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Goodness of fit tests for logistic distribution based on Phi-divergence

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Some goodness of fit tests for logistic distribution based on Phi-divergence are developed. The powers of the introduced tests are compared with some traditional goodness of fit tests including Kolmogorov-Smirnov, Anderson-Darling and Cramer-von Mises tests for logistic distribution using Monte Carlo simulation. It is shown the proposed tests have good performance as compared with their competitors in the literature. A real data set is used for illustration.

Keywords: Logistic distribution; Phi-divergence; Goodness of fit tests; Kolmogorov-Smirnov; Anderson-Darling.

1 Introduction

A random variable X is said to have logistic distribution with location parameter $\mu \in \Re$ and the scale parameter $\sigma > 0$ if it has the cumulative distribution function (CDF) defined as

$$F(x;\mu,\sigma) = \frac{1}{1+e^{-\frac{x-\mu}{\sigma}}}; \quad -\infty < x < \infty. \tag{1}$$

The corresponding probability density function (PDF) is given by

$$f(x;\mu,\sigma) = \frac{e^{-\frac{x-\mu}{\sigma}}}{\sigma\left(1+e^{-\frac{x-\mu}{\sigma}}\right)^2}; \quad -\infty < x < \infty. \tag{2}$$

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Note that the PDF of the logistic distribution is symmetric about its location parameter μ. Throughout this paper, the logistic distribution will be denoted by $Lo(\mu, \sigma)$. For the $Lo(\mu, \sigma)$ distribution, the mean, median and the mode are all equal to μ and the variance is given by $\frac{\sigma^2\pi}{3}$.

Phi-divergence between two probability distributions Q and G was firstly introduced by Csiszar (1963) and is given by

$$L_\phi\left(Q|G\right) = \int \phi\left(\frac{dQ}{dG}\right) dG, \phi \in \Psi, \tag{3}$$

where Q is absolutely continuous with respect to G, Ψ is the class of all convex functions and $\phi(.)$ is an arbitrary convex function, $\phi : [0, \infty) \to \Re \bigcup \{\infty\}$ such that $\phi(1) = 0$ and $\phi''(1) > 0$. It can be shown that if $Q = G$ then $L_\phi\left(Q|G\right) = 0$. For more details about these properties see Csiszar (1967) and Vajda (1989).

Alizadeh Noughabi and Balakrishnan (2016) was the first who utilized Phi-divergence for goodness-of-fit tests for normal, exponential, uniform and laplace distributions which were followed by Zamanzade and Mahdizadeh (2017) for Rayleigh distribution. However, the problem of goodness of fit tests has been already considered by many researchers in the literature including Puig (2000), Zhang and Cheng (2003), Zamanzade and Arghami (2011), Zamanzade and Arghami (2012), Alizadeh Noughabi and Arghami (2016), Zamanzade (2015), Al-Omari and Haq (2016), Al-Omari and Zamanzade (2016), Alizadeh Noughabi and Park (2016), Mahdizadeh and Zamanzade (2017a), Mahdizadeh and Zamanzade (2017b), Al-Omari and Zamanzade (2017) and Zamanzade (2018).

The rest of this paper is organized as follows. In Section 2 the tests of fit based on Phi-divergence for the logistic distribution are presented. In Section 3, critical values of the suggested tests are obtained, and they are compared with the competitor tests in terms of power. A real data example is provided and analyzed in Section 4. Some conclusions are given in Section 5.

2 Tests of fit for logistic distribution based on Phi-divergence

Let f_0 and f be probability density functions under null and alternative hypotheses, respectively. We can construct goodness of fit tests by choosing different convex functions for $\phi(.)$ in $L_\phi\left(Q|G\right)$. Some well-known measures are obtained as follows:

1. Kullback-Liebler (KL) distance is obtained by choosing $\phi(z) = z \log z$ which yields to
$$KL = \int f(x) \log\left(\frac{f(x)}{f_0(x)}\right) dx = E_f\left(log\left(\frac{f(x)}{f_0(x)}\right)\right).$$
2. Hellinger (H) distance is obtained by choosing $\phi(z) = \frac{1}{2}\left(\sqrt{z} - 1\right)^2$ which yields to
$$H = \frac{1}{2}\int\left(\sqrt{f(x)} - \sqrt{f_0(x)}\right)^2 dx = \frac{1}{2}E_f\left[\left(\sqrt{\frac{f_0(x)}{f(x)}} - 1\right)^2\right].$$

3. Jeffreys (J) distance is obtained by choosing $\phi(z) = (z-1)\log z$ which yields to $J = \int (f(x) - f_0(x))\, log\left(\frac{f(x)}{f_0(x)}\right) = E_f\left[\left(1 - \frac{f_0(x)}{f(x)}\right)\log\left(\frac{f(x)}{f_0(x)}\right)\right]$.

4. Total variation (TV) distance is obtained by choosing $\phi(z) = \frac{1}{2}|z-1|$ which yields to $TV = \int |f(x) - f_0(x)|\, dx = E_f\left(\left|1 - \frac{f_0(x)}{f(x)}\right|\right)$.

5. Chi-square (χ^2) distance is obtained by choosing $\phi(z) = (z-1)^2$ which yields to

$$\chi^2 = \int \frac{(f(x) - f_0(x))^2}{f_0(x)} dx = E_f\left[\left(\frac{f(x) - f_0(x)}{f_0(x)f(x)}\right)^2\right].$$

Let $X_1, X_2, ..., X_n$ be a simple random sample size n from the population of interest. Suppose that we are interested in testing the null hypothesis

$H_0 : f(x) = f_0\,(x;\mu,\sigma) = \frac{e^{-\frac{x-\mu}{\sigma}}}{\sigma\left(1+e^{-\frac{x-\mu}{\sigma}}\right)^2};\quad -\infty < x < \infty$ for some $\mu \in \Re$, $\sigma \in \Re^+$,

against the alternative hypothesis

$H_1 : f(x) \neq f_0\,(x;\mu,\sigma)$ for all $\mu \in \Re$, $\sigma \in \Re^+$.

We propose to use the sample version of Phi-divergence as follows

$$L_n = \frac{1}{n}\sum_{i=1}^{n} \frac{f_0\,(x_i;\hat{\mu},\hat{\sigma})}{\hat{f}\,(x_i)}\phi\left(\frac{\hat{f}\,(x_i)}{f_0\,(x_i;\hat{\mu},\hat{\sigma})}\right),$$

where $\hat{\mu}$ and $\hat{\sigma}$ are maximum likelihood estimators of μ and σ, respectively, $\hat{f}(x) = \frac{1}{nh}\sum_{r=1}^{n} k\left(\frac{x-x_r}{h}\right)$ is kernel density estimator of $f(x)$, and the density of standard normal distribution is used as kernel function $k(.)$, and the bandwidth is selected to be as $h = \left(\frac{4s^2}{3n}\right)^{0.2}$, which is proposed by Silverman (1986). The maximum likelihood estimators of μ and σ are obtained using *optim*() function in R statistical software.

Therefore, the null hypothesis which states that the parent distribution follows a logistic distribution is rejected if

$$L_n \geq L_n(1-\alpha),$$

where $L_n(\alpha)$ is the αth quantile of distribution of L_n under null hypothesis.

Consistency of the above test statistic follows from the fact that $\hat{\mu}$ and $\hat{\sigma}$ are both consistent estimator of μ and σ, respectively. Therefore, $f_0\,(x;\hat{\mu},\hat{\sigma}) \to f_0\,(x;\mu,\sigma)$ and $\hat{f}\,(x) \to f\,(x)$ in probability as n goes to infinity. Thus, the consistency of the proposed test statistic follows from the law of large numbers.

3 Simulation study

In what follows, we denote the test statistics obtained using $\phi(.)$ function corresponding to Kullback-Liebler, Hellinger, Jeffrey, Total variation and Chi-square distances as KL_n,

Table 1: Critical values of different goodness of fit tests based on phi-divergence for logistic distribution

n	α	KL_n	H_n	J_n	TV_n	χ^2_n
	0.01	0.158	0.031	0.282	0.297	1.270
10	0.05	0.114	0.022	0.187	0.284	0.453
	0.10	0.091	0.018	0.150	0.277	0.281
	0.01	0.140	0.021	0.200	0.280	1.559
20	0.05	0.103	0.016	0.138	0.254	0.432
	0.10	0.082	0.014	0.115	0.241	0.250
	0.01	0.117	0.016	0.152	0.262	1.468
30	0.05	0.087	0.013	0.110	0.234	0.357
	0.10	0.072	0.011	0.095	0.220	0.215
	0.01	0.100	0.013	0.125	0.244	1.261
40	0.05	0.076	0.011	0.095	0.219	0.319
	0.10	0.063	0.010	0.082	0.204	0.191
	0.01	0.093	0.012	0.111	0.233	1.096
50	0.05	0.068	0.010	0.084	0.207	0.289
	0.10	0.057	0.009	0.073	0.193	0.166

H_n, J_n, TV_n and χ^2_n, respectively. The critical values of these goodness of fit tests are obtained using Monte Carlo simulation with 100,000 repetitions and the results are reported in Table 1.

We note that as the significance level values increase the critical values are decreased.

We now compare the power of different goodness of fit tests based on phi-divergence with some standard goodness of fit tests in the literature for logistic distribution. Let $X_{(1)}, X_{(2)}, \cdots, X_{(n)}$ be an ordered simple random sample of size $n = 10, 20, 50$ from population of interest, let F_0 be CDF of logistic distribution, $\hat{\mu}$ and $\hat{\sigma}$ be MLE of the parameters μ and σ, respectively. The competing goodness of fit test statistics are as follows:

1. The Kolmogorov-Smirnov (Kolmogorov, 1933) test statistic:

$$KS = Max\left\{\underset{1\leq i\leq n}{Max}\left[\frac{i}{n} - F_0\left(X_{(i)};\hat{\mu},\hat{\sigma}\right)\right], \underset{1\leq i\leq n}{Max}\left[F_0\left(X_{(i)};\hat{\mu},\hat{\sigma}\right) - \frac{i-1}{n}\right]\right\}.$$

2. The Anderson-Darling (Anderson and Darling, 1954) test statistic:

$$A^2 = -\frac{2}{n}\sum_{i=1}^{n}\left\{\left(i-\frac{1}{2}\right)\log\left[F_0\left(X_{(i)};\hat{\mu},\hat{\sigma}\right)\right] + \left(n-i+\frac{1}{2}\right)\log\left[1-F_0\left(X_{(i)};\hat{\mu},\hat{\sigma}\right)\right]\right\} - n.$$

3. The Cramer-von Mises (Von Misese, 1932) test statistic:

$$W^2 = \sum_{i=1}^{n}\left[F_0\left(X_{(i)},\hat{\mu},\hat{\sigma}\right) - \frac{2i-1}{2n}\right] + \frac{1}{12n}.$$

In order to compare the power of different goodness of fit tests, we have generated 100,000 simple random samples from the following alternative distributions. We then estimate the power of each test by dividing the numbers of test statistic values are greater than its critical value at significance level $\alpha = 0.05$. The alternative distributions considered in this study are as follows.

- Standard normal distribution, denoted by $N(0,1)$.
- Student's T distribution with 10 degrees of freedom, denoted by $T(10)$.
- Student's T distribution with 3 degrees of freedom, denoted by $T(3)$.
- Standard Cauchy distribution denoted by $C(0,1)$.
- Standard Laplace distribution, denoted by $L(0,1)$.
- Skew normal distribution with shape parameter 0.5, denoted by $SN(0.5)$.
- Skew normal distribution with shape parameter 2, denoted by $SN(2)$.
- Skew normal distribution with shape parameter 3, denoted by $SN(3)$

- Standard exponential distribution, denoted by $Exp(1)$.
- Lognormal distribution with log mean zero and log standard deviation 0.5, denoted by $LN(0.5)$.
- Lognormal distribution with log mean zero and log standard deviation 1, denoted by $LN(1)$.
- Weibull distribution with shape parameter 0.5 and scale parameter 1, denoted by $W(0.5)$.
- Weibull distribution with shape parameter 2 and scale parameter 1, denoted by $W(2)$.
- Standard uniform distribution, denoted by $U(0,1)$.
- Beta distribution with parameters 2 and 2, denoted by $B(2,2)$.
- Beta distribution with parameters 0.5 and 0.5, denoted by $B(0.5,0.5)$.
- Beta distribution with parameters 2 and 1, denoted by $B(2,1)$.
- Beta distribution with parameters 0.5 and 1, denoted by $B(0.5,1)$.

The bolded fonts in the Tables 2-4 are the largest power values based on each distribution. However, the largest values are for the test based on total variation distance for $n = 20, 50$, while for $n = 10$, the maximum values are for Anderson-Darling test.

4 A real data example

In this section, a real data set is considered to explain the suggested tests. The following data set was firstly considered by Bain and Englehardt (1973) which consists of 33 differences in flood levels between two stations on a river. The data are as follows:

```
1.96, 1.97, 3.60, 3.80, 4.79, 5.66, 5.76, 5.78, 6.27, 6.30,6.76, 7.65,
7.84, 7.99, 8.51, 9.18, 10.13, 10.24, 10.25, 10.43, 11.45, 11.48, 11.75,
11.81, 12.34, 12.78, 13.06, 13.29, 13.98, 14.18, 14.40, 16.22, 17.06.
```

We now perform the introduced goodness-of-fit tests for logistic distribution for the above data set. For this purpose, the ML estimates of location and scale parameters of logistic distribution are obtained as $\hat{\mu} = 9.404$ and $\hat{\sigma} = 2.361$. Thus, the values of the considered test statistics (the critical values at significance level $\alpha = 0.05$) are given by

$$KS = 0.097(0.131), A^2 = 0.326(0.654), W^2 = 0.053(0.097), TV = 0.209(0.230),$$

$$KL = 0.038(0.079), H = 0.007(0.011), J = 0.060(0.101), \chi^2 = 0.065(0.184).$$

We observe that none of the values of the above test statistics are larger than their corresponding critical values, and therefore the assumption that data follow logistic distribution is not rejected at the significance level 0.05.

Table 2: Estimated powers for different tests of size 0.05 when $n = 10$

Dist.	KS	AD	CM	KL_n	H_n	J_n	TV_n	χ_n^2
N(0,1)	0.049	0.039	0.047	0.042	0.022	0.022	**0.052**	0.022
T(10)	**0.050**	0.047	0.048	0.047	0.042	0.042	**0.050**	0.042
T(3)	0.084	0.113	0.090	0.118	0.148	**0.149**	0.085	0.147
C(0,1)	0.447	0.524	0.483	0.474	**0.534**	0.533	0.388	0.523
L(0,1)	0.073	0.093	0.077	0.087	**0.112**	**0.112**	0.066	0.109
SN(0.5)	**0.048**	0.037	0.045	0.039	0.021	0.021	0.051	0.021
SN(2)	**0.053**	0.050	0.053	0.050	0.033	0.033	0.051	0.033
SN(3)	0.063	**0.066**	**0.066**	0.068	0.045	0.045	0.051	0.046
Exp(1)	0.232	**0.339**	0.281	0.304	0.219	0.219	0.079	0.220
LN(0.5)	0.112	0.166	0.134	**0.168**	0.143	0.143	0.067	0.144
LN(1)	0.374	**0.508**	0.430	0.468	0.377	0.378	0.139	0.378
W(0.5)	0.780	**0.852**	0.808	0.751	0.597	0.598	0.239	0.594
W(2)	0.063	0.062	0.065	**0.068**	0.036	0.036	0.061	0.037
U(0,1)	0.099	0.094	0.117	**0.119**	0.007	0.007	0.103	0.006
B(2,2)	0.063	0.048	0.064	0.059	0.006	0.006	**0.072**	0.006
B(0.5,0.5)	0.241	0.297	0.317	**0.322**	0.017	0.016	0.168	0.012
B(2,1)	0.102	0.116	**0.123**	0.120	0.030	0.030	0.081	0.029
B(0.5,1)	0.247	**0.312**	0.311	0.279	0.060	0.058	0.107	0.053

Table 3: Estimated powers for different tests of size 0.05 when $n = 20$

Dist.	KS	AD	CM	KL_n	H_n	J_n	TV_n	χ^2_n
N(0,1)	0.052	0.042	0.051	0.039	0.013	0.011	**0.086**	0.008
T(10)	0.052	0.046	0.049	0.047	0.040	0.040	**0.059**	0.040
T(3)	0.108	0.158	0.123	0.194	0.236	0.241	0.035	**0.243**
C(0,1)	0.711	0.785	0.757	0.770	0.799	**0.801**	0.346	0.789
L(0,1)	0.100	0.124	0.112	0.116	0.149	**0.150**	0.019	0.146
SN(0.5)	0.053	0.043	0.052	0.040	0.014	0.012	**0.090**	0.010
SN(2)	0.064	0.067	0.067	0.062	0.036	0.033	**0.092**	0.027
SN(3)	0.083	0.110	0.100	0.105	0.064	0.059	**0.112**	0.044
Exp(1)	0.507	**0.698**	0.561	0.663	0.529	0.501	0.215	0.361
LN(0.5)	0.182	0.337	0.239	**0.353**	0.303	0.293	0.121	0.237
LN(1)	0.712	**0.855**	0.754	0.841	0.763	0.745	0.174	0.635
W(0.5)	0.992	**0.996**	0.988	0.989	0.954	0.946	0.240	0.861
W(2)	0.081	0.103	0.095	0.103	0.050	0.044	**0.149**	0.031
U(0,1)	0.181	0.242	0.245	0.309	0.045	0.019	**0.591**	0.000
B(2,2)	0.089	0.083	0.099	0.105	0.010	0.005	**0.267**	0.000
B(0.5,0.5)	0.488	0.699	0.648	0.742	0.321	0.187	**0.921**	0.001
B(2,1)	0.183	0.284	0.253	0.277	0.089	0.061	**0.409**	0.012
B(0.5,1)	0.516	0.686	0.623	0.646	0.313	0.233	**0.724**	0.029

Table 4: Estimated powers for different tests of size 0.05 when $n = 50$

Dist.	KS	AD	CM	KL_n	H_n	J_n	TV_n	χ^2_n
N(0,1)	0.064	0.058	0.069	0.044	0.035	0.020	**0.159**	0.002
T(10)	0.052	0.048	0.051	0.047	0.044	0.040	**0.073**	0.038
T(3)	0.171	0.259	0.210	0.352	0.366	0.392	0.019	**0.404**
C(0,1)	0.966	**0.983**	0.980	0.982	0.981	0.982	0.465	0.976
L(0,1)	0.175	**0.215**	**0.215**	0.188	0.188	0.200	0.006	0.196
SN(0.5)	0.066	0.060	0.070	0.045	0.035	0.020	**0.160**	0.002
SN(2)	0.091	0.135	0.115	0.105	0.097	0.069	**0.197**	0.016
SN(3)	0.146	**0.282**	0.210	0.233	0.222	0.168	0.302	0.034
Exp(1)	0.973	0.994	0.954	**0.994**	0.992	0.985	0.784	0.629
LN(0.5)	0.432	0.768	0.564	**0.790**	0.784	0.734	0.439	0.405
LN(1)	0.994	0.999	0.993	**0.999**	0.999	0.999	0.494	0.923
W(0.5)	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	0.421	0.997
W(2)	0.148	0.291	0.216	0.272	0.247	0.177	**0.431**	0.021
U(0,1)	0.459	0.748	0.650	0.854	0.873	0.799	**0.979**	0.000
B(2,2)	0.187	0.262	0.253	0.325	0.321	0.223	**0.697**	0.000
B(0.5,0.5)	0.925	0.997	0.987	0.999	**1.000**	0.999	**1.000**	0.000
B(2,1)	0.499	0.785	0.653	**0.794**	0.781	0.681	0.931	0.003
B(0.5,1)	0.967	0.996	0.979	0.997	0.997	0.992	**1.000**	0.030

5 Concluding remarks

In this paper, we proposed general goodness of fit tests using Phi-divergence for the logistic distribution. The critical and power values of the proposed tests are obtained for different sample sizes using Monte Carlo simulation. Our simulation results showed the preference of the proposed procedures in practice.

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