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Comment on "The thermosolutal instability of compressible Hall plasma"

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Recently Sharma and Rani (1988) in a paper, to be referred hereafter as I, have discussed Hall current effects on the instability of a compressible plasma layer of finite thickness in the presence of a uniform vertical magnetic field. In the horizontal direction, the layer extends infinitely and its both upper and lower surfaces are supposed to be free and perfect conductors of heat and solute. Further, the layer is assumed to be heated and soluted from below so that a uniform temperature gradient and a solute gradient are maintained.

Under Spiegal and Veronis' (1960) approximation, using linear theory and normal mode technique, the authors in I have claimed that for the onset of instability via oscillations

$$p_1 > 1, p_1 > q \quad \text{and} \quad p_1 > p_2 \tag{1}$$

must hold. The symbols used in the present note are the same as in I. In the following discussion we shall see that neither this result nor the argument given for its derivation is correct. Further, in the absence of Hall currents, we shall show that if at the onset of instability the conditions (1) hold, the system is necessarily non-oscillatory (the condition $p_1 > 1$ can be dropped). In fact (1) are sufficient conditions for the principle of exchange of stabilities to be valid (in the absence of Hall currents).

Following I, the dispersion relation or the stability, governing equation can be written as

$$R_1 x = \frac{G}{G-1}\left[S_1 x \frac{1+x+ip_1\sigma_1}{1+x+iq\sigma_1} + \frac{(1+x)(1+x+ip_1\sigma_1)[\{(1+x+i\sigma_1)(1+x+ip_2\sigma_1)+Q_1\}^2+M(1+x)(1+x+i\sigma_1)^2]}{(1+x+ip_2\sigma_1)\{(1+x+i\sigma_1)(1+x+ip_2\sigma_1)+Q_1\}+M(1+x)(1+x+i\sigma_1)}\right] \tag{2}$$

where the imaginary part of σ_1 is zero at the onset of instability. For the onset of instability through oscillations, the real and imaginary parts of eq. (2) lead to

$$A_4 c^4 + A_3 c^3 + A_2 c^2 + A_1 c + A_0 = 0 \tag{3}$$

where

$$A_4 = p_2^4 q^2 (1+p_1) b,$$

$$\begin{aligned} A_0 = {} & (1+p_1)b^9 + 2M(1+p_1)b^8 + \{Q_1(3p_1 - p_2 + 2) + M^2(1+p_1)\}b^7 \\ & + \{S_1(p_1 - q)(b-1) + MQ_1(3p_1 + p_2 + 2)\}b^6 \\ & + \{Q_1^2(3p_1 - 2p_2 + 1) + 2MS_1(p_1 - q)(b-1)\}b^5 \\ & + \{2Q_1 S_1(p_1 - q)(b-1) + M^2 S_1(p_1 - q)(b-1) + Q_1^2 M(p_1 - 1)\}b^4 \\ & + \{Q_1^3(p_1 - p_2) + 2MS_1 Q_1(p_1 - q)(b-1)\}b^3 \\ & + Q_1^2 S_1(p_1 - q)(b-1)b^2, \end{aligned} \tag{5}$$

$$c = \sigma_1^2 \quad \text{and} \quad b = 1 + x. \tag{6}$$

Since eq. (3) is a biquadratic in $c > 0$ with real coefficients, the product of its roots A_0/A_4 must be positive. This requires $A_0 > 0$. Therefore, $A_0 > 0$ is a necessary condition for the existence of overstability. The authors in I claim that since under (1) $A_0 > 0$ so it is a necessary condition for the product of roots of eq. (3) to be positive. Actually (1) are sufficient conditions for $A_0 > 0$ (this does not at all mean a sufficient condition for the existence of over-stability). Thus, the argument given by the authors is not correct. Now we shall prove that (1) is not a necessary condition also.

Rewriting eq. (5) as

$$\begin{aligned} A_0 = {} & S_1 x(p_1 - q)(b^3 + b^2 M + Q_1 b)^2 + (1+p_1)b^9 \\ & + 2M(1+p_1)b^8 + [Q_1(3p_1 - p_2 + 2) + M^2(1+p_1)]b^7 + MQ_1(3p_1 + p_2)b^6 \\ & + Q_1(3p_1 - 2p_2 + 1)b^5 + [Mp_1 Q_1^2 + MQ_1\{2 - Q_1 + 2(x + x^2)\}]b^4 \\ & + Q_1^3(p_1 - p_2)b^3 \end{aligned} \tag{7}$$

one can easily conclude that if

$$p_1 > q,\ p_1 > p_2 \quad \text{and} \quad Q_1 < 2 \tag{8}$$

$A_0 > 0$. Thus even if (1) are not satisfied, under conditions (8), $A_0 > 0$. In fact A_0 may be positive even if none of the inequalities in (1) is satisfied. Certainly for this, one has to be careful about the nature of A_3, A_2 and A_1 also. Therefore, (1) can not at all be accepted as necessary conditions for the occurrence of overstability.

In the absence of Hall currents eq. (2) become

$$R_1x=\frac{G}{G-1}\Big[S_1x\,\frac{1+x+ip_1\sigma_1}{1+x+iq\sigma_1}+(1+x)(1+x+i\sigma_1)(1+x+ip_1\sigma_1)$$

$$+Q_1\frac{(1+x)(1+x+ip_1\sigma_1)}{1+x+ip_2\sigma_1}\Big] \tag{9}$$

At the onset of instability (taking σ_1 real), the imaginary part of eq. (9) gives

$$\sigma_1\Big[S_1x\frac{p_1-q}{(1+x)^2+q^2\sigma_1^2}+(1+x)(1+p_1)+Q_1\frac{(1+x)(p_1-p_2)}{(1+x)^2+p_2^2\sigma_1^2}\Big]=0. \tag{10}$$

clearly under (1) $\sigma_1=0$ is necessary. Thus, the system is necessarily non-oscillatory. Hence the principle of exchange of stabilities is valid.

Acknowledgment

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References

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Spiegal E A and Veronis G 1960 *Astrophys. J.* **131** 442

Reply to Comment on "The thermosolutal instability of compressible Hall plasma"

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The preceding comment (Saini and Kumar 1990) has brought to our notice several errors in our paper (Sharma and Rani 1988a) which should be corrected as follows :

The sentence after inequalities (23) should read as :

It can be shown, under (23), that $\sigma_1=0$.

$\kappa<\nu$, $\kappa<\kappa'$ and $\kappa<\eta$ are, therefore, sufficient conditions for validity of principle of exchange of stabilities.

Similarly, in another paper (Sharma and Rani 1988b) we need the following corrections :

(i) The last sentence in 'Abstract' should read as :

The case of overstability is also considered wherein sufficient conditions for validity of principle of exchange of stabilities are obtained.

(ii) The sentence after inequalities (28) should read as :

It can be shown, under (28), that $\sigma_1 = 0$.

$\kappa < \nu$, $\kappa < \kappa'$ and $\kappa < \eta$ are, therefore, sufficient conditions for validity of principle of exchange of stabilities.

References

Saini J C and Kumar Pawan *Indian J Phys.* **64B** (preceding comment)
Sharma K C and Rani Neela 1988a *Indian J. Phys.* **62B** 211
——1988b *ibid* **62B** 181