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NUMERICAL MODELLING OF NON-UNIFORM CORROSION INDUCED CONCRETE CRACK WIDTH

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ABSTRACT

Corrosion of reinforced concrete is one of the major deterioration mechanisms which result in premature failure of the reinforced concrete structures. Crack width is often used as an effective criterion to assess the serviceability of concrete structures. However, research on prediction of corrosion-induced concrete crack width, especially by considering the corrosion as a non-uniform process, has still been scarce. This paper attempts to develop a finite element model to predict the crack width for corrosion-affected concrete structures under realistic non-uniform corrosion of the reinforcement. A non-uniform corrosion model was first formulated as a function of time. To simulate arbitrary cracking in concrete, cohesive elements are inserted in the sufficiently fine mesh which is achieved through a script written in Python. The surface crack width is obtained as a function of service time and verification against experimental results from literature is conducted. Accurate prediction of crack width can allow timely maintenance which prolongs the service life of the reinforced concrete structures.

Keywords: corrosion, crack width, cohesive crack model, reinforced concrete, numerical modelling.

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1 INTRODUCTION

Reinforced concrete (RC) structures have been the most common type of structures used in the civil engineering construction since middle nineteenth century. RC structures have been widely used for buildings, bridges, retaining walls, tunnels, and indeed any physical infrastructure built on and under the ground. Since 1970s, it has become an accepted knowledge that the concrete cover has its limitation on protecting the reinforcing steel from corrosion. As a result, a series of research has been initiated on improving the understanding of the corrosion of steel in concrete (Faustino, et al., 2014, Faustino, et al., 2017, Wilkins and Lawrence, 1980), such as the Concrete in the Oceans research programme in the UK in the 1970s. Furthermore, it appears to be inevitable that RC structures will suffer from reinforcement corrosion in chloride (Cl^-) and carbon dioxide (CO_2) laden environment. Practical experience and experimental observations (Andrade, et al., 1993, Li, 2003, Li, 2005, Otsuki, et al., 2000) suggest that corrosion affected RC structures deteriorate faster in terms of serviceability (e.g., cracking or deflection) than safety (e.g., strength). Therefore, there is a well justified need for a thorough investigation of the cracking process and crack width of concrete, not least bearing in mind that crack width is one of the most important practical parameters for the design and assessment of RC structures.

To model cracking of concrete, some researchers have resorted to analytical approach, mainly due to the accuracy of the solution and the convenience of its practical application (Bhargava, et al., 2006, Li and Yang, 2011, Pantazopoulou and Papoulia, 2001, Yang, et al., 2017). For example, Li and Yang (2011) developed an analytical model for concrete crack width caused by reinforcement corrosion and applied load, by introducing a stiffness reduction factor to account for the post-cracking quasi-brittle behaviour of concrete. The stiffness reduction factor then modifies the differential equation for obtaining the cracked stress and strain components.

Correlations between material corrosion and the structural effects can then be established, e.g., crack width (Li and Yang, 2011), time to surface cracking (Bhargava, et al., 2006), etc. However, the application of analytical modelling in crack propagation in concrete is limited to some special cases, e.g., particular boundary conditions, and the assumption that the crack is smeared and uniformly distributed in the damaged solid to satisfy the requirement on continuous displacement. Some studies have employed complex functions to formulate the stress development under arbitrary boundary conditions (Ning, et al., 2012, Yang, et al., 2015), i.e., non-uniform corrosion rust distribution; however, they have been limited to elastic solution only.

In light of the limitation of analytical modelling on crack propagation in concrete, numerical modelling has brought considerable advantages. Depending on the specific application and the scale of the problem, different numerical techniques may be used, e.g., finite element method (FEM) (Hanson and Ingraffea, 2003, Roesler, et al., 2007), discrete element method (DEM) (Beckmann, et al., 2012), boundary element method (BEM) (Aliabadi and Saleh, 2002, Chahrour and Ohtsu, 1992), particle meshfree method (Rabczuk and Belytschko, 2006, Rabczuk and Zi, 2006, Rabczuk and Belytschko, 2007, Rabczuk, et al., 2007, Rabczuk, et al., 2008, Rabczuk, et al., 2008, Rabczuk, et al., 2010) and peridynamics (Gerstle, et al., 2007, Huang, et al., 2015). Amongst these numerical methods, FEM has received the most research interest in solving corrosion-induced reinforced concrete cracking problems. Amongst these numerical models available in literature, most focuses on uniform corrosion. However, Yuan and Ji (2009) conducted accelerated corrosion test by using an artificial environmental chamber on RC specimens and confirmed that the corrosion rust distribution along the reinforcing bar was non-uniform. According to their results, the half of the reinforcing bar in the cover side was corroded and the distribution of the corrosion rust was in a semi-elliptical shape. It is also evident that the localized corrosion can be equivalent to about four to eight times that of the reinforcement under overall corrosion (González, et al., 1995).

Recently, some researchers have started to model the non-uniform corrosion caused cracking in concrete. For example, Jang and Oh (2010) considered a few of non-uniform distributions of the corrosion rust and calculated the stresses in concrete induced by corrosion expansion. Pan and Lu (2012) modelled the crack propagation in concrete in FEM under non-uniform corrosion expansion and assumed concrete as a heterogeneous material. Moreover, Du et al. (2014) established a damage plasticity model to simulate the non-uniform corrosion induced concrete cracking. A number of parameters were investigated on their effects to the surface cracking. Zhao et al. (2011) proposed a Gaussian distribution for the non-uniform corrosion caused displacement and simulated the cracking behaviour of concrete by smeared crack model. The literature review suggests (1) the non-uniform corrosion models derived so far have seldom been based on experimental test results but more on hypothesis and; (2) no model has been developed in modelling discrete crack propagation and calculating the surface crack width, caused by non-uniform corrosion. Crack width is an important parameter regarding the durability of concrete structures while it is still not quite clear how those underlying factors, e.g., corrosion rate, material/mechanical properties of concrete, may quantitatively affect the development of crack width of the concrete. Experimental results will be very useful to help and understand the degradation mechanism; however, existing experimental data are rather limited with regards to the full understanding of the problem. Therefore, it is well justified that a numerical model be developed to predict the non-uniform corrosion induced concrete crack width after corrosion initiation.

This paper attempts to develop a numerical method to simulate the discrete crack propagation in concrete and predict the surface crack width under the non-uniform corrosion of reinforcement. A non-uniform corrosion model is first formulated as a function of time, based on experimental results. In order to model the arbitrary discrete cracking of concrete, cohesive crack elements are inserted in the sufficiently fine mesh, through an in-house script written in Python. Discussion on choices of the cohesive parameters is presented since they are the key to establishing a rational model with cohesive elements. After the formulation of the model, an example is presented to demonstrate its application. The developed model is then verified by comparing the results with those from experiments. Moreover, a parametric study is carried out to investigate the effects of some basic parameters, e.g., corrosion rate, elastic modulus of concrete, tensile strength of concrete, shape of softening curve, on the development of surface crack width after corrosion initiation.

2 CONSTITUTIVE MODEL FOR CRACKS

The failure of structures is significantly influenced by the properties of the material used. Concrete is considered as a quasi-brittle material, in which the tensile stress gradually decreases after it reaches its peak value while the tensile strain continues to increase. Such the stiffness degradation behaviour is called strain softening. The concept of strain softening evolves from plasticity where the post-peak decline of the tensile stress is considered as a gradual decrease of the tensile strength, i.e., softening. Concrete exhibits the strain softening behaviour is because there is an inelastic zone developed ahead of its crack tip, often referred to as fracture process zone (FPZ), as shown in Figure 1-a. When a crack propagates in concrete, the cracked surfaces may be in contact and are tortuous in nature (Mindess and Diamond, 1982), due to various toughening mechanisms such as aggregate bridging, void formation or microcrack shielding (Shah, et al., 1995). Therefore, the cracked surfaces may still be able to sustain the tensile stress which is characterized by the softening degradation curve.

To model the discrete cracks in concrete, cohesive crack model was first developed by Hillerborg et al. (Hillerborg, et al., 1976). In general, the constitutive relation for the cracks can be described as follows:

$$\sigma = f_{T-S}(\delta) \tag{1}$$

where σ is the local opening stress of the crack, δ is the total displacement of the crack and f_{T-S} is a nonlinear function defining the traction-separation relationship. To prevent mesh sensitivity in FE analysis, the stress-displacement relation is used rather than stress-strain relation. As the distance between the nodes is used as a crack measure rather than a change in strain (which depends on the element length), the mesh dependency is significantly reduced.

The function f_{T-S} defines the complete mechanical behaviour of a cohesive crack, i.e., linear elasticity and nonlinear fracture property. It is schematically shown in Figure 1-b. The tensile stress linearly increases until its maximum value, i.e., tensile strength $f_t^{'}$; such a linearity is determined by a penalty stiffness K_p. After reaching the peak value, the tensile stress decreases, following certain strain softening rules, e.g., linear, bi-linear, exponential, etc. The area underneath the f_{T-S} curve is known as the fracture energy G_f.

The constitutive relation of the cohesive crack is often divided into elasticity and fracture, respectively. The elastic property of a crack is characterised by K_p and $f_t^{'}$, whilst the fracture property of the crack is defined by G_f and the softening shape. As such, the crack opening displacement w, neglecting the elastic displacement, can be used to characterise the softening relation only. The constitutive softening relation, expressed as a function of the crack mouth opening displacement, can be expressed as follows:

$$\sigma = f(w) = f_t^{'} \exp\left(-\frac{f_t^{'}}{G_f} w \right) \tag{2}$$

Equation (2) is based on the exponential softening and related to the real material properties, i.e., the tensile strength $f_t^{'}$ and the fracture energy G_f.

The key to developing a rational model of cohesive cracking is the choices of the parameters, i.e., the penalty stiffness K_p, the tensile strength $f_t^{'}$, the fracture energy G_f and the shape of the softening curve. The concept of penalty stiffness evolves from the elastic stiffness which is obtained by dividing the elastic modulus of the concrete by its thickness. Since cohesive interface is normally very thin or even of zero thickness, the elastic stiffness of the cohesive interface approaches infinitesimally large. This makes sense as the interface should be stiff enough prior to the initiation of crack to hold the two surfaces of the bulk concrete together, leading to the same performance as that of no interface existing. This also meets the general condition of cohesive crack model which assumes that the energy required to create the new surfaces is vanishingly small compared to that required to separate them (Bažant and Planas, 1998). However, the elastic stiffness cannot be too large as it will cause convergence problems due to ill-conditioning of the numerical solver of the FE programmes (ACI Committee 446, 1997). Therefore, the cohesive stiffness becomes a "penalty" parameter, which controls how easily the cohesive interface deforms elastically. As such this stiffness is large enough to provide the same or close response of intact concrete prior to cracking, but not so large as to cause numerical problems.

The tensile strength $f_t^{'}$ of concrete is used as an important index to determine if a cohesive crack is initiated. For Mode I fracture, once the tensile stress at any point of a structure reaches its tensile strength, a crack is initiated and the material of that point starts to degrade. The fracture energy G_f is the energy absorbed per unit area of crack with the unit of N/mm or N/m usually. It can be regarded as the external energy supply required to create and fully break a unit surface area of the cohesive crack. It may be mentioned that there is an argument as if G_f is a material constant, i.e., independent of boundary condition, etc. G_f is used in the energy balance criterion which controls the stable crack propagation. Further, the shape of softening curve, as illustrated in Figure 1-b, is arguably important. Some researchers believe the shape of the softening is less important than the fracture energy (Elices, et al., 2002). However, there are a range of shapes developed in the last two decades, e.g., linear, bilinear and exponential curves, etc.

3 EMBEDED COHESIVE ELEMENTS

To model the arbitrary cracking in concrete, the cohesive elements are embedded in the mesh. The insertion process of cohesive elements can be shown in Figure 2. Figure 2(a) presents the initial mesh generated, which consists of triangle elements only. First, all individual nodes are replaced by certain number of new nodes at the same location. The number of newly created nodes depends on the number of the elements connecting to the original node. For instance, if a node is connected by 4 triangle elements, the node is replaced by 4 newly created nodes; if a node is connected by 8 triangle elements, the node is replaced by 8 newly created nodes. Subsequently, the 4 nodes at the interface between two triangle elements are identified and linked to form a cohesive element. In this way, all the interfaces between the triangle elements are inserted of cohesive elements. The cohesive elements are shown in red in Figure 2. This insertion process was conducted by a script written in Python.

The cohesive elements generated in this way are of zero thickness in geometry. The two nodes of a cohesive element, in the thickness direction, share the same coordinates before loading. The constitutive/calculation thickness of the cohesive elements, however, is not zero. In practice, it is usually set as 1.0 for the convenience of transformation between strain and displacement. After the insertion of cohesive elements, the structure can simulate discrete cracking behaviour. As long as the mesh is sufficiently fine, the embedded cohesive elements form a sensible and reasonable network of potential cracking paths.

The 4-node cohesive element generated from the insertion process has two stress components, i.e., the normal stress and the shear stress, as shown in Figure 2(b). Unlike other solid elements, there are only two stress components for the cohesive elements because the thickness is zero. The cohesive element undergoes linear elastic behaviour prior to the peak load followed by the initiation and evolution of damage, i.e., cracking. The elasticity for the whole cohesive element, in terms of the nominal stresses and nominal displacements, is defined as follows :

$$\sigma = \begin{Bmatrix} \sigma_1 \\ \sigma_2 \end{Bmatrix} = \begin{bmatrix} K_n & 0 \\ 0 & K_s \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} \tag{3}$$

where σ_1 and σ_2 are the normal stress and shear stress respectively, K_n and K_s are the stiffness that relate to the normal stress and shear stress, and δ_1 and δ_2 are the corresponding displacements.

4 NON-UNIFORM CORROSION MODEL

Concrete with an embedded bar subjected to an internal pressure at the interface between the bar and concrete can be modelled as a thick-wall cylinder (Bazant, 1979, Pantazopoulou and Papoulia, 2001, Tepfers, 1979). As shown in Figure 3, D is the diameter of the bar and d_0 is the thickness of the annular layer of concrete pores at the interface between the bar and concrete, often referred to as interfacial transition zone (ITZ). Usually d_0 is constant once concrete has hardened. The inner and outer radii of the cylinder are considered $a = D/2 + d_0$ and $b = C + D/2 + d_0$.

It is known that the corrosion products of reinforcement usually occupy a few times more space than the original steel. The accumulation process of the corrosion products starts from filling in the ITZ, with its thickness d_0, but normally do not produce stresses in concrete. There might be arguments that at this stage stress can be induced; but for simplicity, it is assumed stress-free in this paper. As the corrosion products proceed further in concrete, a band of corrosion products forms, as shown in Figure 3(a). Due to the fact that the chlorides, as well as moisture and oxygen, reach the reinforcement surface at different rates through different sides of the concrete structure, it is very rare to have uniform corrosion around the reinforcement. This is particularly the case for offshore RC structures as the surface facing the ocean waves has much faster ingression rate and hence more severe corrosion on this side. It has been found that the front of corrosion products for the half of rebar facing concrete cover is in a semi-elliptical shape, while corrosion of the opposite half of rebar is negligibly small and can be neglected (Yuan and Ji, 2009). Yuan and Ji (2009), amongst the very limited experimental results, is used as the basis of formulating the time-dependent non-uniform corrosion.

As illustrated in Figure 3(b), there may be three bands accommodating the corrosion products: the semi-elliptical band of corroded steel with maximum thickness d_{co-st}, the porous circular band d_0 and the semi-elliptical rust band with maximum thickness d_m. The front of the corrosion is in a semi-elliptical shape with the semi-major axis equal to $D/2+d_0+d_m$ and the semi-minor axis equal to $D/2+d_0$.

Based on the geometry, the total amount of corrosion products $W_{rust}(t)$ can be related to these band thicknesses, shown as follows:

$$\frac{2W_{rust}(t)}{\pi}\left(\frac{1}{\rho_{rust}}-\frac{\alpha_{rust}}{\rho_{st}}\right)=Dd_0+d_0^2+\frac{D}{2}d_m+d_0d_m \tag{4}$$

where ρ_{rust} is the density of corrosion products, ρ_{st} is the density of steel and α_{rust} is the molecular weight of steel divided by the molecular weight of corrosion products. It varies from 0.523 to 0.622 according to different types of corrosion products (Liu and Weyers, 1998).

By neglecting the small quantities d_0d_m and d_0^2, d_m can be derived as follows:

$$d_m(t)=\frac{4W_{rust}}{\pi D}\left(\frac{1}{\rho_{rust}}-\frac{\alpha_{rust}}{\rho_{st}}\right)-2d_0 \tag{5}$$

$d_m(t)$ in Equation (4) is the maximum corrosion-induced expansion along the interface to the concrete cylinder under which the stress will be initiated in the cylinder. $d_m(t)$ determines the shape of the semi-ellipse which is the boundary condition of the concrete cylinder in deriving stresses and strains in concrete.

In Equation (5), $W_{rust}(t)$ is related to the corrosion rate of the steel rebar and can be expressed as follows (Liu and Weyers, 1998):

$$W_{rust}(t)=\sqrt{2\int_{0}^{t}0.105(1/\alpha_{rust})\pi D i_{corr}(t)dt} \tag{6}$$

where i_{corr} is the corrosion current density in $\mu A/cm^2$, which is widely used as a measure of corrosion rate.

To determine the displacement boundary condition of the concrete cylinder, the function of the semi-ellipse of the corrosion front needs to be derived. It is known that, in rectangular coordinate system, the function for an ellipse can be expressed as follows:

$$\frac{y^2}{\left(\frac{D}{2}+d_0+d_m\right)^2}+\frac{x^2}{\left(\frac{D}{2}+d_0\right)^2}=1 \tag{7}$$

Equation (7) can be transformed in a polar coordinate system as follows,

$$r=\frac{(D+2d_0+2d_m)(D+2d_0)}{\sqrt{(2D+4d_0)^2+16d_m(D+2d_0+d_m)\cos^2\theta}} \tag{8}$$

The displacement boundary condition of the concrete cylinder $\delta(\theta,t)$ can be derived as follows:

$$\delta(\theta,t)=\frac{\left[D+2d_0+\frac{8W_{rust}(t)}{\pi D}\left(\frac{1}{\rho_{rust}}-\frac{\alpha_{rust}}{\rho_{st}}\right)-4d_0\right](D+2d_0)}{\sqrt{(2D+4d_0)^2+32\left[\frac{2W_{rust}(t)}{\pi D}\left(\frac{1}{\rho_{rust}}-\frac{\alpha_{rust}}{\rho_{st}}\right)-d_0\right]\left[D+2d_0+\frac{4W_{rust}(t)}{\pi D}\left(\frac{1}{\rho_{rust}}-\frac{\alpha_{rust}}{\rho_{st}}\right)-2d_0\right]\cos^2\theta}}-\frac{D}{2}-d_0 \tag{9}$$

where $0\le\theta\le\pi$.

5 FINITE ELEMENT SIMULATION

For concrete with embedded reinforcing steel bar, it is widely accepted to be modelled as a thick-wall cylinder (Pantazopoulou and Papoulia, 2001, Tepfers, 1979). Due to the symmetry of the loading and the structure, only half of the cylinder is modelled. Two elements are employed in this study, i.e., 4-node cohesive elements at all interfaces between the triangle elements, and 3-node plane strain element for the bulk concrete. Reduced integration is used for the plane strain element. As a result, the damage evolution of the cohesive element is combined with the elastic deformation of the bulk concrete in the overall response. Very fine mesh is initially generated for the sake of inserting sufficient number of cohesive elements. There are 26,784 solid triangle elements plus 40,011 cohesive elements inserted, for half of the cylinder.

Crack initiation marks the beginning of degradation or damage of concrete at a point. Crack is assumed to initiate when the maximum nominal tensile stress reaches the tensile strength of the concrete for the Mode I fracture – opening mode, expressed as follows,

$$\langle \sigma_1 \rangle = f_t^{'} \tag{10}$$

where $\langle \sigma_1 \rangle = \begin{cases} \sigma_1 & for \quad \sigma_1 > 0 \\ 0 & for \quad \sigma_1 < 0 \end{cases}$

The operation $\langle \sigma_1 \rangle$ is to ensure that a crack will not initiate under compression.

After cracking is initiated, the cohesive element is damaged and the normal stress of this element softens in a manner as defined. The failure of the element is governed by the softening curve. To calculate the residual stress after its peak stress, a damage parameter D is introduced into the stress calculation as follows:

$$\sigma = (1 - D)\sigma_u \tag{11a}$$

$$\sigma_u = K_p \delta \tag{11b}$$

where σ_u is the undamaged stress as shown in Figure 4.

Convergence is usually a problem in the execution of FE programmes for materials exhibiting softening behaviour under implicit scheme as in most FE programmes. Sudden dissipation of energy will make the computation more dynamical. An artificial viscosity is therefore used to overcome the convergence difficulties by making the stiffness matrix of the material positive. This viscosity regularizes the constitutive relation of the cohesive element by modifying the stiffness reduction variable D as follows,

$$\dot{D}_v = \frac{D - D_v}{\mu} \tag{12}$$

where μ is the viscosity parameter which can be specified in the property of cohesive element and D_v is the viscous stiffness degradation variable. Once μ and D are known, D_v can be determined. A small viscosity value μ helps improve the rate of convergence without compromising results.

6 WORKED EXAMPLE

As a demonstration of the application of the developed numerical method and techniques in solving non-uniform corrosion induced concrete cracking, an example is carried out. The values for all the basic parameters are shown in Table 1, together with their sources. The boundary condition to the concrete cylinder caused by corrosion is first calculated from Equation (9). Figure 5 shows the displacement $d_m(t)$, a key parameter controlling the whole inner boundary condition, during the first 10 years' service life. It can be seen that the corrosion does not cause any displacement of concrete, until about 0.2 year. During this initial period, the corrosion products fill the band of the ITZ, of which the process is assumed stress-free. According to the inputs in Table 1, this process takes 0.19 year; afterwards, the corrosion will cause deformation of concrete and hence stresses. The displacement d_m steadily increases over the service time and reaches about 0.33mm after 10 years. Once d_m is known, the whole inner boundary condition of the concrete cylinder is determined.

The mesh grid of numerical example is shown as Figure 6. Under the time-varying expansion caused by corrosion, the concrete cylinder undergoes a short period of elasticity, followed by cracking initiation and propagation. Figure 7 shows the stress plot (i.e., the maximum principal stress) for the concrete cylinder at 6%, 13%, 21%, and 75% of the displacement at 10 years. From the loading stage up to 21% of the 10-year displacement, the stress concentration can be clearly seen which moves from the inner boundary to the outer boundary of the cylinder. Meanwhile, a discrete crack was initiated at about 10^0 counter-clockwise from the bottom point of the inner circular boundary. This discrete crack propagates outwards as the loading increases and finally reach the concrete cover. Moreover, from the insets of Figure 7, it is clearer to see the crack and the distance between the true crack tip and the stress concentration. Such a distance represents the length of the fracture process zone and will be discussed in detail later.

Other than the principal discrete crack propagating from the inner boundary to the outer cover surface of the concrete, there are also a number of small cracks or damage zones around this principal crack. Figure 8 shows the cohesive elements whose damage parameter *D* is between 0.1 and 1.0. It should be mentioned that the value 0 of *D* represents the no damage and 1.0 represents the state of fully damaged/cracked. These damage zones will never be cracked fully, as there is only one discrete crack propagating. In Figure 8, the black lines represent the partially damaged cohesive elements whose damage value is bigger than 0.1 but smaller than 1.0, i.e., $0.1 < D < 1.0$; the green lines represent the fully cracked cohesive elements, i.e., $D = 1.0$. The fully cracked cohesive elements form a discrete crack propogating from inner boundary to the cover surface, whilst the partically damaged cohesive elements around the discrete crack can be treated as damaged zones around the main crack. Such damaged zones are believed considerably affected by the displacement boundary condition applied at the inner boundary. For example, once the discrete crack is initiated at the inner boundary, there may be some sliding between the concrete cylinder and the steel, according to the frictional property of the rust. In modelling, such a phenomenon has not been considered and the sliding is prohibited. How much this may affect the results needs to be investigated and confirmed in future research.

Once the crack approaches the outer cover surface of the concrete, the crack starts to be visible, known as longitudinal crack, on the surface of concrete structures. Since the theoretical thickness of the cohesive element is set to be 1.0, the strain of the cohesive element is equal to its displacement. Upon measuring the distance between the nodes of the last cohesive element at the outer boundary of the cylinder, the surface crack width can be expressed in a function of time, shown in Figure 9. In Figure 9, it can be seen that the crack propagates to the surface at about 1.20 years when the steel loss ratio is about 0.29% after which the crack width gradually increases with time. The abrupt increase in the crack width corresponds to the sudden energy release when the crack approaches the cover surface. At 10 years, the steel loss ratio is about 0.94% and the crack width reaches about 0.33mm.

7 VERIFICATION

To verify the proposed numerical method, the results are compared with those from (Vidal, et al., 2004). Vidal et al. (Vidal, et al., 2004) have experimentally investigated the relationship between the degree of corrosion and the surface crack width; more importantly, non-uniform pitting corrosion was naturally produced over periods of 14 and 17 years. In their research, the results of the "beam B in concrete tensile zone" were taken, because the corrosion distribution (i.e., Figure 8 in (Vidal, et al., 2004)) of this sample is very similar to that proposed in this paper, i.e., the semi-elliptical shape. The inputs from the test (Vidal, et al., 2004) were used in the numerical simulation, as presented in Table 2. The experimental results of crack width were expressed as a function of the pit penetration; however, it is transformed to crack width versus maximum displacement, d_m, for the sake of comparison with the numerical results. The transformation procedure is presented below before the comparison is provided.

The values of the pit penetration and its corresponding crack width are directly extracted from the Figure 10 in (Vidal, et al., 2004) and $\Delta A_s / A_s$ calculated. To correlate the corrosion extent $\Delta A_s / A_s$ to the corrosion front displacement d_m in the developed model of this research, the loss of the cross section of rebar is first determined as follows:

$$\Delta A_s = \frac{\alpha_{rust} W_{rust}}{\rho_{st}} \tag{13}$$

where W_{rust} is a function of the corrosion rate and can be determined via Equation (6). α_{rust} and ρ_{rust} are material constants and have been defined earlier. The corrosion extent $\Delta A_s / A_s$ can therefore be derived as follows:

$$\frac{\Delta A_s}{A_s} = \frac{\alpha_{rust} W_{rust}}{\rho_{st} A_s} \tag{14}$$

Based on Equation (5), d_m can be obtained as a function of the corrosion extent $\Delta A_s / A_s$, presented as follows:

$$d_m(t) = \frac{4\rho_{st}A_s}{\alpha_{rust}\pi D}\left(\frac{1}{\rho_{rust}} - \frac{\alpha_{rust}}{\rho_{st}}\right)\frac{\Delta A_s}{A_s} - 2d_0 \tag{15}$$

The comparisons of the crack width from the developed numerical model and the experiments are illustrated in Figure 10. It can be seen that the simulated results are in good agreement with the experimental values. It is also interesting to see that the crack width growth is in a roughly linear relationship with the maximum corrosion-caused displacement d_m.

8 ANALYSIS AND DISCUSSION

Under the non-uniform expansion, it would be useful to plot the hoop stress distribution along the inner boundary and the outer boundary of the concrete cylinder. The hoop stress determines the cracking initiation state and the inner and outer boundaries represent the two key locations where internal cracking and surface cracking start. It can be seen from Figure 11, the hoop stress changes along the inner boundary. At loading stage of 6% of the 10-year displacement, the whole inner boundary is in tension in the hoop direction except for a small region around the location of 90^0; such a small region at the bottom of the circular boundary is in bi-axial compression state, although the compression in the hoop direction is small. Moreover, the tensile stress is concentrated around the location of 10^0 of the inner boundary, which indicates the location of the start of the crack. As the expansive load is increased and meanwhile the crack is propagated, the hoop stress (tensile) at the location of 10^0 drops to 0 and the hoop stress at location of 90^0 increases up to about 10MPa (in compression). The top half of the boundary of the concrete cylinder is dominantly in tension in the hoop direction. Clearly, there are two regions to be of interest, i.e., one around the location of 10^0 where the tensile stress is concentrated and the other around the location of 90^0 where the compression is localised.

Figure 12 illustrates the hoop stress distribution along the outer boundary of the concrete cylinder. It is very interesting to see that, under non-uniform corrosion expansion, the outer boundary of the concrete is not all in tension in the hoop direction from the beginning of loading. For example, under the 13% of the 10-year displacement, the area around the 10^0 location is in compression; however, it gradually changes to tension and increases to its peak stress before it drops to 0 which marks the cover surface cracking. The three curves in Figure 12 represent the typical stress states of the outer boundary of the concrete: (1) tension-dominated with a small compression concentration area around the location of 10^0, at minor loading level; (2) all in tension with a strong tension concentration around the location of 10^0 at cracking loading level; and (3) nearly stress-free at post-surface cracking loading level.

Figure 13 shows the hoop stress distribution along the discrete crack path in the 10^0 direction outwards. It represents the tensile stress distribution along the discrete crack. It can be seen that, as the loading increases, the peak stress (i.e., tensile strength) moves along the crack. After the peak stress moves over a point, the stress of that point starts to decrease. Such a phenomenon is called strain softening, as explained earlier. From the point where the tensile stress softens to zero to the point where the tensile stress reaches its maximum value, the region is known as Fracture Process zone (FPZ). In Figure 13, it has been schematically demonstrated the length of the FPZ in this problem is 7.05mm. This FPZ has not been fully developed until the loading of 11.4% of the 10-year displacement; afterwards, the constant length of FPZ will move, ahead of the true crack, towards the cover surface, as shown in Figure 7.

To investigate the sensitivity of some underlying factors, a parametric study is conducted. Figure 14 shows the effect of corrosion rate i_{corr} on the crack width as a function of time. Four corrosion rates have been chosen, i.e., the varying corrosion rate (as presented in Table 1), $i_{corr} = 0.5\mu A/cm^2$, $i_{corr} = 1.0\mu A/cm^2$ and $i_{corr} = 5.0\mu A/cm^2$ (broomfield, 2007). It can be seen that, as the corrosion rate increases, the time to surface cracking is advanced significantly. Further, the crack width under higher corrosion rate is considerably larger than that under lower corrosion rate. It is also interesting to find out that the varying corrosion rate used (e.g., in Table 1) is equivalent to a corrosion rate between $i_{corr} = 1.0\mu A/cm^2$ and $5.0\mu A/cm^2$ but more close to $i_{corr} = 1.0\mu A/cm^2$. Nevertheless, the initial sudden increases in the surface crack width for different corrosion rates are comparable.

Figure 15 shows the effect of Young's modulus of concrete on the development of the surface crack width. Quite a large range of Young's modulus was selected, i.e., from 4GPa to 28GPa. It has been found that the change of Young's modulus of concrete does not change much the long-term increase of the crack width; however, it does affect the time to surface cracking and the initial increase of crack width. It is surprising to see the larger the elastic modulus of the concrete, the earlier the surface is cracked. This is mainly because, under the displacement boundary condition, the larger elastic modulus will cause higher stress and thus earlier cracking. Figure 16 shows the effect of the tensile strength of concrete on the surface crack width. Four curves have been produced regarding the crack width development over time for the tensile strengths of 3MPa to 6MPa. It can be found that, as the tensile strength changes, the crack width does not change significantly, especially after the surface cracking. However, there is some effect on the time to surface cracking; for example, when the tensile strength is increased from 3MPa to 6MPa, the time to surface cracking is delayed from about 1.16 years to 1.27 years (i.e., 0.11 year delay). Therefore, the tensile strength and Young's modulus of concrete have very little effect on long-term crack width development, which is consistent with experimental results from (Wang and Zheng, 2009, Williamson and Clark, 2000).

There have been arguments as if the shape of the softening curve of the tensile stress-displacement constitutive relation of concrete will affect the fracture behaviour of concrete structures. In this study, we tested three softening curves, i.e., linear, bi-linear and exponential, as shown in Figure 17(a). Although the three curves are different in their softening shapes, the areas underneath the curves are identical, i.e., the fracture energy is the same. $G_f = 50N/m$ and $f_t = 5MPa$ are used for all softening curves. The kink point for the bi-linear curve is determined from Petersson (Petersson, 1981). Accordingly, $\sigma_s = \frac{f_t}{3}$ and $W_s = \frac{4W_0}{9}$. The development of the crack width over time for these three cases is shown in Figure 17(b). It can be seen that these is almost no difference amongst these curves. Therefore, it is proved that the shape of the softening curve is less important than the actual value of fracture energy, in terms of surface crack width development.

Other than material properties, some key numerical parameters are also investigated to determine their sensibility on the results. Figure 18 shows the effects of the viscosity and the penalty stiffness on the surface crack width development. It can be found that, the smaller the viscosity is, the surface crack time is smaller and more simulation steps/time are needed. However the effect of viscosity becomes insignificant when the viscosity is smaller than 1e-4. For viscosity 1e-6, the calculation has convergence problem after about 9 years. After the trial and error analysis, the viscosity in this study is set 1e-5. As discussed, the stiffness of the cohesive interface should be infinitesimally large in theory. For the penalty stiffness of 10GPa, the elastic deformation of cohesive element has a significant effect on the crack width evolution, as shown in Figure 18b. However, when the penalty stiffness is larger than 100GPa, the crack width development is close to each other, although there are still slight differences in the initial crack width increase. The penalty stiffness in this study is set 1,000GPa. The sudden increase of crack width after formation of surface crack is caused by a deformation of cohesive elements around the reinforcing bar.

9 CONCLUSIONS

In this paper, a numerical model has been developed to predict the crack width caused by non-uniform reinforcement corrosion. A non-uniform corrosion model was first formulated as a function of the time. Under the non-uniform expansion to the concrete cover, a fracture model was established to simulate the crack initiation and propagation. In formulating arbitrary cracking, cohesive crack elements were inserted at all the boundaries of the mesh, based on an in-house script written in Python. A worked example was presented to demonstrate the application of the derived model and comparisons with the experimental results from literature were made. It has been found that the numerical results are in good agreement with the experimental results. A comprehensive parametric study was also carried out to investigate the sensitivities of some key parameters mainly on the crack width growth. It has also been found that the discrete crack tends to be initiated at the 10^0 location and the corrosion rate is the most influencing single parameter that controls both the initiation and the long-term development of the crack width; however, the elastic modulus and the tensile strength of concrete only affect the initial crack width and the time to surface cracking but have little effect on the long-term development. Moreover, the shape of the softening curve from the constitutive stress-displacement relation of concrete has found to have nearly no effect on the crack width development. It can be concluded that the numerical method presented in the paper can predict the concrete crack width induced by the realistic non-uniform reinforcement corrosion.

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REFERENCES

ABAQUS 6.13, User documentation, Dessault systems, 2013.

ACI Committee 446 (1997). "Finite Element Analysis of Fracture in Concrete Structures." Farmington Hills, Mich.

Aliabadi, M. H., and Saleh, A. L. (2002). "Fracture mechanics analysis of cracking in plain and reinforced concrete using the boundary element method." *Eng. Fract. Mech.*, 69(2), 267-280.

Andrade, C., Molina, F. J., and Alonso, C. (1993). "Cover Cracking as A Function of Rebar Corrosion: Part 1 - Experiment Test." *Mater. and Struct.*, 26, 453-454.

Bazant, Z. P. (1979). "Physical model for steel corrosion in concrete sea structures - theory." *Journal of the Structural Division-ASCE*, 105(6), 1137-1153.

Bažant, Z. P., and Planas, J. (1998). *Fracture and Size Effect in Concrete and Other Quasibrittle Materials*, CRC Press, Boca Raton, Florida.

Beckmann, B., Schicktanz, K., Reischl, D., and Curbach, M. (2012). "DEM simulation of concrete fracture and crack evolution." *Struct. Concr.*, 13(4), 213-220.

Bhargava, K., Ghosh, A. K., Mori, Y., and Ramanujam, S. (2006). "Model for cover cracking due to rebar corrosion in RC structures." *Eng. Struct.*, 28(8), 1093-1109.

broomfield, J. P. (2007). *Corroision of steel in concrete-understanding, investigation and repair*, Taylor & Francis Abingdon.

Chahrour, A. H., and Ohtsu, M. (1992). "BEM Analysis of Crack Propagation in Concrete Based on Fracture Mechanics." *Boundary Element Methods: Fundamentals and Applications*, S. Kobayashi, and N. Nishimura, eds., Springer Berlin Heidelberg, Berlin, Heidelberg, 59-66.

Du, X., Jin, L., and Zhang, R. (2014). "Modeling the cracking of cover concrete due to non-uniform corrosion of reinforcement." *Corros. Sci.*, 89, 189-202.

Elices, M., Guinea, G. V., Gomez, J., and Planas, J. (2002). "The Cohesive Zone Model: Advantages, Limitations and Challenge." *Eng. Fract. Mech.*, 69, 137-163.

Faustino, P., Brás, A., and Ripper, T. (2014). "Corrosion inhibitors' effect on design service life of RC structures." *Constr. Build. Mater.*, 53, 360-369.

Faustino, P., Brás, A., Gonçalves, F., and Nunes, Â. (2017). "Probabilistic service life of RC structures under carbonation." *Mag. Concr. Res.*, 69(6), 280-291.

Gerstle, W., Sau, N., and Silling, S. (2007). "Peridynamic modeling of concrete structures." *Nucl. Eng. Des.*, 237(12–13), 1250-1258.

González, J. A., Andrade, C., Alonso, C., and Feliu, S. (1995). "Comparison of rates of general corrosion and maximum pitting penetration on concrete embedded steel reinforcement." *Cem. Concr. Res.*, 25(2), 257-264.

Hanson, J. H., and Ingraffea, A. R. (2003). "Using Numerical Simulations to Compare the Fracture Toughness Values for Concrete from the Size-Effect, Two-Parameter and Fictitious Crack Models." *Eng. Fract. Mech.*, 70, 1015-1027.

Hillerborg, A., Modeer, M., and Petersson, P. E. (1976). "Analysis of crack formation and crack growth in concrete by means of fracture mechanics and finite elements." *Cem. Concr. Res.*, 6(6), 773-781.

Huang, D., Lu, G., and Liu, Y. (2015). "Nonlocal Peridynamic Modeling and Simulation on Crack Propagation in Concrete Structures." *Math. Probl. Eng.*, 2015, 11.

Jang, B. S., and Oh, B. H. (2010). "Effects of non-uniform corrosion on the cracking and service life of reinforced concrete structures." *Cem. Concr. Res.*, 40(9), 1441-1450.

Li, C. Q. (2003). "Life-Cycle Modelling of Corrosion-Affected Concrete Structures: Propagation." *J. Struct. Eng.*, 129(6), 753-761.

Li, C. Q. (2005). "Time Dependent Reliability Analysis of the Serviceability of Corrosion Affected Concrete Structures." *Int. J. Mater. and Struct. Reliab.*, 3(2), 105-116.

Li, C. Q., and Yang, S. T. (2011). "Prediction of concrete crack width under combined reinforcement corrosion and applied load." *J. Eng. Mech.-ASCE*, 137(11), 722-731.

Liu, Y., and Weyers, R. E. (1998). "Modelling the time-to-corrosion cracking in chloride contaminated reinforced concrete structures." *ACI Mater. J.*, 95(6), 675-681.

Mindess, S., and Diamond, S. (1982). "The Cracking and Fracture of Mortar." *Mater. Struct.*, 15(86), 107-113.

Ning, X., Qingwen, R., Joe, P., Robert, Y. L., and Anil, P. (2012). "Nonuniform Corrosion-Induced Stresses in Steel-Reinforced Concrete." *Eng. Mech.*, 138(4), 338-346.

Otsuki, N., Miyazato, S., Diola, N. B., and Suzuki, H. (2000). "Influences of Bending Crack and Water-Cement Ratio on Chloride-Induced Corrosion of Main Reinforcing Bars and Stirrups." *ACI Mater. J.*, 97(4), 454-465.

Pan, T., and Lu, Y. (2012). "Stochastic Modeling of Reinforced Concrete Cracking due to Nonuniform Corrosion: FEM-Based Cross-Scale Analysis."

Pantazopoulou, S. J., and Papoulia, K. D. (2001). "Modeling cover cracking due to reinforcement corrosion in RC structures." *J. Eng. Mech.-ASCE*, 127(4), 342-351.

Petersson, P.-E. (1981). *Crack growth and development of fracture zones in plain concrete and similar materials*, Division, Inst.

Rabczuk, T., and Belytschko, T. (2006). "Application of Particle Methods to Static Fracture of Reinforced Concrete Structures." *Int. J. Fract.*, 137(1-4), 19-49.

Rabczuk, T., and Zi, G. (2006). "A Meshfree Method based on the Local Partition of Unity for Cohesive Cracks." *Comput. Mech.*, 39(6), 743-760.

Rabczuk, T., and Belytschko, T. (2007). "A three-dimensional large deformation meshfree method for arbitrary evolving cracks." *Comput. Method. Appl. M.*, 196(29-30), 2777-2799.

Rabczuk, T., Bordas, S., and Zi, G. (2007). "A three-dimensional meshfree method for continuous multiple-crack initiation, propagation and junction in statics and dynamics." *Comput. Mech.*, 40(3), 473-495.

Rabczuk, T., Zi, G., Gerstenberger, A., and Wall, W. A. (2008). "A new crack tip element for the phantom‐node method with arbitrary cohesive cracks." *Int. J. Numer. Meth. Eng.*, 75(5), 577-599.

Rabczuk, T., Zi, G., Bordas, S., and Nguyen-Xuan, H. (2008). "A geometrically non-linear three-dimensional cohesive crack method for reinforced concrete structures." *Eng. Fract. Mech.*, 75(16), 4740-4758.

Rabczuk, T., Zi, G., Bordas, S., and Nguyen-Xuan, H. (2010). "A simple and robust three-dimensional cracking-particle method without enrichment." *Comput. Methods. Appl. Mech. Eng.*, 199(37-40), 2437-2455.

Ren, W., Yang, Z., Sharma, R., Zhang, C., and Withers, P. J. (2015). "Two-dimensional X-ray CT image based meso-scale fracture modelling of concrete." *Eng. Fract. Mech.*, 133, 24-39.

Roesler, J., Paulino, G. H., Park, K., and Gaedicke, C. (2007). "Concrete Fracture Prediction Using Bilinear Softening." *Cem. Concr. Compos.,* 29, 300-312.

Shah, S. P., Swartz, S. E., and Ouyang, C. (1995). *Fracture Mechanics of Concrete: Applications of Fracture Mechanics to Concrete, Rock, and Other Quasi-brittle Materials*, John Wiley & Sons, Inc., New York.

Tepfers, R. (1979). "Cracking of concrete cover along anchored deformed reinforcing bars." *Mag. Concr. Res.*, 31(106), 3-12.

Timoshenko, S. P., and Goodier, J. N. (1970). *Theory of Elasticity*, Singapore.

Vidal, T., Castel, A., and François, R. (2004). "Analyzing crack width to predict corrosion in reinforced concrete." *Cem. Concr. Res.*, 34(1), 165-174.

Wang, X., and Zheng, J. (2009). "Experimental study of corrosion-induced crack initiation and propagation of reinforced concrete structures." *J. Dalian Univ. Tech.* 49(2), 246-252 (In Chinese).

Wilkins, N. J. M., and Lawrence, P. F. (1980). "Concrete in the Oceans: Fundamental Mechanics of Corrosion of Steel Reinforcements in Concrete Immersed in Sea Water."Slough, UK.

Williamson, S. J., and Clark, L. A. (2000). "Pressure required to cause cover cracking of concrete due to reinforcement corrosion." *Mag. Concr. Res.*, 52(6), 13.

Yang, S., Li, K., and Li, C.-Q. (2017). "Analytical model for non-uniform corrosion-induced concrete cracking." *Mag. Concr. Res.*, 1-10.

Yang, S. T., Li, K. F., and Li, C. Q. "Non-uniform corrosion-induced reinforced concrete cracking: an anlytical approach." *Proc., 15th International Conference on Civil, Structural and Environmental Engineering Computing*, Civil-Comp Press.

Yuan, Y., and Ji, Y. (2009). "Modeling corroded section configuration of steel bar in concrete structure." *Constr. Build. Mater.*, 23(6), 2461-2466.

Zhao, Y., Karimi, A. R., Wong, H. S., Hu, B., Buenfeld, N. R., and Jin, W. (2011). "Comparison of uniform and non-uniform corrosion induced damage in reinforced concrete based on a Gaussian description of the corrosion layer." *Corros. Sci.*, 53(9), 2803-2814.

LIST OF TABLES

Table 1 Values of basic variables used in the example

Description	Symbol	Values	Sources
Inner radius	a	6mm	(Li, 2003)
Outer radius	b	37mm	(Li, 2003)
Effective modulus of elasticity	E_{ef}	7.05 GPa	(Li, 2003)
Poisson's ratio	ν_c	0.18	(Li, 2003)
Shear modulus	G	$E/[2(1+\nu)]$	(Timoshenko and Goodier, 1970)
Density of rust	ρ_{rust}	3600 kg/m^3	(Liu and Weyers, 1998)
Density of steel	ρ_{st}	7850 kg/m^3	(Liu and Weyers, 1998)
Corrosion current density	i_{corr}	$0.3686 ln(t)+1.1305$ μ A/cm^2	(Li, 2003)
Tensile strength	$f_t^{'}$	5MPa	(Li, 2003)
Fracture energy	G_f	50N/m	(Ren, et al., 2015)
Penalty Stiffness	K_p	100GPa	N/A
Viscosity	μ	1e-5	N/A

Table 2 Values of some variables used for comparison and validation

Description	**Symbol**	**Values**
Inner radius	a	6mm
Outer radius	b	22mm
Modulus of elasticity	E	32 GPa
Creep	c_p	2.0
Effective modulus of elasticity	E_{ef}	$\frac{E}{1+c_p}$
Poisson's ratio	ν_c	0.18
Tensile strength	$f_t^{'}$	4.7MPa
Fracture energy	G_f	50N/m

LIST OF FIGURES

(a) softening curves used as inputs; (b) simulated crack widths as a function of time for different softening curves

18. Comparisons of crack width evolution for different values of viscosity and penalty stiffness

(a) Effect of viscosity; (b) Effect of penalty stiffness