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**AN EFFICIENT TORQUE SPEED CHARACTERISTIC CALCULATION METHOD FOR
BRUSHLESS C.A.D USING OPTIMIZATION TECHNIQUES.**

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Abstract: Nowadays, powerful tools enable to simulate the work of inverter fed machine systems. But, they generally require long calculation times and large memory capacities and many parameters. So, they are not well adapted for an iterative design. In this paper, we present an efficient model, based on optimization techniques, for current fed synchronous machines fed by means of current controlled voltage inverter which leads to very short calculation times to determine torque speed characteristic of such a actuator. The limits of the validity of this model are discussed and simulated results are compared to experimental ones for validation.

Keywords: brushless machine, voltage inverter, current control, torque speed characteristic, C.A.D

I-INTRODUCTION

It is a great advantage of C.A.D procedure of modern brushless actuator to include the ability of doing true "simulated experiments" of the designed actuator to establish the final solution without the need of prototype realization. As an example, it is necessary to determine the winding matching with supply voltage at rated point by a good torque-speed calculation.

Today, powerful tools enables working simulation of such machine-converter system, indeed [1]. But, they generally require long calculation time and large memory capacities and many parameters not well adapted for an iterative design which requires short calculation time not to penalise the designer comfort and efficiency. In this paper, we present an efficient model for current fed peripheral magnets synchronous machines fed by means of current controlled voltage inverter which leads to very short calculation times to determine torque speed characteristic of such an actuator.

So, it can be included in a C.A.D procedure to do an efficient simulated experiments of the designed actuator.

This model is based on the use of simple optimization techniques. After the method description, experimental and simulated results are presented for validation.

II-MODELISATION

The considered system consist in a current fed permanent magnet synchronous machine by means of a current controlled voltage inverter. Then, it is composed of three main devices appearing on figure 1:

- the permanent magnet synchronous machine;
- the current controlled voltage inverter;

-the generation device of current references.

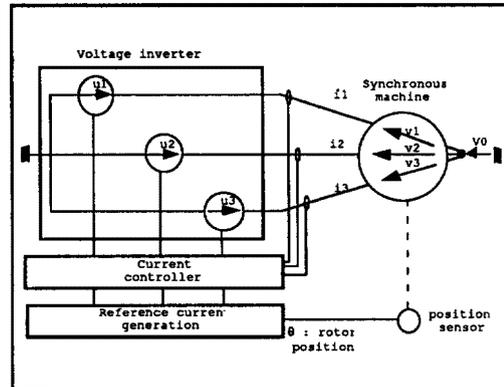


Figure 1: principle scheme of synchronous machine fed by a controlled current voltage inverter

II-1-Machine modelisation

The considered permanent magnet machine are of surface mounted magnet structure types. Indeed, this structure without polar pieces and dampers brings more advantages as simplicity of realization, low cost and high efficiency, which lead to a use in more and more application today, often with rectangular current feeding. This type of machine can be represented by the following equations [2]:

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} L & M & M \\ M & L & M \\ M & M & L \end{pmatrix} \begin{pmatrix} \dot{i}_1 \\ \dot{i}_2 \\ \dot{i}_3 \end{pmatrix} + R \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} + \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}$$

$$T_{em} = \frac{(e_1 i_1 + e_2 i_2 + e_3 i_3)}{\Omega}$$

v_j : the voltage applied to the phase j
 L : self inductance of each phase .
 i_j : the current in the phase j .
 M : mutual inductance between two phases.
 e_j : the e.m.f of the phase j .
 R : resistance of each phase.
 T_{em} : electromagnetic torque
 Ω : mechanical rotation speed.

II-2-Modalisation of the voltage inverter

Considering u_j the "instantaneous average value" of the output voltage across each phase of the permanent magnet synchronous machine, the voltage inverter can be considered as a pure gain so long that u_j remains between two limits $+E$ and $-E$ depending on the inverter type (pwm inverter with linear or hysteresis control, resonant inverter...). For each type of inverter, these limits $+E$ and $-E$ have to be identified as well as the validity area of the model as described in the following.

II-2-1-Resonant inverters. The dynamical behaviour of the output voltage of a resonant inverter both depends on the switch frequency, on the values of the passive circuit elements (capacitances, self-inductances, resistances) and on the type of the control (in frequency, in angle...). The great variety of resonant inverter structures does not allow to give a general expression of the maximum "instantaneous average value" of the output voltage as a function of the DC input voltage.

The resonant inverter has generally a linear dynamical characteristic for frequencies less than the switch frequency. The resonant inverter can be considered as a pure gain so long as the more important harmonics of the voltage references have a frequency lower than the first cutoff frequency of the inverter transfert function.

II-2-2-PWM inverter modalisation. The output voltages of a PWM converter have several levels (V_1, V_2, V_3, \dots). Let consider a load constituted by an inductor L and a resistor R connected to a PWM converter applying two states voltages V_1 and V_2 (fig-2).

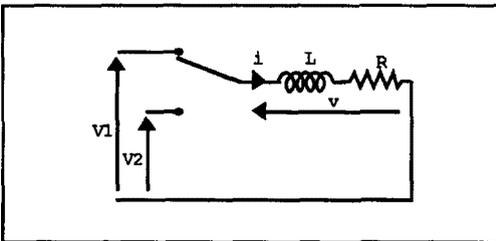


figure 2: R,L load supplied by PWM inverter

During the time within the range $(0, T)$, the voltage verifies:

$$t \in [0, RT] \quad v = V_1$$

$$t \in [RT, T] \quad v = V_2$$

If $i = I_0$ at $t = 0$ then

$$i(T) = \frac{V_2}{R} \left(1 - e^{-\frac{R(1-R)T}{L}} \right) + \frac{V_1}{R} \left(e^{-\frac{R(1-R)T}{L}} - e^{-\frac{RT}{L}} \right) + I_0 e^{-\frac{RT}{L}} \quad (2)$$

If a constant voltage is applied to the load within the range $[0, T]$, the current $i'(T)$ at the time T verifies:

$$i'(T) = I_0 e^{-\frac{RT}{L}} + \frac{V}{R} \left(1 - e^{-\frac{RT}{L}} \right) \quad (3)$$

The difference $\Delta(R) = i(T) - i'(T)$ verifies:

$$\Delta(R) = \frac{V}{R} \left(1 - e^{-\frac{RT}{L}} \right) - \frac{V_2}{R} \left(1 - e^{-\frac{R(1-R)T}{L}} \right) - \frac{V_1}{R} \left(e^{-\frac{R(1-R)T}{L}} - e^{-\frac{RT}{L}} \right) \quad (4)$$

$$\text{If } \frac{RT}{L} \ll 1 \quad \text{then} \quad \begin{cases} 1 - e^{-\frac{RT}{L}} \approx \frac{RT}{L} \\ 1 - e^{-\frac{RT(1-R)}{L}} \approx \frac{RT(1-R)}{L} \end{cases}$$

Considering these approximations, $\Delta(R)$ is equal to zero if:

$$V = R V_1 + (1-R) V_2 \quad (5)$$

So, if the machine time constant is twenty times greater than the switch frequency, the load current behaviour when the supply voltage is equal to V_1 within $[0, RT]$ and V_2 within $[RT, T]$ is practically the same as the current behaviour when the supplied voltage is constant and equal to $R V_1 + (1-R) V_2$. This result can be extended to inverters with number of levels of the output voltage greater than 2.

PWM current linear current control. The principle scheme of a PWM linear current control is presented on Figure-3.

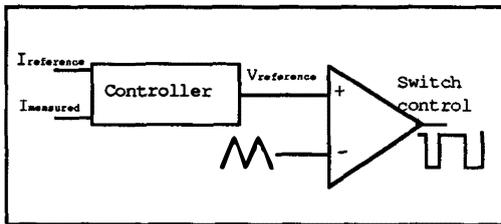


figure 3: PWM linear current control

To have a right working, the voltage references must be slowly variable with respect to the variation of the sawtooth signal and the current controllers are calculated to verify this condition in

practice. Consequently, if the switch frequency is ten times greater than the machine cutoff frequency, the voltage inverter can be considered as a pure gain. In PWM linear current control, the duty cycle must be within the R_{min} and R_{max} values so as to impose a minimum time between the switch off and the switch on. Generally, R_{max} and R_{min} verify the relation:

$$R_{max} = 1 - R_{min} \quad (6)$$

According to the equation (5), the maximum v_{max} and minimum v_{min} average values are equal to:

$$v_{max} = R_{max}.V1 + (1 - R_{max}).V2 \quad (7)$$

$$v_{min} = R_{min}.V1 + (1 - R_{min}).V2$$

The DC input is equal to $2E$. If the modulation is of two levels type $(+E, -E)$, then:

$$v_{max} = (2R_{max} - 1).E \quad (8)$$

$$v_{min} = (2R_{min} - 1).E = -v_{max}$$

If the modulation is of three levels type $(+E, 0, -E)$, then:

$$v_{max} = R_{max}.E \quad (9)$$

$$v_{min} = -(1 - R_{min}).E = -v_{max}$$

Hysteresis current control. The hysteresis current control does not have a control structure as presented in a figure-1 where the voltage inverter appears as a pure gain. The output of hysteresis controllers gives directly a switch state. If the bandwidth is low, the regulation have the same behaviour as a proportionnal linear controller. The duty cycle lies between 0 and 1. So, the maximum instantaneous average values are:

$$v_{max} = E \quad (10)$$

$$v_{min} = -E = -v_{max}$$

II-3-Modelisation of the current control.

We work on a discrete system and the sampling period has been chosen lower than the machine time constant. The voltages are supposed to be constant during this time interval.

Then, the developed model of the inverter is based on the following philosophy: "At each time, the current controlled inverter acts on the output voltage u_j across the j phase in order to minimize the difference between the reference and the actual current in the phase j , taking into account the u_j variation boundaries $+E$ and $-E$."

Of course, this is basically the philosophy of any regulation, but it is here

directly applied leading to the equivalent of hysteresis regulation with a zero bandwidth.

$u_j(t_n)$ is the voltage which must be applied across the j phase and the neutral of the machine to have a current equal to its reference during the time interval $[t_{n-1}, t_n]$. Assuming that the current $i_j(t_{n-1})$ is known and $v_j(t_n)$ is constant during the sampling period $u_j(t_n)$ verifies:

$$v_j(t_n) = R \frac{i_{refj}(t_n) - i_j(t_{n-1}) e^{-\frac{R}{L-M} T}}{1 - e^{-\frac{R}{L-M} T}} + e_j(t_n) + e$$

$$\text{avec } e = -\frac{e_1 + e_2 + e_3}{3} \quad (11)$$

If $u_j(t_n)$ is the j phase voltage of the inverter, according to the previous statement, the three voltages verify the following equations system:

$$\frac{2u_j(t_n) - u_1(t_n) - u_k(t_n)}{3} - \frac{i_j(t_n) - i_j(t_{n-1}) e^{-\frac{R}{L-M} T}}{1 - e^{-\frac{R}{L-M} T}} + e_j(t_n) + e$$

$$\text{with } j \neq k \neq l \text{ and } (j, k, l) = (1, 2, 3) \quad (12)$$

According to (12), the current control of the j phase of the inverter minimizes the criterion $[i_j(t_n) - i_{refj}(t_n)]^2$. According to the relation (11) et (12), this is equivalent to minimize at each time t_n the criterion:

$$f[u_j(t_n)] = (3v_j(t_n) - 2u_j(t_n) + u_1(t_n) + u_k(t_n))^2 \quad (13)$$

by acting on the voltage $u_j(t_n)$ with the constraints: $-E \leq u_j(t_n) \leq E$.

f_j is a strictly convex function and the studied domain (bounded by the constraints) is also convex. So, $u_j(t_n)$ minimizes the criterion and consequently minimizes the difference between the current and its reference in the j phase if:

$$\frac{df_j[u_j(t_n)]}{du_j(t_n)} + \mu_j - \lambda_j = 0 \quad (14)$$

Where λ_j et μ_j are the Kuhn and Tucker factors associated to the two inequalities $u_j(t_n) \geq E$ and $u_j(t_n) \leq -E$.

Considering the balanced three-phase system, it is sufficient to work with two variables v_2 and v_3 . This problem has been solved analytically by using the programming linear methods. The details of calculation of the equations system is presented in the ANNEXE I. Then, the figure 3 and the table 4 provide the "instantaneous average voltages" u_1, u_2, u_3 , as function of v_2 and v_3 . On this statement, a simple algorithm has been developed to calculate voltages, currents and electromagnetic torque.

The only time constant of the simulated actual system which occurs in the model is the machine time constant. So as to have an obvious sampling, the sample period must be chosen ten times greater than the two

following variables:
 -the machine time constant
 -the frequencies of the main harmonics of references.

The sampling period can be compute automatically, not to be given by the operator.

The shortest time constant of the simulated actual device which occurs in a closed loop regulation are very lower than the machine time constant. The sampling period used to simulate a closed loop control must be very lower than the sampling period used in the proposed simulation tools. So, the proposed model allows shorter calculation times than a classical modelisation taking into account the actual characteristics of the current controllers.

The proposed model does not require the calculation of a controller which depend on the characteristics of the studied machine and inverter. The maximum and minimum instantaneous average values of the voltage is sufficient to describe the voltage inverter and the current control.

III-TORQUE-SPEED CHARACTERISTICS

For a given reference current (waveshape, amplitude and phase difference angle versus the emf), the software computes for rising speed values, the average value of the torque produced by the machine in steady state operation and stops the calculation when the torque becomes negative. The torque-speed characteristic is then obtained very quickly (for 50 points characteristics, a 386-PC needs only 1 minute calculation).

The described method has been validated by comparison with tests on an experimental plant using a 30 kHz mosfet PWM inverter feeding a permanent magnet machine with rectangular currents. The figure 5 shows the simulated and experimented currents. The figure 6 shows a comparison of the measured and calculated torque speed characteristics. These results are in very good agreements as can be observed.

IV-CONCLUSION

The presented modelisation of brushless actuators fed with current controlled inverters requires few parameters relative to the actual system and no regulator calculation. The time constant of this model is longer than the time constant of models including a classical current regulation leading to very short computation times.

Comparison with experimental test has shown a very good agreement of results.

This modelisation enables to do an efficient simulated experiment very useful in C.A.D to determine the torque speed characteristic of the whole actuator.

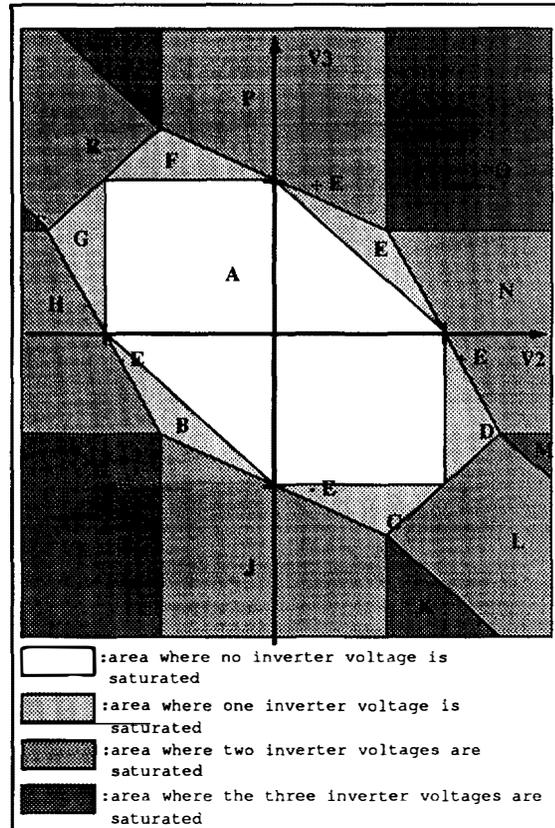


Figure 4: areas map providing the values of u_1, u_2, u_3 as function of v_1, v_2, v_3 .

area	u_1	u_2	u_3
A	v_1	v_2	v_3
B	$+E$	$v_2 - v_1 + E$	$v_3 - v_1 + E$
C	$v_1 - v_3 - E$	$v_2 - v_3 - E$	$-E$
D	$v_1 - v_2 + E$	$+E$	$v_3 - v_2 + E$
E	$-E$	$v_2 - v_1 - E$	$v_3 - v_1 - E$
F	$v_1 - v_3 + E$	$v_2 - v_3 + E$	$+E$
G	$v_1 - v_2 - E$	$-E$	$v_3 - v_2 - E$
H	$+E$	$-E$	$1,5 * v_3$
I	$+E$	$-E$	$-E$
J	$+E$	$1,5 * v_2$	$-E$
K	$+E$	$+E$	$-E$
L	$1,5 * v_1$	$+E$	$-E$
M	$-E$	$+E$	$-E$
N	$-E$	$+E$	$1,5 * v_3$
O	$-E$	$+E$	$+E$
P	$-E$	$1,5 * v_2$	$+E$
Q	$-E$	$-E$	$+E$
R	$1,5 * v_1$	$-E$	$+E$
S	$+E$	$-E$	$+E$

TABLE1: table providing the values of u_1, u_2, u_3 as function of v_1, v_2, v_3 .

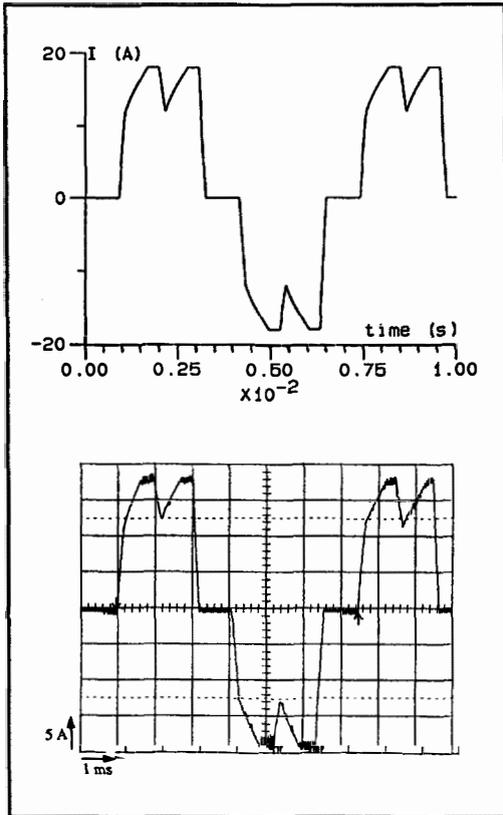


Figure 5: simulated and measured currents in the case of rectangular current references.

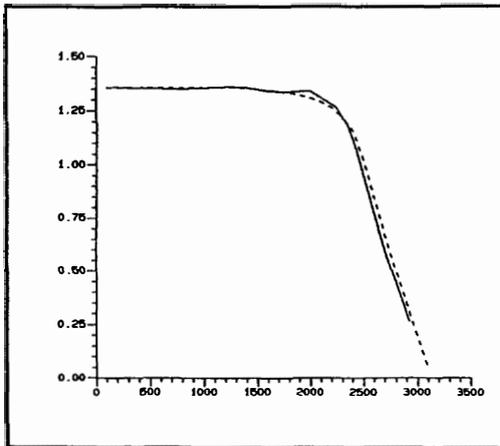


Figure 6: Torque-speed characteristics:
 —: measured
 ---: simulated

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ANNEXE I

To simplify the mathematical expressions, the variable $x_j(t_n)$ will be written x_j .

u_1, u_2, u_3 are solutions of (13) then u_1, u_2, u_3 verify the system of equations:

$$\begin{aligned} 8u_1 - 4u_2 - 4u_3 + \mu_1 - \lambda_1 &= 12v_1 \\ 8u_2 - 4u_3 - 4u_1 + \mu_2 - \lambda_2 &= 12v_2 \\ 8u_3 - 4u_1 - 4u_2 + \mu_3 - \lambda_3 &= 12v_3 \end{aligned}$$

with $-E \leq u_1 \leq E, -E \leq u_2 \leq E, -E \leq u_3 \leq E$
 and $\mu_1, \lambda_1, \mu_2, \lambda_2, \mu_3, \lambda_3$ positive.

v_1, v_2, v_3 verify $v_1 + v_2 + v_3 = 0$

A- If $E \leq v_1 \leq E, -E \leq v_2 \leq E, -E \leq v_3 \leq E$
 then $u_1 = v_1, u_2 = v_2, u_3 = v_3$
 because $\mu_m = \lambda_m = 0$

B- $v_1 > +E, -E \leq v_2 \leq E, -E \leq v_3 \leq E$

Assume that $u_1 = +E$ then $\mu_1 \neq 0, \lambda_1 = 0$ and u_2, u_3 not saturated then $\mu_2 = \lambda_2 = \mu_3 = \lambda_3 = 0$, the system of equations becomes:

$$\begin{aligned} 8E - 4u_2 - 4u_3 + \mu_1 &= 12v_1 \\ 8u_2 - 4u_3 - 4E &= 12v_2 \\ 8u_3 - 4E - 4u_2 &= 12v_3 \end{aligned}$$

then

$$\begin{aligned} u_2 &= v_2 - v_1 + E \\ u_3 &= v_3 - v_1 + E \end{aligned}$$

and $\mu_1 = 4(v_1 + v_2 + v_3)$

taking into account that $v_1 + v_2 + v_3 = 0$:

$$\mu_1 = 0$$

So, if $v_1 > +E, -E \leq v_2 \leq E, -E \leq v_3 \leq E$ the solutions are :

$$\begin{aligned} u_1 &= +E \\ u_2 &= v_2 - v_1 + E \\ u_3 &= v_3 - v_1 + E \end{aligned}$$

C- If $E \leq v_1 \leq E, -E \leq v_2 \leq E, v_3 < -E$

The same reasoning as in B gives:

$$\begin{aligned} u_1 &= v_1 - v_3 - E \\ u_2 &= v_2 - v_3 - E \\ u_3 &= -E \end{aligned}$$

D- If $E \leq v_1 \leq E, v_2 > +E, -E \leq v_3 \leq E$

The same reasoning as in B gives:

$u_1 = v_1 - v_2 + E$
 $u_2 = +E$
 $u_3 = v_3 - v_2 + E$

E-If $v_1 < -E$, $-E \leq v_2 \leq +E$, $-E \leq v_3 \leq +E$
 The same reasoning as in B gives:
 $u_1 = -E$
 $u_2 = v_2 - v_1 - E$
 $u_3 = v_3 - v_1 - E$

F-If $E \leq v_1 \leq +E$, $-E \leq v_2 \leq +E$, $v_3 > +E$
 The same reasoning as in B gives:
 $u_1 = v_1 - v_3 + E$
 $u_2 = v_2 - v_3 + E$
 $u_3 = +E$

G-If $E \leq v_1 \leq +E$, $v_2 < -E$, $-E \leq v_3 \leq +E$
 The same reasoning as in B gives:
 $u_1 = v_1 - v_2 - E$
 $u_2 = -E$
 $u_3 = v_3 - v_2 - E$

H- $v_1 > +E$, $v_2 < -E$, $-E \leq v_3 \leq +E$
 Assume that $u_1 = +E$, $u_2 = -E$ then $\mu_1 \neq 0$, $\lambda_2 \neq 0$, $\lambda_1 = 0$, $\mu_2 = 0$ and u_3 not saturated then $\mu_3 = \lambda_3 = 0$, the system of equations becomes:

$$\begin{array}{rcl} 12E - 4u_3 + \mu_1 & = & 12v_1 \\ -12E - 4u_3 - \lambda_2 & = & 12v_2 \\ 8u_3 & = & 12v_3 \end{array}$$

then
 $u_3 = 1,5v_3$
 taking into account that $v_1 + v_2 + v_3 = 0$
 $\mu_1 = 6(v_1 - v_2) - 12E$
 $\lambda_2 = -6(v_1 - v_2) + 12E$

to the extend that $v_1 > +E$ and $v_2 < -E$ then
 $v_1 - v_2 > 2E$
 so, $\mu_1 > 0$ and $\lambda_2 > 0$

So, if $v_1 > +E$, $v_2 < -E$, $-E \leq v_3 \leq +E$ the solutions are :

$$\begin{array}{l} u_1 = +E \\ u_2 = -E \\ u_3 = 1,5 v_3 \end{array}$$

I- $v_1 > +E$, $v_2 < -E$, $v_3 < -E$
 Assume that $u_1 = +E$, $u_2 = -E$, $u_3 = -E$ then $\mu_1 \neq 0$, $\lambda_2 \neq 0$, $\lambda_3 \neq 0$, $\lambda_1 = 0$, $\mu_2 = 0$, $\mu_3 = 0$, the system of equations becomes:

$$\begin{array}{l} 16E + \mu_1 = 12v_1 \\ 8E - \lambda_2 = 12v_2 \\ 8E - \lambda_3 = 12v_3 \end{array}$$

So, $\mu_1 = 4(3v_1 - 4E)$
 $\lambda_2 = 4(2E - 3v_2)$
 $\lambda_3 = 4(2E - 3v_3)$

v_2 and v_3 are greater than E so λ_2 and λ_3 are positive.
 $v_1 + v_2 + v_3 = 0$ so $v_1 = -v_2 - v_3 > 2E$ then μ_1 is positive.
 So, if $v_1 > +E$, $v_2 < -E$, $v_3 < -E$ then

$$\begin{array}{l} u_1 = +E \\ u_2 = -E \\ u_3 = -E \end{array}$$

J- $v_1 > +E$, $-E \leq v_2 \leq +E$, $v_3 < -E$
 The same reasoning as in H gives:
 $u_1 = E$
 $u_2 = 1,5 * v_2$
 $u_3 = -E$

K- $v_1 > +E$, $v_2 > +E$, $v_3 < -E$
 The same reasoning as in I gives:
 $u_1 = +E$
 $u_2 = +E$
 $u_3 = -E$

L- $-E \leq v_1 \leq +E$, $v_2 > +E$, $v_3 < -E$
 The same reasoning as in H gives:
 $u_1 = 1,5 * v_1$
 $u_2 = +E$
 $u_3 = -E$

M- $v_1 < -E$, $v_2 > +E$, $v_3 < -E$
 The same reasoning as in I gives:
 $u_1 = -E$
 $u_2 = +E$
 $u_3 = -E$

N- $v_1 < -E$, $v_2 > +E$, $-E \leq v_3 \leq +E$
 The same reasoning as in H gives:
 $u_1 = +E$
 $u_2 = -E$
 $u_3 = 1,5 v_3$

O- $v_1 < -E$, $v_2 > +E$, $v_3 > +E$
 The same reasoning as in I gives:
 $u_1 = -E$
 $u_2 = +E$
 $u_3 = +E$

P- $v_1 < -E$, $-E \leq v_2 \leq +E$, $v_3 > +E$
 The same reasoning as in H gives:
 $u_1 = -E$
 $u_2 = 1,5 * v_2$
 $u_3 = +E$

Q- $v_1 < -E$, $v_2 < -E$, $v_3 > +E$
 The same reasoning as in I gives:
 $u_1 = -E$
 $u_2 = -E$
 $u_3 = +E$

R- $E \leq v_1 \leq +E$, $v_2 < -E$, $v_3 > +E$
 The same reasoning as in H gives:
 $u_1 = 1,5 * v_1$
 $u_2 = -E$
 $u_3 = +E$

S- $v_1 > +E$, $v_2 < -E$, $v_3 > +E$
 The same reasoning as in I gives:
 $u_1 = +E$
 $u_2 = -E$
 $u_3 = +E$

T-According to $v_1 + v_2 + v_3 = 0$, the following cases do not exist:
 $-v_1 > E$, $v_2 > E$, $v_3 > E$
 $-v_1 < -E$, $v_2 < -E$, $v_3 < -E$
 $-E \leq v_1 \leq +E$, $v_2 < -E$, $v_3 < -E$
 $-E \leq v_1 \leq +E$, $v_2 > +E$, $v_3 > +E$
 $-v_1 < -E$, $-E \leq v_2 \leq +E$, $v_3 < -E$
 $-v_1 > +E$, $-E \leq v_2 \leq +E$, $v_3 > +E$
 $-v_1 < -E$, $v_2 < -E$, $-E \leq v_3 \leq +E$,
 $-v_1 > +E$, $v_2 > +E$, $-E \leq v_3 \leq +E$.