.4855098605195757 \cdot 10^{-6}T^3 - \frac{1.2080330016937095 \cdot 10^{-1}}{x^*} \\ &B(T,x^*) &= 1.1902674923928015 \cdot 10^{-3} - 1.9211358507172177 \cdot 10^{-2}T + 2.4648094311204255 \cdot 10^{-4}T^2 \\ &\quad -7.5641448594711666 \cdot 10^{-7}T^3 - \frac{2.0668639384228818 \cdot 10^{-2}}{x^*} \\ &C(T,x^*) &= -3.7593072011595188 \cdot 10^{-2} + 9.0993182774415718 \cdot 10^{-4}T - 9.5698412164297149 \cdot 10^{-6}T^2 \\ &\quad +3.7163166416110421 \cdot 10^{-8}T^3 + \frac{1.1026579525210847 \cdot 10^{-2}}{x^*} \\ &D(T,x^*) &= 1.1530844115561925 \cdot 10^{-2} - 1.8083253906466668 \cdot 10^{-4}T + 8.0213604053330654 \cdot 10^{-7}T^2 \\ &\quad -8.5797885383051337 \cdot 10^{-10}T^3 + \frac{1.0243693899717402 \cdot 10^{-3}}{x^*} \\ &E(T,x^*) &= -1.7248695296299649 \cdot 10^{-2} + 1.1294004162437157 \cdot 10^{-2}T - 1.2283640163189278 \cdot 10^{-4}T^2 \\ &\quad +2.7391732258259009 \cdot 10^{-7}T^3 + \frac{6.8505583974029602 \cdot 10^{-2}}{x^*} \\ &F(T,x^*) &= 2.9750968179523635 \cdot 10^{-1} - 3.6681154503992296 \cdot 10^{-3}T + 1.0636473034653114 \cdot 10^{-5}T^2 \\ &\quad +5.8687098466515866 \cdot 10^{-9}T^3 - \frac{5.2028866094191509 \cdot 10^{-3}}{x^*} \\ &G(T,x^*) &= 7.6971988880587231 \cdot 10^{-4} - 2.4605575820433763 \cdot 10^{-5}T + 2.3818484400893008 \cdot 10^{-7}T^2 \\ &\quad -8.8474102392445200 \cdot 10^{-10}T^3 - \frac{1.664056678168968 \cdot 10^{-4}}{x^*} \\ &H(T,x^*) &= -7.7390093776705471 \cdot 10^{-2} + 5.8220163188828482 \cdot 10^{-4}T + 1.2291679321523287 \cdot 10^{-6}T^2 \\ &\quad -7.4690997508075749 \cdot 10^{-9}T^3 - \frac{5.6357941220497648 \cdot 10^{-3}}{x^*} \\ &I(T,x^*) &= -4.7170109625089768 \cdot 10^{-3} + 6.982886534370193 \cdot 10^{-5}T - 3.1738912157036403 \cdot 10^{-7}T^2 \\ &\quad +2.3975538706787416 \cdot 10^{-10}T^3 + \frac{4.2304213386288567 \cdot 10^{-4}}{x^*} \\ &J(T,x^*) &= 1.3696520973423231 \cdot 10^{-3} - 1.6863387574788190 \cdot 10^{-5}T + 2.7959499278844516 \cdot 10^{-8}T^2 \\ &\quad +3.9423927013227455 \cdot 10^{-11}T^3 + \frac{8.6136359966337272 \cdot 10^{-5}}{x^*}. \end{split}$$

263 264 The radius of the critical cluster is given as a function of the mole fraction and the total number of molecules in the cluster:

 $r^*[\mathrm{nm}] = \exp[-22.378268374023630 + 0.44462953606125100x^* + 0.33499495707849131\ln(n^*_{\mathrm{tot}})].$

We also present a parameterization for the threshold total concentration of sulfuric acid 265 that gives particle formation rate $J=1 \text{ cm}^{-3}\text{s}^{-1}$. The threshold concentration depends on the 266 temperature and the relative humidity as given in the following equations. We divided the tem-267 perature range in three parts (155-185 K, where particle formation is in the kinetic range, 190-268 310 K and 310-400 K) to ensure a good quality of the fits. The smallest RH used for fitting 269 is 5% ($S = 5 \cdot 10^{-2}$). We have included sulfuric acid concentrations above 10^{13} cm⁻³ (which 270 were excluded from the parameterizations of J, x, n_{tot} and r^*) to enable the threshold pa-271 rameterization to extend to the whole RH range also at high temperatures. For the (nucleation) 272 range 310-400 K the threshold concentration parameterization is 273

$$\begin{split} \rho_a^{J=1}[1/\text{cm}^3] &= \exp\left[-2.8220714121794250 + 1.1492362322651116 \cdot 10^1 \frac{RH}{100} \right] (5) \\ &- \frac{3.3034839106184218 \cdot 10^3}{T} - \frac{7.1828571490168133 \cdot 10^2}{T} \frac{RH}{100} + 1.4649510835204091 \cdot 10^{-1} T \\ &- 3.0442736551916524 \cdot 10^{-2} \frac{RH}{100} T - 9.3258567137451497 \cdot 10^{-5} T^2 \\ &- 1.1583992506895649 \cdot 10^1 \ln(\frac{RH}{100}) + \frac{1.5184848765906165 \cdot 10^3 \ln(\frac{RH}{100})}{T} \\ &+ 1.8144983916747057 \cdot 10^{-2} T \ln(\frac{RH}{100}) \right], \end{split}$$

and for the (nucleation) range 190-310 K:

$$\begin{split} \rho_a^{J=1}[1/\text{cm}^3] &= \exp\left[-3.1820396091231999 \cdot 10^2 + 7.2451289153199676 \frac{RH}{100} \right] (6) \\ &+ \frac{2.6729355170089486 \cdot 10^4}{T} - \frac{7.1492506076423069 \cdot 10^2}{T} \frac{RH}{100} + 1.2617291148391978 T \\ &- 1.6438112080468487 \cdot 10^{-2} \frac{RH}{100} T - 1.4185518234553220 \cdot 10^{-3} T^2 \\ &- 9.2864597847386694 \ln\left(\frac{RH}{100}\right) + \frac{1.2607421852455602 \cdot 10^3 \ln\left(\frac{RH}{100}\right)}{T} \\ &+ 1.3324434472218746 \cdot 10^{-2} T \ln\left(\frac{RH}{100}\right) \right]. \end{split}$$

For the range 155-185 K where particle formation is kinetic, the threshold concentration parameterization is:

$$\rho_a^{J=1}[1/\text{cm}^3] = 1.1788859232398459 \cdot 10^5 - 1.0244255702550814 \cdot 10^4 \cdot \frac{RH}{100}$$
(7)
+ 4.6815029684321962 \cdot 10^3 \cdot (\frac{RH}{100})^2 - 1.6755952338499657 \cdot 10^2 \cdot T.

The following equations give the parameterization for the total concentration of sulfuric acid at the kinetic limit (as a function of RH and T). These equations are to be used to check if the conditions are such that particle formation is kinetic, in which case the Eqs. (11) and (12) should be used to calculate the particle formation rate instead of Eq. (2). Here we divided the relative humidity range in three parts (RH=1%-100%, 0.01%-1% and 0.0005%

-0.01%) to ensure a good quality of the fits. The kinetic limit sulfuric acid concentration (above

which particle formation is kinetic) for RH=1%-100% is:

$$\rho_{a}^{\rm kin}[1/\rm{cm}^{3}] = \exp\left[7.8920778706888086 \cdot 10^{1} + 7.3665492897447082 \frac{RH}{100}\right]$$
(8)
$$-\frac{1.2420166571163805 \cdot 10^{4}}{T} - \frac{6.1831234251470971 \cdot 10^{2}}{T} \frac{RH}{100} - 2.4501159970109945 \cdot 10^{-2} T - 1.3463066443605762 \cdot 10^{-2} \frac{RH}{100} T + 8.3736373989909194 \cdot 10^{-6} T^{2} - 1.4673887785408892 \ln\left(\frac{RH}{100}\right) - \frac{3.2141890006517094 \cdot 10^{1} \ln\left(\frac{RH}{100}\right)}{T} + 2.7137429081917556 \cdot 10^{-3} T \ln\left(\frac{RH}{100}\right)\right].$$

The kinetic limit sulfuric acid concentration for RH=0.01%-1% is:

$$\rho_{a}^{\rm kin}[1/\rm cm^{3}] = \exp\left[7.9074383049843647 \cdot 10^{1} - 2.8746005462158347 \cdot 10^{1} \frac{RH}{100}\right] \tag{9}$$

$$-\frac{1.2070272068458380 \cdot 10^{4}}{T} - \frac{5.9205040320056632 \cdot 10^{3}}{T} \frac{RH}{100} - 2.4800372593452726 \cdot 10^{-2} T - 4.3983007681295948 \cdot 10^{-2} \frac{RH}{100} T + 2.5943854791342071 \cdot 10^{-5} T^{2} - 2.3141363245211317 \ln\left(\frac{RH}{100}\right) + \frac{9.9186787997857735 \cdot 10^{1} \ln\left(\frac{RH}{100}\right)}{T} + 5.6819382556144681 \cdot 10^{-3} T \ln\left(\frac{RH}{100}\right)\right].$$

The kinetic limit sulfuric acid concentration for RH=0.0005%-0.01% is:

$$\begin{split} \rho_{a}^{\rm kin}[1/{\rm cm}^{3}] &= \exp\left[8.5599712000361677\cdot10^{1}+2.7335119660796581\cdot10^{3}\,\frac{RH}{100}\right] \tag{10} \\ &-\frac{1.1842350246291651\cdot10^{4}}{T} - \frac{1.2439843468881438\cdot10^{6}}{T}\frac{RH}{100} - 5.4536964974944230\cdot10^{-2}\,T \\ &+5.0886987425326087\frac{RH}{100}\,T + 7.1964722655507067\cdot10^{-5}\,T^{2} \\ &-2.4472627526306372\,\ln(\frac{RH}{100}) + \frac{1.7561478001423779\cdot10^{2}\,\ln(\frac{RH}{100})}{T} \\ &+6.2640132818141811\cdot10^{-3}\,T\,\ln(\frac{RH}{100})\right]. \end{split}$$

The neutral particle formation rate in the kinetic range (J_{kin}) can be easily calculated via the following equations [*Merikanto et al.*, 2015]:

$$J_{\rm kin,neutral} = \frac{C}{2} \sqrt{T} (\rho_a^{\rm total})^2 \tag{11}$$

where

$$C = (r_a + r_{\rm ref})^2 \sqrt{8\pi k \left(\frac{1}{m_a} + \frac{1}{m_{\rm ref}}\right)}$$
(12)

where $r_{\rm ref} = r_a = 0.3 \cdot 10^{-9}$ m and $m_{\rm ref} = m_a = 1.661 \cdot 10^{-27}$ kg in the neutral particle

²⁸⁷ formation case.

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4.2 Ion-induced particle formation

In the following we present the parameterizations for ion-induced particle formation rate, 289 the number of molecules in the critical cluster and the critical cluster radius as functions of 290 ρ_a , the total gas phase concentration of sulfuric acid (cm⁻³), T, the absolute temperature, and 291 RH, the relative humidity in %. The kinetic limit sulfuric acid concentration parameteriza-292 tion is given as a function of temperature T and relative humidity RH. Note that the critical 293 cluster mole fraction can be solved from the same equation for both neutral and ion-induced 294 particle formation [see Eq. (6) of Merikanto et al., 2015, and the references therein], and thus 295 the same mole fraction parameterization (Eq. (1)) can be used for both particle formation types. 296

The ion-induced particle formation rate for the assumed negative ion concentration of 1 cm^{-3} is given by the following dependence on $\ln(RH/100)$ and $\ln(\rho_a)$ (the actual particle formation rate is obtained by multiplying with the actual ion concentration in cm⁻³):

$$J = \exp\left[f_{1}^{J}(T) + f_{2}^{J}(T) \cdot \left[\ln(\frac{RH}{100})\right]^{-2} + f_{3}^{J}(T) \cdot \left[\ln(\frac{RH}{100})\right]^{-2} \cdot \ln\rho_{a}$$

$$+ f_{4}^{J}(T) \cdot \left[\ln(\frac{RH}{100})\right]^{-1} \cdot \ln^{-1}\rho_{a} + f_{5}^{J}(T) \cdot \left[\ln(\frac{RH}{100})\right]^{-1} + f_{6}^{J}(T) \cdot \left[\ln(\frac{RH}{100})\right]^{-1} \cdot \ln\rho_{a}$$

$$+ f_{7}^{J}(T) \cdot \left[\ln(\frac{RH}{100})\right]^{-1} \cdot \ln^{2}\rho_{a} + f_{8}^{J}(T) \cdot \left[\ln(\frac{RH}{100})\right]^{-1} \cdot \ln^{3}\rho_{a} + f_{9}^{J}(T) \cdot \ln^{-2}\rho_{a} + f_{10}^{J}(T) \cdot \ln^{-1}\rho_{a}$$

$$+ f_{11}^{J}(T) \cdot \ln\rho_{a} + f_{12}^{J}(T) \cdot \ln^{2}\rho_{a} + f_{13}^{J}(T) \cdot \ln^{3}\rho_{a} + f_{14}^{J}(T) \cdot \ln(\frac{RH}{100}) \cdot \ln^{-2}\rho_{a}$$

$$+ f_{15}^{J}(T) \cdot \ln(\frac{RH}{100}) \cdot \ln^{-1}\rho_{a} + f_{16}^{J}(T) \cdot \ln(\frac{RH}{100}) + f_{17}^{J}(T) \cdot \ln(\frac{RH}{100}) \cdot \ln\rho_{a} + f_{18}^{J}(T) \cdot \ln(\frac{RH}{100}) \cdot \ln^{2}\rho_{a}$$

$$+ f_{19}^{J}(T) \cdot \ln(\frac{RH}{100}) \cdot \ln^{3}\rho_{a} + f_{20}^{J}(T) \cdot \left[\ln(\frac{RH}{100})\right]^{2} \cdot \ln^{-1}\rho_{a} + f_{21}^{J}(T) \cdot \left[\ln(\frac{RH}{100})\right]^{2}$$

$$+ f_{22}^{J}(T) \cdot \left[\ln(\frac{RH}{100})\right]^{2} \cdot \ln\rho_{a} + f_{23}^{J}(T) \cdot \left[\ln(\frac{RH}{100})\right]^{2} \cdot \ln^{2}\rho_{a} + f_{24}^{J}(T) \cdot \frac{RH}{100} \right],$$
(13)

where the coefficients f_N^J are functions of temperature T as follows:

$$f_N^J(T) = a_{1,N} + a_{2,N} \cdot T + a_{3,N} \cdot T^2 + a_{4,N} \cdot T^3 + a_{5,N} \cdot T^{-1},$$
(14)

and the coefficients $a_{1-5,N}$ for each $f_N^J(T)$ are given in Table 1.

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N	$\mathbf{a}_{1,N}$	$a_{2,N}$	$a_{3,N}$	$\mathbf{a}_{4,N}$	$a_{5,N}$
1	$3.0108954259038608 \cdot 10^1$	$6.1176722090512577{\cdot}10^1$	8.7240333618891663·10 ⁻¹	$-4.6191788649375719{\cdot}10^{-3}$	$8.3537059107024481 \cdot 10^{-1}$
2	$1.5028549216690628 \cdot 10^1$	$-1.9310989753720623 \cdot 10^{-1}$	$8.0155514634860480 \cdot 10^{-4}$	$-1.0832730707799128 \cdot 10^{-6}$	1.7577660457989019
3	$\textbf{-2.0487870170216488} \cdot 10^{-1}$	$1.3263949252910405 \cdot 10^{-3}$	$-8.4195688402450274{\cdot}10^{-6}$	$1.6154895940993287 \cdot 10^{-8}$	$3.8734212545203874 \cdot 10^1$
4	1.4955918863858371	$9.2290004245522454 \cdot 10^1$	$-8.9006965195392618{\cdot}10^{-1}$	$2.2319123411013099 \cdot 10^{-3}$	$4.0180079996840852{\cdot}10^{-3}$
5	7.9018031228561085	$-1.1649433968658949 \cdot 10^{1}$	$1.1400827854910951 \cdot 10^{-1}$	$-3.1941526492127755 {\cdot} 10^{-4}$	$-3.7662115740271446{\cdot}10^{-1}$
6	$1.5725237111225979 \cdot 10^2$	-1.0051649979836277	$1.1866484014507624 \cdot 10^{-3}$	$7.3557614998540389{\cdot}10^{-6}$	2.6270197023115189
7	$-1.6973840122470968 \cdot 10^{1}$	$1.1258423691432135 \cdot 10^{-1}$	$-2.9850139351463793 \cdot 10^{-4}$	$1.4301286324827064{\cdot}10^{-7}$	$1.3163389235253725 \cdot 10^1$
8	-1.0399591631839757	$2.7022055588257691 \cdot 10^{-3}$	$-2.1507467231330936 \cdot 10^{-6}$	$3.8059489037584171{\cdot}10^{-10}$	$1.5000492788553410 \cdot 10^2$
9	1.2250990965305315	$3.0495946490079444 \cdot 10^1$	$2.1051563135187106{\cdot}10^1$	$-8.2200682916580878 \cdot 10^{-2}$	$2.9965871386685029 \cdot 10^{-2}$
10	4.8281605955680433	$1.7346551710836445 \cdot 10^2$	$-1.0113602140796010{\cdot}10^1$	$3.7482518458685089 \cdot 10^{-2}$	$-1.4449998158558205 {\cdot} 10^{-1}$
11	$2.3399230964451237 \cdot 10^2$	$-2.3099267235261948 {\cdot} 10^1$	$8.0122962140916354 \cdot 10^{-2}$	$6.1542576994557088{\cdot}10^{-5}$	5.3718413254843007
12	$1.0299715519499360 \cdot 10^2$	$-6.4663357203364136{\cdot}10^{-2}$	$-2.0487150565050316 {\cdot} 10^{-3}$	$8.7935289055530897 \cdot 10^{-7}$	$3.6013204601215229 \cdot 10^1$
13	-3.5452115439584042	$1.7083445731159330{\cdot}10^{-2}$	$-1.2552625290862626 \cdot 10^{-5}$	$1.2968447449182847 \cdot 10^{-9}$	$1.5748687512056560 \cdot 10^2$
14	2.2338490119517975	$1.0229410216045540 \cdot 10^2$	-3.2103611955174052	$1.3397152304977591 \cdot 10^{-2}$	$-2.4155187776460030 \cdot 10^{-2}$
15	3.7592282990713963	$-1.5257988769009816 \cdot 10^2$	2.6113805420558802	$-9.0380721653694363 \cdot 10^{-3}$	$-1.3974197138171082{\cdot}10^{-1}$
16	$1.8293600730573988{\cdot}10^1$	$1.8344728606002992 \cdot 10^1$	$-4.0063363221106751 \cdot 10^{-1}$	$1.4842749371258522 \cdot 10^{-3}$	1.1848846003282287
17	$-1.7634531623032314{\cdot}10^2$	4.9011762441271278	$-1.3195821562746339 \cdot 10^{-2}$	$\textbf{-2.8668619526430859}{\cdot}10^{-5}$	$-2.9823396976393551 \cdot 10^{-1}$
18	$-3.2944043694275727 \cdot 10^{1}$	$1.2517571921051887 \cdot 10^{-1}$	$8.3239769771186714 \cdot 10^{-5}$	$2.8191859341519507{\cdot}10^{-7}$	$-2.7352880736682319{\cdot}10^1$
19	-1.1451811137553243	$2.0625997485732494 \cdot 10^{-3}$	$-3.4225389469233624{\cdot}10^{-6}$	$4.4437613496984567{\cdot}10^{-10}$	$1.8666644332606754 \cdot 10^2$
20	$3.2270897099493567 \cdot 10^1$	$7.7898447327513687 \cdot 10^{-1}$	$-6.5662738484679626 \cdot 10^{-3}$	$3.7899330796456790 \cdot 10^{-6}$	$7.1106427501756542{\cdot}10^{-1}$
21	$-2.8901906781697811 \cdot 10^1$	-1.5356398793054860	$1.9267271774384788 \cdot 10^{-2}$	$-5.3886270475516162{\cdot}10^{-5}$	$5.0490415975693426 \cdot 10^{-1}$
22	$3.3365683645733924 \cdot 10^1$	$-3.6114561564894537{\cdot}10^{-1}$	$9.2977354471929262 \cdot 10^{-4}$	$1.9549769069511355 \cdot 10^{-7}$	-8.8865930095112855
23	2.4592563042806375	$-8.3227071743101084{\cdot}10^{-3}$	$8.2563338043447783 \cdot 10^{-6}$	$-8.4374976698593496{\cdot}10^{-9}$	$-2.0938173949893473 \cdot 10^2$
24	$4.4099823444352317{\cdot}10^1$	2.5915665826835252	$-1.6449091819482634{\cdot}10^{-2}$	$2.6797249816144721 \cdot 10^{-5}$	$5.5045672663909995 \cdot 10^{-1}$

Table 1.
Coefficients
$a_{1-5,N}$
of Eq.
(14)

302 303 The number of molecules in the critical cluster in the ion-induced case can be calculated as a function of $\ln(RH/100)$ and $\ln(\rho_a)$ with the following expression:

$$n_{\text{tot}}^{*} = \left| \left(f_{1}^{n}(T) + f_{2}^{n}(T) \cdot \left[\ln(\frac{RH}{100}) \right]^{-2} \cdot \ln^{-2} \rho_{a} + f_{3}^{n}(T) \cdot \left[\ln(\frac{RH}{100}) \right]^{-2} \right| (15) + f_{4}^{n}(T) \cdot \left[\ln(\frac{RH}{100}) \right]^{-2} \cdot \ln \rho_{a} + f_{5}^{n}(T) \cdot \left[\ln(\frac{RH}{100}) \right]^{-1} \cdot \ln^{-2} \rho_{a} + f_{6}^{n}(T) \cdot \left[\ln(\frac{RH}{100}) \right]^{-1} \cdot \ln^{-1} \rho_{a} + f_{7}^{n}(T) \cdot \left[\ln(\frac{RH}{100}) \right]^{-1} + f_{8}^{n}(T) \cdot \left[\ln(\frac{RH}{100}) \right]^{-1} \cdot \ln \rho_{a} + f_{9}^{n}(T) \cdot \ln^{-2} \rho_{a} + f_{10}^{n}(T) \cdot \ln^{-1} \rho_{a} + f_{11}^{n}(T) \cdot \ln \rho_{a} + f_{12}^{n}(T) \cdot \ln^{2} \rho_{a} + f_{13}^{n}(T) \cdot \ln(\frac{RH}{100}) \cdot \ln^{-2} \rho_{a} + f_{14}^{n}(T) \cdot \ln(\frac{RH}{100}) \cdot \ln^{-1} \rho_{a} + f_{15}^{n}(T) \cdot \ln(\frac{RH}{100}) + f_{16}^{n}(T) \cdot \ln(\frac{RH}{100}) \cdot \ln \rho_{a} + f_{17}^{n}(T) \cdot \left[\ln(\frac{RH}{100}) \right]^{2} + f_{18}^{n}(T) \cdot \left[\ln(\frac{RH}{100}) \right]^{2} \cdot \ln^{-1} \rho_{a} + f_{19}^{n}(T) \cdot \left[\ln(\frac{RH}{100}) \right]^{-3} + f_{20}^{n}(T) \cdot \frac{RH}{100} \cdot \ln \rho_{a} + f_{21}^{n}(T) \cdot \frac{RH}{100} + f_{22}^{n}(T) \cdot \frac{RH}{100} \cdot \ln^{2} \rho_{a} \right) \right|$$

where the coefficients $f_{\boldsymbol{N}}^n$ are functions of temperature \boldsymbol{T} as follows:

$$f_N^n(T) = b_{1,N} + b_{2,N} \cdot T + b_{3,N} \cdot T^2 + b_{4,N} \cdot T^{-1} + b_{5,N} \cdot T^3$$
(16)

and the coefficients $b_{1-5,N}$ for each $f_N^n(T)$ are given in Table 2.

The critical cluster radius for ion-induced particle formation can be evaluated using the

³⁰⁷ following expression:

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$$r^{*} = f_{1}^{r}(T) + f_{2}^{r}(T) \cdot \left[\ln(\frac{RH}{100})\right]^{-2} \cdot \ln^{-1}\rho_{a} + f_{3}^{r}(T) \cdot \left[\ln(\frac{RH}{100})\right]^{-2} \qquad (17)$$

$$+ f_{4}^{r}(T) \cdot \left[\ln(\frac{RH}{100})\right]^{-2} \cdot \ln\rho_{a} + f_{5}^{r}(T) \cdot \left[\ln(\frac{RH}{100})\right]^{-1} \cdot \ln^{-2}\rho_{a} + f_{6}^{r}(T) \cdot \left[\ln(\frac{RH}{100})\right]^{-1} \cdot \ln^{-1}\rho_{a}$$

$$+ f_{7}^{r}(T) \cdot \left[\ln(\frac{RH}{100})\right]^{-1} + f_{8}^{r}(T) \cdot \left[\ln(\frac{RH}{100})\right]^{-1} \cdot \ln\rho_{a} + f_{9}^{r}(T) \cdot \left[\ln(\frac{RH}{100})\right]^{-1} \cdot \ln^{2}\rho_{a}$$

$$+ f_{10}^{r}(T) \cdot \ln^{-2}\rho_{a} + f_{11}^{r}(T) \cdot \ln^{-1}\rho_{a} + f_{12}^{r}(T) \cdot \ln\rho_{a} + f_{13}^{r}(T) \cdot \ln^{2}\rho_{a}$$

$$+ f_{14}^{r}(T) \cdot \ln(\frac{RH}{100}) \cdot \ln^{-2}\rho_{a} + f_{15}^{r}(T) \cdot \ln(\frac{RH}{100}) \cdot \ln^{-1}\rho_{a} + f_{16}^{r}(T) \cdot \ln(\frac{RH}{100})$$

$$+ f_{17}^{r}(T) \cdot \ln(\frac{RH}{100}) \cdot \ln\rho_{a} + f_{18}^{r}(T) \cdot \ln(\frac{RH}{100}) \cdot \ln^{2}\rho_{a} + f_{19}^{r}(T) \cdot \left[\ln(\frac{RH}{100})\right]^{2} \cdot \ln^{-1}\rho_{a}$$

$$+ f_{20}^{r}(T) \cdot \left[\ln(\frac{RH}{100})\right]^{2} + f_{21}^{r}(T) \cdot \left[\ln(\frac{RH}{100})\right]^{2} \cdot \ln\rho_{a} + f_{22}^{r}(T) \cdot \left[\ln(\frac{RH}{100})\right]^{3} \cdot \ln\rho_{a}$$

where the coefficients f_N^r are functions of temperature T as follows:

$$f_N^r(T) = c_{1,N} + c_{2,N} \cdot T + c_{3,N} \cdot T^2 + c_{4,N} \cdot T^3$$
(18)

and the coefficients $c_{1-4,N}$ for each $f_N^r(T)$ are given in Table 3.

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N	$\mathfrak{b}_{1,N}$	$\mathfrak{b}_{2,N}$	$\mathfrak{b}_{3,N}$	$b_{4,N}$	$b_{5,N}$
1	$-4.8324296064013375{\cdot}10^4$	$5.0469120697428906{\cdot}10^2$	-1.1528940488496042	$-8.6892744676239192{\cdot}10^2$	$4.0030302028120469{\cdot}10^{-4}$
2	$-6.7259105232039847{\cdot}10^3$	$1.9197488157452008 \cdot 10^2$	-1.3602976930126354	$-1.1212637938360332 \cdot 10^2$	$2.8515597265933207{\cdot}10^{-3}$
3	$2.6216455217763342 \cdot 10^2$	-2.3687553252750821	$7.4074554767517521 {\cdot} 10^{-3}$	$-1.9213956820114927 \cdot 10^3$	$-9.3839114856129453 \cdot 10^{-6}$
4	3.9652478944137344	$1.2469375098256536{\cdot}10^{-2}$	$\textbf{-9.9837754694045633} \cdot 10^{-5}$	$-5.1919499210175138{\cdot}10^2$	$1.6489001324583862{\cdot}10^{-7}$
5	$2.4975714429096206{\cdot}10^2$	$1.7107594562445172 \cdot 10^2$	$-7.8988711365135289{\cdot}10^{-1}$	$-2.2243599782483177 \cdot 10^1$	$-1.6291523004095427 \cdot 10^{-4}$
6	$-8.9270715592533611 \cdot 10^2$	$1.2053538883338946{\cdot}10^2$	-1.5490408828541018	$-1.1243275579419826 \cdot 10^1$	$4.8053105606904655{\cdot}10^{-3}$
7	$7.6426441642091631{\cdot}10^3$	$\text{-7.1785462414656578} \cdot 10^1$	$2.3851864923199523{\cdot}10^{-1}$	$8.5591775688708395 \cdot 10^{1}$	$-3.7000473243342858 \cdot 10^{-4}$
8	$\textbf{-5.1516826398607911}{\cdot}10^1$	$9.1385720811460558{\cdot}10^{-1}$	$-3.5477100262158974{\cdot}10^{-3}$	$2.7545544507625586{\cdot}10^3$	$5.4708262093640928{\cdot}10^{-6}$
9	$-3.0386767129196176{\cdot}10^2$	$-1.1033438883583569 \cdot 10^4$	$8.1296859732896067{\cdot}10^1$	$1.2625883141097162 \cdot 10^1$	$-1.2728497822219101 \cdot 10^{-1}$
10	$-3.3763494256461472{\cdot}10^3$	$3.1916579136391006 \cdot 10^3$	$-2.7234339474441143\cdot10^{1}$	$-2.1897653262707397{\cdot}10^1$	$5.1788505812259071{\cdot}10^{-2}$
11	$-1.8817843873687068{\cdot}10^3$	4.3038072285882070	$6.6244087689671860{\cdot}10^{-3}$	$-2.7133073605696295 \cdot 10^3$	$-1.7951557394285043 \cdot 10^{-5}$
12	$-1.7668827539244447{\cdot}10^2$	$4.8160932330629913{\cdot}10^{-1}$	$\textbf{-6.3133007671100293} \cdot 10^{-4}$	$2.5631774669873157{\cdot}10^4$	$4.1534484127873519{\cdot}10^{-7}$
13	$-1.6661835889222382 \cdot 10^3$	$1.3708900504682877 \cdot 10^3$	$-1.7919060052198969{\cdot}10^1$	$-3.5145029804436405{\cdot}10^1$	$5.1047240947371224{\cdot}10^{-2}$
14	$1.0843549363030939 \cdot 10^4$	$-7.3557073636139577 \cdot 10^1$	1.2054625131778862	$1.9358737917864391 \cdot 10^2$	$-4.2871620775911338 \cdot 10^{-3}$
15	$-2.4269802549752835{\cdot}10^3$	$1.1348265061941714 \cdot 10^{1}$	$-5.0430423939495157{\cdot}10^{-2}$	$2.3709874548950634{\cdot}10^3$	$1.4091851828620244{\cdot}10^{-4}$
16	$5.2745372575251588{\cdot}10^2$	-2.6080675912627314	$5.6902218056670145 \cdot 10^{-3}$	$-3.2149319482897838{\cdot}10^4$	$-5.4121996056745853 \cdot 10^{-6}$
17	$-1.6401959518360403{\cdot}10^1$	$2.4322962162439640{\cdot}10^{-1}$	$1.1744366627725344 \cdot 10^{-3}$	$-8.2694427518413195{\cdot}10^3$	$-5.0028379203873102{\cdot}10^{-6}$
18	$-2.7556572017167782{\cdot}10^3$	$4.9293344495058264{\cdot}10^1$	$\textbf{-2.6503456520676050} \cdot 10^{-1}$	$1.2130698030982167 \cdot 10^3$	$4.3530610668042957{\cdot}10^{-4}$
19	-6.3419182228959192	$4.0636212834605827{\cdot}10^{-2}$	$\textbf{-1.0450112687842742} \cdot 10^{-4}$	$3.1035882189759656 \cdot 10^2$	$9.4328418657873500{\cdot}10^{-8}$
20	$3.0189213304689042 \cdot 10^3$	$\text{-}2.3804654203861684{\cdot}10^1$	$6.8113013411972942{\cdot}10^{-2}$	$6.3112071081188913{\cdot}10^2$	$-9.4460854261685723 {\cdot} 10^{-5}$
21	$1.1924791930673702 \cdot 10^4$	$-1.1973824959206000 \cdot 10^2$	$1.6888713097971020{\cdot}10^{-1}$	$1.8735938211539585 \cdot 10^2$	$5.0974564680442852 \cdot 10^{-4}$
22	3.6409071302482083·10 ¹	$1.7919859306449623 \cdot 10^{-1}$	$-1.0020116255895206 \cdot 10^{-3}$	$-8.3521083354432303 \cdot 10^3$	$1.5879900546795635 \cdot 10^{-6}$

Table 2. Coefficients $b_{1-5,N}$ of the Eq. (15).

N	$c_{1,N}$	$c_{2,N}$	$c_{3,N}$	$c_{4,N}$
1	$-3.6318550637865524{\cdot}10^{-8}$	$2.1740704135789128 \cdot 10^{-9}$	$-8.5521429066506161 \cdot 10^{-12}$	$-9.3538647454573390 \cdot 10^{-15}$
2	$2.1366936839394922 \cdot 10^{-8}$	$-2.4087168827395623 \cdot 10^{-10}$	$8.7969869277074319 \cdot 10^{-13}$	$-1.0294466881303291 \cdot 10^{-15}$
3	$-7.7804007761164303 \cdot 10^{-10}$	$1.0327058173517932 \cdot 10^{-11}$	$-4.2557697639692428 \cdot 10^{-14}$	$5.4082507061618662{\cdot}10^{-17}$
4	$3.2628927397420860 {\cdot} 10^{-12}$	$-7.6475692919751066 \cdot 10^{-14}$	$4.1985816845259788{\cdot}10^{-16}$	$\textbf{-6.2281395889592719} \cdot 10^{-19}$
5	$2.0442205540818555 \cdot 10^{-9}$	$4.0441858911249830 \cdot 10^{-8}$	$-3.3423487629482825 \cdot 10^{-10}$	$6.8000404742985678{\cdot}10^{-13}$
6	$1.8381489183824627 \cdot 10^{-8}$	$-8.9853322951518919 \cdot 10^{-9}$	$7.5888799566036185 {\cdot} 10^{-11}$	$-1.5823457864755549 \cdot 10^{-13}$
7	$1.1795760639695057 \cdot 10^{-7}$	$-8.1046722896375875 \cdot 10^{-10}$	$9.1868604369041857{\cdot}10^{-14}$	$4.7882428237444610 \cdot 10^{-15}$
8	$-4.4028846582545952{\cdot}10^{-9}$	$4.6541269232626618 {\cdot} 10^{-11}$	$-1.1939929984285194{\cdot}10^{-13}$	$2.3602037016614437{\cdot}10^{-17}$
9	$2.7885056884209128{\cdot}10^{-11}$	$-4.5167129624119121 \cdot 10^{-13}$	$1.6558404997394422 \cdot 10^{-15}$	$-1.2037336621218054{\cdot}10^{-18}$
10	$-2.3719627171699983 \cdot 10^{-9}$	$-1.5260127909292053 \cdot 10^{-7}$	$1.7177017944754134 \cdot 10^{-9}$	$-4.7031737537526395 \cdot 10^{-12}$
11	$-5.6946433724699646 \cdot 10^{-9}$	$8.4629788237081735 \cdot 10^{-9}$	$-1.7674135187061521 \cdot 10^{-10}$	$6.6236547903091862{\cdot}10^{-13}$
12	$-2.2808617930606012 \cdot 10^{-8}$	$1.4773376696847775 \cdot 10^{-10}$	$-1.3076953119957355 \cdot 10^{-13}$	$2.3625301497914000 \cdot 10^{-16}$
13	$1.4014269939947841 \cdot 10^{-10}$	$-2.3675117757377632 \cdot 10^{-12}$	$5.1514033966707879 \cdot 10^{-15}$	$-4.8864233454747856 \cdot 10^{-18}$
14	$6.5464943868885886 \cdot 10^{-11}$	$1.6494354816942769 \cdot 10^{-8}$	$-1.7480097393483653 \cdot 10^{-10}$	$4.7460075628523984{\cdot}10^{-13}$
15	$8.4737893183927871 \cdot 10^{-9}$	$-6.0243327445597118 \cdot 10^{-9}$	$5.8766070529814883 {\cdot} 10^{-11}$	$-1.4926748560042018 \cdot 10^{-13}$
16	$1.0761964135701397 \cdot 10^{-7}$	$-1.0142496009071148 {\cdot} 10^{-9}$	$2.1337312466519190 \cdot 10^{-12}$	$1.6376014957685404{\cdot}10^{-15}$
17	$-3.5621571395968670 \cdot 10^{-9}$	$4.1175339587760905{\cdot}10^{-11}$	$-1.3535372357998504 \cdot 10^{-13}$	$8.9334219536920720{\cdot}10^{-17}$
18	$2.0700482083136289{\cdot}10^{-11}$	$-3.9238944562717421 \cdot 10^{-13}$	$1.5850961422040196{\cdot}10^{-15}$	$-1.5336775610911665 \cdot 10^{-18}$
19	$1.8524255464416206 \cdot 10^{-9}$	$-2.1959816152743264 \cdot 10^{-11}$	$-6.4478119501677012 \cdot 10^{-14}$	$5.5135243833766056 \cdot 10^{-16}$
20	$1.9349488650922679 \cdot 10^{-9}$	$-2.2647295919976428 \cdot 10^{-11}$	$9.2917479748268751 \cdot 10^{-14}$	$-1.2741959892173170 {\cdot} 10^{-16}$
21	$2.1484978031650972{\cdot}10^{-11}$	$-9.3976642475838013 \cdot 10^{-14}$	$-4.8892738002751923 \cdot 10^{-16}$	$1.4676120441783832 \cdot 10^{-18}$
22	$6.7565715216420310 \cdot 10^{-13}$	$-3.5421162549480807 \cdot 10^{-15}$	$-3.4201196868693569{\cdot}10^{-18}$	$2.2260187650412392{\cdot}10^{-20}$

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The sulfuric acid threshold concentration (in cm^{-3}) for kinetic ion-induced particle for-

mation can be expressed as a function of temperature and relative humidity with
$$\begin{bmatrix} BH \end{bmatrix}^{-2}$$

$$\begin{aligned} \ln \rho_{a}^{\rm kin} &= 5.3742280876674478 \cdot 10^{1} - 6.6837931590012266 \cdot 10^{-3} \cdot \left[\ln(\frac{RH}{100})\right] \end{aligned} \tag{19} \\ &-1.0142598385422842 \cdot 10^{-1} \cdot \left[\ln(\frac{RH}{100})\right]^{-1} - 6.4170597272606873 \cdot \ln(\frac{RH}{100}) \\ &-6.4315798914824518 \cdot 10^{-1} \cdot \left[\ln(\frac{RH}{100})\right]^{2} - 2.4428391714772721 \cdot 10^{-2} \cdot \left[\ln(\frac{RH}{100})\right]^{3} \\ &-3.5356658734539019 \cdot 10^{-4} \cdot \left[\ln(\frac{RH}{100})\right]^{4} + 2.5400015099140506 \cdot 10^{-5} \cdot T \cdot \left[\ln(\frac{RH}{100})\right]^{-2} \\ &-2.7928900816637790 \cdot 10^{-4} \cdot T \cdot \left[\ln(\frac{RH}{100})\right]^{-1} + 4.4108573484923690 \cdot 10^{-2} \cdot T \cdot \ln(\frac{RH}{100}) \\ &+6.3943789012475532 \cdot 10^{-3} \cdot T \cdot \left[\ln(\frac{RH}{100})\right]^{2} + 2.3164296174966580 \cdot 10^{-4} \cdot T \cdot \left[\ln(\frac{RH}{100})\right]^{3} \\ &+3.0372070669934950 \cdot 10^{-6} \cdot T \cdot \left[\ln(\frac{RH}{100})\right]^{4} + 3.8255873977423475 \cdot 10^{-6} \cdot T^{2} \cdot \left[\ln(\frac{RH}{100})\right]^{-1} \\ &-1.2344793083561629 \cdot 10^{-4} \cdot T^{2} \cdot \ln(\frac{RH}{100}) - 1.7959048869810192 \cdot 10^{-5} \cdot T^{2} \cdot \left[\ln(\frac{RH}{100})\right]^{-1} \\ &-3.2165622558722767 \cdot 10^{-7} \cdot T^{2} \cdot \left[\ln(\frac{RH}{100})\right]^{3} - 4.7136923780988659 \cdot 10^{-9} \cdot T^{3} \cdot \left[\ln(\frac{RH}{100})\right]^{-1} \\ &+1.1873317184482216 \cdot 10^{-7} \cdot T^{3} \cdot \ln(\frac{RH}{100}) + 1.5685860354866621 \cdot 10^{-8} \cdot T^{3} \cdot \left[\ln(\frac{RH}{100})\right]^{2} \\ &-1.4329645891059557 \cdot 10^{4} \cdot T^{-1} + 1.3842599842575321 \cdot 10^{-1} \cdot T \\ &-4.1376265912842938 \cdot 10^{-4} \cdot T^{2} + 3.9147639775826004 \cdot 10^{-7} \cdot T^{3}. \end{aligned}$$

In the kinetic range the ion-induced particle formation can be described with the following equation [*Merikanto et al.*, 2015]:

$$J_{\rm kin,ion-induced} = C\rho_{\rm pre}\sqrt{T}(\rho_a^{\rm total})$$
⁽²⁰⁾

where C is given as in Eq. 12 with $r_{\rm ref} = r_{\rm ion} = 0.487 \cdot 10^{-9}$ m and $m_{\rm ref} = m_{\rm ion} = 1.661 \cdot 10^{-27}$ kg, and the pre-existing cluster concentration $\rho_{\rm pre}$ is simply the negative ion concentration $\rho_{\rm ion}$.

4.3 How to apply the parameterizations in practice?

315

For simplifying the use of the parameterizations, we are distributing Fortran codes in the Supplementary Materials of this article. The following points should be kept in mind when using the parameterizations.

1. The total particle formation rate is the sum of neutral and ion-induced cases: $J_{\text{tot}} = J_{\text{neutral}} + J_{\text{ion}}$.

Parameterization	Neutral	Ion-induced
Kinetic threshold $ ho_a^{ m kin}[1/{ m cm}^3]$	Eqs. (8), (9), (10)	Eq. (19)
x	Eq. (1)	Eq. (1)
$J_{ m nuc}$	Eq. (2)	Eqs. (13), (14), Table 1
$N_{ m tot}$	Eq. (3)	Eqs. (15), (16), Table 2
r	Eq. (4)	Eqs. (17), (18), Table 3
Threshold $ ho_a^{J=1}[1/{ m cm}^3]$	Eqs. (5), (6), (7)	_

Table 4. List of the parameterizations developed in this article.

321	2. The ion-induced formation rate is limited by the ion pair production rate (I.P.R.): $J_{ion} =$
322	$\min(J_{\mathrm{ion}}, I.P.R.).$
323	3. The ion-induced particle formation rate is directly proportional to the negative ion con-
324	centration: $J_{\rm ion} \propto N_{\rm ion}$. The parameterization has been generated for $N_{\rm ion} = 1~{ m cm}^{-3}$
325	and should be scaled accordingly.
326	4. The particle formation rates are calculated at the (critical) formation size. If the param-
327	eterizations are used in comparison with measurements at larger sizes or in an atmo-
328	spheric model where the smallest size bin is larger than the formation size, the forma-
329	tion rates at those sizes should be evaluated using formulations developed for this pur-
330	pose [Kerminen and Kulmala, 2002; Lehtinen et al., 2007]. For example, to calculate
331	the formation rates at the size of 3 nm: $J_{3nm} = J_{neutral} \cdot KK(r_{neutral}^*) + J_{ion} \cdot KK(r_{ion}^*)$,
332	where KK is the scaling given by, for example, Kerminen and Kulmala [2002] or Lehti-
333	nen et al. [2007].

334

335

Finally, we provide a quick reference of the different parameterizations in Table 4.

336 5 Results

In this section, we compare the new theoretical particle formation rates [*Merikanto et al.*, 2015] against the old [*Vehkamäki et al.*, 2002] and new parameterized particle formation rates and cluster properties in a wide range of conditions.

Furthermore, to test the parameterization in realistic atmospheric modeling conditions, we have also implemented the new neutral particle formation parameterization in the global

aerosol-climate model ECHAM5.5-HAM2 [Stier et al., 2005; Zhang et al., 2012]. This model 342 describes the aerosol population with four soluble and three insoluble log-normal modes, in-343 cluding sulfate, organic carbon, black carbon, dust and sea salt. The simulations start in June 344 1999 with a 6-month spin-up period and end in December 2000: analysis is done only for year 345 2000. The model is nudged against ERA-40 meteorology to improve signal-to-noise ratio. The 346 simulations include anthropogenic aerosol and precursor emissions from ACCMIP [Lamar-347 que et al., 2010], while sea salt, dust and DMS emissions are calculated online. We apply the 348 model in two cases, 1) with the default Vehkamäki et al. [2002] binary nucleation parameter-349 ization and 2) with the new parameterization, and we will present this comparison as well. 350

351

5.1 Neutral parameterization

Figures 1, 2, 3 and 4 show a comparison of the theoretical and the parametrized values for particle formation rate, sulfuric acid mole fraction, total number of molecules and radius of the critical cluster, respectively.

The ratio of the theoretical and the parameterized particle formation rates (Fig. 1) shows 363 that in the whole range of particle formation rates most of the cases are within one order of 364 magnitude $(10^{-1}-10^{1})$ of the perfect fit. Our wider ranges for all the input variables cause larger 365 deviations between the theoretical values and the fit than in the parametrization of Vehkamäki 366 et al. [2002]. For very low and very high theoretical particle formation rates the parameter-367 ization may overestimate the rates by a factor of 10^3 in some cases. These cases are related 368 to the extremities of the validity ranges. For example, for high sulfuric acid concentrations (above 369 10^9 cm^{-3}) sometimes the model predicts very large clusters (n_{tot} around 100) with correspond-370 ing very low particle formation rates, which are overestimated by the parameterization. This 371 overestimation grows with growing sulfuric acid concentration and is particularly clear for con-372 centrations above approximately 10¹¹ cm⁻³, giving the highest overestimations of small par-373 ticle formation rates (cases on the left extremity of Fig. 1 for $J_{\rm theoretical}/J_{\rm param}$ values be-374 low 10^{-2}). However, the largest discrepancies between the theoretical and the parameterized 375 particle formation rates are either at insignificantly low particle formation rates, where the par-376 ticle formation is practically zero, or at extremely high particle formation rates where parti-377 cle formation rate is not the limiting factor for appearance of particles of atmospheric rele-378 vance, but also growth and loss processes play a major role. 379



Figure 1. Comparison between the parameterized and the theoretical values for the neutral particle formation rate $J_{neutral}$.



Figure 2. Comparison between the parameterized and the theoretical values for the critical cluster mole fraction x^* in the neutral case.



Figure 3. Comparison between the parameterized and the theoretical values for the total number of molecules in the critical cluster n_{tot}^* in the neutral case.



Figure 4. Comparison between the parameterized and the theoretical values for the critical cluster radius r^* in the neutral case.

Figure 2 shows the difference between the theoretical and parameterized mole fractions. Fractions below x < 0.5 are best fitted (difference between theory and parameterization ranging between -0.017 and 0.007), but above this value the difference grows. In particular, the mole fractions above x > 0.7 are somewhat overestimated (deviation up to 0.069) by the parameterization. The relative errors are at most around 13% (lowest mole fractions) and stay below 9% at high mole fractions. The best fits are acquired in the range x = 0.33 - 0.45(relative errors around or less than 1%).

The difference between the theoretical and the parameterized total number of molecules n_{tot} in the critical cluster seen in Fig. 3 shows a best fit for small n_{tot} with larger absolute deviations appearing the larger the n_{tot} is. The values of n_{tot} vary from 1 to 200, with the large total numbers corresponding to very small particle formation rates, with a lower limit of J=10⁻⁷cm⁻³s⁻¹. The deviations range between -3 and +5 molecules at n_{tot} =40 and n_{tot} =55, respectively, giving a relative error of less than 10 %.

393 394 The critical cluster radii vary approximately between 0.28 nm and 1.2 nm (Fig. 4) with the ratio of the theoretical and the parameterized radii ranging between 0.98 and 1.07.

Figure 5 shows the behavior of the particle formation rate for several sulfuric acid con-403 centrations as a function of temperature and relative humidity. It can be seen that the differ-404 ence between the Vehkamäki et al. [2002] model (crosses) and parameterization (dashed lines) 405 and the present model (triangles) and parameterization (solid lines) is largest at low temper-406 atures and at low relative humidities. The old and present particle formation rates are nearly 407 superposed above 230-250 K and above RH=1% of relative humidity (see the lowest four pan-408 els of Fig. 5), but differences are seen already at 270 K when RH<1% and at 240K with RH<10%, 409 depending also on the acid concentration. The largest differences between the present and Vehkamäki 410 et al. [2002] results are seen at low acid concentrations (top panels in Fig. 5). The old param-411 eterization is not valid below 190K (absence of dashed lines). The difference between the old 412 and new models is also large at low relative humidities RH < 1%, depending also on the acid 413 concentration. Similarly, in the upper left panel of Fig. 5 the particle formation rates are seen 414 to differ the most below 210 K and particularly so for low relative humidities. Figure 5 demon-415 strates the good quality of our new parameterization, since the solid lines (the parameterized 416 particle formation rates) are well superposed with the triangles representing the theoretical re-417 sults. Comparison of the dashed and solid lines in the plots also reveals the larger validity range 418 of the new parameterization. Note in particular the kinetic range (black lines and triangles), 419



Figure 5. Left: The neutral particle formation rate J as a function of temperature at different total sulfuric 395 acid concentrations and relative humidities. The relative humidities correspond to different colors as marked 396 in the plot. Right: The particle formation rate as a function of relative humidity at different total sulfuric acid 397 concentrations and temperatures. The temperatures correspond to different colors as marked in the plot. The 398 sulfuric acid concentrations (RHA= ρ_a) are given in the plot titles [1/cm³]. The crosses and dashed lines show, 399 respectively, the old theoretical values and the old parameterization [Vehkamäki et al., 2002]. The triangles 400 and solid lines show, respectively, the new theoretical values and the new parameterized rates. The black 401 triangles and solid lines show the kinetic particle formation rates calculated with Eqs. (11) and (12). 402

now accessible with our parameterization. A slight discontinuity can be seen on the upper right
panel at 175 K, with the parameterization (solid lines) exhibiting a dip just before arriving at
the kinetic limit. This is due to difficulties in fitting the functional form of the parameterization at the edge of the validity range.

Figure 6 shows the behavior of the threshold concentration parameterization. The ratio 431 of the theoretical to the parameterized values for the sulfuric acid concentration correspond-432 ing to particle formation rate $1 \text{ cm}^{-3}\text{s}^{-1}$ is 0.99-1.01 in the temperature range 155-185 K (ki-433 netic range), 0.92-1.16 in the range 190-310 K and 0.83-1.13 in the range 310-400 K. It can 434 be seen that at all but the lowest temperatures, as expected, the required sulfuric acid concen-435 tration decreases with decreasing temperature. For a given acid concentration, the saturation 436 vapor pressure decreases and thus the saturation ratio increases with decreasing temperature. 437 This leads to a lower particle formation barrier, and a larger value for the exponential term in 438 the particle formation rate equation. At the same time the particle formation kinetics slows down, 439 but the growth of the exponential term dominates. However, we can see that this is valid only 440 down to temperature of 190 K (see Fig. 7), around which the kinetic regime is entered. Be-441 low this, the necessary acid concentration starts to increase with decreasing temperature. This 442 can be explained by the vanishing particle formation barrier, leaving the kinetics as the de-443 termining factor. Since the kinetic processes slow down with decreasing temperature, a higher 444 acid concentration will be required to maintain the same particle formation rate. This behav-445 ior was not seen in Vehkamäki et al. [2002], since they limited their cluster sizes to $n_{\rm tot}$ > 446 4, whereas we go down to $n_{\rm tot} > 1$, and also model specifically the kinetic range. Note that 447 in Fig. 7 the data points behave in a step-like manner only because of the coarse resolution 448 of the sulfuric acid concentration grid. 449

The kinetic limit is described as the threshold sulfuric acid concentration above which 450 particle formation is kinetic at a certain relative humidity and temperature. This concentration 451 is lowest at low temperatures: for example, it is between 10^0 and 10^4 cm⁻³ at 160 K. The 452 variation of the threshold concentration for a constant temperature arises from the RH-dependence, 453 giving the lowest threshold at highest RH. The kinetic threshold concentration increases to-454 wards higher temperatures reaching 10¹⁸ cm⁻³ at 400 K. The fit for the sulfuric acid concen-455 tration at the kinetic limit is within 10-12% of the theoretical results. The absolute lowest RH 456 data value used is $5 \cdot 10^{-6}$, but the minimum RH value depends on the temperature. Note that 457 RH=0% can not be used because the fitting function contains logarithms of RH. 458



Figure 6. The threshold sulfuric acid concentration parameterization (lines, data with symbols) yielding neutral particle formation rates $1 \text{ cm}^{-3}\text{s}^{-1}$ as function of relative humidity. Upper panel: temperature range from 310 K to 400 K. Lower panel: 190-310 K. The temperatures are as given in the legend.



Figure 7. The threshold sulfuric acid concentration parameterization (lines, data with symbols) yielding neutral particle formation rates $1 \text{ cm}^{-3}\text{s}^{-1}$ in the kinetic range, at temperatures ranging from 155 K to 185 K. The step-wise behavior of the data points is only an artifact of the sulfuric acid grid resolution we used for calculating the data.



Figure 8. Results of the comparison of the old and new parameterizations within the ECHAM5.5-HAM2 model. The leftmost panels show zonal mean particle formation rates calculated with the *Vehkamäki et al.* [2002] parameterization and the middle panels the rates calculated with the new parameterization. The formation rates are presented as function of latitude and pressure. The rightmost panels show the ratio of the particle formation rates given by the new parameterization to the ones calculated with *Vehkamäki et al.* [2002]. The upper panels show results for January and the lower panels for July.

Figure 8 shows the ECHAM5.5-HAM2 model results of the comparison of particle for-465 mation rates calculated with the Vehkamäki et al. [2002] parameterization and the new one. 466 Both Vehkamäki et al. [2002] and new parameterization produce an overall similar zonal pat-467 tern (Fig. 8) with high average nucleation rates of 10-100 cm⁻³ s⁻¹ in the UTLS and gen-468 erally low rates in the lower atmosphere, which is also found in earlier studies [Makkonen et al., 469 2009; Yu et al., 2010; Lee et al., 2013]. During NH winter, nucleation rates up to 0.1-1 cm⁻³ 470 s⁻¹ can extend below 800 hPa between 30-70°N due to cold continental temperatures in Siberia 471 and Northern America. 472

A gap in nucleation rate is visible in both January and July simulation with Vehkamäki 473 et al. [2002] parameterization around 100 hPa between 15°S-15°N, since the parameteriza-474 tion was limited to critical clusters of at least 4 molecules. This gap has been filled in the sim-475 ulations with new parameterization. Hence, the dominant effect of using the improved param-476 eterization is seen as an increase of global nucleation rates by a factor of 100 around 100 hPa, 477 thanks to the wider validity range of the new parameterization to smaller critical clusters ($n_{\rm tot}$ < 478 4) down to the kinetic range $(n_{\rm a}^* < 1)$. However, even lower atmosphere (800-1000 hPa) nu-479 cleation rates are increased by a factor of 2-3 (January) and 2-6 (July) with the new param-480 eterization. 481

482

5.2 Ion-induced parameterization

Figures 9, 10 and 11 show a comparison of the theoretical and the parametrized values for particle formation rate, total number of molecules and radius of the critical cluster, respectively, in the ion-induced case.

Fig. 9 reveals that in general the parameterization describes well the ion-induced particle formation rate, but seems to overestimate the rate in particular at the lowest and the highest theoretical formation rates. In most of the range the parameterized rates are within two orders of magnitude of the perfect fit with some excursions up to 3-4 orders of magnitude (largest overestimation by the parameterization being $3 \cdot 10^4$ and largest underestimation $8 \cdot 10^2$).

The parameterization for the number of molecules in the critical cluster (Fig. 10) overestimates the values at most by about 40% and underestimates the values by almost 50% in the smallest cluster sizes in some isolated cases, but overall the differences are smaller.



Figure 9. Comparison between the parameterized and the theoretical values for the ion-induced particle formation rate J_{ion} .



Figure 10. Comparison between the parameterized and the theoretical values for the total number of molecules in the critical cluster n_{tot} in the ion-induced case.



Figure 11. Comparison between the parameterized and the theoretical values for the critical cluster radius r^* in the ion-induced case.

The critical radius parameterization behaves well (Fig. 11) with slight overestimations of the radius at smallest sizes and underestimation at the largest sizes. The maximum deviations remain below 8%.

The parameterization for the sulfuric acid concentration at the kinetic limit is within 15% 503 of the perfect fit most of the time, with highest values at the lowest and highest threshold con-504 centration values. Naturally, as in the neutral case, the kinetic limit sulfuric acid concentra-505 tion increases with increasing temperature and with decreasing relative humidity: the concen-506 tration is the highest at high temperatures and low relative humidities. The range of the ki-507 netic limit sulfuric acid concentration spans a large range from 10^{-10} cm⁻³ at T=160 K and 508 RH=100% to 10¹⁶ cm⁻³ at T=400 K and RH=0.1%. The concentrations are always significantly 509 lower than the corresponding values for the neutral case. 510

Figure 12 shows a detailed comparison between the theoretical ion-induced particle formation rates (symbols) and the parameterized ones (lines). The kinetic range is shown in black. The ion-induced nucleation rate parameterization (solid lines) follows well the theory (triangles). The temperature at which the kinetic ion-induced range (black solid lines and triangles) is reached increases with increasing sulfuric acid concentration and increasing relative humidity, as for the neutral case.

524 6 Conclusions

We have developed new parameterizations based on an improved model [Merikanto et al., 525 2015] for two-component particle formation of sulfuric acid and water in the neutral and ion-526 induced cases. The new neutral parameterization widens the validity range of the Vehkamäki 527 et al. [2002] parameterization to lower relative humidities and higher sulfuric acid concentra-528 tions, and to a larger temperature range. Parameterizations are given for particle formation rate 529 $(J > 10^{-7} \text{ cm}^{-3} \text{s}^{-1})$, radius and composition of the critical cluster, and number of molecules 530 $(n_{\rm tot} > 1)$ in the critical cluster. Above the sulfuric acid concentrations given by parameter-531 ized kinetic limit, a simple kinetic regime formula for the formation rate can be used. It should 532 be noted that, as in Vehkamäki et al. [2002], the low temperature results are based on extrap-533 olations of the thermodynamic parameters below 230 K. 534

The neutral parameterization for *J* deviates slightly more from the theoretical values than the one of *Vehkamäki et al.* [2002] but the largest discrepancies are found at very high or very low particle formation rates with little relevance for correctly predicting whether particle for-

-37-



Figure 12. Left: The ion-induced particle formation rate J_{ion} as a function of temperature at different total sulfuric acid concentrations and relative humidities. The relative humidities correspond to the colors as marked in the plot. Right: The particle formation rate as a function of relative humidity at different total sulfuric acid concentrations and temperatures. The temperatures correspond to the colors as marked in the plot. The sulfuric acid concentrations (RHA= ρ_a) are given in the plot titles [1/cm³]. The triangles and solid lines show, respectively, the theoretical values and the parameterization in the nucleation range. The black triangles and lines show, respectively, the kinetic range values calculated with Eq. (20).

mation is occurring or not. The other parametrized quantities follow the theoretical predictions well with errors mostly below 10% (12% at maximum). The new theory and the new parameterization differ the most from the ones of *Vehkamäki et al.* [2002] at low temperatures and relative humidities, and at low acid concentrations. We have also developed a parameterization for the threshold sulfuric acid concentration for $J = 1 \text{ cm}^{-3}\text{s}^{-1}$ and for the threshold sulfuric acid concentration above which particle formation is kinetic.

The ion-induced parameterization behaves overall as well as the neutral one, with the parameterization mainly overestimating by 3-4 orders of magnitude the particle formation rates at very low and high theoretical formation rates. For the critical cluster radius the relative differences are always below 8%, but for the number of molecules in the critical cluster the differences can reach almost 50%. The kinetic limit is well described by the parameterization, the relative difference being always below 15%.

We recommend using the new model and the improved parameterizations from now on 550 instead of Vehkamäki et al. [2002]. The model performance has been compared to state-of-the-551 art particle formation measurements [Duplissy et al., 2015]. The neutral parameterization widens 552 the validity range of the Vehkamäki et al. [2002] parameterization, approaches correctly the 553 one-component limit and the kinetic range limit has been parameterized as well. A new ion-554 induced parameterization has been produced for a large range of conditions, and it also includes 555 the kinetic range limit. This development opens new opportunities for Terrestrial atmosphere 556 studies, and also for the Venus cloud community, which can use the new parameterization in 557 the range of conditions encountered within the Venus cloud formation region. 558

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Fortran codes of the parameterizations are given in the Supplementary Material.

560

A: Behavior of the two-component nucleation theory at the one-component limit

⁵⁶¹ When starting to develop the new parameterization, we were particularly interested in ⁵⁶² very low relative humidities, and preliminary calculations revealed formation of near-pure sul-⁵⁶³ furic acid clusters ($x \ge 0.99$) in these conditions. Thus we have performed both theoretical ⁵⁶⁴ and numerical tests to ensure correct behavior of the two-component particle formation model ⁵⁶⁵ at the one-component limit.

The general expression for nucleation rate J is

$$J = \frac{|\lambda_1|}{2\pi kT} \rho^* \frac{1}{\sqrt{|\det \frac{W^*}{2\pi kT}|}},$$
 (A.1)

where ρ^* is the number concentration of crtical clusters in the vapor phase. When using a self-

consistent cluster distribution the concentration ρ^* known to reduce correctly to the one-component

case at the one-component limit [Wilemski and Wyslouzil, 1995], but the behavior of the ki-

netic term $|\lambda_1|/\sqrt{|\det W^*|}$ needs to be examined, where $|\lambda_1|$ is the negative eigenvalue of

the product matrix R^*W^* . The matrix R^* contains the condensation coefficients that describe

the collisions of clusters of size n_a , n_w with the critical cluster:

$$R_{aa}^* = \sum_{n_a=1, n_w=0}^{N_c} n_a^2 (r^* + r)^2 \sqrt{8\pi kT \left(\frac{1}{m^*} + \frac{1}{m}\right)} \rho(n_a, n_w)$$
(A.2)

$$R_{ww}^* = \sum_{n_a=0, n_w=1}^{N_c} n_w^2 (r^* + r)^2 \sqrt{8\pi kT \left(\frac{1}{m^*} + \frac{1}{m}\right)} \rho(n_a, n_w)$$
(A.3)

$$R_{aw}^* = R_{wa}^* = \sum_{n_a=1, n_w=1}^{N_c} n_a n_w (r^* + r)^2 \sqrt{8\pi kT \left(\frac{1}{m^*} + \frac{1}{m}\right)} \rho(n_a, n_w), \qquad (A.4)$$

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where N_c stands for the largest hydrate accounted for. The matrix W^* contains the second derivatives of the formation energy with respect to the number of molecules in the cluster (W_{aa}, W_{aw}, W_{ww}), which can be written as follows:

$$W_{aa} \equiv \frac{d^{2}\Delta\varphi}{dn_{a}^{2}} = \frac{-v_{a}(x^{*})^{2}\sigma}{2\pi r^{*4}} + \left(kT\frac{d\mathcal{A}_{a}}{dx_{a}}\frac{1}{\mathcal{A}_{a}} + 2\frac{dv_{a}(x^{*})}{dx_{a}}\frac{\sigma}{r^{*}} + 2v_{a}(x^{*})\frac{d\sigma}{dx_{a}}\frac{1}{r^{*}}\right)\frac{n_{w}^{*}}{(n_{a}^{*} + n_{w}^{*})^{2}}$$
(A.5)
$$W_{ww} \equiv \frac{d^{2}\Delta\varphi}{dn_{w}^{2}} = \frac{-v_{w}(x^{*})^{2}\sigma}{2\pi r^{*4}} + \left(kT\frac{d\mathcal{A}_{w}}{dx_{w}}\frac{1}{\mathcal{A}_{w}} + 2\frac{dv_{w}(x^{*})}{dx_{w}}\frac{\sigma}{r^{*}} + 2v_{w}(x^{*})\frac{d\sigma}{dx_{w}}\frac{1}{r^{*}}\right)\frac{n_{a}^{*}}{(n_{a}^{*} + n_{w}^{*})^{2}}$$
(A.6)

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$$W_{aw} = W_{wa} \equiv \frac{d^2 \Delta \varphi}{dn_w dn_a} = \frac{-v_a(x^*)v_w(x^*)\sigma}{2\pi r^{*4}} + \left(kT\frac{d\mathcal{A}_w}{dx_w}\frac{1}{\mathcal{A}_w} + 2\frac{dv_w(x^*)}{dx_w}\frac{\sigma}{r^*} + 2v_w(x^*)\frac{d\sigma}{dx_w}\frac{1}{r^*}\right)\frac{n_w^*}{(n_a^* + n_w^*)^2}$$
(A.7)

where A_a and A_w are, respectively, the gas phase activities of acid and water.

Here $\rho(n_a, n_w)$ is the concentration of clusters containing n_a sulfuric acid molecules and n_w water molecules, m^* and r^* are the mass and the radius of the critical cluster, and mand r are the mass and the radius of the cluster colliding with the critical cluster.

The eigenvalues $\lambda_{1,2}$ of the product matrix R^*W^*

$$\begin{pmatrix} R_{ww} & R_{wa} \\ R_{aw} & R_{aa} \end{pmatrix} \begin{pmatrix} W_{ww} & W_{wa} \\ W_{aw} & W_{aa} \end{pmatrix}$$
(A.8)

are

$$\lambda_{1,2} = \frac{1}{2} \left(\alpha \pm \sqrt{\alpha^2 - 4 \det R^* \det W^*} \right) \tag{A.9}$$

where we use the shorthand notation $\alpha = W_{ww}R_{ww} + W_{aa}R_{aa} + 2W_{wa}R_{wa}$. When the 580 concentration of sulfuric acid, $\rho_a = \rho(1,0)$, is much higher than that of water, $\rho_w = \rho(0,1)$ 581 (i.e., $\rho_a >> \rho_w$), the cluster consist almost solely of sulphuric acid, $x \ge 0.99$. The forma-582 tion free energy surface around the critical cluster area forms a steep valley with almost ver-583 tical walls and the bottom of the valley running along the n_a -axis. When moving across the 584 valley on a line parallel to the n_w -axis, the slope of the free energy surface changes fast from 585 a large negative number to a large positive number when passing the critical size. Thus, mag-586 nitude of the second derivate $|W_{ww}|$ is large, approaching infinity for a truly one-component 587 system. When moving along the bottom of the valley up to the critical size and down after 588 that, the the slopes changes less rapidly, $|W_{ww}| >> |W_{aa}|$. Also, when comparing the al-589 most infinite slopes in the n_w direction for different values of n_a , it can be concluded that these 590 slopes do not depend strongly on the location along the n_a -axis, and thus $|W_{wa}|$ has a mod-591 est values and $|W_{ww}| >> |W_{wa}|$. Condensation coefficient R_{aa} contains terms represent-592 ing the collisions of the critical cluster with small clusters with acid only, as well as those with 593 both acid and water. If there is much more acid than water molecules in the systems, the acid 594 only cluster dominate over the clusters with both acid and water molecules as well as over clus-595 ter with only water molecules. Coefficient R_{ww} contains terms representing with water clus-596 ters only and water-acid clusters, and coefficient R_{aw} contains terms representing only water-597 acid clusters, and thus it can be concluded that for nearly pure acid case $R_{aa} >> R_{wa}$, $R_{aa} >>$ 598 R_{ww} . 599

In this case the determinants of the matrices can be approximated as $\det W = W_{ww}W_{aa}$ -600 $W_{wa}^2 \approx W_{ww} W_{aa}$ and det $R = R_{ww} R_{aa} - R_{wa}^2 \approx R_{ww} R_{aa}$. Parameter α can be written 601 as $\alpha = R_{ww}(W_{ww} + R_{aa}/R_{ww} \cdot W_{aa} + 2R_{wa}/R_{ww} \cdot W_{wa}) \approx W_{ww}R_{ww} + W_{aa}R_{aa}$. This is 602 justified as R_{wa} and R_{ww} contain almost similar terms terms apart from the the term propor-603 tional to $\rho(0,1) = \rho_w$, which is very small compared to the terms corresponding to clusters 604 with acid molecules in them. Another difference between R_{wa} and R_{ww} is the multiplication 605 of the terms with $n_a n_w$ vs. multiplication with n_w^2 , but this does not change the order of mag-606 nitude of the summation results. Thus R_{wa}/R_{ww} is of the order of 1, and $|W_{wa}|$ is small com-607 pared to $|W_{ww}|$, whereas R_{aa}/R_{ww} is large and thus $R_{aa}/R_{ww} \cdot W_{aa}$ can not safely be ne-608 glected although $|W_{ww}| >> |W_{aa}|$. 609

Inserting these approximations into Eq. (A.9) we get an approximate expression for the eigenvalues:

$$\lambda_{1,2} \approx \frac{1}{2} \left[W_{ww} R_{ww} + W_{aa} R_{aa} \pm \left(W_{ww} R_{ww} - W_{aa} R_{aa} \right) \right].$$
(A.10)

This results in simple expressions for the two eigenvalues of the product matrix R^*W^*

$$\lambda_{1,2} = \begin{cases} W_{aa} R_{aa} < 0 \\ W_{ww} R_{ww} > 0. \end{cases}$$
(A.11)

and we can evaluate the term $|\lambda_1|/\sqrt{|\det W^*|}$ of the nucleation rate equation (A.1):

$$\frac{|\lambda_1|}{\sqrt{|\det W^*|}} \approx \frac{-W_{aa}R_{aa}}{\sqrt{|W_{ww}W_{aa}|}} = R_{aa}\sqrt{\frac{|W_{aa}|}{W_{ww}}}.$$
(A.12)

In the cases of two-component nucleation at the one-component limit we thus get:

$$J_{2-\rm comp}^{1-\rm limit} = R_{aa} \sqrt{\frac{|W_{aa}^{2-\rm comp}|}{W_{ww}^{2-\rm comp}}} \rho^*$$
(A.13)

since the $2\pi kT$ terms in equation (A1) cancel in the two-component case. The standard onecomponent Zeldovich factor reads [*Vehkamäki*, 2006]

$$J_{1-\text{comp}} = R_{aa} \sqrt{\frac{|W_{aa}^{1-\text{comp}}|}{2\pi kT}} \rho^*,$$
(A.14)

It can now be seen that

$$\frac{J_{2-\text{comp}}^{1-\text{limit}}}{J_{1-\text{comp}}} = \sqrt{\frac{2\pi kT}{W_{ww}}} \sqrt{\frac{W_{aa}^{2-\text{comp}}}{W_{aa}^{1-\text{comp}}}},$$
(A.15)

which indicates that the presence of water affects the nucleation rate even when approaching the only-acid limit.

This can be understood as follows: For numerical reasons the sulfuric acid mole frac-612 tion in the critical cluster can not be set to exactly one (in our study we used x = 0.99 at 613 most), and as a consequence, water has necessarily a role in the process. Mathematically, in 614 the hypothetical case of x = 1, some elements of matrices R^* and W^* would be zero or in-615 finite and these matrices would thus be ill-behaved. In CNT the main contribution to the nu-616 cleation rate comes from the net flow of clusters through the critical cluster size in the direc-617 tion of the eigenvector corresponding to the negative eigenvalue λ_1 . The cluster flows on paths 618 parallel to this main path, passing by the critical cluster size, are however taken into account 619 by integrating over all these secondary paths giving rise to a factor $\sqrt{(2\pi kT/W_{ww})}$ seen in 620 Eq. (A.15). As the second derivative W_{ww} does not approach $2\pi kT$ even though $\rho_a >> \rho_w$, 621 this factor does not reduce to unity at the one-component limit. It must also be noted that be-622 cause water concentration can not be equal to zero in the two-component theory, the second 623 derivative W_{aa} does not exactly have its one-component value $v_a^2\sigma_a/(2\pi r^{*4})$ either. 624

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