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# Stochastic Programming Approach for TEP Optimization Considering RES Integration in Electricity Market

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**Abstract** — Transmission expansion planning (TEP) is helping the system operator to decide the optimal solution for building new lines and in the same time to increase the reliability and safety of the existing power system. The proposed problem is a mixed-integer nonlinear programming problem (MINLP) and it is solved using stochastic programming. Stochastic programming is applied when uncertain environment occurs, in this case the uncertain environment refers to the production of renewable energy sources (RES) and its dependence on the short-term weather conditions. Stochastic Optimization weights all scenarios considered in this paper in order to obtain an expected total cost. The expected total cost includes the cost associated with the construction of new transmission lines, generation cost and load shedding cost.

**Index Terms** -- decision support optimization, electricity market, renewable energy sources, stochastic programming, transmission expansion planning.

## I. NOMENCLATURE

### Indices:

$d$  – demands  
 $h$  – hydro energy generation units  
 $t$  – thermal energy production units  
 $w$  – wind energy production units  
 $l$  – transmission lines  
 $b$  – buses

### Sets:

$d_b$  – demand supplied from bus  $b$   
 $h_b$  – hydro energy production unit located at bus  $b$   
 $t_b$  – thermal energy production unit located at bus  $b$   
 $w_b$  – wind energy production unit located at bus  $b$   
 $l_p$  – transmission lines possible to be build

$r(l)$  – receiving node  
 $s(l)$  – sending node  
 $s$  – complete set of scenarios  
 $\tau$  – analysis horizon

### Parameters:

$\pi_s$  – probability of scenarios  
 $C_l$  – maximum capacity for transmission line  $l$  [MW]  
 $B_l$  – susceptance of transmission line  $l$  [p.u.]  
 $G_h$  – generation capacity of hydro unit  $h$  [MW]  
 $G_t$  – generation capacity of thermal unit  $t$  [MW]  
 $G_w$  – generation capacity of wind farm  $w$  [MW]  
 $L_d^{\tau,s}$  – consumption of demand  $d$  [MW]  
 $c_{LS}^d$  – load shedding cost of demand  $d$  [€/MWh]  
 $c_h$  – cost of production of hydro unit  $h$  [€/MWh]  
 $c_t$  – cost of production of thermal unit  $t$  [€/MWh]  
 $c_w$  – cost of production of wind unit  $w$  [€/MWh]  
 $BI_p^{ib}$  – annualized budget for investing in construction of new transmission lines [Euro]

$CI_p^{im}$  – annualized cost for investing in new transmission lines  $l$  [Euro]

### Binary variables:

$u_l^\tau$  – Binary variable that is 1 if a new transmission line  $l$  is built, and 0 if no new transmission line  $l$  is built, for each time period.

### Variables:

$LS_d^{\tau,s}$  – load shedding at demand  $d$  [MW]  
 $P_t^{\tau,s}$  – power produced by thermo unit  $t$  [MW]  
 $P_w^{\tau,s}$  – power produced by wind unit  $w$  [MW]  
 $P_h^{\tau,s}$  – power produced by hydro unit  $h$  [MW]  
 $PF_l^{\tau,s}$  – power flow through transmission line  $l$  [MW]

$\theta_b^{\tau,s}$  – voltage angle at bus  $b$  [rad]  
 $\gamma$  – time period [hours].

## II. INTRODUCTION

In Romania, there is a pronounced trend of installing renewable energy sources (determined by the green certificates incentives), especially wind and photovoltaic, therefore the need of knowing weather forecast is necessary in exploitation time of wind power systems (WPP) and photovoltaic installations (PV) and obviously when the placement of the new RESs is determined. Although it is hard to estimate the forward amount of energy in time that can be extracted from these sources and errors may appear [1], [2]. In order to integrate the renewable energy sources in the power grid and avoid overloads it is necessary to take into consideration infrastructure reinforcements [3]. The transmission expansion planning (TEP) has the purpose to ensure economic trade and engineering reliability and should provide reliability and safety of the power grid. The engineering reliability objective and economical trade is very important in TEP problem. The decision of which new lines should be built must satisfy forthcoming loads while power quality is maintained. The engineering reliability objective is referring to reliability in supply the demands which should be provide when constructing new lines while economical trade is referring to the minimization of generating costs and load shed costs [4].

In this paper, the expansion transmission planning is applied on a large-scale transmission network for integrating RESs in the existing grid [5], [6]. The final solution of this problem establishes the number of lines, and where new electrical lines should be built so that the power supply and quality are preserved considering four scenarios. The construction of new electrical lines is based on a given investment budget, and analysis of its variation on the optimal solution are considered in this paper [7], [8]. TEP can provide safety in the power grid and should lower the load shed costs. The load shed is increasing if the consumers are not supplied, with the construction of the new lines load shed cost is decreasing with the decrease of the number of the consumers that are not supplied. It is also important to note that the inherent computational difficulty of optimal TEP makes it necessary to limit the size of the candidate list in order to keep the problem tractable. In very large systems, even a reduced list of interesting candidates (as opposed to all feasible candidates) can be excessive for optimization [9].

## III. MATHEMATICAL FORMULATION OF THE PROBLEM

In this paper, it is proposed a mixed-integer nonlinear programming problem that is solved using stochastic programming. Stochastic programming is applied for large-scales models and when uncertain environment occurs, in this case the uncertain environment refers to the production of RESs and its dependence on the short-term weather conditions [1]. The mathematical model for the considered problem is a nonlinear one considering the products between binary variables and continuous variables. The problem is reformulated as a mixed-integer linear programming problem [10].

The problem is formulated under the assumption in which the losses are neglected. It is considered that voltage magnitudes are approximately constant in the system and that voltage angle differences are small enough between two connected nodes. This hypothesis allows the power flow equations to be linearized. In order to linearize the mathematical model, it is possible to replace the nonlinear constraints by introducing exact equivalent mixed-integer linear sets and a large enough constant  $M$  [7]. The TEP problem presented in this article was solved using GAMS (General Algebraic Modeling System) optimization software.

For solving the proposed problem, four scenarios with different probabilities are considered ( $s_1=0,26$ ;  $s_2=0,11$ ;  $s_3=0,34$  and  $s_4=0,29$ ) which refers to the RESs and their dependence on short-term weather conditions [1]. Also, there are considered two different time periods ( $t_1$  and  $t_2$ ) but the investment decision is considered to be made at the beginning of each time period considered. The planning horizon is divided into two different time periods that are considered to be equivalent to two different years and due to this fact the optimal solution of the mathematical model described is a dynamic one. According to the threshold budget, the optimal solution provides the number of new transmission lines that are possible to be built in order to minimize the costs for constructing the new lines, the cost for operating the generating units and the load shed cost [1]. The budget is varying between 10 million euros and 55 million euros, and obviously, the optimal solution is different for each threshold budget considered.

### A. Linearized Mathematical Model

The objective function of the mathematical model presented in this paper is minimizing the total cost of building new electrical transmission lines, which contains the operating cost of production units, investment cost and the load shedding cost.

$$\begin{aligned}
 & [MIN] \sum_{\tau \in I_p} \sum_p CI_p^{im} u_i^{\tau} \\
 & + \gamma \left\{ \sum_s \pi_s \left[ \begin{aligned} & \left[ \sum_t c_t P_t^{\tau,s} + \sum_d c_{LS}^d LS_d^{\tau,s} \right] + \\ & \left[ \sum_w c_w P_w^{\tau,s} + \sum_d c_{LS}^d LS_d^{\tau,s} \right] + \\ & \left[ \sum_h c_h P_h^{\tau,s} + \sum_d c_{LS}^d LS_d^{\tau,s} \right] \end{aligned} \right] \right\} \quad (1)
 \end{aligned}$$

Subjected to constrains:

$$\begin{aligned}
 & \sum_{l \in I_p} CI_p^{im} u_l^{\tau} \leq BI_p^{ib}, \forall s \in S, \forall \tau \\
 & u_l^{\tau} = \{0,1\}, \forall s \in S
 \end{aligned} \quad (2)$$

In equation (2) the cost associated with the construction of the new lines in each scenario must be smaller than the threshold budget.

$$\begin{aligned} & \sum_{t \in I_b} P_t^{\tau,s} + \sum_{h \in h_b} P_h^{\tau,s} + \sum_{w \in W_b} P_w^{\tau,s} - \sum_{l \in S(l)=n} PF_l^{\tau,s} + \\ & + \sum_{l \in r(l)=n} PF_l^{\tau,s} = \sum_{d \in d_n} (L_d^{\tau,s} - LS_d^{\tau,s}), \forall s \in S, \forall \tau \end{aligned} \quad (3)$$

In equation (3) is presented the balance between consumption and production and the power flow.

DC power flow is defined by the susceptance of the transmission line and is multiplied by the subtraction between voltage angles at the receiving points and sending points for each scenario.

$$PF_l^{\tau,s} = B_l (\theta_{s(l)}^{\tau,s} - \theta_{r(l)}^{\tau,s}), \forall l \in l_p, \forall s \in S, \forall \tau \quad (4)$$

DC power flow is limited by the capacity of the transmission lines in any scenario as the following:

$$-C_l \leq PF_l^{\tau,s} \leq C_l, \forall l \in l_p, \forall s \in S, \forall \tau \quad (5)$$

$$-u_l^r C_l \leq PF_l^{\tau,s} \leq u_l^r C_l, \forall l \in l_p, \forall s \in S, \forall \tau \quad (6)$$

$$\begin{aligned} & -(1 - u_l^r) M \leq PF_l^{\tau,s} - B_l (\theta_{s(l)}^{\tau,s} - \theta_{r(l)}^{\tau,s}) \leq \\ & \leq (1 - u_l^r) M, \forall l \in l_p, \forall s \in S, \forall \tau \end{aligned} \quad (7)$$

The production of each generator cannot be higher the capacity of the generators:

$$0 \leq P_t^{\tau,s} \leq G_t, \forall t, \forall s \in S, \forall \tau \quad (8)$$

$$0 \leq P_h^{\tau,s} \leq G_h, \forall t, \forall s \in S, \forall \tau \quad (9)$$

$$0 \leq P_w^{\tau,s} \leq G_w, \forall t, \forall s \in S, \forall \tau \quad (10)$$

Load shedding of demand d cannot be higher than the consumption of demand d.

$$0 \leq LS_d^{\tau,s} \leq L_d^{\tau,s}, \forall t, \forall s \in S, \forall \tau \quad (11)$$

Voltage angle must be between  $-\pi$  and  $\pi$ .

$$-\pi \leq \theta_b^{\tau,s} \leq \pi, \forall t, \forall s \in S, \forall \tau \quad (12)$$

Voltage angle for reference node (node 5):

$$\theta_b^{\tau,s} = 0, \text{ reference node} \quad (13)$$

### B. Real Test Network

The stochastic programming method is applied on a real test network from south-east of Romania, in Dobrogea area, where are connected many RESs. The topology of the considered power system it is shown in Fig.1. This grid contains 18 generating units (10 thermal generating units, 7 wind generating units and 1 hydro generating unit), 29 existing lines and 16 demands. The new transmission lines that are possible to be built are  $l_6, l_{11}, l_{13}, l_{19}$  and  $l_{26}$  and their number depends on the threshold budget, the growth of the demand and energy production which is increasing in every year.

The energy production is steadily increasing from the first to the second time period, and continues to grow with each scenario considered. The demands are constant in each scenario but they are increasing from the first year to the second. The number of the new transmission lines that are possible to be build is varying in each year. The considered threshold budget in this paper is varying between 10 million euros and 55 million euros. In the first time period the threshold budget is 30 million euros and in the second time period it is 25 million euros.

The data for thermal generating unit and hydro generating unit is not varying and it is taken from [8]. The data for demands which is considered in the two choose time periods it is shown in Fig. 2. The consumption is increasing in each year. The data for wind generating unit which is considered in first and the second scenario is presented in Fig. 3. The data for wind generating unit which is considered in third and the fourth scenario is presented in Fig. 4. It can be seen that the production is increases from the first scenario to the last one.

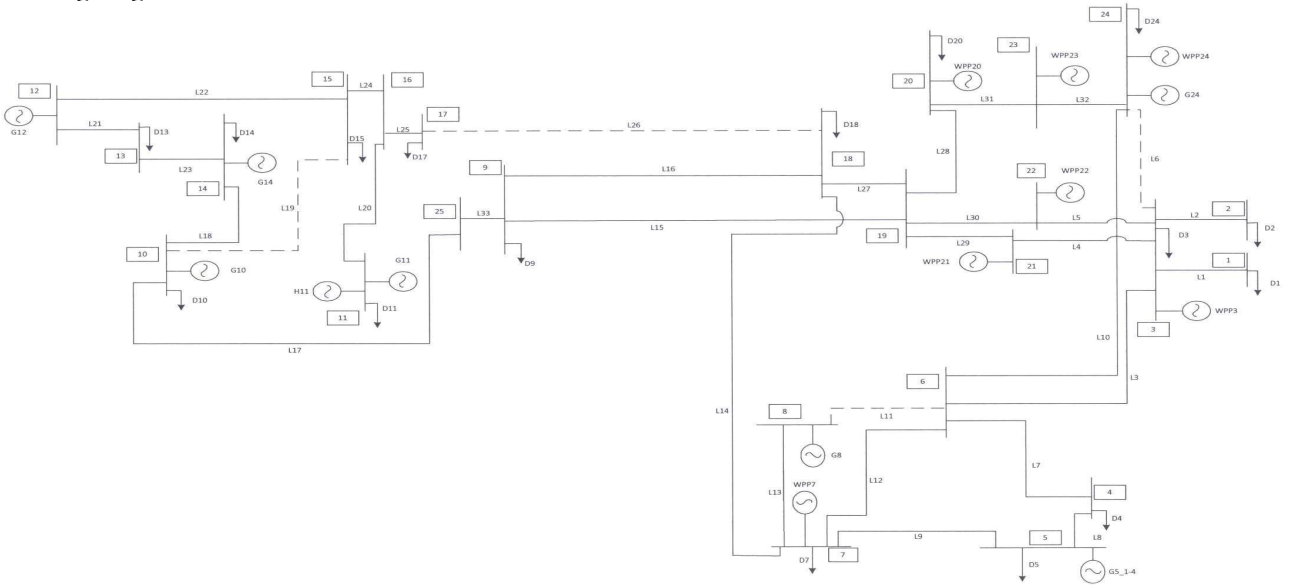


Figure 1. Considered transmission network

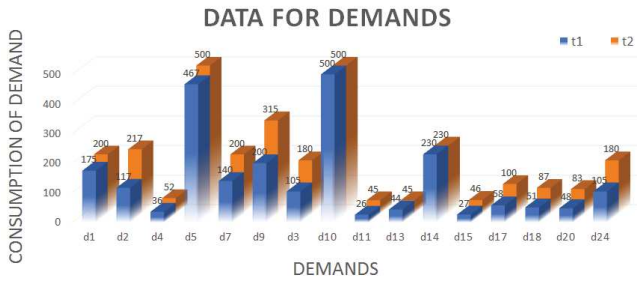


Figure 2. Data for demands in  $t_1$  and  $t_2$

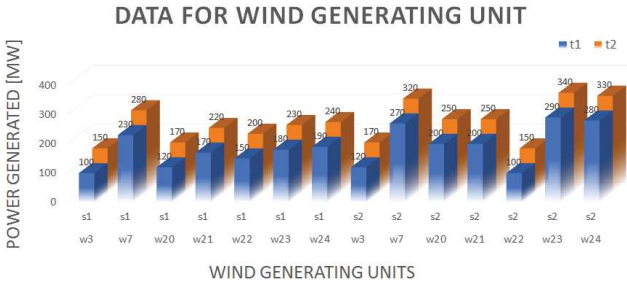


Figure 3. Data for wind generating unit in  $s_1$  and  $s_2$

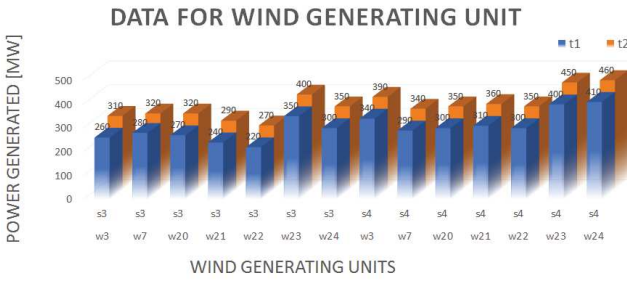


Figure 4. Data for wind generating unit in  $s_3$  and  $s_4$

#### IV. CASE STUDY RESULTS

The case study refers to TEP problem applied to Dobrogea system where are installed a lot of RESs. It is necessary to take into consideration the interconnection of RESs into the existing power system in order to avoid the overloads [11]. After solving the proposed mathematical model, the system operator should be able to decide which is the best reinforcement of the existing power system in order to maintain safety and reliability of the power system.

In Fig. 5 and Fig. 6 it can be seen that different number of transmission lines are built depending on the threshold budget and on the time periods. The threshold budget is varying between 10 million euros and 55 million euros taking into account both time periods. It is considered that the investment budget is varying between 10 million euros and 30 million euros in the first time period and between 10 million euros and 25 million euros on the second time period.

In Fig. 5 it can be seen that for the first time period, with the investment budget of 10 million euros only line  $l_{19}$  is possible to be built and with the investment budget of 30 million euros three lines are possible to be built  $l_{13}$ ,  $l_{19}$  and  $l_{26}$ . In the second time period it is considered that all three lines

$l_{13}$ ,  $l_{19}$  and  $l_{26}$  are built in the order to obtain fewer consumers shed that leads to a lower value of the objective function. Therefore, the decisions that are made in the second time period are made following the construction of the three required lines from the first time period ( $l_{13}$ ,  $l_{19}$  and  $l_{26}$ ) thus only  $l_6$  and  $l_{11}$  can be built in this time period. For the second time period, the maximum investment budget is 35 million euros and only two new transmission lines are possible to be built ( $l_6$  and  $l_{11}$ ). In Fig. 6, it can be seen that for an investment budget of 10 million euros, only line  $l_6$  it is possible to be built and with the investment budget of 25 million euros  $l_6$  and  $l_{11}$  are possible to be built.

Different new transmission lines and different number of new transmission line will be built according to the threshold budget.

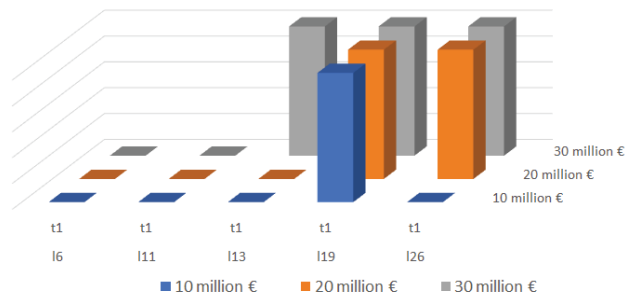


Figure 5. The construction of the new transmission lines in the first time period

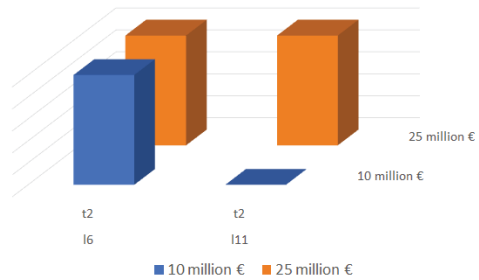


Figure 6. The construction of the new transmission lines in the first time period

TABLE I. CASE STUDY RESULTS FOR THE FIRST TIME PERIOD

I [mill. €]	Lines to be build [0/1]					Z [mill. €]
	L6	L11	L13	L19	L26	
10	0	0	1	0	0	58
20	0	0	1	1	0	52
30	0	0	1	1	1	51

In both tables, Table I and Table II, the value of the objective function is decreasing while the investment budget is increasing.

The number of the new lines that are required to be built is increasing also due to the increase of the investment budget that allows the construction of new transmission lines in each time period considered.



TABLE II. CASE STUDY RESULTS FOR THE SECOND TIME PERIOD

I [mill. €]	Lines to be built [0/1]		Z [mill. €]
	L6	L11	
0	0	0	97
10	1	0	88
25	1	1	85

As showed in table II, with no investment budget in the second time period, the value of the objective function is bigger than in the first time period due to the increase of the demand and power generated. It is considered that in this situation the network has undergone no modification regarding the constructions of new lines over the first period of time. This fact is due to the increase of the demand and power generated and it requires another investment budget allocated for the expansion of the existing grid. Fig. 7 and Fig. 8 shows the power generated in both time periods, in the case that the objective function has the minimum value presented in this paper (51 million euros and 85 million euros, respectively). It can be seen that the power produced is increasing in the second time period due to the increase of the demand and the power installed.

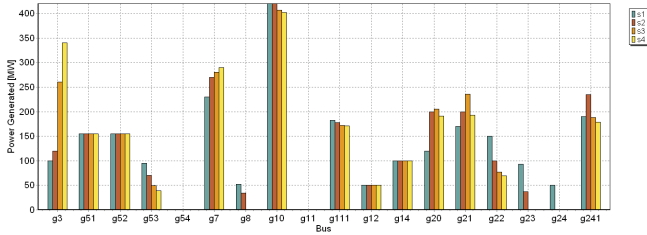


Figure 7. Power generated [MW] in the first time period in each considered scenario ( $S_1, S_2, S_3, S_4$ ).

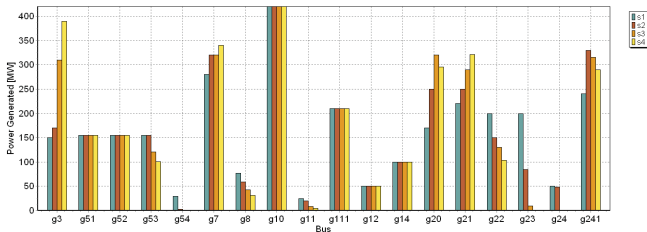


Figure 8. Power generated [MW] in the first time period in each considered scenario ( $S_1, S_2, S_3, S_4$ ).

## V. CONCLUSIONS

In this paper, the mathematical model is a mixed-integer nonlinear programming problem that is reformulated as a mixed-integer linear programming problem and it is solved using stochastic programming. The objective function is minimizing the cost of building new electrical transmission lines, the operating cost of production units, and the load shedding cost. The optimal solution of the resolved mathematical model is different in each year, reinforcements of the existing grid are necessary to be made due to the growth of the consumption and for providing safety and reliability of the grid. In order to avoid overloads, the aging of the infrastructure and the growth of demand. The pronounced

trend of installing RESs is contributing to the necessity of infrastructure reinforcements. The solution of the problem should allow the transmission system operator to make the optimal decision for the reinforcement of the power system in order to maintain the system reliability and satisfy the forthcoming loads along with maintaining the power quality.

The number of new transmission lines that are required to be built is different in the first time period than in the second due to the steady growth of the consumption and the variation of the threshold budget that is between 10 million euros and 55 million euros. Along with the increase of the investment budget, the number of the new transmission lines that are possible to be built is increasing too and the load shed is decreasing for there are less consumers that are not fed.

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