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ORIGINAL ARTICLE

Using mathematics to solve real world problems: the role of enablers

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Abstract The purpose of this article is to report on a newly funded research project in which we will investigate how secondary students apply mathematical modelling to effectively address real world situations. Through this study, we will identify factors, mathematical, cognitive, social and environmental that "enable" year 10/11 students to successfully begin the modelling process, that is, formulate and mathematise a real world problem. The 3-year study will take a design research approach in working intensively with six schools across two educational jurisdictions. It is anticipated that this research will generate new theoretical and practical insights into the role of "enablers" within the process of mathematisation, leading to the development of principles for the design and implementation for tasks that support students' development as modellers.

Keywords Applications · Mathematical modelling · Real world problem · Mathematisation · Formulation · Implemented anticipation · Anticipatory metacognition · Enablers

Introduction

The ability to apply mathematics to the real world underpins many aspects of personal, civic and work life and is an issue of rising international concern for educational policy

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makers and researchers. As Paulos (2000) notes, an inability to use mathematics limits an individual's career aspirations, social well-being and financial security. The growing profile of international comparative assessments such as the Programme for International Student Assessment (PISA) and Programme for International Assessment of Adult Competencies (PIAAC), which have components aimed at ascertaining the capacity of individuals to apply mathematics to life-like problems, is a reflection of governments' interests in the mathematical capability of their citizenry. The outcomes of these assessments are increasingly shaping government policy reform in education (Geiger et al. 2015a, b).

Concomitantly, the capacity to use mathematics to model real world phenomena and make predictions is a vital capability within science, technology, engineering and mathematics (STEM) careers, a sector that contributes substantially to maintaining the productivity of the nation in an increasingly globalised world characterised by rapid technological and economic change (Office of the Chief Scientist 2012). In response to these changing global demands, governments such as South Korea, Vietnam, Germany, Japan and China have all built economic policy with reliance on a central plank of STEM capability development (New York Academy of Sciences 2015). For students to choose and have access to careers within STEM, and related professions, they must be confident with, and competent in, applying mathematics to the real world (English 2016). At the same time, there is growing concern, worldwide, over students' capability, engagement and participation in the STEM disciplines (Marginson et al. 2013).

Within the Australian context, the important capability of applying mathematics to problems in different real world contexts is captured at the very beginning of the National Curriculum Statement (ACARA 2016):

> mathematics aims to ensure that students are confident, creative users and communicators of mathematics, able to investigate, represent and interpret situations in their personal and work lives and as active citizens (p.1).

The purpose of our recently funded Australian Research Council project is to address this issue by identifying, refining and applying teaching approaches that help secondary students learn how to use mathematics to solve real world problems through the processes of mathematical modelling. As a particular focus, we will investigate the factors, mathematical, cognitive, social and environmental, that enable year 10/11 students to successfully begin the modelling process as a precursor to its successful conclusion. This involves developing mathematical representations of a real world problem—an activity that involves mathematisation and formulation (Niss 2010). In attending to this challenge, this project will address the following specific aims:

i). Describe the nature of anticipatory metacognition and identify and describe enablers necessary for students to translate problems involving real world situations into mathematical models;
ii). Design tasks that support the development of students' anticipatory metacognition or allow for the identification of issues that are problematic for that development; and
iii). Develop trial and refine teaching practices that support the growth of students' anticipatory metacognition while working on effective modelling tasks.

The planned research will address these aims and lead to the development of an *Integrated Modelling Task and Pedagogy Framework* (*IMTPF*) that incorporates principles of effective task design and guidelines for classroom implementation (i.e. supportive pedagogies, necessary resources, other environmental and social factors). Thus, through this study, we will generate new theoretical insights (enablers of mathematisation, anticipatory metacognition) and practical strategies (tasks, pedagogies) needed to progress students' success with modelling real world problems.

Background

International assessments such as the Programme for International Student Assessment (PISA) provide clear evidence that other countries are outperforming Australia within STEM-related subjects. For example, across 2003–2015 PISA results, Australia was ranked 20th for mathematical literacy in 2015, down from 19th in 2012, 13th in 2009 and 8th in 2006. Of even greater concern, PISA results show that 22% of Australian 15-year-old students did not meet the international proficiency level 2 for mathematical literacy—indicative of the level of competence necessary to use mathematics effectively in real-life situations. Further, 45% were below the nationally agreed baseline of level 3 (Thomson et al. 2016a). A similar decline is also evident in PISA results for scientific literacy and in Australia's performance in TIMMS (Thomson et al. 2016b). These results are particularly concerning, as PISA test items are designed to assess attributes that contribute to the capacity of students to apply their knowledge to real-life situations. If the root cause of these results remains unchallenged, Australia faces the prospect of limited life opportunities for individuals, and diminished effectiveness of our work force, resulting in a potential down-turn in our nation's growth and prosperity.

The difficulties secondary students experience in applying mathematics to real-life or context-based tasks are a long-standing problem in educational research (e.g. Wijaya et al. 2014). Features known to influence students' capacity to mathematise include teachers' expertise and pedagogical knowledge in modelling (Blum 2011), teachers' and students' dispositions toward and beliefs about mathematics (Kaiser and Maaβ 2007), and the skilful deployment of digital tools by teachers and students (e.g. Brown 2015; Geiger et al. 2010). Researchers are in agreement, however, that the main area of difficulty for those learning how to model is the transformation of a real world situation into a mathematical form in order that mathematical techniques can be brought to bear (Gould and Wasserman 2014). This view, long established in the modelling community (e.g. Treilibs 1979) is also supported by PISA data, which indicates formulation is the most difficult process for Australian students when attempting to solve problems drawn from the real world (Stacey and Turner 2015), and so is fundamental to successful modelling.

It should be noted, however, that performance on test items of the PISA type identifies symptoms only, and then not systematically. The goals of this project go far deeper, to develop abilities which cover attributes, some of which may be tested by means of such items, but much more. Continued monitoring of outcomes from assessment programs provides useful evidence of effectiveness, but is insufficient on its own. PISA and similar items may assess individual competencies, but overall competence requires a synthesis of these that can only be addressed in extended settings using authentic modelling tasks.

To date, few pedagogical solutions have been proposed to the problem of how students learn to mathematise consistently. Our contention is that the construct of *implemented anticipation* (Niss 2010), a metacognitive/cognitive process in which students anticipate, within the act of modelling, what is useful mathematically in subsequent steps, and also in decision-making and carrying through of actions that bring those following steps to fruition, is central to students' ability to mathematize (Stillman et al. 2015). We use the term *anticipatory metacognition* (e.g. Galbraith 2015) to describe the associated metacognitive aspect. Our earlier work with anticipatory metacognition (e.g. Stillman et al. 2010) has shown promise in dealing with the difficulties students experience with mathematisation, indicating further research is worth pursuing. Consequently, enablers of anticipatory metacognition, that is, the mathematical, cognitive, physical and digital resources necessary for students to transform real world situations into mathematical models and then utilise them, are also of prime importance.

The process of mathematisation is often presented as the entire mathematical modelling cycle (e.g. OECD 2009, p. 105), resulting in a level of complexity that makes it difficult to identify fundamental enablers of mathematisation and to communicate these effectively to students and teachers. Metacognition related to engaging with a mathematics problem is often portrayed as "any knowledge or cognitive activity" (Flavell et al. 2002, p. 164) that accounts for an individual's awareness, regulation and monitoring of their progress and feelings. In this study, we will focus on anticipatory, and not just reflective, processes—a metacognitive stance through the concept of "anticipation". Our previous work (e.g. Stillman et al. 2015) has provided empirical evidence for the existence of Niss' (2010) hypothesised foreshadowing and feedback loops used by successful modellers during the process of mathematisation.

Conceptual framework

The open-ended nature of mathematical modelling is evident in the modelling task example, below, which is drawn from earlier work on designing modelling tasks (e.g. Stillman 2010, p. 307). The task demonstrates the demanding nature of mathematisation. This requires, at least, insight into what mathematical knowledge is appropriate for the problem and how this knowledge could be used in a real-life authentic situation. In this case, mathematical modelling featured centrally in related court cases and provided evidence that the victim was thrown from the cliff (Fig. 1).

As a result, a further investigation was initiated, eventually resulting in the conviction of the accused. This conviction has now been overturned, following further argument around the modelling inferences made by the prosecution "expert" witness. The case demonstrates the level of mathematical literacy expected by our society for potential jurors in our justice system.

Modelling is typically described as a cyclic activity that can be represented as in Fig. 2. This diagram is an analytical representation of the key components of the modelling process—a simplification of modelling activity in action. Entries A–G represent stages in the modelling process with the thicker arrows indicating transitions

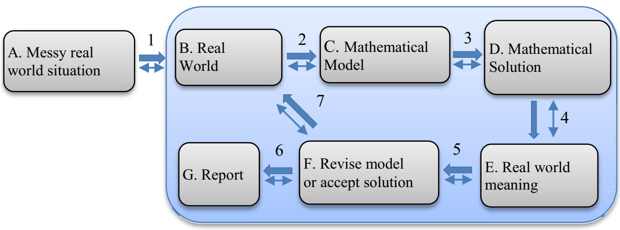
Caroline Byrne, an Australian model, was found at the bottom of a cliff at The Gap in Sydney in the early hours of 8 June 1995. Given that the cliff is 29 metres high and her body was found 11.8 metres from the base of the cliff, determine if she fell, jumped or was thrown.

Fig. 1 Modelling task example

in activity. Formulation of a mathematically feasible problem from the real situation occurs through making assumptions and identifying features essential to addressing a real world problem that must be posed. These are then incorporated into a formulation of the mathematical model designed to solve the problem. Solution of the model takes place in the mathematical domain that includes relevant mathematical knowledge methods and artefacts (e.g. diagrams or graphs). Mathematical outputs must then be interpreted in terms of the original real situation. Interpreted outputs provide answers to questions posed about the real situation or, if unsatisfactory for this purpose, stimulate further modelling. The kinds of cognitive activity that modellers utilise as they attempt to transition from one stage to the next are shown in the descriptors 1–7 in Fig. 2. The double-headed arrows indicate the presence of reflective metacognitive activity that acts on these cognitive activities. This can involve looking forwards or backwards with respect to stages in the modelling (Stillman 2011); hence, the bi-directionality.

Within this cycle, the capability to anticipate future steps and make decisions is vital. Anticipating was used by Niss (2010) within a theoretical model of the mathematisation process. He coined the term, “implemented anticipation” (p. 55), as successful mathematisation involves both anticipating what will need to be done mathematically in subsequent steps, and using that anticipation in consequent decision making involving the carrying through of actions that bring those steps to fruition. We contend that such activity employs an anticipatory form of metacognition in which the following processes are employed:

1. Implementing decisions about what features are essential as well as generating a related problem statement (B) by anticipating their usefulness in mathematising a mathematically feasible problem from the real situation (A → B in Fig. 2).
2. Anticipating mathematical representations and mathematical questions that, from previous experience, have been effective when forming a mathematical model (B → C); thus, invoking metacognitive knowledge in an anticipatory manner.
3. Awareness of the utility of the selected mathematisation and resulting model (C), in the future problem solving processes, to provide a mathematical solution (D) to the questions posed by the mathematisation; therefore, anticipated mathematical procedures and strategies are used in problem solving after mathematisation is complete.



1. Understanding, structuring, simplifying, interpreting context
2. Assuming, formulating, mathematizing
3. Working mathematically
4. Interpreting mathematical output
5. Comparing, critiquing, validating
6. Communicating, justifying (if model is deemed satisfactory)
7. Revisiting the modelling process (if model is deemed unsatisfactory

Fig. 2 The process of mathematical modelling (Stillman 2011)

The foreshadowing of the results of future actions being "projected back onto current actions" (Niss 2010, p. 55) generates a "sense of direction" that is crucial in modelling (Maaß 2006). Niss (2010) proposed four enablers of successful anticipatory metacognition in that modellers need to (1) believe a valid use of mathematics is modelling real phenomena, (2) possess relevant mathematical knowledge, (3) be capable of using this when modelling and (4) have perseverance and confidence in their mathematical capabilities (p. 57). The necessity and sufficiency of these and other enablers, for example the role of digital tools (Geiger et al. 2010), require further research (Stillman and Brown 2014) and will be a focus within this project.

Design and methods

A design-based methodology was chosen for the study because this approach is suited to applied research that develops contextualised theories of learning and teaching in tandem with practical approaches to solving educational problems across multiple settings. Cobb and his colleagues (Cobb et al. 2003) have identified several requirements in planning for design-based research. These are outlined in Table 1 with an indication of how the proposed study will address each.

The research plan for this study aligns with these methodological considerations and consists of the documentation of student cases within the contexts in which they learn and develop. Methods include participant observation, interviewing and collection of written and non-written data (survey questionnaires, teaching materials, student artefacts, video/audio recordings, video-stimulated recall).

Participants

Participants will include six teachers (three from each state) and three successive cohorts of intact year 10–11 classes (approximately 150 students per year; total 450) from Queensland and Victorian secondary schools. These two states provide

Table 1 Principles of design-based research applied to the project

Principles of design-based research	Relevance to the study
1. Theoretical intent should be clarified	Identification and refinement of enablers of anticipatory metacognition in mathematical modelling
2. Goals or desired outcomes should be specified	Development of an Integrated Modelling Task and Pedagogy Framework (IMTPF) that incorporates principles of effective task design and guidelines for classroom implementation
3. Starting points should be identified	Working with students and teachers on pilot tasks designed to provoke the use of enablers
4. Conjectures should be developed and tested concerning how teaching practice might change and how this change can be identified	The study will be implemented via cycles of in-class trials, refinement and retrial in developing the IMTPF.

contrasting curriculum contexts—enabling important inputs when considering the scaling up of results of the research. For example, while mathematical modelling has been a major focus of Queensland mathematics syllabuses for at least 25 years, the focus on modelling in Victoria has been more subtle. Consequently, a higher level of experience with teaching modelling can be expected of Queensland teachers. Years 10 and 11 students have been selected as mathematising have been confirmed, (e.g. via PISA (Stacey and Turner 2015)), as a difficulty in the previous year group (year 9). Further, this level of schooling allows for engagement with a level of modelling challenge that requires metacognitive capabilities (Stillman 2004). From each student cohort, additional data will be gathered from selected pairs of students via video-stimulated recall sessions. Teachers will be purposively selected (Burns 2000), on the basis of their expertise in teaching modelling.

Data collection and analysis methods

Data collection methods will consist of a Conceptions of Learning and Dispositions toward Mathematics Questionnaire (CDM), lesson observations, student and teacher interviews, open paired video-stimulated recall sessions and directed paired video-stimulated recall sessions. These are now described in brief.

Conceptions of learning and dispositions toward mathematics questionnaire CDM will be developed as part of the research and administered at the beginning and end of each phase of the project in order to gauge students' conceptions of learning and dispositions toward using mathematics. In designing the questionnaire, we will draw on previous surveys developed by Wood et al. (2012) and Cai and Melino (2011). Likert items will be subject to descriptive statistical analysis in order to determine changes to students' conceptions of learning across a single school year. Thematic analysis via NVivo will be conducted on open-ended responses.

Lesson observations, student and teacher interviews Using methods developed previously (Geiger et al. 2015a, b), lesson observation field notes, pre- and post-lesson interviews with teachers, post-lesson interviews with small groups of students and student work samples will be used to gain insight into the effectiveness of both task and pedagogical design for students attempting to work on demanding applications of mathematics. The analysis of interview excerpts, field notes and student artefacts will be integrated into accounts of individuals' teaching practice and of students' deployment of enablers of mathematisation.

Open paired video-stimulated recall Two pairs of students per class will be videotaped during the first round of school visits for each phase of the project. As soon as possible after the school visits, researchers will convene a video-stimulated recall session where video recordings of students' approaches to modelling real world problems will be overlaid with students' descriptions of their own activity (drawing on Jorgensen and Lowrie 2012). The commentaries will be analysed for the anticipatory nature of decision making with respect to (1) essential features of the real world situation, (2) choice of mathematical artefacts for representation of that situation and (3) choice and use of mathematical techniques.

Directed paired video-stimulated recall In a similar fashion, two pairs of students per class will be videotaped during the second round of school visits in each phase of the project. During follow-up video-stimulated recall sessions, students will view sections of recordings selected by researchers that represent critical moments in students' attempts to solve a modelling problem. After playing a section of video students will be asked to respond to prompts based on Witzel and Reiter's (2012) problem-centred interview protocol. This analysis will aid in confirming the existence of conjectured enablers and assist in identifying additional enablers.

Design

The research design involves four types of research and enabling activity as follows: (1) whole day meetings between teachers and researchers for immersion experiences, establishing goals, planning task trials, identifying pedagogies that support students' development in mathematizing, and evaluating progress; (2) teacher and student activity that will be captured through video techniques to establish enablers of successful student mathematisation; (3) day-long visits to schools to investigate the progress of task development and implementation via lesson observations, student work samples and interviews with teachers and students; and (4) collection and analysis of data for evidence of students' modelling proficiency and mathematics achievement. The project will be conducted in three phases corresponding to the 3 years of project funding. In phase 1, the goal is to gain insight into the thinking of students as they mathematise and to elaborate upon the nature of anticipatory metacognition and the enablers that support it. Phases 2 and 3 are geared to test developing theory and influence students' and teachers' development.

Phase 1 (year 1) Phase 1 involves six teachers in six schools working with intact classes. The focus will be on "able" students (teacher identified) in order to establish reasonable expectations for what students can achieve as modellers at years 10/11. The purpose of this phase is to map knowledge and understanding of (a) how "able" students mathematise, (b) which enablers are necessary and sufficient for successful modelling with "able" students and (c) the features of modelling tasks and mentoring that support the development of students' anticipatory metacognition or assist with the identification of issues that are problematic for this development.

This phase will involve three whole day meetings, two rounds of school visits and two rounds of video-stimulated recall sessions, where pairs of students and their teacher will be interviewed while viewing a video recording of the students working on a modelling problem. In the first workshop, the researchers will do the following:

- Explain and illustrate the modelling cycle to the participant teachers
- Describe the role of mathematisation within the cycle and the critical nature of enablers of students' ability to mathematise; present case studies drawn from our previous research on modelling in order to illustrate effective modelling teaching practice

- Introduce teachers to the first common modelling task (CMT) (developed out of successful activities employed in previous research projects, e.g. Galbraith et al. 2010; Geiger et al. 2010)
- And plan for upcoming school visits by researchers.

Teachers will implement the first CMT before the second whole day workshop. Lessons in which these tasks are implemented will be observed and video-recorded during school visits. Additionally, two pairs of students from each class will be video recorded (using two additional video cameras) at close range while working on this task. Teachers will be interviewed before a lesson, in order to document their aims and identify the intended enablers of mathematisation. Teachers and focus groups of students will be interviewed after each observation to gauge the perceived effectiveness of tasks in terms of student interest and learning and the extent to which the intended enablers featured. As soon as practical after each school visit, pairs of students and their teachers will take part in an open paired video-stimulated recall session in order to gain insight into students' approaches to mathematisation. Whole class video recordings, teacher and student interviews and paired video-recorded stimulated recall sessions will be analysed in order to identify and describe enablers that support anticipatory metacognition, for example, the way digital tools are used to promote the process of mathematisation. Initial findings will inform the development of a draft modelling task and technology integrated pedagogical framework (IMTPF).

A second cycle of similar research activity will complete phase 1 with data gathering mirroring that of the first round of school visits and including teacher and student interviews, as well as whole class and paired-student video recording. These visits will be followed by directed paired video-stimulated recall sessions (instead of open paired video-stimulated recall). Analysis of these data for identification and description of additional enablers will help refine the draft IMTPF.

After the first and second whole day meetings, the CDM questionnaire will be administered to students. A third whole day meeting (end of year) will be conducted for the purpose of seeking teacher feedback on the refined draft IMTPF.

Phase 2 (year 2) Phase 2 involves the phase 1 teachers with new cohorts of students. The specific purpose of this phase is to test the effectiveness of the draft IMTPF for designing modelling tasks and supportive pedagogies for mathematically able students, and to further refine the framework. This phase will also involve three whole day meetings, two rounds of school visits and two rounds of video-stimulated recall sessions (one open and one directed). Between meetings, cycles of action and data collection will then be repeated as for phase 1 with the focus again on able modellers. With the experience of phase 1, teachers should be more attuned to the nuances of students' anticipatory capabilities when they attempt to mathematise, and so will be able to provide greater support for this process.

At the first whole day workshop, teachers will collaborate with researchers to review the IMTPF, develop new school based tasks and discuss pedagogies appropriate for these tasks, in preparation for implementation in classrooms. During the second whole day workshop, teachers and researchers will share insights gained from implementing their modelling tasks and their views on enablers that support students' attempts to mathematise, and plan for the implementation of an additional modelling task. This task will be one where students themselves pose a problem developed from a real world situation of personal interest, formulate it and then mathematise the situation into a mathematical model. This freedom to choose the situation to model is crucial to establishing empirical evidence for the first aspect of Niss's (2010) "implemented anticipation" and to generate student performance data with respect to mathematizing. A third whole day meeting will be conducted to validate and enhance the IMTPF.

Phase 3 (year 3) Teachers from phase 1 and 2 will work with a new cohort of year 10/11 students. Phase 3 will adopt a similar cyclic structure to phases 1 and 2, except that the focus of open and directed video-stimulated recall sessions will be on less able mathematics students. These students will be selected in order to determine if the IMTPF, developed with able modellers, is effective in assisting teachers to design modelling tasks and supportive pedagogies for a wider range of students. Phase 3 will also consist of three whole day meetings and two rounds of school visits. Within the first and second whole day meetings, teachers and researchers will develop tasks and discuss pedagogies for implementation in classrooms as in phase 2. The project will conclude with a final whole day project workshop where findings of the project will be presented, including the difference between the repertoire and use of enablers for more and less able students. Final input will also be invited from teachers for refinement of the IMTPF. Cycles of data collection between whole day meetings will be conducted as in phase 2.

Advances in knowledge and anticipated benefits

Falling participation and under-performance in the STEM disciplines in Australia has created serious concerns about Australia's capacity to sustain a knowledge-based economy and society (Australian Industry Group 2015). Mathematical modelling underpins many of the advances made in science and manufacturing as well as areas such as communications, environmental change, transport and resources. Mathematical literacy is the foundation for successful participation in all STEM disciplines, including mathematics, and for the ACARA ideals of citizens able to use mathematics to enrich their lives personally, and as responsible citizens. We concur with the writers of the Californian STEM Taskforce Report (2014) who suggest mathematically literate students know "how to analyse, reason, and communicate ideas effectively (and how to) mathematically pose, model, formulate, solve, and interpret questions and solutions in science, technology, and engineering" (p. 9), all elements of mathematical modelling. Thus, through this project we seek to address the challenge of falling student interest and performance in the study of mathematics and participation in the STEM disciplines by focusing on what we see as an essential attribute of the mathematical expertise of the future, that is, mathematical modelling.

The anticipated outcomes of the project will contribute to new theoretical and practical knowledge about how students' can learn to apply mathematics to real world problems. Theory about the teaching and learning of applications of mathematics via modelling will be extended by investigating the nature of anticipatory metacognition and the role of enablers of mathematisation, This approach represents a new direction, different from previous studies which have looked at "in the moment" and reflective metacognitive activity. Further, bringing focus to the role of enablers, rather than the entire mathematical modelling cycle, allows for a more targeted approach to developing insight into those elements that are essential to effective mathematical modelling. As we aim to investigate anticipatory, and not just reflective or online processes, potential findings will allow for the implementation of more proactive activity while modelling.

At a practical level, the project will provide new understandings about how teaching practice and student learning can be changed through the implementation of tasks and pedagogies designed to promote the capability to mathematise. Insights gained though the project will lead to the development of task design principles and teaching practices, embodied in the IMTPF, that support students' mathematisation and hence enhance their real world problem solving ability. The development of the IMTPF will provide direct support to teachers intending to assist their students to learn how to apply mathematics to the real world and also identify issues that limit the development of this capability. Support will include exemplar tasks and guidelines for how teachers can develop their own tasks that support development of anticipatory metacognition. There will be additional advice on how tasks should be implemented in classrooms. In this sense, the aims of the project align with two key findings of the STEM: Country Comparison Report (Marginson et al. 2013), that (a) it is important to broaden STEM engagement and achievement, and (b) schools should promote inquiry, reasoning, and creativity and design in STEM curricula. This project addresses (a) by developing tasks that situate mathematical learning within real world scenarios relevant and of interest to students and (b) by supporting teachers to identify and present students with open-ended problems originating from real world phenomena that require novel thinking and the use of their mathematical resources in creative ways.

The project is situated in different curriculum and experiential contexts in order to make judgments about the transferability of findings across educational jurisdictions across the nation. Thus, outcomes of the project will include advice on the development of professional learning programs aimed at enhancing aspects of teaching and learning mathematical modelling and it is up-scaling nationally.

References

ACARA (2016). *Australian Curriculum: Mathematics Aims* Retrieved 27 Sept 2016 from http://www.australiancurriculum.edu.au/mathematics/aims.

Australian Industry Group (2015). *Progressing STEM skills in Australia*. Sydney: Author.

Blum, W. (2011). Can modelling be taught and learnt? Some answers from empirical research. In G. Kaiser, W. Blum, R. Borromeo Ferri, & G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling* (pp. 15–30). Dordrecht: Springer.

Brown, J. (2015). Complexities of digital technology use and the teaching and learning of function. *Computers & Education, 87*, 112–122.

Burns, R. (2000). *Introduction to research methods* (4th ed.). Sydney: Longman.

Cai, J., & Melino, F. J. (2011). Metaphors: a powerful means for assessing students' mathematical disposition. In D. J. Brahier & W. R. Speer (Eds.), *Motivation and disposition: pathways to learning mathematics* (pp. 147–156). Reston, VA: NCTM.

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher, 32*(1), 9–13.

English, L. D. (2016). Advancing mathematics education research within a STEM environment. In K. Makar, S. Dole, J. Visnovska, M. Goos, A. Bennison, & K. Fry (Eds.), *Research in mathematics education in Australasia 2012–2015* (pp. 353–372). Singapore: Springer.

Flavell, J., Miller, P., & Miller, S. (2002). *Cognitive development* (4th ed.). Upper Saddle River, NJ: Prentice Hall.

Galbraith, P. (2015). "Noticing" in the practice of modelling as real world problem solving. In G. Kaiser & H.-W. Henn (Eds.), *Werner Blum und seine Beiträge zum Modellieren im Mathematikunterricht: Realitätsbezüge im Mathematikunterricht* (pp. 151–166). Wiesbaden: Springer.

Galbraith, P., Stillman, G., & Brown, J. (2010). Turning ideas into modeling problems. In R. Lesh, P. L. Galbraith, C. R. Haines, & A. Hurford (Eds.), *Modeling students'mathematical competencies* (pp. 133–144). New York: Springer.

Geiger, V., Faragher, R., & Goos, M. (2010). CAS-enabled technologies as 'agents provocateurs' in teaching and learning mathematical modelling in secondary school classrooms. *Mathematics Education Research Journal, 22*(2), 48–68.

Geiger, V., Goos, M., & Dole, S. (2015a). The role of digital technologies in numeracy teaching and learning. *International Journal of Science and Mathematics Education, 13*(5), 1115–1137. doi:10.1007/s10763-014-9530-4.

Geiger, V., Goos, M., & Forgasz, H. (2015b). A rich interpretation of numeracy for the 21st century: a survey of the state of the field. *ZDM–Mathematics Education, 47*(4), 531–548. doi:10.1007/s11858-015-0708-1.

Gould, H., & Wasserman, N. H. (2014). Striking a balance: students' tendencies to oversimplify or overcomplicate in mathematical modelling. *Journal of Mathematics Education at Teachers' College, 5*(1), 27–34.

Jorgensen, R., & Lowrie, T. (2012). Digital games for learning mathematics: possibilities and limitations. In J. Dindyal, L. P. Cheng, & S. F. Ng (Eds.), *Mathematics education: expanding horizons* (pp. 378–384). Adelaide: MERGA.

Kaiser, G., & Maaβ, K. (2007). Modelling in lower secondary mathematics classroom—problems and opportunities. In W. Blum, P. Galbraith, H.-W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education* (pp. 99–108). New York: Springer.

Marginson, S., Tytler, R., Freeman, B. & Roberts, K. (2013). STEM country comparisons: international comparisons of science, technology, engineering and mathematics (STEM) education. Report for the Australian Council of Learned Academies. Retrieved from www.acola.org.au on 5 Feb 2017.

Maaß, K. (2006). What are modelling competencies? *ZDM –Mathematics Education, 38*(2), 113–142.

New York Academy of Sciences. (2015). *The global STEM paradox*. New York: Author.

Niss, M. (2010). Modeling a crucial aspect of students' mathematical modeling. In R. Lesh, P. L. Galbraith, C. R. Haines, & A. Hurford (Eds.), *Modeling students' mathematical competencies* (pp. 43–59). New York: Springer.

OECD. (2009). Mathematics framework. In *OECD PISA 2009 assessment framework* (pp. 83–123). Paris: OECD Publishing.

Office of the Chief Scientist. (2012). *Mathematics, engineering and science in the national interest*. Canberra: Commonwealth of Australia.

Paulos, J. A. (2000). *Innumeracy: mathematical illiteracy and its consequences*. London: Penguin.

Stacey, K., & Turner, R. (2015). PISA's reporting of mathematical processes. In K. Beswick, T. Muir, & J. Wells (Eds.), *Proceedings of the 39th conference of IGPME* (Vol. 4, pp. 201–208). Hobart: PME.

STEM Task Force Report. (2014). *Innovate: a blueprint for science, technology, engineering, and mathematics in California public education*. Dublin, CA: Californians Dedicated to Education Foundation.

Stillman, G. (2004). Strategies employed by upper secondary students for overcoming or exploiting conditions affecting accessibility of applications task. *Mathematics Education Research Journal, 16*(1), 41–70.

Stillman, G. (2010). Implementing applications and modelling in secondary school: issues for teaching and learning. In B. Kaur & J. Dindyal (Eds.), *Mathematical applications and modelling* (pp. 300–322). Singapore: World Scientific.

Stillman, G. (2011). Applying metacognitive knowledge and strategies in applications and modelling tasks at secondary school. In G. Kaiser, W. Blum, R. Borrromeo Ferri, & G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling* (pp. 165–180). Dordrecht: Springer.

Stillman, G., Brown, J., & Galbraith, P. (2010). Identifying challenges within transition phases of mathematical modelling activities at year 9. In R. L. Lesh, P. L. Galbraith, C. L. Haines, & A. Hurford (Eds.), *Modeling students mathematical modeling competencies* (pp. 385–398). New York: Springer.

Stillman, G., & Brown, J. (2014). Evidence of "implemented anticipation" in mathematising by beginning modellers. *Mathematics Education Research Journal, 26*(4), 763–789. doi:10.1007/s13394-014-0119-6.

Stillman, G., Brown, J., & Geiger, V. (2015). Facilitating mathematisation in modelling by beginning modellers in secondary schools. In G. A. Stillman, W. Blum, & M. S. Biembengut (Eds.), *Mathematical modelling in education, research and practice: cultural, social, and cognitive influences* (pp. 93–103). Cham: Springer.

Thomson, S., De Bortoli, L., & Underwood, C. (2016a). *PISA 2015: a first look at Australia's results*. Melbourne: ACER.

Thomson, S., Wernert, N., O'Grady, E., & Rodrigues, S. (2016b). *TIMSS 2015: a first look at Australia's results*. Melbourne: ACER.

Treilibs, V. (1979). Formulation processes in mathematical modelling. – Thesis submitted to the University of Nottingham for the degree of Master of Philosophy.

Wijaya, A., Van den Heuvel-Panhuizen, M., Doorman, M., & Robitzsch, A. (2014). Difficulties in solving context based PISA mathematics tasks: an analysis of students' errors. *The Mathematics Enthusiast, 11*(3), 555–584.

Witzel, A., & Reiter, H. (2012). *The problem-centred interview*. London: Sage.

Wood, L., Petocz, P., & Reid, A. (2012). *Becoming a mathematician: an international perspective*. Dordrecht: Springer.