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Reliability Measurement for Multistate Manufacturing Systems with Failure Interaction

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Abstract

Reliability is one of the important factors for manufacturing system. Most researches assume that the failure is independent and the components only have two states, which will lead to inaccurate results. In this paper, a reliability model is proposed considering both failure interaction and multi-state property of the manufacturing system. Starting with a two-component system, a function of state probability under the impact of failure interaction is established after the analysis of failure interaction. Then the multi-component system is decomposed into several subsystems and the failure interaction coefficient is estimated in each subsystem with a Copula function and the Grey model method. Finally, the reliability model is realized with the performance generating function which is derived with the UGF technique and failure interaction coefficients. An example of a cylinder engine manufacturing system is studied, and the result is closer to the practical data.

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Keywords: multistate manufacturing system; reliability measure; failure interaction; universal generating function; grey model

1. Introduction

Reliability describes the probability of a system completing its expected function during an interval of time, which gives an assessment of the overall performance of a system. An accurate reliability measure for a system guarantees its functionality, efficiency and safety. With the development of its scale and complexity, manufacturing system puts forward higher requirements for reliability analysis.

Most of the reliability analyses are based on two simplified assumptions: First, consider the subsystems and the whole system as a binary system, which means the system has a fully operational state and a completely failed state. Second, no failure interaction exists between the components. Failure interaction here refers to a prevailing phenomenon that one subsystem`s failure or degradation will affect the failure process of other related subsystems.

The study of multistate systems (MSS) started in the mid-1970s[2], since then lots of research have been carried out in this area. Methods adopted to reliability analysis of MSS mainly include: Monte Carlo Simulation[3,4], Stochastic Process Analysis[5], Universal Generating Function(UGF)[6] and so on. Other methods were also developed. Ding et al.[7] developed the fuzzy universal generating function considering a multistate system where performance rates and corresponding state probabilities are presented as fuzzy values. Lisnianski[8] extended the classical reliability block diagram method to a repairable multistate system based on the combined random processes and the universal generating function technique. Taboada et al.[9] developed a custom genetic algorithm to solve multiple objective multistate reliability optimization design problems. Qian et al.[10] proposed a new discretized modeling process on Bayesian belief networks basis for the reliability of multi-state mechanical systems. The study mentioned above focus on the multistate property of MSS to extend the study on reliability

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analyses. However, reliability can be influenced by various factors. Ping et al.[11] considered joint buffer station in a multi-state manufacturing network. Infinite and finite buffer volume was discussed and the study indicated that the assumption of infinite buffer volume will overestimate the system reliability.

Failure interaction commonly exists in multistate systems, which may also influence the performance of the system. During the operation of a manufacturing system, failures of a unit caused by corrosion, ageing, wearing or shock damages like improper maintenance or overwork may increase the load of other units, then affects the failure characteristics of the other units and eventually leads to the failure of the units. Attention has been paid to the failure mechanism and failure interaction in the systems` reliability as well.

Nakagawa and Murthy[12] divided failure interaction into three types, which is widely used in this area. According to this category, researchers expanded failure interaction into many aspects. Lai and Chen[13] developed an optimal periodical replacement policy for a multi-unit system subject to failure rate interaction between units. Sun et al.[14,15] introduced the concepts of interactive failure, developed an analytical model and presents five approaches to estimate the interactive failures. Gao et al. [16] established a reliability model of system and a quasi-periodic dynamic Preventive Replacement to research the coexists of type I and II failure interaction. Qi et al.[17] presented two periodical maintenance cost models for a two-state series system and a three-state series system respectively based on the three types of failure interactions.

Considering the failure interaction gives rise to the analysis of the system, however the result always yields the practical data. The studies mentioned above focusing on the failure interaction studied issue on reliability and maintenance either in a multistate two-component system or a binary multi-component system. Study focusing on multistate multi-component system with failure interaction is still in infancy.

This paper for the first time considers failure interaction in a multi-component multistate manufacturing system(MSMS) to derive the reliability. Moreover, based on a previous study on the influence of the degeneration[18]. The reliability is given with two failure mechanisms, namely failure interaction and the degeneration. To define the failure interaction, the copula function and Grey model is used to find the parameters as a new approach. The rest of the paper is organized as follows. The next section gives an analysis of a two components multistate manufacturing system with failure interaction. Markov chain is used to represent all the states and their transitions. Section 3 proposes a decomposition method of the system based on fault correlation of the components and figures out the failure interaction coefficient using Copula function and grey model method. Then the reliability model is constructed after giving the performance generating function of one component and the whole system by using the UGF technique and mapping relationship. Section 4 dedicates to a case study on a three processes engine cylinder manufacturing system. The paper concludes that considering failure interaction in multistate system will give a lower reliability.

2. Failure Transition in A Two-component MSMS

2.1. Definition and Assumption

Consider a system with two machines. Any machine can have $l+1$ different states in its lifecycle, represented by the set $L=\{l,l-1,\cdots,0\}$, where l denotes the new state and 0 denotes the completely failure state. Very mild assumptions are required for the sequential study. These are follows:

- All general faults are maintained immediately after it occurs and no maintenance time is considered.
- Whenever a system failure occurs, it is detected immediately and only one unit fails naturally.
- All failed units are correctively maintained but cannot be repaired as good as new.
- All performance analyses are done only when the system is at a steady state.
- The state transition caused by failure interaction is instantaneous.

2.2. Failure transition representation

According to the failure interaction category [12], the failure transition for two machines is illustrated in Figure.1.

When a machine fails due to the fault of itself, such as corrosion, ageing and so on, it can induce the failure of the related component refer to Machine II in Figure.1 with a probability of *p*; or it increases the failure rate of machine II with a probability *1-p* and gives rise to the failure of machine II with the accumulation of the damage. While the failure of machine II will induce an instantaneous failure to machine I with a probability of q.

When machine I fails due to an external shock damage, the following two situation should be considered: The shock damage is quite small that only a general failure occurs on machine I. The hazard will increase the failure rate of machine II with a probability of p` and lead to failure when the total damage exceeds a specified level. While the failure of machine II caused by the damage will induce instantaneous damage to machine I with a probability of $q$` and lead to the failure of machine I. Otherwise, if the shock damage gives rise to a ‘dramatic destroy’ on machine I it will lead to instantaneous failure. The hazard will induce an instantaneous failure of machine II with a probability of *1-p*` and result in failure of the system eventually.

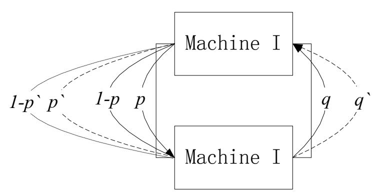

Fig.1 Failure transition between two components

2.3. Markov model for state transition

In its lifecycle [0, t), all machines will degrade from state *l* to its completely failure state *0*. The machine can transfer to any lower state from the start state due to the influence of itself or the failure interaction. Considering the state degeneration and failure interaction, the state transition of a two machine system is illustrated in Figure.2, where $\mu_{i,j}$ and $\lambda_{j,i}$ refer to the transition rate between different states.

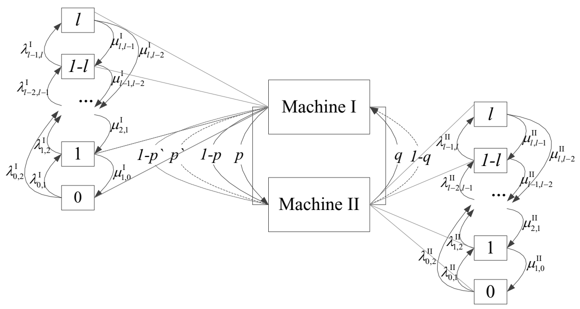

Fig.2 State transition considering the failure interaction

According to the above analysis, one can find that the process of state transition with failure interaction is a stochastic process in continuous time interval [0, t) and finite state space $L(t)=\{l,l-1,\cdots,0\}$, which gives use of non-homogeneous Markov chain. Based on Figure.2, the state transition matrix of component is derived:

$$A^{I}(t)=\begin{bmatrix} \eta^{I}_{(l,l-1)}(t)\partial^{II\to I}_{(l.l-1)} & \eta^{I}_{(l,l-2)}(t)\partial^{II\to I}_{(l,l-2)} & \cdots & \eta^{I}_{(l,0)}(t)\partial^{II\to I}_{(l,0)} \\ 0 & \eta^{I}_{(l-1,l-2)}(t)\partial^{II\to I}_{(l-1,l-2)} & \cdots & \eta^{I}_{(l-1,0)}(t)\partial^{II\to I}_{(l-1,0)} \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \eta^{I}_{(0,0)}(t)\partial^{II\to I}_{(0,0)} \end{bmatrix} \quad (1)$$

Where:

- $\partial^{II\to I}$ = failure interaction coefficient of machine I respect to machine II;
- $\eta^{I}_{(i,j)}(t)$ = probability of state transition and

$$\eta^{I}_{(i,j)}(t)=\lim_{\Delta t\to 0}\frac{\Pr\{L(t+\Delta t)=j\,|\,L(t)=i\}}{\Delta t} \quad (2)$$

2.4. State probability analysis considering the failure interaction

For any multistate manufacturing system, the failure rate of components will increase due to the operation of the system. The probability of each state is a result of both internal and external factors of the component.

In allusion to the state transition process of a two units system in Figure.1, supposing machine I and II have a service year of t_I and t_{II}. The failure of machine I will follow a non-homogeneous Poisson process with mean-value function $\Theta_{I(t)}=\int_0^t u(x)dx$ and an intensity ratio of $u(t)=r_I(t)+p\lambda(t)$, where $r_I(t)$ is failure rate of machine I and $\lambda(t)$ is the shock damage ratio function. And the state probability for machine I is derived in paper[18].

While the state probability of machine II is decided by itself and the failure of machine I. When machine I fails, it will influence the performance of machine II with a probability of p, making machine II work in a lower state. Suppose the failure rate of machine II is $r_{II}(t)=(1+\partial)^k r_I(t)$ with ∂ represents the failure interaction coefficient. Then we can derive the probability of each state for machine II:

$$\begin{aligned} p_{(II,i)} &= \Pr(t_{II}(t);N_I(t)) \\ &= \Pr\{X_{II}(t)=i\},i=0,\cdots,l;\Pr\{N_I(t)\ge k\} \\ &= \int_0^t\cdots\int_0^{\tau_2}\exp\left[-\int_0^{\tau_1}\sum_{i=0}^{l_m-1}\eta^{II}_{(l,i)}(s)ds\right]\eta^{II}_{(l,l-2)}(\tau_2) \\ &\prod_r^{n-1}\left\{\exp\left[-\int_{\tau_i}^{\tau_{r+1}}\sum_{i=0}^{x-1}\eta^{II}_{(l,i)}(s)ds\right]\eta^{II}_{(l,l+1)}(\tau_{r+1})\right\} \\ &\exp\left[-\int_{\tau_n}^{\tau_i}\sum_{i=0}^{l_m-1}\eta^{II}_{(l,i)}(s)ds\right]\eta^{II}_{(l,i)}\exp\left[-\int_{\tau_i}^{t}\sum_{j=0}^{i=1}\eta^{I}_{(i,j)}(s)ds\right] \\ &d\tau_1\cdots d\tau_{n=1};\sum_{j=k}^{\infty}\frac{(\Theta_{I(t)})^j\exp(-\Theta_{I(t)})}{j!} \end{aligned} \quad (3)$$

Where:

- $X_{II}(t)$ = function of failure rate respect to time;
- $N_I(t)$ = machine I`s failure number in time interval [0,t);
- n = state number that machine I passed from state l_m to state i;
- k = the k[th] failure.

3. Reliability analysis of Multi-Component MSMS

To measure the reliability of a MSMS with failure interaction, decomposition of the system is needed so that the failure interaction in one subsystem can be discussed. Based on the interaction analyses, a Copula function of failure interaction coefficient is constructed and grey model method will be used to determine the failure interaction coefficient. Then, the reliability model with failure interaction is realized.

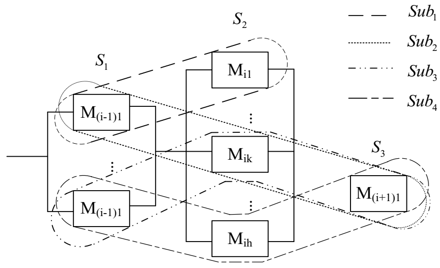

Fig.3 A three-process manufacturing system

Consider a multistate manufacturing system with three processes S_1, S_2 and S_3, showed in Fig.3. Any component of a process has l different states corresponding to different performance rates. The lifetime of all components are random variables with distribution function $F_1(t)$, $F_2(t)$, … , $F_n(t)$ and the components start from the new state at $t=0$.

3.1. Decomposition of the Multi-component manufacturing system

All materials flow from the first process to the following procedures. Any components of a previous process`s failure will impact the performance of the components in the following process. A decomposition of the system is shown in Fig.3 based on the interaction caused by material flow. The whole system is thus divided into four subsystems: Sub_1(composed of $M_{(i-1)1}$ and M_{i1}), Sub_2(composed of $M_{(i-1)1}$, M_{ik} and $M_{(i+1)1}$), Sub_3(composed of $M_{(i-1)d}$, M_{ik} and $M_{(i+1)1}$) and Sub_4 (composed of $M_{(i-1)d}$, M_{ih} and $M_{(i+1)1}$).

3.2. Estimation the failure interaction.

Copula is a function that joins or couples multivariate distribution functions to their one-dimensional marginal distribution functions and describes correlation between the random variables. The relevance of distributional copulas for failure interaction is mainly because of Sklar`s theorem, which states that for a given copula C and marginal distribution functions, the joint distribution can be obtained via:

$$F(x_1, x_2, \cdots, x_n) = C(F(x_1), F(x_2), \cdots, F(x_n)) \quad (4)$$

If $F(x_1), F(x_2), \cdots, F(x_n)$ are continuous, then C is unique; otherwise, C is uniquely determined on random variables of F.

In allusion to a multistate manufacturing system if we have the failure rate functions and the joint distribution function of two interaction components, the failure interaction coefficient can be calculated by choosing a proper Copula function followed by estimating it parameters.

Consider a MSMS with h subsystems, each subsystem is comprised of several machines. Set the failure rate function of the component as $P(x_i), i = 2, \cdots n$ and the joint distribution function as $P(x_1, x_2, \cdots, x_n)$. According to the Copula function and Skalr`s theorem, a Copula function $C(P(x_1), \cdots, P(x_n); \xi)$ can be obtain, and it satisfy the following relationship:

$$P(x_1, x_2, \cdots, x_n) = C(P(x_1), P(x_2), \cdots, P(x_n); \xi) \quad (5)$$

For any manufacturing system, the failure interaction exists only between some components while the others remain independent. So the failure rate function should write as:

$$P(x) = \prod_{i=1}^{m} C_i(x; \partial_i) \prod_{j=1}^{n} P_j(x) \quad (6)$$

Where:

- $C_i(x)$ = general failure rate of components with failure interaction;
- $P_j(x)$ = failure rate of dependent component;
- ∂_i = interaction coefficient of component i.

Traditionally, copula parameters are determined by Moment Estimation Method or Maximum Likelihood Estimation Method[19]. The method of moment estimation is fairly simple but the result may not necessarily sufficient, while the method of maximum likelihood estimation is to the contrary.

Grey system theory is a new methodology that focuses on problems involving small data and poor information. It deals with uncertain systems with partially known information through sequence operators, excavating, and extracting useful information from what is available and gives a result with high accuracy after simpler calculation. The failure interaction coefficient is derived from the failure rate and the correlation between components, which exactly meet the requirement for using Grey Model method. The typical procedure of estimating parameters with Grey model is the GM(1,1) model[20].

3.3. Reliability model with UGF technique

As mentioned previously, the target system in this paper can be decomposed into four subsystems. Take subsystem 2 as an example. When $M_{(i-1)1}$ fails, it will have an impact on machine M_{ik}, leading to the increase of its failure rate and the change of the performance and state probabilities. Suppose the increased failure rate of M_{ik} is $\tilde{p}_i$, the increased performance of each state is $\tilde{g}_i$. Then the performance generating function of M_{ik} consider the failure interaction can be derived using the UGF technique:

$$\begin{aligned} u'_{ik}(z) &= \sum_{i=1}^{k_i} p_i z^{g_i} + \tilde{u}_{ik}(z) \\ &= \sum_{i=1}^{k_i} p_i z^{g_i} + \sum_{i=1}^{k_i} \tilde{p}_i z^{\tilde{g}_i} = \sum_{i=1}^{k_i} p_i z^{g_i} + \sum_{i=1}^{k_i} \frac{p_i}{1-\partial} z^{g_i} \end{aligned} \quad (7)$$

Where:

- $\tilde{u}_{ik}(z)$ = variation of the generating function;
- p_i = probability of the machine at state i;
- g_i = performance of the machine at state i;
- ∂ = failure interaction coefficient.

For the whole system a mapping from components performance to system can be realized as follow:

$$\begin{aligned} U(z) &= u_{s_1}(z) \underset{\varphi}{\otimes} u_{s_2}(z) \underset{\varphi}{\otimes} u_{s_3}(z) \underset{\varphi}{\otimes} u_{s_4}(z) \\ &= [\sum_{i=1}^{h} p_{i1} z^{g_{i1}} + \tilde{u}_{i1}(z)] \underset{\varphi}{\otimes} [\sum_{i=1}^{h} p_{i2} z^{g_{i2}} + \tilde{u}_{i2}(z)] \\ &\underset{\varphi}{\otimes} [\sum_{i=1}^{h} p_{i3} z^{g_{i3}} + \tilde{u}_{i3}(z)] \underset{\varphi}{\otimes} [\sum_{i=1}^{h} p_{i4} z^{g_{i4}} + \tilde{u}_{i4}(z)] \end{aligned} \quad (8)$$

According to the definition of reliability, the reliability model of the system is established:

$$\begin{aligned} R_s(t) &= \delta(U(z), \varpi) \\ &= \delta_s(p_{M(i,1)1} z^{g_{M(i,1)1}} \cdot p_{M(i,2)1} z^{g_{M(i,2)1}} \cdot p_{M(i,3)1} z^{g_{M(i,3)1}} \\ &+ \cdots + \sum_{j=1}^{k_j} \frac{p_{(M_{(i-1,1)1} \to M_{(i,1)1})j}}{1-\partial_{(i-1,i)}} z^{g_{(M(i-1,1)1 \to M(i,1)1)j}} \cdots \\ &\sum_{j=1}^{k_f} \frac{p_{(M_{(i,)} \to M_{(i+1,1)1})f}}{1-\partial_{(i,i+1)}} z^{g_{(M(i,) \to M(i+1,1)1)f}}, \varpi) \cdot \sum_{i=0}^{k_m} p_i 1(r_i - \varpi \geq 0) \end{aligned} \quad (9)$$

Where:

- ϖ = required performance level;
- $1(\Theta)$ = characteristic function, it takes 1 when $r_i \geq \varpi$;
- r_i = performance value.

4. Numerical Example

The manufacturing system illustrated in Fig.4 is used to machine an engine cylinder with three processes. Each process has several machines of the same characteristic. The reliability of each machine is the result of operation time, working load and maintenance. Due to the different processing time of each process, the utilization of each machine is not the same, which leads to a different degeneration in each machine. The probability of state transition due to its degeneration is given in Table 1.

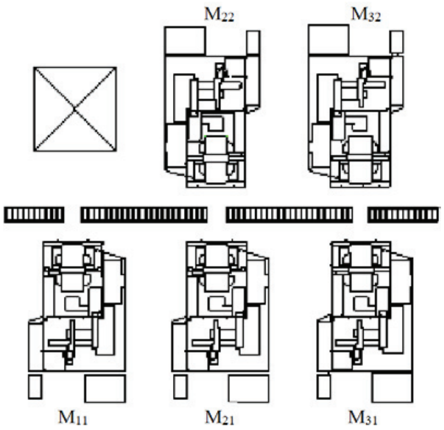

Fig.4 Engine cylinder manufacturing system

Table 1. Probability of the state transition respect to degeneration

M_{11}	M_{21}	M_{22}	M_{31}	M_{32}
$\eta_{(3,2)}^{M_{11}}=0.002$				
$\eta_{(3,1)}^{M_{11}}=0.001$	$\eta_{(2,1)}^{M_{21}}=0.021$	$\eta_{(2,1)}^{M_{22}}=0.021$	$\eta_{(2,1)}^{M_{31}}=0.030$	$\eta_{(2,1)}^{M_{32}}=0.030$
$\eta_{(3,0)}^{M_{11}}=0.002$	$\eta_{(2,0)}^{M_{21}}=0.004$	$\eta_{(2,0)}^{M_{22}}=0.004$	$\eta_{(2,0)}^{M_{31}}=0.001$	$\eta_{(2,0)}^{M_{32}}=0.001$
$\eta_{(2,1)}^{M_{11}}=0.007$	$\eta_{(1,0)}^{M_{21}}=0.002$	$\eta_{(2,0)}^{M_{22}}=0.002$	$\eta_{(1,0)}^{M_{31}}=0.002$	$\eta_{(1,0)}^{M_{32}}=0.002$
$\eta_{(2,0)}^{M_{11}}=0.003$				
$\eta_{(1,0)}^{M_{11}}=0.001$				

When M_{11} fails, it will induce the failure of M_{21} and M_{22} with a probability of p_{11} , p_{12} respectively, and lead to the failure of the system eventually; or it increases the failure rate of M_{21} and M_{22} with the probability of $1-p_{11}$ and $1-p_{12}$, and leads to the failure of M_{21} and M_{22} only if the damage accumulates to a proper level. While the failure of M_{21} and M_{22} will induce a failure of M_{11} with a probability of q_{21} and q_{22} . The same failure transition exists between M_{21} respect to M_{31} and M_{32}, M_{22} respect to M_{31} and M_{32}. The probability of the failure transition is given in Table 2.

Table 2. Probability of failure transition

M	$M_{11} \updownarrow M_{21}$	$M_{11} \updownarrow M_{22}$	$M_{21} \updownarrow M_{31}$	$M_{21} \updownarrow M_{32}$	$M_{22} \updownarrow M_{31}$	$M_{22} \updownarrow M_{32}$
p_{ij}	0.738	0.563	0.704	0.913	0.690	0.553
$1-p_{ij}$	0.261	0.437	0.296	0.087	0.310	0.447
q_{ji}	0.105	0.317	0.141	0.502	0.226	0.151

According to the decomposition proposed in section 3.1, the engine cylinder manufacturing system can be divided into four subsystems: Sub_1(comprised of M_{11}, M_{21} and M_{31}), Sub_2(comprised of M_{11}, M_{21} and M_{32}), Sub_3(comprised of M_{11}, M_{22} and M_{31}), Sub_4(comprised of M_{11}, M_{22} and M_{32}).

Then the function of failure interaction coefficient can be obtained from formula (6):

$$P(X)=C_1(x;\theta_{M_{11}\rightleftarrows M_{21}})C_1(x;\theta_{M_{11}\rightleftarrows M_{22}})\cdots C_1(x;\theta_{M_{22}\rightleftarrows M_{32}})$$
$$\times 0.738\times\cdots\times 0.151$$

Solving the function by using grey model method, the failure interaction coefficients are obtained in Table 3.

Table 3. Failure interaction coefficient

	$\theta_{M_{11}\rightleftarrows M_{21}}$	$\theta_{M_{11}\rightleftarrows M_{22}}$	$\theta_{M_{21}\rightleftarrows M_{31}}$	$\theta_{M_{21}\rightleftarrows M_{32}}$	$\theta_{M_{22}\rightleftarrows M_{31}}$	$\theta_{M_{22}\rightleftarrows M_{32}}$
Value	0.359	0.602	0.015	0.223	0.116	0.407

Utilizing the probability of state transition, failure interaction coefficient and the probability of failure transition, the performance and probability at each state of the machine are obtained in Table 4:

Table 4. Performance and probability at each state

Machine	State	performance	Probability
M_{11}	3	55	0.293
	2	40	0.450
	1	35	0.116
	0	10	0.141
M_{21}/M_{22}	2	60	0.413
	1	50	0.279
	0	30	0.308
M_{31}/M_{32}	2	55	0.098
	1	40	0.630
	0	30	0.272

Based on the calculation above, the generating function of each machine considering failure interaction can be obtained from equation (7):

$$u_{M_{11}}(z)=1.486z^{35}+2.283z^{40}+0.530z^{35}+0.715z^{10}$$
$$u_{M_{21}}(z)=2.870z^{60}+2.736z^{50}+2.765z^{30}$$
$$u_{M_{22}}(z)=2.288z^{60}+2.154z^{50}+2.183z^{30}$$
$$u_{M_{31}}(z)=1.967z^{55}+2.499z^{40}+2.141z^{30}$$
$$u_{M_{32}}(z)=1.468z^{55}+2.000z^{40}+2.135z^{30}$$

Then the generating function of the system is obtained from formula (8):

$$U_s(z)=u_{M_{11}}(z)\underset{\varphi}{\otimes}u_{M_{21}}(z)\underset{\varphi}{\otimes}u_{M_{22}}(z)\underset{\varphi}{\otimes}u_{M_{31}}(z)\underset{\varphi}{\otimes}u_{M_{32}}(z)$$
$$=0.00106\times z^{110}+0.011\times z^{95}+\cdots+0.000008z^{15}$$

According to the definition of reliability and formula (9), the reliability of the system is:

$$R_s(t)=\delta_s(U(Z),\varpi)$$
$$=\delta_S(0.00106\times z^{110}+0.011\times z^{95}+\cdots+0.000008z^{15},35)$$
$$=0.617$$

If no failure interaction is considered, the generating function should be:

$$U_s(z) = (0.293z^{55} + 0.450z^{40} + 0.116z^{35} + 0.141z^{10}$$
$$\underset{\varphi}{\otimes}(0.413z^{60} + 0.279z^{50} + 0.0.308z^{30})$$
$$\underset{\varphi}{\otimes}(0.413z^{60} + 0.279z^{50} + 0.308z^{30})$$
$$\underset{\varphi}{\otimes}(0.098z^{55} + 0.630z^{40} + 0.272z^{30})$$
$$\underset{\varphi}{\otimes}(0.098z^{55} + 0.630z^{40} + 0.272z^{30})$$
$$= 0.004 \times z^{162} + 0.002 \times z^{89} + \cdots + 0000036 \times z^{27}$$

Then the reliability of the system is:

$$R_s(t) = \delta_s(U(Z), \varpi)$$
$$= \delta_S(0.004 \times z^{162} + 0.002 \times z^{89} + \cdots + 0000036 \times z^{27}, 35)$$
$$= 0.730$$

Comparing the results, it can be found that considering the failure interaction will give a lower result in reliability analysis, which also proves that manufacturing system is a complex system with various failure mechanism, considering only degeneration by itself will inflate the error of the reliability analysis and influence the prediction of performance of the system eventually. Taking into account two failure mechanism in the established model complicates the computation but gives a more accurate result.

5. Conclusion

This paper established a new reliability analysis model considering the failure interaction in the multistate manufacturing system. The model considered two failure mechanism of a manufacturing system, namely failure interaction and degeneration of the components. To present the influence of failure interaction, failure interaction coefficient was introduced, and derived with a copula function and Grey model method.

A case study was proposed to validate the model. The result indicates that despite the failure interaction will overestimate system reliability, which also shows that the proposed method is more accurate and reasonable. The proposed model also provides a basis for maintenance policy study. To achieve a more practical insight, preventive maintenance policy considering the failure interaction will be further study in the future.

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