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Etch-pit formation-a convective diffusion process

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Etch-pit formation is one of the numerous physical phenomena which can be understood as convective diffusion processes. The general theory of such processes is given by Levich (Levich 1962 ; see also Daccord 1987). Preliminary investigations in this direction were carried out by the present authors a few years ago.

We begin by writing down the differential equations for convective-diffusion processes (ν is the fluid velocity, c the concentration of the etchant, other symbols are standard) :

$$\frac{\partial \nu}{\partial t}+(\nu.\nabla)\nu=-\frac{\nabla p}{\rho_{sol.}}+\nu\nabla^2\nu \qquad \text{(Navier-Stokes eqn.)} \qquad (1)$$

$$\frac{\partial c}{\partial t}+(\nu.\nabla)c=D\nabla^2 c \qquad \text{(eqn. for convective diffusion)} \qquad (2)$$

We choose a coordinate system with origin at the vertex of the pit in the SSNTD. The track will be assumed to be perpendicular to the flat surface of the detector, and the (+) ve direction of the Z-axis oriented back along the track. We can now specify the boundary conditions :

(i) $c \xrightarrow[(z\to\infty)]{} c_0$

(ii) $D\left(\frac{\partial c}{\partial n}\right)=Kc$, on the reaction front (walls of the pit),

where

$$K=K_0[1+\alpha I],$$

$I=I(\vec{r}, t)=$ ionization density in the detector, and K_0, α are two constants.

An approximate solution of this convective-diffusion problem can be worked out in the quasi-stationary limit :

$$\frac{\partial \nu}{\partial t}\simeq 0, \; \frac{\partial c}{\partial t}\simeq 0. \qquad (3)$$

Actually, inside the pit, $v \cong 0$, since the track etch rate $V_t \sim$ few μm/hr.

Thus we are essentially left to solve the differential eqn. $\nabla^2 c = 0$, i.e.,

$$\frac{\partial^2 c}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial c}{\partial \rho} + \frac{\partial^2 c}{\partial z^2} = 0 \tag{4}$$

in the cylindrical polar coordinates.

Now, on general physical grounds, we expect a narrow pit, conical in shape in a first approximation. Since a cone is characterized by a constant value of the ratio (ρ/z), the following appears to be a plausible solution of the problem :

Eqn. (4) remains unchanged when the variables are transformed as follows ;

$$\rho \rightarrow \lambda \rho', \; z \rightarrow \lambda z' \tag{5}$$

In view of this, a solution of eqn. (4) may be sought such that a combination of variables ρ and z remains unchanged by transformations (5) ; such a combination is

$$\eta = \frac{\rho}{z}$$

Setting $c = c(\eta)$. we can write eqn. (4) in the form

$$(1+\eta^2)\frac{d^2 c}{d\eta^2} + \left(2\eta + \frac{1}{\eta}\right)\frac{dc}{d\eta} = 0$$

This equation can be solved easily (for $\eta > 0$). We have

$$\frac{dc}{d\eta} = \frac{A}{\eta(1+\eta^2)^{1/2}} = \frac{Az^2}{\rho(\rho^2+z^2)^{1/2}}$$

where $A(<0)$ is to be fixed from the boundary condition (ii).

Finally, for fixed z,

$$c(\rho, z) = c(\eta) = \int_{\eta_0}^{\eta} \left(\frac{dc}{d\eta}\right) d\eta + c(\eta_0) \; [\rho_0 \gtrsim 0.1 \; \mu\text{m}]$$

Now the rate (dR/dt) at which the radius (R) of the pit, for a given h (depth, referred to the point of entry of the particle into the SSNTD), changes with time, is proportional to the diffusional flux rate (j_{diff}) of the etchant on to the surface of the pit. Again

$$j_{diff}(R, z) = D\frac{\partial c}{\partial n}\Big|_{R,z} = -\frac{DA}{R}$$

and

$$\frac{dR}{dt} \propto \sqrt{1+\frac{R^2}{z^2}} \cdot \frac{D\,|A|}{R} \tag{6}$$

Since, on physical grounds, $z=\int V_t\,dt-h$ (where V_t is the track etch rate) is much larger than R, eqn. (6) reduces to

$$\frac{dR}{dt}\sim\frac{1}{R}$$

Thus

$$R\sim t^{1/2}\ \text{(for fixed } h)$$

Actually, this solution does not correspond exactly to a conical etch pit ; but we should remember that a cone is an approximation for the shape of the pit.

If $\frac{\partial^2 c}{\partial z^2}$ be negligible, then the eqn. for convective-diffusion, near the walls of the pit, can be written as

$$V_t\,\frac{\partial c}{\partial z}=D\left(\frac{\partial^2 c}{\partial \rho^2}+\frac{1}{\rho}\,\frac{\partial c}{\partial \rho}\right) \tag{7}$$

Scaling arguments show that we may express $c(\rho,\,z)$ as a function of $\zeta=\frac{\rho}{z^{1/2}}$. This time the differential eqn. (7) reduces to

$$\frac{d^2c}{d\zeta^2}+\left(k\zeta+\frac{1}{\zeta}\right)\frac{dc}{d\zeta}=0$$

where

$$k=\frac{V_t}{2D}\leqslant 1.$$

Thus

$$\frac{dc}{d\zeta}=\frac{Ae^{-\frac{k}{2}\zeta^2}}{\zeta}$$

Finally,

$$\frac{dR}{dt}\propto\frac{1}{z^{1/2}}\left(1+\frac{R^2}{4z^2}\right)\frac{dc}{d\zeta}\bigg|_{R,z}\ \xrightarrow[(z>>R)]{}\ \frac{dR}{dt}\sim\frac{1}{R}$$

Consequently we again get

$$R\sim t^{1/2}\ \text{(at fixed } h).$$

Since $z\sim t$ (for fixed h), therefore $R\sim z^{1/2}$. If the initial boundary condition is something like $c\simeq 0$ on $\zeta=\zeta_0$, i.e. $c(\zeta_0)\simeq 0$, then, because $R\sim z^{1/2}$, $c(\zeta_0)\simeq 0$ at all later times. Thus $R^2\simeq\zeta_0^2 z$ represents the etch pit for large times.

We, therefore, conclude that in either case the etch pit should look somewhat like a paraboloid rather than a cone.

References

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