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Fictitious photon mass in radiative Bhabha scattering

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Abstract : Fictitious photon mass spectrum in radiative Bhabha scattering at 70 Gev c.m. energy is studied as a function of cosine of the scattering angle. The curve is found to possess a peak in the vicinity of 90° angle of scattering to the lower side.

Keywords : Electron-positron, virtual photon, Bhabha scattering.

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1. Introduction

In the collision of an electron (e^-) with a positron (e^+), the e^+ is considered as an unoccupied state of negative energy, which leads to a process different from the one which is obtained if e^+ is treated as an independently charged particle in a state of positive energy whose behaviour is described by the Dirac equation (Dirac 1930, 1934). This difference is due to the effect of exchange between the e^- observed initially and the virtual electrons in the state of negative energy. In the original formulation of Bhabha scattering (Bhabha 1935), e^- and e^+ are considered in the states a^0_+ and b^0_+ of positive energies, respectively. After the scattering process, the e^- is found in a state a'_+ and the e^+ in a state b'_+. According to Dirac's theory (Dirac 1930, 1934) the two states of the e^+, b^0_+ correspond to two unoccupied states of negative energy which are denoted by a^0_- and a'_-, respectively. The initial e^- in the state a^0_+ is denoted by suffix 1 while the e^+ is denoted by suffix 2. The unoccupied state a_- represents the e^+. After scattering, the e^-_1 goes over to the final state a'_+ and the e^-_2 jumps into the unoccupied state a'_- leaving the state a^0_- unoccupied, which then appears as the scattered e^+. This is the normal scattering process. The exchange effect takes place in such a way that a similar final state is observed if the e^-_1 jumped into the unoccupied state a'_-, and the e^-_2 jumped into the final state a'_+. In Bhabha scattering the process is clearly considered in such

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a way that the e^- and the e^+ annihilate each other and simultaneously an e^- and e^+ pair is created. Thus the scattering cross section contains extra terms in addition to the annihilation terms as given below.

The effective differential cross section dQ, in the center of mass (c.m.) system, for e^-e^+ scattering through an angle between θ and $\theta+dQ$ is given by (Bhabha 1935).

$$dQ=\frac{\pi}{4}\frac{e^4}{m^2c^4\gamma}\left[\frac{2(1+\cos^4\theta/2)}{\sin^4\theta/2}+(1+\cos^2\theta)-\frac{(1+\cos\theta)^2}{\sin^2\theta/2}\sin\theta d\theta\right] \tag{1.1}$$

where, m is the mass of the colliding e^- (or e^+), c is the velocity of light and $\gamma=E/MC^2$, where E is the energy of the colliding $e^-(e^+)$, and e is the electronic charge.

In eq. (1.1), the first term corresponds to ordinary scattering process which may be directly obtained if the e^+ is treated as an independently charged particle in a state of positive energy. The other two terms correspond to exchange effects. The second term is due to the annihilation of the initial e^-e^+ pair and the simultaneous creation of the new e^-e^+ pair. The third term represents the interference between the direct scattering and the latter process.

The development of quantum electrodynamics (QED) has led to the formulation of the same expressions for Bhabha scattering (Hang and Tai 1969), which has now been widely studied in e^-e^+ colliding beam experiments. This final state is characterized by a collinear pair of showering particles and it tests the photon propagator for both the time-like and the space-like values of momentum transfer (Schwitters and Strauch 1978). The Bhabha scattering is studied at large angles as a test of QED for large space-like and time-like values of virtual photon mass. The QED test is based on the measurements of the shape of angular distribution of the scattered e^-e^+ showers.

The interaction between the e^- and the e^+ is mediated via the exchange of only virtual photon up to 10 Gev c.m. energy. Above 10 Gev, the virtual intermediate vector bosons also begin to be exchanged. In the first approximation of perturbation theory the virtual photon lines occurring in the Feynman diagrams are regarded as corresponding to a fictitious real (Berestetskii *et al* 1982) particle, a vector boson, which interacts with an electron in the same way as a photon does. In order to study various processes, viz, electron form factors, radiative corrections, etc. some divergent integrals usually appear (Beresteskii *et al* 1982). These divergences are found to be closely related to infrared catastrophe, which points out the fact that the cross section for any process involving charged particles including scattering of electrons has no significance in itself but only when simultaneous emission of any number of soft photons is taken into account. To remove these divergences, some cut off has to be applied. In order to obtain the optimum result, it is necessary for the inital "cut off" of the divergent integrals to be taken in the same manner in all the cross sections in the sum. This cut off of divergent integrals is applied by means of a fictitious finite mass λ of the virtual photon, which is used in the photon propagatar to eliminate the infrared divergences (Berends *et al* 1973a, 1973b, Berestetskii *et al* 1982). This is why a method has to

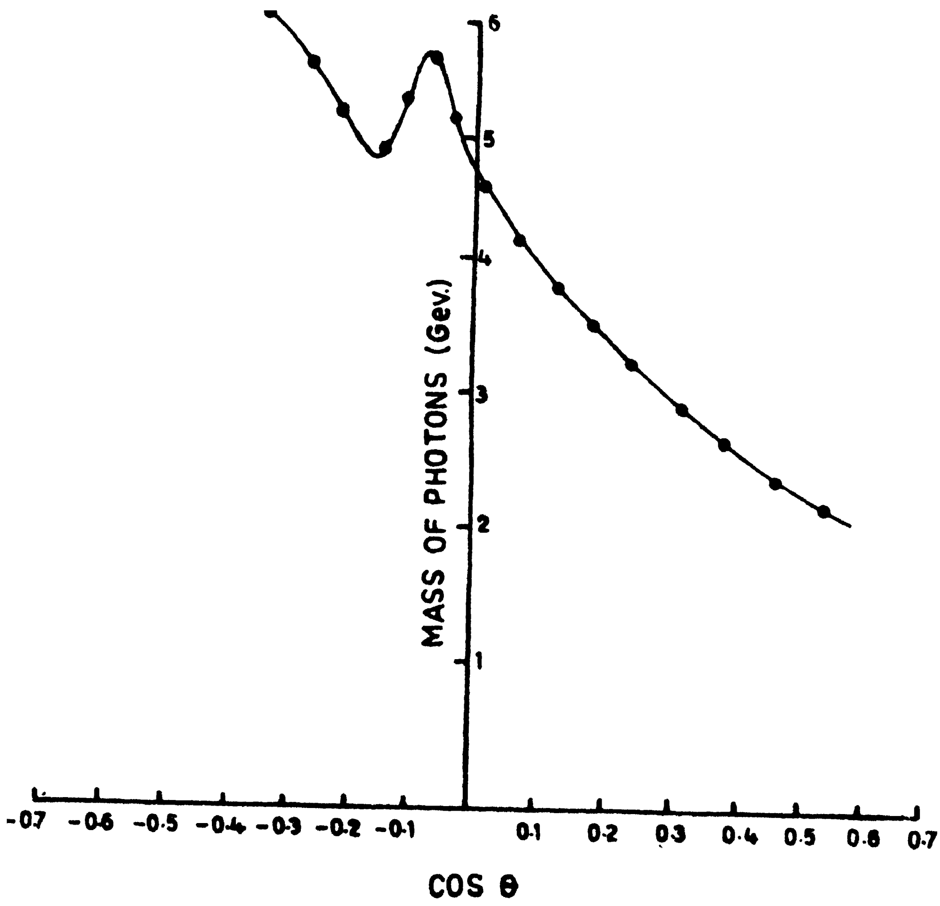

Figure 1. Behaviour of photon-mass as a function of cos θ.

be found out to eliminate the mass of the virtual photon. In this work an attempt has been made to estimate the fictitious mass of the virtual photon at 70 Gev c.m. energy. The experimental data are taken from the work of KEK group (Tobimatsu and Shimuzu 1986).

2. Method of calculation

The soft photon emission cross section is given by (Tobimatsu and Shimuzu 1986)

$$\left(\frac{d\sigma}{d\Omega}\right)_{soft} = \left\{\frac{d\sigma}{d\Omega}\right\}_0 R(k_0, \lambda). \tag{2.1}$$

$\left(\frac{d\sigma}{d\Omega}\right)_0$ is the lowest order cross section both in s and t channels in which z^0 exchange is included and is defined by

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \frac{\alpha^2}{4s}\left(\tau_0^{(s)} + \tau_0^{(t)} + \tau_0^{(\mathrm{Int})}\right), \tag{2.2}$$

where

$$\tau_0^{(s)} = (1+\cos^2\theta) + \frac{2s(s-M_z^2)}{D(s)}[C_v^2(1+\cos^2\theta) + 2C_A^2\cos\theta]$$

$$+\frac{s^2}{D(s)}[C_v^2+C_A^2)^2(1+\cos^2\theta) + 8C_v^2C_A^2\cos\theta], \tag{2.3}$$

$$\tau_0^{(t)} = 2\left\{\frac{s^2}{t^2}(1+\cos^2\theta\ 2) + \frac{2s^2}{t(t-M_z^2)}\left[(C_v^2-C_A^2)+(C_v^2+C_A^2)\cos^4\frac{\theta}{2}\right]\right.$$

$$\left.+\frac{s^2}{(t-M_z^2)^2}\left[(C_v^2-C_A^2)+(C_v^2+C_A^2)^2+4C_v^2C_A^2\cos^4\frac{\theta}{2}\right]\right\}, \tag{2.4}$$

$$\tau_0^{(\mathrm{Int})} = 4\cos^4\frac{\theta}{2}\left\{\frac{s}{t} + \left(\frac{s^2(s-M_z^2)}{tD(s)} + \frac{s}{t-M_z^2}\right)(C_v^2+C_A^2)\right.$$

$$\left.+\frac{s^2(s-M_z^2)}{D(s)(t-M_z^2)}(C_v^2+C_A^2)^2+4C_v^2C_A^2\right\}, \tag{2.5}$$

and

$$D(s) = (s-M_z^2)^2 + M_z^2\Gamma_z^2. \tag{2.6}$$

In these expressions the square of c.m. energy is denoted by s and the scattering angle by θ, respectively. M_z and Γ_z are the mass and width of the intermediate vector boson Z^0, respectively. α is the fine structure constant. If the boson mass is denoted by w and the mass of Higgs particle is denoted by M_H, we have

$$C_v = M_z^2\left(3-\frac{4M_w^2}{M_z^2}\right)/(4M_w\sqrt{M_z^2-M_w^2}),$$

$$C_A = M_z^2/(4M_w\sqrt{M_z^2-M_w^2}). \tag{2.7}$$

$R(k_0, \lambda)$ is given by

$$R(k_0, \lambda) = R_{ii} + R_{ff} + R_{if} \tag{2.8}$$

where

$$R_{ii} = \frac{2\alpha}{\pi}\left[\left(\log\left(\frac{s}{m_e^2}\right)-1\right)\log\left(\frac{2k_o}{\lambda}\right)-\frac{1}{4}\log^2\left(\frac{s}{m_e^2}\right)\right.$$

$$\left.+\frac{1}{2}\log\left(\frac{s}{m_e^2}\right)-\frac{\pi^2}{6}\right],$$

$$R_{ff} = R_{ii},$$

$$R_{if} = \frac{2\alpha}{\pi}\left[-4\log\left(\frac{\cos\theta/2}{\sin\theta/2}\right)\log\left(\frac{2K_o}{\lambda}\right)+\mathrm{Sp}\,(\cos^2\theta/2)\right.$$

$$\left.-\mathrm{Sp}\,(\sin^2\theta/2)-2\log\left(\frac{\cos\theta/2}{\sin\theta/2}\right)\log\,(\cos\theta/2.\,\sin\theta/2)\right], \tag{2.9}$$

and Sp is Spence function defined by

$$\mathrm{Sp}(Z) = -\int_0^Z \frac{dt}{t}\log\,(1-t). \tag{2.10}$$

$R(k_o, \lambda)$ is the radiative correction to Bhabha scattering due to multiple soft photon emissions while δ_v is the virtual radiative correction which is caused by the emission of virtual photons. These photons, obviously cause e^+e^- to interact electromagnetically. The radiative correction are classified into two groups ;

(a) virtual radiative corrections,

(b) real photon emissions.

For the sake of convenience the latter is also divided into two, soft and hard photon emissions. The virtual correction requires the calculation of all one loop diagrams. In QED, the corrections have been completely calculated including hard photon emission in a series of works by Berends and Gastmans (1973, 1978) and Berends et *al* (1974).

In eq. (2.9), the fictitious photon mass λ has been introduced, the value of which has to be computed. For one loop corrections, the lowest order cross section is given by (Tobimatsu and Shimuzu 1986)

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{virtual}} = \left(\frac{d\sigma}{d\Omega}\right)_0 \delta_v(\lambda) \tag{2.11}$$

where $\delta_v(\lambda)$ is the radiative correction due to virtual photons, which may consist of vertex, box and vacuum polarization, and the value of which is given by (Calva 1973)

$$\delta_v = 1-\frac{4\alpha}{\pi}\left\{\frac{23}{18}-\frac{11}{16}\log\left(\frac{2E}{m_e}\right)-\frac{11}{12}\cdot\frac{8+(1+x)^3}{(3+x^2)^2}\log\left(\frac{1-x}{2}\right)\right\}. \tag{2.12}$$

Here $x = \cos\theta$ and E is the beam energy. Combining above equations we obtain the corrected cross section as follows,

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_0 (1+\delta_{Tot}), \tag{2.13}$$

Table I. Fictitious photon mass.

$\cos\theta$	sp $(\cos^2\theta/2)$	sp $(\sin^2\theta/2)$	δ_v	δ_{total}	$R\ (k_c, \lambda)$	λ (GeV)
−0.925	−0.0378575	−1.3468148	−1.131	−0.048	1.083	4.3235×10^{-3}
−0.875	−0.0635046	−1.2970590	−1.101	−0.053	1.048	4.5119×10^{-3}
−0.825	−0.0894795	−1.2484184	−1.082	−0.059	1.023	4.6850×10^{-3}
−0.775	−0.1158322	1.2008608	1.068	−0.062	1.006	4.7040×10^{-3}
−0.725	−0.1425377	−1.1543548	1.056	−0.066	0.099	4.8025×10^{-3}
−0.675	−0.1696219	1.1088694	−1.046	−0.067	0.979	4.7471×10^{-3}
−0.625	−0.2058877	−1.0643743	−1.038	−0.068	0.97	4.6523×10^{-3}
−0.575	−0.2249825	−1.0208398	−1.030	−0.070	0.96	4.6533×10^{-3}
0.525	−0.2532889	0.9782366	1.024	−0.071	0.953	4.5408×10^{-3}
−0.475	−0.282033	0.9365362	−1.018	−0.072	0.946	4.0653×10^{-3}
0.425	−0.3112314	−0.8957106	−1.013	0.073	0.94	4.7397×10^{-3}
−0.375	−0.3409009	−0.8557324	1.008	−0.073	0.935	4.2017×10^{-3}
0.325	0.3710589	−0.8165748	1.003	−0.072	0.931	4.0291×10^{-3}
0.275	−0.4015043	−0.7782115	0.999	−0.073	0.926	5.3585×10^{-3}
−0.225	0.4329133	0.7406169	−0.994	−0.074	0.92	3.8539×10^{-3}
−0.175	−0.4646474	−0.7037658	−0.99	−0.073	0.917	3.6683×10^{-3}
0.125	−0.4969438	0.667634	0.987	−0.074	0.913	3.5338×10^{-3}
0.075	0.5298287	0.6321973	0.983	−0.074	0.909	3.4046×10^{-3}
0.025	−0.5633171	−0.5974326	0.979	−0.073	0.906	4.2728×10^{-3}
0.025	−0.5974326	0.5633171	−0.975	−0.073	0.902	3.1180×10^{-3}
0.075	0.6321973	0.5298287	−0.971	−0.072	0.899	2.9697×10^{-3}
0.125	−0.6676343	−0.4969458	−0.968	−0.072	0.896	2.7854×10^{-3}
0.175	0.7037658	−0.4646474	−0.964	−0.071	0.893	2.6536×10^{-3}
0.225	−0.7406169	0.4329133	−0.96	−0.068	0.892	2.4387×10^{-3}
0.275	−0.7782115	0.4015043	−0.955	−0.067	0.888	2.3205×10^{-3}
0.325	−0.8165748	−0.3710589	0.951	−0.065	0.886	2.1419×10^{-3}
0.375	−0.8557384	0.3409009	−0.0946	−0.063	0.883	1.9922×10^{-3}
0.425	0.8957106	−0.3112314	0.942	−0.059	0.883	1.7673×10^{-3}
0.475	−0.9365362	−0.2820330	−0.936	−0.058	0.878	1.6338×10^{-3}
0.525	−0.9782366	0.2532889	−0.931	−0.055	0.876	1.4466×10^{-3}
0.575	−1.0208398	0.2249826	−0.925	−0.049	0.876	1.2749×10^{-3}
0.625	−1.0643745	0.2058877	−0.919	−0.045	0.874	1.1045×10^{-3}
0.625	−1.1088694	−0.1696219	0.912	−0.039	0.873	9.1922×10^{-4}
0.725	−1.1543548	0.1425377	−0.904	−0.032	0.872	7.3960×10^{-4}
0.775	1.2008608	−0.1158322	−0.895	−0.024	0.871	5.6572×10^{-4}
0.825	−1.2484184	0.0894793	0.884	−0.012	0.872	3.8609×10^{-4}
0.875	−1.297059	−0.0635046	−0.869	$\pm$0.001	9.87	3.8609×10^{-4}
0.925	−1.3468148	−0.0578575	−0.849	+0.021	0.87	1.0153×10^{-4}

where δ_{Tot} is the total radiative correction term. Using the value δ_{Tot} from KEK data, the fictitious photon mass is computed by the eq. (2.9).

3. Results and discussions

The results of the present calculations for Bhabha scattering are presented in Table 1. The experimental value of the virtual radiative correction effect δ_v and contribution due to all the types of emitted radiations (KEK data) are presented in columns 4, 5 and 6 of the Table, respectively. The radiative correction due to soft photons is presented in column 6 of the Table (KEK data).

The value of radiative correction decreases as the angle of scattering increases. The fictitious photon mass λ computed corresponding to radiative correction $R(k_o, \lambda)$ is presented in column 4 of the Table 1. In general, the value of fictitious photon mass decreases monotonously with increase in the value of the scattering angle. However, near 90° angle of scattering the mass undergoes a substantial variation showing rise and fall in its value. The fictitious photon mass is related to cut off applied to remove divergences as mentioned in Section 1. Obviously, the mass must be related to the cut off used, and the behaviour of the mass as a function of cos θ, probably reflects the nature of experimental data as obtained by KEK group.

4. Conclusion

The fictitious photon mass decreases monotonously with increase in the value of scattering angle.

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