BOREAL ENVIRONMENT RESEARCH 15: 245–254 © 2010
ISSN 1239-6095 (print) ISSN 1797-2469 (online) Helsinki 30 April 2010

Applicability of the FLake model to Lake Balaton

Miklós Vörös[1)]*, Vera Istvánovics[2)] and Tamás Weidinger[3)]

[1)] Hungarian Meteorological Service, Kitaibel P. u. 1, H-1024 Budapest, Hungary (*voros.m@met.hu)
[2)] Water Research Team of the Hungarian Academy of Sciences, Műegyetem rakpart 3, H-1111 Budapest, Hungary
[3)] Department of Meteorology, Eötvös Loránd University, Pázmány Péter s. 1/A, H-1117 Budapest, Hungary

Received 18 Feb. 2009, accepted 18 Oct. 2009 (Editor in charge of this article: Kai Myrberg)

Vörös, M., Istvánovics, V. & Weidinger, T. 2010: Applicability of the FLake model to Lake Balaton. *Boreal Env. Res.* 15: 245–254.

Owing to its uniquely large surface area/depth ratio, the temperature of Lake Balaton is highly sensitive to atmospheric events. In the ALADIN weather prediction model, which is used operationally in Hungary, lakes are not properly represented, as their temperature is initialized from interpolated values of sea surface temperature and considered to be constant for the duration of the forecast. The FLake model as a lake parameterization offers a more detailed but still computationally inexpensive solution to this deficiency. We investigated whether the model could be applied to Lake Balaton. We tested the performance of a standalone version of the FLake model by using observations and atmospheric model data. In the off-line simulations, FLake performed well on Lake Balaton with default settings in predicting surface temperatures, and less well in capturing the bottom water temperatures and stratification.

Introduction

The role of lakes in numerical weather prediction (NWP) and climate modeling is rising with the ever-increasing spatial resolution of the models. For NWP at low resolutions (40–50 km), the classification of surfaces as “continents” and “oceans” is satisfactory since most European lakes occupy only a small portion of a gridbox. At higher resolutions the number of boxes with a significant lake coverage increases, and even pure lake gridboxes might appear. Consequently, the effect of lakes needs to be taken into account as the exchange of radiation, moisture, heat, and momentum is different at air–lake and air–ground interfaces.

Lake Balaton is a large (594 km^2), shallow (mean depth = 3.3 m, max. depth = 12.2 m) lake in the western part of Hungary. It is a popular tourist attraction, and it exerts a noticeable influence over the microclimate up to 2–15 km downwind of the lake, in terms of temperature, wind characteristics, cloudiness and precipitation (Lóczy 1920, Bartha *et al*. 1986). Concrete effects show great local variation; the only generally accepted universal effect (valid for all points surrounding the lake) is a smoothing of the daily temperature cycle (Lóczy 1920). Lake Balaton has four basins, and the circulation of each is largely independent of the others (Shanahan *et al*. 1986). Therefore, for modeling purposes it can be thought of as four different lakes of different depths (Fig. 1).

Previous studies found that Lake Balaton is polymictic, and has no thermal stratification in the classical sense (Entz 1950). In calm, sunny

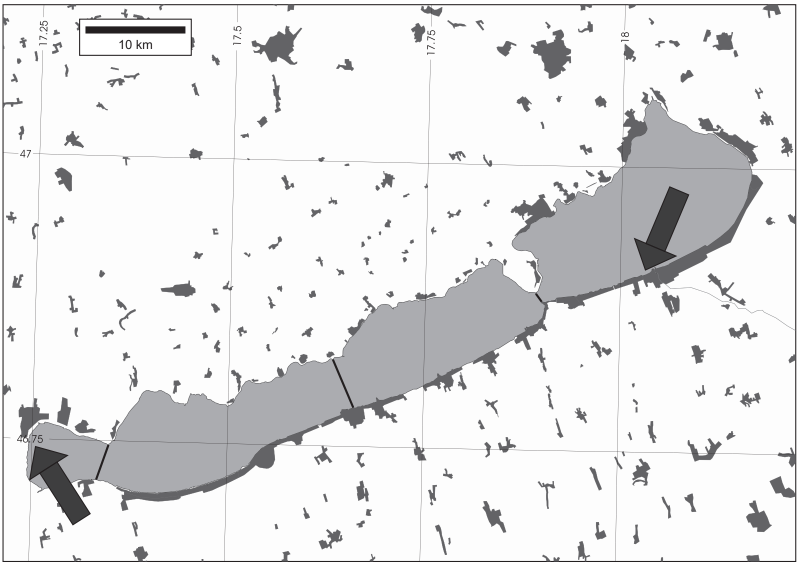

Fig. 1. Map of Lake Balaton, showing the four basins separated by straight lines, and observation sites at Keszthely (left arrow), and Siófok (right arrow).

summer conditions, however, the water surface can occasionally be up to 4.4 °C warmer than the bottom (Entz and Sebestyén 1940). Inverse microstratification of the water temperature can be observed under ice (Entz 1950). A wind of 3 to 4 $m s^{-1}$ is sufficient to mix the lake, break up microstratification and induce sediment resuspension (Entz and Sebestyén 1940, Luettich *et al*. 1990). The diurnal temperature cycle has the same phase at all depths (Endrődi and Kissné 1964). These historical temperature measurements involved a single thermometer used on an extracted water sample, or the instrument itself lowered into the water and read after retrieval, and thus might underestimate the extent of temperature stratification. Their temporal resolution is insufficient for our purposes.

Lake models that predict the physical state of a lake (temperature, ice cover) serve an input for numerical weather forecasts by setting bottom boundary conditions and additionally provide important information to the public. The Freshwater Lake (FLake) parameterization scheme (Mironov 2008) is a computationally efficient lake model developed for NWP and climate modeling. It relies on the assumption of a universal water temperature-depth profile (Mironov 2008: fig. 1). Owing to its availability, low computational cost and promising predictive ability (Dutra *et al*. 2006), the model could satisfy the needs of both the operative forecasting system of the Hungarian Meteorological Service (HMS) and academic research.

FLake is a part of the SURFEX surface scheme, used in the AROME NWP model (Ducrocq *et al*. 2005), that is considered a potential successor to the ALADIN (Aire Limite Adaptation dynamique Developpement InterNational) model (Horányi *et al*. 1996), at kilometric resolutions. Plans to use SURFEX within ALADIN are under way.

Albedo is an important parameter of the FLake model. Previous studies (Weingartner 1968, Dávid *et al*. 1974) found the albedo of Lake Balaton to vary, mainly between 8% and 12%, throughout the year. For this study we used a mean value of 10%.

The ALADIN model, used to provide the atmospheric forcing for our investigations, is a limited area version of the IFS/ARPEGE global model (Déqué *et al*. 1994) developed by the ALADIN (http://www.cnrm.meteo.fr/aladin/) and RC LACE (www.rclace.eu) NWP consortia, and is the operationally used short-range weather forecasting model of the HMS. The model is run at 8.5 km horizontal resolution, with 300-s timestep, outputs are created every 15 min for up to 48 hours, four times a day. In its current setup of ALADIN, the temperature of lakes is kept fixed during the whole time of the atmospheric simulation, and initialized by interpolating the sea surface temperatures of the nearest sea gridpoints.

In order to test whether FLake could adequately simulate the behavior of Lake Balaton, we needed more detailed temperature/depth profiles to check the assumptions of the model, and simulated time series to allow comparison with observations.

Material and methods

We used two independent sets of data, described below.

Seasonal monitoring station in Keszthely

The Water Research Team of the Hungarian Academy of Sciences operates an automatic monitoring station between April and October in the westernmost part (Fig. 1) of Lake Balaton (Istvánovics *et al*. 2004). The station is situated 100 m offshore, and operates as a component of the GLEON (Global Lake Ecological Observatory Network) community. In this study, we used simultaneous water temperature readings at (2 to 7) annually varying depths of the water and the bottom sediment (1.6 m water depth), using multiple stationary and a single floating thermometer, and additionally solar radiation, 2-m air temperature, and 10-m wind. Readings represent 15-min averages. The floating thermometer was shielded from direct solar radiation by the float itself. Owing to the high turbidity of the lake we judged it unnecessary to apply shielding to the others. We obtained supplementary humidity data from a nearby (2600 m) inland station of the HMS (WMO ID: 12 920), and long-wave radiation values from the six hourly ECMWF analyses and +3 h forecasts from the nearest gridpoint.

Permanent weather station in Siófok

Water temperature readings were taken by the HMS at its onshore weather station in Siófok (WMO ID: 12 935) (Fig. 1). A thermometer was placed at 1-m depth, 20 cm from the bottom, 20 m from the shore. Readings represent 10-min averages.

Occurrence of thermal stratification

To examine the occurrence and type of thermal stratification, we turned to the data from Keszthely.

We looked at the temperature difference between the surface water (the floating thermometer on the surface) and the fixed thermometer at the bottom. Between the spring of 2003 and the autumn of 2006 there were 56 699 readings when both thermometers registered valid values. The mean temperature difference was 0.47 °C, with a median 0.3 °C, minimum –1.4 °C, and maximum 5.7 °C. The probability density and distribution functions of the temperature difference (Fig. 2) show that the temperature difference was below or at –1 °C in 0.25%, over 1 °C in 20%, 1.6 °C in 8.8%, and over 2 °C in 6% of the readings. Negative temperature differences appeared mostly during cooling periods: at night, early spring, and late autumn. Inverse stratification is expected to be commoner in winter and especially under ice, but we have no data for these conditions.

Given the average depth of the lake at the Keszthely site, a 1.6 °C temperature difference would mean a 1 °C m^{-1} temperature gradient indicative of the thermocline (Jellison and Melack 1993). For the purposes in this study we considered the water column stratified if the temperature difference between its top and bottom exceeded 1.6 °C. We found 732 cases of stratification (Fig. 3), 342 (46 %) of which lasted for one hour or less (at most seven readings), 186 (25%) for at least six hours, 77 (11%) for at least 12 hours, 20 (2.7%) for at least one day, 11 (1.5%) for two days, and one for more than a week. The mean duration of the stratified state was 5.1 hours (computed from one-hour groups).

This brings us to the conclusion that Lake Balaton in spring, summer and autumn shows direct water temperature stratification in a non-negligible percentage of cases, with the surface water being warmer than the bottom of the lake. Most of these stratified situations are short-lived; however, in calm situations stratification can be preserved for more than a day, or even a week. Short term (1–12 hours) stratification seems to be owed to rapid daytime heating under calm conditions, with the effect also seen in computer

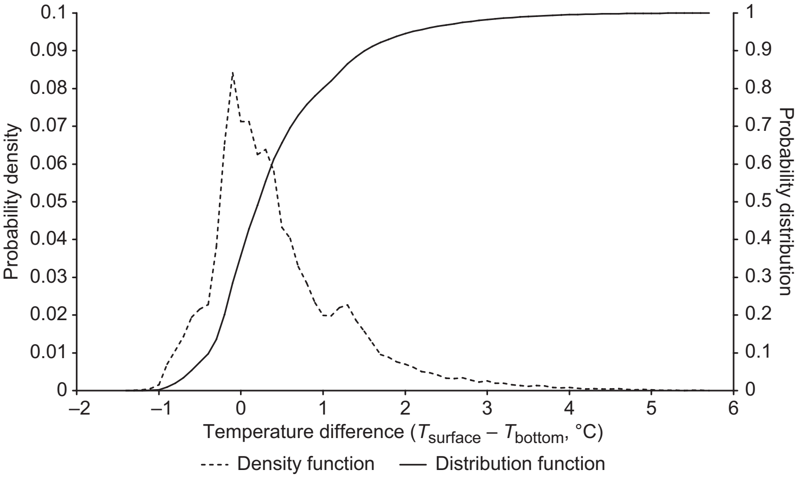

Fig. 2. Probability density and distribution of the temperature difference between the lake surface and the lake bottom at Keszthely for 2003–2006.

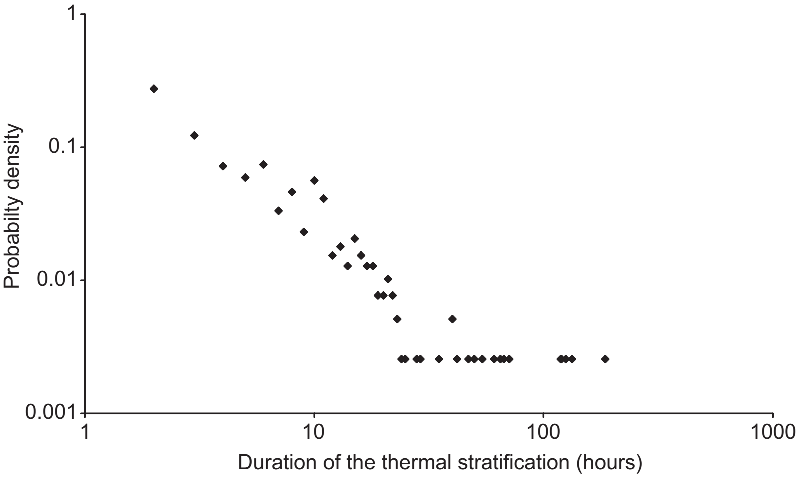

Fig. 3. Probability density of the duration of thermal stratification situations at Keszthely for 2003–2006 — only the cases lasting more than one hour are considered.

animations of the temporal evolution of temperature profiles.

The vertical spacing of the thermometers turned out to be too large to allow us to determine the shape of the temperature–depth profile, or to decide if it fits the universal shape required by FLake. As stratification is generally weak, however, the exact shape of the profile is unlikely to affect the performance of the model.

Results

We carried out two kinds of computer simulations to test the performance of FLake for Lake Balaton. We ran the simulations using a standalone, off-line version of FLake created from the freely available FLake library (http://www.flake.igb-berlin.de/) and a simple, FORTRAN interface. The following input data are required to run FLake: long-wave radiation, short-wave radiation, 2-m air temperature, 2-m humidity (dewpoint temperature), 10-m wind speed.

We initialized the experiments starting from a uniform, non-stratified state of the lake. In the first experiment, atmospheric forcing was derived from measurements at Keszthely. In the second one, we used data from Siófok for initialization and forcing from the forecast fields of the ALADIN model.

We ran the baseline experiments using the default settings of the library, set the locally observed water depths, and disabled the thermally active sediment module. This is the most rudimentary setup of FLake.

In both experiments, we examined the sen-

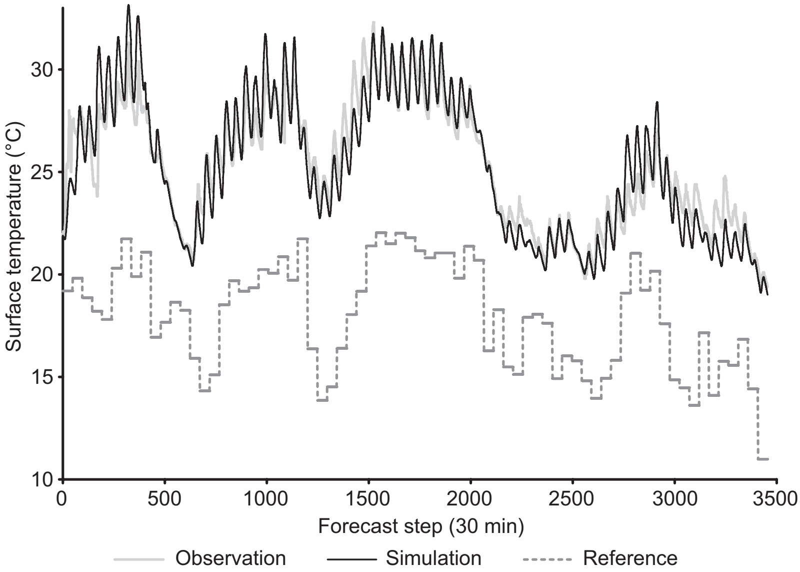

Fig. 4. Observation-driven FLake experiment: forecast and observed lake surface temperatures for Keszthely 21 June 2006–31 August 2006. Reference is the surface temperature of the 00 UTC ARPEGE analysis of each day kept constant for 24 hours.

sitivity of the system to the modeled lake depth and modeled thermally active sediment depth, as we expected the strongest response from these two parameters. Other parameters (optical properties, wind fetch, etc.) and uncertainties of the forcing may also, however, play a role. Thus the optimal values of the sensitivity parameters should be treated with caution.

Pure observation-driven simulation

In this experiment, we studied the behavior of Lake Balaton in the 72-day interval between 21 June 2006 and 31 August 2006. We used the measured atmospheric values complemented by the ECMWF long-wave radiation estimate as atmospheric forcing, and applied 30-min timesteps.

Plotting the time series of the observed and modeled lake surface temperatures (Fig. 4) shows an excellent fit (Table 1). As a reference, we compared the results with the current operational setup in which lake temperatures are prescribed from the driving ARPEGE model (as interpolated values from nearby sea points). We used the 00 UTC analysis of ARPEGE throughout all the days as an estimate for both the surface and the bottom water temperatures (Table 1).

The time series of the observed and modeled temperature errors (Fig. 5) indicate that the rudimentary setup of the model is able to predict the surface temperatures better than bottom temperatures, with both predictions surpassing the reference. The modeled bottom temperature changed much slower than the observed values, resulting in an excessively high temperature difference between the surface and bottom layers:

Table 1. Observation-driven FLake experiment: errors of the forecasted lake surface (mixed layer) and bottom temperatures (°C) for Keszthely 6 June 2006–31 August 2006. We used the ARPEGE surface temperature as an estimate of both the bottom and surface temperatures.

	Model surf./ obs. surface	Model bottom/ obs. bottom	Reference/ obs. surface	Reference/ obs. bottom
Mean error (ME)	–0.05	–2.08	–7.34	–6.61
Mean absolute error (MAE)	0.86	2.21	7.34	6.61
Root mean square error (RMSE)	1.14	2.98	7.62	6.87

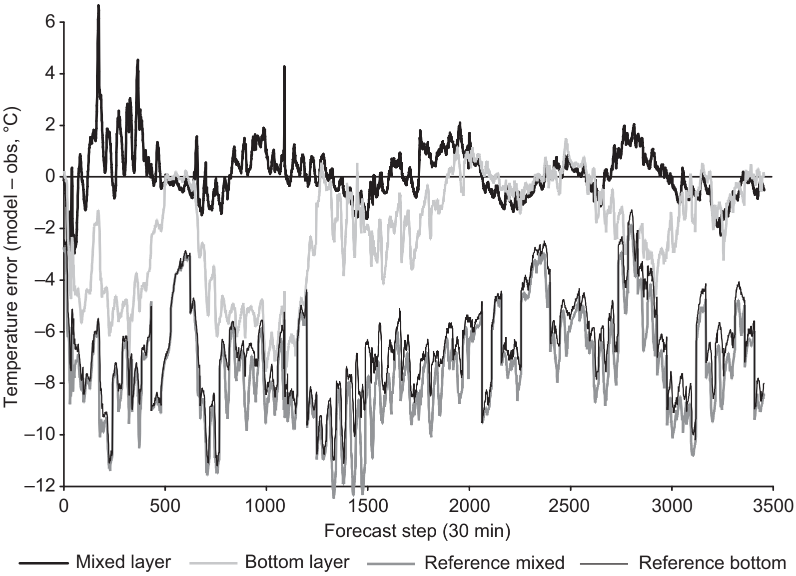

Fig. 5. Observation-driven FLake experiment: time evolution of the error of the forecast lake surface (mixed layer) and bottom temperatures for Keszthely 6 June 2006–31 August 2006. We used the ARPEGE surface temperature as an estimate of both the bottom and surface temperatures. Mixed layer error should be compared with the Mixed reference and Bottom layer error with Bottom reference.

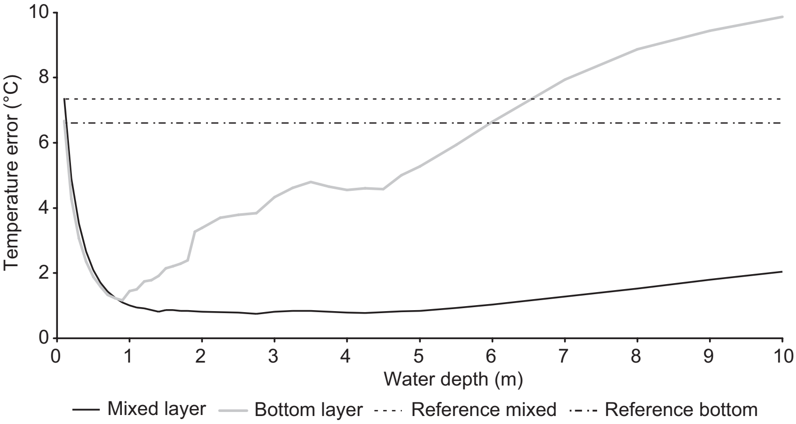

Fig. 6. Observation-driven FLake experiment: mean absolute error of mixed layer and bottom layer temperature forecasts as a function of modeled water depth. Reference is the constant temperature approximation from the ARPEGE NWP model.

model (mean difference ($T_{\text{surface}} - T_{\text{bottom}}$) = 2.76 °C, median = 2.07 °C, min = 0 °C, max. = 10.61 °C), observation (mean difference = 0.74 °C, median = 0.5 °C, min = –0.3 °C, max. = 4.2 °C). In the model the mean modeled mixed layer height was 1.2 m (of the 1.6 m total water depth), and the lake was fully mixed in 942 (27%) of the total 3456 time steps. When not fully mixed, the model thermocline had a mean gradient of (4.31 °C m^{-1}).

We performed a sensitivity check on the modeled lake depth by running the experiment with a range of water depths from 0.1 to 10 m, and comparing the mean absolute model errors (Fig. 6). We found the quality of surface water temperature to be rather insensitive to lake depth between 1 and 6 m. Within these bounds virtually no difference can be seen, but outside this range the forecast deteriorates quickly. The bottom water temperature error is much more sensitive to the modeled water depth, and has a sharp optimum at 0.9 m. Below this value the error of the surface and bottom temperatures run together — as the modeled lake is fully mixed. For greater depths the water column is no longer thoroughly mixed, and the bottom temperature forecast becomes unreliable.

We also tested the model performance by enabling the thermally active sediment module, and running a series of simulations trying to find optimal values for lake depth and thermally active bottom sediment depth. We ran the model for all pairs of water and sediment depths: 22 values of water depth between 0.25 m and 5.5 m, and 12 values of bottom sediment depth between 0.01 m and 100 m. We found that 20-m sediment depth and 1.7-m water depth yielded the lowest surface temperature MAE = 0.71 °C, at the cost of increased bottom temperature MAE = 2.78 °C. It seems that the requirements of minimizing surface and bottom temperature errors are in conflict, and the reason is the underestimation of mixing in Lake Balaton. We could not resolve this conflict by tuning the wind fetch parameter.

The simplest setup of the FLake was therefore able to model the surface temperature of Lake Balaton accurately. The evolution of bottom water temperature and therefore the stratification properties of the lake were not captured well by the model. Although some improvement could be reached by tuning various model parameters, on the basis of these experiences we find it more consistent and practical to stay with the default settings and not to use the active sediment module.

Atmospheric model-driven simulation

In the second experiment, we studied the behavior of the model on a shorter timescale. We initialized the model from the water temperature measurements in Siófok. Lake characteristics were the same as during the first experiment. For atmospheric forcing, we used the operational ALADIN forecasts. We ran 35 forecasts up to +48 hours with 15-min time steps, starting from 00 UTC every day between 16 October 2008 and 19 November 2008. We considered the reference system to be the actual measured water temperatures at the starting time kept constant throughout the model run.

As measurements were taken close to the bottom of the lake and also the shore, we consider the measured values more representative of the mean water temperature than of an ideal 1-m depth. Thus we compared the simulated mean lake water temperature with measurements taken at 1-m depth.

Since in this case we dealt with a series of forecasts, instead of a single model run, we had to define the initial conditions for all runs. We used two different approaches, the re-initialization (Reinit), and the cycling (Cycle) method. In the Reinit method, we initialized the sediment and water temperatures in each forecast to the observed value, which resulted in a uniform temperature profile. By means of this method, the individual forecasts were independent of one another. In the Cycle method, only the first forecast was initialized with the observed temperature value, and all others continued the previous forecast (from its +24 h state). Cycle forecasts are not independent, and have no feedback from observations, and thus have a greater theoretical chance of developing a significant bias. This method mimics the use of a data assimilation suite, albeit one without measurements.

Time series of the forecast errors of Reinit (Fig. 7) model runs show that modeled water temperatures remain bounded by the extremes of the constant temperature reference, and Cycle (Fig. 8) runs return to the same bounds within 12 hours. The mean error of the forecasts shows smaller diurnal variations than the mean error of the reference (Table 2) in both methods. Whereas performance scores of the Cycle method are slightly poorer than those of the Reinit method, the former is able to operate without feedback from observations.

Using both methods, we examined the sensitivity of the model to the modeled water depth by repeating the above experiment using different lake depths. The mean error and mean absolute error of the forecast (Fig. 9) present a mixed image. It is clear as expected that the Reinit method shows less sensitivity with regard to the model parameters. It also shows minimum bias at 1.6–1.7 m, and minimum mean absolute error in the 2.0–3.0 m depth interval, outperforming the reference at all modeled depths above 1.0–1.2 m. The Cycle method has a minimum bias at the actual 1.2–1.4 m lake depth, which remains better than the reference in the 1.2–1.6 m range; its mean absolute error is lowest between 1.8 and 2.0 m, but it never surpasses the reference mean absolute error, even though it comes close.

We examined the sensitivity of the daily temperature cycle on the modeled water depth by taking the mean error of the runs at every forecast step. If the model correctly predicts the diurnal temperature cycle the mean forecast error should be the same for all time-steps, and so we compared the standard deviation of the mean

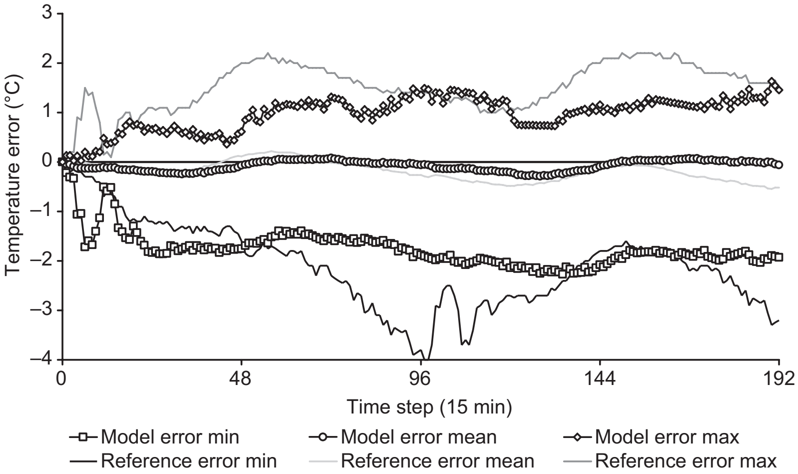

Fig. 7. ALADIN atmospheric model-driven experiment: the error spread of forecast mean lake temperatures for Siófok from 35 consecutive forecasts, as a function of the model time-step (15 min). Reinit method: all forecasts are initialized from the observed lake temperature. Model error mean, Model error min, and Model error max are the mean and extremes of the errors of the modeled lake temperature. Reference mean, Reference min. and Reference max. are the mean and the extremes of the errors of the constant temperature reference.

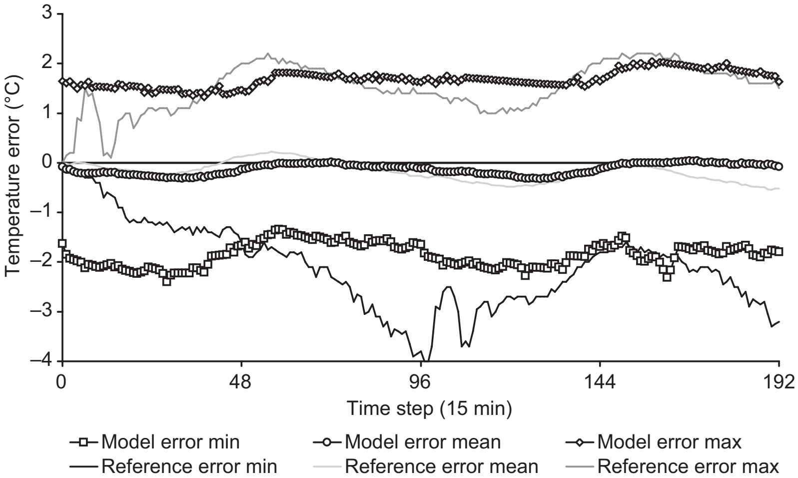

Fig. 8. ALADIN atmospheric model-driven experiment: the error spread of forecast mean lake temperatures for Siófok from 35 consecutive forecasts, as a function of the model time-step (15 min). Cycle method: the first forecast is initialized from the observed lake temperature, all others from the previous forecast. Model error mean, Model error min, and Model error max are the mean and extremes of the errors of the modeled lake temperature. Reference mean, Reference min. and Reference max. are the mean and the extremes of the errors of the constant temperature reference.

forecast error for different water depths (Fig 10). Again the Reinit method has a lower sensitivity, showing minimum standard deviation of the mean error, and thus best fit between 1.6–2.0 m. The Cycle method provides the most fitting diurnal cycle at 2.0 m modeled water depth.

Finally we turned on the thermally active sediment module. We were able to find sets of model parameters (water depth = 2.6 m, sediment depth = 0.2 m) that gave results superior to the ones presented above (smaller ME and MAE at the same time, not shown here), but found the 0.2 m sediment depth unconvincingly small.

We can conclude that the model, with its default settings and locally measured lake depth, provides a satisfactory performance even com-

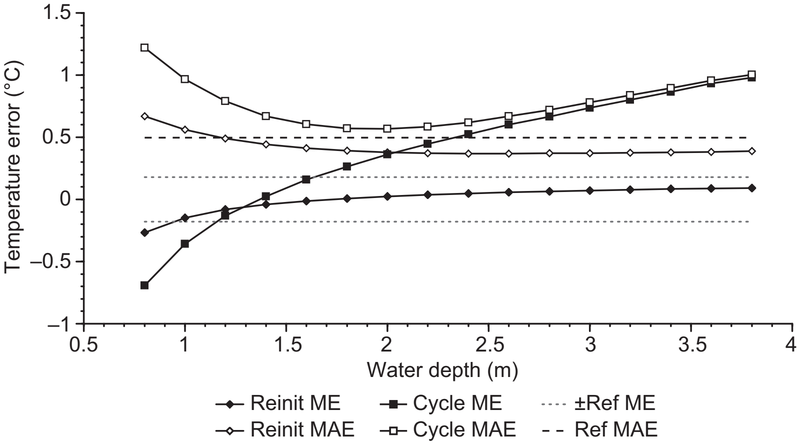

Fig. 9. ALADIN atmospheric model-driven experiment: mean error and mean absolute error of the temperature forecasts for Siófok from 35 consecutive forecasts as a function of the modeled water depth. Reference is the constant temperature approximation. The terms Reinit and Cycle are explained in captions of Fig. 7 and Fig. 8, respectively.

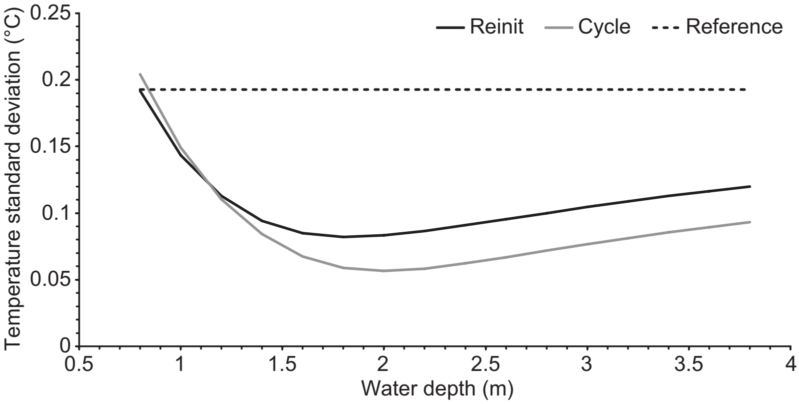

Fig. 10. ALADIN atmospheric model-driven experiment: standard deviation of the mean forecast error of the temperature forecasts for Siófok from 35 consecutive forecasts as a function of the modeled water depth. Reference is the constant temperature approximation. Lower standard deviation indicates a better captured diurnal temperature cycle. The terms Reinit and Cycle are explained in captions of Fig. 7 and Fig. 8, respectively.

pared with a really strict reference. By optimizing the model parameters we could produce better temperature forecasts, but either the gain was too small to justify deviating from the observed parameter values, or the optimal parameters were not realistic.

Summary

We investigated the thermal stratification of Lake Balaton using measurements at Keszthely. Pronounced and enduring stratification happens rarely, in line with previous findings, but microstratification is relatively common.

Table 2. ALADIN atmospheric model-driven experiment: mean error, mean absolute error and root mean square error of the temperature forecasts for Siófok from 35 consecutive forecasts. Reference is the constant temperature approximation. Reinit: all forecasts are initialized from the observed lake temperature; Cycle: the first forecast is initialized from the observed lake temperature, all others from the previous forecast.

	Reinit (°C)	Cycle (°C)	Reference (°C)
ME	–0.08	–0.13	–0.18
MAE	0.49	0.79	0.50
RMSE	0.65	0.89	0.76

The vertical resolution of our observations is insufficient to decide how well the assumed temperature profile in FLake matches that of Lake Balaton, but as the typical temperature difference between the water surface and bottom is small, the exact shape of the modeled profile seems irrelevant for NWP applications.

We ran computer simulations using the standalone version of the FLake lake model to examine its ability to forecast the thermal behavior of the lake.

Both pure observed and modeled forcing resulted in promising results; the model is able to outperform the constant temperature approximation, similar to the one currently used in the operational ALADIN model in Hungary.

Observation-driven experiments indicate that, at times, FLake underestimates the depth of the mixed layer of Lake Balaton to some extent; in other words, it overestimates the occurrence and extent of stratification.

The temperature error of the modeled lake remained within the same bounds when the model was initialized from observations at the beginning of each run and when it was initialized from the previous forecast. This indicates that the FLake model is able to capture the important processes of the lake well enough to be used in a data assimilation cycle, even without any observations.

On the basis of these off-line experiments, we consider the FLake parameterization to be a good candidate to provide an improved description of water surface temperature evolution, which is expected to improve the representation of surface-air interactions in NWP applications. We can proceed to on-line experiments, learning more about the interaction of FLake and the atmospheric model.

Acknowledgments: We are very grateful for the help of László Kullmann, András Horányi, Ekaterina Kurzeneva, Rui Salgado, and Patick Le Moigne.

References

Bartha I., Böjti B. & Horváth Á. 1986. A balatoni viharjelzések optimalizálása a jelzőrendszer korszerűsítése után. *Időjárás* 90: 371–380.

Dávid A., Major Gy. & Takács O. 1974. Hazai állóvizeink albedójának vizsgálata. *Időjárás* 78: 46–50.

Déqué M., Dreveton M.C., Braun A. & Cariolle D. 1994. The ARPEGE/IFS atmosphere model: a contribution to the French community climate modelling. *Climate Dynamics* 10: 249–266.

Ducrocq V., Bouttier F., Malardel S., Montmerle T. & Seity Y. 2005. The AROME project. *Houille Blanche* 2: 39–43. [In French with English abstract].

Dutra E., Stepanenko V., Miranda P.A., Viterbo P., Mironov D. & Lykosov V.N. 2006. *Evaporation and seasonal temperature changes in lakes of the Iberian Peninsula*. Available at http://nwpi.krc.karelia.ru/flake/papers/dutraetal_sevilha2006.pdf

Endrődi G. & Kissné T.E. 1964. Water temperatures and thermal balances of Lake Balaton. *Időjárás* 68: 226–231.

Entz B. 1950. Some physical and chemical conditions of the water of Lake Balaton, investigated from September 1948 to April 1949 (temperature, transparency, dissolved oxygen, pH and organic substances). *Archiva Biologica Hungarica* XIX: 69–81.

Entz B. & Sebestyén O. 1940. A Balaton élete. *A Magyar Biológiai Kutatóintézet munkái* XII: 2–22.

Horányi A., Ihász I. & Radnóti G. 1996. ARPEGE/ALADIN, a numerical weather prediction model for central Europe with the participation of the HMS. *Időjárás* 100: 277–301.

Istvánovics V., Osztoics A. & Honti M. 2004. Dynamics and ecological significance of daily internal load of phosphorus in shallow Lake Balaton, Hungary. *Freshwater Biology* 49: 232–252.

Jellison R. & Melack J.M. 1993. Meromixis in hypersaline Mono Lake, California. 1. Stratification and vertical mixing during the onset, persistence, and breakdown of meromixis. *Limnology and Oceanography* 38: 1008–1019.

Lóczy L. 1920. *A Balaton földrajzi és társadalmi állapotainak leírása*. M. Kir. Udvari Könyvnyomdája.

Luettich R.A., Harleman D.R.F. & Somlyódy L. 1990. Dynamic behavior of suspended sediment concentrations in a shallow lake perturbed by episodic wind events. *Limnology and Oceanography* 35: 1050–1067.

Mironov D.V. 2008. Parameterization of lakes in numerical weather prediction. Description of a lake model. *COSMO Technical Report* 11: 41.

Shanahan P., Harleman D.R.F. & Somlyódy L. 1986. Wind-induced water motion. In: Somlyódy L. & van Straten G. (eds.), *Modeling and managing shallow lake eutrophication*, Springer-Verlag, Berlin, pp. 204–255.

Weingartner F. 1968. Az albedó területi eloszlása Magyarországon. *Beszámolók az 1968-ban végzett tudományos kutatásokról* XXXV: 296–306.