Permit Market Auctions with Allowance Reserves

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Abstract

This article investigates pollution permit auctions that incorporate allowance reserves. In these auctions the sale of a fixed quantity of permits is supplemented by an additional permit reserve. This reserve automatically releases permits if a sufficiently high price is triggered. The main justifications for implementing an allowance reserve are to reduce price volatility as well as assisting in cost containment. We show—paradoxically—that incorporating an allowance reserve into a permit auction can decrease firms' payoffs, increase the clearing price, and increase compliance costs. This has implications for all major cap-and-trade markets, including the US Regional Greenhouse Gas Initiative.

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1 Introduction

Cap-and-trade markets are now a common regulatory instrument to control pollution. Behind this enthusiastic adoption is the core economic rationale of least-cost pollution control: the aggregate costs of pollution control are minimized by allowing firms to trade a fixed number of pollution permits. Although regulation is apparently least-cost, current regulators have real concerns about the prohibitive costs to industry. The potential for price volatility—and therefore increased (and uncertain) industry costs—has placed the issue of cost containment at the center of policy debates (Tatsutani and Pizer, 2008). Many regulators have implemented cost-containment procedures to reduce firms' compliance costs and ensure strong industry support. One such mechanism is the so-called *allowance reserve*, which provides the market with a fixed reserve of permits that can be released if a threshold permit price is triggered (Murray et al., 2009).[1] This mechanism is now common practice in major cap-and-trade markets, which include the Cost Containment Reserve (CCR) of the US Regional Greenhouse Gas Initiative (RGGI), the Market Stability Reserve (MSR) of the European Union Emissions Trading Scheme (EU-ETS), and the Allowance Price Containment Reserve (APCR) of the Californian Cap-and-Trade Program (AB-32).

Within these cap-and-trade markets, a second structural change has been observed; the process of initial permit allocation is moving away from free allocation towards auctioning. Until very recently, the auctions that did exist tended to follow a very standard format: a multi-unit auction with a fixed number of permits. Importantly, however, many of these auction formats have been modified to include the existence of an allowance reserve. Thus in some schemes—such as RGGI—the auction system now has endogenity in the supply of permits: firms' bids can now activate the release of additional permits.[2] Given the addition of an allowance reserve, it is *a priori* unclear how the auction equilibria will change. Having an additional supply of permits realized on reaching a trigger price may have consequences for firms' equilibrium bidding strategies, revenue generation, and the functioning of the permit market. Given that both the auctioning of permits and the presence of an allowance reserve are prevalent in contemporary cap-and-trade markets, it is important to consider how these two institutional structures interact and the implications for cost containment.

In this article we investigate a permit auction under the presence of an allowance reserve. We model a permit auction in which the supply of permits can be increased if the clearing price reaches a threshold trigger price. We provide firms' equilibrium bidding strategies, as well as the implications for the regulator's revenue and auction clearing prices. We find that for a class of symmetric subgame-perfect equilibria, implementing an auction with allowance reserve may, in fact, decrease firms' expected payoffs and increase the clearing price relative to an auction without allowance reserve. Intuitively, the introduction of an allowance reserve distorts firms' demand schedules. As a result, for clearing prices below the trigger price, the allowance reserve is not activated. This could lead to a lower level of permit allocation (and identical price) relative to an standard auction: firms' payoffs are strictly lower under the implementation of an allowance reserve. This scenario is very likely to occur: as we prove in this article, clearing prices above the trigger price do not constitute an equilibrium. Indeed, further support

[1]This is also known as a 'soft' price ceiling (Fell et al., 2012). Other cost containment processes—such as 'hard' price ceilings and price collars—have also been considered. Cost containment can be established by allowing offsets to enter the market as well as the introduction of banking and borrowing mechanisms (Fell and Morgenstern, 2010).

[2]This is also similar to the allowance reserve proposed in H.R. 2454 (Waxman-Markey Bill). Other schemes have subtle differences. As documented in Fell (2015), the European Commission decided to: (i) 'backload' permits by reducing the number of permits for sale at auction in Phase III of the scheme. Allowances were planned to be reduced by 400 million in 2014, 300 million in 2015 and 200 million in 2016, as well as (ii) creating the Market Stability Reserve (MSR). The MSR is triggered when the number of permits in circulation falls outside a pre-defined range. In particular, allowances are added to the reserve (taken away from future auctions) if the total surplus of permits is higher than 833 million allowances. Similarly, allowances are released from the reserve if the total surplus falls below 400 million allowances. In the Californian Cap-and-Trade Program AB-32, the reserve can be sold to firms for a predetermined (three-tiered) price (Borenstein et al., 2015).

arises as we additionally show that there exists only one equilibrium where the allowance reserve is actually used, and this occurs when the clearing price is equal to the trigger price. Interestingly, in this one equilibrium the allowance reserve is completely exhausted, which suggests we are unlikely to observe partially used reserves in reality. Our model predictions fit the experiences of the US Regional Greenhouse Gas Initiative (RGGI).

In our framework the regulator sets a reserve price, quantity of allowance reserve, and a threshold trigger price: all three parameters are set prior to the auction and are common knowledge to all firms. If the initial clearing price is larger than the trigger price, then the reserve quantity is released until either the clearing price becomes equal to the trigger price or the whole reserve exhausts. To investigate this, we advance the framework of a multi-unit uniform-price auction, where bidders simultaneously submit their demand schedules. In such a setting the regulator aggregates the demand schedules then determines the clearing price. This core process is frequently used in the auctioning of financial securities, pollution permits, and energy products (Ausubel and Cramton, 2004). The popularity of uniform-price auctions continues even though it is well known that they have inefficient equilibria: bidders' optimal strategies may result in a reduction of their actual demand functions and consequently the clearing price is lower than the equivalent Walrasian equilibrium price (e.g., Wilson, 1979; Back and Zender, 1993; Wang and Zender, 2002; Ausubel et al., 2014). The majority of literature on multi-unit auctions focuses on fixed supply, yet a small sub-field now considers variable supply (Hansen, 1988; Lengwiler, 1999; Back and Zender, 2001; LiCalzia and Pavan, 2005; McAdams, 2007; Montero, 2008; Damianov and Becker, 2010). Although this sub-field of literature allows the auction supply to vary, the choice of quantity is generally unrestricted and is often chosen to maximize the seller's payoff. In contrast to this literature, we provide an alternative framework with two distinct features. First, in our setting the reserve quantity for sale is restricted due to policy (and political) design. Thus, unlike the existing literature, the choice of additional supply cannot be determined after the bids have been submitted. Second, the additional supply is based on conditionality: the additional quantity will only be released for a sufficiently large aggregate demand schedule. Thus we provide the first analysis of a multi-unit auction with a trigger price and fixed reserve. Using our framework, we provide a paradoxical result; namely, introducing a cost containment reserve may actually *decrease* firms' payoffs and *increase* the price of permits and the associated costs of compliance. As many permit auctions are used for their price-discovery benefits, this distortion has the potential to generate incorrect price signals to market participants.[3]

Our model has similar structural characteristics to the auction process of the Regional Greenhouse Gas Initiative (RGGI).[4] This market incorporates 9 Northeastern US states and controls greenhouse gases emitted from the power sector. Since its inception in 2008, quarterly auctions (with a reserve price) have initially allocated the permits. From the beginning of 2014, the auction mechanism was augmented to include a Cost Containment Reserve (CCR) (RGGI, 2013). Initially the allowance reserve was set at 5 million permits then increased and sustained at 10 million permits each year thereafter (the allowance reserve is not transferable between years). The auction trigger price required to release the permits was set at \$4 per CO_2 allowance in 2014 and \$6 per CO_2 allowance in 2015. This trigger price increases to \$8 and \$10 per CO_2 allowance for the years 2016 and 2017, respectively, and increases by 2.5% per annum thereafter. As can be seen from Table 1, the CCR has been fully exhausted in two auctions with the clearing prices fitting our predictions. The question then arises: how effective is the allowance reserve in controlling costs within the auction mechanism?

[3]As explained by Ellerman et al. (2000), the use of annual US EPA auctions for the Acid Rain Program helped to stimulate the permit market by generating early price signals, which could be used to assist in bilateral trade as well as providing new entrants with a secure method of permit allocation. Further evidence has been provided—for a range of auction types—that show permit auction clearing prices track underlying market conditions accurately (Burtraw et al., 2010a).

[4]For the preliminary auction design of RGGI see Holt et al. (2007).

Table 1: RGGI auction results 2014-2015.

Auction	Permits offered	CCR released volume (% of reserve)	Clearing price in $ per allowance
Auction 30 (Q4 2015)	15,374,274	0	$7.50
Auction 29 (Q3 2015)	15,374,294	10,000,000 (100%)	$6.02
Auction 28 (Q2 2015)	15,507,571	0	$5.50
Auction 27 (Q1 2015)	15,272,670	0	$5.41
Auction 26 (Q4 2014)	18,198,685	0	$5.21
Auction 25 (Q3 2014)	17,998,687	0	$4.88
Auction 24 (Q2 2014)	18,062,384	0	$5.02
Auction 23 (Q1 2014)	18,491,350	5,000,000 (100%)	$4.00

Source: Adapted from: https://www.rggi.org/market/co2_auctions/results

An extensive debate exists over the effectiveness of allowance reserves. Proponents of these allowance reserves show that, under the presence of uncertainty, allowance reserves reduce the net present value of compliance costs. In particular Fell et al. (2012) shows, by using a stochastic dynamic framework, that an expansion of an allowance reserve reduces net present value of control costs, but at a diminishing rate.[5] Further, on investigating the reserve within the EU-ETS, Fell (2015) finds that a reserve can help stabilize permit allocation price levels in a more cost-effective manner than scheduled permit allocation reductions.[6] Due to the lowering of the net present value of compliance costs, it has thus been argued—from a political economy perspective—that an allowance reserve "may be able to bridge differences between environmental advocates seeking a cap on emissions and industrial interests concerned about costs" (Murray et al., 2009, p. 90). From this perspective, then, the concept of a reserve has been developed as a politically viable version of a 'hard' price collar (safety valve) (Burtraw et al., 2010b; Fell and Morgenstern, 2010; Grüll and Taschini, 2011; Fell et al., 2012).[7] Yet there continues to be a controversial debate over the use of a reserve. For example, Borenstein et al. (2014, pp. 86-88) argues—in the context of the California AB-32 market—that in order to fully control price volatility the regulator should, instead, be ready to expand the reserve in order to limit the clearing price (akin to having a 'hard' price ceiling).[8] Indeed, as seen above in the context of RGGI, the allowance reserve has been completely exhausted each year, which may not effectively contain costs within the auction.

Throughout this debate, attention has—justifiably—focused on the ability of the allowance reserve to dampen cost shocks and aggregate compliance costs within the permit market. As such it has abstracted from the complex interactions between the allowance reserve and the auction mechanism. Yet given that

[5]An important design issue is the source of permits within the allowance reserve. It is clear that, holding demand constant, if the permits are sourced from the current cap, then clearing prices may rise under the presence of an allowance reserve (Schatzki, 2012). Yet as this article proves, demand schedules endogenously change due to the inclusion of an allowance reserve. Murray et al. (2009) suggests permits can be sourced from future caps ("system borrowing") or allowing them to vanish if not used (thus establishing a range of potential long term caps).

[6]Recently, however, Perkis et al. (2016), Shobe et al. (2014), and Holt and Shobe (2016) conducted laboratory experiments and found that 'soft' ceilings often did not perform well.

[7]Hybrid approaches were first analyzed by Roberts and Spence (1976) and developed by Pizer (2002). Recent approaches on price controls have incorporated issues relating to strategic behavior (Stocking, 2012) and compliance (Holland and Moore, 2013; Hasegawa and Salant, 2014).

[8]For an analysis of price volatility within the California AB-32 market see Borenstein et al. (2015) and for a discussion of the issues involved within permit auction design see Lopomo et al. (2011).

auctions provide a major component of information dissemination (such as price discovery) within the market, it is important to consider the potential impact of an allowance reserve. In this article, then, we address this gap by focusing our attention on how an allowance reserve has the potential to alter the auction equilibria.

Our contribution is twofold. First, we establish a novel auction framework that incorporates an allowance reserve with a trigger price. We add to the literature by incorporating an auction supply mechanism that is both limited in size and has the characteristics of conditionality. This, therefore, extends the scope and potential applicability of multi-unit auctions with endogenous supply. Second, we advance the policy debate on allowance reserves by critically evaluating the potential consequences of a reserve within a permit auction. This article, then, provides the first theoretical analysis of this connection. We are thus able to suggest potential auction design improvements for existing cap-and-trade markets that incorporate allowance reserves.

The article is structured as follows. In Section 2 we provide a formal model of the auction mechanism with and without an allowance reserve. In Section 3 we detail the relative merits and drawbacks of an allowance reserve within a permit auction. Section 4 extends the model to asymmetric marginal valuations as well as decreasing valuations. Section 5 provides some policy implications for the RGGI auctions. Section 6 provides some concluding remarks. All proofs are provided in the Appendix.

2 The model

2.1 Preliminaries

Consider a cap-and-trade market that consists of an index set of firms $I = \{1, 2, \dots, n\}$. The regulator of this market has an aggregate emissions target of $Q \in \mathbb{R}_+$ and initially allocates a corresponding number of pollution permits via a permit auction.[9] Suppose firms have a marginal value of v for each pollution permit, which is common knowledge to all firms but not the regulator.[10] This assumption is made for tractability of the analysis, however we show later in the article (Section 4) that our main results still hold when we allow for asymmetric permit valuations and decreasing marginal valuations. Although the regulator does not know v, it knows v is distributed according to some distribution function $F(.)$ with the support $[\underline{v}, \bar{v}]$. F is twice differentiable and has a continuous density $f < \infty$.

The regulator sells the Q permits via an uniform-price sealed-bid auction with a reserve price $r > 0$. Similar to existing cap-and-trade markets, the regulator also has an allowance reserve of permits, which can be released to the auction if the clearing price reaches a certain threshold. Formally, denote the *allowance reserve* as $\breve{Q} \equiv \bar{Q} - Q > 0$, where $\bar{Q}$ is the maximum supply of permits. The regulator commits to the following permit supply schedule: if the clearing price is higher than a trigger price, denoted $\tilde{p}$, then the regulator releases more permits (up to $\breve{Q} > 0$) in order to reduce the market-clearing price towards $\tilde{p}$.

Our auction game is structured as follows. First, the regulator announces $\{Q, \bar{Q}, \tilde{p}, r\}$. Second, firms submit permit demand schedules. There are two possible scenarios that can occur: (i) the clearing price is $p^* \leq \tilde{p}$ and (ii) the clearing price $p^* > \tilde{p}$. In the first case, all bids higher than the clearing price would

[9] As our focus here is on how existing institutional auction designs affect participating firms, we abstract from the regulator's objective function. We do, however, provide analysis of revenue maximization later in the article.

[10] The marginal value v is the avoided marginal costs of reducing pollution. We thus follow the realistic construction of marginal abatement costs (e.g., Mckinsey and Company, 2016) and assume that firms have long-run marginal abatement cost curves that are globally convex but discontinuous because of alternative technology classes that increase in (marginal) cost. In other words, the marginal abatement cost function is a step function where marginal costs are constant for specific technology classes but discontinuously jump when adopting new technologies.

receive their submitted quantities and pay p^*: the allowance reserve is not utilized. In the second case, the regulator releases more permits until either the clearing price becomes equal to $\tilde{p}$ or the whole extra permits exhausts.

2.2 An auction mechanism without an allowance reserve

Each firm $i \in I$ submits a demand schedule $d_i(p,v) : [r,\infty) \to [0,Q]$, which is decreasing and left continuous. The aggregate demand is the sum of individual demand functions, such that $D(p,v) = \sum_{i=1}^{n} d_i(p,v)$. Given demand schedules and a quantity of permits Q for sale in auction, the clearing price is,

$$p^* = \max\left\{p|D(p) \geq Q; p \geq r\right\}. \tag{1}$$

If there is no price higher than r that satisfies the above condition, then the clearing price becomes the reserve price r. In this case, at least some of the total supply of permits will not be allocated. Now it is possible that the aggregate demand becomes discontinuous or flat at some points including p^*. If the demand is flat at p^* then we have excess demand at the clearing price and the extra demand must be rationed. In particular, a discontinuity in each firm's demand at any price p can be defined by $\Delta d_i(p,v) \equiv d_i(p,v) - \lim_{p' \downarrow p} d_i(p',v)$, where p' is any price greater than p. Aggregating over all firms determines this discontinuity for the total demand schedule, which we denote by $\Delta D(p,v) \equiv \sum_{i=1}^{n} \Delta d_i(p,v)$. Using this we can derive firm i's permit allocation,

$$q_i^*(p^*,v) = d_i(p^*,v) - \big(D(p^*,v) - Q\big)\frac{\Delta d_i(p^*,v)}{\Delta D(p^*,v)}. \tag{2}$$

Firm i's equilibrium initial allocation of permits equals their permits received from their individual demand schedule at the clearing price $d_i(p^*,v)$ minus the *pro rata* reduction of permits due to excess demand (if any). That is, the ratio on the right hand side is the total excess demand $(D(p^*,v) - Q)$ multiplied by the proportion of discontinuity (excess demand) for firm i relative to all firms $\frac{\Delta d_i(p^*,v)}{\Delta D(p^*,v)}$. Given the clearing price and the equilibrium quantity, each firm pays the total amount of $p^* q_i^*(p^*,v)$ for all permits received. Therefore, the payoff function for firm $i \in I$, given a clearing price p^*, is

$$\pi_i = (v - p^*) q_i^*(p^*,v). \tag{3}$$

We first begin by analyzing the equilibrium demand schedules when the supply of permits is fixed at the level Q. For a fixed supply of permits, if the regulator sets a reserve price $r \leq \underline{v}$, then there are equilibria of the uniform-price auction in which the regulator could do the same by canceling the auction and setting a fixed price $\underline{v}$ for all units (Back and Zender, 1993). To avoid such equilibria we focus on those cases with $r > \underline{v}$. The following proposition is adapted from Back and Zender (1993).

Proposition 1 (Back and Zender (1993)). *For any price $p^* \in [r,v]$, there is a symmetric pure-strategy subgame-perfect equilibrium with a clearing price p^* in which firms have the following demand schedule,*

$$d_i(p) = \begin{cases} 0 & \textit{if} \quad p > p^\dagger, \\ Q\frac{(p^\dagger - p)}{n(p^\dagger - p) + p - p^*} & \textit{if} \quad p^* < p \leq p^\dagger, \\ \frac{Q}{n-1} & \textit{if} \quad r \leq p \leq p^*, \end{cases} \tag{4}$$

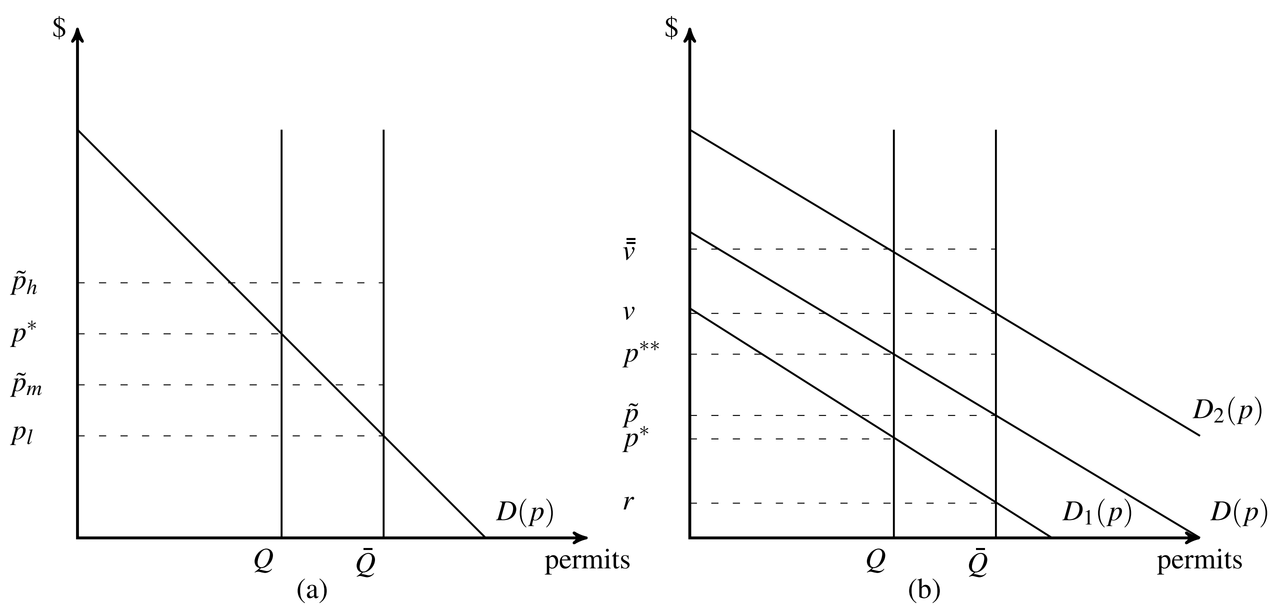
where $p^\dagger = \frac{(n-1)\bar{v}}{n} + \frac{p^*}{n}$ and each firm receives $\frac{Q}{n}$ permits.

Proposition 1 suggests that there exists a continuum of equilibrium prices that can lie between the reserve price and v. There are some notable points about this result. First, as shown in Back and Zender (1993), even if each bidder has her own independent private signal regarding the marginal value v, we still have the same set of equilibria and the result remains unchanged. Indeed, this class of equilibria is independent of bidders private signals. Second, since any price between $[r, v]$ could characterize an equilibrium with the same final permit allocation $\frac{Q}{n}$, any rational firm prefers to pay less.[11] Therefore, the most likely prediction is the lowest possible price, that is, the reserve price r. In fact, a clearing price equal to r is Pareto dominant equilibrium for all firms. The last point raises a major concern for those auctions without a reserve price (or a small reserve price) as the clearing price can potentially end up very low. We now turn to consider how these classes of equilibria change when a regulator implements an allowance reserve.

2.3 An auction mechanism with an allowance reserve

Suppose the regulator announces their target level Q of permits for the auction followed by the commitment to increase the supply with the allowance reserve $\breve{Q}$ if the clearing price is greater than $\tilde{p}$. This mechanism design is similar to the mechanism within the Regional Greenhouse Gas Initiative (RGGI). One clear observation is that if $\tilde{p}$ is greater than or equal to v, nothing will change: firms would never pay a price above their marginal valuations. From this point forward we suppose $\tilde{p} < v$ unless it is explicitly stated otherwise.

Figure 1: Aggregate submitted demand and the allowance reserve



As shown in Figure 1(a) suppose the aggregate demand is $D(p)$. Then given the permits Q, a clearing price exists at p^*. There are three possibilities for where $\tilde{p}$ lies with respect to p^*. If $\tilde{p}$ is above p^* (for

[11] Our aim in this article is to focus on the consequences for the clearing price when an allowance exists, as such our focus is not on the individual firm permit allocation. If it was, then it would be possible to incorporate financial capacity constraints for all firms, which would generate asymmetric post-auction permit holdings.

example, $\tilde{p}_h$), then no further supply would be released and firms with bids higher than the clearing price would receive their permits and pay p^*. If $\tilde{p}$ is below p^*, two possible cases can happen: (i) $p_l \leq \tilde{p} < p^*$ (for example, $\tilde{p}_m$), where p_l is the price in which the aggregate demand is equal to $\bar{Q}$ and (ii) $\tilde{p} < p_l$. In the first case, the quantity of permits sold in the auction increases to a point within the interval $[Q, \bar{Q}]$ and the clearing price becomes $\tilde{p}$. Note that if $\tilde{p} = p_l$ the allowance reserve is fully exhausted at $\bar{Q}$. In the second case, the quantity of permits sold will increase to $\bar{Q}$ (the reserve is fully exhausted) but the reserve quantity is not enough to reduce the clearing price to $\tilde{p}$ and the clearing price becomes greater than $\tilde{p}$. This illustrates the potential complexity of this supply scheme.

To further assist in our understanding of the supply scheme, observe Figure 1(b), which depicts a number of potential demand schedules. As shown in Figure 1(b), $D(p)$ is the case where submitted demand and supply intersect at a point above $\tilde{p}$, but below v, $D_1(p)$ is a case where demand is "too low" with a clearing price p^* below $\tilde{p}$, and $D_2(p)$ is a case where demand is "too high". As can be observed, p^{**} is the highest threshold price in which the addition of an allowance reserve can reduce the clearing price to $\tilde{p}$. It is important to mention that p^{**} can be larger than v but as long as v is above $\tilde{p}$ the analysis remains the same. When the demand is "too high", such as $D_2(p)$, then $\bar{\bar{v}}$ is the highest threshold price in which the addition of an allowance reserve would result in the clearing price being equal to firms' valuations v. Of course, for prices above $\bar{\bar{v}}$ firms' demands are zero.

We now provide an equilibrium characterization of the auction with allowance reserve.

Theorem 1. *Suppose the regulator commits to a pair $(r, \tilde{p})$ such that, $r < \tilde{p}$, and a supply of Q permits which can increase up to $\bar{Q} > Q$, if the initial clearing price is greater than $\tilde{p}$. For any price $p^* \in [r, \tilde{p}]$ there exists a symmetric pure-strategy subgame-perfect equilibrium in which the final clearing price is p^* and each firm has the following demand schedule:*

$$d_i(p) = \begin{cases} 0 & \text{if} \quad p > \bar{\bar{v}}, \\ \frac{\bar{Q}}{n}\frac{(n-1)\bar{v}+p^{**}-np}{(n-1)\bar{v}+p-np} & \text{if} \quad p^{**} < p \leq \bar{\bar{v}}, \\ \frac{Q^\dagger}{n} & \text{if} \quad \tilde{p} \leq p \leq p^{**}, \\ \frac{Q}{n}\frac{(n-1)\bar{v}+p^{*}-np}{(n-1)\bar{v}+p-np} & \text{if} \quad p^* < p < \tilde{p}, \\ \frac{Q}{n-1} & \text{if} \quad r \leq p \leq p^*, \end{cases} \tag{5}$$

where $Q^\dagger = Q + \lambda(\bar{Q} - Q)$ with $\lambda \in (0, 1]$, $\bar{\bar{v}}$ is the highest price in which buyers have positive demand, and p^{**} is the maximum price in which the allowance reserve permits are enough to reduce the clearing price to $\tilde{p}$. Also firm i receives $\frac{Q}{n}$ if $p^* < \tilde{p}$ and $\frac{\bar{Q}}{n}$ if $p^* = \tilde{p}$.

Theorem 1 characterizes a continuum of equilibria that includes any clearing price between $[r, \tilde{p}]$. There are numerous demand functions that can characterize equilibria like the one in (5). In fact, one can show any decreasing left continuous function $\gamma(p)$ with $\gamma(p^{**}) = \frac{\bar{Q}}{n}$, can characterize similar symmetric equilibria as long as the residual supply left for a bidder is flat enough for prices greater than p^{**} and steep enough for prices lower than p^{**}. Such characterization can be observed in Section 4 where we extend the model to asymmetric values. However, for all these functions the set of equilibrium clearing prices and the quantity each firm receives would remain the same.

Comparing Theorem 1 with Proposition 1 highlights the structural differences when an allowance reserve is incorporated into the auction. The introduction of the allowance reserve augments the firms' demand schedules in such a way as to separate between the emissions target Q and the allowance reserve $\breve{Q}$. Further, Theorem 1 proves that $p^* > \tilde{p}$ cannot constitute an equilibrium in the current setting. To observe a clearing price greater than $\tilde{p}$, it must be the case that the whole reserve quantity is released and is not enough to reduce the price to $\tilde{p}$. Such cases would happen for clearing prices above p^{**}—the maximum price in which the extra permits are enough to reduce the clearing price to $\tilde{p}$ — which are too high for firms to increase their payoffs.[12]

According to Theorem 1, each firm i receives $\frac{Q}{n}$ for $p^* < \tilde{p}$ and $\frac{\bar{Q}}{n}$ for $p^* = \tilde{p}$. The implication of this result is significant. This shows that—in any equilibria—if we observe a clearing price of $\tilde{p}$ then it is the case that all the extra quantity is used. Indeed, in all other equilibria, the allowance reserve is left completely unused with a clearing price below $\tilde{p}$. Consider, for a moment, the observable real world outcomes of the RGGI auctions with an allowance reserve. As we have seen in Table 1, Auction 23 (Q1 2014) and 29 (Q3 2015) resulted in the CCR being released. In both cases, the CCR was entirely exhausted and the clearing price is in line with our predictions (i.e., equal to the trigger price). This is, in a sense, good news for the regulator: the aim of the allowance reserve is to prevent prices being greater than $\tilde{p}$. However, as we will prove in Section 3, obtaining this target via an allowance reserve is not necessarily desirable. The result in Theorem 1 gives us an important clue to analyze when there is a higher chance of observing a clearing price equal to $\tilde{p}$ or below.

Lemma 1. *Given a value v, if there exists a $\hat{p} \in [r, \tilde{p})$ such that $\hat{p} < v - \frac{\bar{Q}}{Q}(v - \tilde{p})$, then an equilibrium clearing price $\tilde{p}$ is Pareto dominated for firms by any clearing price less than or equal to $\hat{p}$.*

The intuition of Lemma 1 suggests even when the reserve is released there are conditions where firms' payoffs are lower. Given firms' payoffs are lower under a released allowance reserve it suggests firms bid in such a way to avoid activating the allowance reserve. This then means that we are more likely to observe equilibria with smaller clearing prices, which makes the extra supply schedule analogous to an auction without an allowance reserve.

There are some comparative statics of this result which are important to mention. First, it is clear that the greater the $\tilde{p}$, the higher the chance that the extra supply becomes unused. However, when $\frac{\bar{Q}}{Q}$ becomes larger, then there is a greater chance of having a clearing price equal to $\tilde{p}$ and the whole reserve supply being used. This obviously holds for the case with $v > \tilde{p}$. Remember, however, that v is not known to the regulator who sets $\tilde{p}$ and it is possible to have $\tilde{p}$ greater than v. In that case it is clear that the equilibrium is $p^* \in [r, v]$ and the extra supply would never be used.

3 Evaluating the relative merits of allowance reserves in permit auctions

In this section we compare the outcomes of auctions with and without an allowance reserve. We begin analyzing if the auction with an allowance reserve and trigger price could potentially reduce the cost to firms, given the current setting. As our aim is to consider how the existing institutional auction design may affect firms, there is no requirement to model the objective function of the regulator. Indeed, in reality, it is unclear as to the regulator's objective. There can be a host of possible objectives, for example, to maximize revenue (to provide public goods or in the case of RGGI, energy efficiency schemes), to maximize social welfare, improve allocative efficiency or to simply contain abatement costs (e.g., Cramton and Kerr, 2002). To provide some highlight, we do consider the optimal auction design for a regulator that is concerned about revenue maximization.

[12]It is important to mention that since $\tilde{p}$ is exogenous and is set by the regulator without the knowledge of v it can be set greater than v. Then the equilibrium demand will be identical to the case without the allowance reserve.

3.1 Cost containment

The primary goal of an allowance reserve is to contain firms' cost within a cap-and-trade market. Due to unexpected cost shocks in the economy, the permit market may experience levels of increased price volatility and potentially higher levels of compliance costs. Our purpose in this subsection is to consider the implications of the allowance reserve on firms' cost (payoffs) within the auction. To do this we want to hold target-level environmental stringency constant and compare firms' payoffs when they can obtain $\bar{Q}$ permits without an allowance versus an allowance reserve that first distributes Q then $\breve{Q}$ if the initial clearing price reaches the trigger price.[13] The following proposition shows that, in many cases, the allowance reserve actually reduces firms' payoffs.

Proposition 2. *For a permit allocation of $\bar{Q}$, there exists a class of symmetric equilibria in which the auction without an allowance reserve is Pareto superior for firms relative to the auction with an allowance reserve.*

Proposition 2 shows that any equilibria with $p^* \in [r, \tilde{p})$ are Pareto superior to the auction with an allowance reserve. Thus Proposition 2 holds for a large class of equilibria. The only class of equilibria in the auction that remains to be discussed is $p^* \in [\tilde{p}, v]$. Note that, for an auction without an allowance reserve, Proposition 1 suggests the most likely prediction is the Pareto superior clearing price $p^* = r$. Thus even for a potentially large continuum of prices, the equilibrium clearing price is likely to be pushed down towards the reserve price: Proposition 2 is therefore still applicable. Only if the equilibrium clearing price in the auction remains Pareto inferior for firms (i.e., $p^* \in [\tilde{p}, v]$) will payoffs be higher under an auction with allowance reserve. As we will illustrate in Section 5, the evidence from RGGI auctions appears to confirm our predictions in Proposition 2.[14]

One possible explanation for Proposition 2 is that the permit supply in the auction is larger than or equal to the supply in the allowance reserve regime; thus, firms' payoffs are higher (prices are lower). There is a problem with such intuition: it implicitly assumes that firms' demand schedules are invariant to changes in the supply schedule. Yet this is incorrect. We have shown in Theorem 1 and Proposition 2, that the demand schedules endogenously change when an allowance reserve is included. To highlight these differences further, we can compare regimes with potentially different environmental stringency ($\bar{Q}$ (Q) for an auction with(out) an allowance reserve).

Corollary 1. *For every $\hat{p} \in [r, \tilde{p})$ such that $\hat{p} < v - \frac{\bar{Q}}{Q}(v - \tilde{p})$, an auction with an allowance reserve is weakly dominated by an auction without an allowance reserve and quantity Q.*

We now show for the same class of equilibria discussed above, the impact of an allowance reserve on the aggregate cost of pollution reduction, which is derived by calculating the area under the demand schedule and to the right of $\bar{Q}$.

Corollary 2. *For a permit allocation of $\bar{Q}$, there exists a class of symmetric equilibria in which the auction with an allowance reserve has higher aggregate pollution abatement costs relative to an auction without an allowance reserve.*

Corollary 2 shows that for our class of symmetric equilibria, pollution reduction costs are lower when no allowance reserve is implemented. The inclusion of an allowance reserve may therefore not contain costs within a permit auction.

[13] We also provide conditions in Corollary 1 under which the auction with tighter environmental stringency $Q < \bar{Q}$ weakly dominates the auction with an allowance reserve and emissions $\bar{Q}$.

[14] We abstract from introducing exogenous shocks (uncertainty) to the demand schedule. The inclusion of cost shocks is common practice in the cost-containment literature, as without cost shocks there would be no requirement for an allowance reserve. Note, however, that introducing cost shocks here will not alter our results. For an objective comparison, regimes with and without allowance reserves should be subject to identical cost shocks. As such, this has the consequence of altering the demand schedules for both cases in an identical manner.

3.2 Clearing prices and maximizing auction revenue

The regulatory objective of cost containment is clearly important for the development of cap-and-trade markets. Yet the generation (and maximization) of auction revenue is also another objective that we must consider. Indeed, the generation of periodic and stable revenue is becoming increasingly important: one can observe from many permit markets that auction revenues are earmarked to provide investment in new clean technologies, energy efficiency programs, and so on (see, for example, RGGI, 2015).

The next proposition characterizes a condition under which the regulator could guarantee a minimum clearing price.

Proposition 3. *If $r > \bar{v} - \frac{\bar{Q}}{Q}(\bar{v} - \tilde{p})$ then there exists a class of symmetric equilibria where:*

- *in the auction with an allowance reserve the clearing price $\tilde{p}$ is Pareto superior for firms,*
- *in the auction without an allowance reserve the clearing price of r is Pareto superior for firms.*

Therefore, there is a higher probability to observe larger clearing prices in an auction with an allowance reserve compared to an auction without an allowance reserve with all available units $\bar{Q}$.

For the allowance reserve regime, Proposition 3 shows that there are two effects on firms' payoffs when the equilibrium clearing price is $\tilde{p}$. First, there is a negative effect because firms pay a higher price. Second, there is a positive effect, which is the release of the extra quantity of permits and thus a larger share for each firm. Proposition 3 shows a class of equilibria under which the net effect on payoffs is positive: accordingly, firms prefer a clearing price of $\tilde{p}$. Also the above proposition suggests that the regulator can implement $\{r, \tilde{p}, Q, \bar{Q}\}$ in such a way—even with incomplete information on v—that encourages firms to increase their submitted demands in an auction with an allowance reserve. In this case, unlike the auction without allowance reserve, where the seller has no instruments except the reserve price, the regulator would expect a higher clearing price and therefore larger revenue if the marginal value is greater than a threshold $\tilde{v}$, which is determined by $r = \tilde{v} - \frac{\bar{Q}}{Q}(\tilde{v} - \tilde{p})$.

The above result investigates the conditions where the regulator—by choice of an appropriate reserve price—can generate a larger clearing price with the inclusion of an allowance reserve. This result, however, can be interpreted from a broader perspective. As in most permit auctions, the reserve price (and any per annum increase) is usually fixed at the beginning of auction design. For example under RGGI the reserve price was set at the outset and increases at 2.5% per annum indefinitely. In absence of any routine review process, there will exist inertia within the regulatory system, where the value of reserve price is not always desirable.[15] Thus it is feasible that the inequality $r > \bar{v} - \frac{\bar{Q}}{Q}(\bar{v} - \tilde{p})$ may hold due to adjustments and shocks to v (and its bounds $[\underline{v}, \bar{v}]$) rather than deliberate regulatory design. Thus it is perfectly feasible to observe higher clearing prices when an allowance reserve is implemented simply by the existence of an 'unresponsive' reserve price. For the RGGI application, the 2016 variables are $\tilde{p} = \$8$ per CO_2 allowance, $r = \$2.10$ per CO_2 allowance, $\breve{Q} = 10$ million allowances, and $Q \approx 15$ million allowances. Thus using the above inequality, shows that if $\bar{v} > \$16.85$ per CO_2 allowance then the results in Proposition 3 hold. This, in the current institutional context, is quite plausible.

Although the choice of reserve price is the most obvious transmission mechanism that alters the clearing price, the regulator can also adjust the allowance reserve without changing the maximum level of permits $\bar{Q}$.

Corollary 3. *For any positive quantity $\bar{Q}$ and $\tilde{p} < \bar{v}$, the regulator can adjust the size of the allowance reserve such that $r > \bar{v} - \frac{\bar{Q}}{Q}(\bar{v} - \tilde{p})$.*

[15]Review processes, of course, exist but inertia will continue until the process is complete and recommendations are implemented. Even then, market conditions will continually alter.

Corollary 3 shows the importance of the allowance reserve level, or the ratio of $\frac{\bar{Q}}{Q}$, in this environment. In fact, if the regulator wants to maximize revenue, it—only with the adjustment of the reserve quantity—can generate the conditions for a larger clearing price. Again, using the RGGI application for the 2016 parameters, it can be easily shown that the regulator can obtain the desired outcome if $\frac{\bar{Q}}{Q} > \frac{\bar{v}-2.10}{\bar{v}-8}$. As the current ratio is $\frac{\bar{Q}}{Q} \approx 1.67$, it is quite plausible that the results in Corollary 3 could hold in this context.

3.2.1 Optimal trigger price

One of the key instruments in the current setting is the trigger price. In fact, in reality, trigger prices may be more adjustable than any other variable from the regulator's perspective. An interesting question is what would be the optimal trigger price if the regulator wanted to maximize revenue? Although there are several possibilities for the equilibrium clearing price in the current setting, we use a max-min approach to identify the optimal trigger price. In particular, we consider the worst possible clearing price in the event that the extra supply is not released and then maximize the seller's objective function. When the extra supply is released there is only one equilibrium clearing price which is $\tilde{p}$. The following proposition characterizes an optimal trigger price, given the current reserve allowance scheme.

Proposition 4. *Suppose the regulator's objective is to maximize revenue under the reserve allowance scheme. Then the optimal trigger price is the one that satisfies*

$$\frac{1-F(M)}{f(M)} = M, \tag{6}$$

where $M = \frac{\bar{Q}\tilde{p}-Qr}{\check{Q}}$.

Proposition 4 shows that to maximize revenue the optimal trigger price should be set such that the allowance reserve $\check{Q}$ is equal to the hazard rate, where $M = \frac{\bar{Q}\tilde{p}-Qr}{\check{Q}}$. Note that from RGGI 2016 parameters ($Q = 10$, $\check{Q} = 10$, $r = 2.10$), if we suppose $\bar{v} = \$28$ per ton of CO_2 allowance and v is distributed uniformly on $[0, 28]$, then the optimal trigger price becomes $\tilde{p} \approx \$8$. Thus, it is feasible that the actual optimal trigger price in RGGI is maximizing the regulator's auction revenue. Further note that the condition in Proposition 3 is also satisfied with these numbers and therefore, the regulator not only optimizes the trigger price to maximize revenue but also sets the parameters such that $\tilde{p}$ becomes the Pareto superior clearing price for firms. Here we fix a reserve price and derive the optimal condition for the trigger price. Now, however, consider a case where the trigger price is set together with the reserve price to maximize the regulator's revenue.

Proposition 5. *Suppose the regulator's objective is to maximize revenue under the reserve allowance scheme, then the reserve price must be set equal to the trigger price, and the condition in* (6) *becomes,*

$$\tilde{p} = \frac{1-F(\tilde{p})}{f(\tilde{p})}. \tag{7}$$

Proposition 5 is intuitive. First, the reserve price cannot be larger than the trigger price. Second, as the reserve price increases then the higher the chance that the extra quantity would be released and the clearing price becomes the trigger price. Therefore, a reserve price equal to the trigger price is the optimal choice of the regulator to increase revenue. This result appears to be out of line with the experiences in RGGI. Note that both the reserve price and the trigger price were set at different stages of the regulation process: the reserve price was set at the creation of the scheme and the trigger price was chosen approximately seven years later. Thus, in this case, Proposition 4 provides the realistic experience of RGGI where the trigger price is set given a fixed reserve price. We now provide further discussion on the RGGI auctions.

4 Asymmetric valuations

In this section we investigate how sensitive our results are to the assumption of symmetric values among firms. First, let us start by assuming that there are two types of firms, low type and high type. Each low-type firm has a marginal value equal to v_l while each high-type firm has a marginal value equal to v_h for permits with $0 < v_l < v_h < \bar{v}$. Types and values are common knowledge among firms but the regulator only knows l number of firms are low type and the rest are high types. The following Proposition extends the result of Proposition 1.

Proposition 6. *When there are two types of firms,*

- *for any price $p^* \in [0, v_l]$, there is a class of symmetric equilibria where both types demand permits.*
- *for any price $\hat{p}^* \in (v_l, v_h]$, there is a class of symmetric equilibria where only high-type firms demand permits and low-type firms demand zero.*

As in Proposition 1, we continue to observe that demand reduction still exists, and there are equilbria in which the clearing price is very low. However, there is a class of equilibria where the price is too high for low-type firms, and they exit the auction. One can still show the condition under which the demand reduction equilibrium remains Pareto dominant for all firms.

We now want to extend the results of Theorem 1 to a case with asymmetric values. Suppose the regulator chooses a trigger price $\tilde{p}$ and an allowance reserve $\breve{Q}$ as we defined earlier in the article. Since the regulator does not know v_l and v_h, there are three possibilities for which $\tilde{p}$ could be compared to v_l and v_h. First, if $\tilde{p} > v_h$, then nothing changes compared to a case without allowance reserve. Second, if $v_l < \tilde{p} < v_h$, then obviously low-type firms would never receive any amount of the extra supply. In this case, the class of equilibria with clearing prices lower than v_l exist and is similar to the one in Proposition 6. For prices above v_l the result of Theorem 1 remains unchanged but with one difference that only high-type firms are active. Finally, if $\tilde{p} < v_l < v_h$ the following Proposition characterizes the new set of equilibria.

Proposition 7. *If $\tilde{p} < v_l < v_h$ then, for any price $p^* \in [0, \tilde{p}]$, there exist a class of symmetric equilibria such that:*

- *for equilibrium prices $p^* < \tilde{p}$ no allowance reserve is used,*
- *for an equilibrium price $p^* = \tilde{p}$, the allowance reserve is completely exhausted.*

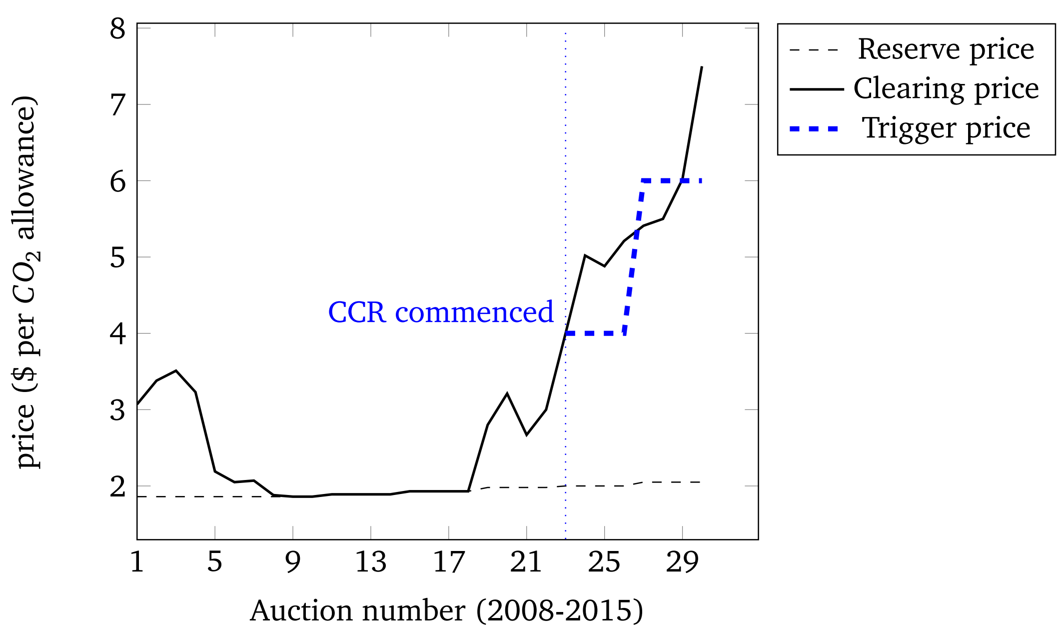
This section extends the assumption of the original model to a case where firms have asymmetric values for permits. As results show, there are no significant changes in the set of equilibria. In particular, the introduction of allowance reserve would not change the result of Theorem 1. Therefore, the allowance reserve is used in only one instance (and fully exhausted), where the equilibrium clearing price is equal to the trigger price. It is important to note that this analysis can easily be extended to more than than two types without any loss in results. One major point is that with many asymmetric types, the result of Theorem 1 still holds and the allowance reserve is only used in an equilibrium price equal to the trigger price.

Another major implication is to note that even if firm's marginal values are not constant for all units, we can extend the result of Theorem 1. To see this, suppose firms have access to multiple abatement technologies in the short term and therefore their demand for permits are diminishing but in a discrete fashion. This demand can be shown as an step function, and each step of this function can be treated as a separate firm with a different type. Therefore, even if firms have decreasing marginal values, the set of equilibria with the maximum clearing price equal to the trigger price remains unchanged. In particular, the only instance where the allowance reserve is used, is the one with a clearing price equal to the trigger price and the reserve allowance is used completely. This is also intuitive: if firms increase their demand beyond $\tilde{p}$, they have incentives to increase it up to a point where the entire allowance reserve is released because the price they pay is not affected while they receive more permits and consequently higher payoffs.

5 Policy implications: RGGI auction prices

In this section we further relate our results and predictions to the US Regional Greenhouse Gas Initiative (RGGI). As discussed earlier in this article, RGGI has a similar structure to what we have modeled here. In fact, it can be readily stated that for 2016: $\tilde{p} = \$8$ per CO_2 allowance, $\check{Q} = 10$ million allowances, and $r = \$2.10$ per CO_2 allowance. Figure 2 provides the reserve and clearing prices for all 30 RGGI auctions from 2008-2015 (inclusive). Also, as indicated in Figure 2, the Cost Containment Reserve (CCR) was established after Auction 22. As can be observed from Figure 2, the clearing price was identical to the reserve price from Auction 9 to 18 (inclusive). In many of these cases there was excess supply of permits in the auction. However, there did exist instances in which the clearing price was equal to the reserve with no excess supply (for example, auction 11). Our analysis appears to be consistent when the Cost Containment Reserve (CCR) was established: when the CCR was used, the entire reserve was exhausted and the clearing price was consistent with the regulator's threshold trigger price (Theorem 1).

Figure 2: Auction clearing prices for RGGI



Source: Adapted from: http://rggi.org/news/releases-archive

Note that from Figure 2 a dramatic increase in the price of permits was observed from Auction 21 onwards. The establishment of the CCR is in an ideal position for us to consider the true cost containment feature of an allowance reserve. Simple observation shows that even when the CCR was active, prices continued to rise exponentially. Factors that may have affected the clearing price within this time-frame are, for example, US legislation on regulating greenhouse gases, the realization of any potential aggregate shocks as well as the tightening of the cap (reduced in 2014 and reducing by 2.5% per annum for 2015-2020). [16] Further, other possible explanations do exist. For example, if the trigger price is reached, this may indicate that prices are likely to be at least as high in future auctions. Thus firms may have an incentive to 'hoard' permits and, as a result, the entire CCR is exhausted.

Given our predictions in Theorem 1, another possible factor can be suggested for the increase in clearing prices. Along with the tightening of the aggregate cap by 2.5% per annum, the regulator has also established a precedent of significant increases in the trigger price: fixed percentage increases of the trigger prices were set at 50% (2014-2015), 33% (2015-2016), 25% (2016-2017), then 2.5% thereafter. Thus our analysis suggests that such large increases in the trigger price are potentially placing upward pressure on the clearing price. Observing the regulator's choice of increasing trigger prices, it illustrates the potential for a paradoxical result: the regulator, in an attempt to contain costs may, in fact, increase auction compliance costs. As Proposition 3 shows, under realistic conditions, the regulator's choice of a high trigger price will turn out to be the firms' Pareto superior clearing price and this is likely to be higher than a corresponding clearing price in an auction without allowance reserve.

We would advocate, given our predictions, that the regulator either set Q or $\bar{Q}$ but discontinue the allowance reserve in its current format. As we proved earlier, setting a cap of $\bar{Q}$ is likely to be more effective in reducing the clearing price than setting Q with an allowance reserve $\breve{Q}$. If the regulator's main objective is to strictly control prices, then we suggest that focus is placed on choosing an optimal reserve price. An alternative method is to provide an unlimited allowance reserve. A major drawback, however, is that an unlimited reserve is void of any quantitative restriction on pollution.

6 Concluding remarks

The purpose of this article is to investigate permit auctions with allowance reserves. A permit allowance reserve is a fixed stock of permits that can be released by the regulator if the clearing price reaches a threshold trigger price. Allowance reserves are justified due to their apparent assistance in reducing price volatility within the permit market and, as a consequence, reducing the net present value of compliance costs (e.g., Murray et al., 2009; Fell et al., 2012).

In this article we model a uniform-price multi-unit auction with and without a permit allowance reserve. We show that there is a class of symmetric subgame-perfect equilibria in which all firms have lower payoffs when the auction format includes a permit allowance reserve. Consequentially, and paradoxically, an allowance reserve with the aim of reducing firms' costs (increasing payoffs) may actually do the opposite. The existence of the allowance reserve distorts firms' equilibrium bidding strategies in such a way that it is entirely possible that the market clearing price is higher under the inclusion of an allowance reserve. This suggests that the justifications for using a cost containment reserve may have to be revised in light of our results. In terms of the US Regional Greenhouse Gas Initiative (RGGI), our results suggest that the Cost Containment Reserve (CCR), should, in many cases, be discontinued in its current format. As the auction mechanism provides the market with price signals, a distortion within these prices may have substantial consequences for the RGGI auction and secondary market.

[16] For a comprehensive analysis of factors affecting RGGI emissions and clearing prices see Murray et al. (2014).

We do not call for the complete disbandment of all cost containment reserves. Indeed, a point to note is that our analysis has focused on the auction. Thus it may, in the vein of Murray et al. (2009), still be feasible to justify cost containment reserves within the market. However, we suggest that any regulator be wary of the potential side effects of an allowance reserve within the auction mechanism.

Appendix

Proof of Proposition 1

Proof. See Back and Zender (1993) Theorem 1. □

Proof of Theorem 1

Proof. Suppose all other firms except firm i follow the demand schedule in (5). Then the residual supply for firm i is the difference between the total available quantity and the sum of demands of all other $(n-1)$ firms. We know that the total quantity could be any amount greater or equal to Q and up to $\bar{Q}$, depending on the initial clearing price. If the clearing price is less than $\tilde{p}$, then the total quantity would be Q, and if the clearing price is greater but still less than p^{**} then a proportion of the extra quantity would be released. Finally, if the clearing price is greater or equal to p^{**} then the total quantity $\bar{Q}$ would be released. Given this supply schedule the residual supply for firm i, $x_i(p)$, is as follows.

$$x_i(p) = \begin{cases} \bar{Q} & \text{if} \quad p > \bar{\bar{v}}, \\ \frac{\bar{Q}}{n}\left(\frac{\bar{v}-p^{**}}{\bar{v}-p}\right) & \text{if} \quad p^{**} < p \leq \bar{\bar{v}}, \\ \frac{Q^{\dagger}}{n} & \text{if} \quad \tilde{p} \leq p \leq p^{**}, \\ \frac{Q}{n}\left(\frac{\bar{v}-p^{*}}{\bar{v}-p}\right) & \text{if} \quad p^{*} < p < \tilde{p}, \\ \frac{Q}{n} & \text{if} \quad p = p^{*}, \\ 0 & \text{if} \quad r \leq p < p^{*}. \end{cases} \tag{8}$$

If the price is higher than $\bar{\bar{v}}$, no other firm demands any quantity and therefore the residual quantity for firm i is the whole quantity. This is because even if the reserve quantity is released the final price never falls below v. If the price is between p^{**} and $\bar{\bar{v}}$ then the residual supply shown above is the result of the subtraction of the demand on $(n-1)$ bidders (line two of (5)) from the total quantity $\bar{Q}$ for such prices. Firm i's payoff is therefore,

$$\pi_i = (v-p)\frac{\bar{Q}}{n}\left(\frac{\bar{v}-p^{**}}{\bar{v}-p}\right),$$

which is decreasing in p for prices above p^{**}. Therefore, this term is maximized at $p = p^{**}$, where each firm receives a quantity of $\frac{\bar{Q}}{n}$. The final clearing price would also fall to $\tilde{p}$.

For prices between $\tilde{p}$ and p^{**}, the reserve quantity will be released and always reduces the final clearing price to $\tilde{p}$. In fact, in this scenario firm i can submit a demand anywhere between $[0, \frac{\bar{Q}}{n}]$, but quantity $\frac{\bar{Q}}{n}$ is the most desirable quantity. Now if $p^* < p < \tilde{p}$ the calculation for residual supply left for bidder i gives us the equation in line four of (8). Then firm i's payoff is,

$$\pi_i = (v-p)\frac{Q}{n}\left(\frac{\bar{v}-p^{*}}{\bar{v}-p}\right).$$

Again one can show firm i's payoff is decreasing in p and the term is maximized at $p = p^*$. When $p = p^*$ firm i's best response is to capture up to its share $\frac{Q}{n}$. Firm i can demand any quantity between $[0, \frac{Q}{n}]$, but since the price is p^* for any quantities, the firm obtains the maximum quantity, which is, $\frac{Q}{n}$. At any price lower than p^* all permits would be demanded and no extra unit is available. Therefore, there are two possible equilibria with final clearing prices of p^* and $\tilde{p}$ and each firm receives $\frac{Q}{n}$ and $\frac{\bar{Q}}{n}$ respectively. □

Proof of Lemma 1

Proof. The condition in this Lemma is the result of the comparison of the firms' payoffs with a clearing price less than $\tilde{p}$ and the best possible scenario that can happen with the release of the extra permits (the full permit supply $\bar{Q}$ to be released with the clearing price of $\tilde{p}$). So we need to have

$$(v-\hat{p})\frac{Q}{n} > (v-\tilde{p})\frac{\bar{Q}}{n},$$

and therefore, if

$$\hat{p} < v - \frac{\bar{Q}}{Q}(v-\tilde{p}),$$

then the buyers' payoffs are higher for all clearing prices less or equal to $\hat{p}$.

□

Proof of Proposition 2

Proof. From Theorem 1 we know when there is an allowance reserve with a trigger price $\tilde{p}$, there exists a $p^* \in [r, \tilde{p}]$ that characterizes the equilibrium, and the allocated quantities are $\frac{Q}{n}$ if $p^* < \tilde{p}$ and $\frac{\bar{Q}}{n}$ if $p^* = \tilde{p}$. Now if the regulator advertises all the permits, that is, to advertise $\bar{Q}$ from the beginning with a reserve price r and no trigger price, then we can change the quantity in Proposition 1 to $\bar{Q}$ and the same result applies but with a $\bar{Q}$ instead of Q. Therefore, we have the following demand function with each firm receiving $\frac{\bar{Q}}{n}$ and the clearing price equal to p^*.

$$d_i(p) = \begin{cases} 0 & \text{if} \quad p > p^\dagger, \\ \bar{Q}\frac{(p^\dagger - p)}{n(p^\dagger - p) + p - p^*} & \text{if} \quad p^* < p \le p^\dagger, \\ \frac{\bar{Q}}{n-1} & \text{if} \quad r \le p \le p^*, \end{cases} \tag{9}$$

where $p^\dagger = \frac{(n-1)\bar{v}}{n} + \frac{p^*}{n}$.

Since for this equilibrium the only condition for p^* is to be larger than r and smaller than v, then for any trigger price $\tilde{p}$ greater than r there exist a class of equilibrium clearing prices greater than r and smaller than $\tilde{p}$ with equilibrium quantities of $\frac{\bar{Q}}{n}$ for each firm. We focus on this class of equilibria and show they are Pareto superior to the set of equilibria with an allowance reserve for firms. First, consider a firm who pays a price p and receives a quantity Q with the following payoff:

$$\pi_i = (v-p)\frac{Q}{n}. \tag{10}$$

Clearly this payoff is increasing in the quantity a firm receives and decreasing in the price she pays.

Let us now consider the auction with an allowance reserve. There are two kinds of equilibria for the allowance reserve case. One with a p^* lower than $\tilde{p}$ and a quantity of $\frac{Q}{n}$ for each firm. The other is with clearing price $\tilde{p}$ and a quantity $\frac{\bar{Q}}{n}$. The first kind is dominated by the auction with no allowance with the total quantity of $\bar{Q}$ because the set of clearing prices are the same but sold permits are strictly larger for the auction with no allowance. Therefore, firms payoffs are larger. The second kind is also dominated by the auction with no allowance equilibria because the permits sold are the same but the clearing prices of the auction with no allowance is strictly less therefore, buyers payoffs are larger. Thus, there exist a class of equilibria for an auction with no allowance reserve where it is strictly better for firms compared to those equilibria of the auction with allowance reserve.

□

Proof of Corollary 1

Proof. Consider two possible equilibria for the auction with an allowance: (i) $p^* \in [r, \tilde{p})$ and (ii) $p^* = \tilde{p}$. In case (i) the allowance reserve is not activated and both clearing price and quantity of permits are identical to the auction without an allowance reserve. For case (ii) when $\hat{p} \in [r, \tilde{p})$ such that $\hat{p} < v - \frac{\bar{Q}}{Q}(v - \tilde{p})$ is satisfied, using Lemma 1 we can conclude that firms have larger payoffs in an auction without an allowance reserve even though they receive lower quantities. □

Proof of Corollary 2

Proof. First, focus on the class of equilibria of Proposition 2 for an auction without an allowance reserve. Denote $D(0, v)$ as the aggregate demand of the unconstrained level of emissions. From Proposition 2 if $p^* \in [r, \tilde{p})$, then total emissions released would be equal to Q in an auction with an allowance reserve, and $\bar{Q}$ in an auction without an allowance reserve. Given $D(0, v)$ and the demand schedule is downward sloping, it follows that the aggregate pollution abatement costs are higher under an auction with an allowance reserve. If $p^* = \tilde{p}$ in an auction with an allowance reserve then equilibria of both auctions would result in the same total quantity of $\bar{Q}$. However, in an auction without the allowance reserve the class of equilibria we discuss constitutes a clearing price strictly lower than $\tilde{p}$. Therefore, given $D(0, v)$ and the demand schedule is downward sloping, it follows that the aggregate pollution abatement costs are higher under an auction with an allowance reserve. □

Proof of Proposition 3

Proof. First, rewrite the condition in Lemma 1 for the smallest possible equilibrium clearing price, the reserve price, which is the worst outcome for the regulator:

$$r < v - \frac{\bar{Q}}{Q}(v - \tilde{p}). \tag{11}$$

Now fix $\tilde{p}$ and quantities, then the right hand side would reach its minimum at $\bar{v} - \frac{\bar{Q}}{Q}(\bar{v} - \tilde{p})$. So if the regulator sets the reserve price such that it becomes greater than the minimum then by continuity, there must be some $v < \bar{v}$, which also makes the right hand side less than r. Therefore, if $r > \bar{v} - \frac{\bar{Q}}{Q}(\bar{v} - \tilde{p})$, then there exist a $\tilde{v} < \bar{v}$ such that we have,

$$r = \tilde{v} - \frac{\bar{Q}}{Q}(\tilde{v} - \tilde{p}). \tag{12}$$

For any value $v \in [\tilde{v}, \bar{v}]$, firms prefer the clearing price $\tilde{p}$ with the share of $\frac{\bar{Q}}{n}$ permits. So there is a higher chance of observing $\tilde{p}$ as the clearing price compared to any other clearing price $p^* \in [r, \tilde{p})$. For the case without allowance reserve, it has already been shown that for $p^* \in [r, v]$, the Pareto superior clearing price is r. □

Proof of Corollary 3

Proof. The reserve quantity by definition is $\bar{Q} - Q$. If the regulator keeps almost all quantities as the reserve then $\frac{\bar{Q}}{Q}$ goes to infinity. Since $\bar{v}$ is a positive finite number then we must have $r > \bar{v} - \frac{\bar{Q}}{Q}(\bar{v} - \tilde{p})$ for any positive r. Now by continuity, there exists a $\hat{Q} < \bar{Q}$, such that $r = \bar{v} - \frac{\bar{Q}}{\hat{Q}}(\bar{v} - \tilde{p})$. Therefore, the regulator can set the reserve quantity to any amount greater than $\bar{Q} - \hat{Q}$ and guarantee the condition in Proposition 3. □

Proof of Proposition 4

Proof. Given than the lowers clearing price is r in the even that the extra supply is not released we can write the following objective function for a revenue maximizing regulator.

$$\max_{\tilde{p}} \quad U_R = Prob\left(r > v - \frac{\bar{Q}}{Q}(v - \tilde{p})\right)\tilde{p}\bar{Q} + \left[1 - Prob\left(r > v - \frac{\bar{Q}}{Q}(v - \tilde{p})\right)\right]rQ. \tag{13}$$

We can rewrite the above objective function as,

$$U_R = Prob\left(v > \frac{\bar{Q}\tilde{p} - Qr}{\breve{Q}}\right)\tilde{p}\bar{Q} + \left[1 - Prob\left(v > \frac{\bar{Q}\tilde{p} - Qr}{\breve{Q}}\right)rQ\right] \tag{14}$$

This is equal to,

$$U_R = (1 - F(\frac{\bar{Q}\tilde{p} - Qr}{\breve{Q}}))\tilde{p}\bar{Q} + F(\frac{\bar{Q}\tilde{p} - Qr}{\breve{Q}})rQ. \tag{15}$$

Then the first-order condition becomes,

$$(1 - F(\frac{\bar{Q}\tilde{p} - Qr}{\breve{Q}}))\bar{Q} - \frac{\bar{Q}}{\breve{Q}}f(\frac{\bar{Q}\tilde{p} - Qr}{\breve{Q}})\tilde{p}\bar{Q} + \frac{\bar{Q}}{\breve{Q}}f(\frac{\bar{Q}\tilde{p} - Qr}{\breve{Q}})rQ = 0. \tag{16}$$

This gives us,

$$\frac{(1 - F(\frac{\bar{Q}\tilde{p} - Qr}{\breve{Q}}))}{f(\frac{\bar{Q}\tilde{p} - Qr}{\breve{Q}})} = \frac{\bar{Q}\tilde{p} - Qr}{\breve{Q}}. \tag{17}$$

□

Proof of Proposition 5

Proof. According to the objective function in (14), the derivative with respect to r is always positive. Thus, r must be set as large as possible, that is, equal to $\tilde{p}$. □

Proof of Proposition 6

Proof. For any price $p^* \in [0, v_l]$, imagine a demand schedule $\gamma(p)$ with following three conditions:

(I) $\gamma(p^*) = \frac{Q}{n}$

(II) $\{\gamma(p) | x(p) > (v_h - p)x'(p) \text{ for } p > p^*\}$

(III) $\{\gamma(p) | x(p) < (v_l - p)x'(p) \text{ for } p < p^*\}$

where $x(p)$ is the residual supply left for a given firm when all other firms follow $\gamma(p)$. We claim $\gamma(p)$ characterizes the class of equilibria according to the first part of the Proposition.

One can easily check a demand schedule similar to the one in Proposition 1 satisfies all three conditions. The payoff of a representative firm is,

$$\pi_i(p) = (v_i - p)x(p). \tag{18}$$

Differentiating this payoff with respect to p yields,

$$\pi_i'(p) = -x(p) + (v_i - p)x'(p). \tag{19}$$

According to condition (II), the above term is negative for prices higher than p^* and positive for prices lower than p^*. Thus p^* is the best response of firm i. Remember that we did not limit the type of firm i and this proof works for both types.

The proof of the second part is quite similar but now for prices above v_l low-type firms leave the market. Therefore, in equilibrium, the quantity share for each firm becomes $\frac{Q}{h}$, where h is the number of high type firms. Also condition (III) can now change to $\{\gamma(p) | x(p) < (v_h - p)x'(p) \text{ for } p < \hat{p}^*\}$.

□

Proof of Proposition 7

Proof. The first part of the proof is similar to Proposition 6 given that $\tilde{p}$ is less than v_l. So each firm receives a quantity equal to $\frac{Q}{n}$ in equilibrium. Now imagine a case where the price goes above $\tilde{p}$. According to the rules of the game the allowance reserve or the extra supply would increase until either the price goes back to $\tilde{p}$ or the extra supply finishes. With a similar argument to the one in Theorem 1 we can argue that a demand schedule that results in releasing only a part of the extra supply cannot be an equilibrium. The argument is based on the fact that if firms increase their demands marginally then the price would remain the same but the quantity they receive increases. Thus their total payoff increases if they increase the demand. Define p^{**} as the highest price such that the extra supply can reduce the price to $\tilde{p}$. A demand schedule $\gamma(p)$ constitutes an equilibrium if,

(I) $\gamma(p^{**}) = \frac{\bar{Q}}{n}$

(II) $\{\gamma(p) | x(p) > (v_h - p)x'(p) \text{ for } p > p^{**}\}$

Rewrite the payoff function for a representative firm

$$\pi_i(p) = (v_i - p)x(p). \tag{20}$$

Differentiating this payoff with respect to p yields,

$$\pi_i'(p) = -x(p) + (v_i - p)x'(p). \tag{21}$$

A representative firm i would have no incentive to deviate from $\gamma(p)$. According to (II), for prices higher than p^{**} their payoff is decreasing and for prices below p^{**} their payoff is increasing. □

References

Ausubel, L. M. and Cramton, P. (2004). 'Auctioning many divisible goods', *Journal of the European Economic Association*, **2**, pp. 480–493.

Ausubel, L. M., Cramton, P., Pycia, M., Rostek, M. and Weretka, M. (2014). 'Demand reduction and inefficiency in multi-unit auctions', *The Review of Economic Studies*, **81**, pp. 1366–1400.

Back, K. and Zender, J. F. (1993). 'Auctions of divisible goods: On the rationale for the treasury experiment', *The Review of Financial Studies*, **6**, pp. 733–764.

Back, K. and Zender, J. F. (2001). 'Auctions of divisible goods with endogenous supply', *Economics Letters*, **73**, pp. 29–34.

Borenstein, S., Bushnell, J., Wolak, F. A. and Zaragoza-Watkins, M. (2014). 'Report of the market simulation group on competitive supply/demand balance in the California allowance market and the potential for market manipulation', Working Paper, Energy Institute at Haas.

Borenstein, S., Bushnell, J., Wolak, F. A. and Zaragoza-Watkins, M. (2015). 'Expecting the unexpected: Emissions uncertainty and environmental market design', Working Paper 20999, National Bureau of Economic Research.

Burtraw, D., Goeree, J. K., Holt, C. A., Myers, E. C., Palmer, K. L. and Shobe, W. (2010a). 'Price discovery in emissions permit auctions', Discussion Paper 10-32, Resource for the Future.

Burtraw, D., Palmer, K. and Kahn, D. (2010b). 'A symmetric safety valve', *Energy Policy*, **38**, pp. 4921 – 4932.

Cramton, P. and Kerr, S. (2002). 'Tradeable carbon permit auctions: How and why to auction not grandfather', *Energy Policy*, **30**, pp. 333 – 345.

Damianov, D. S. and Becker, J. G. (2010). 'Auctions with variable supply: Uniform price versus discriminatory', *European Economic Review*, **54**, pp. 571–593.

Ellerman, A. D., Joskow, P. L., Schmalensee, R., Montero, J.-P. and Bailey, E. M. (2000). *Markets for clean air: The US Acid Rain Program*: Cambridge University Press (New York).

Fell, H. (2015). 'Comparing policies to confront permit over-allocation', Discussion Paper 15-17, Resources for the Future.

Fell, H., Burtraw, D., Morgenstern, R. D. and Palmer, K. L. (2012). 'Soft and hard price collars in a cap-and-trade system: A comparative analysis', *Journal of Environmental Economics and Management*, **64**, pp. 183 – 198.

Fell, H. and Morgenstern, R. (2010). 'Alternative approaches to cost containment in a cap-and-trade system', *Environmental and Resource Economics*, **47**, pp. 275–297.

Grüll, G. and Taschini, L. (2011). 'Cap-and-trade properties under different hybrid scheme designs', *Journal of Environmental Economics and Management*, **61**, pp. 107 – 118.

Hansen, R. G. (1988). 'Auctions with endogenous quantity', *The RAND Journal of Economics*, **19**, pp. 44–58.

Hasegawa, M. and Salant, S. (2014). 'Cap-and-trade programs under delayed compliance: Consequences of interim injections of permits', *Journal of Public Economics*, **119**, pp. 24 – 34.

Holland, S. P. and Moore, M. R. (2013). 'Market design in cap and trade programs: Permit validity and compliance timing', *Journal of Environmental Economics and Management*, **66**, pp. 671 – 687.

Holt, C. A., Shobe, W., Burtraw, D., Palmer, K. L. and Goeree, J. K. (2007). 'Auction design for selling CO_2 emission allowances under the Regional Greenhouse Gas Initiative', Report, Prepared for the New York State Energy Research and Development Authority, Albany.

Holt, C. A. and Shobe, W. M. (2016). 'Price and quantity collars for stabilizing emission allowance prices: Laboratory experiments on the EU ETS Market Stability Reserve', *Journal of Environmental Economics and Management*, **76**, pp. 32 – 50.

Lengwiler, Y. (1999). 'The multiple unit auction with variable supply', *Economic Theory*, **14**, pp. 373–392.

LiCalzia, M. and Pavan, A. (2005). 'Tilting the supply schedule to enhance competition in uniform-price auctions', *European Economic Review*, **49**, pp. 227–250.

Lopomo, G., Marx, L. M., McAdams, D. and Murray, B. (2011). 'Carbon allowance auction design: An assessment of options for the United States', *Review of Environmental Economics and Policy*, **5**, pp. 25–43.

McAdams, D. (2007). 'Adjustable supply in uniform price auctions: Non-commitment as a strategic tool', *Economics Letters*, **95**, pp. 48–53.

Mckinsey and Company (2016). 'Greenhouse gas abatement cost curves', , Mckinsey and Company. http://www.mckinsey.com/business-functions/sustainability-and-resource-productivity/our-insights/greenhouse-gas-abatement-cost-curves.

Montero, J.-P. (2008). 'A simple auction mechanism for the optimal allocation of the commons', *The American Economic Review*, **98**, pp. 496–518.

Murray, B. C., Newell, R. G. and Pizer, W. A. (2009). 'Balancing cost and emissions certainty: An allowance reserve for cap-and-trade', *Review of Environmental Economics and Policy*, **3**, pp. 84–103.

Murray, B. C., Maniloff, P. and Murray, E. (2014). 'Why have greenhouse emissions in RGGI states declined? An econometric attribution to economic, energy market, and policy factors', Working Paper 2014-04, Duke Environmental and Energy Economics Working Paper Series.

Perkis, D., Cason, T. and Tyner, W. (2016). 'An experimental investigation of hard and soft price ceilings in emissions permit markets', *Environmental and Resource Economics*, **63**, pp. 703–718.

Pizer, W. A. (2002). 'Combining price and quantity controls to mitigate global climate change', *Journal of Public Economics*, **85**, pp. 409 – 434.

RGGI (2013). 'Summary of RGGI model rule changes'. available at http://rggi.org/docs/ProgramReview/_FinalProgramReviewMaterials/Model_Rule_Summary.pdf.

RGGI (2015). 'Investment of RGGI proceeds through 2013'. available at: http://rggi.org/docs/ProceedsReport/Investment-RGGI-Proceeds-Through-2013.pdf.

Roberts, M. J. and Spence, M. (1976). 'Effluent charges and licenses under uncertainty', *Journal of Public Economics*, **5**, pp. 193 – 208.

Schatzki, T. (2012). 'Can options for cost containment raise costs?'. mimeo.

Shobe, W., Holt, C. and Huetteman, T. (2014). 'Elements of emission market design: An experimental analysis of California's market for greenhouse gas allowances', *Journal of Economic Behavior & Organization*, pp. 402 – 420.

Stocking, A. (2012). 'Unintended consequences of price controls: An application to allowance markets', *Journal of Environmental Economics and Management*, **63**, pp. 120 – 136.

Tatsutani, M. and Pizer, W. A. (2008). 'Managing costs in a US greenhouse gas trading program: A workshop summary', Working paper 08-23, Resources for the Future.

Wang, J. J. and Zender, J. F. (2002). 'Auctioning divisible goods', *Economic Theory*, **19**, pp. 673–705.

Wilson, R. (1979). 'Auctions of shares', *The Quarterly Journal of Economics*, pp. 675–689.