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Effect of internal absorbing boundaries on minimum critical volume of fissionable material

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Abstract : Considering diffusion of neutrons through parallelopiped of height '*l*' and cross sectional region bounded by two parallelogram, sides of the inner and outer parallelogram being '*a*' and '*b*' respectively, the neutron diffusion equation is solved. The solution is obtained by using integral transform finite difference method with equilateral triangular mesh. By taking different inner and outer side values the relative size of inner absorbing material is changed and the minimum critical volume is worked out for different relative sizes of absorbing material. A graph indicating variation of V_{mini} with relative size of absorbing material is plotted. It shows that as relative size of absorbing material increases V_{mini} also increases.

Keywords : Fissionable material, critical volume, effect of internal absorbing boundaries, neutron diffusion equation, finite difference method.

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1. Introduction

The problem of neutron scattering through fissionable material is of technological importance because of its use in the preservation of fissionable material, explosion in the form of atom bomb or developing a chain reaction. The critical height and critical volume of the material is of primary importance in all these problems. The critical mass of the fuel material can be determined from the critical volume and the mass of the fuel material per unit volume. The critical size of the fissionable material depends upon the geometry of the material. The critical size for the geometry such as cubes, cylinders and spheres are treated in the standard text book by Srivastava (1975). The critical size for prismatic solid cross section as equilateral triangle and isosceles right triangle are treated by Ugile (1980), Patil and Patil (1982) respectively. The neutron diffusion with neutron absorbing material inside fissionable material has been solved by Patil and Pathan (1980) for elliptic cylindrical geometry and by Nehate (1988) for cylinder of equilateral triangular cross section. Here we consider similar problem of determination of critical volume for a parallelopiped having a cross section of equal sides parallelogram with angle 60°. To solve the diffusion equation and the lowest eigen value, the finite sine transform and finite difference method with triangular mesh is used.

2. Statement of the problem

Consider a fissionable material body of the shape of a parallelopiped of height 'l' and cross section as parallelogram of equal sides 'b' each with 60° angle between them. Let neutron absorbing material of similar geometry but of size 'a' be inside the fissionable material. Let the inner parallelogram boundary surface be B_1 and outer parallelogram boundary surface be B_2. Let there be no neutron sources inside the volume of the solid and 'l' includes extrapolation length.

According to the simple reactor model based on one velocity group as discussed by Wayland (1966), the diffusion of neutrons is governed by

$$D\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right]N+\left[\frac{K_c-1}{\lambda}\right]VN=\frac{\partial N}{\partial t} \tag{2.1}$$

where

$N=N(x, y, z, t)$, the neutron density

$D=$ the thermal diffusion coefficient for neutrons

$V=$ the thermal neutron speed

$\lambda_c=$ absorption mean free path

$K_c=$ average neutrons produced per absorption

Here the cartesian coordinate system is used with the bottom parallelogram as the xy plane and Z-axis alongwith the height, the origin being at the corner of the 60° angle, X-axis taken along one side of the bottom plane and Y-axis in the bottom plane but perpendicular to X-axis and making an angle of 30° with the other of the bottom plane.

Since the material is bare one, the neutron density will be nearly zero at the boundary surfaces. Hence the boundary conditions will be

$$\left.\begin{aligned} N\,|_{B_1}&=0\\ N\,|_{B_2}&=0\end{aligned}\right. \tag{2.2}$$

and

$$\left.\begin{aligned} N\,|_{z=0}&=0\\ N\,|_{z=l}&=0\end{aligned}\right. \tag{2.3}$$

3. Solution and minimum critical volume (Gerald 1970, Garg et al 1986)

Applying finite sine transform of Kernel $\sin\frac{s\pi z}{l}$ with respect to 'z' variable and obtaining the solution by method of separations in x and y and then using the inverse sine transform leads to the solution of the problem as

$$N(x, y, z, t) = \sum \sum_{k} A_k \frac{2}{\pi} \sin\left(\frac{s\pi}{l}\right)^2 U_k(x, y)$$

$$\exp\left[\left\{-D\left(\alpha^2 + \frac{s^2\pi^2}{l^2}\right) + \left(\frac{K_o - 1}{\lambda_o}\right)V\right\}t\right] \tag{3.1}$$

A_k values are such that the summation over k of right hand side eq. (3.1) converges to initial conditions and α^2 are the eigen values of the equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\psi_n = -\alpha^2\psi_n \tag{3.2}$$

It can be easily worked out from eq. (3.1) that for sustained chain reaction the condition is

$$\frac{\pi}{l}^2 + \alpha^2_{\text{mini}} = B^2 \tag{3.3}$$

where

$$B^2_m = \left(\frac{K_o - 1}{\lambda_e D}\right)V \tag{3.4}$$

The minimum critical volume for this geometry then becomes

$$V_{\text{mini}} = \frac{3}{2}\pi(f_0^2 - f_i^2)\beta^2_{\text{mini}}/B^3 \tag{3.5}$$

where β^2_{mini} is the lowest eigen value of eq. (3.2) with

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \text{ replaced by } \{1_1^1 - 6_1^1 1\}$$

and $f_0 = b/h$ and $f_i = a/h$, h being the size of mesh element.

4. Numerical calculations

Let us now consider the evaluation of V_{mini} for different relative sizes of absorbing material of the parallelopiped with parallelogram shaped sides. Since 'a' and 'b' respectively indicate the side of the inner and outer sides of the parallelogram, the ratio a/b can be taken as a measure of the relative size of absorbing material in the fissionable body. We consider below the various cases of relative sizes of absorbing material, for which we have followed the following steps.

(i) The cross section is sketched with the mesh elements and function numbers are allotted at the nodes.

(ii) The equations relating the different nodal values of neutron density are obtained. From these equations, the matrix whose lowest eigen value is to be found out is derived.

(iii) The lowest eigen value of the matrix is determined by converting firstly the matrix into upper Heisenberg form and then using double QR method.

(iv) The lowest eigen value is determined.

(v) V_{mini} is calculated.

5. Conclusion

Table 1 gives the values of V_{mini} calculated for given geometry for values of

Table I. Calculated values of V_{mini} for given geometry.

a/b	f_0	V_{mini}
0.0	1	0.00
0.0	2	$113.04/B^3$
0.0	3	$127.17/B^3$
0.0	4	$158.76/B^3$
0.0	5	$164.93/B^3$
0.0	6	$169.71/B^3$
0.0	7	$170.37/B^3$
0.0	8	$172.25/B^3$
0.0	9	$172.63/B^3$
0.2	5	$402.98/B^3$
0.2	10	$433.00/B^3$
0.2	15	$435.62/B^3$
0.2	20	$435.76/B^3$
0.6	5	0.00
0.6	10	$1091.26/B^3$
0.6	15	$1168.93/B^3$
0.6	20	$1190.55/B^3$
0.6	25	$1198.19/B^3$

$a/b = 0$, 0.2, 0.6 with different number of segments f_0 used for numerical calculation. It shows the convergence with increase of f_0.

Table 2 shows the variation of minimum critical volume against the relative size of absorbing material of the prism with parallelogram cross section. The plot of this variation is shown in Figure 1. It is seen clearly that as relative size

Table 2. Variation of V_{mini} with relative srze of absorbing material.

	a/b	V_{mini}
1	0.000	$172/B^3$
2	0.091	$400/B^3$
3	0.200	$433/B^3$
4	0.272	$558/B^3$
5	0.384	$774/B^3$
6	0.538	$973/B^3$
7	0.666	$1363/B^3$

of absorbing material increases the minimum critical volume increases. This increment is more rapid at high degree of relative size of absorbing material.

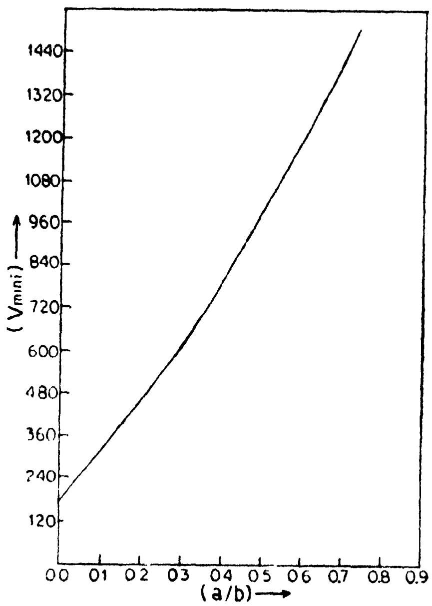

Figure 1. Variation of V_{mini} as a function of a/b.

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