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Attitudes of Pre-Service Mathematics Teachers towards Modelling: A South African Inquiry

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ABSTRACT

This study explores the attitudes of mathematics pre-service teachers, based on their initial exposure to a model-eliciting challenge. The new Curriculum and Assessment Policy Statement determines that mathematics students should be able to identify, investigate and solve problems via modelling. The unpreparedness of mathematics teachers in teaching modelling is widely recognized. Modelling was thus included for the first time in the mathematics education curriculum of a South African university. Based on their modelling exposure, the participants revealed their attitudes via an *Attitudes towards Mathematics Modelling Inventory*. The Mann Whitney U-test detected significant differences between gender and achievement groups. Most participants displayed positive attitudes towards modelling, even after this brief exposure. The main implications of the study are that the modelling competencies of mathematics pre-service-teachers could be strengthened during their formal education by lecturers that adopt an appropriate modelling pedagogy that takes cognizance of their attitudes, while gradually building their confidence

Keywords: Teacher attitudes towards modelling, Mathematical modelling[1], Mathematics pre-service teachers, Mathematics teacher education, Mathematics achievement and gender

BACKGROUND CONTEXT AND PURPOSE

Authentic problem solving is progressively used to great effect in enhancing students' mathematical competencies and mathematics teachers' pedagogical content knowledge (PCK), as well as their mathematical content knowledge (MCK) (Buchholtz & Mesrogli, 2013). What is especially comforting is that the relationship between mathematical modelling, increasingly emphasised in schools' mathematics curricula in several countries over the last decade or so, including South Africa (CAPS, 2011), and authentic learning has been proven beyond doubt (Campbell, Oh, Maughn, Kiriazis & Zuwallack, 2015; Kang & Noh, 2012). Not only is it the responsibility of mathematics teacher education programmes to support pre-service teachers in developing their own MCK and PCK 'repertoires' and to get them actively engaged

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State of the literature

- Mathematics education programmes increasingly engage pre-service teachers in authentic problem solving. The latter advances teachers' mathematical and pedagogical content knowledge.
- Modelling, the generation of mathematical representations geared at solving authentic problems, is progressively incorporated into school curricula. However, mathematics teachers are generally underprepared to teach and to appreciate modelling, if they were not actively involved in model-eliciting tasks during their pre-service education.
- Pre-service teachers' conceptions on how mathematics should be taught mainly relate to experiences during their formal education and play an important role in their own and their students' attitudes.

Contribution of this paper to the literature

- Pre-service teachers' initial attitudes toward mathematical modelling are positively shaped by an appropriate modelling pedagogy, based on a combination of group work and interactive questioning practices and strategies.
- Pre-service mathematics teachers should ideally be engaged in model-eliciting challenges during their formal education, aimed at enhancing their pedagogical and content knowledge, while also gradually strengthening their dispositions and especially their confidence.
- A number of well-considered teaching and learning principles were deduced from the inquiry, which might be used by mathematics teachers and educators in establishing a conducive culture towards the understanding, teaching and appreciation of mathematical modelling in their classrooms.

in knowledge-in-action, but they also need to shape and refine their attitudes, beliefs and dispositions.

Beliefs and attitudes are the primary watchdogs for mathematics teachers' professional classroom behavior and they profoundly impact on decision making in any mathematics classroom. Pre-service teachers' dispositions, seen as a combination of their beliefs about the nature of and attitudes towards the learning of mathematics (Cooke, 2015), have a pertinent influence on their initial teaching strategies and practices. Beswick (2012) has linked Ernest's (1989) three categories of teachers' beliefs about the nature and learning of mathematics to Van Zoest, Jones and Thornton's (1994) views on the teaching of mathematics. Her connection (Beswick, 2012, p. 129–130) firstly reveals that if mathematics is seen as "an accumulation of facts, skills and rules" (Ernest, 1989, p. 250), learning is typically viewed as a passive reception of knowledge and teaching will then primarily focus on content and optimal student achievement. If mathematics is regarded as a "pre-existing knowledge awaiting discovery" (Beswick, 2012, p. 129), learning typically involves active meaning-making, while teaching will usually be geared at an understanding of the content. However, in Ernest's (1989) third category of teachers' beliefs, mathematics is regarded as process, rather than product based, learning is aimed at problem-solving and teaching is geared at maximizing students' interests and learning. Olanoff, Lo and Tobias (2014) actively campaign for teacher education

programmes that strive to cultivate mathematics teachers that subscribe to the latter category, essentially viewing mathematics as a process, with authentic problem solving at the core of teaching and learning.

With problem-solving and modelling featuring prominently in many mathematics teacher education curriculums nowadays, the ideal is that qualified teachers should increasingly model modelling in their classrooms. However, Karali and Durmus (2015), Ng (2013) and Ikeda (2013) caution against the un- and under preparedness of teachers in respect of a thorough comprehension of and also the teaching of modelling. The open-endedness of model-eliciting activities as well as the nurturing of a classroom culture conducive towards modelling are relentless challenges for them. Soon and Cheng (2013) are of the opinion that teachers may not be able to appreciate the benefits and importance of developing their students' mathematical modelling competencies if they themselves were not adequately exposed to such tasks and activities.

Papageorgiou (2009, p. 7) reports that many mathematics students "separate their mathematical knowledge in formal school mathematics and informal 'everyday' mathematics, and are then unable to connect the two". Such students are then regularly unable to solve problems, which demand both 'everyday' and school mathematics. Many mathematics students commonly believe that answers to problems are either right or wrong (Schoenfeld, 1989 and 2007), while Wilkie (2016) discovers that there even seems to be resistance among the majority of high school students against the use of more than one strategy for problem solving. The aforementioned makes it difficult for students to connect mathematical challenges to everyday activities, which often causes higher levels of anxiety and worse performances than what would reflect their abilities.

The connection between students' attitudes towards and achievement in mathematics has been acknowledged and confirmed many times (Demirel & Dağyar, 2016; Ma & Kishor, 1997; Young-Loveridge, Bicknell & Mills, 2012; Zakaria, Zain, Ahmad & Erlina, 2012; among others). The attitude-achievement connection manifests in both directions: firstly, higher achievers tend to have more positive attitudes toward mathematics and secondly, students' attitudes towards mathematics determine their level of engagement, the quality of their learning and eventually also their performance.

The role that *gender* plays in mathematics achievement at secondary school and higher education levels has been the focus of research for many years. Two American economists, Niederle and Vesterlund (2010, p. 130) report as follows: "Over the past 20 years the fraction of males to females who score in the top five percent in high school math has remained constant at two to one". In searching for potential reasons for this achievement discrepancy, they refer to the research of childhood educationists, Berenbaum, Martin, Hanish, Briggs & Fabes (2008), who highlight two important gender differences already visible at a young age. The first is that boys have and develop superior spatial orientation skills and the second is that

"boys tend to engage in play that is more movement oriented and therefore grow up in more spatially complex environments" (Niederle & Vesterlund, 2010, p. 130).

This achievement 'gap' seems to perpetuate into higher education, and even manifests in formal mathematics teacher education programmes. For example, the Australians, Uusimaki and Nason (2004, p. 370), conducting research on the negative beliefs and anxieties of pre-service mathematics teachers in Australia, ascribe participants' negative beliefs and anxiousness to personality factors, which include "unwillingness to ask questions due to shyness, low self-esteem and for females viewing mathematics as a male domain". They conclude (2004, p. 374) that the origin of the majority of these pre-service teachers' negative beliefs about mathematics and accompanying anxiety could be attributed to prior school experiences and more specifically to underprepared and non-supportive teachers. The latter relates directly to the focus of this inquiry, namely to pro-actively (already during their formal education) detect possible gender- and achievement-based attitudinal differences of pre-service mathematics teachers in respect of a complex theme like mathematical modelling.

The *purpose* of the paper is thus to explore the attitudes of a group of third year mathematics pre-service teachers at a South African university towards mathematical modelling, based on their initial exposure to model-eliciting challenges. The two *research questions* that the paper attempts to answer are:

- What is the nature of pre-service teachers' attitudes towards mathematical modelling?
- Are there differences between the attitudes of the two genders, as well as between higher and lower performing participants?

The research project, of which this inquiry forms part, strives to deduce a set of guidelines aimed at the effective integration of mathematical modelling into the formal education of pre-service (Grade 10–12) mathematics teachers.

LITERATURE PERSPECTIVES

Theoretical framework underlying this inquiry

Based on their vast experience as educators of and as mathematics teachers, the authors are of the opinion that the pre-service education of mathematics teachers in the current South African higher and school education context, has a fundamental influence on their initial practices, beliefs, attitudes and teaching. The first element of the theoretical framework that underlies this inquiry is the *Learning to Teach Secondary Mathematics (LTSM)* framework (Peressini, Borko, Romagnano, Knuth & Willis, 2004, p. 68), grounding its views on learning-to-teach activities and processes in mathematics by two statements viewed through a situative lens. The first assertion is that how a mathematics student acquires a particular set of knowledge and skills and the specific teaching context (situation) in which it happens profoundly influence what is eventually learned (Greeno, Collins & Resnick, 1996). The second

claim, attributed to Adler (2000), herself an experienced South African mathematics educator, is that teachers' knowledge, beliefs and attitudes interact with teaching-learning situations, having the implication that mathematics teacher education is "usefully understood as a process of increasing participation in the practice of teaching, and through this participation, a process of becoming knowledgeable in and about teaching" (p. 37).

The second element of this study's theoretical framework is attributed to the views of Schackow (2005) on *teacher beliefs and attitudes*. Beliefs are seen as the subjective ways in which teachers understand their role(s) in a classroom, their students, the potential contributors to and determinants of learning, the teaching environment, and the goals of education (Schackow, 2005, p. 12). Mathematics teachers' conceptions of how mathematical themes should be taught are deeply rooted, usually relate to their own experiences as mathematics students (mostly during their formal education) and are not easy to change. However, mathematics teachers' beliefs are primarily rational in nature, supporting Dede's (2006) discovery that teachers' beliefs and attitudes can change as a result of appropriate experiences during their pre-service education.

In summary, the theoretical framework, acting as the lens through which this inquiry is viewed, primarily, in respect of a theme like mathematical modelling, determines that the pre-service education of mathematics teachers should:

- be geared at their participation in the mathematical practices underlying the theme
- influence how they approach, understand and teach the theme and
- ultimately impact on their own beliefs about and attitudes towards the theme.

Literature perspectives on mathematical modelling[1]

Conceptualisation

A model is a visualisation of something that cannot be directly observed via a description or a resemblance (Kang & Noh, 2012). Lesh and Doerr (2003) regard models as theoretical or conceptual systems that are used in an abstract form for a specific purpose. Models are social initiatives and should be reusable in different situations (Greer, 1997). Whereas the end-product is known as a *model*, the cognitive activities preceding it, which involve and require reasoning can be labelled as *modelling*.

Modelling is a cyclical process involving (1) the creation of a provisional model, which stems from (2) a series of interactive activities, which should be (3) continually tested and refined in order to improve or verify it (Kang & Noh, 2012). The modelling process can, at any stage, incorporate various forms of language, like computer programmes, sketches, drawings, tables, spreadsheets, and others.

M*athematical modelling* is the process of generating mathematical representations in attempting to solve real life problems (English, Fox & Watters, 2005; Greer, 1997; Ikeda, 2013). A mathematical modelling cycle typically consists of four sequential phases (Balakrishnan,

Yen & Goh, 2010, p. 237–257), namely "mathematisation" (representing a real-world problem mathematically), "working with mathematics" (using appropriate mathematics to solve the problem), "interpretation" (making sense of the solution in terms of its relevance and appropriateness to the real-world situation) and "reflection" (examining the assumptions and subsequent limitations of the suggested solution). These representations are then, according to Ang (2010), validated, applied and continuously refined.

Model-eliciting tasks versus mathematical applications

The International Community for the Teaching of Mathematical Modelling and Applications (ICTMA, in Stillman, Gailbrath, Brown & Edwards, 2007, p. 689), fittingly distinguishes mathematics applications from modelling. Applications attempt to link mathematics to reality: "*Where can I use this particular piece of mathematical knowledge?*" Model-eliciting tasks focus on the antithesis, linking reality to mathematics: "*Where can I find some mathematics to help me with this problem?*" Galbraith and Clatworthy (1990), later supported by Kang and Noh (2012) and (Ng, 2013), acknowledge three different levels of model-eliciting tasks. Traditional problem solving fits the description of a so-called level 1-problem. Such problems are already carefully defined, no additional data is required to formulate a model and the problems require specific mathematical procedures. Problems at level 2 have a slight vagueness as insufficient information needed to successfully complete the task is given. Level 3-problems are the most authentic and open-ended type, characterized by unstructuredness and a challenging level of complexity.

Exposure of pre-service teachers to mathematical modelling

Model-eliciting tasks have the proven ability to develop students' reasoning, communication, problem solving and problem posing abilities (Kang & Noh, 2012; Ng, 2013). Such activities consequently improve decision-making capabilities as they link classroom mathematics to real life situations.

Research in Singapore (Ng, 2013; Tan & Ang, 2012), in the United States of America (Ball, 2000), in Australia (Stillman, 2012) and also in South Africa (Julie, 2002; Julie & Mudaly, 2007) reveal that pre-service teachers' exposure to problem solving, their attitudes towards and beliefs about mathematics are factors that either enhance or restrict their involvement in model-eliciting activities. Their limited exposure to model-eliciting tasks is identified as a cause in their lack of readiness to implement such tasks as many teachers perceive mathematics to be formula-based (Ng, 2013). Pre-service teachers' appreciation of the contribution that modelling might make towards the teaching and learning of mathematics, would increase if they, according to Soon and Cheng (2013) become more familiar with the principles underlying modelling pedagogy, as introduced later on.

Liljedahl et al. (2009), Kang and Noh (2012), Julie (2002), Julie and Mudaly (2007) and Ng (2013) all recognise and emphasise the vital preparatory role of pre-service teacher education programmes, which should ideally expose students to the core content of modelling, to model-

eliciting tasks at various levels (as well as their approach and solution) and eventually to the pedagogy of modelling.

Elements of a modelling pedagogy

Although the content and pedagogical content knowledge needed by teachers in order to effectively teach mathematics themes have been emphasized time after time (compare the 'Background context and purpose' section above), there has been a scarcity of studies on the elements of an appropriate modelling pedagogy. Effective teachers of mathematical modelling engage students in a variety of practices. Campbell et al. (2015, p. 161), in their research on an appropriate modelling pedagogy in science, outline *eight such practices* that are also applicable to mathematics, namely:

> (1) Asking questions; (2) Developing and using models; (3) Planning and carrying out investigations; (4) Analyzing and interpreting data; (5) Using mathematics and computational thinking; (6) Constructing explanations; (7) Engaging in argument from evidence; (8) Obtaining, evaluating, and communicating information.

A quarter of a century ago, Schoenfeld (1992) proposes several so-called *learner strategies* aimed at problem solving (and indirectly also modelling). Schukajlow, Kolter and Blum (2015, p. 1242) highlight that six years earlier, Weinstein and Mayer (1986) have already experimented with four such learner strategies, namely "organization strategies" (to connect various chunks of information with each other), "elaboration strategies" (to link information supplied to prior knowledge), "rehearsal strategies" (to identify the most essential information) and "metacognitive strategies" (to plan a problem solution procedure). Schukajlow et al. (2015, p. 1251), having confronted Grade 9 mathematics students with model-eliciting challenges, conclude that a so-called "solution plan", which basically provides them with a "scaffold" (a supporting solution process framework, which incorporates the four abovementioned learner strategies), should be a key feature of an effective modelling pedagogy.

Tan & Ang (2012, p. 715) have proposed a modelling pedagogy, consisting of a range of so-called "decision procedures" assisting pre-service teachers in converting their modelling teaching strategy into a "series of modelling learning tasks". Their suggested teaching strategy consists of a series of *five questions* (each building on its predecessor), which lecturers/teachers should be asking in striving for optimal learning about modelling. The five questions and their explanations are:

1. *Which level of learning experience?* –Decide which level (1 = acquiring basic modelling skills; 2 = applying a known model to a new situation or 3 = building a new model) of mathematical modelling learning experience students should ideally gain.

2. *What are the skills/competencies and problem*? – List all the specific skills and competencies (mathematical and modelling) that are targeted through the learning experience; also state the problem to be solved.

3. *Where is the Mathematics?* – Make a list of the mathematical concepts, formulae or equations that are needed.
4. *How to solve the problem/model?* – Prepare possible (credible) solutions to the problem.
5. *What learning outcomes are generated?* – List the learning outcomes ideally generated by the modelling experience.

The aforementioned five questions (Tan & Ang, 2012), in combination with the learning practices (Campbell, Oh, Maughn, Kiriazis & Zuwallack, 2015), the learner strategies (Weinstein & Mayer, 1986) and solution plan (Schukajlow, Kolter & Blum, 2015), are regarded as elements of an appropriate modelling pedagogy.

Literature perspectives on student attitudes towards and achievement in mathematics

Student attitudes towards mathematics

Attitudes form a central part of a person's identity. The affective domain of learning typically features three dimensions: *emotions*, *attitudes* and *beliefs (*Papageorgiou, 2009, p. 5). Attitudes are defined by Philipp (2007, p. 259) as manners of acting, feeling, or thinking that show one's disposition or opinion. Attitudes change more slowly than emotions, but they change more quickly than beliefs. Attitudes, like emotions, may involve positive or negative feelings, and they are felt with less intensity than emotions. Attitudes are more cognitive than emotions but less cognitive than beliefs.

Leong and Alexander (2014, p. 611) indicate that attitudes may involve positive or negative feelings "of moderate intensity and reasonable stability". Cooke (2015) adds a fourth dimension to the affective domain of learning, namely *dispositions*, which are not the same as emotions, attitudes or beliefs. She quotes from Katz and Raths (1985), who describe a disposition as an attribute that "incorporates a measure of competence of a skill that had been chosen to be used, not a measure of a skill that a person had but chose not to use" (Cooke, 2015, p. 2). The enjoyment of mathematics could be regarded as a positive learning disposition, as it contains built-in elements such as the "desire, enthusiasm, confidence, and willingness, not out of necessity" (Cooke, 2015, p. 2) to indulge in mathematical tasks or challenges.

Pre-service teachers' attitudes towards mathematics pertinently influence their approach to their own teaching (during their education), how they might teach during their first few years as teachers, as well as the nature of the initial teaching and learning culture in their classrooms (Amato, 2004). Mathematics learning and teaching experiences stemming from their own school years are the main determinants of pre-service teachers' attitudes, although Amato (2004, p. 26) reports that experiences gained during their formal education positively or negatively shape their attitudes. Karali and Durmus (2015, p. 809) offers valuable advice in respect of pre-service teachers' exposure to modelling during their formal education,

viewing the latter as "an arduous process, in which attitudes and skills of teachers and pre-service teachers" are regarded as very important.

The influence of mathematics teaching and achievement on student attitudes

The quality of mathematics teaching and the nature of teacher attitudes seem to have a pertinent influence on students' attitudes towards mathematics and eventually also on their achievement. Yara (2009) confirms that teachers with positive attitudes towards the subject likewise stimulate favourable attitudes in their students. Henderson and Rodrigues (2008) regard the main source of negative learner attitudes toward mathematics as inappropriate teaching practices and teacher attitudes. Ma and Wilkins (2002) put the vital role of teacher attitudes into perspective, by stating that students who believe that teachers have high expectations of them tend to have a more positive attitude towards mathematics.

Sloan (2010, p. 243) has focused her research on pre-service mathematics teachers and discovers that teachers who are not really comfortable with the subject area usually have less positive attitudes toward mathematics, preferably teach in a procedural ("teaching algorithms") manner, and generally tend to focus less on mathematical concepts, reasoning and problem solving strategies.

The empirical investigation of this inquiry is based on the assumption that pre-service teachers' attitudes toward a mathematical theme like modelling might be influenced by their initial experiences of the lecturer's approach to the theme of modelling. Whether there are dissimilarities between the attitudes of pre-service teachers from different gender and achievement groups will also be explored.

RESEARCH DESIGN AND METHODOLOGY

Research paradigm and method

The research paradigm refers to a researcher's worldview, as reflected in a matrix of beliefs, perceptions and underlying assumptions (Foucault, 1972), which guides her/him in approaching the research problem. The main research paradigm underlying the nature of this enquiry relates to an attempt to measure pre-service mathematics teachers' attitudes towards a modelling activity. The investigation was thus conducted from a **post-positivist** stance (Heppner & Heppner, 2004). The post-positivist paradigm is a milder form of positivism, basically following the same principles, but allowing for more engagement between the researchers and the participants, by using a survey as data collection instrument. The authors assume that an external reality exists independently from this inquiry, and although this reality cannot be known fully, attempts at measuring it in an objective manner might be possible caption

Participants

The participants were a convenient sample (students in the one author's class) of 50 pre-service Grade 10 to 12 mathematics teachers, in the third year of their full-time study at the University of Johannesburg during the second semester (July to November) of 2015. **Table 1** below displays elements of their demographic profile. The majority are *male* (63%), *black* (almost 80%), *indigenous language* speaking (also almost 80%), *22 years or younger* (57%), and have scored *70% or more* in their Grade 12 year for mathematics (63%).

Table 1. Demographic profile elements of participants

Profile elements		N	%
Gender (n=49)	Female	18	63.3
	Male	31	36.7
Ethnic group (n=48)	Asian, Indian, Coloured	4	8.3
	Black	39	81.3
	Don't want to indicate	2	4.2
	White	3	6.3
Home language (n=49)	Afrikaans	4	8.2
	English	6	12.2
	Indigenous African	39	79.6
Age in years (n=47) (Mean = 22.4 yrs)	20 to 22 years	28	59.6
	23 to 25 years	15	31.9
	26 years or older	4	8.5
Final Maths mark in Gr 12 (n=48) (Median = 70-79%)	49% or lower	2	4.2
	50 – 59%	4	8.3
	60 – 69%	11	22.9
	70 – 79%	23	47.9
	80% or higher	8	16.7

Data collection, capturing and analysis

A self-designed questionnaire was used to collect information from the participants on *the day after* their exposure to the modelling activity (see 'The model-eliciting activity' section below). Section A of the questionnaire contained demographical items, as outlined in **Table 1** above. Collected demographical data were captured in a Microsoft Excel worksheet and then analysed via the frequencies, cross-tabulations and descriptive statistics options of the Statistical Package for the Social Sciences (SPSS, version 23).

The *Attitudes towards Mathematics Inventory* (ATMI, Tapia & Marsh, 2004) is an internationally recognized instrument, used for gaining learner attitudes towards mathematics. Schackow (2005) tailored the ATMI towards mathematics pre-service- and practicing teachers, thus making it appropriate for this study. The focus of section B of the questionnaire was geared towards participants' attitudes towards modelling, hence known as the *Attitudes towards mathematical modelling inventory* (ATMMI). The items and dimensions of

the original ATMI were kept intact. The ATMMI used in this inquiry contains four dimensions, namely *value* (whether mathematical modelling knowledge and skills are worthwhile and necessary, 10 items), *enjoyment* (whether mathematical problem-solving and modelling challenges are enjoyable, 10 items), *self-confidence* (expectations about doing well in respect of mathematical modelling and how easily modelling is mastered, 15 items) and *motivation* (the desire to learn more about mathematical modelling and to teach it, 5 items). Each of the 40 items uses a Likert-type response scale, ranging from 1 (*Strongly disagree*) to 5 (*Strongly agree*). When reverse coding (which applies to approximately a third of the items) was done, all item responses in each of the four dimensions are added, yielding total scores for value and enjoyment (maximum 50 each), self-confidence (maximum 75) and motivation (maximum 25). Analyses of data, including normality testing, reliability measures and testing for attitudinal differences between subgroups of participants, were also performed via SPSS.

Ethical measures and participants' consent

After the goal of the inquiry, the nature of the data collection instrument and their rights and responsibilities as respondents have been explained to them, individual written consent was obtained from all participants to safeguard the confidentiality of collected data and the anonymity of each pre-service teacher.

Validity and reliability measures

The creators of the ATMI, Tapia and Marsh (2004, p. 18–19) report that the survey shows a high degree of internal consistency (Cronbach's alpha = .88), while its factor structure "covers the domain of attitudes towards mathematics, providing evidence of content validity". The authors conducted a pilot study at the start of the second semester on the 'new' ATMMI, to determine and fine-tune its sight validity. Cronbach's alpha coefficients were hence calculated in respect of each of the four dimensions, as well as in respect of the participants' total ATMMI scores. The coefficients are portrayed in **Table 2** below, revealing high internal consistency (reliability) measures of the instrument.

Table 2. Reliability coefficients for the Attitudes towards Mathematical Modelling Inventory (ATMMI)

ATMMI dimension	Cronbach's alpha
Enjoyment (10 items)	.891
Value (10 items)	.857
Self-confidence (15 items)	.925
Motivation (5 items)	.872
Total ATMMI (40 items)	.893

The model-eliciting activity

The lecturer's approach to the model-eliciting activity was carefully planned. It was initially based on the modelling design guidelines suggested by Kenyon, Davis and Hug (2011). In addition, elements of an appropriate modelling pedagogy, as outlined in the relevant section in the literature review above, were also utilized. This incorporated the five questions

of Tan and Ang (2012), the learning practices of Campbell et al. (2015), the learner strategies of Weinstein and Mayer (1986) and the solution plan of Schukajlow et al. (2015).

The session lasted for almost two hours during a scheduled time table slot. The pre-service teachers have never been exposed to modelling or to its teaching before. They were divided into ten relatively comparable groups, each containing four or five members, based on their mathematics performance in Grade 12 and in the first semester's module. Proportional stratified sampling was employed to randomly assign them to the groups, in such a way that each group had at least a high(er), a moderate and a low(er) achiever.

A brief presentation on the purpose and nature of the inquiry, what modelling entails and expected student actions in respect of a typical model-eliciting activity (illustrated via the abovementioned five questions and the so-called solution plan) served as an introduction. The model-eliciting activity, labelled "World Cup Rugby 2015" (adapted from Stewart, 2013) is based on the content of the curriculum. It entails a level three challenge (compare the section, entitled *Model-eliciting tasks versus mathematical applications* above), being open-ended and incomplete. Participants were expected to make recommendations to the South African Rugby Union on the maximum number of official rugby balls that can be transported via a crate in an airplane to England for the World Rugby Tournament in September/October 2015.

The ten groups were expected to report on the strategies and methods that they employed, and to come up with possible solutions and to critique their suggested solutions. The whole activity, as well as group interactions were carefully monitored by the researchers and each group recorded their strategies, processes and suggested solutions on a predesigned worksheet.

FINDINGS AND DISCUSSION

Participants' attitudes towards mathematical modelling

Sweeting (2011, p. 53–54) categorises teacher attitudes towards mathematics on five levels, which she respectively labels as "strongly negative, negative, neutral, positive and strongly positive". Using her categorisation in this inquiry, strongly positive scores on the *enjoyment* dimension (maximum 50) would be 41 or more. Likewise, strongly positive scores on the *value* dimension (maximum 50) would also be minimum 41, on the *self-confidence* dimension (maximum 75) minimum 61 and on the *motivation* dimension (maximum 25) 21 or more. A strongly positive *ATMMI total* (incorporating all four dimensions – maximum 200) would be 161 or above.

The researchers expected the overwhelming majority of the participants (all of them are studying to become mathematics teachers), to portray positive attitudes towards modelling. However, the challenges generated by model-eliciting activities, and because they have been exposed to it for the very first time, might have had an influence on the participants'

confidence, beliefs and eventually also their attitudes. **Table 3** below provides a breakdown of participants' scores.

Table 3. Distribution and descriptive statistics of participants' ATMMI scores

ATMMI dimensions	Scoring intervals	N=	%
Enjoyment [N=48]	40 or lower	19	39.6
[M = 42.10; SD = 6.33]	41–50	29	60.1
Value [N=38]	40 or lower	18	47.4
[M=39.63; SD=6.73]	41-50	20	52.6
Self-confidence [N=44]	50 or lower	19	43.2
[M=51.32; SD=11.79]	51-60	12	27.3
	61-75	13	29.5
Motivation [N=49]	15 or lower	11	22.4
[M=17.61; SD=4.82]	16-20	23	46.9
	21-25	15	30.7
ATMMI total score [N=36]	140 or lower	11	30.6
[M=152.25; SD=24.27]	141–160	8	22.2
	161 or higher	17	47.2

About 60% of the group displayed strongly positive attitudes in respect of the enjoyment they got from the modelling task, while the percentage of strongly positive participants in respect of the value, self-confidence and motivation dimensions were 53%, 30% and 31% respectively. Just less than half of the participants (47%) exhibited a strongly positive overarching attitude towards modelling. In other words, almost 70% of the pre-service teachers are positive in respect of the enjoyment and value that they gained from the model-eliciting experience, while almost 80% are motivated to learn more about modelling.

The findings in respect of a considerable majority of participants, whose *enjoyment*-, *value*- and *motivational* levels were positively enhanced by their modelling experience, are in synergy with results stemming from an inquiry by Karali and Durmus (2015), involving primary school mathematics pre-service teachers in Turkey. The Turkish participants were impressed with the authenticity of the modelling activities to which they were exposed, as they promote an awareness of mathematics in the real world. In addition, they emphasised that model-eliciting activities "are effective in the acquisition of higher-level thinking skills, analytical thinking skills and strategy development" (p. 811).

On a less convincing note, just more than half (57%) of the pre-service teachers in this inquiry regarded their *confidence* in approaching and handling model-eliciting activities as 'above board'. This finding relates to research conducted by Ng (2013, p. 341), involving 48 in-service and 57 pre-service Singapore mathematics teachers, who had no experience of the implementation of modelling tasks. The Singapore pre-service teachers lacked confidence in verbally presenting their modelling arguments, though they were more assertive in outlining their written mathematical solutions. The latter result was interestingly enough, although not

unexpectedly reversed in respect of the in-service teachers. Ng (2013, p. 347) suggests that many teachers' formula-based perceptions of problem-solving should be eradicated, which will give them more confidence in systematically approaching modelling activities "with a non-exhaustive list of solutions which can use various mathematical representations".

Aiming to determine possible influences on their beliefs, MCK and confidence levels, 130 Australian first year pre-service primary school mathematics teachers, were exposed to a modelling intervention, as reported by Maasepp and Bobis (2014). The two Australians concluded that a modelling intervention is only successful in building pre-service mathematics teachers' confidence, if the university lecturer, who facilitates the intervention, possesses certain desirable characteristics and applies a certain strategy. These qualities and strategy include "the ability to develop a positive rapport with pre-service teachers" (Maasepp & Bobis, 2014, p. 105) and a conducive (non-threatening) pedagogical strategy. Soon and Cheng (2013) integrated a two hour per week modelling learning experience into the formal education programme of a group of 24 pre-service secondary school mathematics teachers in Singapore. As their participants gradually (week after week, over a period of six weeks) applied the principles of their carefully planned modelling pedagogy, their attitudes towards and content knowledge of modelling and also of mathematics in general were positively enhanced.

It seems reasonable to deduce that the lecturer's attributes, strengthened by the pedagogical approach she followed (see the sections '*Elements of a modelling pedagogy*' and '*The model-eliciting activity*' above) might have pertinently contributed to the participants' positive attitudes towards modelling. The first component of the empirical investigation thus confirms the researchers' expectation that pre-service teachers' initial attitudes toward mathematical modelling can indeed be positively shaped by a well-considered and appropriate modelling pedagogy. The question whether there are dissimilarities between the attitudes of pre-service teachers from different gender groups and on different achievement levels will lastly be explored.

Testing for significant differences between subgroups of participants

The Mann-Whitney U test, as non-parametric statistical technique was used to analyse differences between the medians of the responses of participants in the two gender groups and also on two performance levels, based on their score in mathematics in Grade 12, which correlated highly positively with their achievement in their third year mathematics modules. The Mann-Whitney test is considered appropriate, because the participants' responses aren't normally distributed, are measurable on an ordinal scale, are comparable in size and independent (responses from one subgroup don't affect the responses of another subgroup) (Milencović, 2011, p. 74).

Tables 4 and **5** below present the test statistics and ranks in respect of pre-service teachers' overarching attitudes, as presented by their total ATMMI scores, with *gender* and *mathematics achievement* as grouping variables. It is a disappointment that scores of only 36 of

the participants could be used for this comparison, as only the data of fully completed questionnaires were utilised.

Table 4. Test statistics for pre-service teachers' overarching ATMMI scores

	Gender [a]	Achievement in mathematics [b]
Mann-Whitney U	82.500	93.000
Wilcoxon W	187.500	198.000
Z	-2.322	-1.981
Asymp. Sig. (2-tailed)	.020 [c]	.048 [c]
Exact Sig. (1-tailed)	.019 [c]	.049 [c]

Test statistics' grouping variables: *Gender* and *Achievement in mathematics*

a Female pre-service teachers' attitudes are compared to those of *male* pre-service teachers

b Pre-service teachers, who scored *69% or less* for mathematics are compared to pre-service teachers, who scored *70% or more* for mathematics

c Significant at the 95% level of confidence

Table 5. Ranks in respect of total ATMMI scores

Grouping variables	Groups	N=	Mean rank	Sum of ranks
Gender	Female	14	13.39	187.50
[N=36]	Male	22	21.75	478.50
Achievement in mathematics	69% or less	14	14.14	198.00
[N=36]	70% or more	22	21.27	468.00

The Mann-Whitney test findings firstly indicate that female mathematics pre-service teachers in this study (*Mdn* =135) have a significantly lower (at the 95% confidence level) overarching attitude towards mathematical modelling than their male counterparts (*Mdn* = 168.5), U = 82.50, p = .020. Cohen's effect size (r = .39) is in the medium to high interval (Milencović, 2011, p. 77), which implies that the finding has moderate (to high) practical significance. The test findings secondly indicate that mathematics pre-service teachers in this study, who have scored 69% or less for mathematics (*Mdn* =141.0) have a significantly lower (at the 95% confidence level) overarching attitude towards mathematical modelling than their counterparts, who have scored 70% or more (*Mdn* = 168.5), U = 93.0, p = .038. Cohen's effect size (r = .393) is also in the medium to high interval (Milencović, 2011, p. 77), which implies that this finding also has moderate (to high) practical significance. Dissimilarities between between the attitudes of pre-service teachers from different gender groups and at different achievement levels towards mathematical modelling are thus the pertinent findings of this inquiry.

Haroun, Ng, Abdelfattah and Al-Salouli (2016) provide a brief overview of gender-related studies in mathematics education over the past three decades in several countries. They report that while in some countries, male students' achievement in mathematics is significantly higher than that of female students, the exact opposite is true for other countries. Lindberg, Hyde, Petersen and Linn (2010, p. 1127) draw a more 'neutral' picture, based upon

their meta-analysis of data from 242 studies in the USA, conducted between 1990 and 2007, involving more than 1.2 million participants. Their conclusive verdict is that female and male students perform likewise in mathematics. Lindberg et al. (2010, p. 1129) summarise their broad meta-analysis by concluding that "gender is not a strong predictor of mathematics performance." The findings of Haroun et al. (2016) and Lindberg et al. (2010) are supported by Ajai and Imoko (2015) via their study to determine possible achievement differences between the two genders, involving 428 senior secondary students from the African country of Nigeria in problem-based learning. Their conclusion (Ajai & Imoko, 2015, p. 48) is that "female students outperformed their male counterparts, though the difference is not statistically significant", which verifies that female students have reached parity with male students in respect of achievement in mathematics.

In highlighting factors that were found to contribute to gender differences in mathematics achievement, Haroun, et al (2016, p. S384) cite four pertinent contributors, namely "stereotypes of mathematics as a male domain", teachers' expectations, teachers' gender and students' prior teaching and learning experiences in mathematics. The teachers' gender-factor was intensively scrutinised by Haroun, et al (2016, p. S389-S395), as they involve 197 primary school teachers from four cities in Saudi Arabia in their inquiry. They detected a strong, positive relationship between a teacher's gender and her/his students' performance and attitudes, with female teachers having more impact in this regard than their male counterparts.

Christensen, Knezek and Tyler-Wood (2015) quote from a study (initially reported on by Sadler, Sonnert, Hazari & Tai, 2012) involving 6,000 American high school students, which found that the odds of male students pursuing a Science, Technology, Engineering or Mathematics (STEM) career are almost three times higher than for females. Almost 40% of the pre-service mathematics teachers, who participated in this inquiry are female, which is a positive notion. Christensen, et al (2015, p. 900) further elaborate on female learning preferences and emphasise their need to learn in a more social context and also to rather study matters that connect to "the real world". It's exactly these female learning preferences that leave the authors with a sense of optimism. Although female pre-service teachers in this study did display a significantly lower all-encompassing attitude towards mathematical modelling than their male counterparts, this attitudinal difference might potentially be short-lived. If pre-service teachers are exposed more intensively and frequently to mathematical modelling, as a theme in their formal teacher education curriculum, especially female teacher attitudes might be boosted and their attitudes might even naturally evolve into dispositions (Cooke, 2015, p. 2). The reasons underlying the latter hopeful prospect are to be found in the collaborative and social nature of student engagement in mathematical modelling, as well as the authentic and real world character of modelling tasks.

The inquiry's second finding, namely that better achievers display more positive attitudes towards modelling has also been affirmed by a recent study among German mathematics students by Schukajlow, Krug and Rakoczy (2015). They conducted an

experiment, involving 144 grade 9 mathematics students, aiming to determine the influence of prompting for multiple versus singular solutions to model-eliciting tasks on student achievement. They (also) find that students' prior knowledge and achievement in mathematics are prominent factors that influence the quality of their learning and their attitudes. Their inference is that higher mathematics achievers have greater affinity for solving problems, report more self-efficacy in doing mathematics, and often experience more enjoyment while working on mathematical tasks than lower achievers. They draw the conclusion (Schukajlow et al., 2015, p. 411) that students' "experience of competence" is a strong predictor of their modelling performance. The implication of this promising finding is that teaching-learning situations that "improve students' experience of competence contribute to the achievements of cognitive and affective goals in mathematics education" (Schukajlow et al., 2015, p. 412).

In a study, involving 38 pre- and 48 in-service Austrian mathematics teachers, Kuntze, Siller and Vogl (2013) aimed to determine what self-perceptions of their PCK these teachers hold. They detected that especially the pre-service teachers didn't view their modelling PCK in a positive light and that deficits in their MCK, which also have a pertinent influence on their performance in mathematics, are regarded as an important reason. Likewise, the attitudinal differences exhibited by the two *achievement* groups above sanction the argument of Sloan (2010) that pre-service mathematics teachers who are not really comfortable with mathematics typically have less positive attitudes toward themes, which demand less procedural knowledge and substantially more conceptual, reasoning and problem solving competencies, typically featured in modelling tasks.

The findings endorse the dire need for a concerted and prolonged initiative to develop pre-service teachers' modelling PCK as well as their "modelling-specific self-efficacy" (Kuntze, Siller & Vogl, 2013, p. 323–324) during their formal education. This is one of the intentions of the overarching project of which this inquiry forms part, which in the long run might address the diverse attitudes towards modelling that pre-service teachers at different achievement levels display.

IN CONCLUSION

The relationship between mathematical modelling, which has recently been incorporated into the secondary schools' mathematics curricula of several countries (including that of South Africa) and authentic learning has been proven. Another strong relationship between positive attitudes towards and achievement in mathematics has also been well documented (compare Dowker, Ashcraft & Krinzinger, 2012; Durandt & Jacobs, 2013; Ismail & Anwang, 2009; Khatoon & Mahmood, 2010; Schukajlow, Krug and Rakoczy, 2015; Sweeting, 2011, and others).

The research questions that this study attempted to find answers to were two-fold, namely:

- what is the nature of pre-service teachers' attitudes towards mathematical modelling and
- are there differences between the attitudes of the two gender groups, and between higher and lower performing pre-service teachers?

The group of almost 50 third year mathematics pre-service teachers was exposed to a model-eliciting activity (as part of their formal curriculum) for the very first time. An interrogation of their attitudes towards modelling revealed that most of them enjoyed and valued the activity, and are motivated to further their modelling knowledge and skills. However, almost half the group still lack confidence in handling model-eliciting challenges. A further analysis indicated that female pre-service teachers, as well as pre-service teachers who have scored below 70% in mathematics, displayed less conducive attitudes towards mathematical modelling, than their male or higher achieving peers.

The theoretical lens through which this inquiry is viewed, the Learning to Teach Secondary Mathematics framework (Peressini et al., 2004), *firstly* asserts that how a student acquires a particular set of knowledge and skills and the specific teaching context in which it happens fundamentally influence what they eventually learn. It *secondly* stipulates that mathematics teachers' knowledge, but especially their beliefs and attitudes, are shaped through increased participation in the practice of teaching itself. The challenge is clear: mathematics pre-service-teachers need to acquire modelling knowledge and skills during their formal education. It can be effected through a formal programme and via an appropriate modelling pedagogy, which, in addition to enhancing their mathematical content knowledge, also shape their pedagogical knowledge. Such a programme should ideally be based on a well-considered set of guidelines for implementation that take cognisance of and gradually build pre-service mathematics teachers' confidence.

Pre-service mathematics teachers' attitudes (which might perhaps unconsciously become their beliefs) are resilient to change, because they were developed and shaped over many years of their experience on the 'receiving end', as mathematics students. Maasepp and Bobis (2014, p. 103) recommend that to affect desirable attitudinal changes in pre-service teachers, they should "observe and experience many positively charged teaching and learning experiences throughout their teacher education program". It is highly dubious that a once-off intervention, aimed at strengthening pre-service teachers' mathematical modelling competencies, can boost their confidence and change their attitudes and dispositions. For pre-service mathematics teachers to eventually grasp and also to effectively fulfil their nurturing role as modellers of modelling in their classrooms, continual exposure to and reflection on modelling activities and model-eliciting tasks are essential throughout their years of formal education.

This South African inquiry re-emphasised the crucial role of teacher education programmes in defining and strengthening pre-service teachers' content knowledge, but more importantly their initial pedagogical content knowledge. The latter is affirmed in an exemplary manner by very recent research conducted by Haroun et al. (2016) in Saudi Arabia. In attempting to explain why female mathematics students outperform male students in their country, Haroun et al. (2016) discover the solution in the teaching strategy and approach of their female mathematics teachers. Single sex schools are the norm in Saudi Arabia, so male mathematics students would typically be exposed to male teachers, and female students to female teachers only. Haroun, et al. (2016, p. S394) report that female mathematics teachers are more observant in respect of their students' reasoning strategies and potential misconceptions, and seem more likely to understand the content from the perspective of their learners, thereby directly supporting their students' mathematical development and also improving students' attitudes towards the subject.

Although content knowledge is essential in grasping a fairly complex theme like mathematical modelling, the pedagogical content knowledge related to the teaching of modelling seems even more important. It is therefore crucial that pre-service teachers acquire the essentials of the teaching and learning of mathematical modelling through a formal education programme. Such a programme must however also strengthen pre-service teachers' attitudes towards modelling. Well-equipped mathematics teachers, with a positive frame of mind towards modelling will be much better able to connect with their students and to support their learning.

END NOTE

1. ***Modelling*** (with a double l) denotes the typical manner in which the term is written in the South African educational context. Although its spelling is different from ***modeling*** (with a single l), which is mostly used in the American and European educational contexts, the meaning is identical.

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